Simplest Little Higgs Revisited: the $ZH\eta$ vertex

Shi-Ping He

Peking University

2018.8.17(Tianjin)

24th International Summer Institute on Phenomenology of Elementary Particle Physics and Cosmology (SI2018)

Contents

- SLH   introduction
- A possible mistake
- Our several works
- Outlook and conclusions

1. SLH introduction

1.1 Higgs mass naturalness problem

C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016).

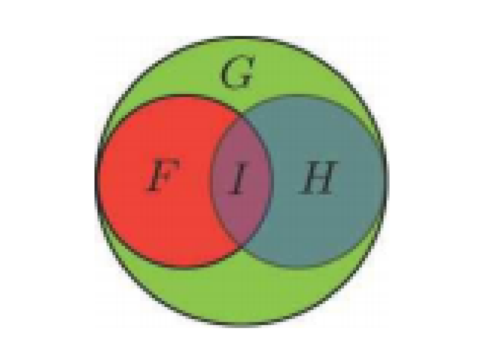
$$m^2(Q) = m^2(\mu) + \delta m^2,$$

$$\delta m^2 = \sum_i g_i (-1)^{2S_i} \frac{\lambda_i^2 m_i^2}{32\pi^2} \log(\frac{Q^2}{\mu^2}),$$

Solutions

- Supersymmetry
- Composite Higgs model
- Higgsless model: technicolor
- Extra dimension
- Relaxation, Clockwork, Higgsplosion, Hyperbolic Higgs…

Higgs as a pseudo-Nambu-Goldstone boson



$$G \rightarrow H$$

Hsin-Chia Cheng, *Karlsruhe 2007, SUSY 2007* 114-121

Little Higgs theories: collective symmetry breaking → 10 TeV

Littlest Higgs Model
Minimal Moose Model
Simplest Little Higgs (SLH)

1.2 Model set up

Gauge group

$$SU_C(3) \otimes SU_L(3) \otimes U_X(1)$$

$\Downarrow$ f

$$SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$$

$\Downarrow$ v

$$SU_C(3) \otimes U_Q(1)$$

- Field representations (anomaly free)

$$\Phi_1, \Phi_2 = (1,3,-\frac{1}{3})$$

10 Goldstones

$$\Phi_1 = e^{i\frac{\Theta'}{f}} e^{i\frac{t_\beta \Theta}{f}} \begin{pmatrix} 0 \\ 0 \\ f c_\beta \end{pmatrix}, \quad \Phi_2 = e^{i\frac{\Theta'}{f}} e^{-i\frac{\Theta}{f t_\beta}} \begin{pmatrix} 0 \\ 0 \\ f s_\beta \end{pmatrix}$$

$$\Theta = \frac{\eta}{\sqrt{2}} + \begin{pmatrix} 0_{2\times 2} & h \\ h^+ & 0 \end{pmatrix}, \quad h = \begin{pmatrix} \frac{v + H - i\chi}{\sqrt{2}} \\ h^- \end{pmatrix}$$

$$\Theta' = \frac{\zeta}{\sqrt{2}} + \begin{pmatrix} 0_{2\times 2} & k \\ k^+ & 0 \end{pmatrix}, \quad k = \begin{pmatrix} \frac{\sigma - i\omega}{\sqrt{2}} \\ k^- \end{pmatrix}$$

Fermion	$Q_{1,2}$	Q_3	u_{Rm}, T_{Rm}	d_{Rm}, D_{Rm}, S_{Rm}	L_m	N_{Rm}	e_{Rm}
Q_x charge	0	1/3	2/3	$-1/3$	$-1/3$	0	-1
$SU(3)$ rep.	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$

$$Q_1 = (d_L, -u_L, iD_L)^T, d_R, u_R, D_R$$

$$Q_2 = (s_L, -c_L, iS_L)^T, s_R, c_R, S_R$$

$$Q_3 = (t_L, b_L, iT_L)^T, t_R, b_R, T_R$$

$$L_m = (\nu_L, l_L, iN_L)_m^T, l_{Rm}, N_{Rm} (m = 1,2,3)$$

The Lagrangian

$$L_{Higgs} = (D_\mu \Phi_1)^+ (D^\mu \Phi_1) + (D_\mu \Phi_2)^+ (D^\mu \Phi_2), D_\mu = \partial_\mu - igA_\mu{}^a T^a + ig_x Q_x B_\mu{}^x, g_x = \frac{gt_W}{\sqrt{1 - t_W{}^2 / 3}}$$

$$L_{potential} = -\mu^2 (\Phi_1{}^+ \Phi_2 + \Phi_2{}^+ \Phi_1) + \lambda (\Phi_1{}^+ \Phi_2)(\Phi_2{}^+ \Phi_1)$$

$$L_{Yukawa}^{lepton} = i\lambda_N{}^m \overline{N}_{Rm} \Phi_2{}^+ L_m + i \frac{\lambda_l{}^{mn}}{\Lambda} \bar{l}_{Rm} \varepsilon_{ijk} \Phi_1{}^i \Phi_2{}^j L_n{}^k + h.c.$$

$$L_{Yukawa}^{quark} = i\lambda_1{}^t \overline{u}_{R3}^1 \Phi_1{}^+ Q_3 + i\lambda_2{}^t \overline{u}_{R3}^2 \Phi_2{}^+ Q_3 + i \frac{\lambda_b{}^{mn}}{\Lambda} \overline{d}_{Rm} \varepsilon_{ijk} \Phi_1{}^i \Phi_2{}^j Q_3{}^k$$

$$+ i\lambda_1{}^{dn} \overline{d}_{Rn}^1 Q_n{}^T \Phi_1 + i\lambda_2{}^{dn} \overline{d}_{Rn}^2 Q_n{}^T \Phi_2 + i \frac{\lambda_u{}^{mn}}{\Lambda} \overline{u}_{Rm} \varepsilon_{ijk} \Phi_1{}^{*,i} \Phi_2{}^{*,j} Q_n{}^k + h.c.$$

$$L_{matter} = \overline{L}_m i\gamma^\mu D_\mu L_m + \bar{l}_{Rm} i\gamma^\mu D_\mu l_{Rm} + \sum_{i=1,2,3} \overline{Q}_i i\gamma^\mu D_\mu Q_i + \sum_{q=u,c,t,T,d,D,s,S,b} \overline{q}_R i\gamma^\mu D_\mu q_R$$

$$L_{gauge} = -\frac{1}{4} B_{x,\mu\nu} B_x{}^{\mu\nu} - \frac{1}{4} A_{\mu\nu}{}^a A^{a,\mu\nu}, A_{\mu\nu}{}^a = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + gf^{abc} A_\mu{}^b A_\nu{}^c$$

1.3 Previous Researches

The Little Higgs from a simple group
David E. Kaplan (Johns Hopkins U.), Martin Schmaltz (Boston U.). Feb 2003. 31 pp.
Published in **JHEP 0310 (2003) 039**
BUHEP-03-03
DOI: 10.1088/1126-6708/2003/10/039
e-Print: **hep-ph/0302049** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
详细记录 - Cited by 323 records 250+

W. Kilian, D. Rainwater, J. Reuter, Phys. Rev. D71 (2005) 015008.

Kingman Cheung, Jeonghyeon Song, Phys. Rev. D76 (2007) 035007.

Kingman Cheung, Jeonghyeon Song, Poyan Tseng, Qi-Shu Yan, Phys. Rev. D78 (2008) 055015.

Ran Lu, Qing Wang, Chin. Phys. Lett. 24 (2007) 3371-3373.

Otto C. W. Kong, Phys. Rev. D70 (2004) 075021.

J. A. Casas, J. R. Espinosa, I. Hidalgo, JHEP 0503 (2005) 038.

W. Kilian, D. Rainwater, J. Reuter, Phys. Rev. D74 (2006) 095003.

B. Grinstein, R. Kelley, P. Uttayarat, JHEP 0909 (2009) 040.

The Simplest little Higgs
Martin Schmaltz (Boston U.). Jul 2004. 19 pp.
Published in **JHEP 0408 (2004) 056**
BUHEP-04-09
DOI: 10.1088/1126-6708/2004/08/056
e-Print: **hep-ph/0407143** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
详细记录 - Cited by 231 records 100+

F. del. Aguila, J. I. Illana, M. D. Jenkins, JHEP 1103 (2011) 080.

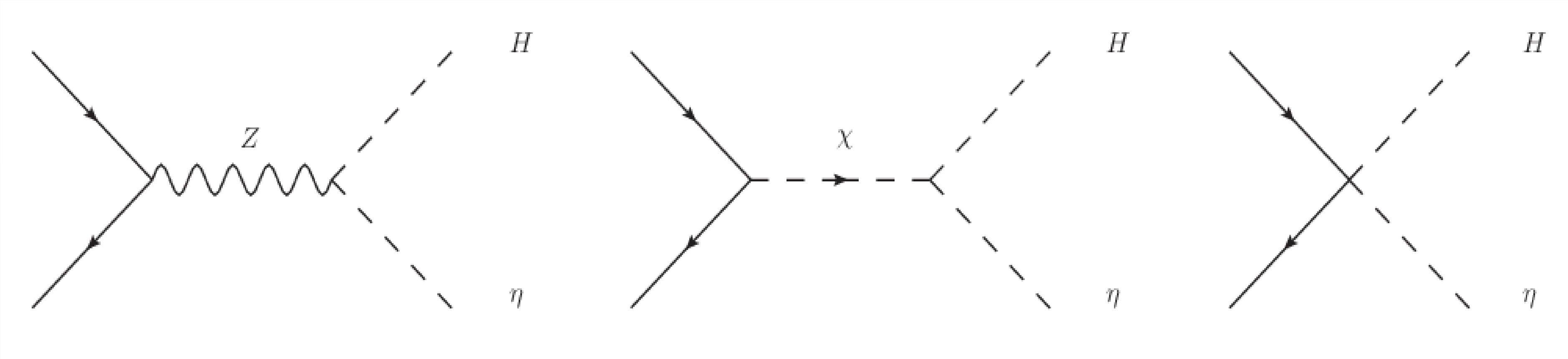
Tao Han, H. E. Logan, Lian-Tao wang, JHEP 0601 (2006) 099.

2. A possible mistake

W. Kilian, D. Rainwater, J. Reuter,
Phys. Rev. D71 (2005) 015008.

$$\mathcal{L}_{ZH\eta} = \frac{m_Z}{\sqrt{2}F} N_2 Z_\mu (\eta \partial^\mu H - H \partial^\mu \eta)$$

W. Kilian, D. Rainwater, J. Reuter,
Phys. Rev. D74 (2006) 095003.



However, Ying-nan Mao finds gauge invariance of the amplitude is not guaranteed.

There must be something wrong! Where is it?

Goldstone identification

- Kinetic mixing terms

$\partial^{\mu}\eta\partial_{\mu}\chi$ ⇒ Diagonalization and normalization

- VS two-point functions

$Z_{\mu}\partial^{\mu}\eta$ ⇒ Gauge fixing

} The correct way

Any step is lost ✗ ⇒ Paradoxes or inconsistencies

Another example

Qing-Hong Cao, Gang Li, Ke-Pan Xie, Jue Zhang, Phys. Rev. D97 (2018) 115036.

SM plus a singlet charged scalar S with representation of (1,1,-1)

Dim-5 bosonic poerators

$$(D_\mu \tilde{h})^\dagger (D^\mu h) S$$

Vanishes because of anti-symmetric contractions

$$\tilde{h}^\dagger (D^\mu h)(D_\mu S)$$

Logic one

$$S^- \to W^- Z(\gamma), W^- H$$

Canonically normalization
& gauge fixing

Logic two

$$0 = (D_\mu \tilde{h})^\dagger (D^\mu h) S$$

$$= \partial_\mu (\tilde{h}^\dagger D^\mu h S) - \tilde{h}^\dagger (D^\mu h)(D_\mu S) - \tilde{h}^\dagger (D^\mu D_\mu h) S$$

Consistent!

Bosonic operators can be eliminated through EOM and surface terms. Thus, S can only decay into fermions.

3. Our works

Shi-Ping He, Ying-nan Mao, Chen Zhang, Shou-hua Zhu, Phys. Rev. D97 (2018) 075005.
Shi-Ping He, Ying-nan Mao, Chen Zhang, Shou-hua Zhu, arXiv:1804.11333.

→ $ZH\eta$ vertex

Kingman Cheung, Shi-Ping He, Ying-nan Mao, Chen Zhang, Yang Zhou, Phys. Rev. D97 (2018) 115001.

→ Scalar potential analysis

Naturalness discussion

3.1 Goldstone identification

$$\begin{pmatrix} \eta \\ \varsigma \\ \chi \\ \omega \end{pmatrix} \Rightarrow \begin{pmatrix} \dfrac{1}{\cos(\gamma+\delta)} \\ -\dfrac{\sin^2\delta\ t_\beta - \sin^2\gamma / t_\beta}{\cos(\gamma+\delta)} \\ \dfrac{v[\cos 2\delta\ t_\beta - \cos 2\gamma / t_\beta]}{\sqrt{2} f \cos(\gamma+\delta)} \\ \dfrac{\sin 2\delta\ t_\beta + \sin 2\gamma / t_\beta}{2\cos(\gamma+\delta)} \end{pmatrix} \eta \quad (\gamma = \frac{v t_\beta}{\sqrt{2} f}, \delta = \frac{v}{\sqrt{2} f t_\beta})$$

3.2 the $ZH\eta$ vertex

$$L \supset c^{as}_{ZH\eta} Z^{\mu}(\eta\partial_{\mu}H - H\partial_{\mu}\eta) + c^{s}_{ZH\eta} Z^{\mu}(\eta\partial_{\mu}H + H\partial_{\mu}\eta)$$

$$c^{as}_{ZH\eta} = -\frac{g}{4\sqrt{2}c_W^3 t_{2\beta}}\xi^3, \xi = \frac{v}{f}$$

$$c^{s}_{ZH\eta} = \frac{g}{\sqrt{2}c_W t_{2\beta}}\xi + \frac{g}{24\sqrt{2}c_W s_{2\beta}}[\frac{8}{s_{2\beta}t_{2\beta}} + 3c_{2\beta}(8 + \frac{6}{c_W^2} - \frac{1}{c_W^4})]\xi^3$$

$$Z^{\mu}(\eta\partial_{\mu}H - H\partial_{\mu}\eta)$$

Anti-symmetric type

$$Z^{\mu}(\eta\partial_{\mu}H + H\partial_{\mu}\eta)$$

symmetric type

→ No physical meaning !

3.3 Effective field theory analysis

Up to Dim 6 CP conserved Gauge invariant Vertex related

$$O_1 = i(\partial_\mu \eta) h^+ \overleftrightarrow{D}_\mu h, O_2 = (h^+ D_\mu h)(h^+ D^\mu h)$$

$$L \supset L_{SM} + \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{c_1}{f} O_1 + \frac{c_2}{f^2} O_2$$

$$\Rightarrow L \supset \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \frac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) + \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta)$$

$$+ c_1 \xi (\partial_\mu \chi)(\partial^\mu \eta) - m_z Z_\mu \partial^\mu (\chi + c_1 \xi \eta)$$

$$+ \frac{g}{2c_W} Z_\mu (\chi \partial^\mu H - H \partial^\mu \chi) - \frac{g}{c_W} c_1 \xi H Z_\mu \partial^\mu \eta$$

⇓

$$L \supset - \frac{g}{2c_W} c_1 \xi Z_\mu (\eta \partial^\mu H + H \partial^\mu \eta)$$

3.4 Influence on the phenomenology

$$v = 246GeV,\ f \geq 8TeV,\ \frac{v}{f} \leq 3.1\%$$

arXiv:180?.?????, in preparation.

Coupling order need to be reconsidered.

η production and decay

Kingman Cheung, Shi-Ping He, Ying-nan Mao,
Chen Zhang, Yang Zhou,
Phys. Rev. D97 (2018) 115001.

$$m_\eta \sim (450,750)GeV$$

Production channels

$$gg \to \eta$$
$$pp \to T\overline{T} \to t\eta / \bar{t}\eta + X$$
$$pp \to Tj + \overline{T}j \to t\eta j + \bar{t}\eta j$$
$$pp \to t\bar{t}\eta$$

$pp \to Z' \to H\eta / \eta Y$ v^2/f^2 suppressed.

Decay channels (kinematically allowable)

$$\eta \to tt, gg, \gamma\gamma$$

$\eta \to ZH$ v^6/f^6 suppressed.

4. Outlook and conclusions

- Point out a mistake in the previous studies

- Get the self-consistent anti-symmetric type $ZH\eta$ vertex

- We will perform a detailed η phenomenology

Thanks!

Backup 1

- Parametrization of the gauge bosons

$$A_\mu^a T^a = \frac{A_\mu^3}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix} + \frac{A_\mu^8}{2\sqrt{3}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0 & W_\mu^+ & Y_\mu^0\\ W_\mu^- & 0 & X_\mu^-\\ \bar{Y}_\mu^0 & X_\mu^+ & 0\end{pmatrix}$$

$$T^1 = \frac{1}{2}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, T^2 = \frac{1}{2}\begin{pmatrix}0 & -i & 0\\ i & 0 & 0\\ 0 & 0 & 0\end{pmatrix}, T^3 = \frac{1}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}, T^4 = \frac{1}{2}\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 0\end{pmatrix}$$

$$T^5 = \frac{1}{2}\begin{pmatrix}0 & 0 & -i\\ 0 & 0 & 0\\ i & 0 & 0\end{pmatrix}, T^6 = \frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}, T^7 = \frac{1}{2}\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & -i\\ 0 & i & 0\end{pmatrix}, T^8 = \frac{1}{2\sqrt{3}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix}$$

Rotation of gauge boson fields at $O\left(\frac{v^2}{f^2}\right)$

$$
\begin{pmatrix} A^3 \\ A^8 \\ B^x \end{pmatrix} = \begin{pmatrix} \sqrt{3-t_w^2}(1-t_w^2)\frac{v^2}{8f^2} & c_w & -s_w \\ \sqrt{1-\frac{t_w^2}{3}}[1+t_w^2(1-t_w^2)\frac{v^2}{8f^2}] & \frac{s_w t_w}{\sqrt{3}}[1-\frac{(1-t_w^2)(3-t_w^2)}{s_w^2}\frac{v^2}{8f^2}] & \frac{s_w}{\sqrt{3}} \\ -\frac{t_w}{\sqrt{3}}[1-(1-t_w^2)(3-t_w^2)\frac{v^2}{8f^2}] & s_w\sqrt{1-\frac{t_w^2}{3}}[1+\frac{1-t_w^2}{c_w^2}\frac{v^2}{8f^2}] & c_w\sqrt{1-\frac{t_w^2}{3}} \end{pmatrix} \begin{pmatrix} Z_0' \\ Z_0 \\ A \end{pmatrix}
$$

$$
\begin{pmatrix} Z_0' \\ Z_0 \end{pmatrix} = \begin{pmatrix} 1 & \delta_z \\ -\delta_z & 1 \end{pmatrix} \begin{pmatrix} Z' \\ Z \end{pmatrix} (\delta_z = -\frac{(1-t_w^2)\sqrt{3-t_w^2}}{c_w}\frac{v^2}{8f^2})
$$

Backup 2

CP odd scalar kinetic terms of $\eta, \varsigma, \chi, \omega$

$$\begin{pmatrix} 1 & 0 & \frac{\sqrt{2}}{t_{2\beta}}\xi - \frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3}\xi^3 & -\sqrt{2}\xi + \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 \\ 0 & 1 & -\frac{1}{\sqrt{2}}\xi + \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2}\xi^3 & -\frac{2\sqrt{2}}{3t_{2\beta}}\xi^3 \\ \frac{\sqrt{2}}{t_{2\beta}}\xi - \frac{7c_{2\beta}+c_{6\beta}}{6\sqrt{2}s_{2\beta}^3}\xi^3 & -\frac{1}{\sqrt{2}}\xi + \frac{5+3c_{4\beta}}{12\sqrt{2}s_{2\beta}^2}\xi^3 & 1-\frac{5+3c_{4\beta}}{12s_{2\beta}^2}\xi^2 & \frac{2}{3t_{2\beta}}\xi^2 \\ -\sqrt{2}\xi + \frac{5+3c_{4\beta}}{3\sqrt{2}s_{2\beta}^2}\xi^3 & -\frac{2\sqrt{2}}{3t_{2\beta}}\xi^3 & \frac{2}{3t_{2\beta}}\xi^2 & 1 \end{pmatrix}$$

VS two-point functions $F_{VS} V^{\mu} \partial_{\mu} S$

$$\begin{pmatrix} F_{Z\eta} & F_{Z\varsigma} & F_{Z\chi} & F_{Z\omega} \\ F_{Z'\eta} & F_{Z'\varsigma} & F_{Z'\chi} & F_{Z'\omega} \\ F_{Y\eta} & F_{Y\varsigma} & F_{Y\chi} & F_{Y\omega} \end{pmatrix} (\rho = \sqrt{\frac{1+2c_{2W}}{1+c_{2W}}}, \kappa = \frac{c_{2W}}{2c_W^2\sqrt{3-t_W^2}})$$

$$= gf \begin{pmatrix} \frac{1}{\sqrt{2}c_W t_{2\beta}}\xi^2 & -\frac{1}{2\sqrt{2}c_W}\xi^2 & \frac{1}{2c_W}\xi - \frac{5+3c_{4\beta}}{24c_W s_{2\beta}^2}\xi^3 & \frac{1}{3c_W t_{2\beta}}\xi^3 \\ \frac{\rho}{t_{2\beta}}\xi^2 & \frac{\sqrt{2}}{\sqrt{3-t_W^2}} - \frac{1+2c_{2W}}{2\sqrt{2}c_W^2\sqrt{3-t_W^2}}\xi^2 & \kappa\xi - \frac{\kappa(5+3c_{4\beta})}{12s_{2\beta}^2}\xi^3 & -\frac{1}{3c_W^2\sqrt{3-t_W^2}\,t_{2\beta}}\xi^3 \\ -\xi + \frac{5+3c_{4\beta}}{6s_{2\beta}^2}\xi^3 & -\frac{2}{3t_{2\beta}}\xi^3 & \frac{\sqrt{2}}{3t_{2\beta}}\xi^2 & \frac{1}{\sqrt{2}} \end{pmatrix}$$