The Threshold Effects of Fermionic Electroweak Multiplet Dark Matter Model

Jin-Wei Wang (王金伟)

Key Laboratory of Particle Astrophysics,

Institute of High Energy Physics, CAS

PhysRevD.97.035021 [arXiv:1711.05622]

Tianjin, August 21, 2018

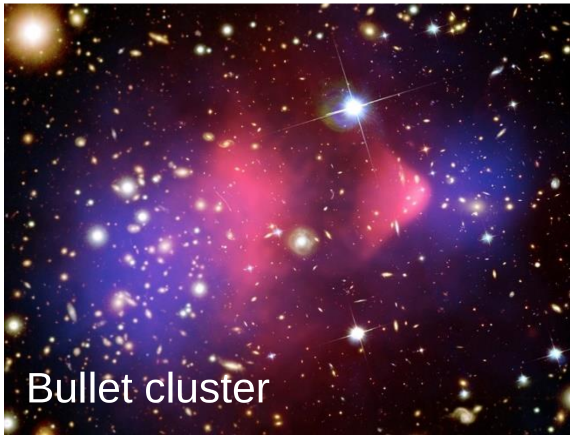
IHEP

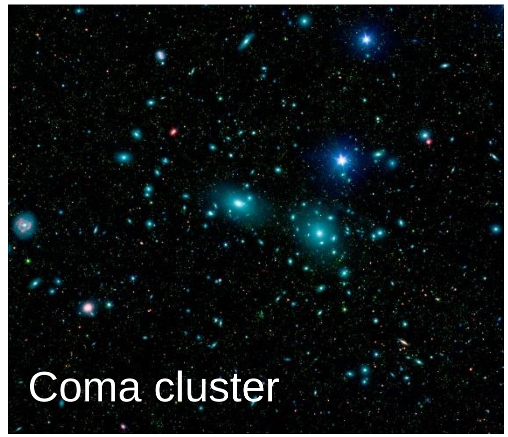
Outline

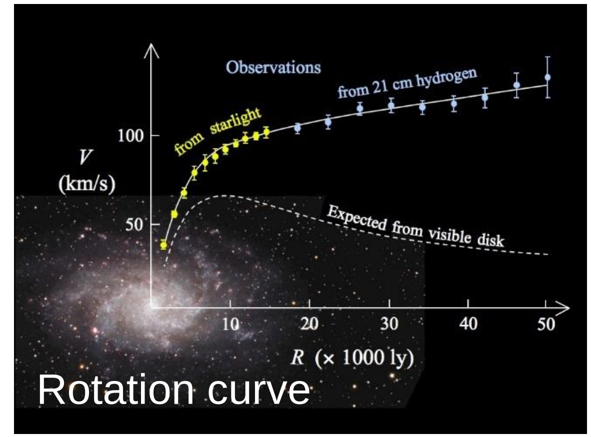
- Introduction and motivation
- Matching schemes and threshold effects
- Triplet-Quadruplet fermionic dark matter model
- Results
- Summary

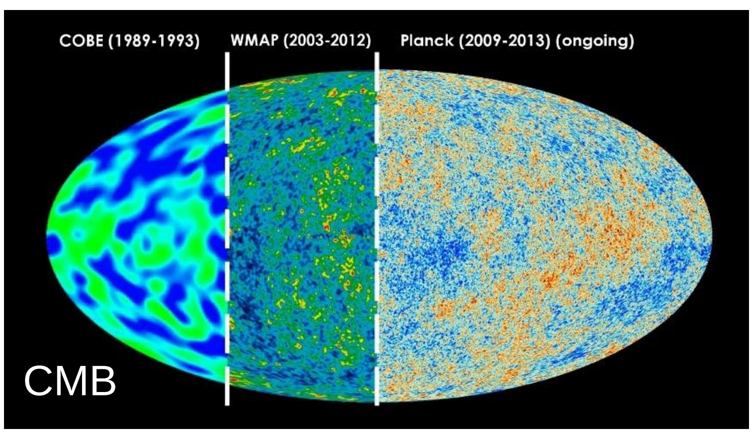
Dark matter in the Universe

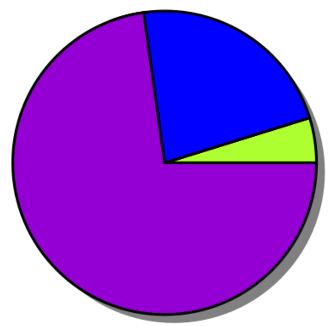
- The astrophysical and cosmological observations have provided compelling evidences of the existence of **dark matter (DM)**.











Planck 2015
[1502.01589]

Cold DM (25.8%)

$$\Omega_c h^2 = 0.1186 \pm 0.0020$$

Baryons (4.8%)

$$\Omega_b h^2 = 0.02226 \pm 0.00023$$

Dark energy (69.3%)

$$\Omega_\Lambda = 0.692 \pm 0.012$$

WIMP models

Weakly interacting massive particles (WIMPs) are very compelling DM candidates. WIMPs are typically introduced in the extensions of the SM.

- **Supersymmetry**: the lightest neutralino ($\tilde{\chi}_1^0$);
- **Universal Extra Dimensions**: the lightest KK particles ($B^{(1)}$, $W^{3(1)}$ or $\upsilon^{(1)}$);

For DM phenomenology, it is quite natural to construct WIMP models by extending the SM with a dark sector consisting of $SU(2)_L$ multiplet, whose neutral components could provide a viable DM candidate

- 1 multiplet in a high dim rep. : **minimal DM model** [Cirelli et al., 0512090]
  - (DM stability is explained by an 'accidental symmetry')
- 2 types of multiplets: an artificial Z_2 symmetry is usually needed
  - **Singlet-doublet DM model** [0510064, 0705.4493, 1109,2604]
  - **Doublet-triplet DM model** [1403.7744, 1707.03094]
  - **Triplet-quadruplet DM model** [1601.01354, **1711.05622**]

  … …

Matching and Running

To extrapolate a theory from the electroweak scale to high energies, we require two ingredients:

(1) the initial values of couplings at low energy scale;

(2) the RGE running of all parameters.

There are two different matching schemes:

For the tree level matching: $\begin{cases} \beta_{SM} & for \ \ \Lambda < \Lambda_{BSM} \\ \beta_{SM} + \beta_{BSM} & for \ \ \Lambda > \Lambda_{BSM} \end{cases}$

For the loop level matching: consider the loop corrections of BSM to the initial value, and then we use the complete RGE $\beta_{SM} + \beta_{BSM}$

We would like to calculate and compare these two different schemes.

Matching and Running

All MS bar parameters have gauge-invariant renormalisation group equations and are gauge invariant. Strumia et .al [arXiv:1307.3536]

So we shall always work in msbar scheme and determine the MS bar parameters in terms of physical observables:

Observables <=> OS parameters <=> MS bar parameters

This can be easily done by using the following relations:

$$\theta_0 = \theta_{\rm OS} - \delta\theta_{\rm OS} = \theta(\bar{\mu}) - \delta\theta_{\overline{\rm MS}} \quad \text{or} \quad \theta(\bar{\mu}) = \theta_{\rm OS} - \delta\theta_{\rm OS} + \delta\theta_{\overline{\rm MS}}$$

The divergent parts are canceled with each other, while for the finite part there is a higher-order discrapancy, but this can be ignored for one-loop level.

$$\theta(\bar{\mu}) = \theta_{\rm OS} - \left.\delta\theta_{\rm OS}\right|_{\rm fin} + \Delta_\theta$$

Matching and Running

Input values of the SM observables:

$$
\begin{aligned}
M_W &= 80.384 \pm 0.014 \text{ GeV} && \text{Pole mass of the } W \text{ boson} \\
M_Z &= 91.1876 \pm 0.0021 \text{ GeV} && \text{Pole mass of the } Z \text{ boson} \\
M_h &= 125.15 \pm 0.24 \text{ GeV} && \text{Pole mass of the higgs} \\
M_t &= 173.34 \pm 0.76 \pm 0.3 \text{ GeV} && \text{Pole mass of the top quark} \\
V \equiv (\sqrt{2} G_\mu)^{-1/2} &= 246.21971 \pm 0.00006 \text{ GeV} && \text{Fermi constant for } \mu \text{ decay} \\
\alpha_3(M_Z) &= 0.1184 \pm 0.0007 && \overline{\text{MS}} \text{ gauge SU}(3)_c \text{ coupling (5 flavours)}
\end{aligned}
$$

the SM fundamental parameters ($\lambda, m, y_t, g_2, g_Y$) can be defined by those observables:

$$
\lambda_{\rm OS} = \frac{G_\mu}{\sqrt{2}} M_h^2, \qquad m_{\rm OS}^2 = M_h^2.
$$

$$
y_{t_{\rm OS}} = 2\left(\frac{G_\mu}{\sqrt{2}} M_t^2\right)^{1/2}, \quad g_{2_{\rm OS}} = 2\left(\sqrt{2}\, G_\mu\right)^{1/2} M_W, \quad g_{Y_{\rm OS}} = 2\left(\sqrt{2}\, G_\mu\right)^{1/2} \sqrt{M_Z^2 - M_W^2} \; .
$$

Matching and Running

the one-loop correction of SM parameters: A. Sirlin and R. Zucchini, Nucl. Phys. B 266 (1986) 389

$$\delta^{(1)}\lambda_{\rm OS} = \frac{G_\mu}{\sqrt{2}}M_h^2\left\{\Delta r_0^{(1)} + \frac{1}{M_h^2}\left[\frac{T^{(1)}}{v_{\rm OS}} + \delta^{(1)}M_h^2\right]\right\} \qquad \delta^{(1)}m^2_{\rm OS} = 3\frac{T^{(1)}}{v_{\rm OS}} + \delta^{(1)}M_h^2$$

$$\delta^{(1)}y_{t_{\rm OS}} = 2\left(\frac{G_\mu}{\sqrt{2}}M_t^2\right)^{1/2}\left(\frac{\delta^{(1)}M_t}{M_t} + \frac{\Delta r_0^{(1)}}{2}\right) \qquad \delta^{(1)}g_{2_{\rm OS}} = \left(\sqrt{2}\,G_\mu\right)^{1/2}M_W\left(\frac{\delta^{(1)}M_W^2}{M_W^2} + \Delta r_0^{(1)}\right)$$

$$\delta^{(1)}g_{Y_{\rm OS}} = \left(\sqrt{2}\,G_\mu\right)^{1/2}\sqrt{M_Z^2 - M_W^2}\left(\frac{\delta^{(1)}M_Z^2 - \delta^{(1)}M_W^2}{M_Z^2 - M_W^2} + \Delta r_0^{(1)}\right)$$

these corrections only depend on mass corrections of higgs and gauge boson and Δr_0

we need to calculate the loop corrections to the muon decay process:

$$\frac{G_\mu}{\sqrt{2}} = \frac{1}{2v_0^2}(1+\Delta r_0) \qquad \Delta r_0^{(1)} = V_W^{(1)} - \frac{A_{WW}^{(1)}}{M_W^2} + \frac{\sqrt{2}}{G_\mu}\mathcal{B}_W^{(1)} + \mathcal{E}^{(1)}$$

V_W is the vertex contribution; A_{WW} is the W self-energy at zero momentum;

$\mathcal{B}_W$ is the box contribution; $\mathcal{E}$ is the term due to renormalization of exernal legs;

All these terms are computed at zero external momenta.

Triplet-quadruplet DM model PhysRevD.97.035021 [arXiv:1711.05622]

Dark sector Weyl fermions ($SU(2)_L \times U(1)_Y$):

$$T = \begin{pmatrix} T^+ \\ T^0 \\ -T^- \end{pmatrix} \in (\mathbf{3}, 0), \quad Q_1 = \begin{pmatrix} Q_1^+ \\ Q_1^0 \\ Q_1^- \\ Q_1^{--} \end{pmatrix} \in \left(\mathbf{4}, -\frac{1}{2}\right), \quad Q_2 = \begin{pmatrix} Q_2^{++} \\ Q_2^+ \\ Q_2^0 \\ Q_2^- \end{pmatrix} \in \left(\mathbf{4}, +\frac{1}{2}\right)$$

Gauge invariant kinetic terms, mass terms and Yukawa couplings:

$$\mathcal{L}_T = iT^\dagger \bar{\sigma}^\mu D_\mu T - \left(m_T a_{ij} T^i T^j + \text{h.c.}\right)$$

$$\mathcal{L}_Q = iQ_1^\dagger \bar{\sigma}^\mu D_\mu Q_1 + iQ_2^\dagger \bar{\sigma}^\mu D_\mu Q_2 - \left(m_Q b_{ij} Q_1^i Q_2^j + \text{h.c.}\right)$$

$$\mathcal{L}_{HTQ} = y_1 c_{ijk} Q_1^i T^j H^k + y_2 d_{ijk} Q_2^i T^j \widetilde{H}^k + \text{h.c.}$$

There are four independent parameters: m_T, m_Q, y_1, y_2

State mixing

Before the EWSB the mass matrix are:

$$\mathcal{M}_N = \begin{pmatrix} m_T & 0 & 0 \\ 0 & 0 & m_Q \\ 0 & m_Q & 0 \end{pmatrix}, \qquad \mathcal{M}_C = \begin{pmatrix} m_T & 0 & 0 \\ 0 & 0 & -m_Q \\ 0 & -m_Q & 0 \end{pmatrix}$$

Before rewrite the gauge eigenstates into mass eigenstates:

$$\begin{pmatrix} T^0 \\ Q_1^0 \\ Q_2^0 \end{pmatrix} = \mathcal{N} \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \\ \chi_3^0 \end{pmatrix}, \qquad \begin{pmatrix} T^+ \\ Q_1^+ \\ Q_2^+ \end{pmatrix} = C_L \begin{pmatrix} \chi_1^+ \\ \chi_2^+ \\ \chi_3^+ \end{pmatrix}, \qquad \begin{pmatrix} T^- \\ Q_1^- \\ Q_2^- \end{pmatrix} = C_R \begin{pmatrix} \chi_1^- \\ \chi_2^- \\ \chi_3^- \end{pmatrix}$$

(1) 3 Majorana fermions, 3 singly charged fermions, 1 doubly charged fermion.

(2) The triplet and quadruplets are decoupled, and there are some interesting properties for the couplings to higgs, W, and Z boson.

(3) If Z_2 symmetry is conserved, χ_1^0 will be the excellent DM candidate.

Beta Functions

First of all, we would like to calculate the β function of g_1, g_2, g_3;

(1) $\beta(g_1) = \beta^{SM}(g_1) + \frac{g_1^3}{(4\pi)^2}\left(\sum_i \frac{4}{3} n_f Q_i^2\right) = \beta^{SM}(g_1) + \frac{g_1^3}{(4\pi)^2}\left(\frac{4}{3}\times 4\times\left(-\frac{1}{2}\right)^2 + \frac{4}{3}\times 4\times\left(\frac{1}{2}\right)^2\right)\times\frac{1}{2} = \beta^{SM}(g_1) + \frac{g_1^3}{(4\pi)^2}\frac{4}{3} = \frac{1}{16\pi^2}\left(\frac{41}{6}+\frac{4}{3}\right)g_1^3;$

(2) $\beta(g_2) = \beta^{SM}(g_2) + \left(\frac{-g_2^3}{(4\pi)^2}\left(-\sum_i \frac{4}{3} n_f C(r)\right)\right) \xrightarrow[C(4)=tr[t^3t^3]=5]{C(3)=tr[t^3t^3]=2} \beta^{SM}(g_2) + \frac{g_2^3}{(4\pi)^2}\left(\frac{4}{3}\times 2 + \frac{4}{3}\times 5 + \frac{4}{3}\times 5\right)\times\frac{1}{2} = \frac{1}{16\pi^2}\left(-\frac{19}{6}+\frac{24}{3}\right)g_2^3;$

(3) $\beta(g_3) = \beta^{SM}(g_3) = \frac{1}{16\pi^2}(-7)g_3^3;$

For other Yukawa couplings we use the formulas:

$$\frac{\partial g}{\partial \ln\mu} = \beta(g) \qquad \beta(g) = M\frac{\partial}{\partial M}\left(-\delta_g + \frac{1}{2} g \sum_i \delta_{Z_i}\right)$$

For higgs quartic couplings we use effective potential:

$$\left(\frac{1}{4}\beta(\lambda) - \gamma\lambda\right)\phi_c^4 = -\mu_R \frac{\partial V_{3-4}}{\partial \mu_R} \qquad \gamma = \frac{1}{2} M \frac{\partial}{\partial M}\delta_h$$

we use PyR@TE compute the two loop beta functions

For 3-4 model:

$$V_{eff,3-4} = -\frac{2}{64\pi^2} m_-^4(\phi_c)\left[\ln\frac{m_-^2(\phi_c)}{\mu_R^2} - \frac{3}{2}\right] - \frac{2}{64\pi^2} m_+^4(\phi_c)\left[\ln\frac{m_+^2(\phi_c)}{\mu_R^2} - \frac{3}{2}\right]$$

$$m_- = \frac{(M_T+M_Q) - \sqrt{(M_T+M_Q)^2 - 4\left[M_T M_Q - \phi_c^2\left(A^2 y_1^2 + B^2 y_2^2\right)\right]}}{2}, \quad m_+ = \frac{(M_T+M_Q) + \sqrt{(M_T+M_Q)^2 - 4\left[M_T M_Q - \phi_c^2\left(A^2 y_1^2 + B^2 y_2^2\right)\right]}}{2}$$

Mass Corrections

The mass correction of higgs:

$$\delta_{3-4}^{(1)} m_h^2 = -\frac{6}{16\pi^2}\left\{\left(\frac{(y_1-y_2)^2}{6}\left[(m_Q+m_T)^2-m_h^2\right]+\frac{(y_1+y_2)^2}{6}\left[(m_Q-m_T)^2-m_h^2\right]\right)B_0(m_h,m_Q,m_T)+\left(\frac{y_1^2+y_2^2}{3}\right)\left[A_0(m_Q)+A_0(m_T)\right]\right\}$$

The mass correction of W and Z boson:

$$\Sigma_T^{ww}(m_w^2) = \frac{(loop-diagrams)_{finite}}{i} = \frac{1}{16\pi^2}(5g_2^2)f(m_w,m_Q,m_Q)+\frac{1}{16\pi^2}(g_2^2)f(m_w,m_T,m_T)$$

$$\Sigma_T^{zz}(m_w^2) = \frac{(loop-diagrams)_{finite}}{i} = \frac{1}{16\pi^2}(g_1^2\sin^2\theta+5g_2^2\cos^2\theta)f(m_w,m_Q,m_Q)+\frac{1}{16\pi^2}(g_2^2\cos^2\theta)f(m_w,m_T,m_T)$$

in which: $$f(m_w,m,m)=2\times\left[-m_w^2B_0(m_w,m,m)-4B_{00}(m_w,m,m)+2A_0(m)\right]$$

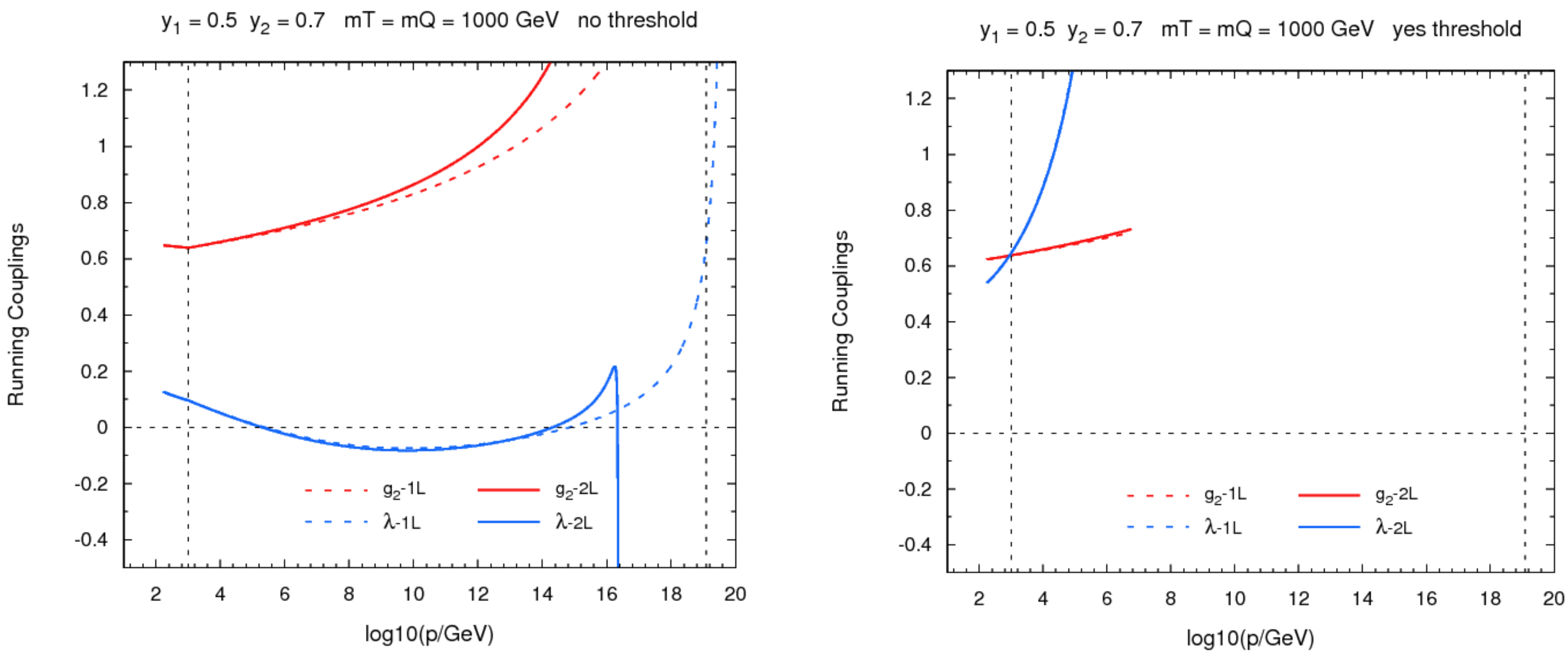
The mass correction of W and Z boson only depend on m_T, m_Q.

Running Couplings

For a banch mark pont: $y_1 = 0.5,\ \ y_2 = 0.7,\ \ \ m_T = m_Q = 1TeV.$

We will compare: (1) tree/loop level matching scheme;

(2) one/two loop RGE;



The stability of vacuum

The present value of the vacuum-decay probability p_0 is

$$p_0 = 0.15 \frac{\Lambda_B^4}{H_0^4} e^{-S(\Lambda_B)}$$

in which:

$$S(\Lambda_B) = \frac{8\pi^2}{3|\lambda(\Lambda_B)|}$$

$$\beta_\lambda(\Lambda_B) = 0$$

$$H_0 \approx 67.4\,\mathrm{km/sec\ Mpc}$$

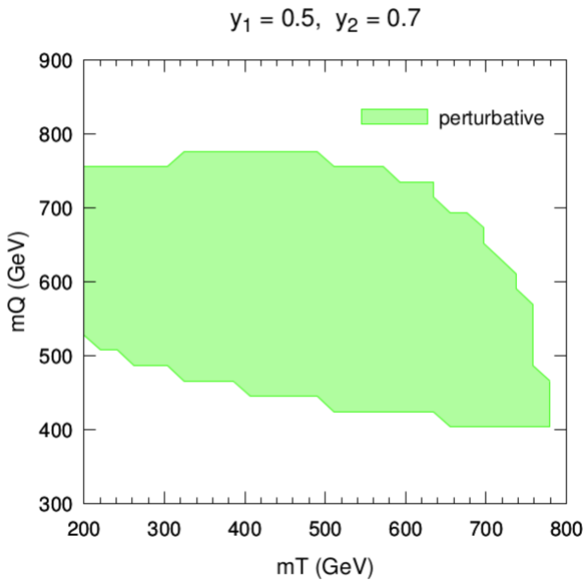
The conditions for different status of vacuum:

(1) stable: if $\lambda(\Lambda_B) > 0$ until Planck scale;

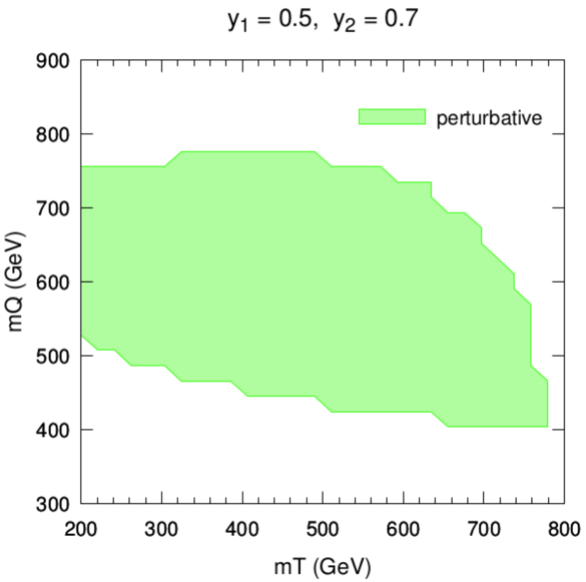
(2) metastable: if $\lambda(\Lambda_B) < 0$ and $p_0 < 1$;

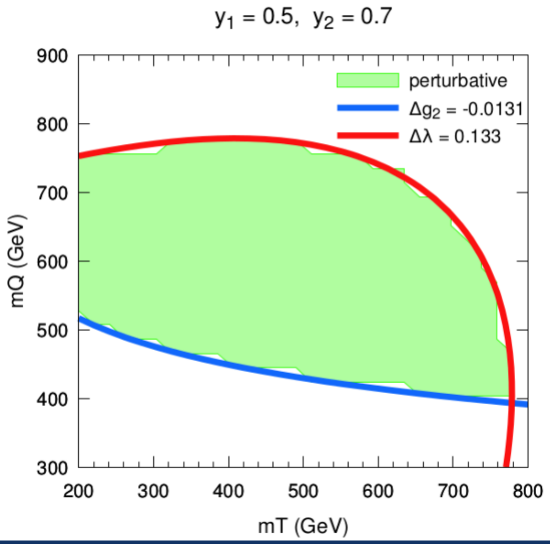
(3) unstable: if $\lambda(\Lambda_B) < 0$ and $p_0 > 1$;

Perturbation constrains

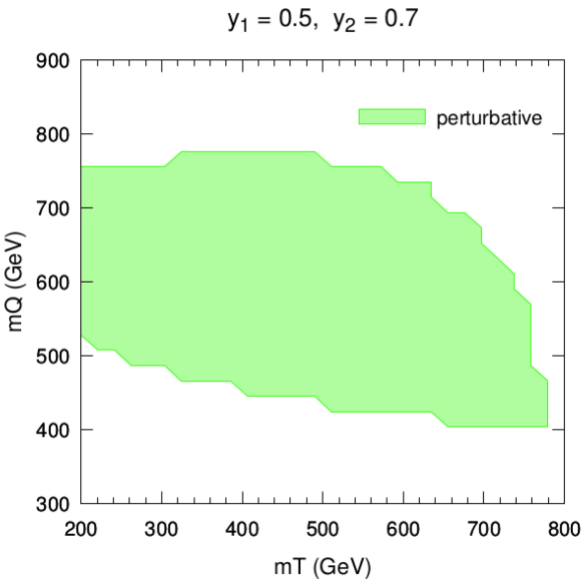


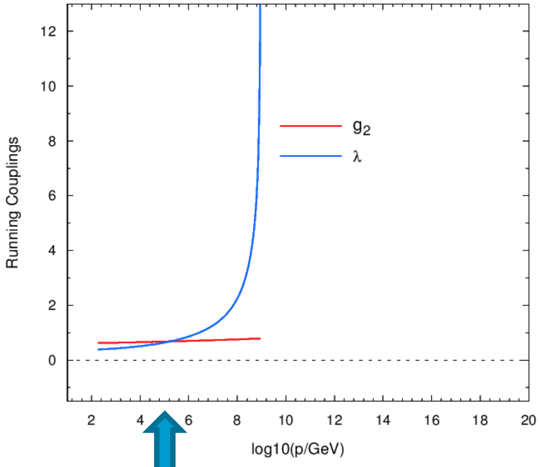
Perturbation constrains

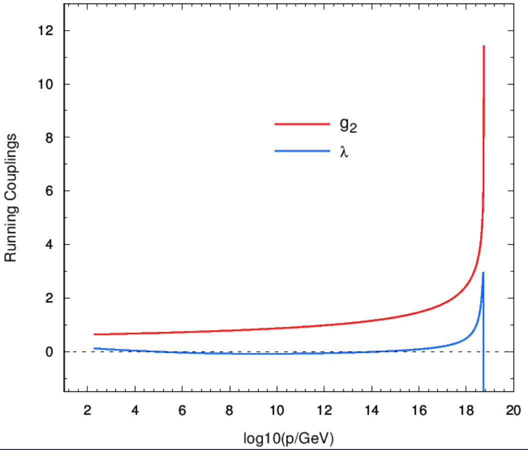


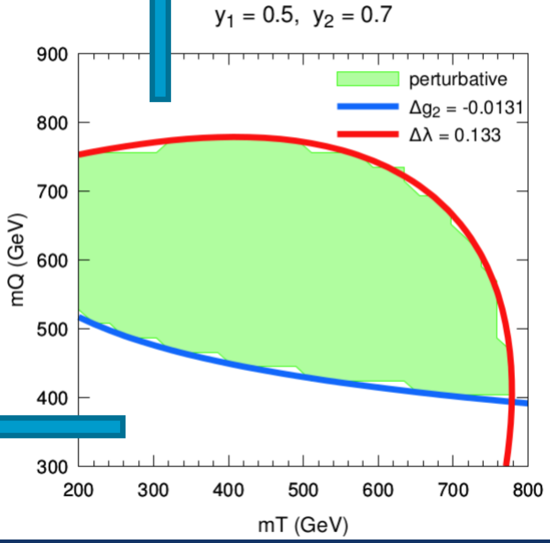


Perturbation constrains

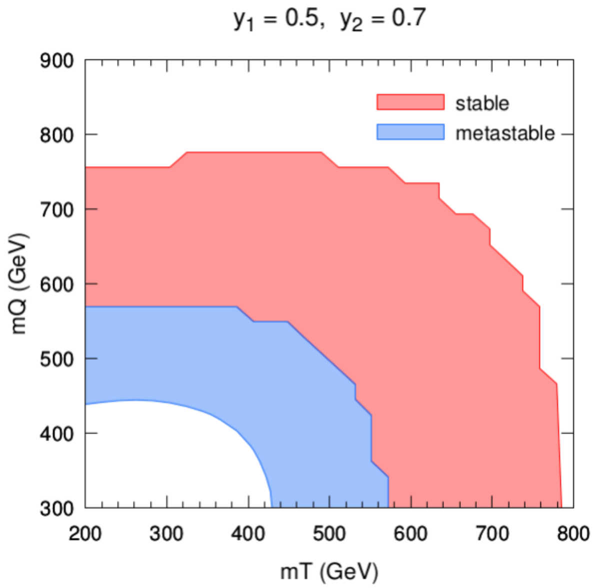




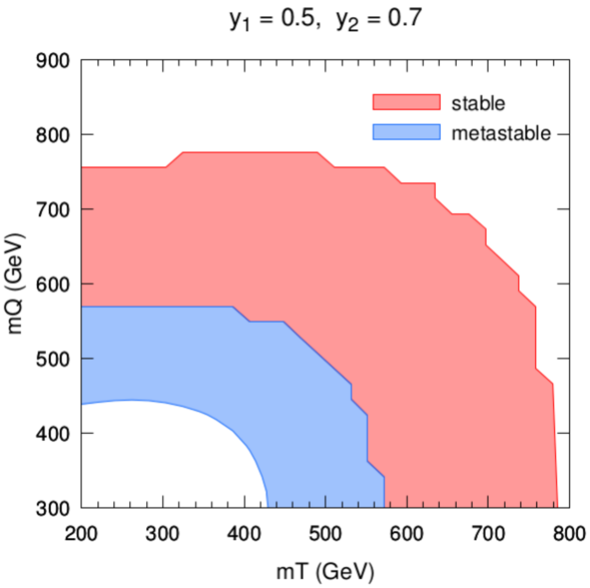


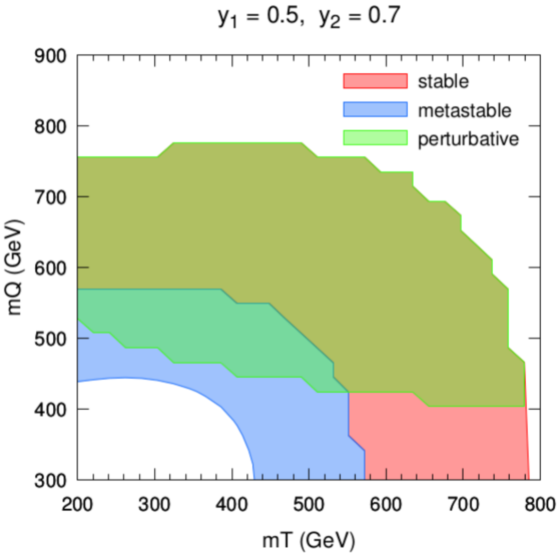


Constrains of vacuum status

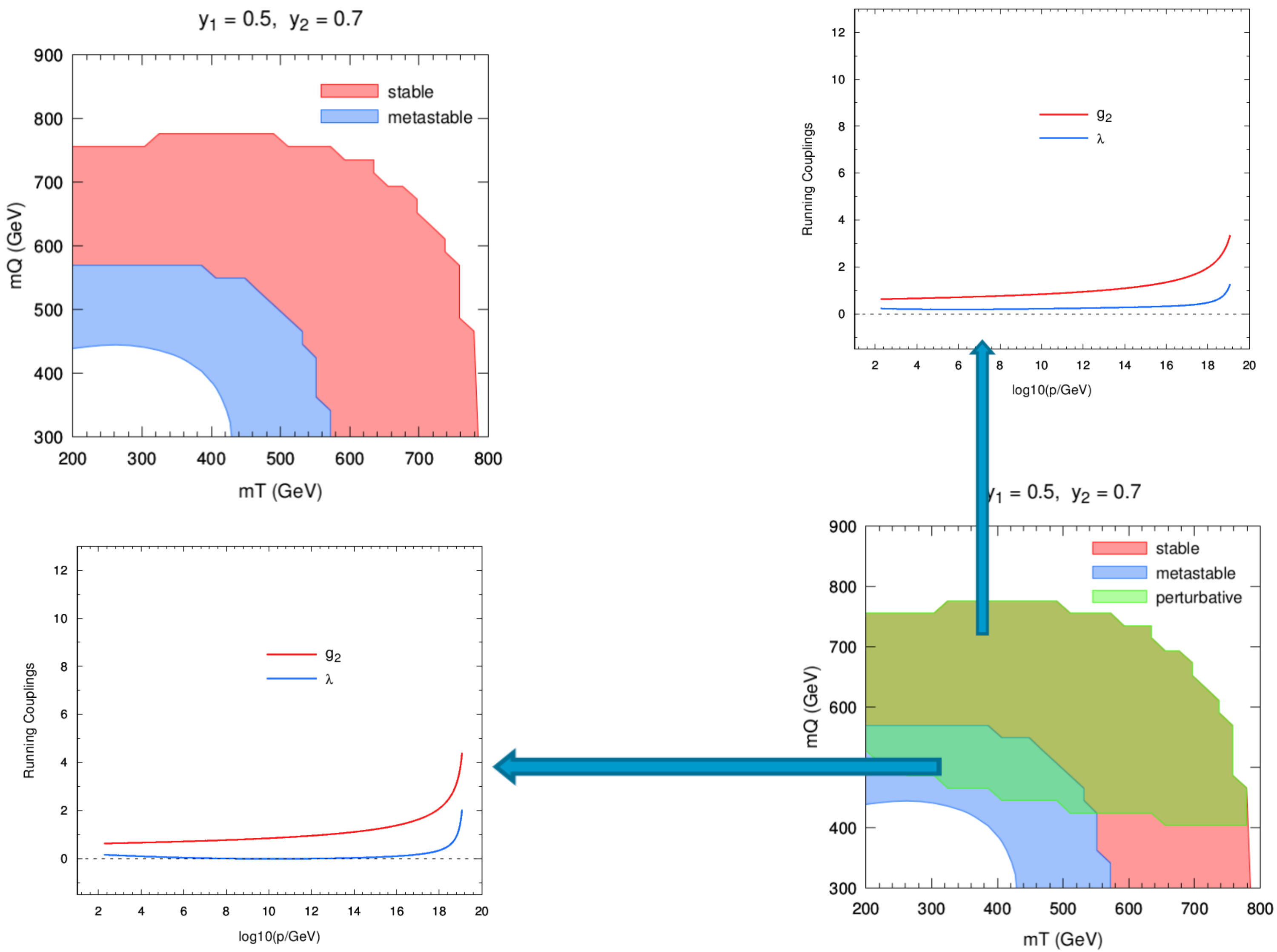


Constrains of vacuum status





Constrains of vacuum status



Summary

- I introduce the motivations of the DM research, and investigate the status of vacuum through RGE;
- Introduce the matching scheme and threshold effects, and use triplet-quadruplet dark matter model to illustrate the calculation method ;
- Evaluate the effects of one/two loop RGE and tree/loop level of matching schemes;
- According to our calculations, the threshold effects are significant, and it deserves the more careful calculation.

Summary

- **I introduce the motivations of the DM research, and investigate the status of vacuum through RGE;**
- **Introduce the matching scheme and threshold effects, and use triplet-quadruplet dark matter model to illustrate the calculation method ;**
- **Evaluate the effects of one/two loop RGE and tree/loop level of matching schemes;**
- **According to our calculations, the threshold effects are significant, and it deserves the more careful calculation.**

Thank you