Top quark pair production

Li Lin Yang
Peking University

第十三届TeV物理工作组学术研讨会

Basic facts about the top

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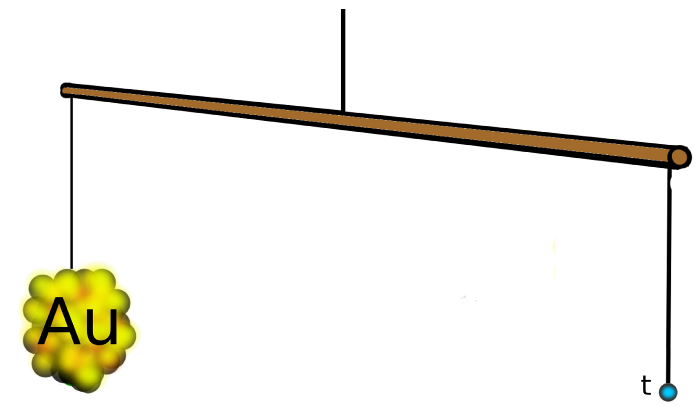
Large mass $m_t \approx 173 \text{ GeV}$



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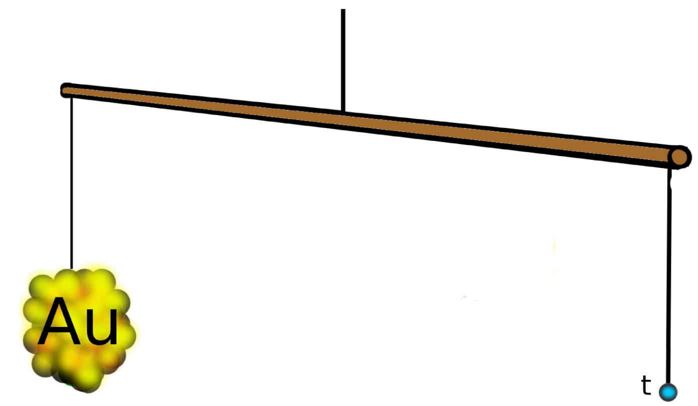

Strong Yukawa coupling $y_t \sim 1$

Fermion mass origin

Hierarchy problem

Vacuum stability

Basic facts about the top

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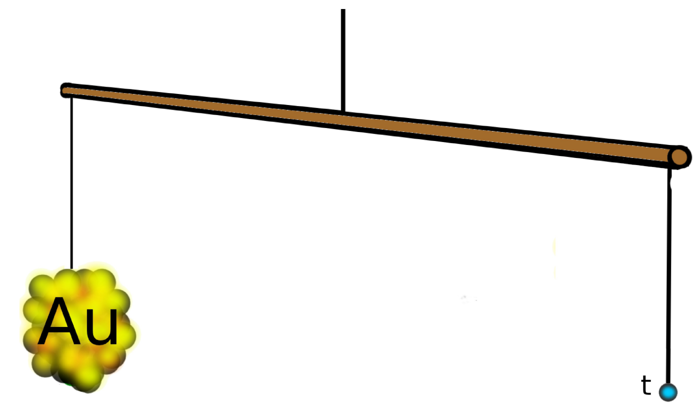


Strong Yukawa coupling $y_t \sim 1$

Fermion mass origin

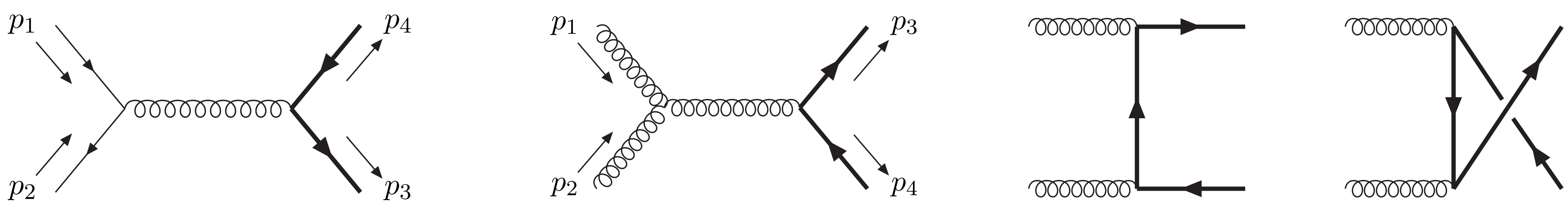
Hierarchy problem

Vacuum stability

Short lifetime $\tau \sim 5 \times 10^{-25}\ \mathrm{s}$

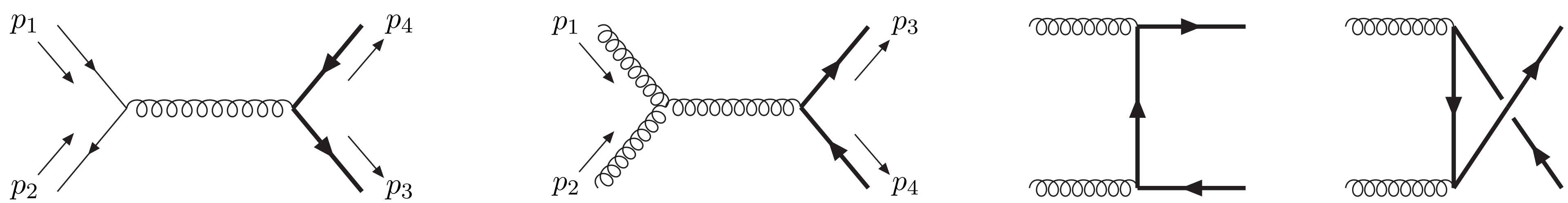
Decays before hadronization: pQCD dominates!

Top quark pair production



A standard candle for the LHC and future colliders

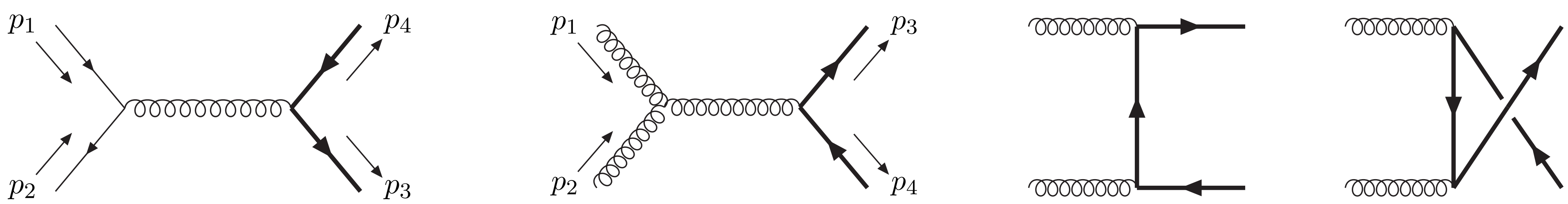
Top quark pair production



A standard candle for the LHC and future colliders

* Test of the SM at the energy frontier

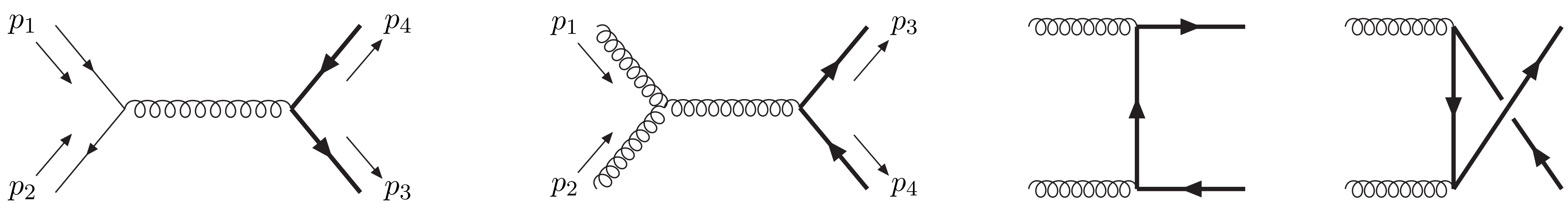
Top quark pair production



A standard candle for the LHC and future colliders

- Test of the SM at the energy frontier
- Possible signals of new physics

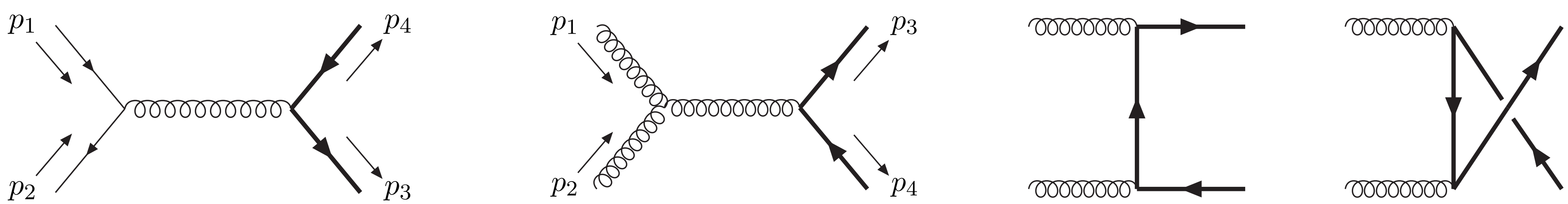
Top quark pair production



A standard candle for the LHC and future colliders

* Test of the SM at the energy frontier
* Possible signals of new physics
* Major background to many searches

Top quark pair production

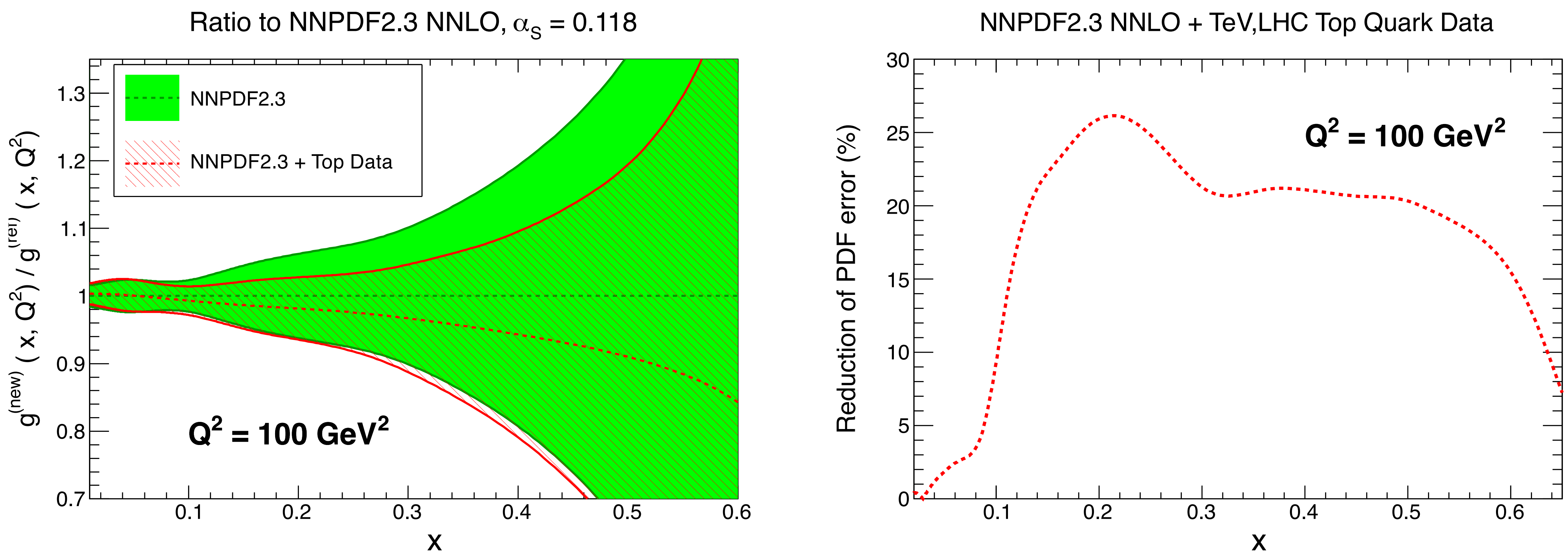


A standard candle for the LHC and future colliders

* Test of the SM at the energy frontier
* Possible signals of new physics
* Major background to many searches
* **Precise theoretical and experimental results have already enabled us to gain useful information!**

Gluon PDF

Top quark pair production can provide information about the gluon parton distribution functions



Czakon, Mangano, Mitov, Rojo: 1303.7215

Note: only used 7 and 8 TeV data!

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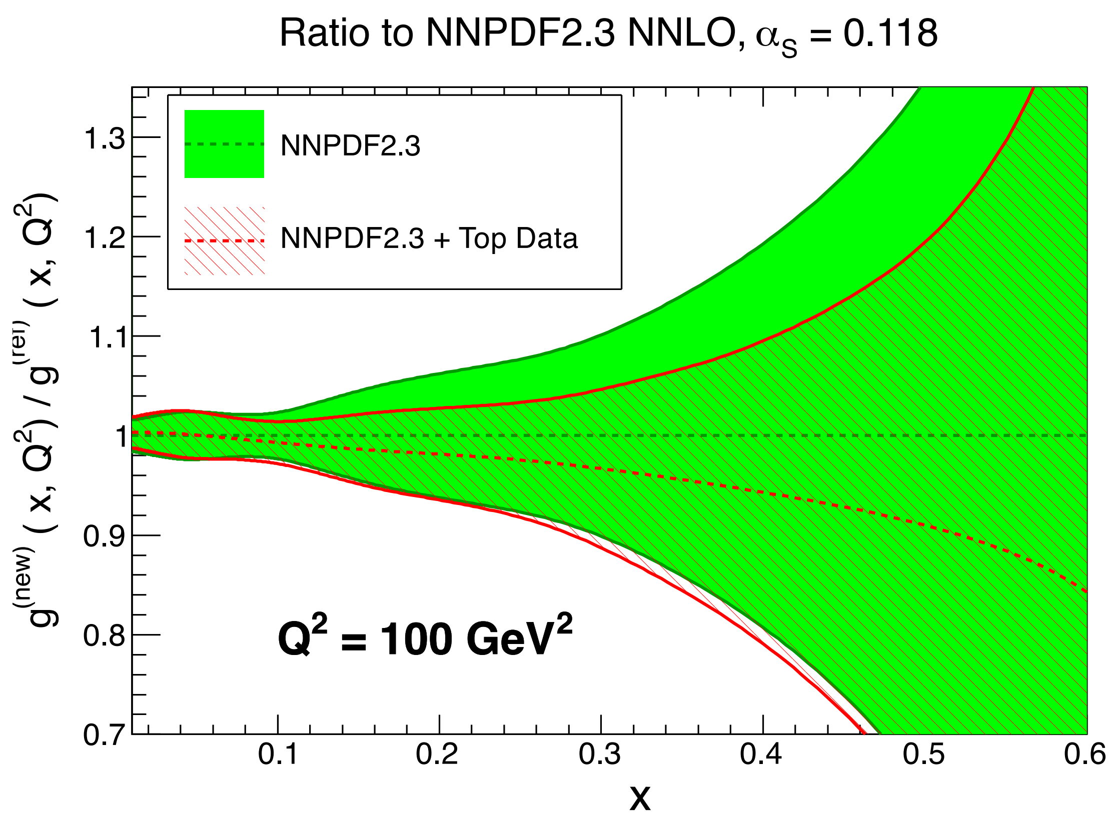

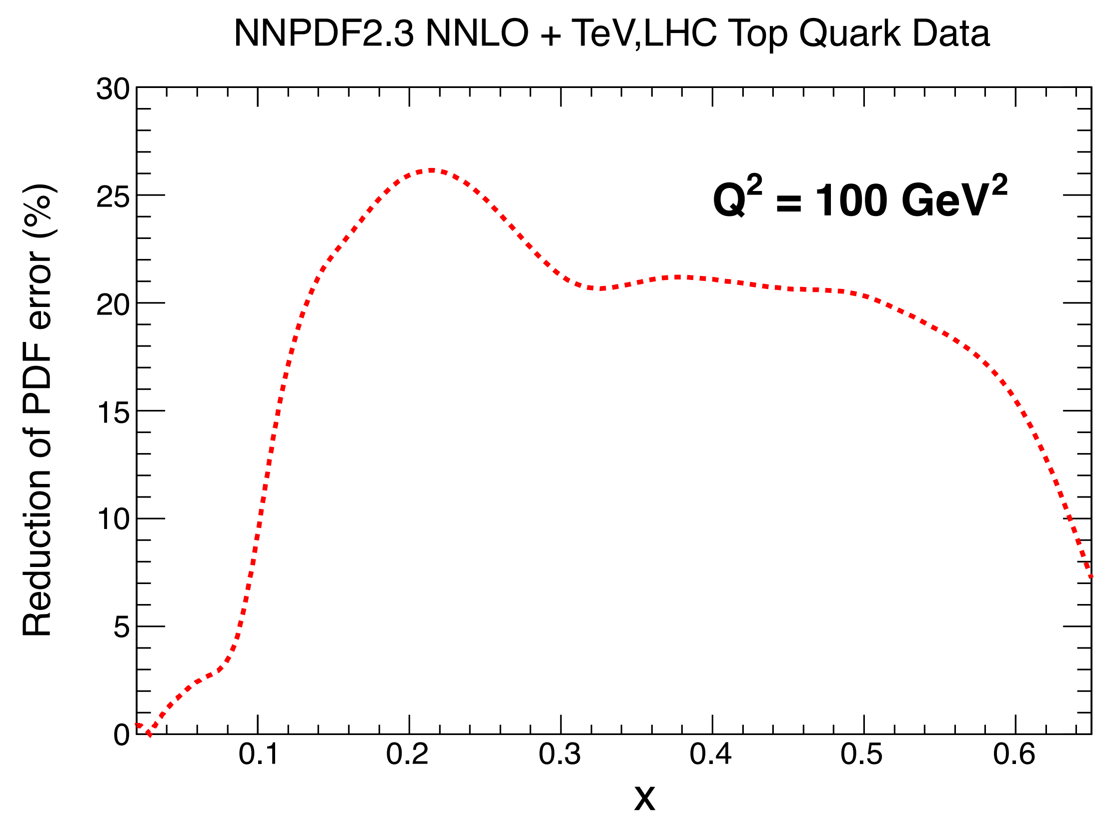

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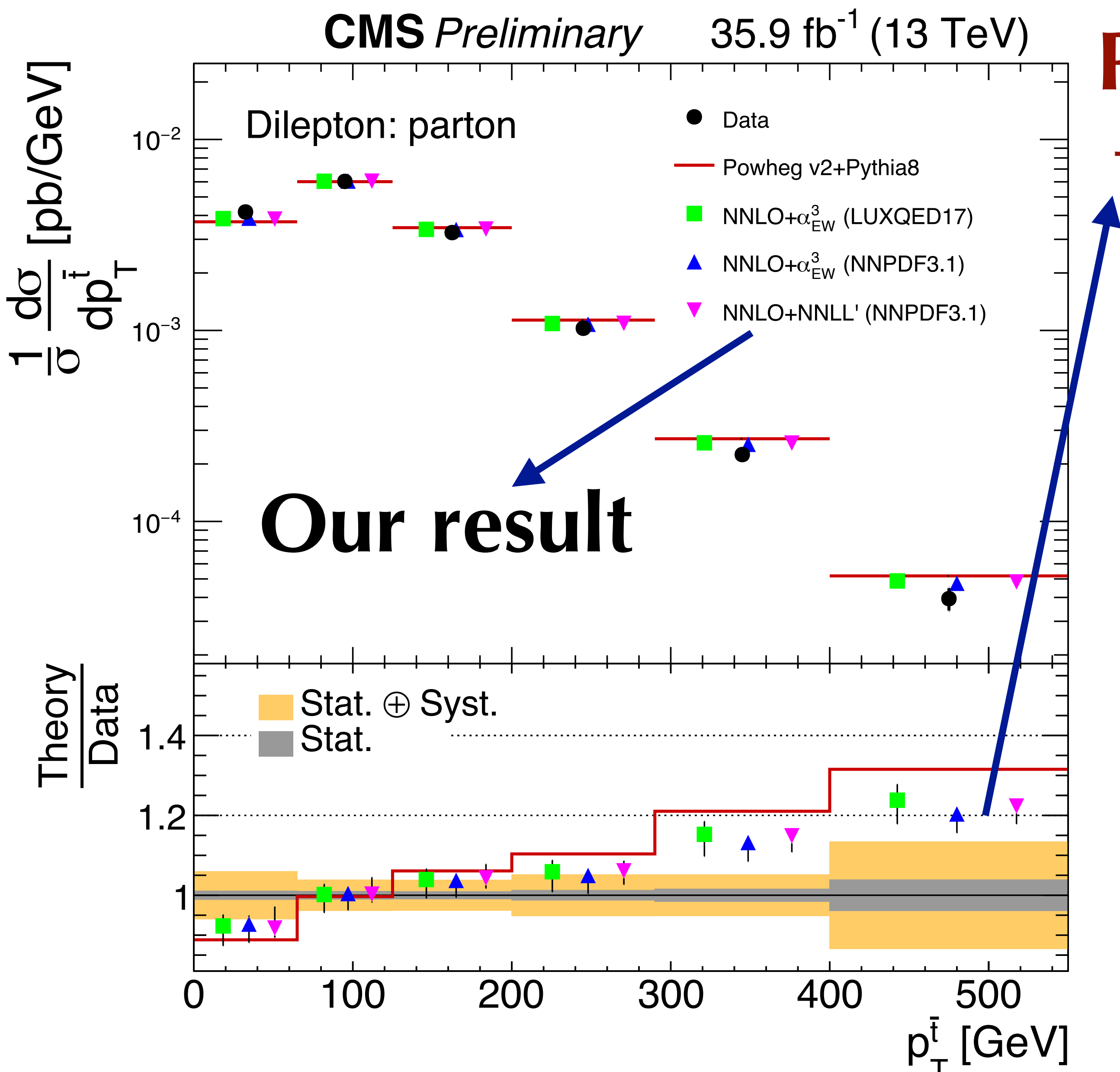
Note: only used 7 and 8 TeV data!

Ongoing: CTEQ analysis with 8 and 13 TeV data

Deviation?



CMS PAS TOP-17-014

Persistent shape difference in the transverse momentum spectrum

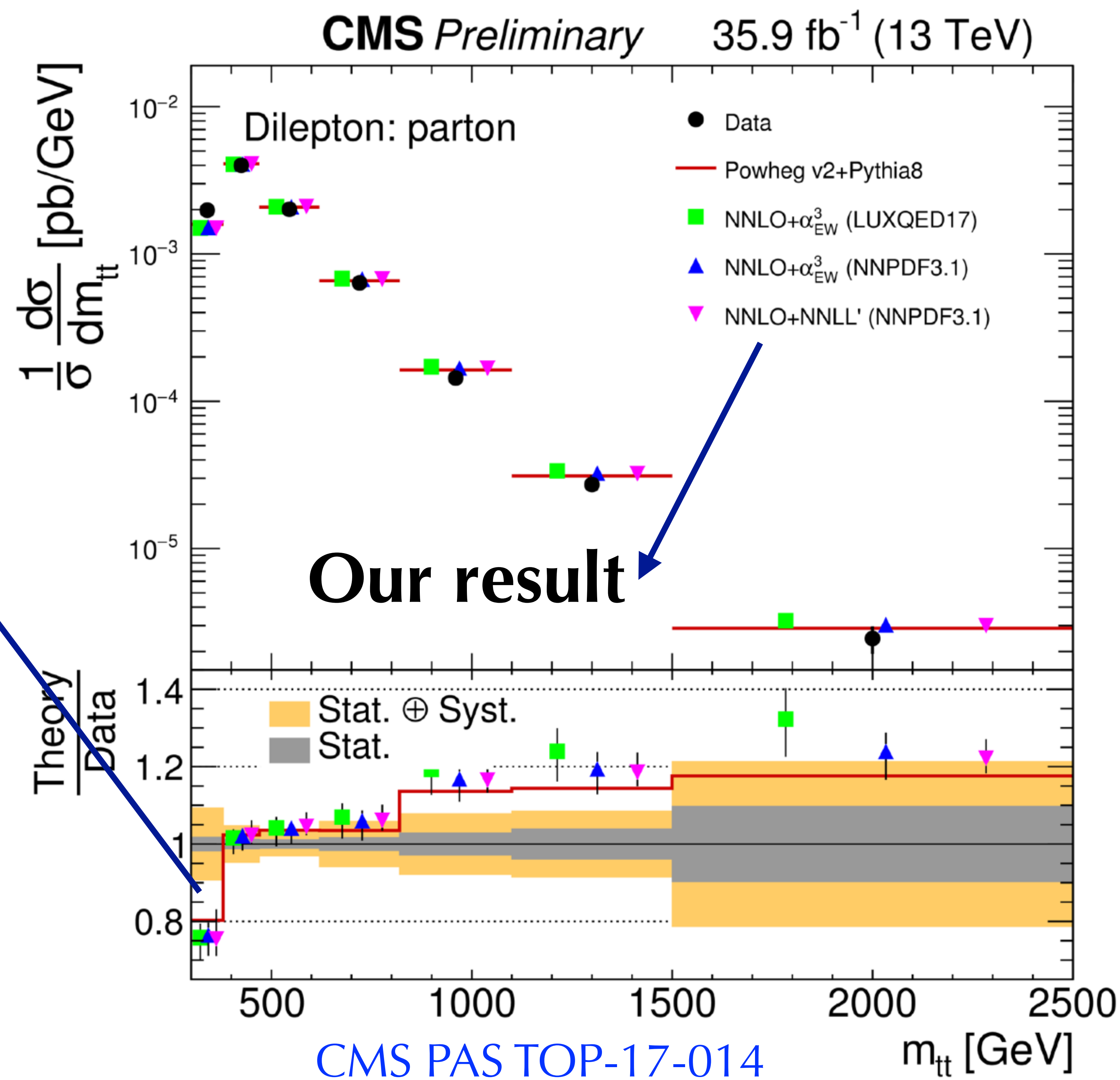
The high precision of the theoretical calculations and the experimental measurements allows to see this difference clearly!

Deviation?

What's going on here?

Top quark mass?

Threshold effect?



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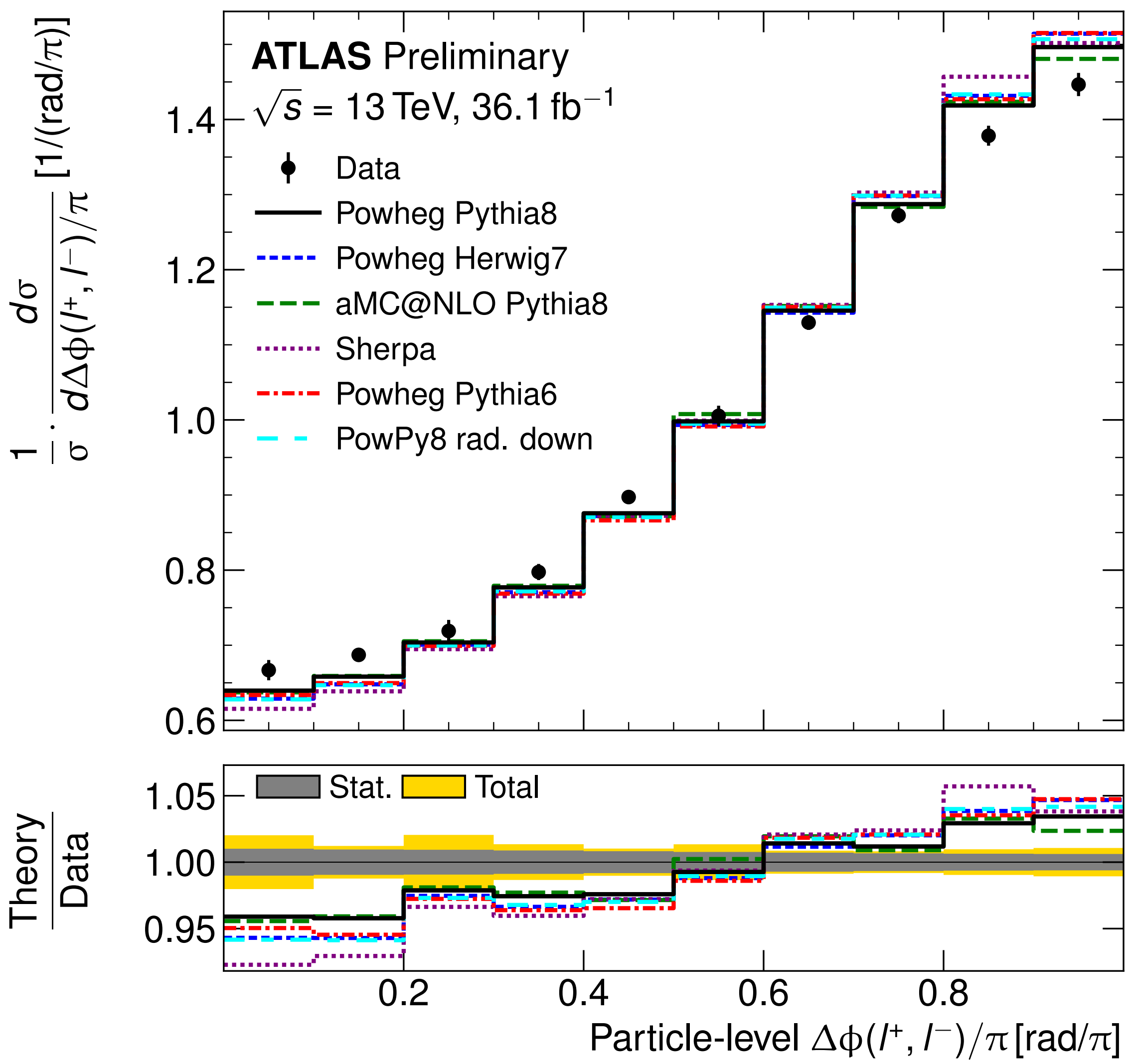
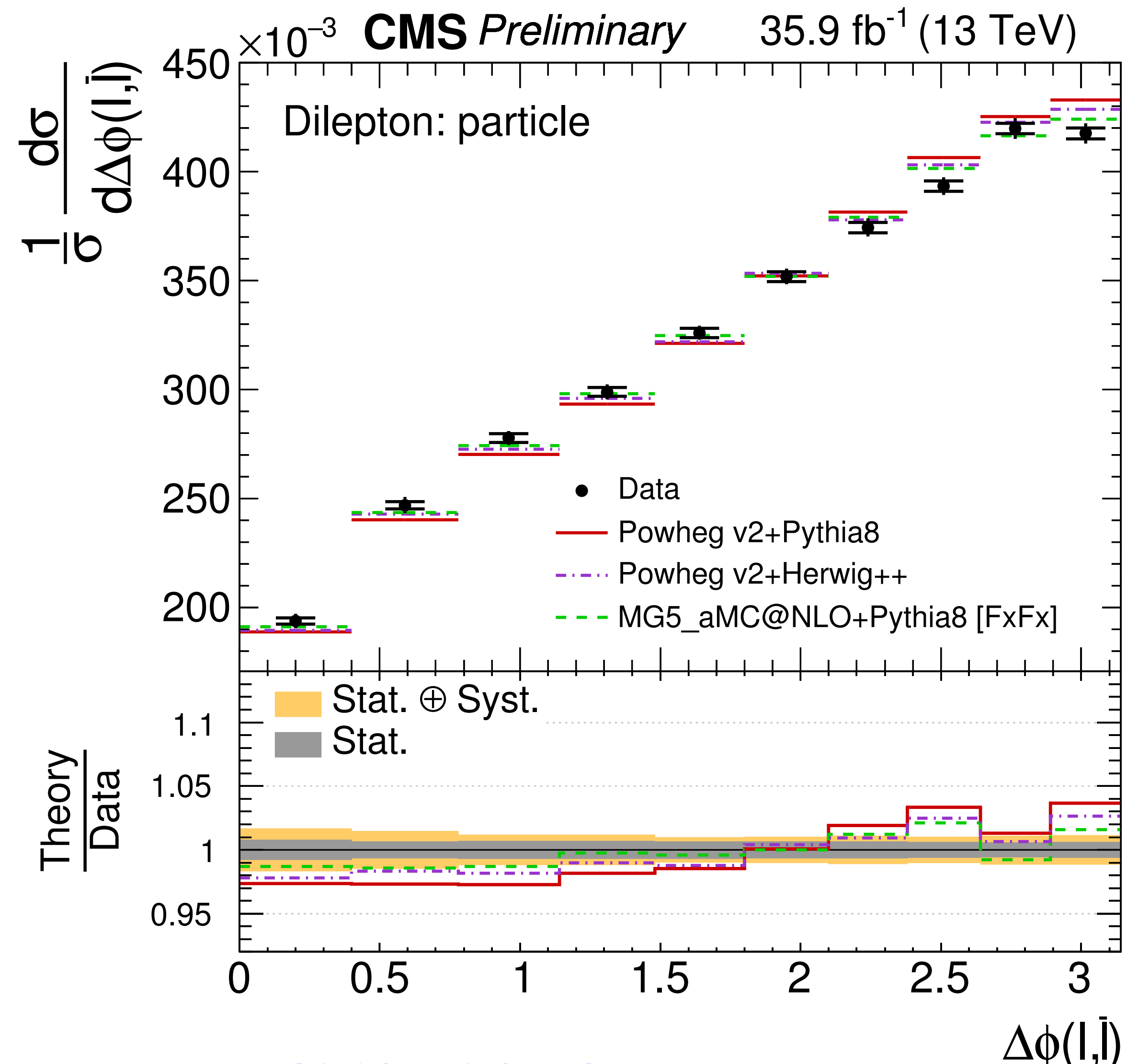


ATLAS-CONF-2018-027



CMS PAS TOP-17-014

See talk of Xu-Ai Zhuang (and of Hua-Qiao Zhang?)

In this talk, I'm going to introduce the state-of-the-art QCD prediction for top quark pair production…

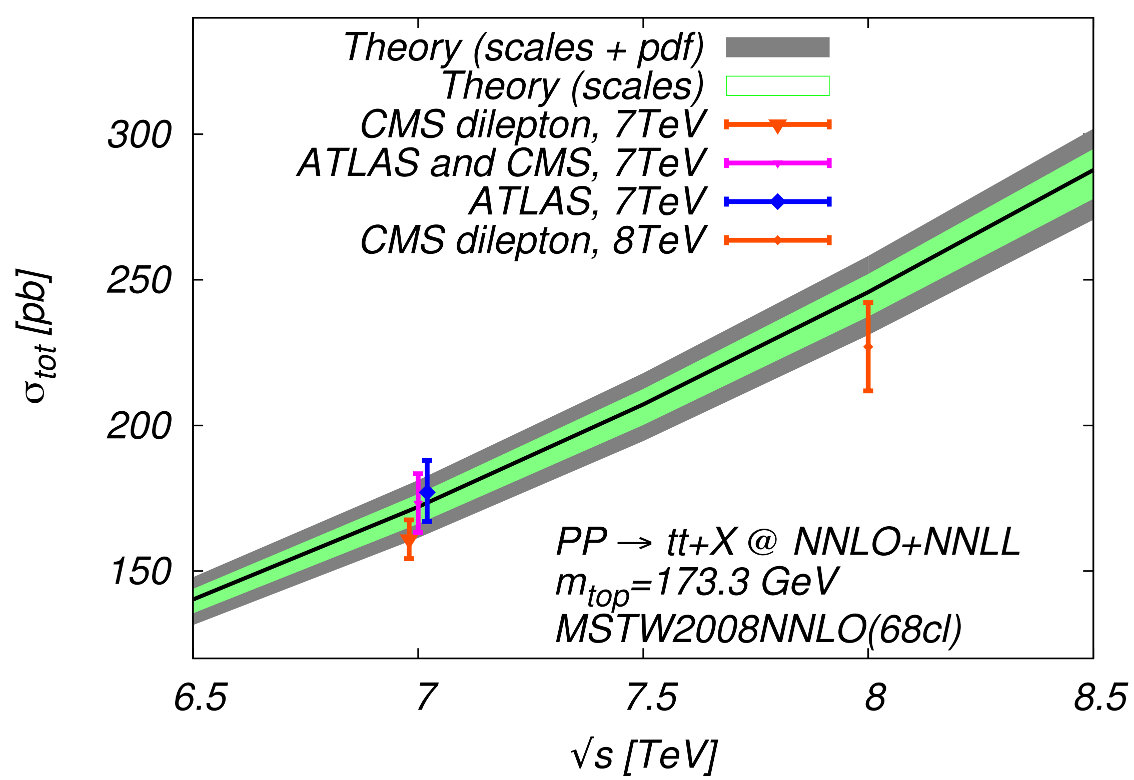
In this talk, I'm going to introduce the state-of-the-art QCD prediction for top quark pair production…

…and some ongoing developments

NNLO QCD for top pair

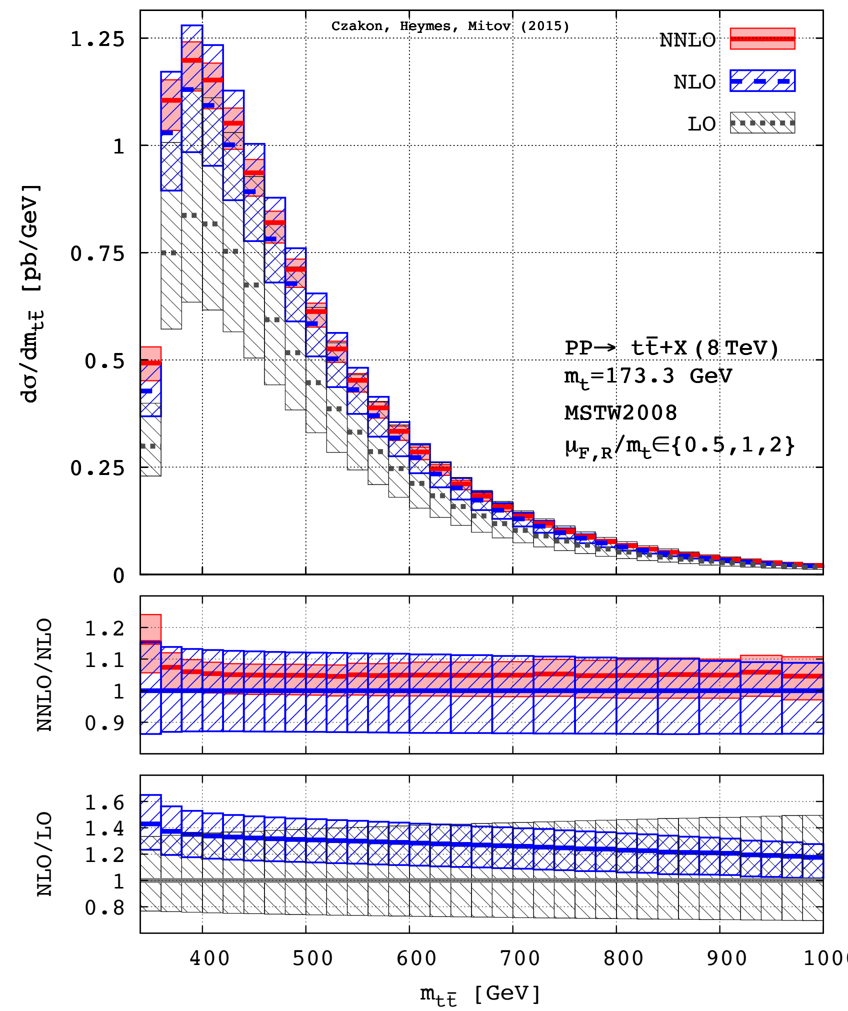
Total cross section

Baernreuther, Czakon, Mitov: 1204.5201;
Czakon, Fiedler, Mitov: 1303.6254



Differential distributions

Czakon, Heymes, Mitov: 1511.00549



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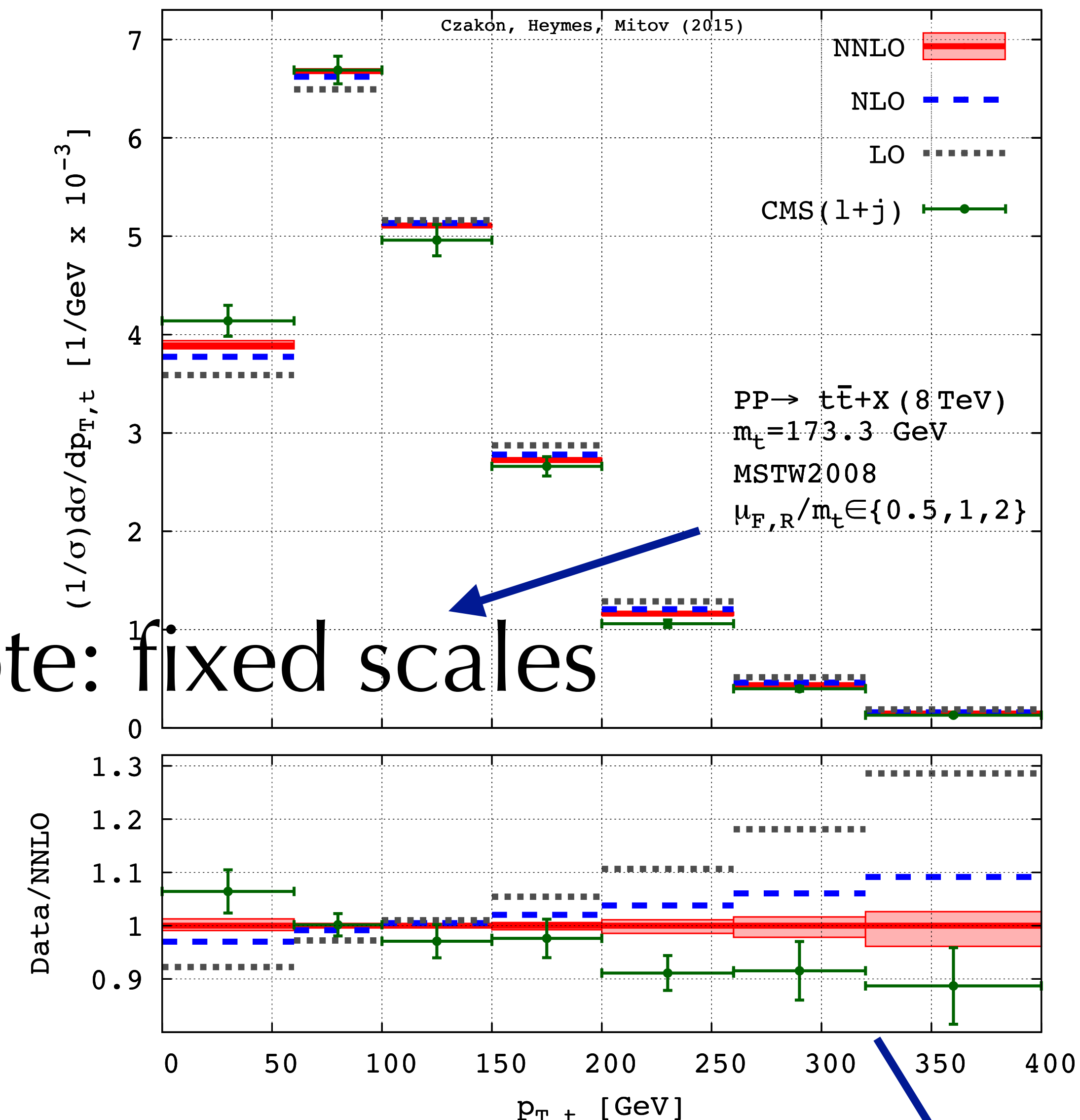
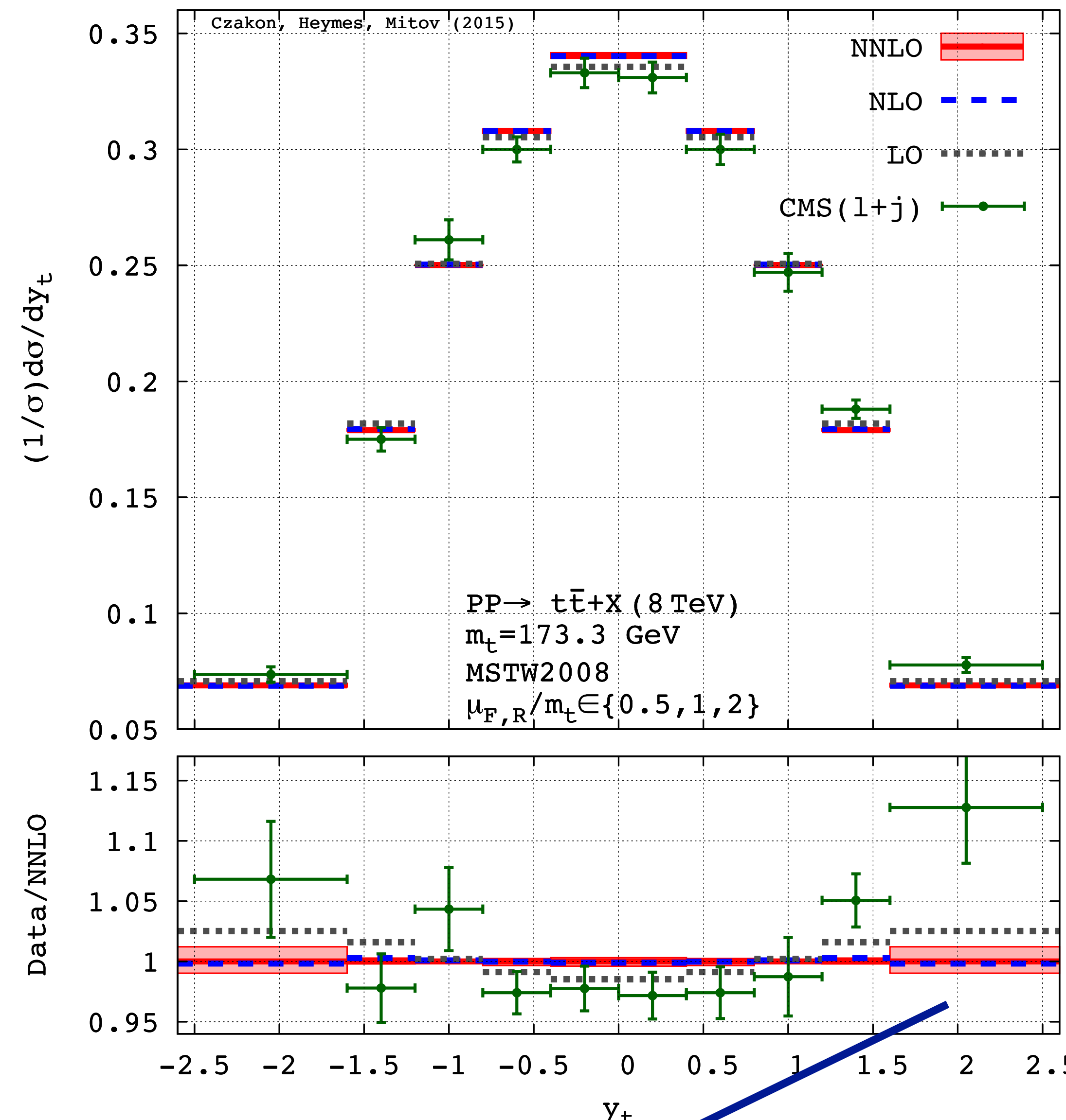


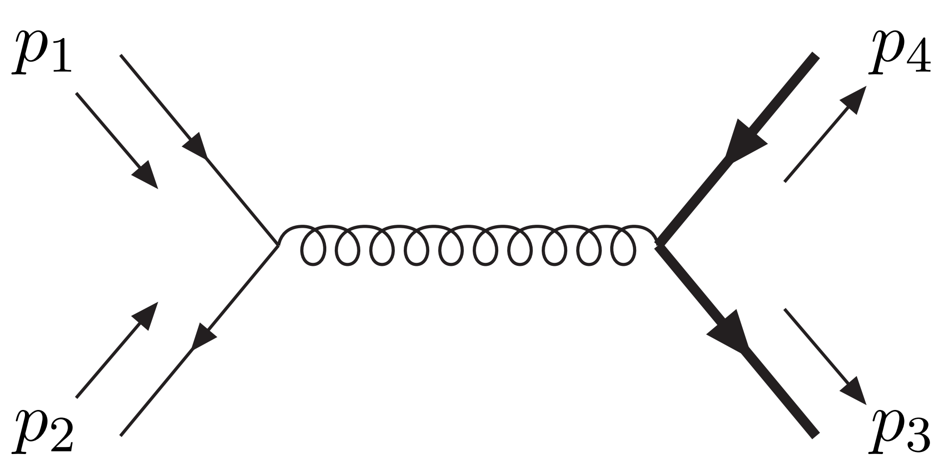
Note: fixed scales



Some tension at high energy (boosted kinematics)

Kinematics

The difficulty for fixed-order calculations: multiple-scale process with complicated kinematics!



Many kinematic variables:

- top quark mass
- p_T of top
- p_T of anti-top
- rapidity of top
- rapidity of anti-top
- Invariant mass M_{tt}
- …

Kinematics

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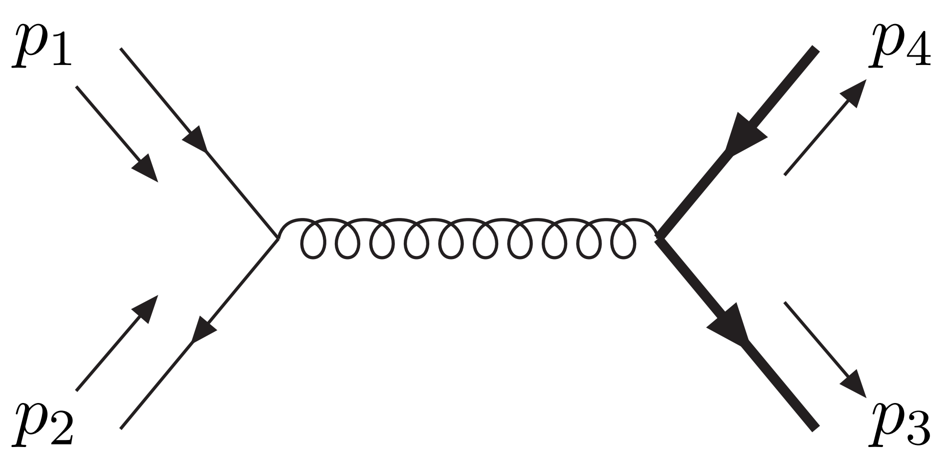


Many kinematic variables:

top quark mass
p_T of top
p_T of anti-top
rapidity of top
rapidity of anti-top
Invariant mass M_{tt}
...

Which (combination) should be used for the renormalization/factorization scales?

NNLO with dynamic scale

Czakon, Heymes, Mitov: 1606.03350

Determine optimal "scale scheme" by minimizing higher order corrections

$$\mu_0 \sim m_t \,,$$

$$\mu_0 \sim m_T = \sqrt{m_t^2 + p_T^2} \,,$$

$$\mu_0 \sim H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} \,,$$

$$\mu_0 \sim H_T' = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} + \sum_i p_{T,i} \,,$$

$$\mu_0 \sim E_T = \sqrt{\sqrt{m_t^2 + p_{T,t}^2}\sqrt{m_t^2 + p_{T,\bar{t}}^2}} \,,$$

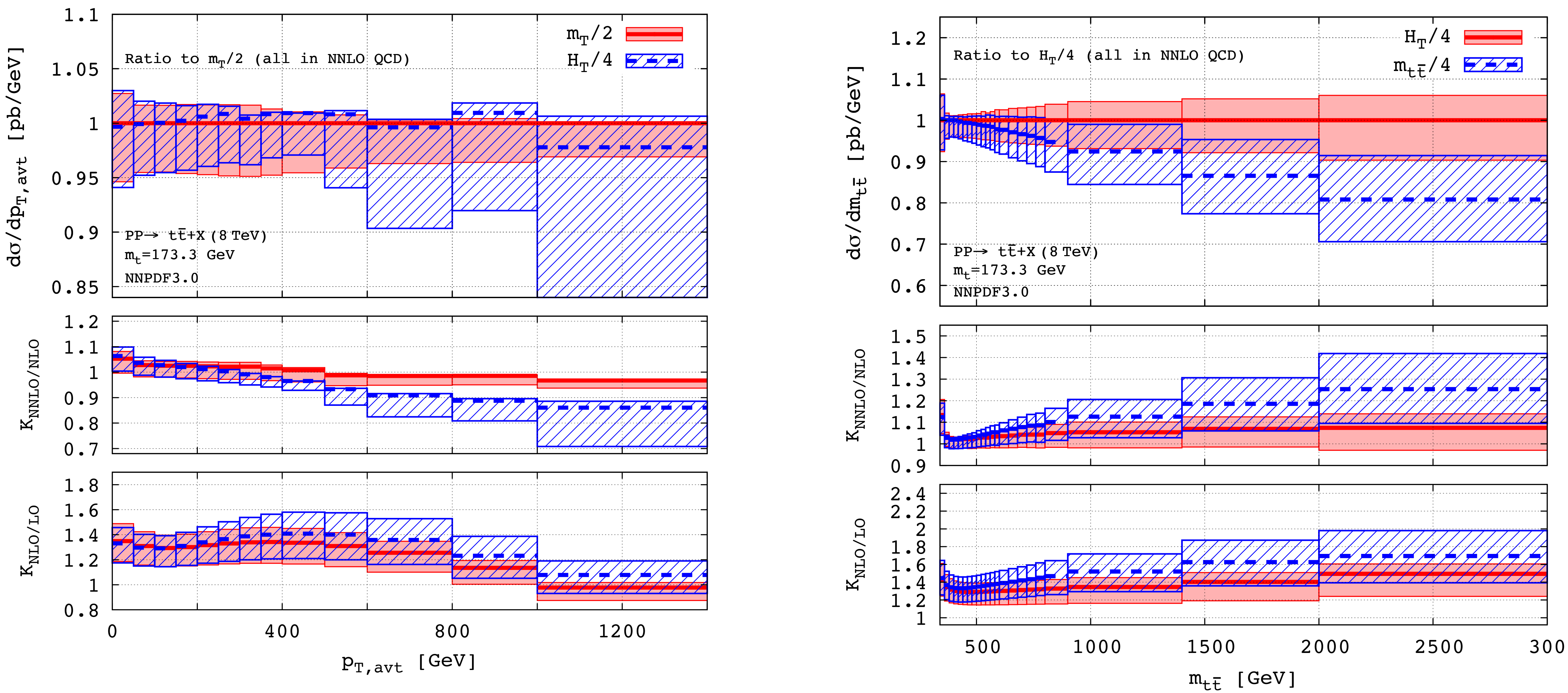
$$\mu_0 \sim H_{T,\mathrm{int}} = \sqrt{(m_t/2)^2 + p_{T,t}^2} + \sqrt{(m_t/2)^2 + p_{T,\bar{t}}^2} \,,$$

$$\mu_0 \sim m_{t\bar{t}} \,,$$

$$\Rightarrow \quad \mu_0 = \begin{cases} \frac{m_T}{2} & \text{for :} \quad p_{T,t},\ p_{T,\bar{t}} \text{ and } p_{T,t/\bar{t}} \,, \\ \frac{H_T}{4} & \text{for :} \quad \text{all other distributions} \,. \end{cases}$$

NNLO with dynamic scale

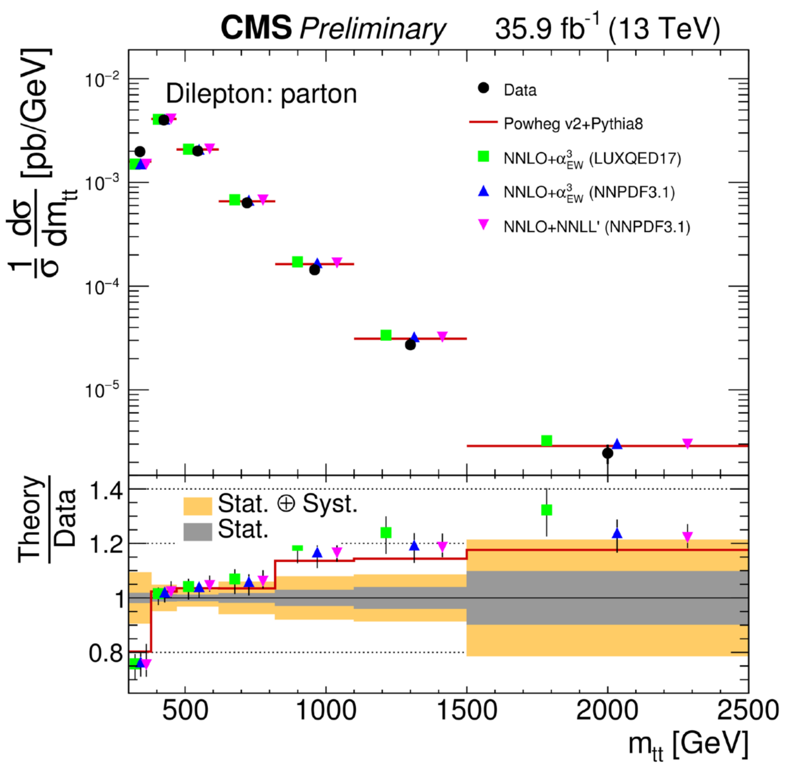
Czakon, Heymes, Mitov: 1606.03350



Vastly different behaviors with different scheme choices
(especially in the boosted region)

Our philosophy

We should study different regions of phase space separately, and combine them to have a good description for all regions!



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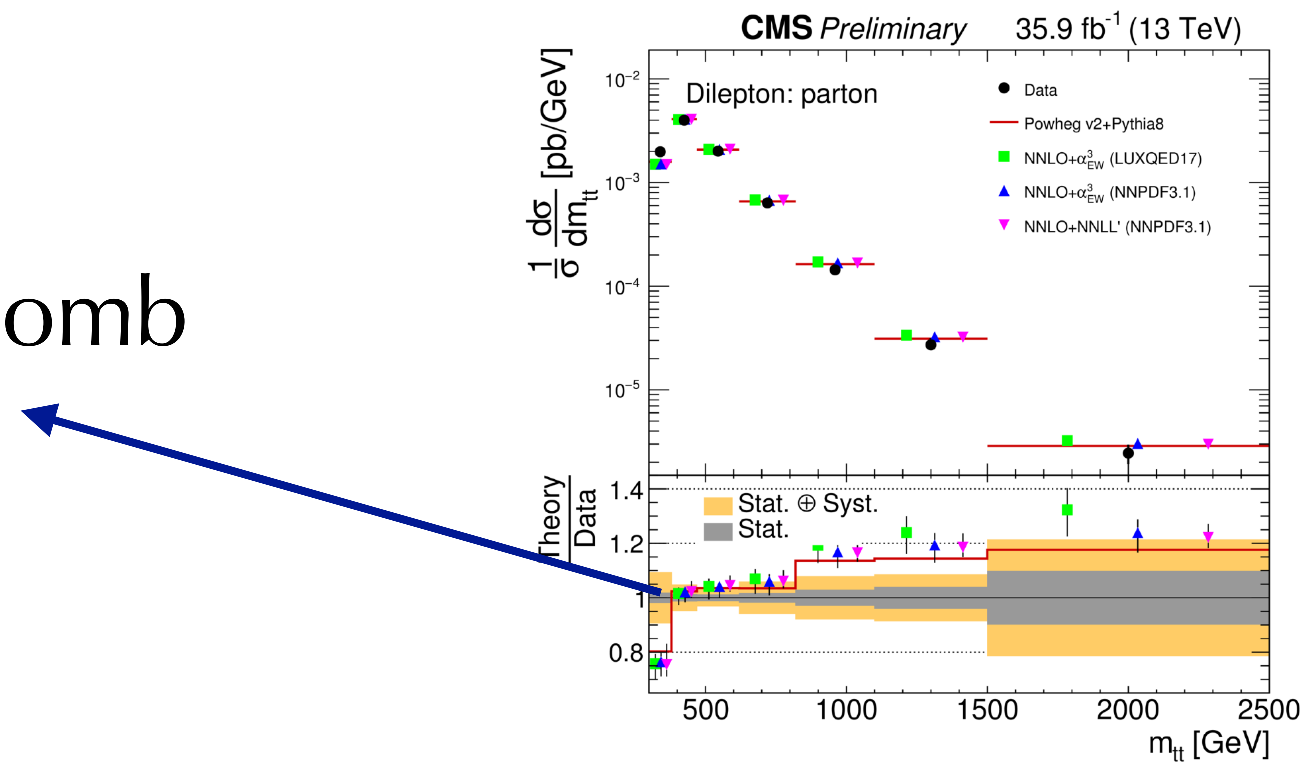
Threshold region
fixed-order+soft+Coulomb
(ongoing)



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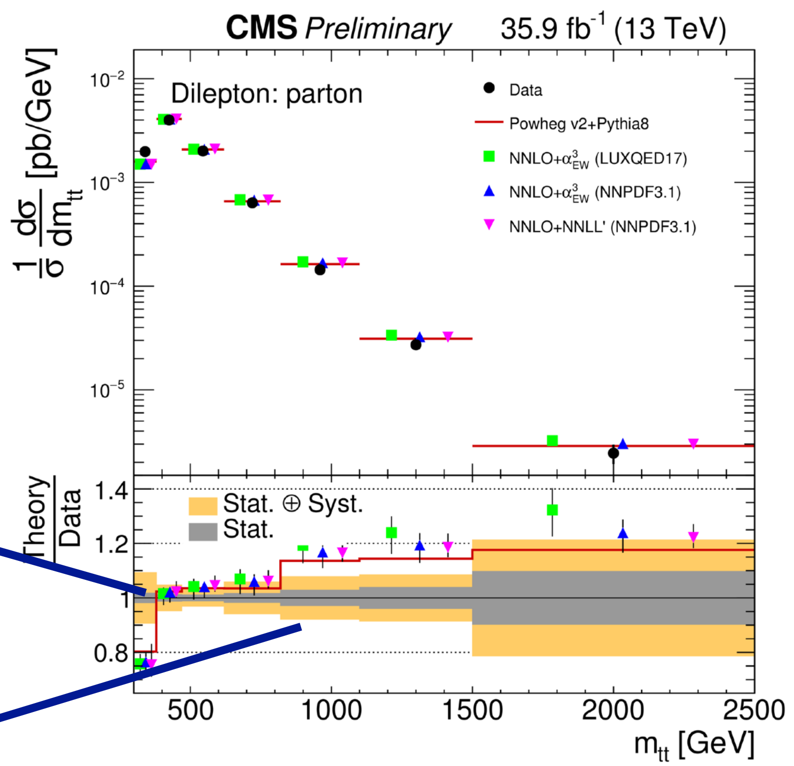

Intermediate region
fixed-order+soft

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

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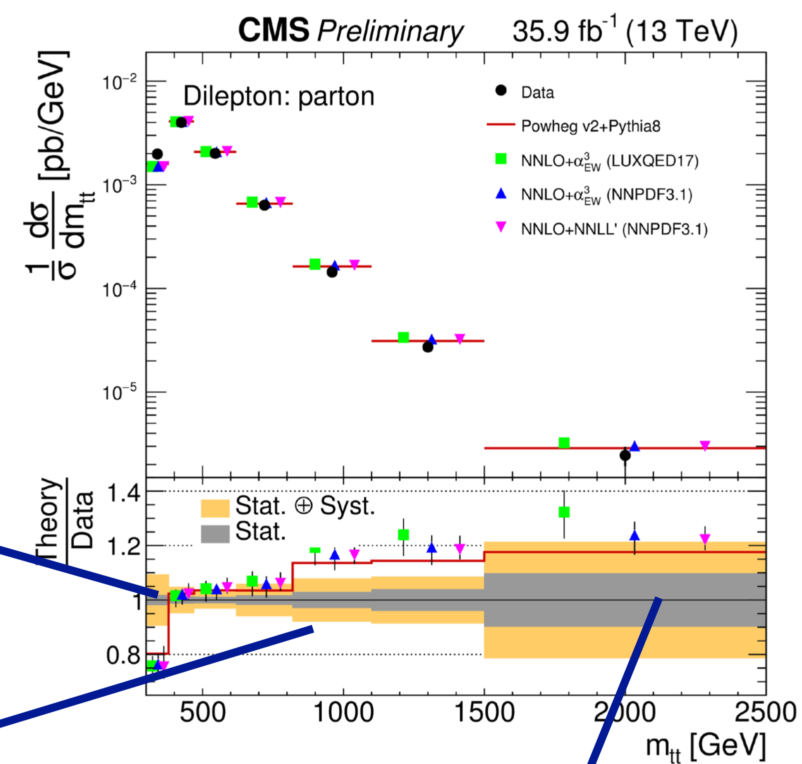


Threshold region

fixed-order+soft+Coulomb (ongoing)

Intermediate region

fixed-order+soft

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Boosted region

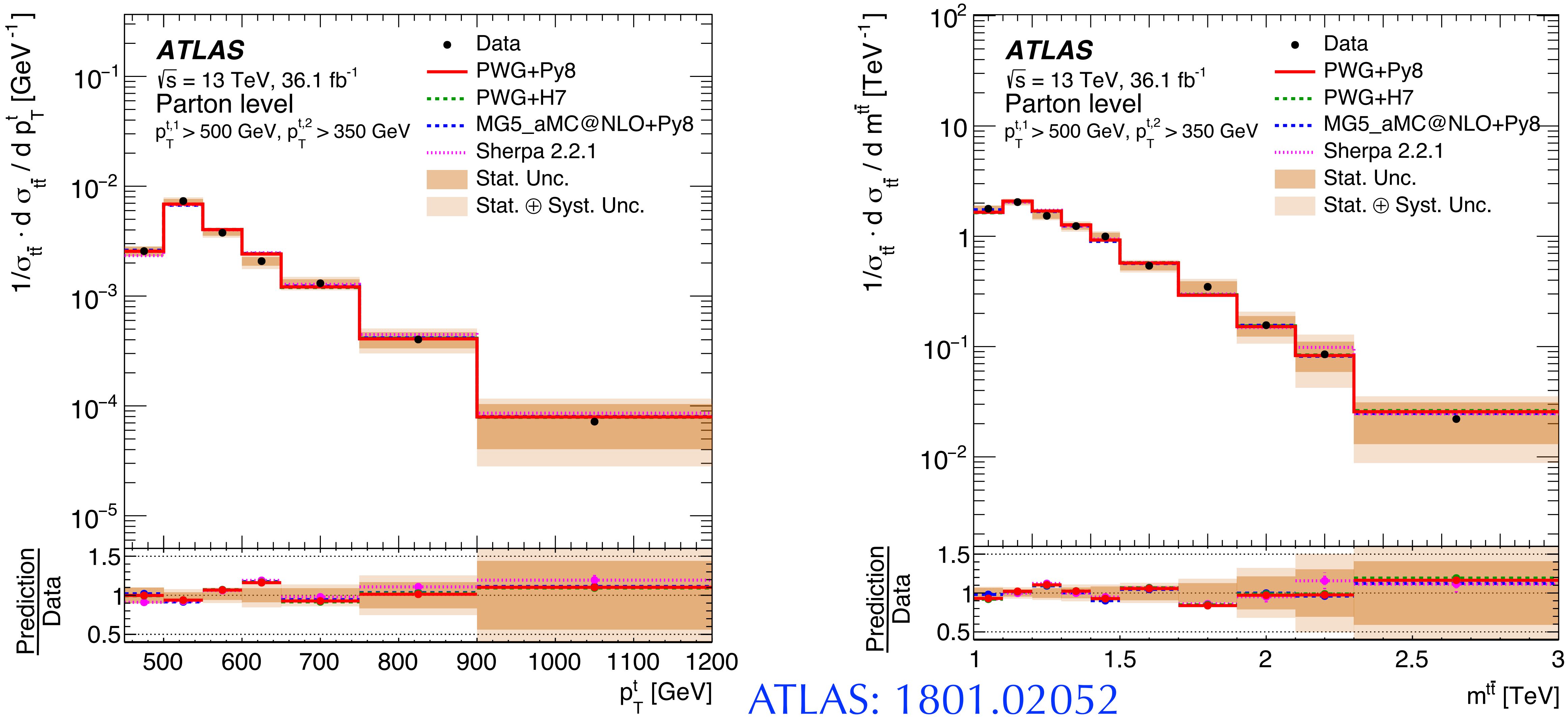
fixed-order+soft+quasi-collinear

Pecjak, Scott, Wang, **LLY**: 1601.07020

Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623

Boosted top quarks

Sensitive to new physics, interesting in its own right!



ATLAS: 1801.02052

Actively being probed by LHC experiments

Producing boosted tops

Hard extra emissions suppressed

↓

soft gluons

↓

$$\ln \frac{\hat{s} - M_{t\bar{t}}^2}{M_{t\bar{t}}^2}$$

Producing boosted tops

Hard extra emissions suppressed

↓

soft gluons

↓

$$\ln \frac{\hat{s} - M_{t\bar{t}}^2}{M_{t\bar{t}}^2}$$

Top quark nearly massless

↓

quasi-collinear gluons

↓

$$\ln \frac{m_t^2}{M_{t\bar{t}}^2}$$

Producing boosted tops

Hard extra emissions suppressed

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quasi-collinear gluons

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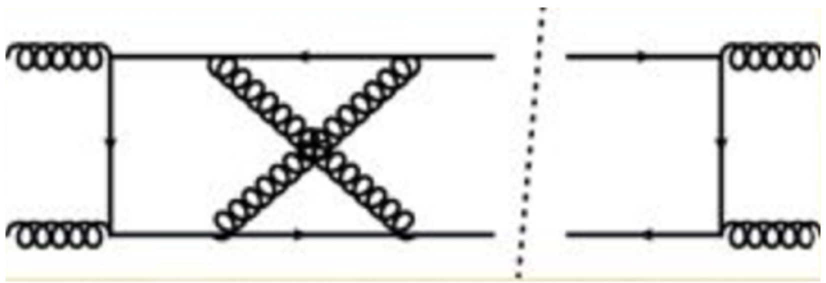
$$\ln \frac{m_t^2}{M_{t\bar{t}}^2}$$

Need to resum both!

Ferroglia, Pecjak, **LLY**: 1205.3662

Soft gluon resummation

Hard function



Kidonakis, Sterman: hep-ph/9705234

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Evolving from the scale of hard scatterings

Soft gluon resummation

Hard function

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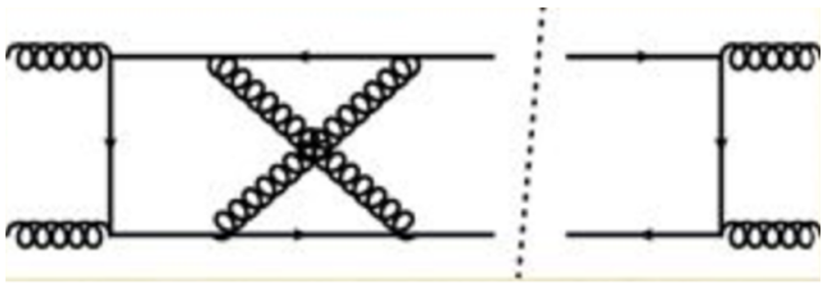

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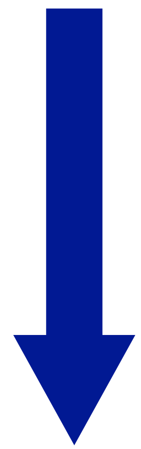
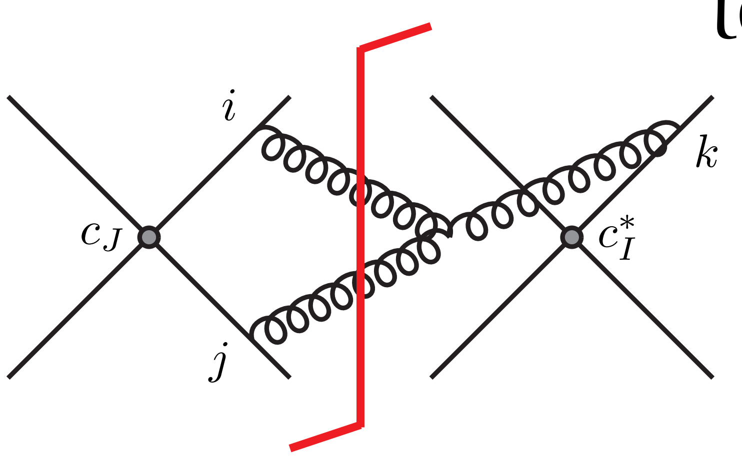


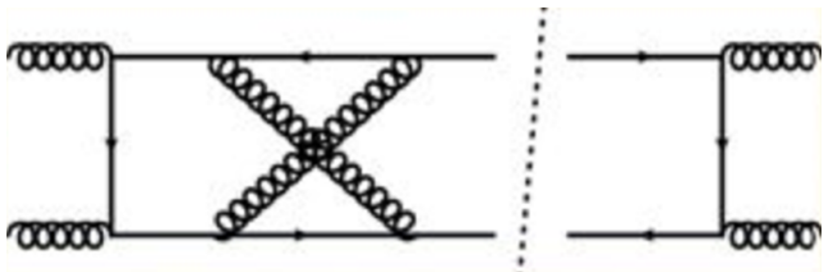
to the scale of soft interactions



Soft function

Soft gluon resummation

Hard function



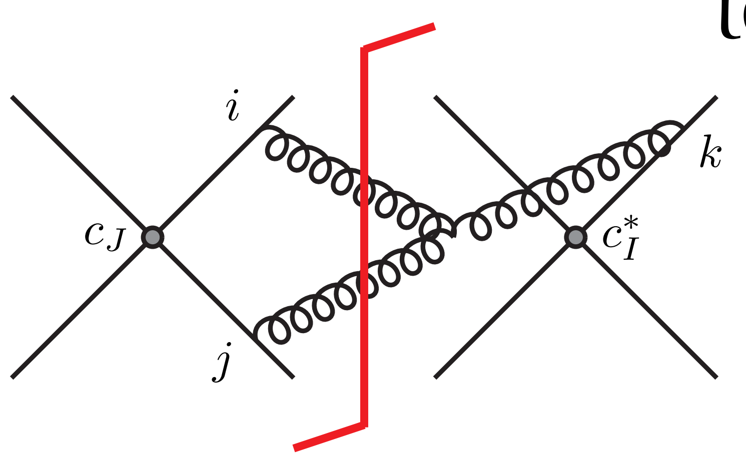
Kidonakis, Sterman: hep-ph/9705234

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Evolving from the scale of hard scatterings

Governed by IR structure

to the scale of soft interactions



Soft function

IR anomalous dimension

Becher, Neubert: 0904.1021

Ferroglia, Neubert, Pecjak, **LLY**: 0907.4791; 0908.3676

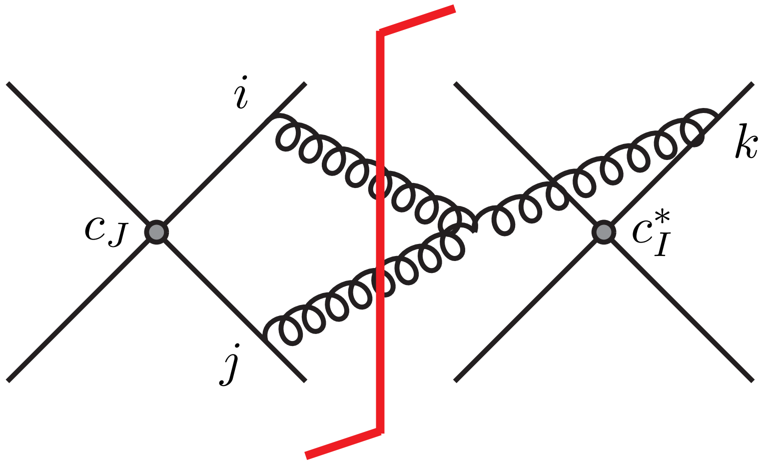
$$\begin{aligned}\boldsymbol{\Gamma} = &\sum_{(i,j)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) \\ &- \sum_{(I,J)} \frac{\boldsymbol{T}_I \cdot \boldsymbol{T}_J}{2}\, \gamma_{\rm cusp}(\beta_{IJ},\alpha_s) + \sum_I \gamma^I(\alpha_s) \\ &+ \sum_{I,j} \boldsymbol{T}_I \cdot \boldsymbol{T}_j\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{m_I \mu}{-s_{Ij}} \qquad (2 \\ &+ \sum_{(I,J,K)} i f^{abc}\, \boldsymbol{T}_I^a \boldsymbol{T}_J^b \boldsymbol{T}_K^c\, F_1(\beta_{IJ},\beta_{JK},\beta_{KI}) \\ &+ \sum_{(I,J)} \sum_k i f^{abc}\, \boldsymbol{T}_I^a \boldsymbol{T}_J^b \boldsymbol{T}_k^c\, f_2\Big(\beta_{IJ}, \ln\frac{-\sigma_{Jk}\, v_J \cdot p_k}{-\sigma_{Ik}\, v_I \cdot p_k}\Big)\end{aligned}$$

$$F_1(\beta_{12},\beta_{23},\beta_{31}) = \frac{\alpha_s^2}{12\pi^2} \sum_{i,j,k} \epsilon_{ijk}\, g(\beta_{ij})\, r(\beta_{ki})$$

$$\begin{aligned} r(\beta) &= \beta \coth\beta\,, \\ g(\beta) &= \coth\beta \left[\beta^2 + 2\beta\, \ln(1-e^{-2\beta}) - \mathrm{Li}_2(e^{-2\beta}) + \frac{\pi^2}{6}\right] \\ &\quad - \beta^2 - \frac{\pi^2}{6}\,. \end{aligned} \qquad (5)$$

3-parton correlations

The soft function



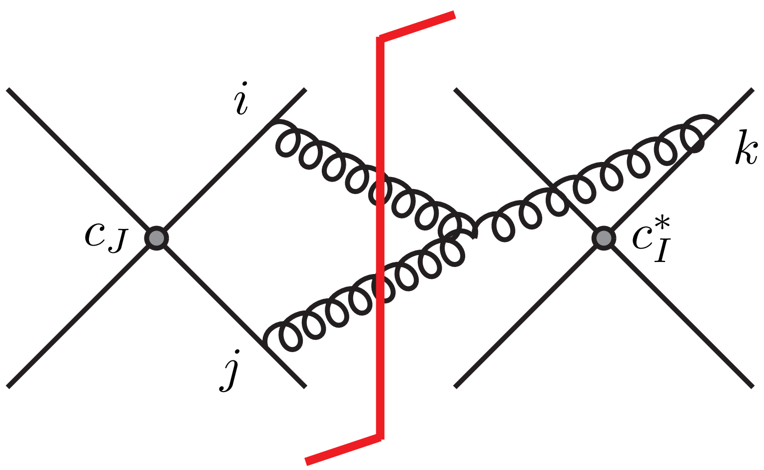
Known at NLO

Ahrens, Ferroglia, Neubert, Pecjak, **LLY**: 1003.5827

Known at NNLO in the massless limit (except an off-diagonal 3-parton piece)

Ferroglia, Pecjak, **LLY**: 1207.4798

The soft function



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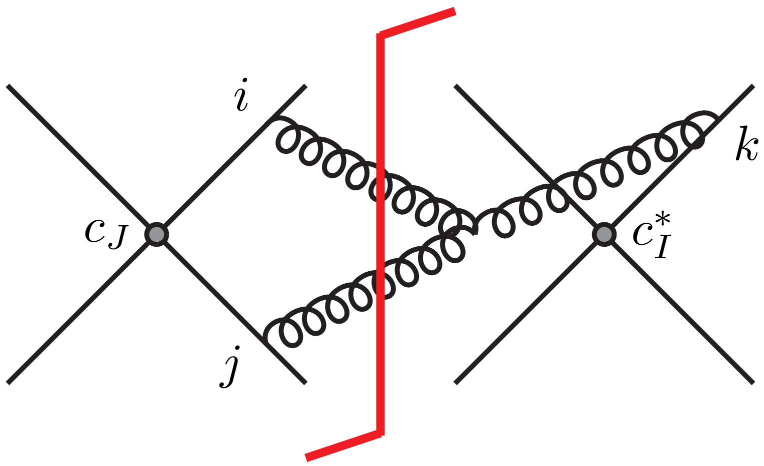
Recent calculation at NNLO with massive tops

Wang, Xu, **LLY**, Zhu: 1804.05218

The soft function



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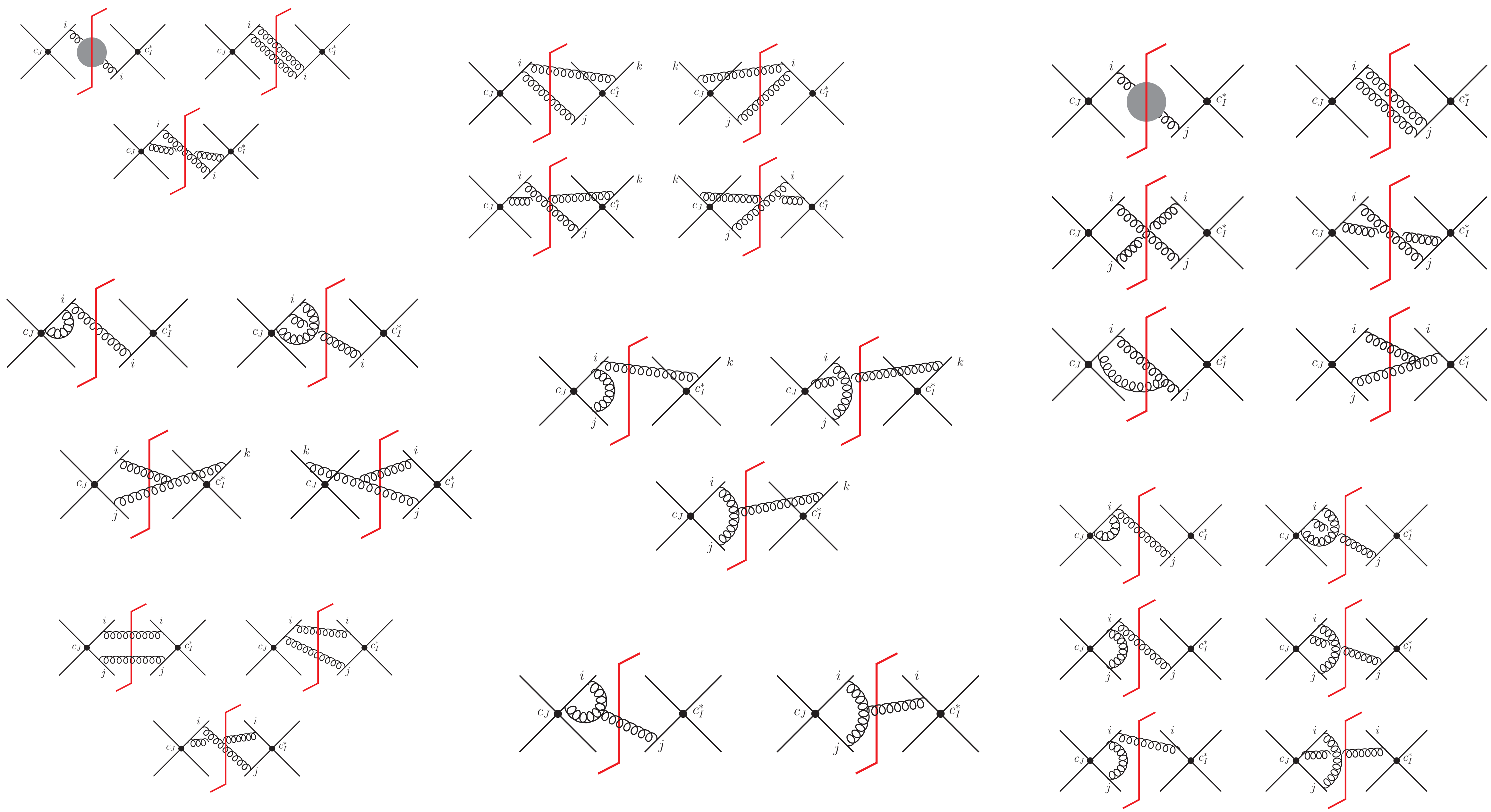
Systematic techniques →
- Inverse unitarity
- IBP identities
- Differential equations

NNLO diagrams

Wang, Xu, **LLY**, Zhu: 1804.05218



Solving integrals

Wang, Xu, **LLY**, Zhu: 1804.05218

~60 master integrals

matrices

Differential equations

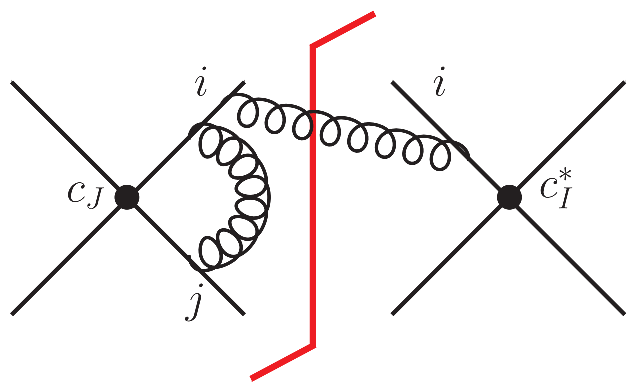
$$\partial_\beta \vec{f}(\epsilon, \beta, \cos\theta) = \epsilon \left(\frac{A}{\beta - 1} + \frac{B}{\beta} + \frac{C}{\beta + 1} + \frac{D}{\beta - 1/\cos\theta} + \frac{E}{\beta + 1/\cos\theta} \right) \vec{f}(\epsilon, \beta, \cos\theta)$$

Solution in terms of generalized polylogarithms

Difficult part: boundary conditions

The boundary conditions

We choose the boundary to be $\beta \equiv \sqrt{1 - \frac{4m_t^2}{M_{t\bar{t}}}} \to 0$

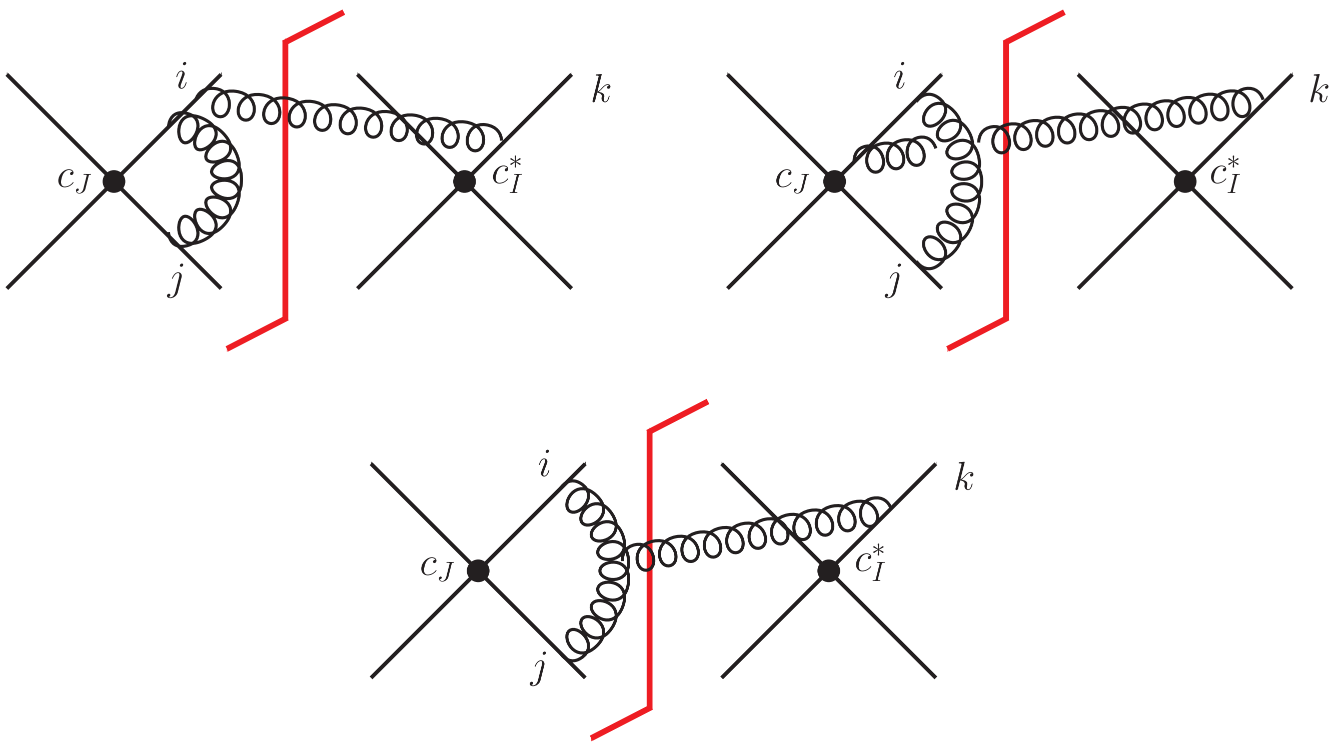


Some virtual-real integrals develop Coulomb/Glauber-type singularities in this limit

Carefully extract the asymptotic behavior, e.g.

$$g_6^{(4)}(\epsilon, \beta \to 0, y) \approx \frac{(e^{-2i\pi\epsilon} - 1)\, \beta^{2\epsilon}\, \Gamma(1-2\epsilon)\, \Gamma(1+\epsilon)}{4^{1-2\epsilon}\, \Gamma(1-\epsilon)}$$

The 3-parton terms



$$i f^{abc} \boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c$$

- Off-diagonal purely-imaginary contributions
- Do not enter the NNLO cross section
- Not calculated in the massless case

Ferroglia, Pecjak, **LLY**: 1207.4798

A piece of final result

Wang, Xu, **LLY**, Zhu: 1804.05218

$$
\begin{aligned}
\tilde{\boldsymbol{s}}_{22}^{q\bar{q},(2)}(0,\beta,y)\Big|_{T_F N_l} &= \frac{16(7\beta^2-126\beta+127)}{243\beta}G_1 + \frac{8(5\beta^2+90\beta+53)}{81\beta}\big(G_{-1,-1}-G_{-1,1}-2G_{0,-1}\big) \\
&- \frac{16(7\beta^2+126\beta+127)}{243\beta}G_{-1} + \frac{8(5\beta^2-90\beta+53)}{81\beta}\big(G_{1,-1}-G_{1,1}+2G_{0,1}\big) \\
&+ \frac{8(\beta^2+18\beta+1)}{27\beta}\big(-G_{-1,-1,-1}+G_{-1,-1,1}+2G_{-1,0,-1}-2G_{-1,0,1}-G_{-1,1,-1}+G_{-1,1,1} \\
&+2G_{0,-1,-1}-2G_{0,-1,1}-4G_{0,0,-1}\big) + \frac{8(\beta^2-18\beta+1)}{27\beta}\big(4G_{0,0,1}+2G_{0,1,-1}-2G_{0,1,1} \\
&-G_{1,-1,-1}+G_{1,-1,1}+2G_{1,0,-1}-2G_{1,0,1}-G_{1,1,-1}+G_{1,1,1}\big) \\
&+ \frac{32}{243}\Big[28G_{-1/y}+98G_{1/y}+30\big(2G_{0,-1/y}+G_{-1/y,-1}+G_{-1/y,1}-2G_{-1/y,-1/y}\big) \\
&+105\big(2G_{0,1/y}+G_{1/y,-1}+G_{1/y,1}-2G_{1/y,1/y}\big)+18\big(4G_{0,0,-1/y}+2G_{0,-1/y,-1}+2G_{0,-1/y,1} \\
&-4G_{0,-1/y,-1/y}-G_{-1/y,-1,-1}+G_{-1/y,-1,1}+2G_{-1/y,0,-1}+2G_{-1/y,0,1}-4G_{-1/y,0,-1/y} \\
&+G_{-1/y,1,-1}-G_{-1/y,1,1}-2G_{-1/y,-1/y,-1}-2G_{-1/y,-1/y,1}+4G_{-1/y,-1/y,-1/y}\big) \\
&+63\big(4G_{0,0,1/y}+2G_{0,1/y,-1}+2G_{0,1/y,1}-4G_{0,1/y,1/y}-G_{1/y,-1,-1}+G_{1/y,-1,1}+2G_{1/y,0,-1} \\
&+2G_{1/y,0,1}-4G_{1/y,0,1/y}+G_{1/y,1,-1}-G_{1/y,1,1}-2G_{1/y,1/y,-1}-2G_{1/y,1/y,1}+4G_{1/y,1/y,1/y}\big) \\
&-\frac{332}{3}-\frac{5\pi^2}{2}+6\zeta_3\Big]\,, \qquad (84)
\end{aligned}
$$

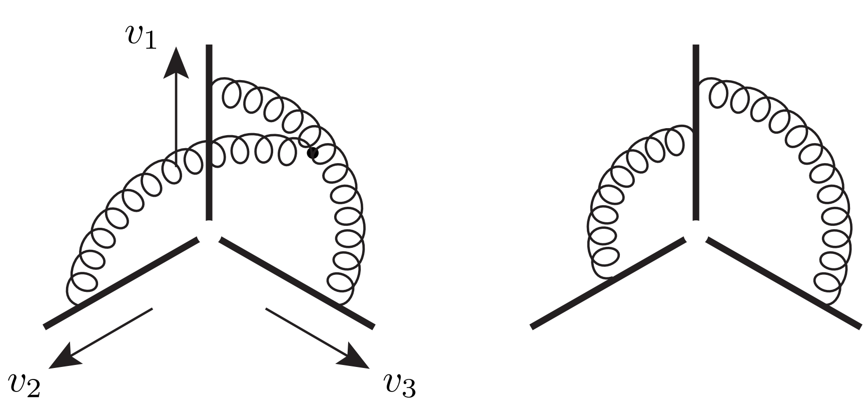
It is remarkable that all the results can be written **analytically** in terms of multiple polylogarithms

Allows fast numerics!

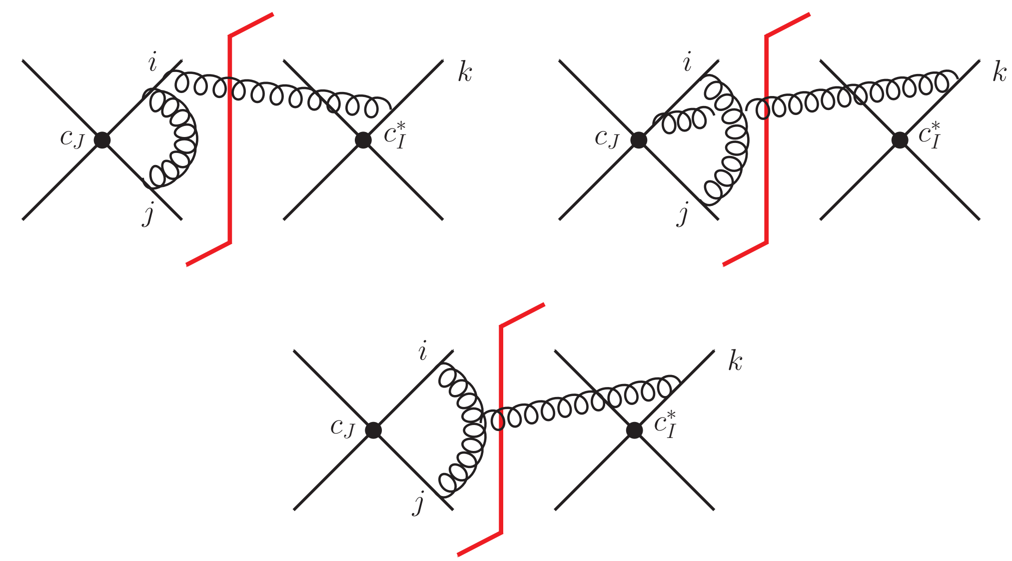
Validation: IR structure

The soft divergence generated from real emissions should be the same as the virtual amplitude! (required by KLN theorem)

virtual



real

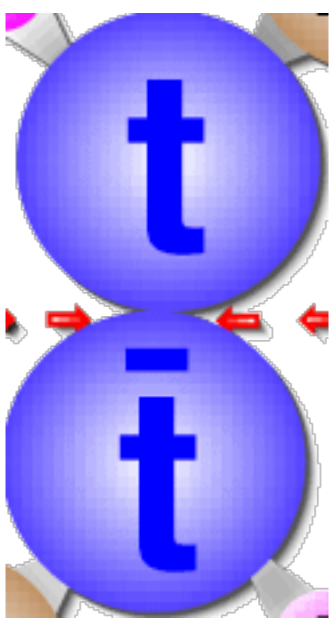


$$+\sum_{(I,J)}\sum_k if^{abc}\,\boldsymbol{T}_I^a\,\boldsymbol{T}_J^b\,\boldsymbol{T}_k^c\,f_2\Big(\beta_{IJ},\ln\frac{-\sigma_{Jk}\,v_J\cdot p_k}{-\sigma_{Ik}\,v_I\cdot p_k}\Big)$$

3-parton correlations: non-trivial cross-check!

Validation: threshold limit

It is interesting to check the threshold limit where the top quarks are produced at rest



Color singlet: same as Drell-Yan and Higgs production

Belitsky: hep-ph/9808389

Color octet Czakon, Fiedler: 1311.2541

Note: singlet-octet mixing terms do NOT vanish in the threshold limit!

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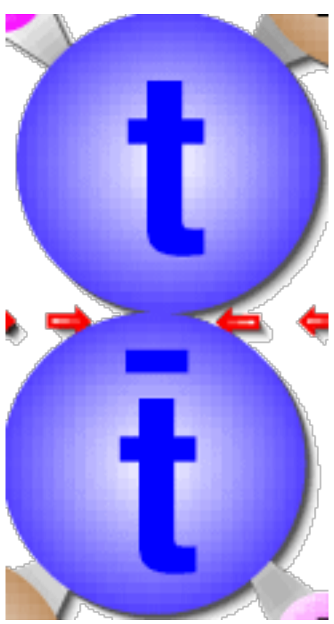

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Implications for near-threshold production?

Validation: boosted limit

In the limit where the top quarks are highly boosted

↓

Factorization Ferroglia, Pecjak, **LLY**: 1205.3662

$$\boldsymbol{S}_{\text{massive}}(s,t,m_t,N) \to \boldsymbol{S}_{\text{massless}}(s,t,N)\, S_D^2(m_t/N)$$

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Also obtain the missing 3-parton piece for free

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Ferroglia, Pecjak, **LLY**: 1207.4798

Also obtain the missing 3-parton piece for free

Allows to extract the soft fragmentation function

Soft and small-mass factorization

Ferroglia, Pecjak, **LLY**: 1205.3662

In Mellin space: $Q \sim \sqrt{s}, \sqrt{-t} \gg Q/N \gg m_t \gg m_t/N$

$$\hat{\sigma}(N, \mu_f) \sim \mathrm{Tr}\big[\boldsymbol{H}(L_h, \mu_f)\, \boldsymbol{S}(L_s, \mu_f)\big] C_D^2(L_c, \mu_f)\, S_D^2(L_{sc}, \mu_f)$$

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hard log

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collinear log
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$$\ln\frac{m_t^2}{\bar{N}^2\mu_f^2}$$

soft-collinear log
(emergent)

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There exist 3 factorization formulas involving C_D an S_D

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Mitov, Moch: hep-ph/0612149

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Ferroglia, Pecjak, **LLY**: 1205.3662
Wang, Xu, **LLY**, Zhu: 1804.05218

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Fragmentation function

Korchemsky, Marchesini: hep-ph/9210281
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All consistent at NNLO!

Soft and small-mass resummation

Massless hard function

Massless soft function

$$\boldsymbol{H}(L_h, \mu_h \sim Q)$$

$$\boldsymbol{S}(L_s, \mu_s \sim Q/\bar{N})$$

$$\mu_f$$

RG flow

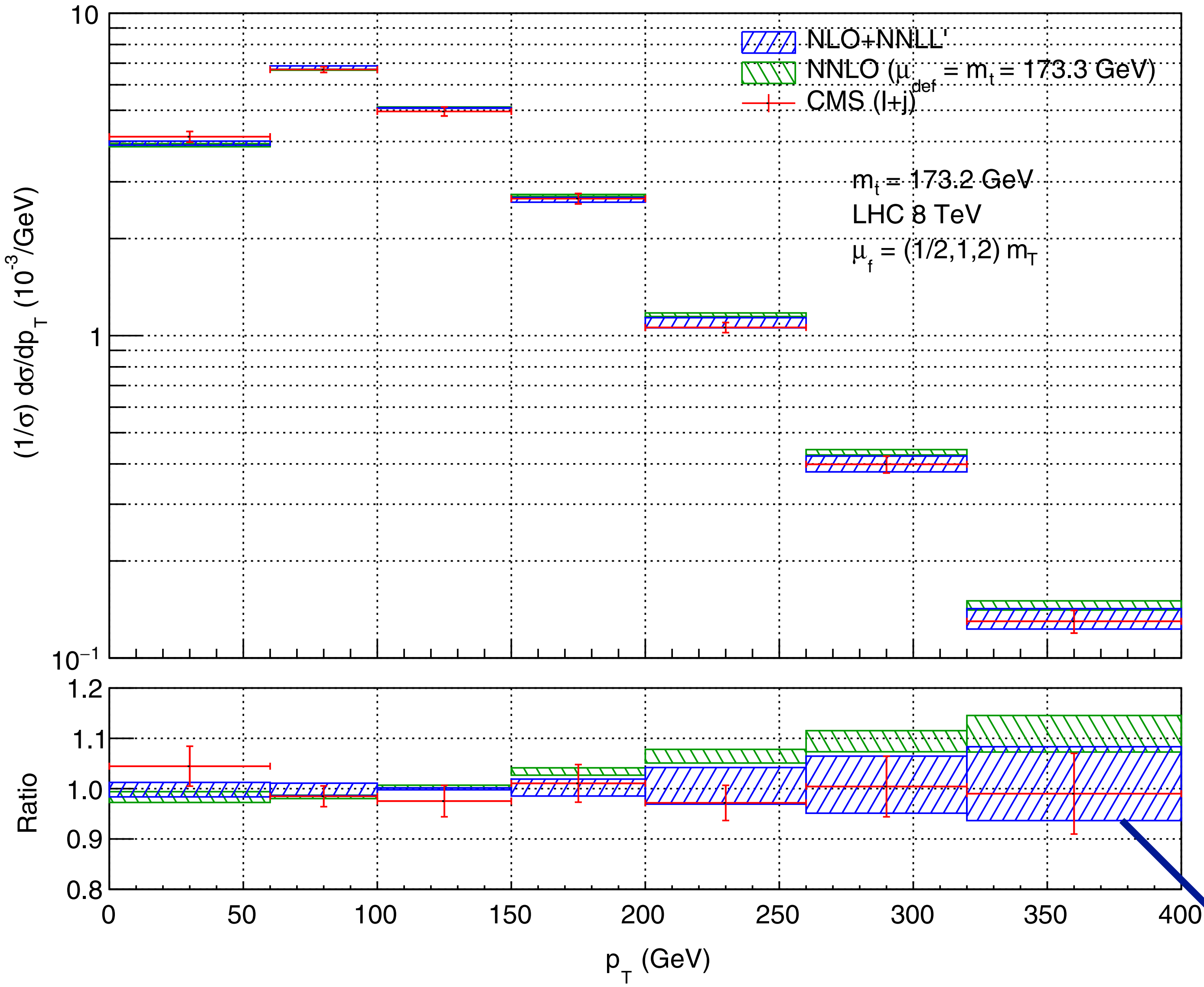
$$C_D(L_c, \mu_c \sim m_t)$$

$$S_D(L_{sc}, \mu_{sc} \sim m_t/\bar{N})$$

All ingredients known at NNLO (for NNLL′ resummation)

NLO+NNLL′

Pecjak, Scott, Wang, **LLY**: 1601.07020



Resummation
softens the spectrum

NNLO+NNLL′

A joint effort of the NNLO group and the resummation group

Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623

$$d\sigma^{\text{(N)NLO+NNLL}'} = d\sigma^{\text{NNLL}'_{b+m}} + \left(d\sigma^{\text{(N)NLO}} - d\sigma^{\text{NNLL}'_{b+m}} \Big|_{\substack{\text{(N)NLO} \\ \text{expansion}}} \right)$$

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↓

$$d\sigma^{\text{NNLL}'_b} + \left(d\sigma^{\text{NNLL}_m} - d\sigma^{\text{NNLL}_m}\Big|_{m_t \to 0} \right)$$

↓ soft & small mass resummation

↓ match to soft resummation

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match to NNLO

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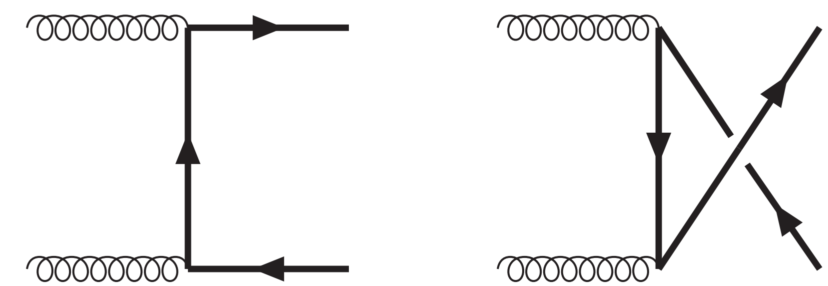
Careful to avoid double-counting!

Scale choices

In the boosted limit, t- and u-channel propagators push the effective hard scale to



$$-t_1\big|_{m_t\to 0} \approx \frac{M_{t\bar{t}}^2}{2}(1-\cos\theta) + m_t^2\cos\theta \xrightarrow{\cos\theta\to 1} p_T^2 + m_t^2 \equiv m_T^2 = H_T^2/4\,,$$

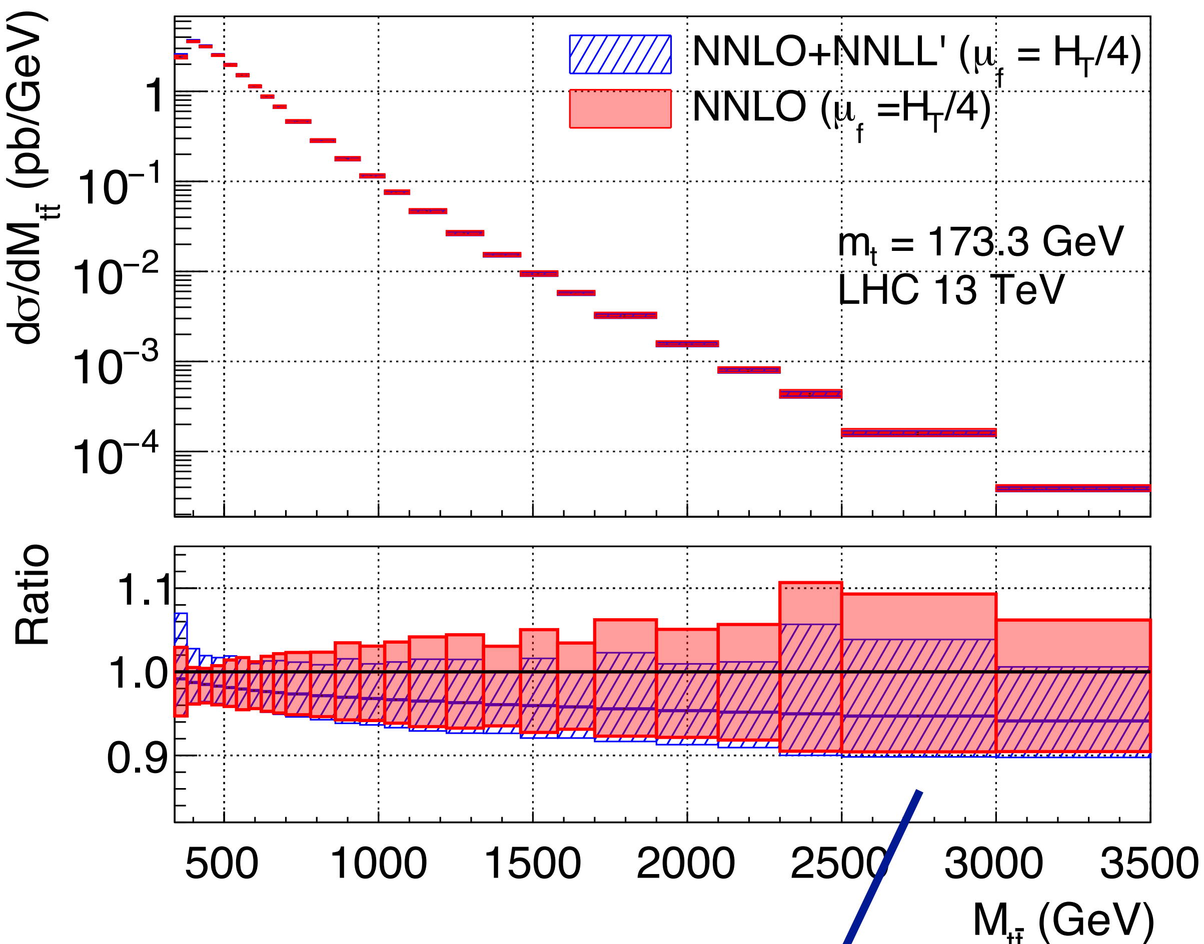
$$-u_1\big|_{m_t\to 0} \approx \frac{M_{t\bar{t}}^2}{2}(1+\cos\theta) - m_t^2\cos\theta \xrightarrow{\cos\theta\to -1} m_T^2 = H_T^2/4\,.$$

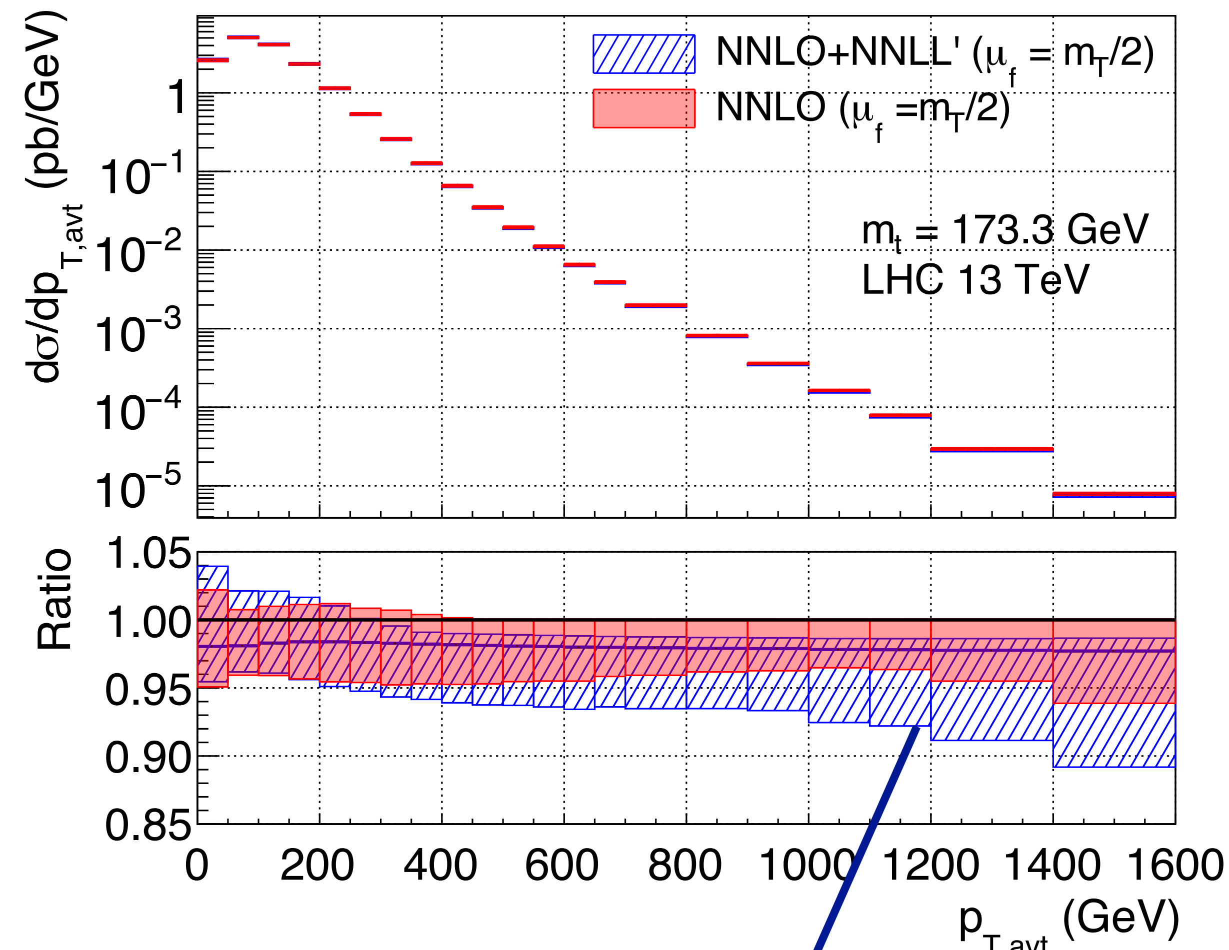
$$\frac{\mathcal{H}_{gg}^{\rm NLO}(\mu_h)}{\mathcal{H}_{gg}^{\rm LO}(\mu_h)}\bigg|_{t_1\to 0} = 1+\frac{\alpha_s(\mu_h)}{36\pi}\left[-78\ln^2\left(\frac{-t_1}{\mu_h^2}\right)+24\ln\left(\frac{-t_1}{\mu_h^2}\right)(3+2\ln x_t)+37\pi^2-168\right]$$

Support the findings of Czakon, Heymes, Mitov: 1606.03350

NNLO+NNLL′

Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623



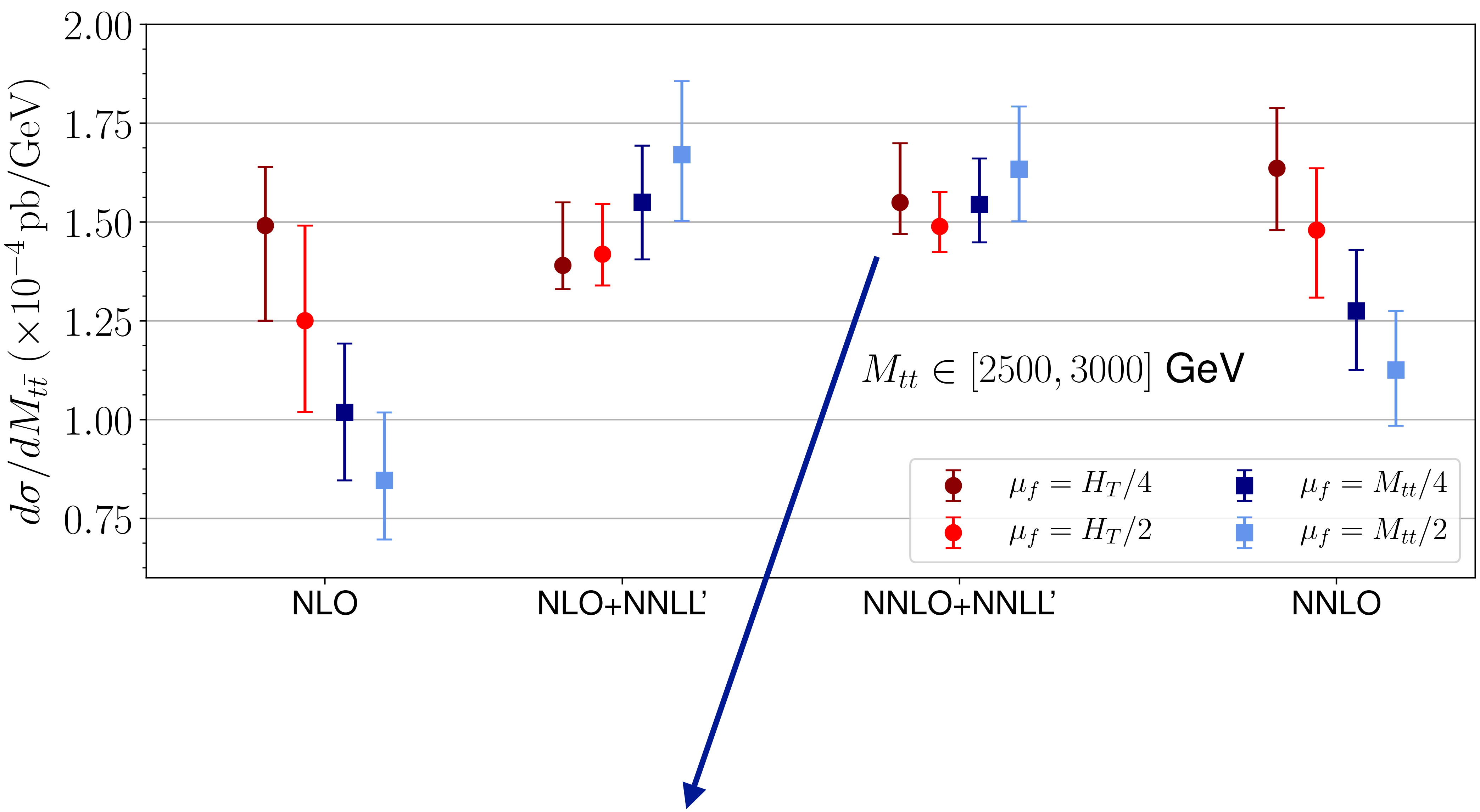


Resummation reduces scale variation

Resummation softens the spectrum

NNLO+NNLL′

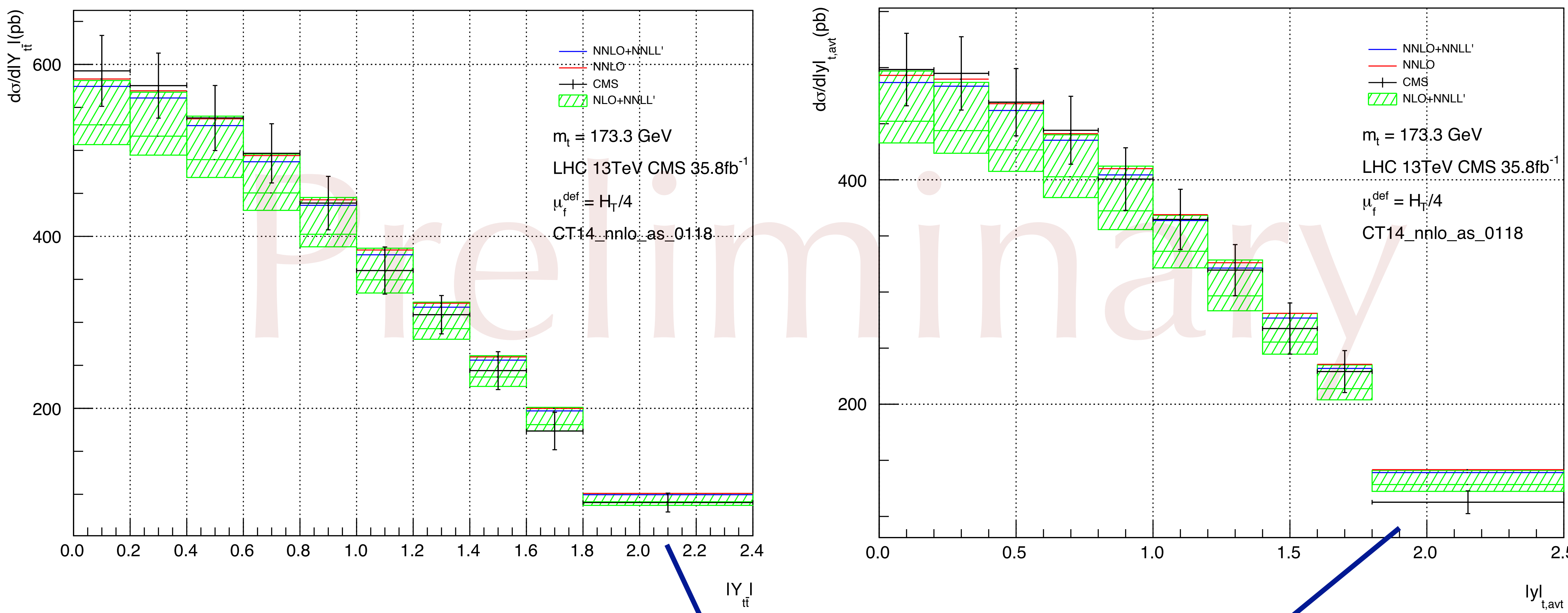
Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623



Matched result insensitive to scale scheme choices

Ongoing: rapidity distributions

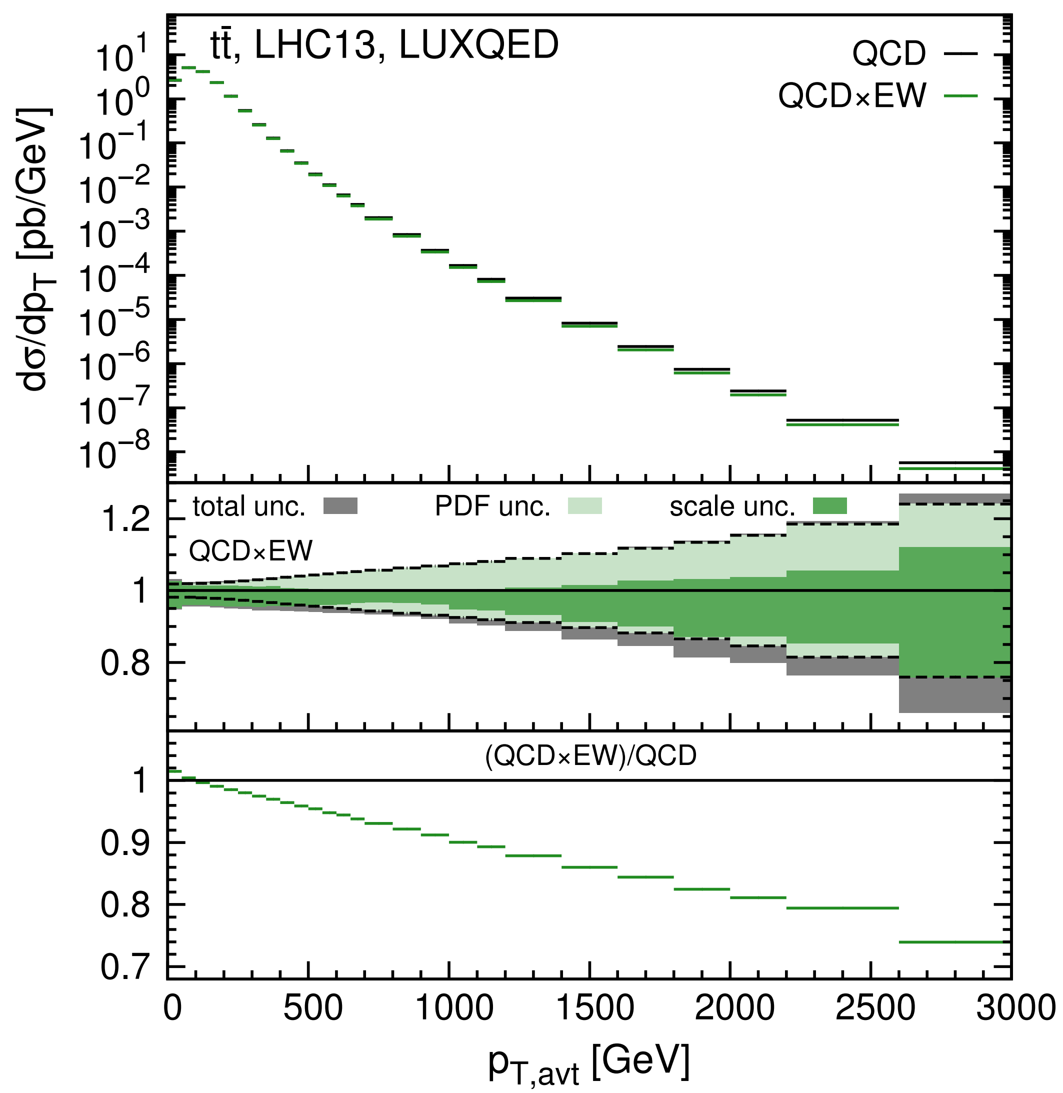
Pecjak, Scott, Wang, **LLY**: to appear



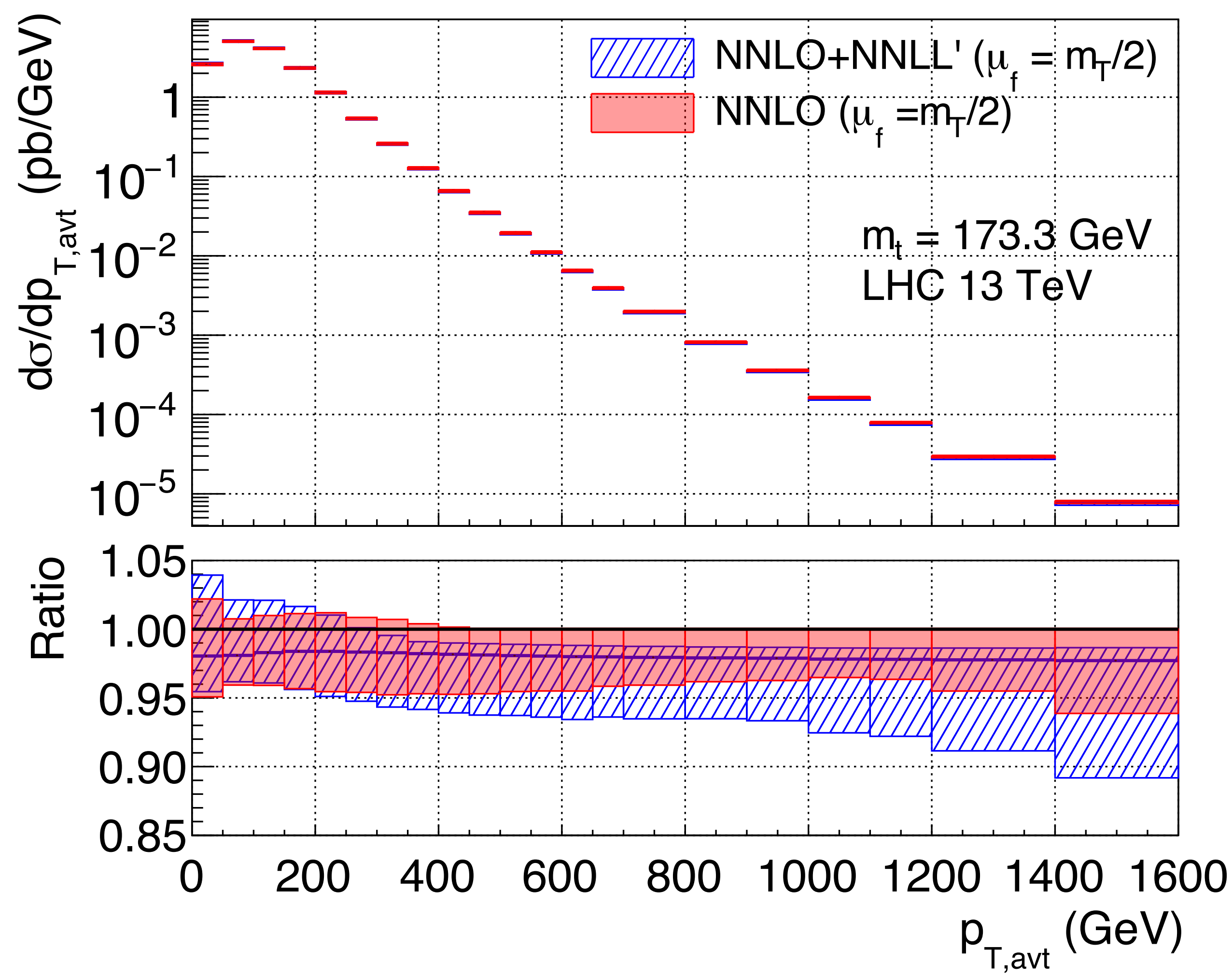
Sensitive to gluon PDF at large x

Ongoing: combination with electroweak corrections

Both EW and resummation effects soften the p_T spectrum



Czakon et al.: 1705.04105



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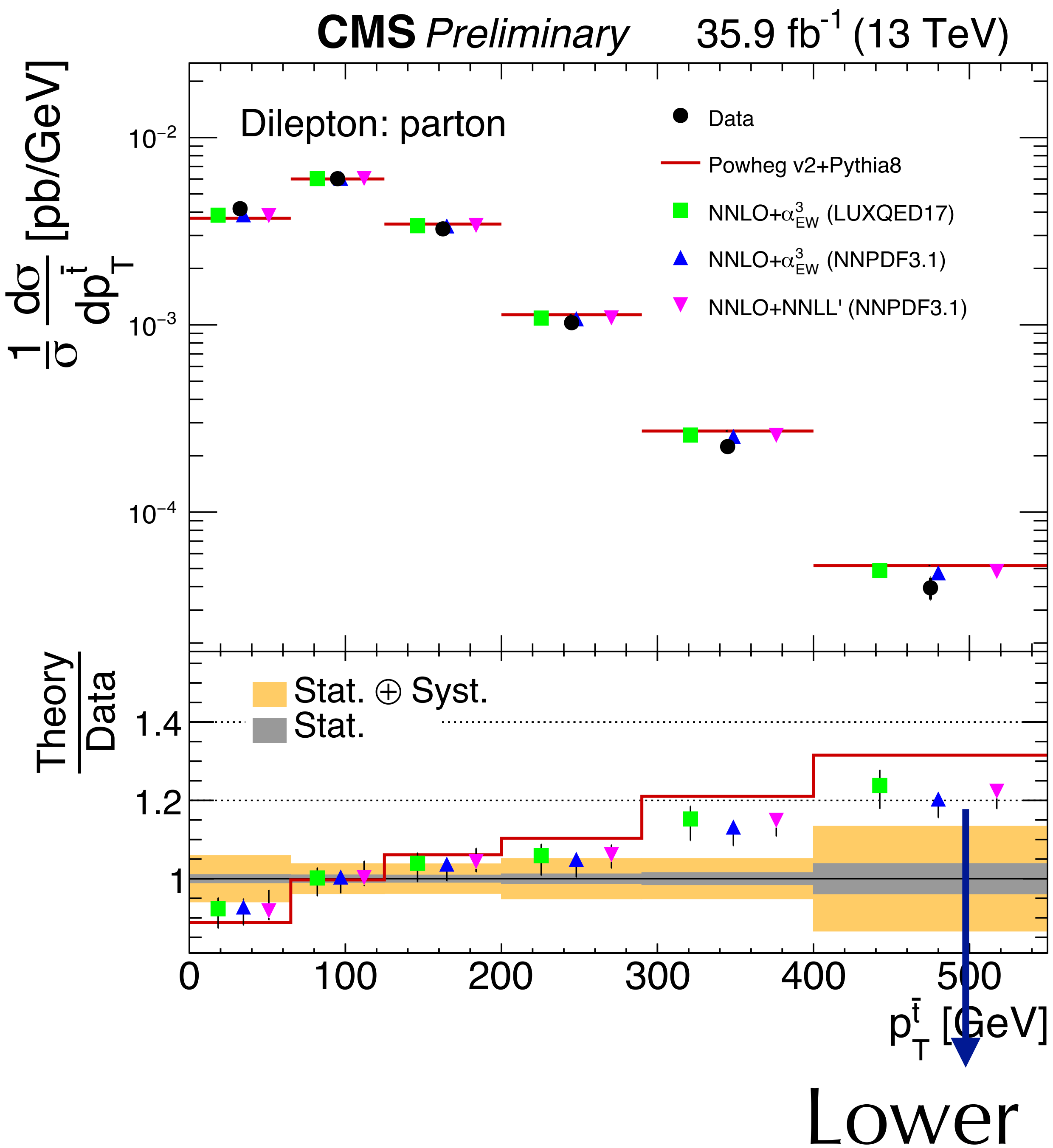
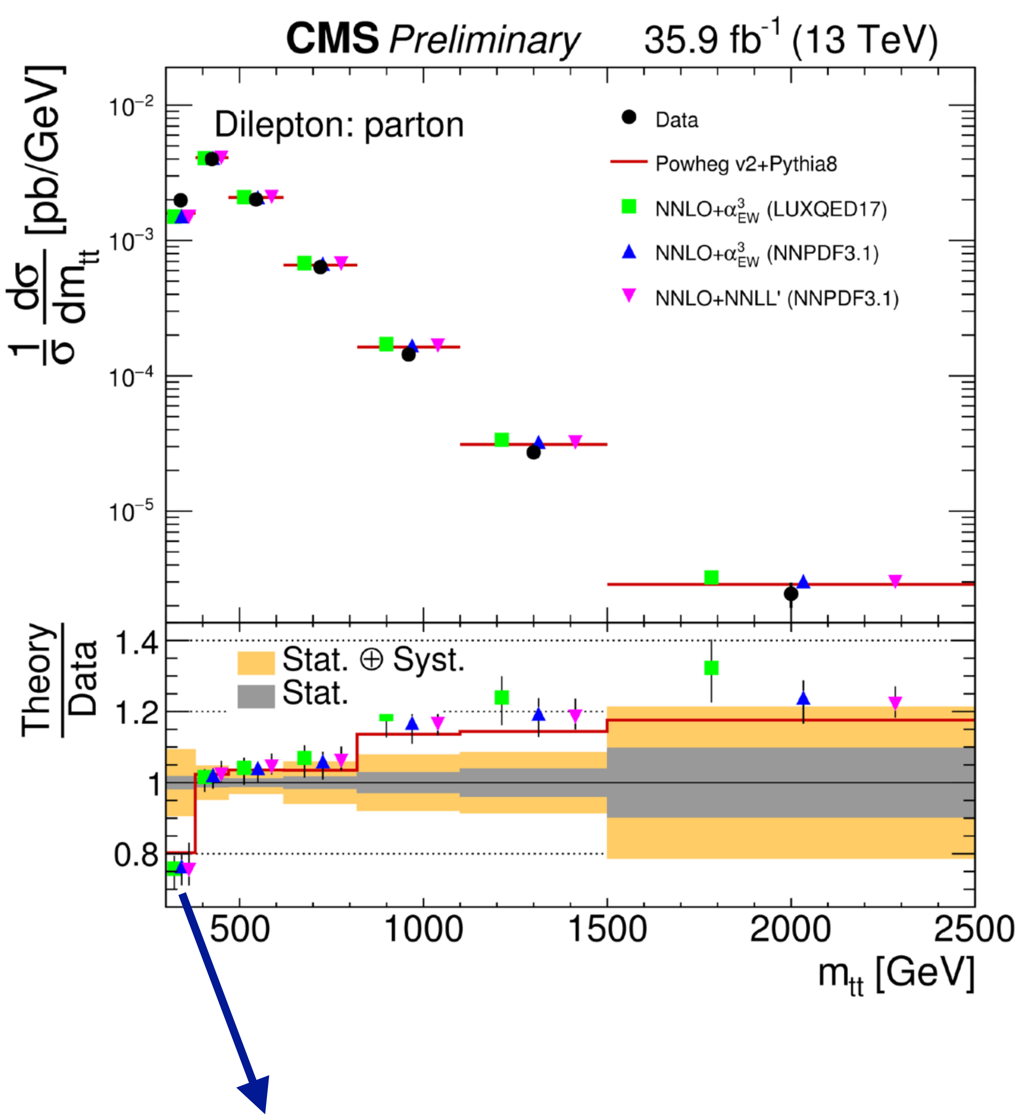


NNLO+NNLL′+EW should be better consistent with data!

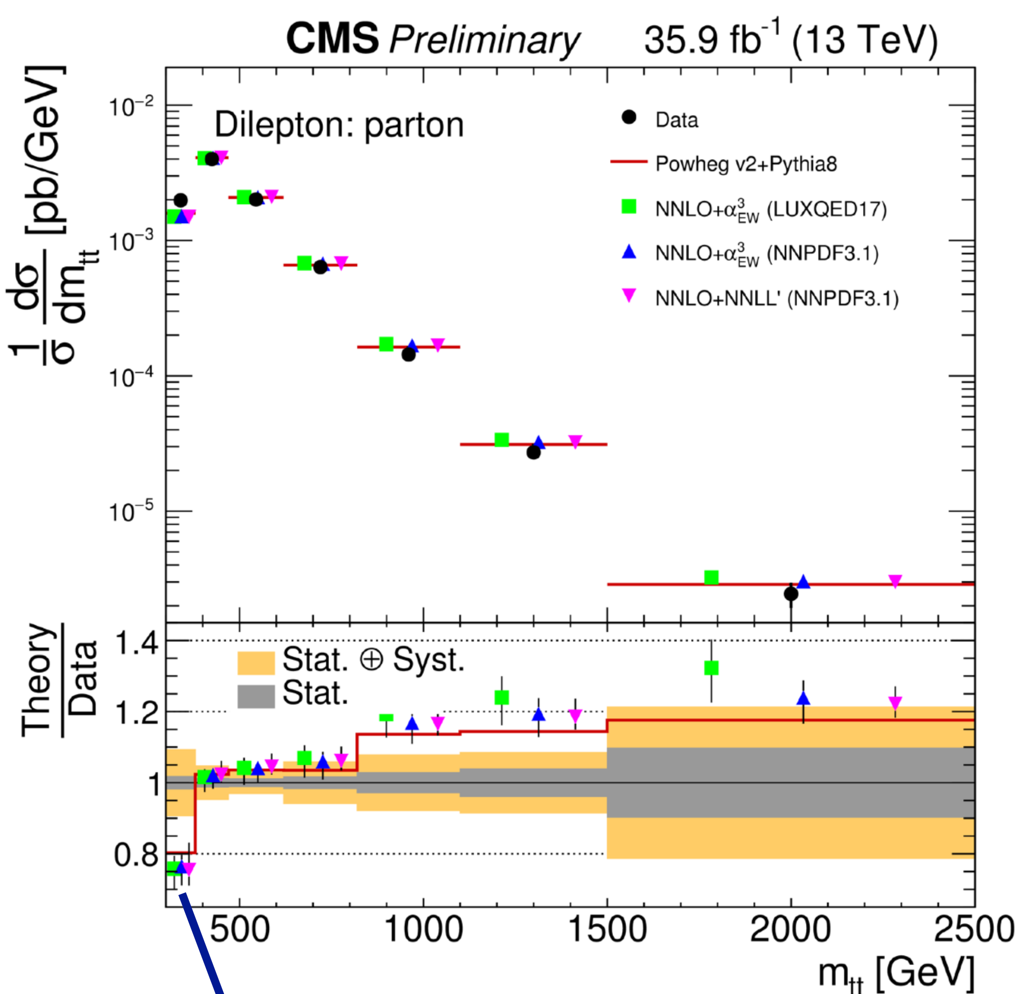
Stay tuned

Ongoing: near-threshold



Threshold region sensitive to
Coulomb gluons

Ongoing: near-threshold

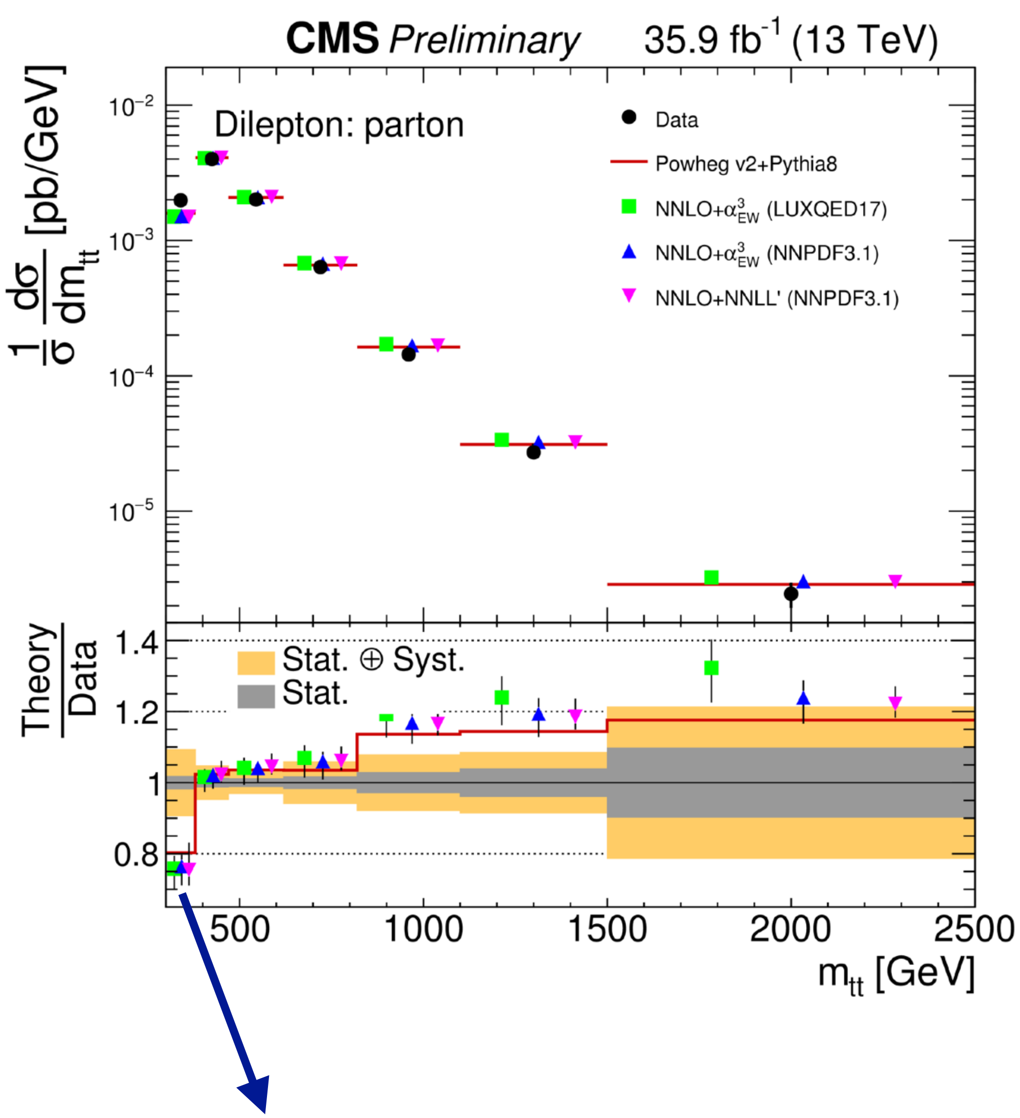


Threshold region sensitive to Coulomb gluons

Historically, Coulomb gluons have been studied only for the **total cross section**

Moch, Uwer: 0804.1476
Beneke, Falgari, Klein, Schwinn: 1109.1536

Ongoing: near-threshold



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Requires new framework to study the M_{tt} distribution!

Summary and outlook

* Top quark production is important
* The most precise QCD calculation: NNLO+NNLL′
* Ongoing:
    * Rapidity distributions
    * Combination with NLO electroweak corrections
    * Near-threshold production

Thank you!