Hadron Physics:
DSEs meet Lattice QCD

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第十三届TeV物理工作组学术研讨会，2018年8月19日

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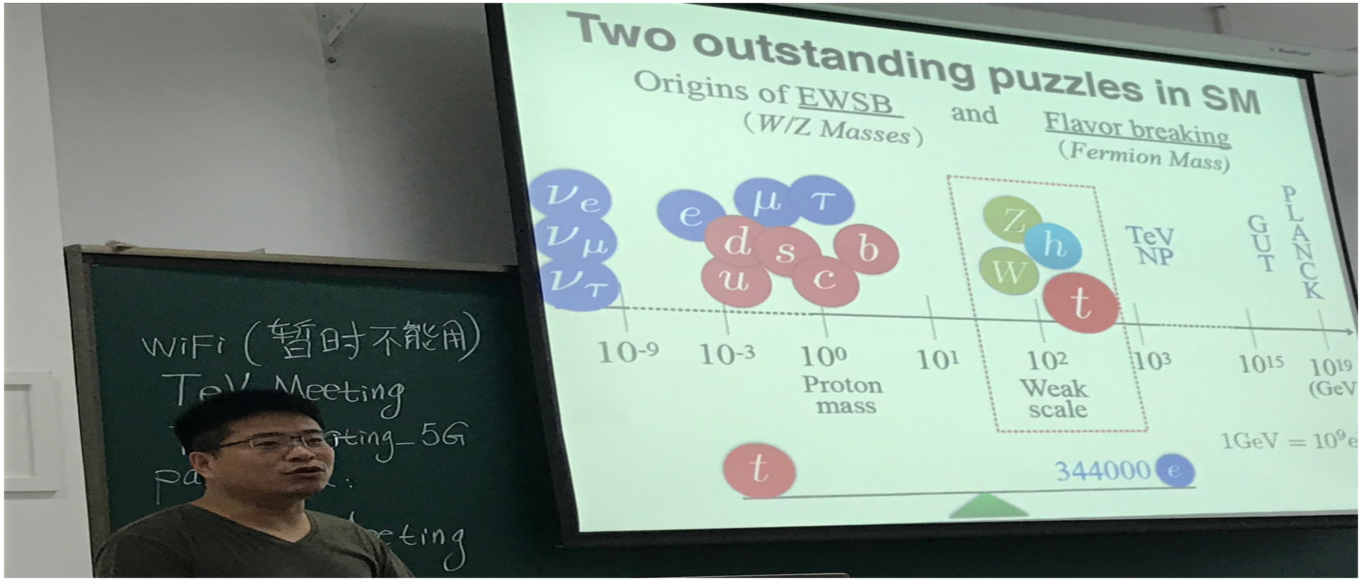

Two outstanding puzzles in SM

Origins of EWSB (*W/Z Masses*) and Flavor breaking (*Fermion Mass*)

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Outline

- Hadron Physics and key problems
- One parameter framework
- PDAs
- Electromagnetic Form Factor
- Conclusion

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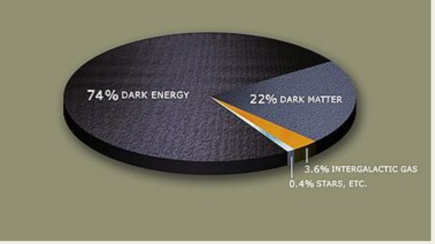
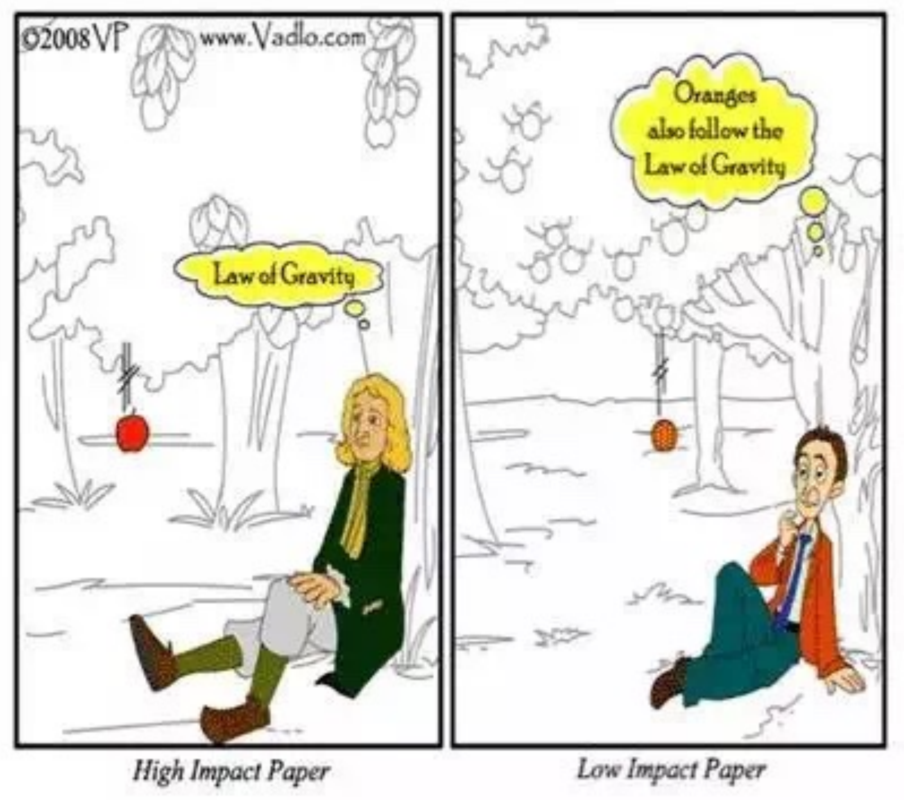


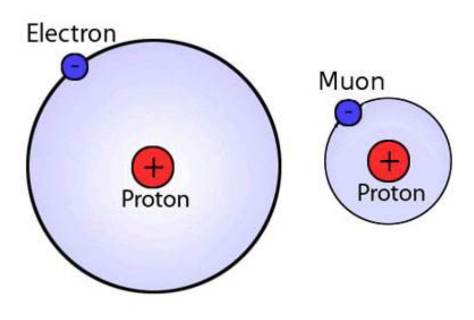
TOP OPEN QUESTIONS IN PHYSICS

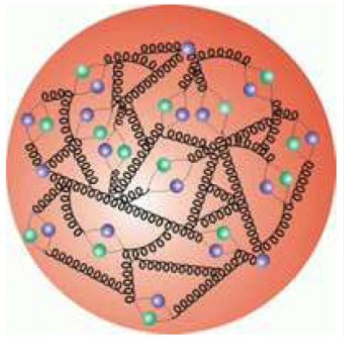
- ……
- Can we quantitatively understand quark and gluon confinement in quantum chromodynamics and the existence of a mass gap?

Quantum chromodynamics, or QCD, is the theory describing the strong nuclear force. Carried by gluons, it binds quarks into particles like protons and neutrons. Apparently, the tiny subparticles are permanently confined: one can't pull a quark or a gluon from a proton because the strong force gets stronger with distance and snaps them right back inside.



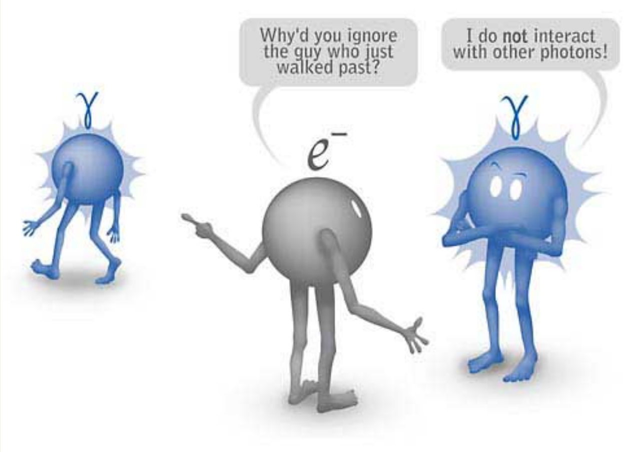
束缚态，场论

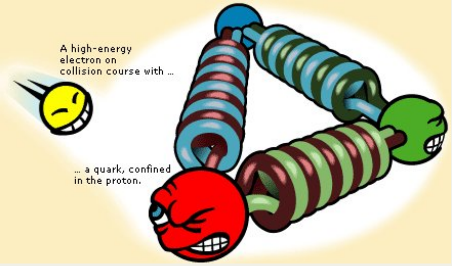




QED

QCD





Mass scale…m_e

Mass scale…m_p=1GeV

Trace anomaly

- All renormalisable four-dimensional theories possess a trace anomaly;
- The size of the trace anomaly in QED must be great deal smaller than that in QCD.

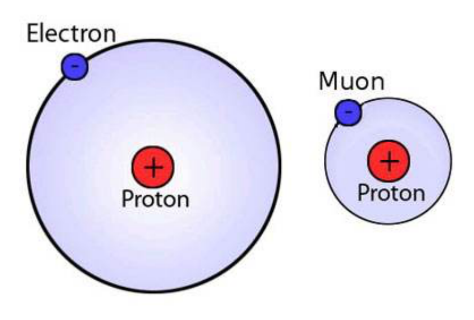
$$\int d^4x\, G_{\mu\nu}G_{\mu\nu} \sim \int d^4x d^4y\, A_\mu(x) D^{-1}_{\mu\nu}(x-y) A_\nu(y)\,,$$

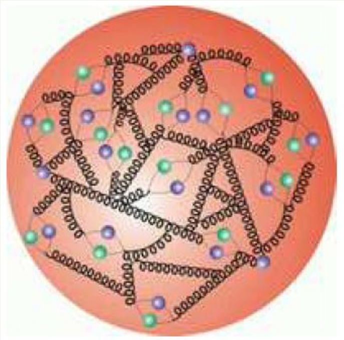
$$\begin{cases} \Pi_{\rm QED}(k^2=0)=0 \\ k^2\Pi_{\rm QCD}(k^2)\big|_{k^2=0} \sim \Lambda^2_{\rm QCD} \end{cases}$$

- QCD trace anomaly expresses a significant mass-scale.

(J.M.Cornwall, PRD26, 1453(1982))

束缚态，场论





QED

QCD

Field theory Successful:

- Nonrelativistic quantum mechanics to handle bound state;
- Perturbation theory to handle relativistic effects

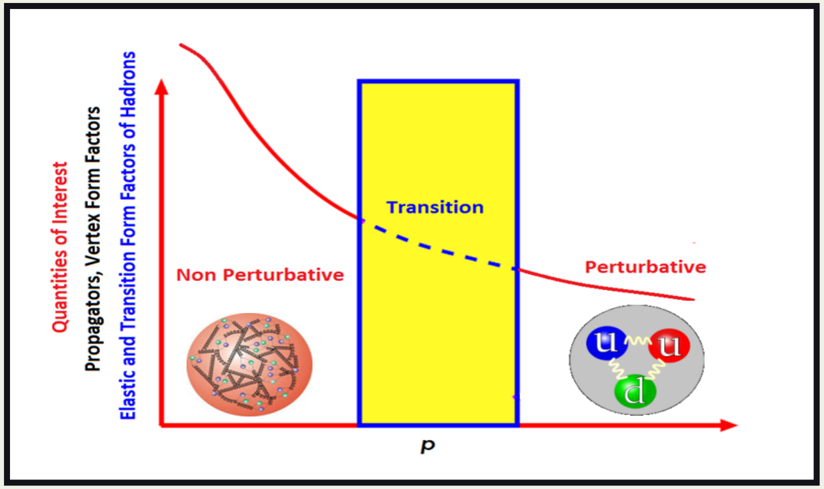
Field theory not Successful yet:

- Unlimited growth of the running coupling constant in the infrared region;
- Confinement;
- Dynamical Chiral Symmetry Breaking;
- Possible nontrivial vacuum structure

强子束缚态问题

Constituent quark model-> intuitive understanding of many low energy observables.

Minimum number of constituents required



Feynman’s parton model-> intuitive understanding of high-energy phenomena.

Constituent picture; Probabilistic interpretation of distribution functions

QCD vacuum is very complicated medium
Individual quarks and gluons are lost in the sea

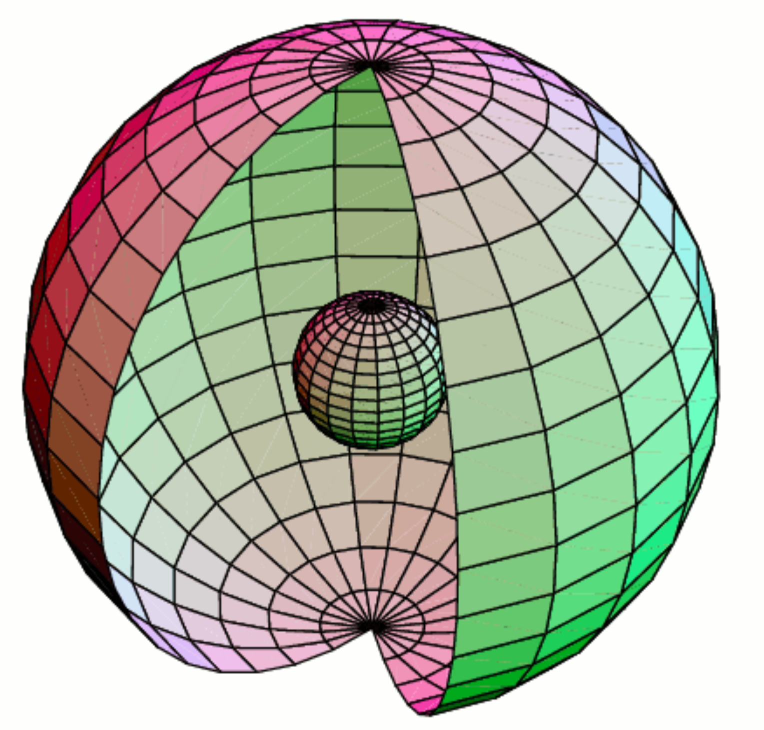
Both the constituent quark model and the parton model are put in peril by QCD with a complicated vacuum structure.

The Problem with QCD

我唯一知道的事，就是我什么都不知道——苏格拉底

What is the interaction throughout more than 98% of the proton's volume?

- *The study of nonperturbative QCD is the puriew of …*



Hadron Physics

粒子物理和核物理之桥梁，交叉学科

Emergent phenomena, 方法和哲学

- Confinement and DCSB are emergent phenomena

  Not revealed by any amount of staring at Lagrangian for quantum chromodynamics;
  They determine the character of the QCD's spectrum, the structure and interactions of bound states

- Can one understand confinement and DCB in terms of properties of the degrees-of-freedom used to formulate QCD?

  E.g., is it pointless to attempt to predict the nucleon's form factor on a domain that is not yet accessible?

If YES:
Must rely on the vase array of effective field theories, developed for different systems, in order, to express and understand the consequences of confinement and DCSB, without identifying their source

If NO:
Must develop nonperturbative calculational methods to define and tackle QCD

1) Lattice-regularized QCD
2) Continuum methods in quantum filed theory
3) Combination of all the above

Currently, each approach has strengths and weaknesses
So 3) is probably the best:
Combine all available methods to fullest extent reasonably possible.

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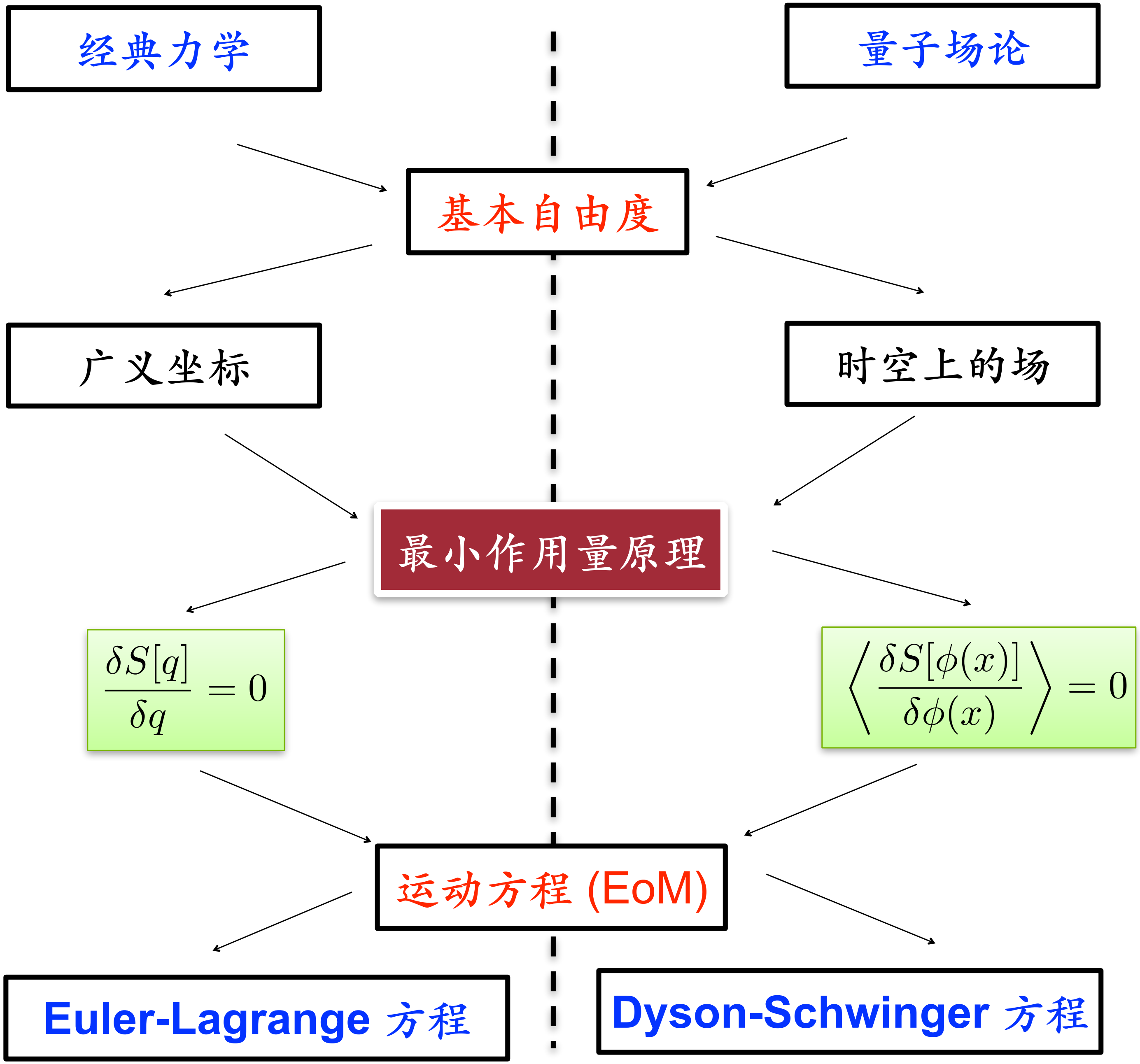
Dyson-Schwinger Equations

- It has long been known that from the field equations of quantum field theory one can derive a system of coupled integral equations interrelating all of a theory's Green functions:

Dyson, F. J. (1949), "The S Matrix In Quantum Electrodynamics," Phys. Rev. 75, 1736.
Schwinger, J. S. (1951), "On The Green's Functions Of Quantized Fields: 1 and 2," Proc. Nat. Acad. Sci. 37 (1951) 452; ibid 455.

- It is an intrinsically nonperturbative complex, which is vitally important in proving the renormalisability of quantum field theories. At its simplest level the complex provides a generating tool for perturbation theory.



经典力学

量子场论

基本自由度

广义坐标

时空上的场

最小作用量原理

$$\frac{\delta S[q]}{\delta q}=0$$

$$\left\langle\frac{\delta S[\phi(x)]}{\delta\phi(x)}\right\rangle=0$$

运动方程 (EoM)

Euler-Lagrange 方程

Dyson-Schwinger 方程

From Sixue Qin

QCDs Dyson-Schwinger Equations

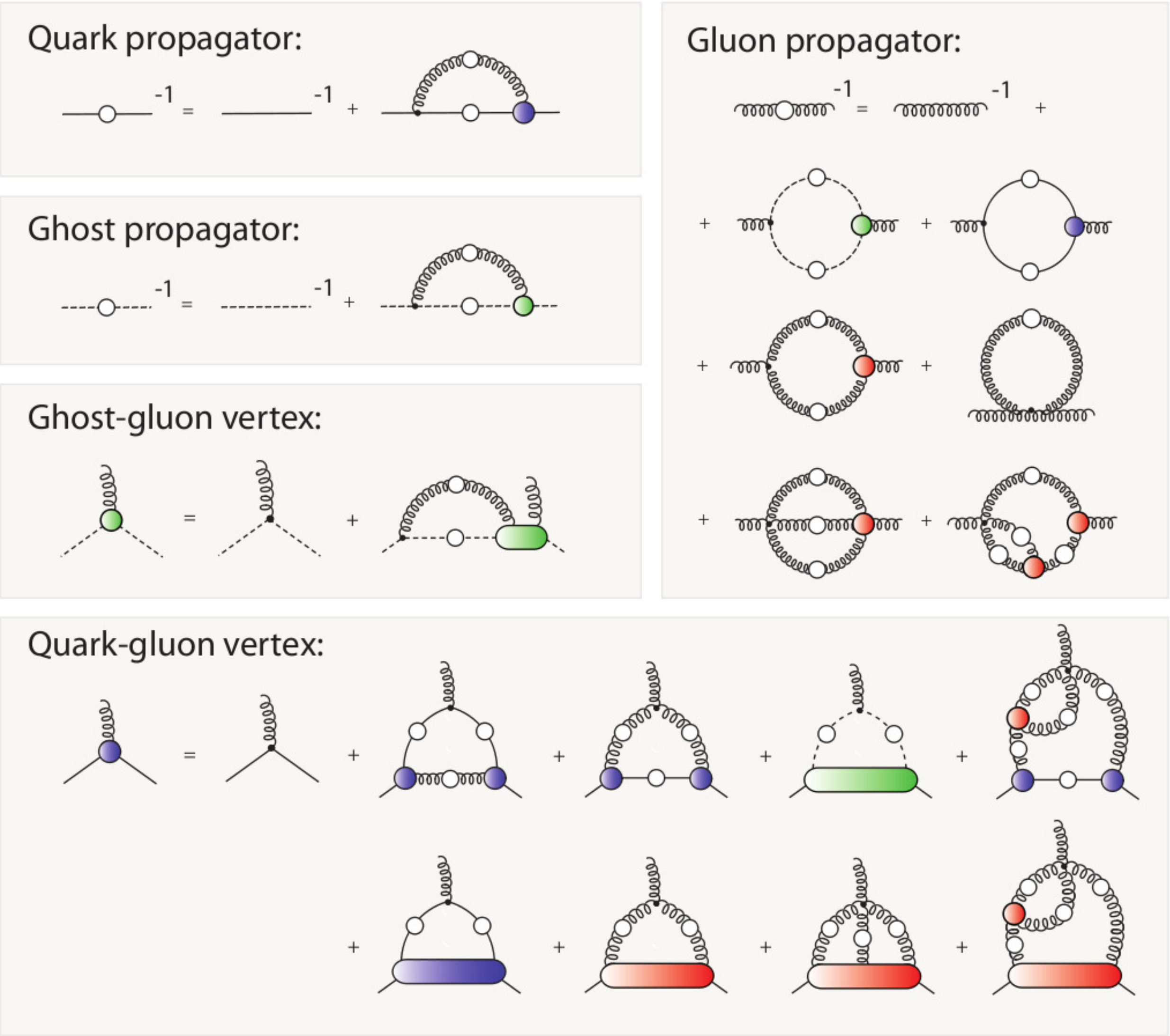
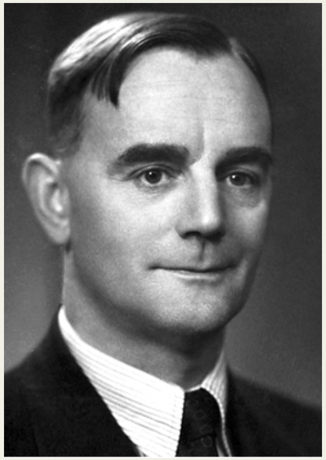


Image courtesy of Gernot Eichmann

Story of π

Why Pion-----Messager of QCD

- Pion is Massless...

- In October 1934**, Hideki Yukawa** predicated the existence of a “heavy quantum” meson, exchanging nuclear force between neutrons and protons.
- It was discovered by **Cecil Powel** in 1949 in cosmic ray tracks in a photographic emulsion.
- Pion was nicely accommodated in the Eight Fold way of **Murray Gell-Mann** in 1961.
- **Yoichiro Nambu** associated it with CSB in 1960.

Pion's dichotomy
Goldstone boson and Bound State

Maris, Roberts and Tandy, Phys. Lett. **B420**(1998) 267-273

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\begin{aligned}\Gamma_{\pi^j}(k;P) &= \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P)+\gamma\cdot P F_\pi(k;P)\\ &\quad +\gamma\cdot k\, k\cdot P\, G_\pi(k;P)+\sigma_{\mu\nu}\,k_\mu P_\nu\, H_\pi(k;P)\Big]\end{aligned}$$

- Dressed-quark propagator $S(p)=\dfrac{1}{i\gamma\cdot p\,A(p^2)+B(p^2)}$

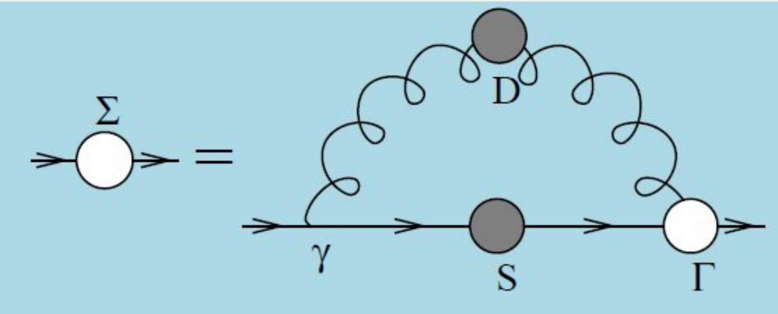
- Axial-vector Ward-Takahashi identity entails(chiral limit)

$$f_\pi E(k;P|P^2=0)=B(k^2)+(k\cdot P)^2\frac{d^2B(k^2)}{d^2k^2}+\dots$$

- Given the dichotomy of pion the fine-tuning should not play any role in an explanation of pion properties;
- Descriptions of pion within frameworks that cannot faithfully express symmetries and their breaking patterns(such as constituent-quark models) are unreliable;
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.

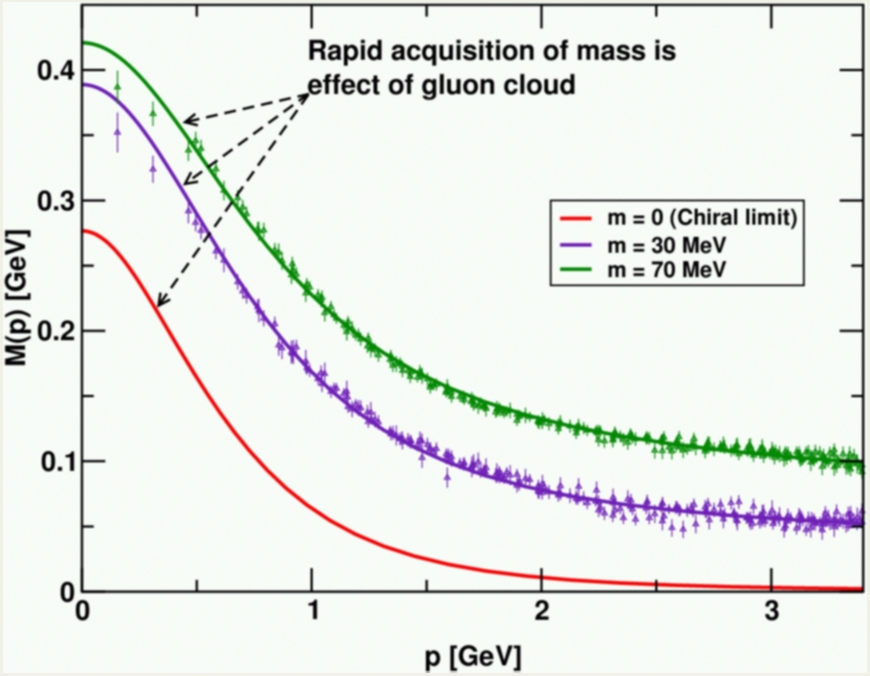
Dynamical Chiral Symmetry Breaking

- Is a crucial emergent phenomenon in QCD
- Expressed in hadron wave functions not in vacuum condensates
- Contemporary theory indicates that it is responsible for more than 98% of the visible mass in the Universe; namely, given that classical massless-QCD is a conformally invariant theory, then DCSB is the origin of *mass from nothing*.



- ***Dynamical***, not spontaneous
  - Add nothing to QCD ,

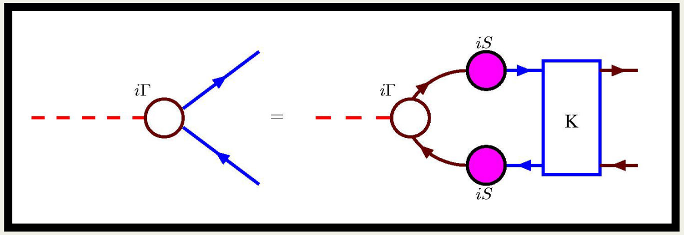
    No Higgs field, nothing! Effect achieved purely through quark+gluon dynamics.

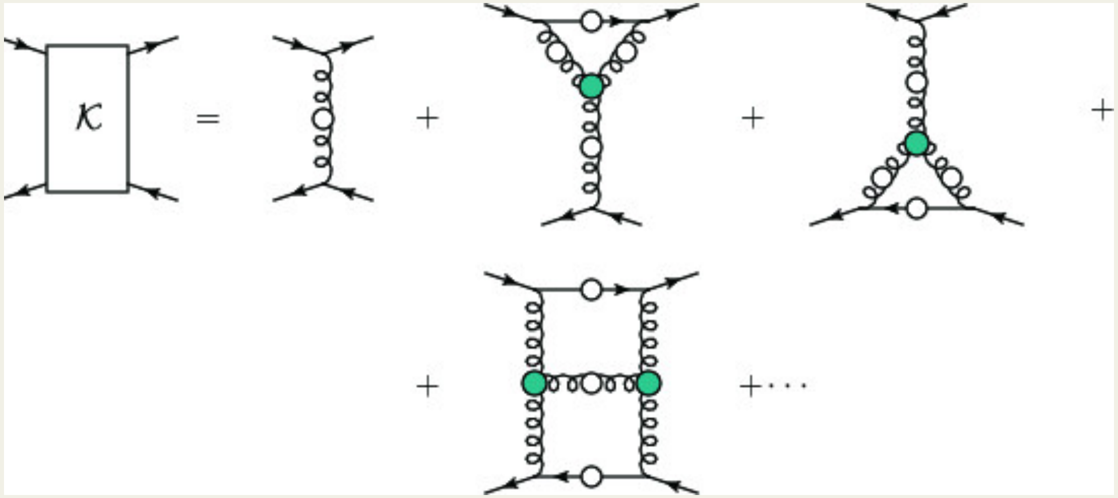


Quark, das ist alles Quark！（夸克［胡扯］，全是夸克［胡扯］！）

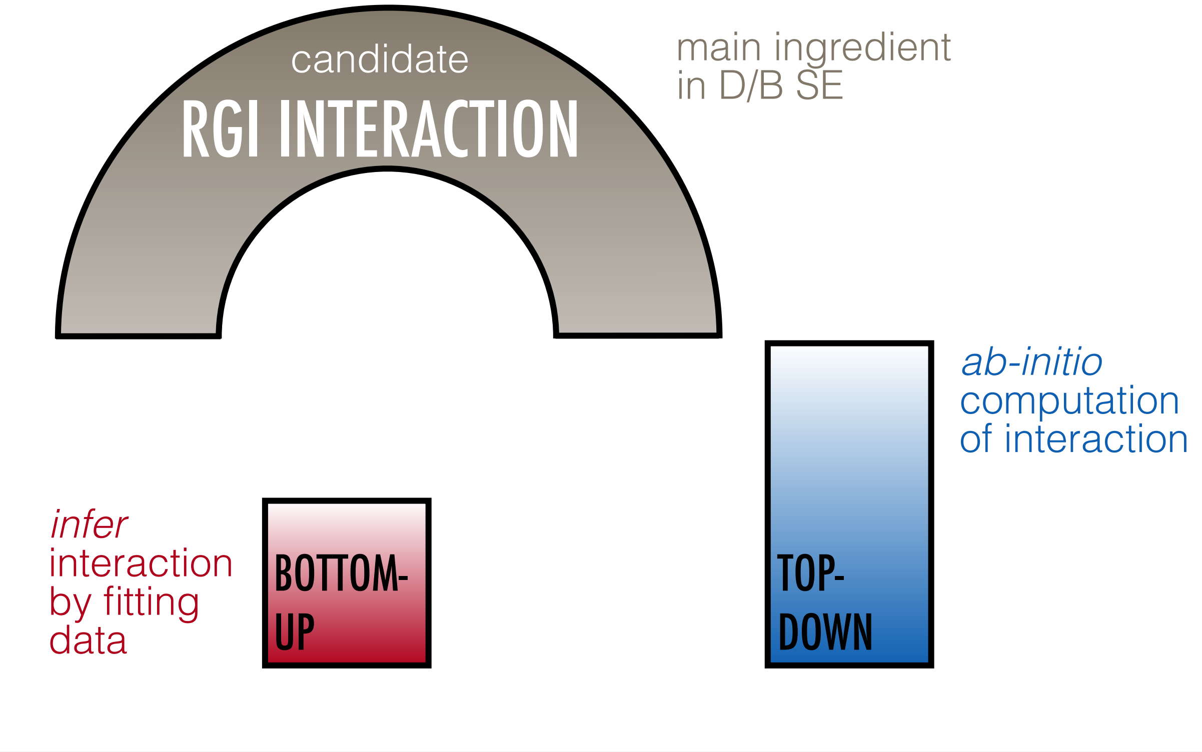
Dyson-Schwinger Equation scope

Bethe-Salpeter Equations for meson bound state





RGI interaction



candidate

RGI INTERACTION

main ingredient
in D/B SE

ab-initio
computation
of interaction

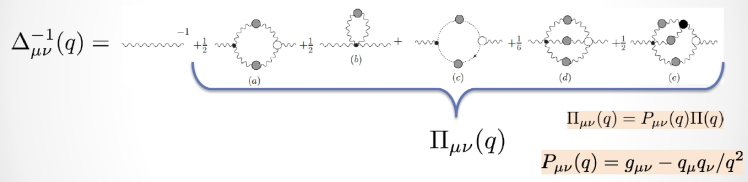
infer
interaction
by fitting
data

BOTTOM-UP

TOP-DOWN

- Bottom-up scheme – infer interaction by fitting data within a truncation of the matter sector DSEs that are relevant to bound-state properties.

- Top-down approach – *ab initio* computation of the interaction via direct analysis of the gauge-sector gap equations



$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu/q^2$$

D.Binosi

J. Skullerud, etal, hep-ph/0303176

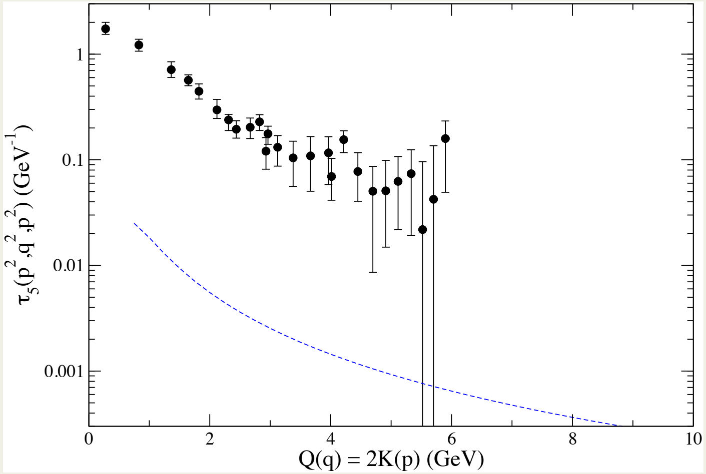
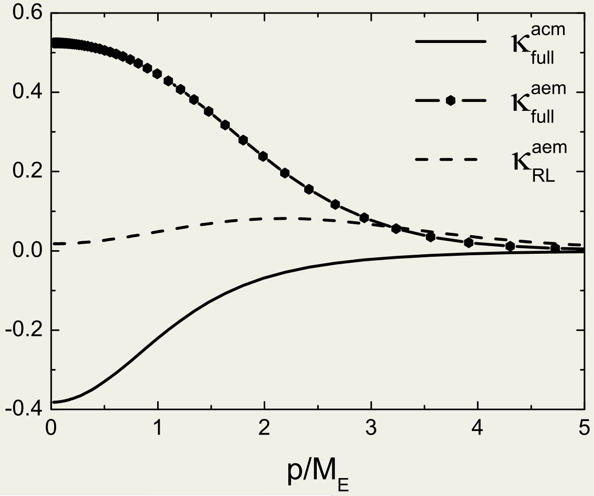


Figure 10: The renormalised form factor τ_5 at the symmetric point as a function of the gluon momentum q. The data shown are those surviving a cylinder cut with radius 2 units of spatial momentum in q. Also shown is the one-loop form of (4.1).

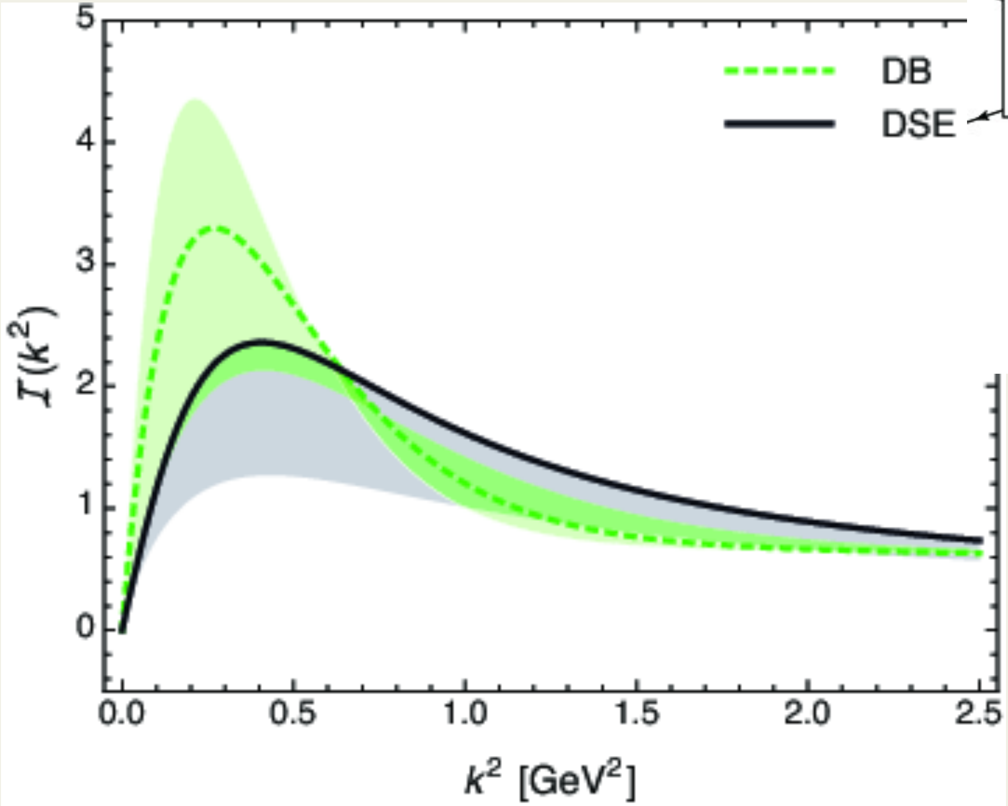
$$\Gamma^{\mathrm{acm}_5}_{\mu}(p_f,p_i;k) = \sigma_{\mu\nu}k_\nu\,\tau_5(p_f,p_i,k)\,.$$

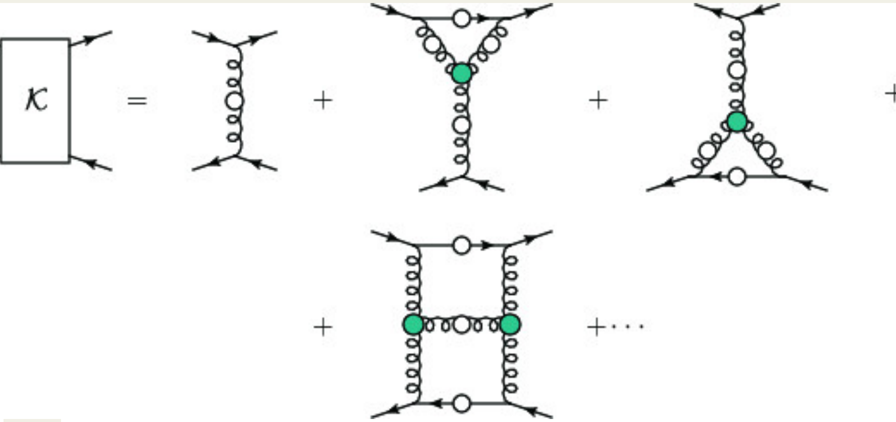


Lei Chang and C. D. Roberts, Phys. Rev. Lett.103 (2009) 081601;
Lei Chang, Yu-xin Liu and C. D. Roberts, Phys. Rev. Lett.106 (2011) 072001

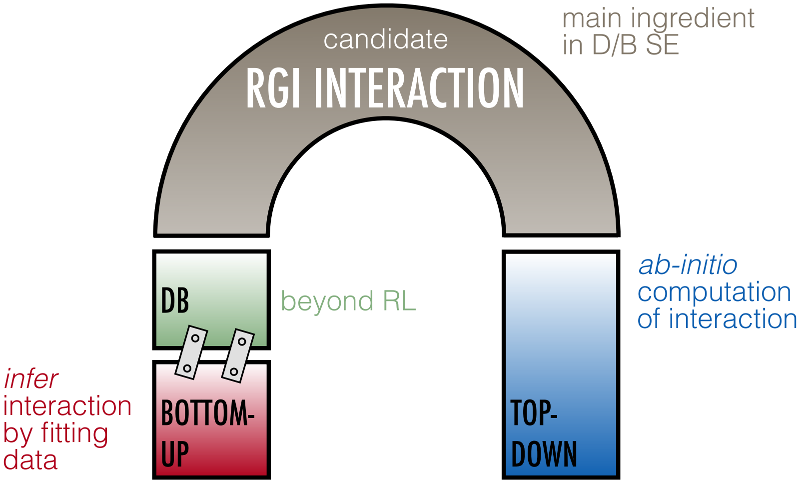
Bridging a gap between continuum-QCD & ab initio predictions of hadron observables

D. Binosi (Italy), L. Chang (Australia), J. Papavassiliou (Spain),
C. D. Roberts (US), arXiv:1412.4782 [nucl-th] ,
Phys. Lett. B 742 (2015) 183





Lei Chang and C. D. Roberts, Phys. Rev. Lett.103 (2009) 081601;
Lei Chang, Yu-xin Liu and C. D. Roberts, Phys. Rev. Lett.106 (2011) 072001



candidate
RGI INTERACTION
main ingredient in D/B SE
DB
beyond RL
BOTTOM-UP
infer interaction by fitting data
TOP-DOWN
ab-initio computation of interaction

– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using the sophisticated DSE truncation

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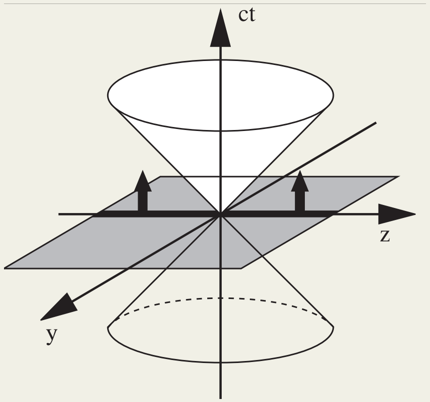
Pion's dichotomy
Goldstone boson and Bound State----On the light front

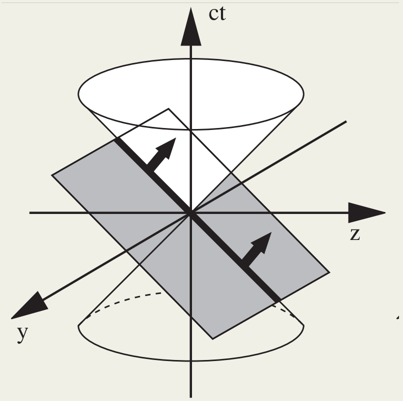
equal-time dynamics vs light-front dynamics

Dirac 1949

$$t \equiv x^0$$



$$t \equiv x^+ = x^0 + x^3$$



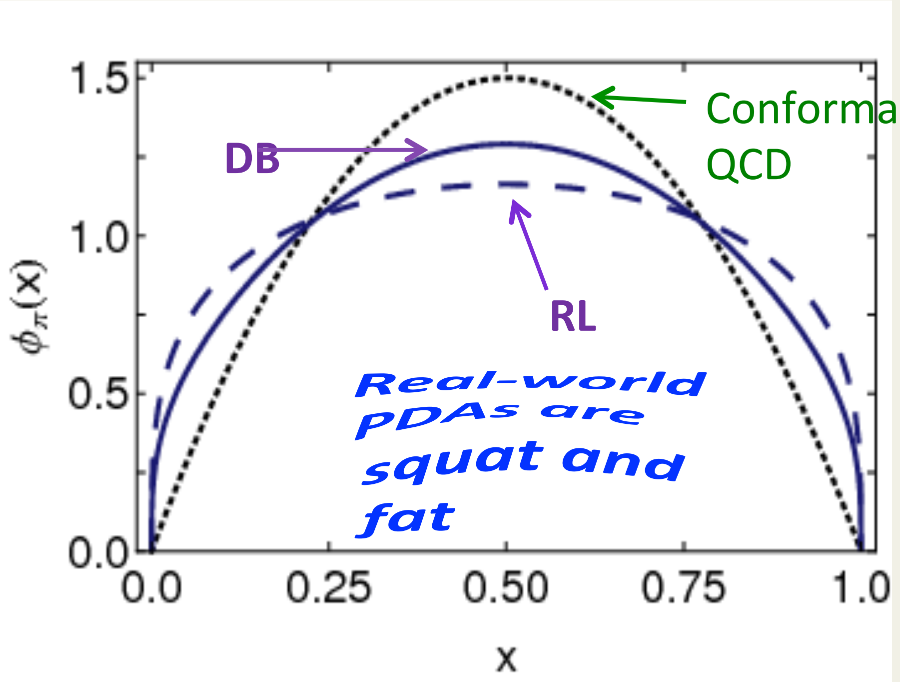
Valence quark picture

Definitive of a hadron – it's how we tell a proton from a neutron

Expresses charge; flavour; baryon number; and other Poincaré-invariant macroscopic quantum numbers

Parton physics involves time-dependent dynamics

Twist-2 and twist-4 PDAs

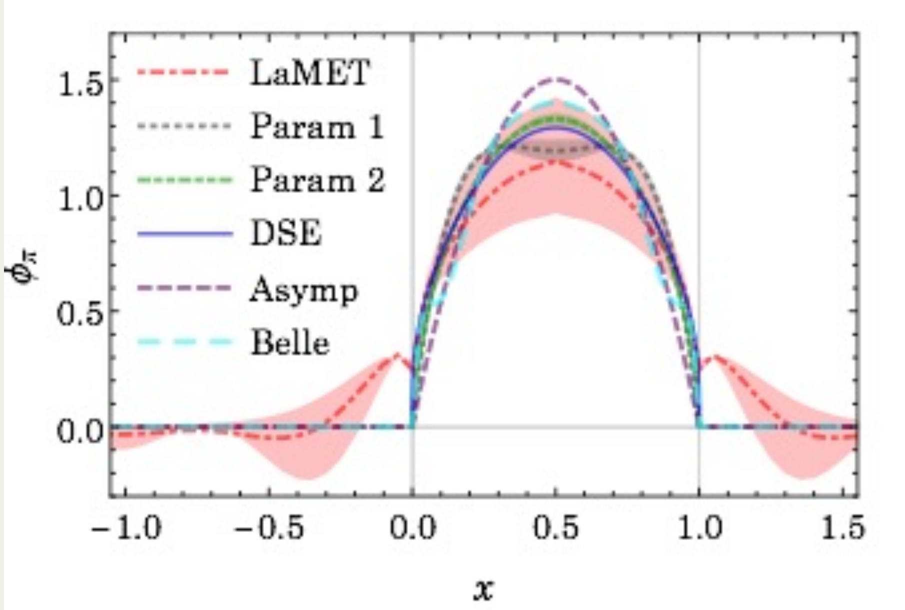


Real-world PDAs are squat and fat

- Continuum-QCD prediction: marked broadening of $\varphi_\pi(x)$, which owes to DCSB
- Scale evolution quite slow

Imaging dynamical chiral symmetry breaking: pion wave function on the light front,
Lei Chang, *et al.*,
Phys. Rev. Lett. 110 (2013) 132001

Quark quasi Distribution Amplitudes----lQCD Progress

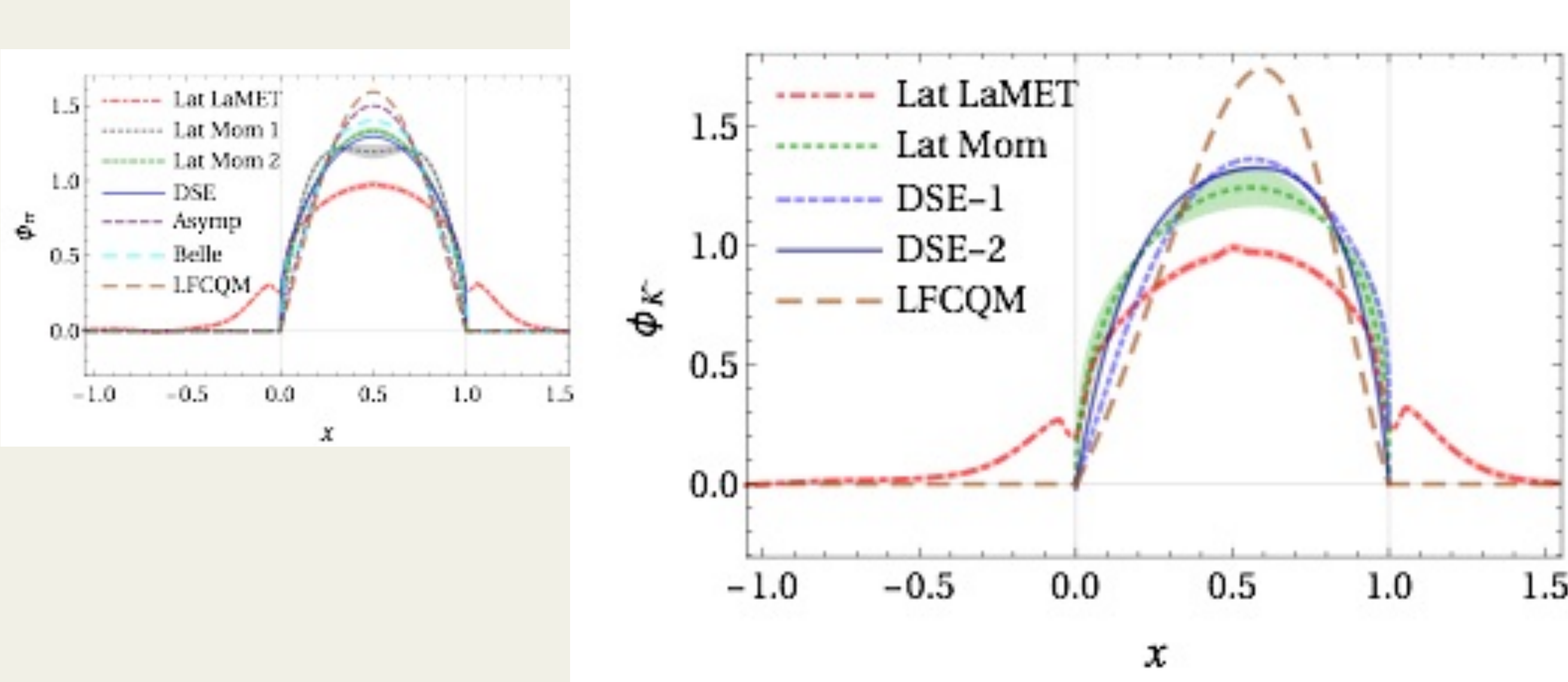


Pion Distribution Amplitude from Lattice QCD

Jian-Hui Zhang,[1,*] Jiunn-Wei Chen,[2,3,†] Xiangdong Ji,[4,5,‡] Luchang Jin,[6,§] and Huey-Wen Lin[7,8,¶]

DSE & lQCD predictions are practically indistinguishable;
Favor no-humped behavior

Quark quasi Distribution Amplitudes----lQCD Progress



Kaon Distribution Amplitude from Lattice QCD and the Flavor SU(3) Symmetry

J-W Chen, et al (LP^3), arXiv:1712.10025

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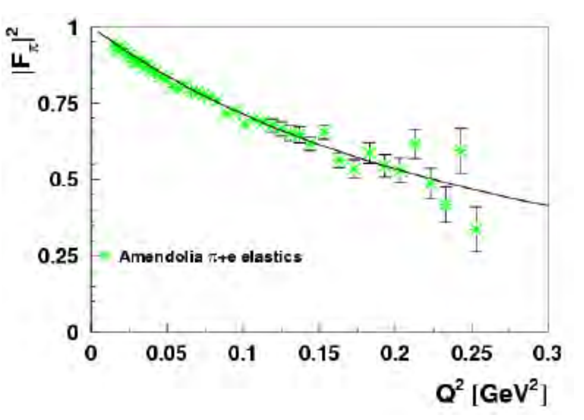
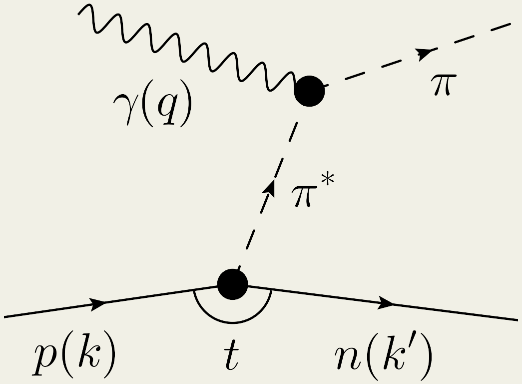
Measurement of the π⁺ Form Factor

- At low Q^2, Fπ can be measured directly via high energy elastic π⁺ scattering from the atomic electrons
  - CERN SPS used 300 GeV pions to measure form factor up to Q^2=0.25GeV^2
    (Amedolia et al, NPB277, 168 (1986))
  - These data used to constrain the pion charge radius: rπ=0.657±0.012 fm



- At larger Q^2, Fπ must be measured indirectly using the "pion cloud" of the proton in exclusive pion electroproduction: p(e, e' π⁺)n
  - at small −t, the pion pole process dominates the longitudinal cross section, σ_L
    (L. Favart, et al, Eur. Phys. J. A 52 (2016) 158)
  - In the Born term model, Fπ appears as

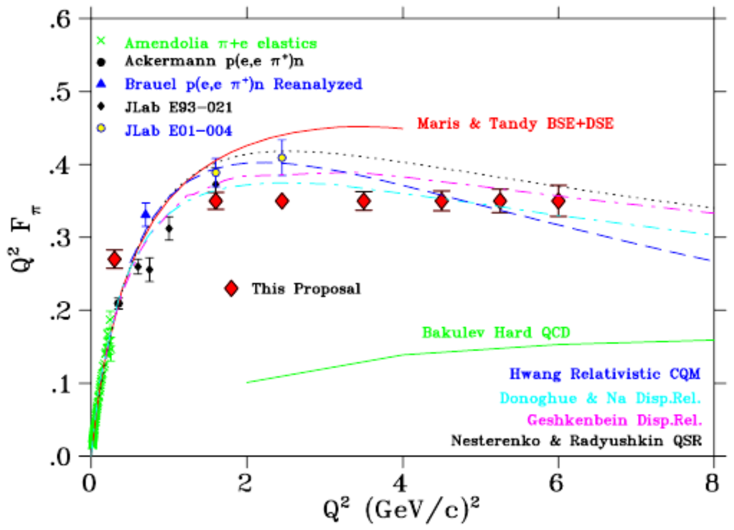
$$\frac{d\sigma_L}{dt} \propto \frac{-t}{(t-m_\pi^2)} g_{\pi NN}^2(t) Q^2 F_\pi^2(Q^2,t)$$



Sullivan process, in which a nucleon's pion cloud is used to provide access to the pion's elastic form factor

Experimental studies over the last decade have given confidence in the electroproduction method yielding the physical pion form factor----Tanja Horn

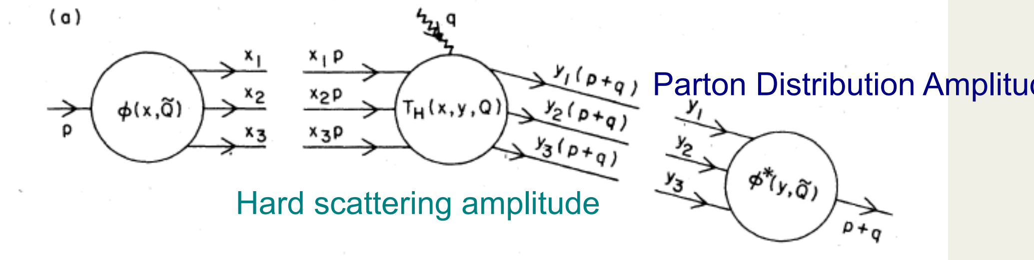
Measure pion elastic form factor in space-like region



Jefferson Lab PAC 30 Proposal

Measurement of the Charged Pion Form Factor to High Q^2

July 7, 2006



Parton Distribution Amplitu

Hard scattering amplitude

(G.R. Farrar and D.R.Jackson, PRL43 (1979) 246;
P. Lepage and S. Brodsky, PLB 87 (1979) 359)

$$Q^2F_\pi(Q^2) \overset{Q^2\gg\Lambda^2_{\rm QCD}}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_\pi(x)$$

Performing asymptotic valence-quark distribution amplitude 6x(1-x)

$Q^2F=0.15$ at $Q^2=4GeV^2$

A factor 2.7 smaller than the empirical value $0.41GeV^2$ quoted at $Q^2=2.45GeV^2$

A factor 3 smaller than the case of BSE

- What the exact perturbative behavior in QCD;
- The most interesting question then, as far as Jlab is able to address, is the description of form factor in the gap between the soft and hard regions;
- DSE prediction...Maris-Tandy(2000)
- What the valence quark structure in the pion with the present experimental scale?
- Connect the valence quark distribution and pion electromagnetic form factor.

Pion electromagnetic form factor at spacelike momenta

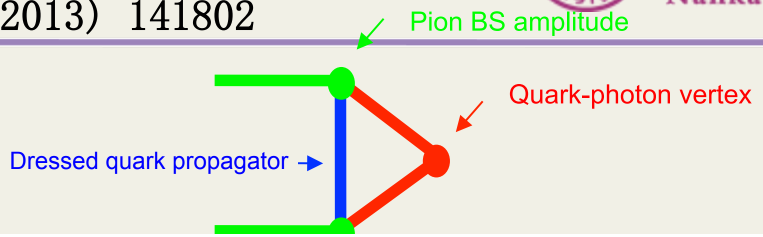
Lei Chang, *et al.*,

arXiv:1307.0026 [nucl-th], Phys. Rev. Lett. 111 (2013) 141802

- Direct, symmetry-preserving computation of pion form factor predicts maximum in $Q^2F_\pi(Q^2)$ at $Q^2=6\text{GeV}^2$
- The QCD prediction can be expressed as

$$Q^2F_\pi(Q^2) \overset{Q^2\gg\Lambda^2_{\rm QCD}}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\,\frac{1}{x}\,\varphi_\pi(x)$$

- PDA Broadening has enormous impact on understanding $F_\pi(Q^2)$
- Find consistency between the direct pion form factor calculation and the QCD hard-scattering formula – if DSE pion PDA is used…15%



Pion BS amplitude

Quark-photon vertex

Dressed quark propagator

$$K_\mu F_\pi(Q^2) = N_c tr_D \int_{dk}^{\Lambda} \chi_\mu(k+p_f,k+p_i)\Gamma_\pi(k+p_i/2;p_i)S(k)\Gamma_\pi(k+p_f/2;-p_f)$$

where Q is the incoming photon momentum and $p_{f,i} = K \pm Q/2$.

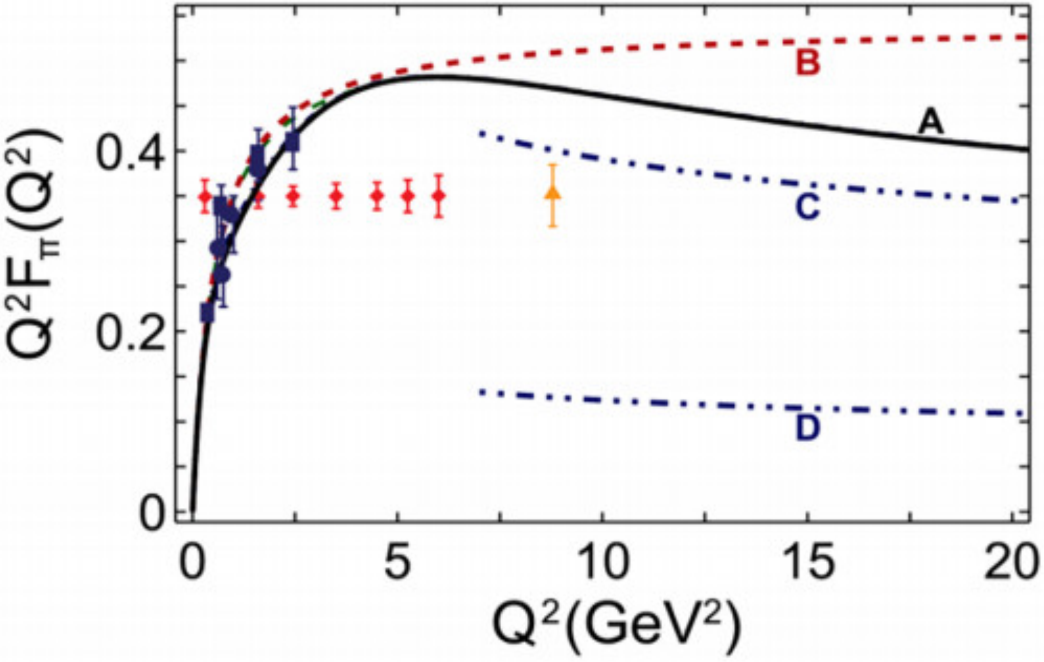
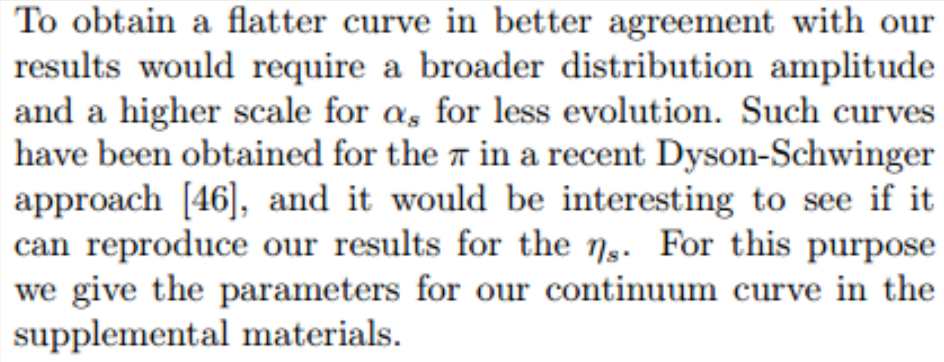


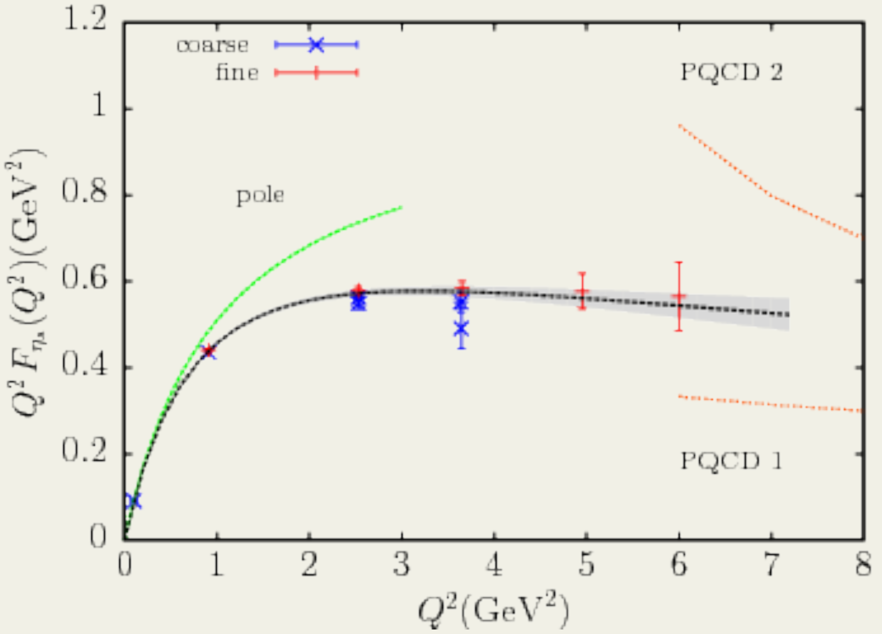
Figure 2.2: *Existing (dark blue) data and projected (red, orange) uncertainties for future data on the pion form factor. The solid curve (A) is the QCD-theory prediction bridging large and short distance scales. Curve B is set by the known long-distance scale—the pion radius. Curves C and D illustrate calculations based on a short-distance quark-gluon view.*

Pseudo-pion form factor
When Lattice meets DSEs

- J. Koponen *et al(HPQCD)*: arXiv:1701.04250 [hep-lat]



To obtain a flatter curve in better agreement with our results would require a broader distribution amplitude and a higher scale for α_s for less evolution. Such curves have been obtained for the π in a recent Dyson-Schwinger approach [46], and it would be interesting to see if it can reproduce our results for the η_s. For this purpose we give the parameters for our continuum curve in the supplemental materials.



Preliminary DSE result for *s*-massive pseudo-π

… Internally consistent calculation, producing *s*-massive PDA ∝ $[x(1-x)]^{0.8}$ to ^{1}

… Independent confirmation of reality and impact of dilated PDAs on meson form factors

	$m_{s\bar{s}}$	$f_{s\bar{s}}$
DSE 2018	0.69 GeV	0.133 GeV
Lattice 2017	0.6885(2)	0.1281(39)

Pseudo-pion form factor
When Lattice meets DSEs

- A. J. Chambers, *et al(QCDSF/UKQCD/CSSM Collaborations)*:

PRD 96, 114509(2017)

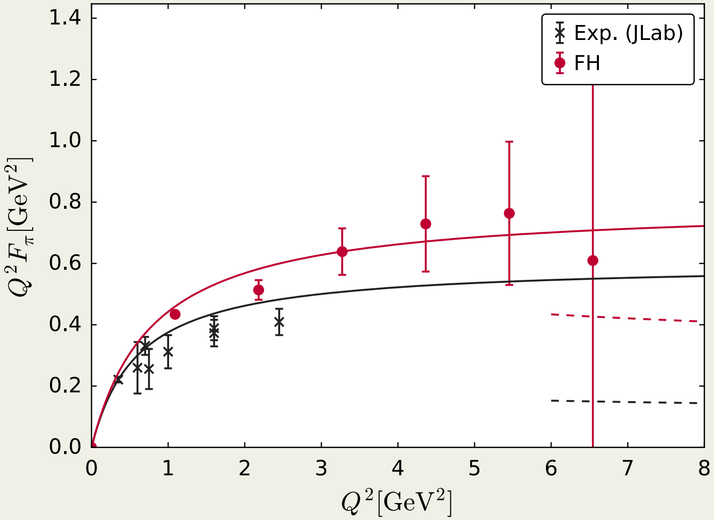
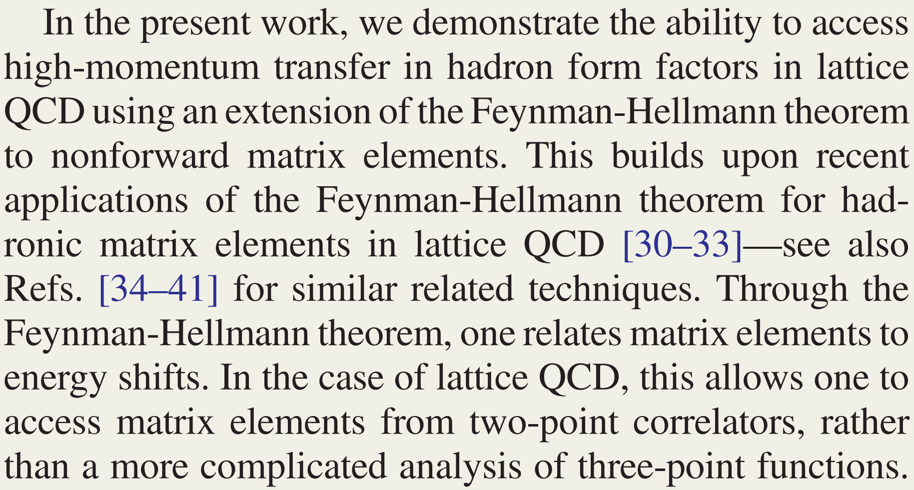


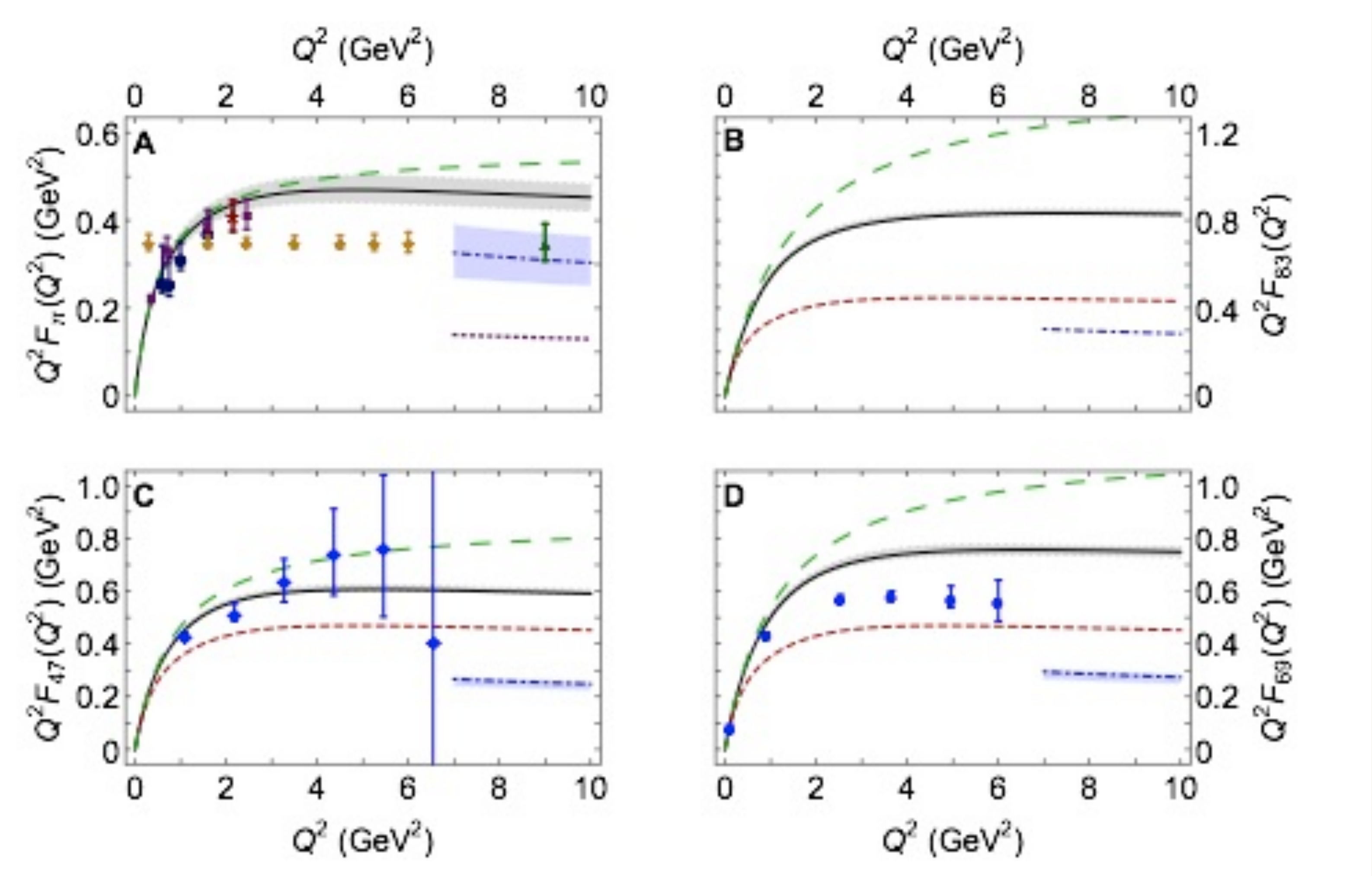
FIG. 5. Scaled pion form factor Q^2F_π from the Feynman-Hellmann technique and from experiment [16]. The solid lines are the vector meson dominance at the relevant pion masses, and the dotted lines are the asymptotic values predicted by perturbative QCD (see [17] for a discussion of this value and its limitations).



In the present work, we demonstrate the ability to access high-momentum transfer in hadron form factors in lattice QCD using an extension of the Feynman-Hellmann theorem to nonforward matrix elements. This builds upon recent applications of the Feynman-Hellmann theorem for hadronic matrix elements in lattice QCD [30–33]—see also Refs. [34–41] for similar related techniques. Through the Feynman-Hellmann theorem, one relates matrix elements to energy shifts. In the case of lattice QCD, this allows one to access matrix elements from two-point correlators, rather than a more complicated analysis of three-point functions.

	$m_{s\bar{s}}$	$f_{s\bar{s}}$
DSE	0.47 GeV	0.11 GeV
Lattice 2017	0.47	0.11(39)

Muyang Chen, *etal*, in preparation



Outline

- Hadron Physics and key problems
- One parameter framework
- PDAs
- Electromagnetic Form Factor
- Conclusion

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THANKS FOR YOUR ATTENTION

Emergent Mass *vs*. Higgs Mechanism

Parton distribution amplitudes of S-wave heavy-quarkonia
Minghui Ding, et al, Phys. Lett. B **753** (2016) pp. 330-335

- When does Higgs mechanism begin to influence mass generation?
- limit $m_{quark} \to \infty$

  $\varphi(x) \to \delta(x-½)$
- limit $m_{quark} \to 0$

  $\varphi(x) \sim (8/\pi)\,[x(1-x)]^{½}$
- Transition boundary lies just above $m_{strange}$
- Comparison between distributions of light-quarks and those involving strange-quarks is obvious place to find signals for strong-mass generation