# Electroweak Vacuum Stability

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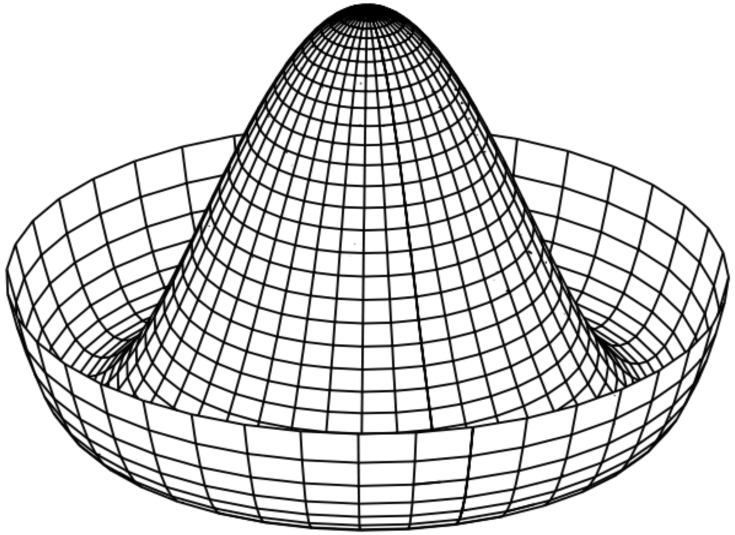
# Outline

- **Spontaneous symmetry breaking(SSB)**
- **Effective potential and running couplings**
- **Matching and running**
- **Boundary conditions**
- **Our results**
- **Summary**

# 1 Spontaneous symmetry breaking(SSB)



$$\mathcal{L}_{\rm Higgs} = (D_\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi) \qquad (1)$$

$$V(\Phi) = m^2|\Phi|^2 + \lambda|\Phi|^4, \quad \Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(\phi + i\chi) \end{pmatrix} \qquad (2)$$

$$V(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \qquad (3)$$

tree-level potential

$$\langle\phi\rangle = v \simeq 246\ \text{GeV},\ M_H = 2\lambda v^2,\ M_W = \frac{gv}{2},\ M_Z = \frac{\sqrt{g'^2 + g^2}\, v}{2},\ M_f = \frac{y_f v}{\sqrt{2}} \qquad (4)$$

$v$: the vacuum expectation value(vev) of the Higgs field

$\lambda$: the Higgs quartic coupling,

$g'$:the U(1) gauge coupling, $g$ :the SU(2) gauge coupling

$y_f$:the Yukawa couplings of fermions,

$M_H$, $M_W$, $M_Z$, $M_f$ :the pole mass

Eq.(4) is valid at tree-level.

# 2 Effective potential and running couplings

The tree-level potential would be modified in higher orders by quantum corrections, and becomes effective potential.

$$V(\phi) \rightarrow V_{\text{eff}}(\phi) = V(\phi) + \Delta V(\phi) \quad \text{[Coleman et al, PRD.7.1888]} \qquad (5)$$

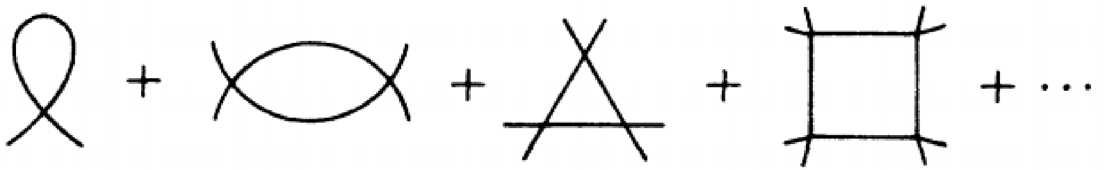


Fig.2. The one-loop approximation for the effective potential.

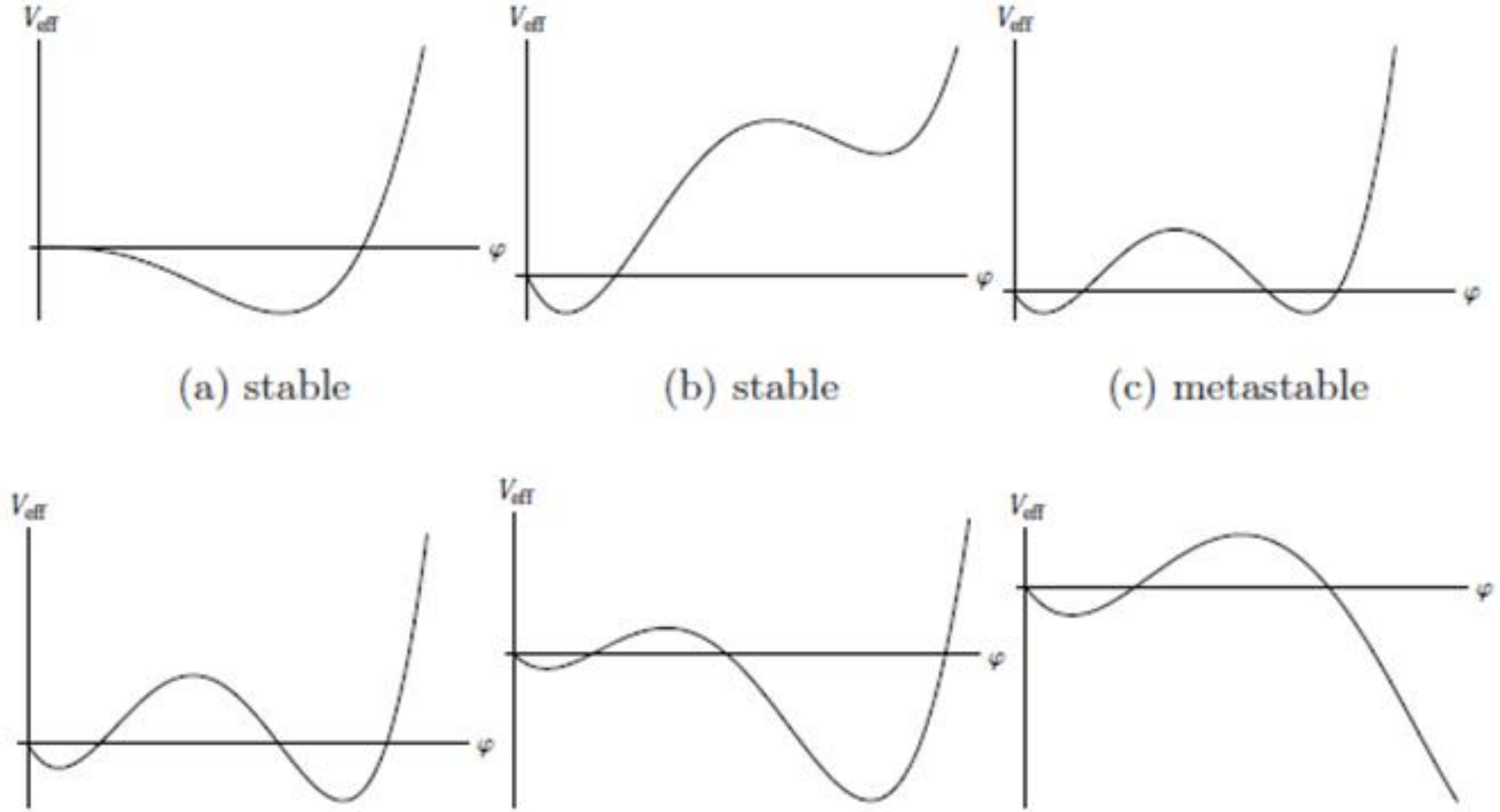
For the same reason, the running couplings of the Standard Model(SM) would also be modified by quantum corrections, such as

$$\lambda(\mu) = \lambda_0 + \Delta\lambda(\mu) \qquad (6)$$

$\lambda_0$ :the tree-level Higgs quartic coupling

$\mu$ :the renormalization scale.

The shape of the effective Higgs potential
Spontaneous symmetry breaking
Vacuum stability




(a) stable (b) stable (c) metastable

(d) metastable (e) unstable (f) unstable

# 3 Matching and running [Pikelner et al, arXiv: 1601.08143]

(i) The renormalization group (RG) evolution of the running parameters.
(ii) The initial conditions that relate to the physical observables.
For the consistent use of L-loop RG evolution, one should take into account at least (L − 1)-loop matching.

The SM running parameters defined in $\overline{\mathrm{MS}}$ renormalization scheme at the renormalization scale μ:

$$g_s(\mu), g(\mu), g'(\mu), y_t(\mu), y_b(\mu), \lambda(\mu), v(\mu) \tag{7}$$

$g_s(\mu)$ :the strong gauge coupling, $y_t(\mu)$, $y_b(\mu)$ :Yukawa couplings of t and b

Choice of input parameters

$$\alpha_s(M_Z), G_F, M_W, M_Z, M_H, M_t, M_b \tag{8}$$

$\alpha_s(\mu) = g_s^2(\mu)/(4\pi)$ :the $\overline{\mathrm{MS}}$ strong-coupling constant, $G_F$ :Fermi's constant.
$\mu = M_Z$ , $n_f = 5$ quark flavors are considered active.

## 3.1 Matching [Pikelner et al, arXiv: 1601.08143]

These initial conditions, which are determined by the so-called threshold corrections, are usually taken at some lower energy scale, which is typically of the order of the masses of the weak gauge bosons or the top quark.

If the input parameters in Eq. (8) is given, then the $\overline{\mathrm{MS}}$ couplings can be obtained at the matching scale $\mu_0$ . The corresponding matching relations are parametrized as

$$\begin{aligned} g^2(\mu_0) &= 2^{5/2} G_F M_W^2 [1 + \delta_W(\mu_0)], \\ g^2(\mu_0) + g'^2(\mu_0) &= 2^{5/2} G_F M_Z^2 [1 + \delta_Z(\mu_0)], \\ \lambda(\mu_0) &= 2^{-1/2} G_F M_H^2 [1 + \delta_H(\mu_0)], \\ y_f(\mu_0) &= 2^{3/4} G_F^{1/2} M_f [1 + \delta_f(\mu_0)], \end{aligned} \tag{9}$$

where $f = t, b$.

In Eq. (9), $\delta_x(\mu)$ are complicated functions of the parameters in Eq. (8) and $\mu_0$, which may be expanded as perturbation series,

$$\delta_x(\mu) = \sum_{i,j} \left(\frac{\alpha(\mu)}{4\pi}\right)^i \left(\frac{\alpha_s(\mu)}{4\pi}\right)^j Y_x^{i,j}(\mu) \tag{10}$$

The expansion coefficients $Y_x^{i,j}(\mu)$ are generally available for $i, j = 1, 2$ , which corresponds to two-loop matching. Beyond that, the pure QCD corrections are known through four loops and are given by $Y_f^{0,3}(\mu)$ and $Y_f^{0,4}(\mu)$

[Marquard et al, arXiv: 1502.01030]

## 3.2 Running [Pikelner et al, arXiv: 1601.08143]

The running of the $\overline{\mathrm{MS}}$ parameters in Eqs. (7) is governed by the RG equations,

$$\mu^2 \frac{dx}{d\mu^2} = \beta_x \quad , \; x = g_s, g, g', y_t, y_b, \lambda \tag{11}$$

with the respective $\beta$ functions $\beta_x$. Given the values of the parameters $x(\mu_0)$ at some initial scale $\mu_0$, Eqs. (11) allow us to find their values at some high scale $\mu$.

The functions $\beta_x$ have been known through four loops in QCD for a long time [arXiv: hep-ph/9701390, arXiv: hep-ph/9703278, arXiv: hep-ph/9703284, arXiv: hep-ph/0405193, arXiv: hep-ph/0411261] and have recently been computed through three loops in the full SM [arXiv: 1201.5868, arXiv: 1205.2892, arXiv: 1208.3357, arXiv: 1210.6873, arXiv: 1212.6829, arXiv: 1303.2890, arXiv: 1310.3806]. In the case of $\beta_\lambda$, even the mixed four-loop correction of order $\mathcal{O}(g_s^6 y_t^4)$ is available.

# 3.3 mr(Mathching & Running) [https://github.com/apik/mr] [Pikelner et al, arXiv: 1601.08143]

The C++ program library mr
Perform analysis at NNLO EW level.
Take into account the full two-loop threshold corrections and the full three-loop RG equations.

- evaluation of the coefficients $Y_x^{i,j}(\mu)$ in Eq. (10), for given input initial values.
- evaluation of the $\overline{\text{MS}}$ couplings according to Eq. (9)
- evolution of the $\overline{\text{MS}}$ parameters in the scale $\mu$ using the RG equations in Eq. (11).

Problems not mentioned above
The gauge-dependence of effective potential,
different definitions of v and thus different manipulations of tadpole contributions,
decoupling relations of effective theory and the full theory and so on.

# 4 Boundary conditions

For large field values ( $\phi \gg v$), the potential is very well approximated by its RG-improved tree-level expression [Degrassi et al, arXiv: 1205.6497]

$$V_{eff}(\phi) = \frac{\lambda(\mu)}{4}\phi^4 \qquad (12)$$

with $\mu = \mathcal{O}(\phi)$ .

As a consequence, the stability of the electroweak vacuum is related to the behavior of the running Higgs self-coupling constant at large values of the renormalization scale, namely if at some point

$$\lambda(\mu) < 0 \qquad (13)$$

there can be a minimum, which is much deeper than our vacuum, so that our universe is not stable.

[Bednyakov' s lecture, 2016]

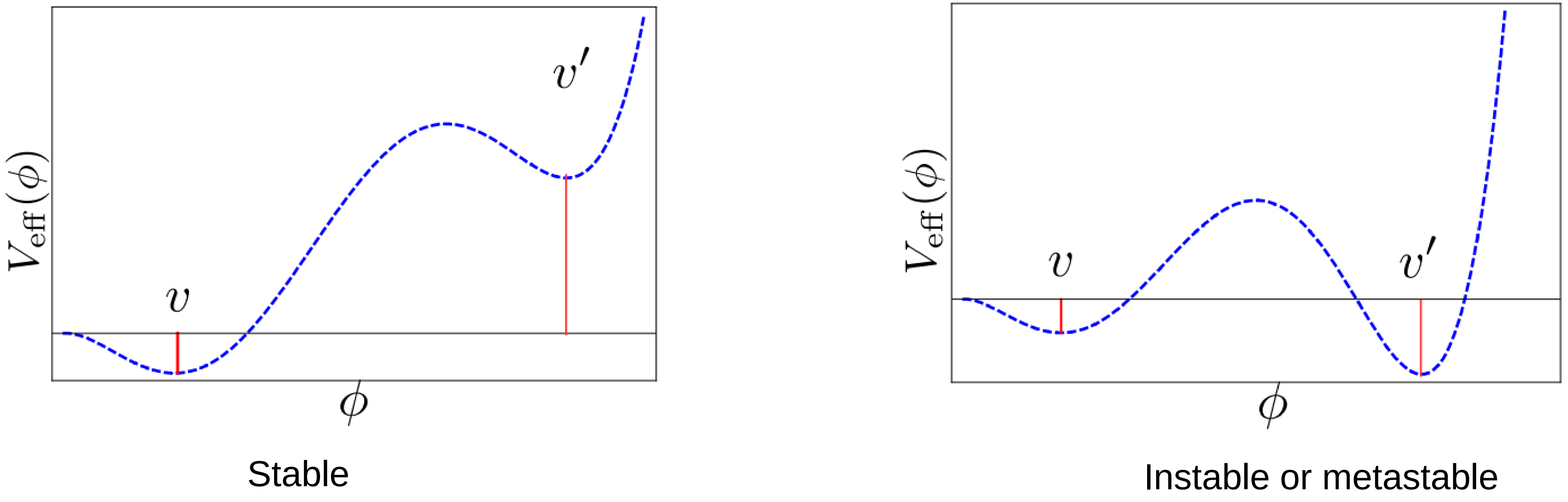



Fig.3. Different shapes of effective potential.

Critical situation: $V_{\text{eff}}(v) = V_{\text{eff}}(v')$

[Degrassi et al, arXiv: 1205.6497]

There are three different boundary conditions to determine the values of Higgs and top mass, which ensure vacuum stability

① Choose a scale $\Lambda$ which makes $\lambda(\Lambda) = 0$, so that for a given Higgs mass we can get a corresponding top mass.

This is the condition of absolute stability of the potential.

② Find the scale $\Lambda$ where

$$\lambda(\Lambda) = \beta_\lambda(\Lambda) = 0 \quad , \quad \beta_\lambda = \frac{d}{d\ln\mu}\lambda(\mu) \tag{14}$$

In practice, the determination of $M_H$ obtained by the condition ② differs by about 0.1 GeV from the one determined by ①, this difference between them is much smaller than the current theoretical and experimental precisions of the Higgs mass. [Bezrukov et al,arXiv: 1205.2893]

③ A more elaborated approache is based on "full" effective potential

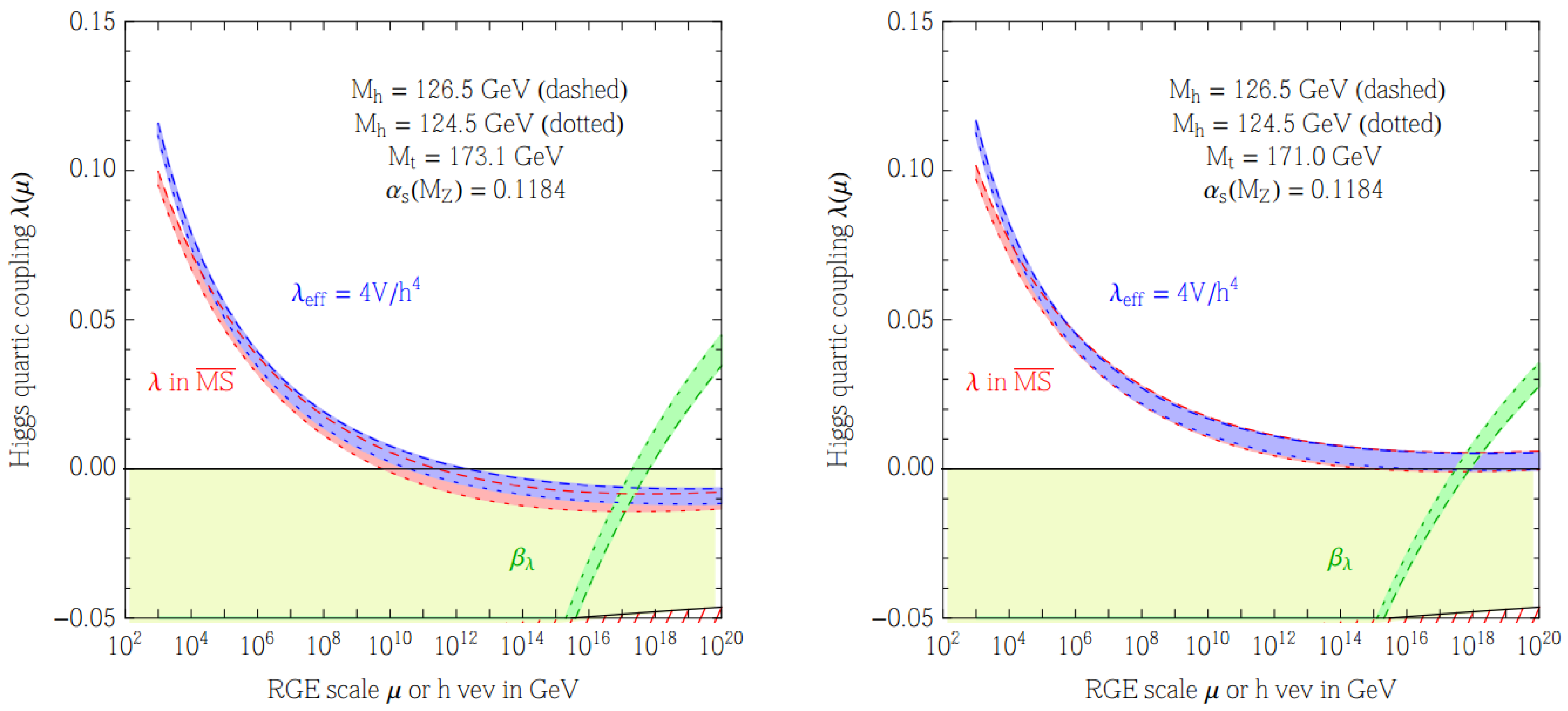
Iacobellis et al, arXiv: 1604.06046

$$V_{eff}(\phi) = \frac{\lambda_{eff}(\mu = \phi)}{4}\phi^4 \tag{15}$$

Ford et al, arXiv: 9210033

and

$$\lambda_{eff}(\mu = \phi) = 0 \tag{16}$$




Degrassi et al, arXiv: 1205.6497

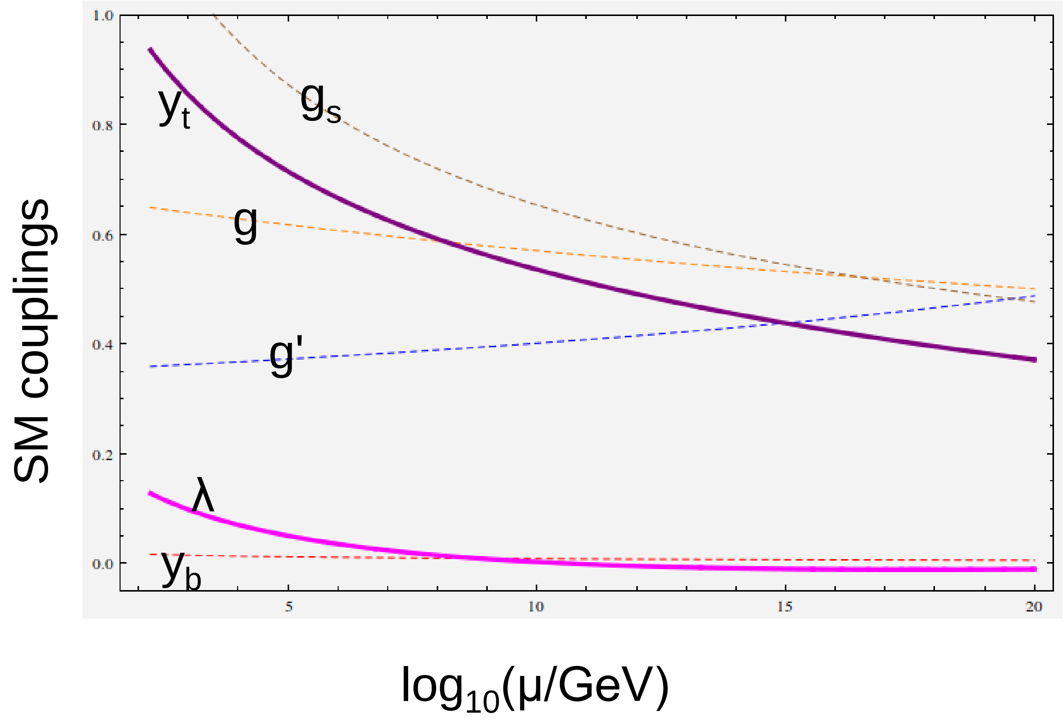
Fig.4. Evolution of the Higgs coupling $\lambda(\mu)$ and its beta function, eq. (14), as a function of the renormalization scale, compared to the evolution of the effective coupling $\lambda_{eff}(\mu)$, defined in eq. (15), as a function of the field value. Left: curves plotted for the best-fit value of Mt. Right: curves plotted for the lower value of Mt that corresponds to $\lambda(M_{pl}) = 0$.

Note that the difference $\lambda_{eff}(\mu) - \lambda(\mu)$ gets suppressed at large field values, we just show our results for the boundary condition 1, namely

$$\lambda(\Lambda) = 0 \quad (17)$$

# 5 Our results

## 5.1 SM RG evolution of the couplings



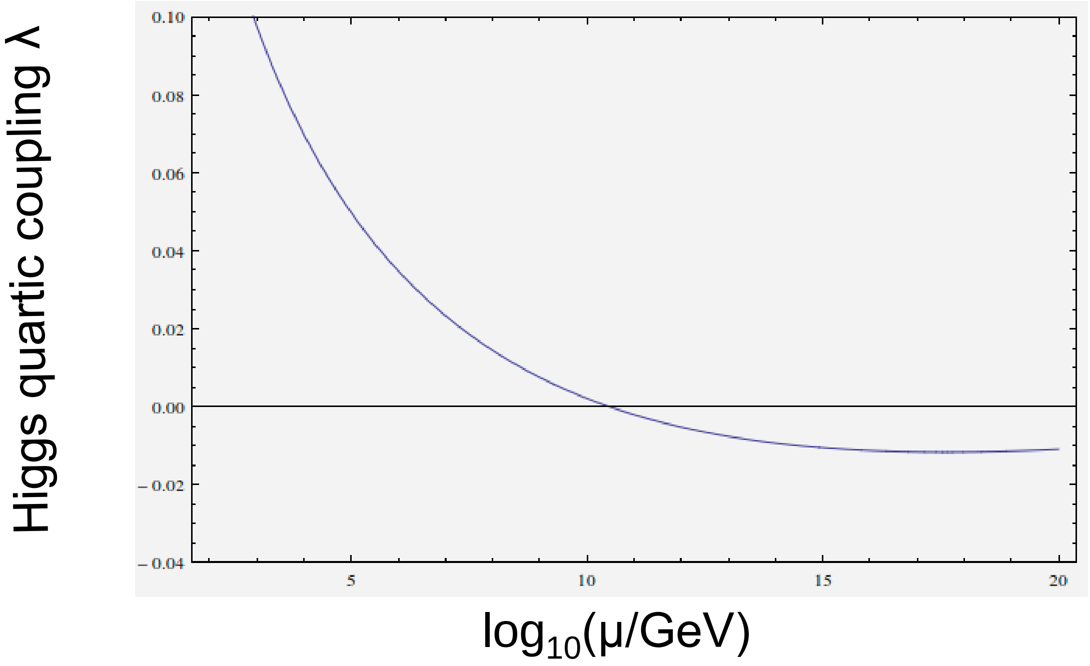
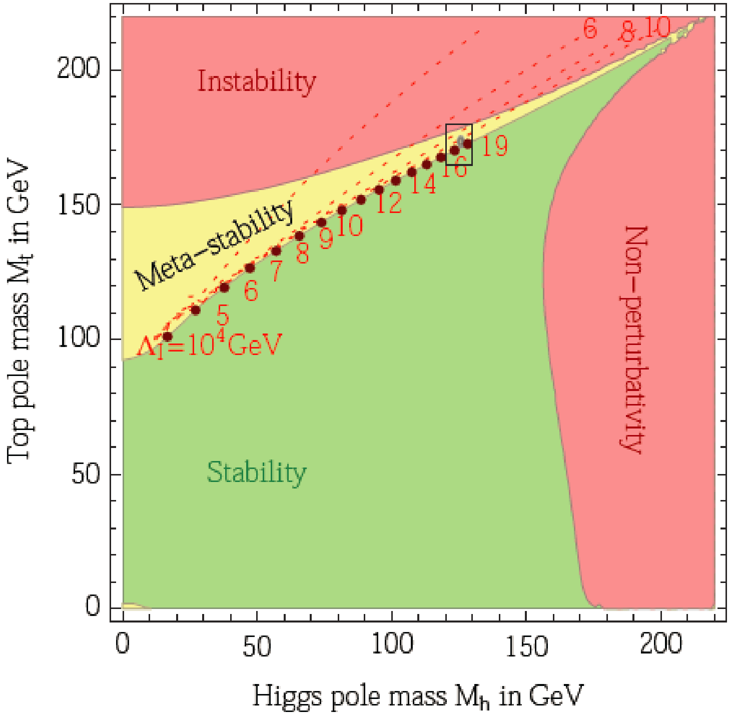
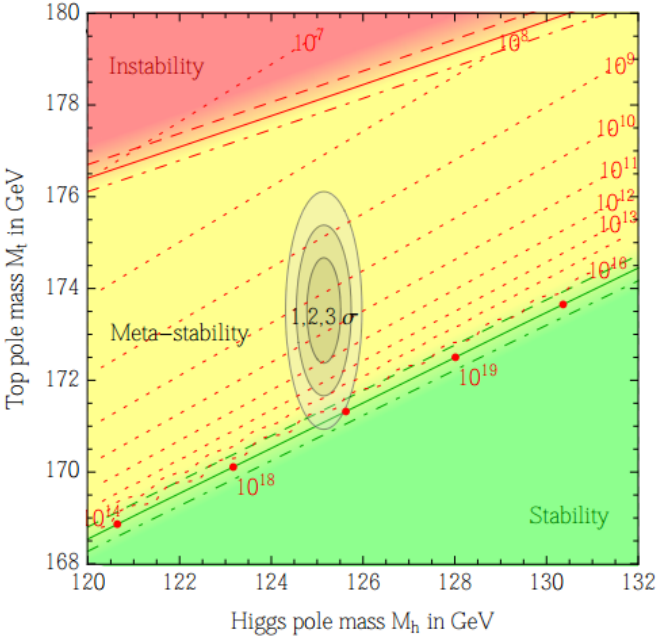




Fig.5. ***Left***: SM RG evolution of the gauge couplings g', g, $g_s$, of the top and bottom Yukawa couplings $y_t$ and $y_b$ , and of the Higgs quartic coupling λ. All couplings are defined in the $\overline{\mathrm{MS}}$ scheme. ***Right***: RG evolution of λ, which gets zero at μ=10^10-10^11GeV.

# 5.2 SM phase diagram in terms of masses

Buttazzo et al, arXiv: 1307.3536



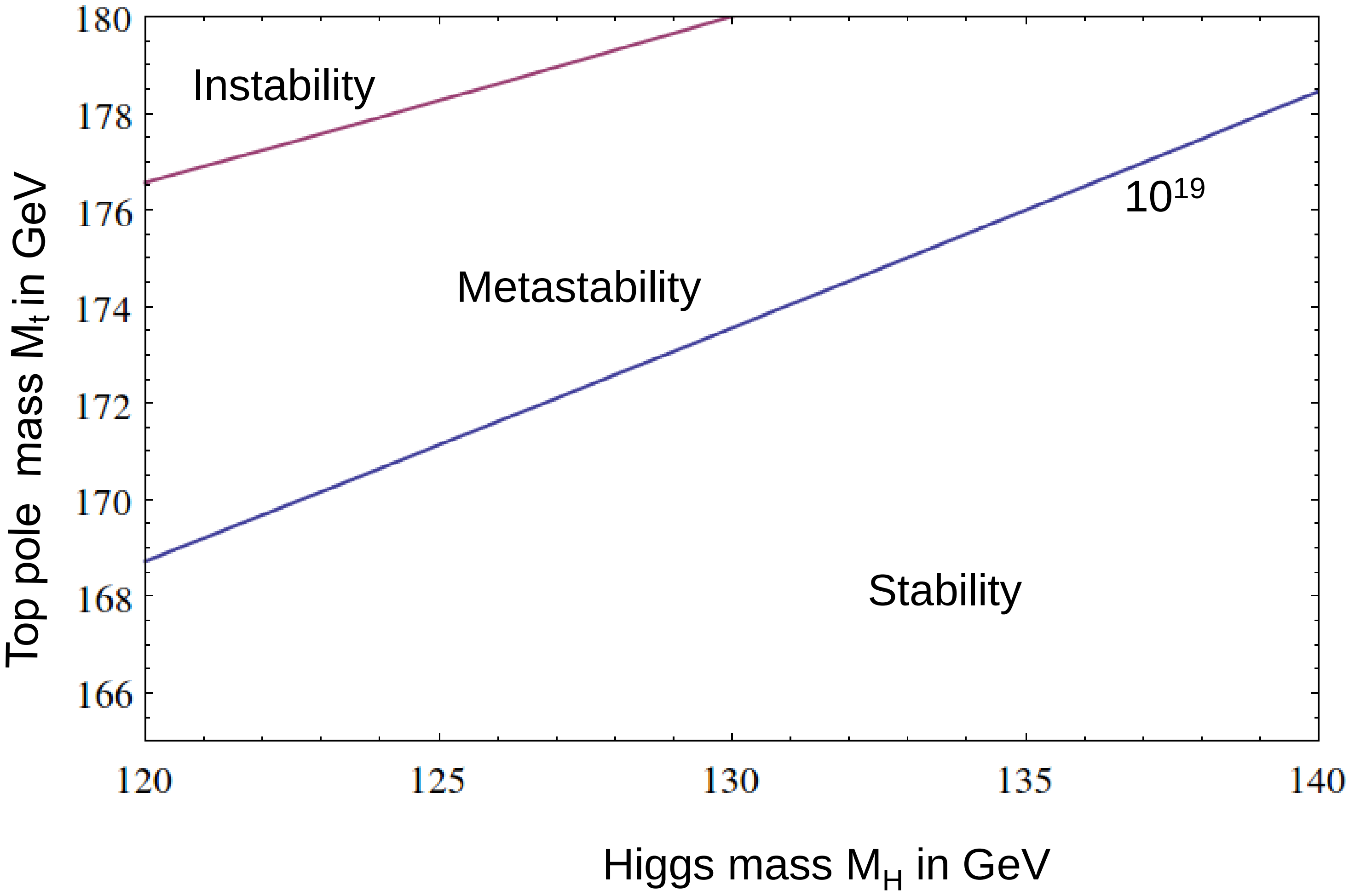
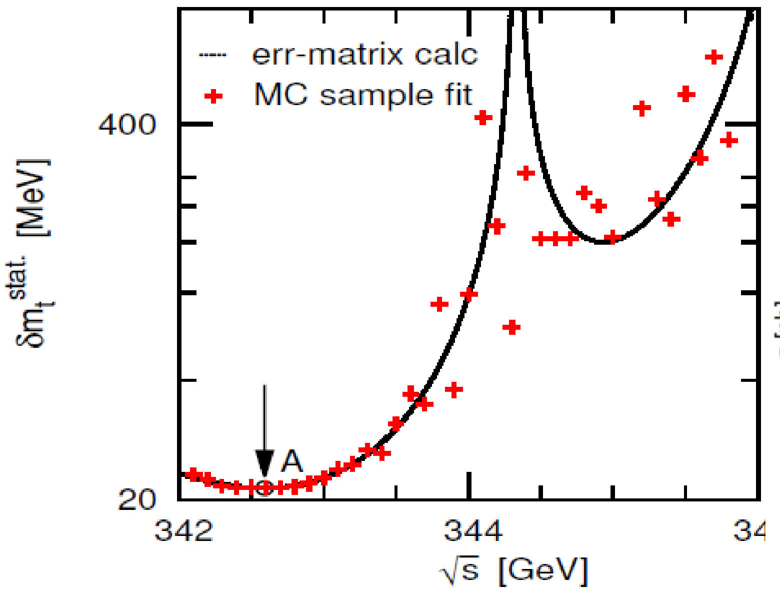
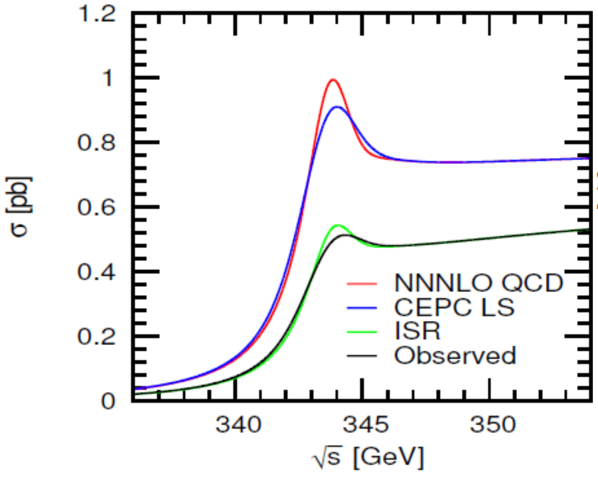
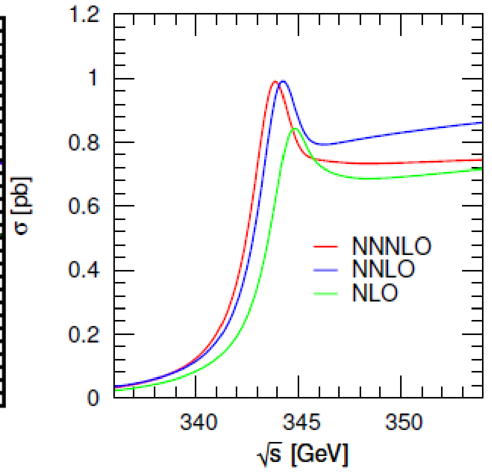



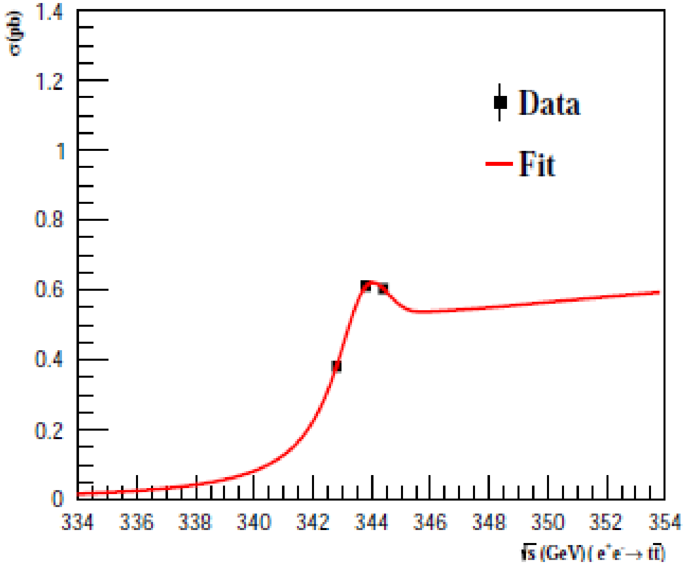

Fig.6. SM phase diagram in the $M_t$-$M_H$ plane. For the current experimental values of the top and Higgs masses, our universe lives in a metastable state, near the edge of stability.

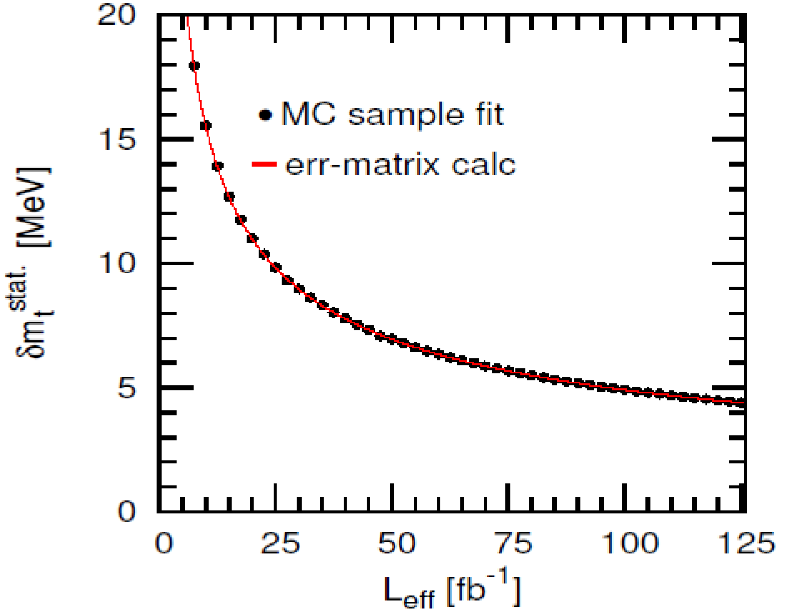
# Top mass measurement simulation at CEPC




| parameters | results | mt | width | $\alpha_s$ |
|---|---|---|---|---|
| 1 | central value | 171.500 GeV | - | - |
| | stat.err | 4 MeV | - | - |
| 2 | central value | 171.500 GeV | 1.314 GeV | - |
| | stat.err | 4 MeV | 9 MeV | - |
| 3 | central value | 171.515 GeV | 1.354 GeV | 0.11918 |
| | stat.err | 9 MeV | 21 MeV | 0.0004 |

## 5.3 Lifetime of the EW vacuum

Branchina et al, arXiv: 1407.4112

For a given potential $V(\phi)$, the general procedure to obtain the tunnelling time $\tau$ is to look first for the bounce solution (tree level) to the Euclidean equation of motion, and to compute then the quantum fluctuations on the top of it. For the Higgs potential $V(\phi) = \lambda\phi^4/4$, once the running of the quartic coupling is taken into account, this amounts to the following minimization formula

$$\tau = T_U \min_{\mu} \mathcal{T}(\mu) \qquad (18)$$

where $T_U$ is the lifetime of our universe, and $\mathcal{T}(\mu)$

$$\mathcal{T}(\mu) \sim T_U^{-4} \mu^{-4} e^{\frac{8\pi^2}{3|\lambda_{eff}(\mu)|}} \qquad (19)$$

In Fig.6., the stability line (boarder between the stability and the metastability regions) is obtained for those values of $M_H$ and $M_t$ such that Eq.(16) is satisfied, the instability line (boarder between the metastability region, $\tau > T_U$, and the instability region, $\tau < T_U$) for those values of $M_H$ and $M_t$ such that $\tau = T_U$ .

The lifetime of our universe is

$$T_U \simeq 13.8 \times 10^9 \text{ years} \tag{20}$$

we have

$$\tau \simeq 10^{742}\ T_U \tag{21}$$

Or equivalently, the probability of quantum tunnelling out of the EW vacuum is given, in semi-classical approximation, by

Isidori et al, arXiv: 0104016

$$P \simeq T_U^4\ \mu^4\ e^{\frac{-8\ \pi^2}{3\ |\lambda\ (\mu)|}} \tag{22}$$

we have

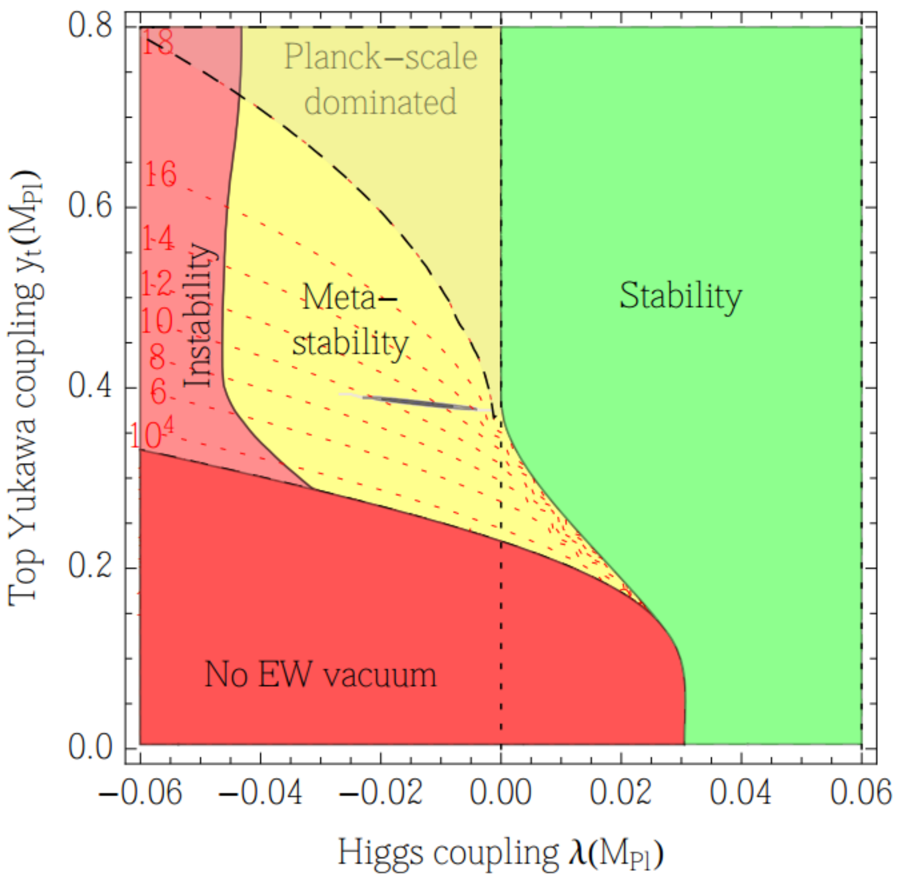
$$P \simeq 10^{-742} \tag{23}$$

The probility that our universe decay into the true EW vacuum is nearly zero.

# 5.4 SM phase diagram in terms of couplings

## 5.4.1 Couplings renormalised at Planck scale

Buttazzo et al, arXiv: 1307.3536



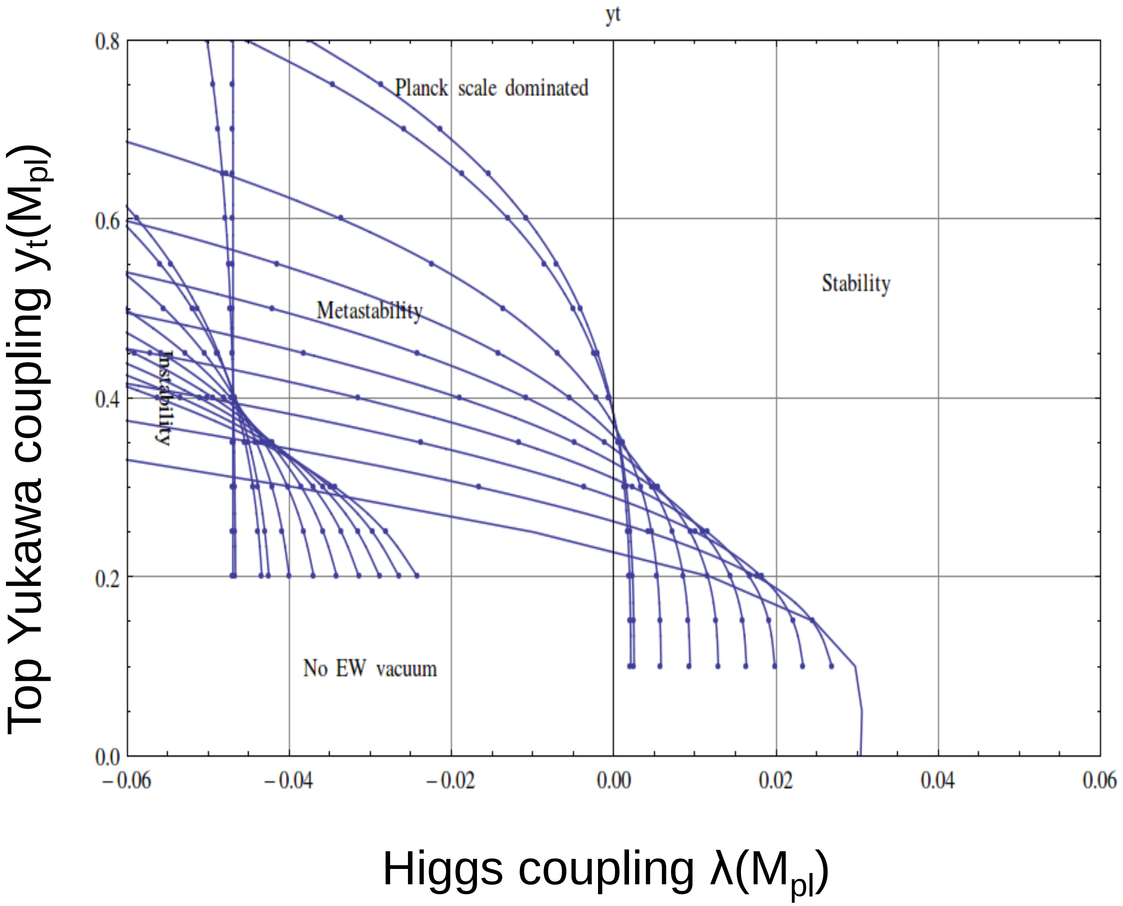
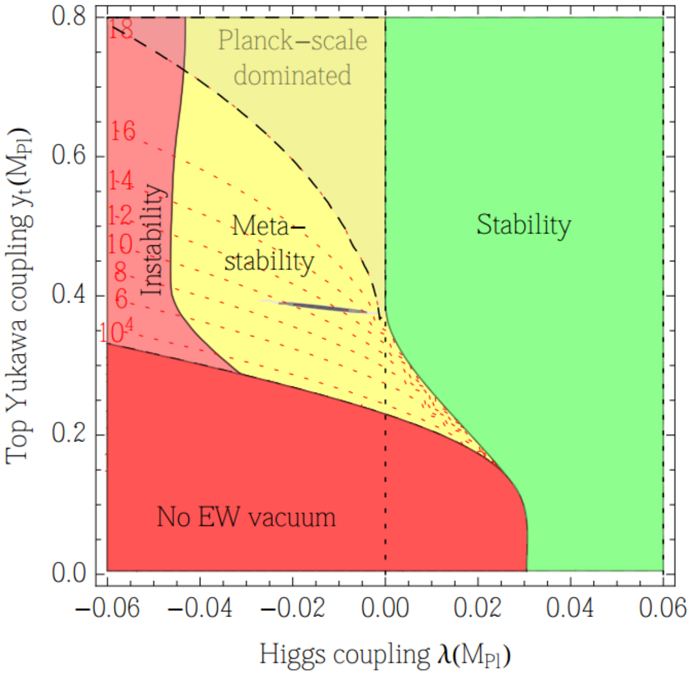
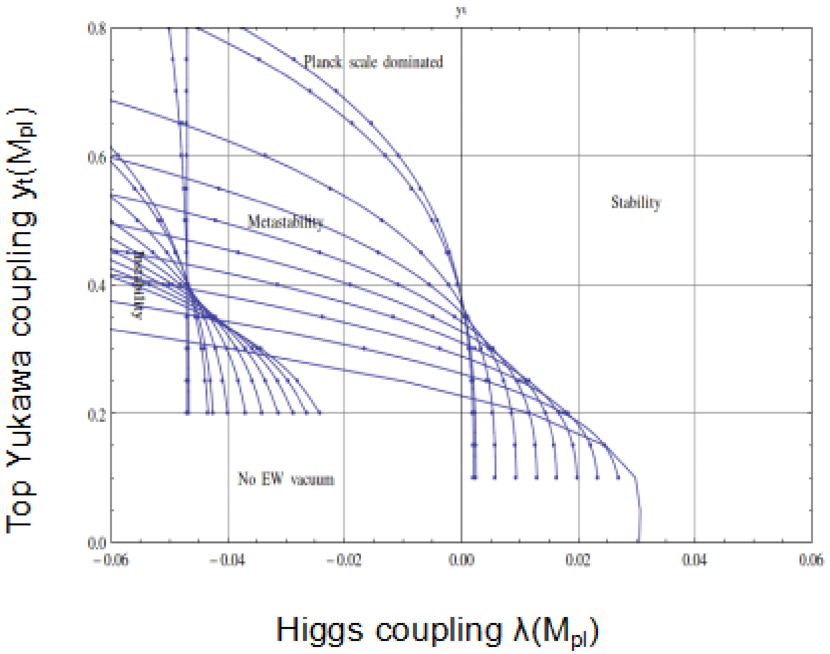




Fig.7. SM phase diagram in terms of quartic Higgs coupling $\lambda$ and top Yukawa coupling $y_t$ renormalised at the Planck scale. Right side is our result, we have chosen $M_t$ as the EW scale .




The "no EW vacuum" corresponds to a situation in which λ is negative at the weak scale, and therefore the usual Higgs vacuum does not exist.

The stability line (boarder between the stability and the metastability regions) is obtained for that $\lambda(M_{pl})=0$.

The metastability line is obtained for that $\lambda(\mu)=0$, μ changes form $10^4$ to $10^{18}$ GeV. The difference between the stability line and metastability line is just because our different choices of μ, and when $\mu>10^{18}$GeV, we call it "Planck-scale dominated" region.

For different μ, we have different values of λ, which can be determined by Eq.(19). For example, when $\mu=10^{11}$GeV, $\lambda\approx-0.054$. μ changes from $10^{11}$ GeV to $M_{pl}$, we get the instability region.

# Observation of $t\bar{t}H$ production

The CMS Collaboration, arXiv: 1804.02610

The observation of Higgs boson production in association with a top quark-antiquark pair is reported, the combined best fit signal strength normalized to the standard model prediction is $1.26^{+0.31}_{-0.26}$ .

The ratio between the normalization of the $t\bar{t}H$ production process and its SM expectation , defined as the signal strength modifier $\mu_{t\bar{t}H}$ , is a freely floating parameter in the fit

$$\mu_i = \frac{\sigma_i}{(\sigma_i)_{SM}} \tag{24}$$

A way to parametrize the couplings of the Higgs boson to the SM particles is given by using the values $\kappa_t$ and $\kappa_V$ , which are defined as the ratio of the actual coupling strengths to the SM predictions for the top quark and the massive vector bosons, respectively, and

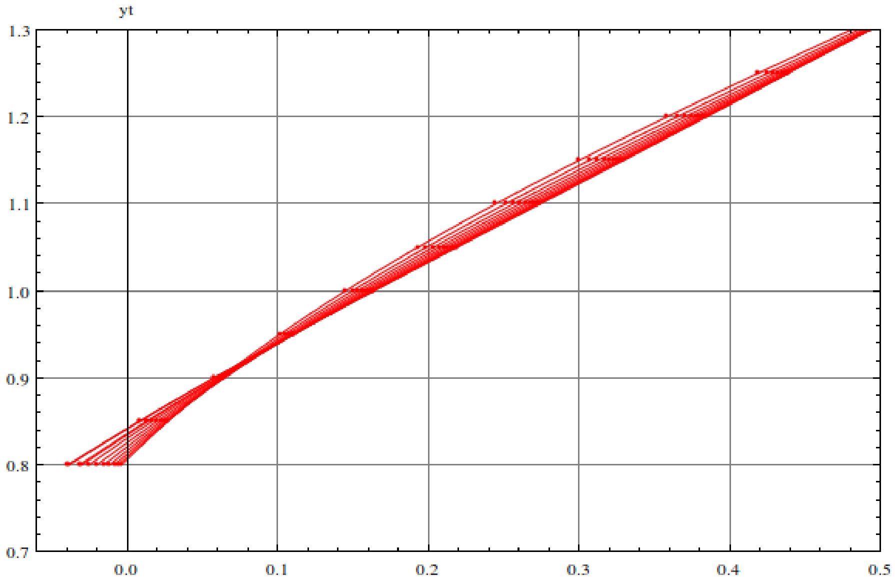
$$\kappa_i^2 = \frac{\sigma_i}{\sigma_i^{SM}} \tag{25}$$

so we can make a naive translation and get

$$\mu_{t\bar{t}H} = \kappa_t^2 = 1.26^{+0.31}_{-0.26} \;\Rightarrow\; \kappa_t = 1.12^{+0.14}_{-0.12}\,,\quad \kappa_t \in (1.00,\, 1.26) \tag{26}$$

# 5.4.2 Couplings renormalized at EW scale

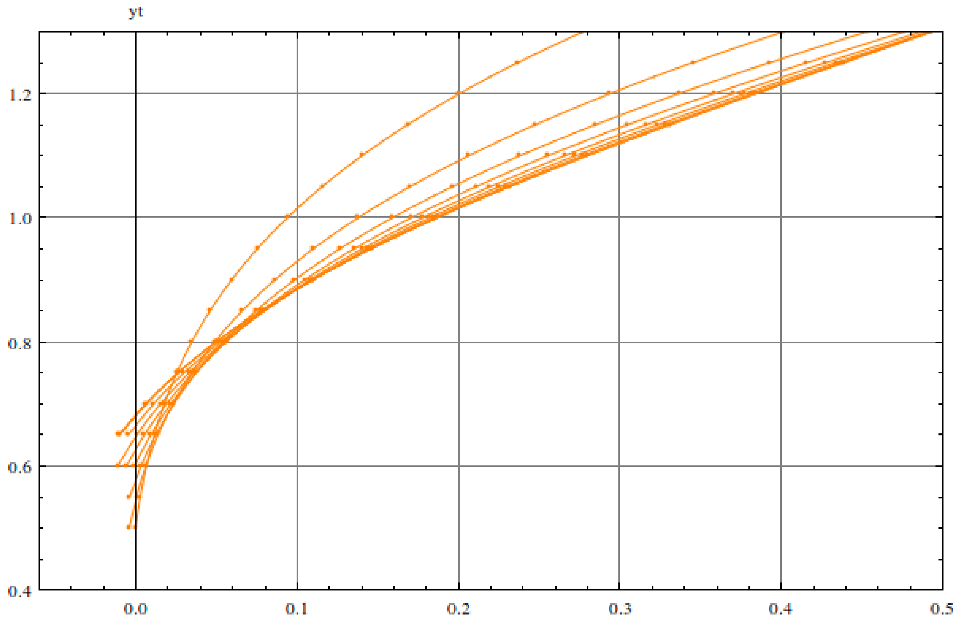
Top Yukawa coupling $y_t(M_t)$




Higgs coupling $\lambda(M_t)$

Instability line(μ change from 10^11 GeV to $M_{pl}$)

Top Yukawa coupling $y_t(M_t)$



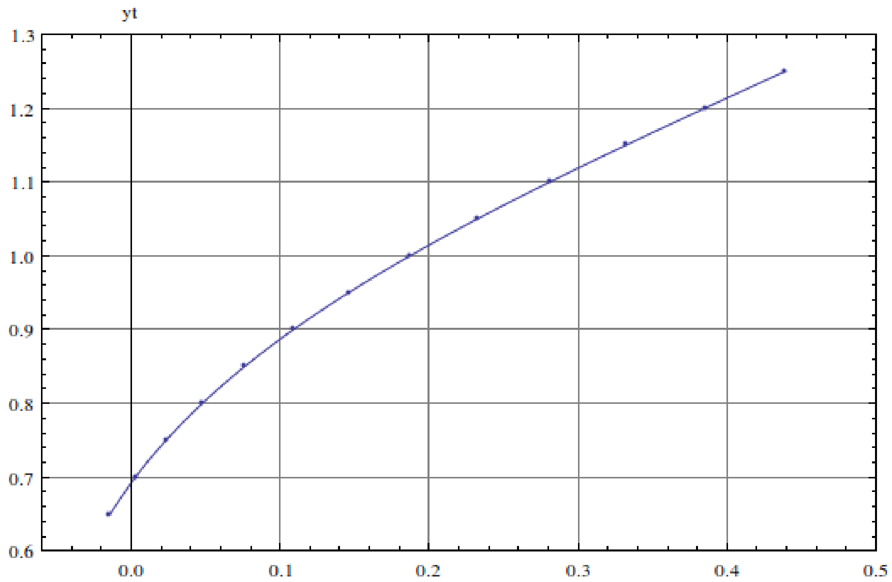

Higgs coupling $\lambda(M_t)$

Metastability line(μ change form 10^4 to 10^18 GeV)

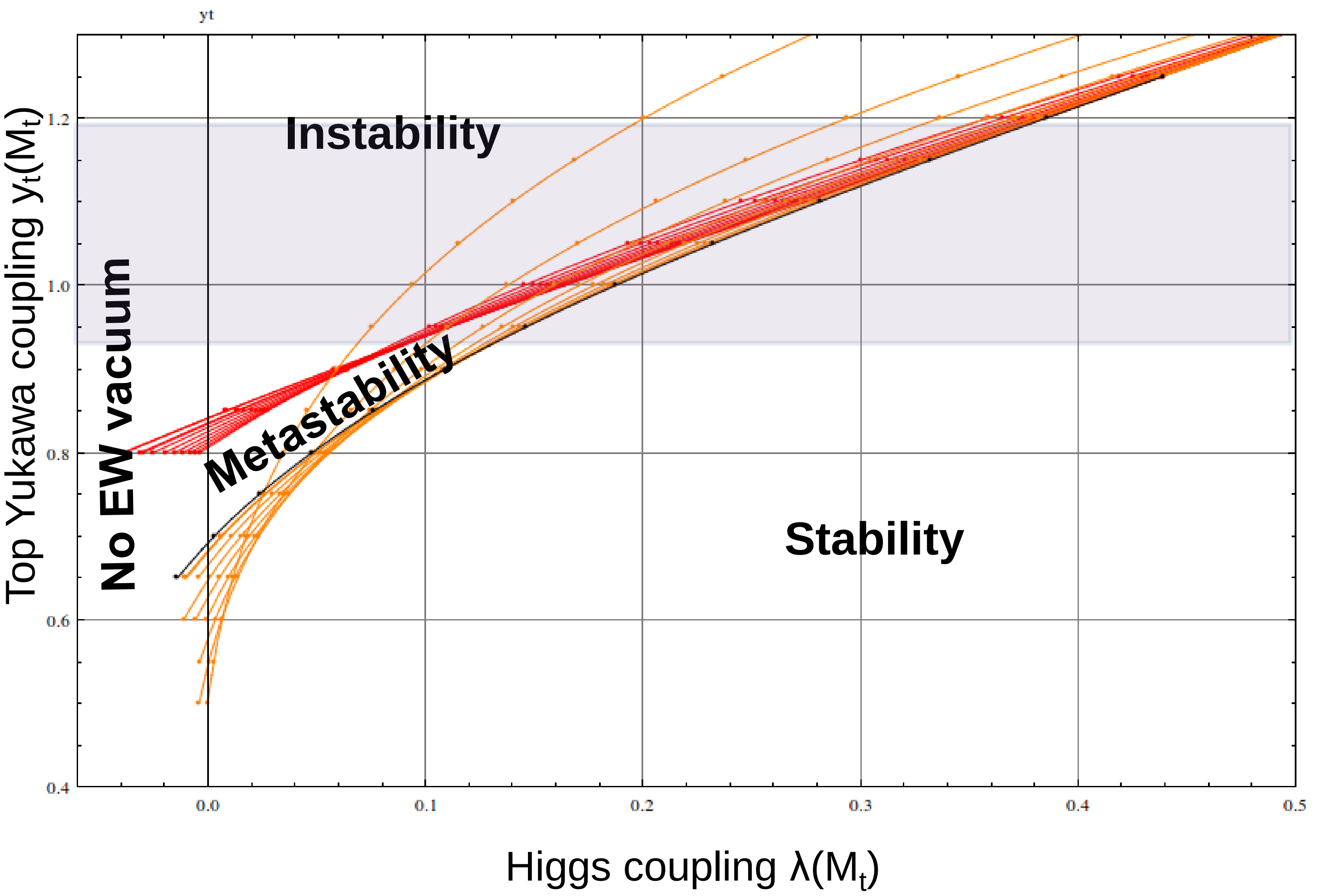
Fig.8

Stability line(μ=$M_{pl}$)

Top Yukawa coupling $y_t(M_t)$




Higgs coupling $\lambda(M_t)$



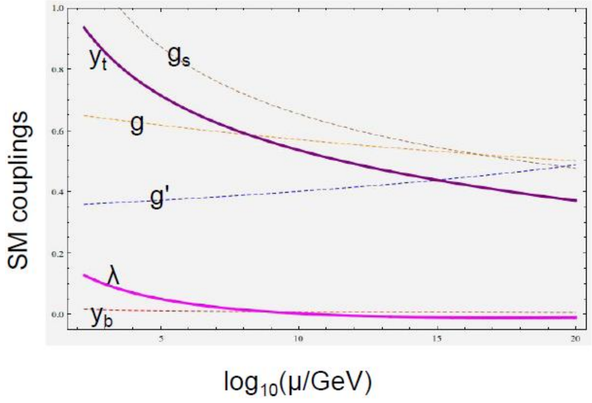




Fig.9 SM phase diagram in terms of quartic Higgs coupling λ and top Yukawa coupling $y_t$ at the EW scale. Because those couplings are at the EW scale, the region where $\lambda<0$ is "No EW vacuum".

# 6 Summary

(1)SM electroweak vacuum is possibly metastable. Precise top mass measurement from CEPC is highly expected.

(2)We show 3 phase diagrams here. Mt vs MH, Yt vs $\lambda$ at Plank scale, and Yt vs $\lambda$ at top mass scale.

(3)Top Yukawa coupling strength from LHC ttH experiment can be used to constrain the vacuum stability.

**Thank you**