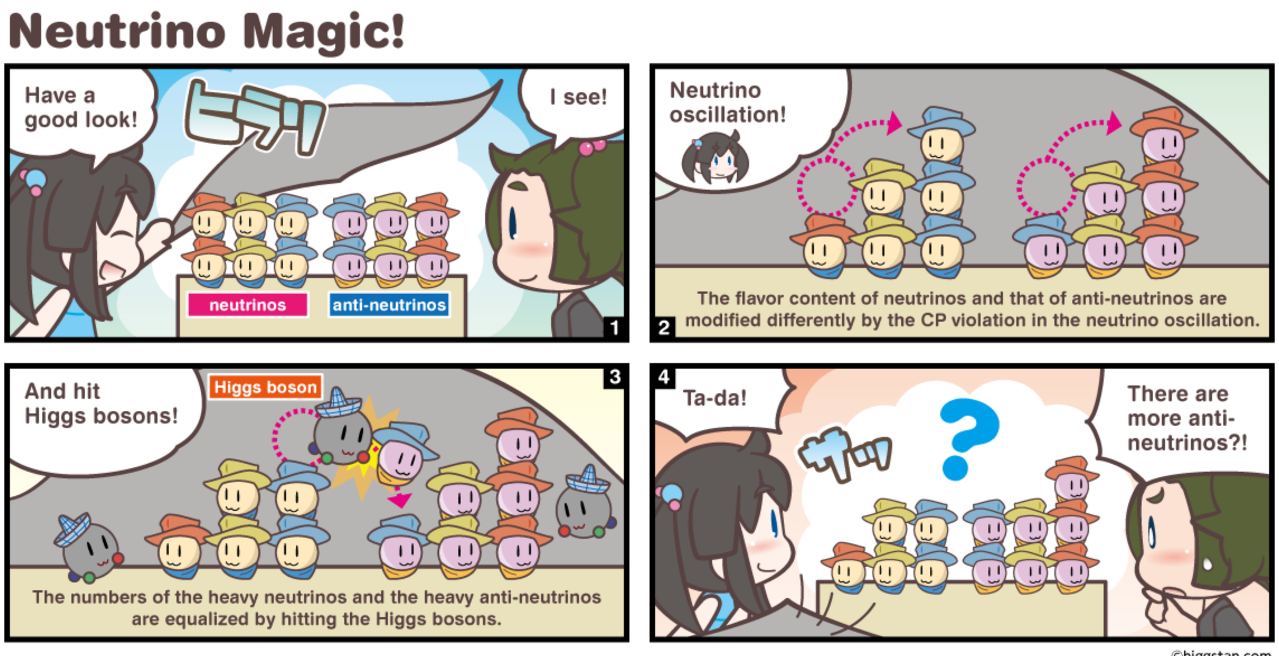
1807.06582 with Ryuichiro Kitano, and Yuta Hamada

Leptogenesis from "Observation"

Wen Yin, IHEP → KAIST from Sep



More anti-neutrinos than neutrinos?
Starting with the same numbers of neutrinos and anti-neutrinos, some magic under the cloth created an imbalance between them. This CP violating phenomenon, if it has really happened in the early Universe, give the reason for the Universe being made of matter rather than anti-matter.

comic by Yuki Yamamoto, higgstan.com

@SI2018, 14, Aug, Tianjin, China

Contents

1. Introduction

How to generate the baryon asymmetry?

Sakharov's conditions

**Baryon/Lepton number violation*

**C and CP violation*

**Out of thermal equilibrium*

Unfortunately, the SM does not sufficiently satisfy…

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A clear New Physics! Neutirno mass

The SM (probably) is an effective theory with Majorana neutrino mass term

$$-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH + \text{h.c.}$$

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Strong phase in CP violation

$$|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2 \neq 0$$

CP conjugate

we get

$$\overline{\mathcal{M}}(\delta_{CP}) = \mathcal{M}(-\delta_{CP}) \neq \mathcal{M}(\delta_{CP})^*$$

There is a ***<u>Strong Phase</u>***, which does not change under CP transf.

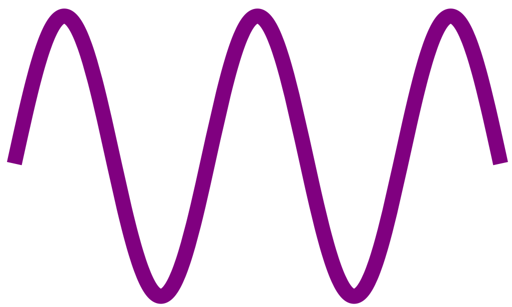
e.g. e^{imt} Quantum oscillation

Neutrino Oscillation Provides CP violation.

(If PMNS matrix has CP phase)

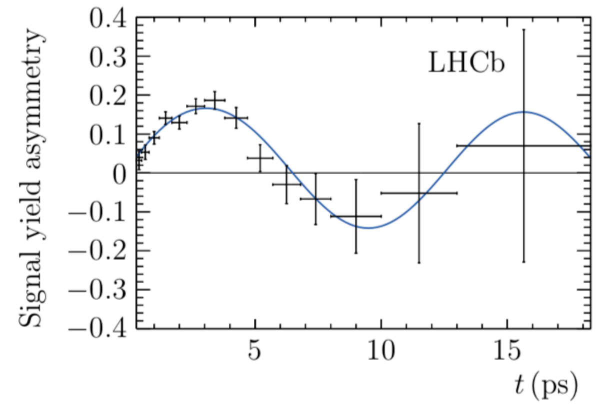
c.f. B-meson oscillation 1503.07089

$\nu_i - \bar{\nu}_i$



strong phase

$$\frac{n_B - n_{\bar{B}}}{n_B + n_{\bar{B}}}$$



In fact, the CP violation from neutrino oscillation is ***observed*** at >90% CL

T2K Collaboration, 1701.00432

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Neutrino oscillation can provide CP violation

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**C and CP violation* ✓

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**Out of thermal equilibrium* When ? (temperature should be high.)

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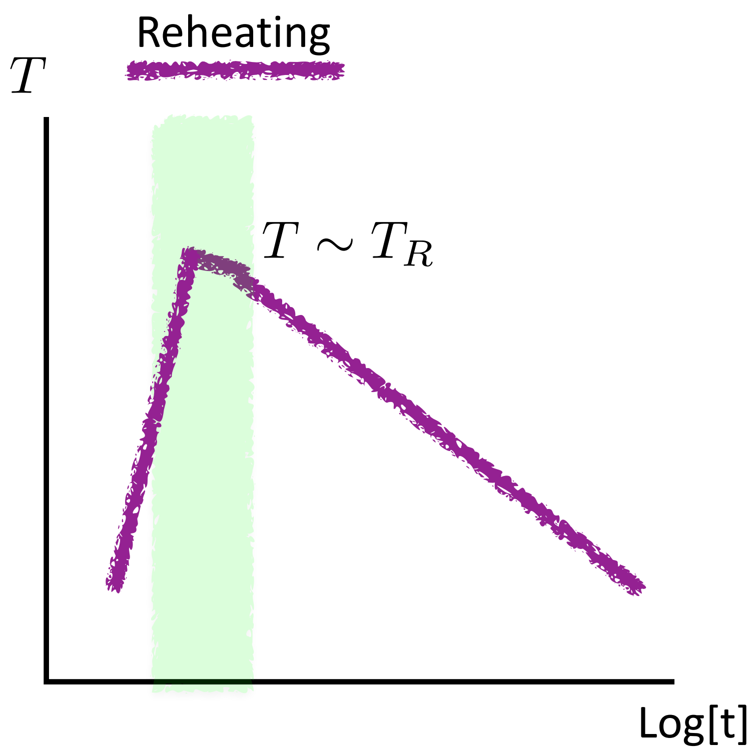
$$-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH + \mathrm{h.c.}$$

Thermalization: one way process

At high temperature, some particles are generally out of equilibrium

CASE A



$$T \sim T_R$$

Particles from inflaton decays are being thermalized

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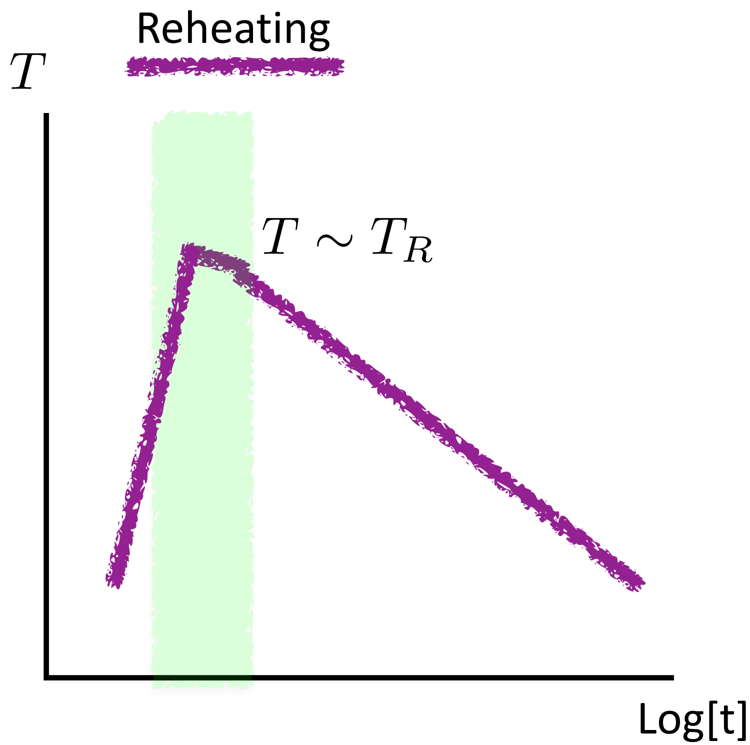

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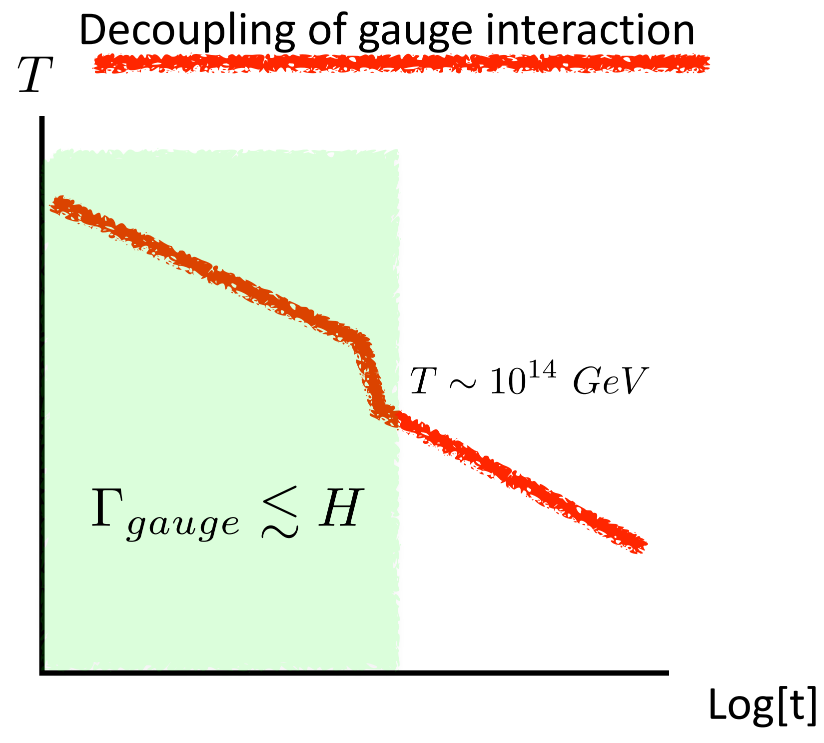
CASE B

$$\Gamma_{gauge} \propto T \quad H \propto T^2$$

$$T_R \gg T \gtrsim 10^{14}\ GeV$$

Particles lack of “force” are being thermalized.



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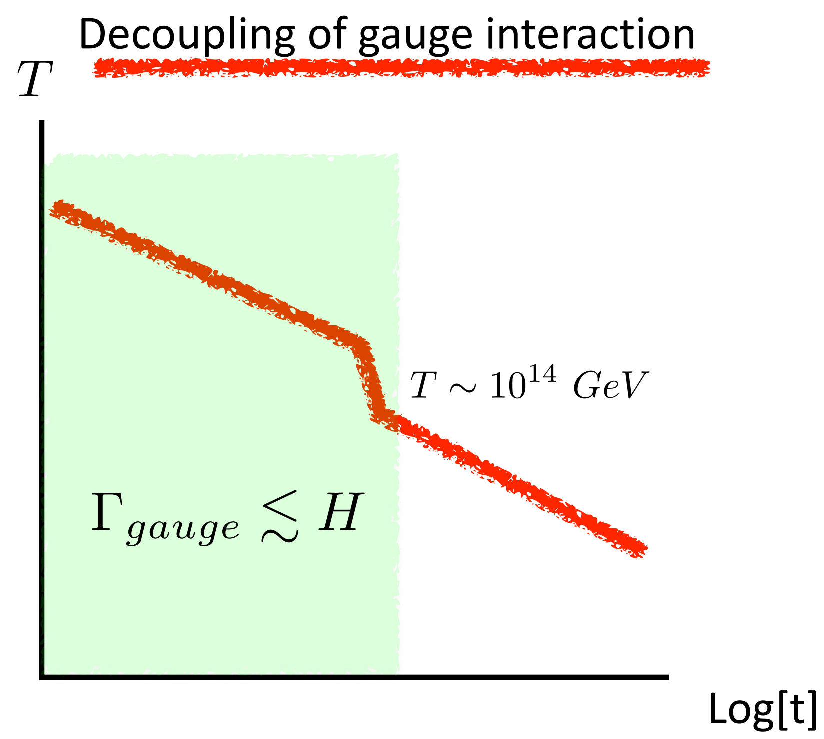


**Out of thermal equilibrium* ✓

Setup

Kitano, Hamada, WY 1807.06582

*Baryon/Lepton number violation $-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH + \mathrm{h.c.}$

*C and CP violation Neutrino oscillation

*Out of thermal equilibrium Thermalization

What we found

Kitano, Hamada, WY   1807.06582

Enough baryon asymmetry created during thermalization from neutrino osc. within SM with $-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH$

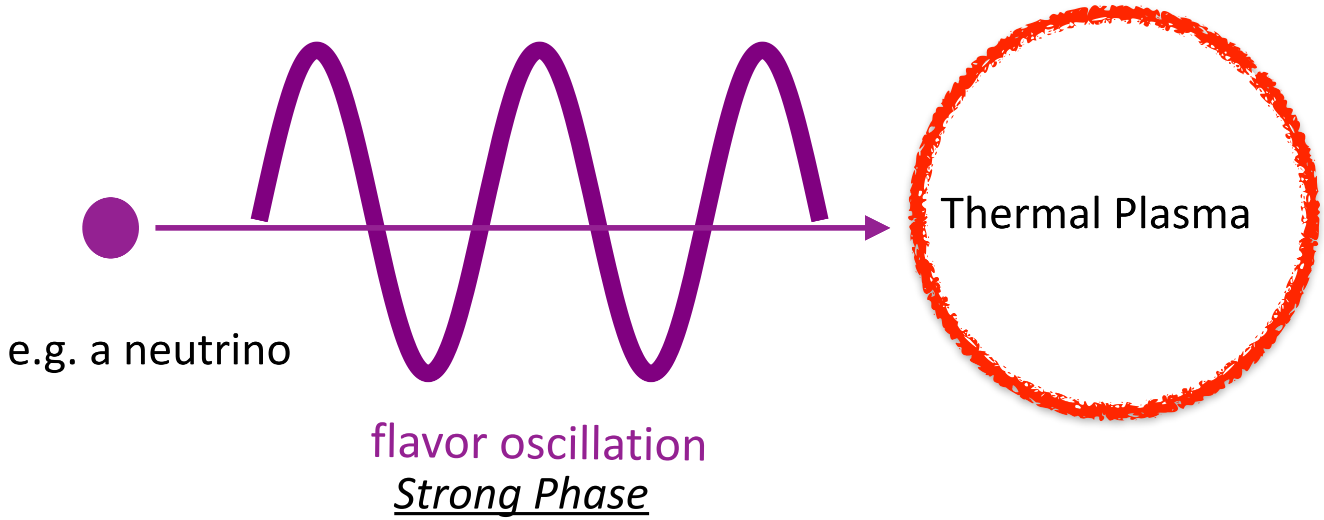
- Baryogenesis could be possible in any UV models lead to the effective SM.

e.g. from integrating out heavy particles
Yanagida 79; Gell-Mann, et al 79; Minkowski, 77;
Glashow, 80; Mohapatra and Senjanovic, 80;
WY, 1808.00440 as a consequence of charge quantization.

- Testable in neutrino exps.

2. Quantum Mechanics in Thermalization

Thermalization: isolated particles get scattered into thermal plasma.



Flavor oscillation of leptons from the inflaton decay explains the lepton asymmetry in a different model.

Kitano, Hamada, 1609.05028

Kinetic Equation (Extended Boltzmann Eqs)

Kitano, Hamada, WY 1807.06582

density matrix for left-handed leptons $\rho(\mathbf{p}) \equiv \rho_{ij}(\mathbf{p})$ $i, j = e, \mu, \tau$

Sigl, Raffelt, 1993

$$i\frac{d\rho(\mathbf{p})}{dt} = [\Omega(\mathbf{p}), \rho(\mathbf{p})] - \frac{i}{2}\{\Gamma^d_{\mathbf{p}}, \rho(\mathbf{p})\} + \frac{i}{2}\{\Gamma^p_{\mathbf{p}}, 1 - \rho(\mathbf{p})\},$$

Interaction terms (with CP phase)

$$\left(i\frac{d\overline{\rho}(\mathbf{p})}{dt} = - [\Omega(\mathbf{p}), \bar{\rho}(\mathbf{p})] - \frac{i}{2}\{\Gamma^d_{\mathbf{p}}, \overline{\rho}(\mathbf{p})\} + \frac{i}{2}\{\Gamma^p_{\mathbf{p}}, 1 - \overline{\rho}(\mathbf{p})\}, \right)^*$$

Hamiltonian: $$\Omega_{ij}(\mathbf{p}) \simeq \frac{y_i^2 T^2}{16|\mathbf{p}|}\delta_{ij} + 0.046(\kappa^*\kappa)_{ij}\frac{T^4}{|\mathbf{p}|}, \quad \text{for } |\mathbf{p}| \gtrsim T.$$

This is absent in ordinary Boltzmann eqs.

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Different   Interaction terms (with CP phase)

$$\left(i\frac{d\bar{\rho}(\mathbf{p})}{dt} = -[\Omega(\mathbf{p}), \bar{\rho}(\mathbf{p})] - \frac{i}{2}\{\Gamma^d_{\mathbf{p}}, \bar{\rho}(\mathbf{p})\} + \frac{i}{2}\{\Gamma^p_{\mathbf{p}}, 1-\bar{\rho}(\mathbf{p})\}, \right)^*$$

strong phase

Hamiltonian: $$\Omega_{ij}(\mathbf{p}) \simeq \frac{y_i^2 T^2}{16|\mathbf{p}|}\delta_{ij} + 0.046(\kappa^*\kappa)_{ij}\frac{T^4}{|\mathbf{p}|}, \quad \text{for } |\mathbf{p}| \gtrsim T.$$

This is absent in ordinary Boltzmann eqs.

Flavor dependent lepton asymmetry, but $\mathrm{tr}[\rho - \bar{\rho}] = 0$

Kinetic Equations to be solved

Two scales approximation, $p \sim m_\phi, p \sim T$

$$i\frac{d\rho_{\mathbf{k}}}{dt} = [\Omega_{\mathbf{k}}, \rho_{\mathbf{k}}] - \frac{i}{2}\{\Gamma^d_{\mathbf{k}}, \rho_{\mathbf{k}}\},$$

and eqs for CP conjugate process
and right-handed leptons

$$i\frac{d\delta\rho_T}{dt} = [\Omega_T, \delta\rho_T] - \frac{i}{2}\{\Gamma^d_T, \delta\rho_T\} + i\delta\Gamma^p_T,$$

$$\Omega_{ij}(\mathbf{p}) \simeq \frac{y_i^2 T^2}{16|\mathbf{p}|}\delta_{ij} + 0.046(\kappa^*\kappa)_{ij}\frac{T^4}{|\mathbf{p}|}, \quad \text{for } |\mathbf{p}| \gtrsim T.$$

$$\left(\Gamma^d_{\mathbf{k}}\right)_{ij} \simeq C\alpha_2^2 T\sqrt{\frac{T}{|\mathbf{k}|}}\delta_{ij} + \frac{9y_t^2}{64\pi^3|\mathbf{k}|}T^2(\delta_{i\tau}\delta_{\tau j}y_\tau^2 + \delta_{i\mu}\delta_{\mu j}y_\mu^2) + \frac{21\zeta(3)}{32\pi^3}(\kappa^*\cdot\kappa)_{ij}T^3,$$

$$\left(\Gamma^d_T\right)_{ij} \simeq C'\alpha_2^2 T\delta_{ij} + \frac{9y_t^2}{64\pi^3}T(\delta_{i\tau}\delta_{\tau j}y_\tau^2 + \delta_{i\mu}\delta_{\mu j}y_\mu^2) + \frac{21\zeta(3)}{32\pi^3}(\kappa^*\cdot\kappa)_{ij}T^3,$$

$$\begin{aligned}(\delta\Gamma^p_T)_{ij} &\simeq C\alpha_2^2 T\sqrt{\frac{T}{|\mathbf{k}|}}\,(\rho_{\mathbf{k}})_{ij} - C'\alpha_2^2 T\,(\delta\bar{\rho}_T)_{ij} \\ &+ \frac{3\zeta(3)}{8\pi^3}(\kappa^*\cdot(\bar{\rho}_{\mathbf{k}} - 3/4\rho_{\mathbf{k}})^t\cdot\kappa)_{ij}T^3 + \frac{3\zeta(3)}{8\pi^3}(\kappa^*\cdot(\delta\bar{\rho}_T - 3/4\delta\rho_T)^t\cdot\kappa)_{ij}T^3.\end{aligned}$$

Some formula can be also found in Akhmedov,et al. 9803255; Abada, et al. 0601083.; Asaka, et al. 1112.5565.

3. Leptogenesis via Neutrino Oscillation

CASE A: Inflaton to Leptons

Inflaton $\longrightarrow$

$$V^e|\,l_e\rangle + V^\mu|\,l_\mu\rangle + V^\tau|\,l_\tau\rangle\,,$$
$$(V^e)^*|\,\bar{l}_e\rangle + (V^\mu)^*|\,\bar{l}_\mu\rangle + (V^\tau)^*|\,\bar{l}_\tau\rangle$$

Initial condition

inflaton mass scale

$$\rho_{\mathbf{k}}|_{t=t_R} = \bar{\rho}_{\mathbf{k}}|_{t=t_R} = \mathcal{N} V_i V_j^*,$$

$$\mathcal{N} = \frac{3}{4}\frac{T_R}{m_\phi}B,$$

temperature scale

$$\delta\rho_T|_{t=t_R} = \delta\bar{\rho}_T|_{t=t_R} = 0.$$

(means thermalized.)

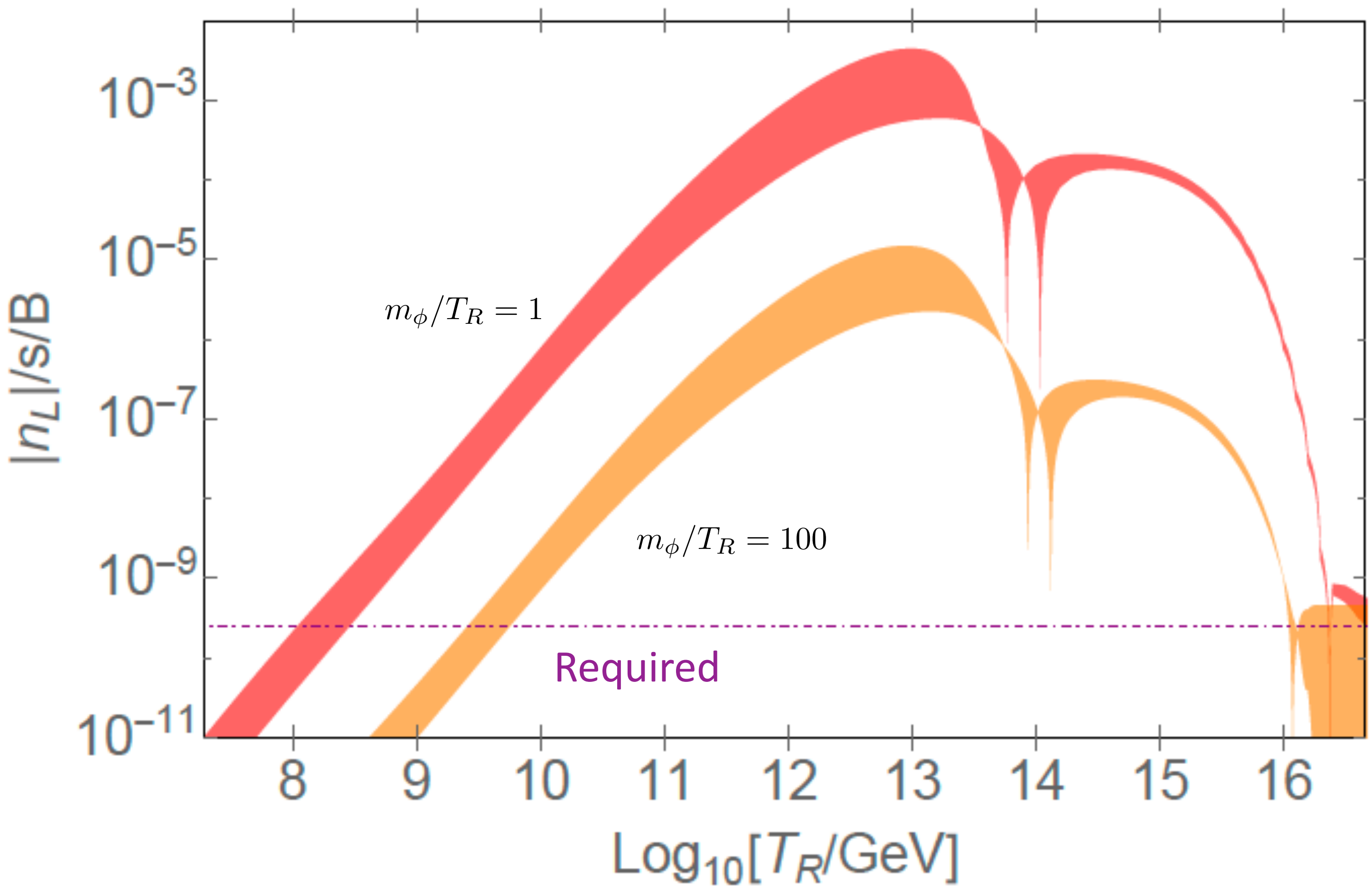
Parameters

$$T_R, m_\phi, B, V_i$$

and those for neutrino oscillation.

Solution (one massless neutrino)

Dirac phase=-pi/2, Majorana phase=0.3 pi, V={1,1,1}, Inverted Hierarchy



Baryon asymmetry could be explained by ***<u>any model</u>***, with

$$\mathcal{L} = \mathcal{L}_{SM} + \kappa llHH + \ldots \quad @ \ \mu_{RG} > 10^8\,\mathrm{GeV}$$

Trick of CASE A

(Mechanism for reheating temp<10^13GeV)

Thermalization of the inflaton decay products

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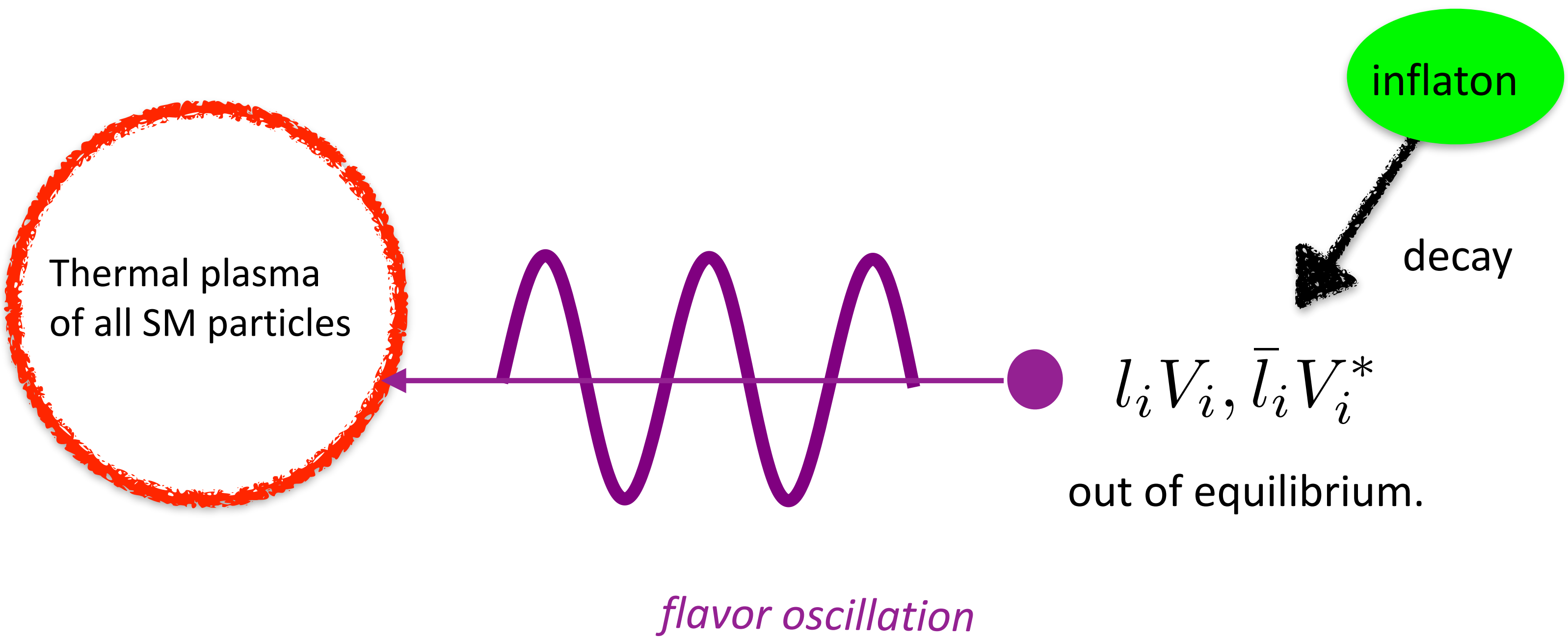
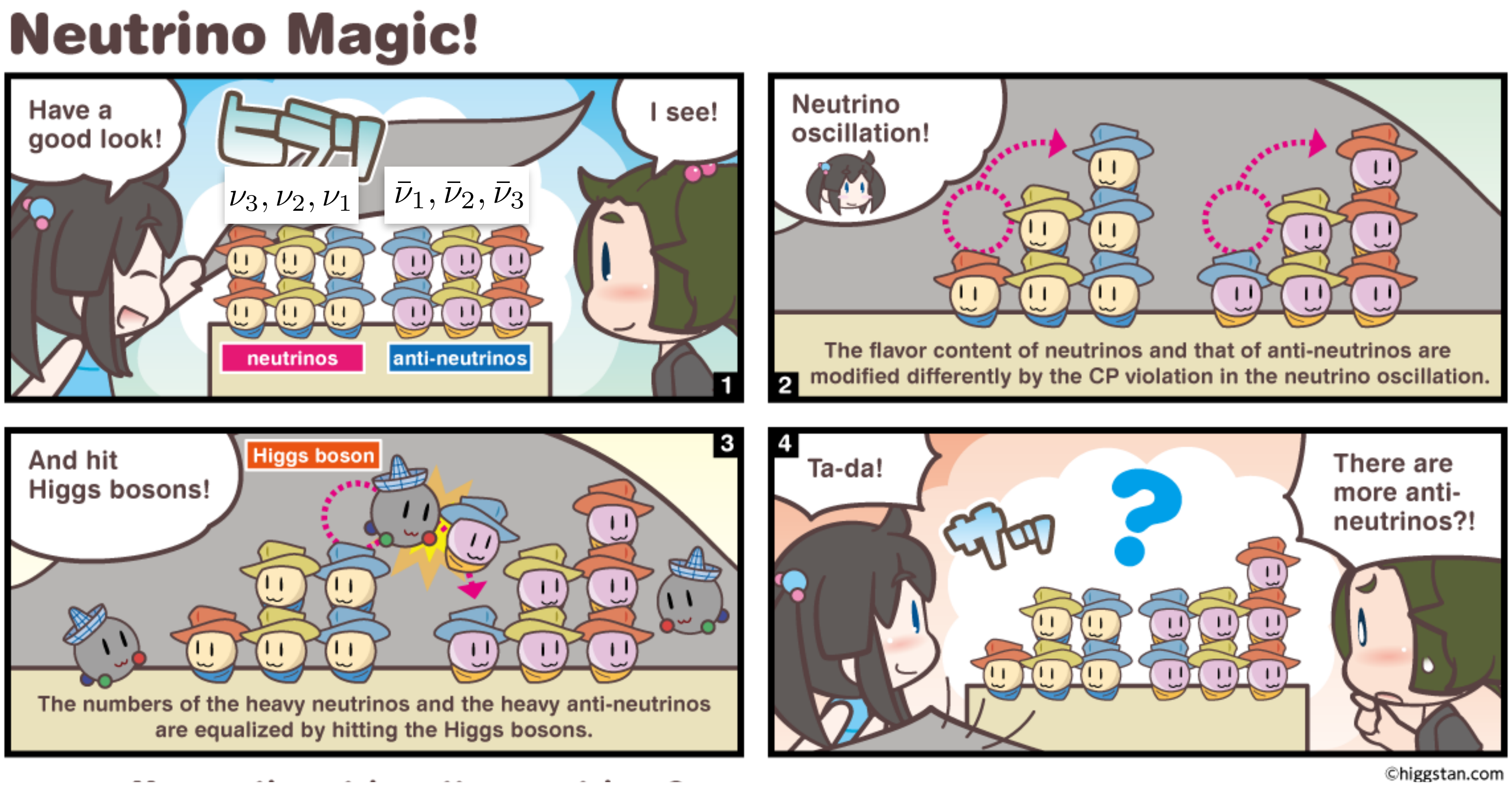


Neutrino Magic

(Mechanism for reheating temp<10^13GeV)



Frame 3: ***<u>Flavor dependent</u>*** *<u>washout effect</u>* through llHH,

This tends to eliminate the asymmetry of heavier neutrinos.

$$\frac{n_L}{s} \sim -2\times 10^{-6}\cdot \xi_{CP}\cdot B\cdot \left(\frac{T_R}{10^{11}\ \mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{13}\ \mathrm{GeV}}\right)^{-1}, \qquad (T_R \gtrsim 10^{11}\ \mathrm{GeV}),$$

and

$$\frac{n_L}{s} \sim -2\times 10^{-6}\cdot \xi_{CP}\cdot B\cdot \left(\frac{T_R}{10^{11}\ \mathrm{GeV}}\right)^3 \left(\frac{m_\phi}{10^{13}\ \mathrm{GeV}}\right)^{-1}, \qquad (T_R \lesssim 10^{11}\ \mathrm{GeV}),$$

CASE B: Inflaton dominantly to Higgs boson

<u>*Thermalization is through the Higgs boson*</u>

Inflaton $\longrightarrow$ Higgs, Higgs* $\longrightarrow$ $l, \bar{l}$

Initial conditions

$$(\rho_{\mathbf{k}})_{ij} = (\bar{\rho}_{\mathbf{k}})_{ij} = \mathcal{N} \cdot \frac{\frac{21\zeta(3)}{32\pi^3}(\kappa^* \kappa)_{ij} T_R^3}{C\alpha_2^2 T_R \sqrt{\frac{T_R}{m_\phi}}} \sim 7 \times 10^{-2} \cdot \mathcal{N} \left(\frac{m_\phi}{10^{15}\ \mathrm{GeV}}\right)^{1/2} \left(\frac{T_R}{10^{13}\ \mathrm{GeV}}\right)^{3/2}.$$

$$\rho_T\big|_{t=t_R} = \bar{\rho}_T\big|_{t=t_R} = 0$$

Leptons are not thermalized initially

Parameters

$$T_R, m_\phi,$$

and those for neutrino oscillation.

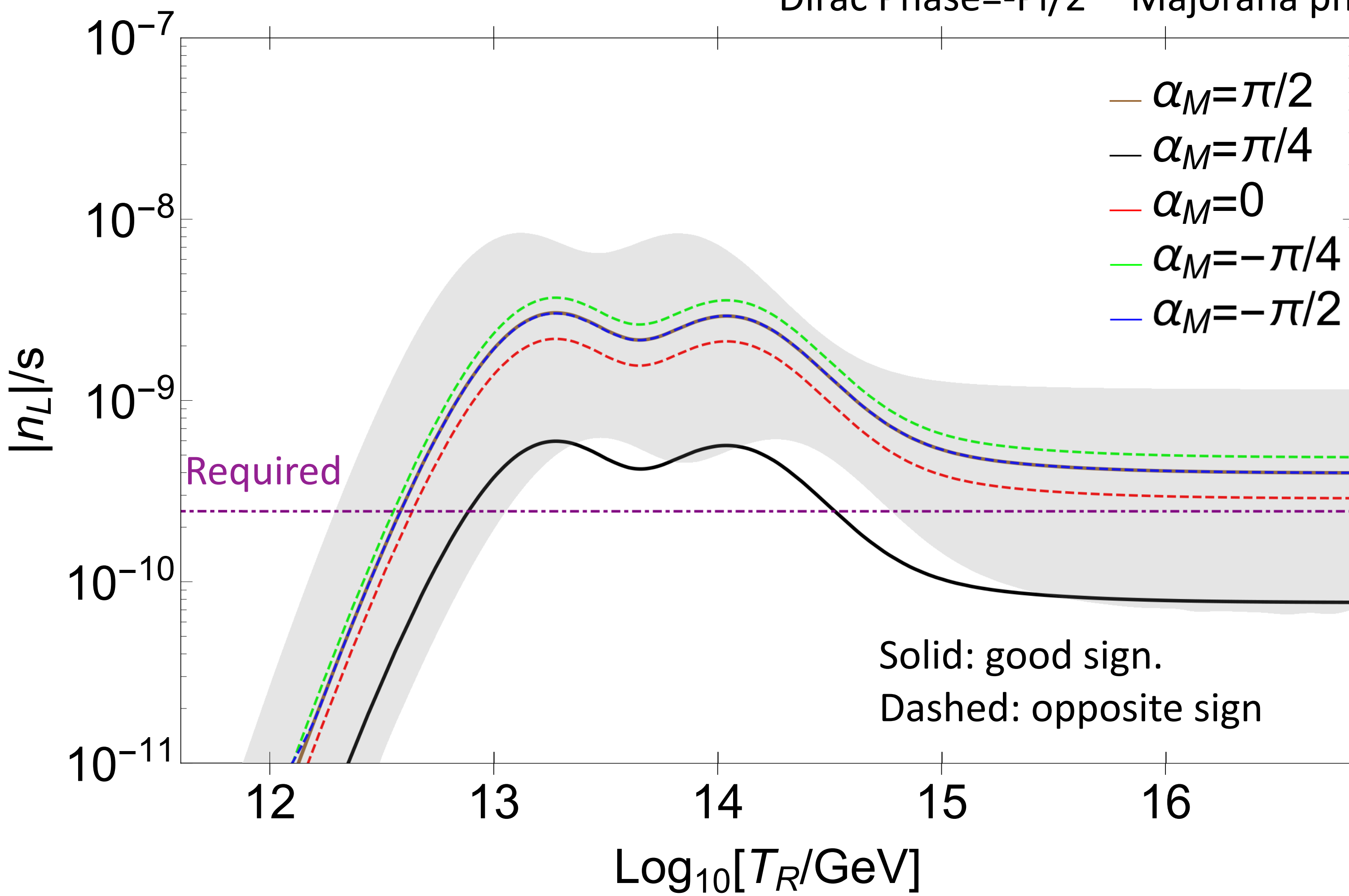
Solution Inverted Hierarchy

Inflaton mass =100 T_R

$\phi \rightarrow \text{Higgs}$

Inverted Hierarchy, two massive neutrinos.

Dirac Phase=-Pi/2 Majorana phase



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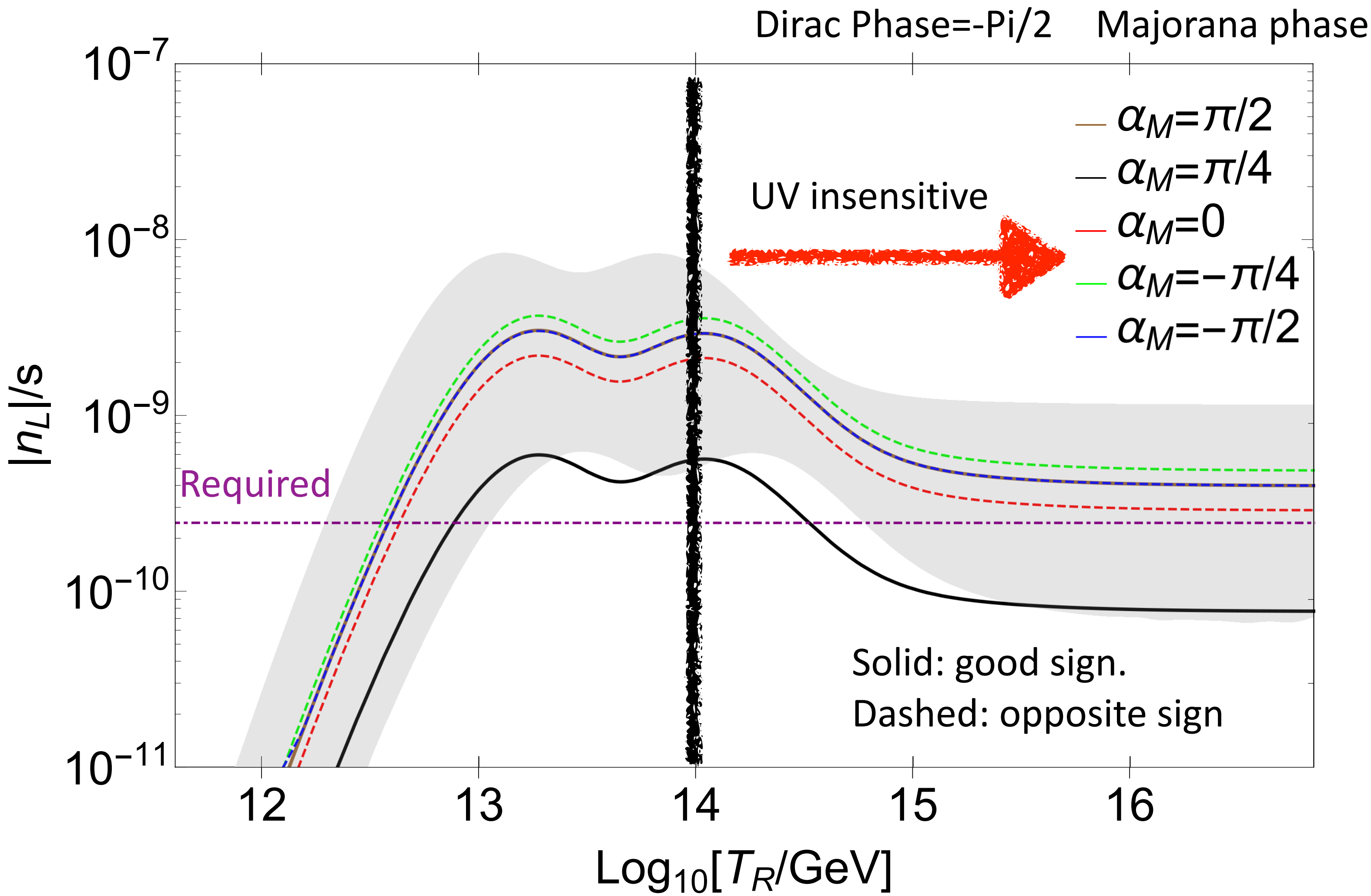
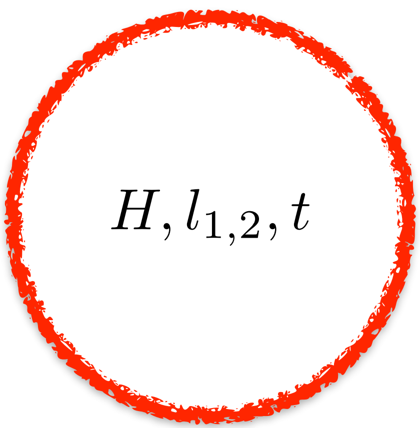


Trick of CASE B

$T_R \gg T \gtrsim 10^{14}\mathrm{GeV}$ Gauge interaction rates are slower than expansion.

Thermal Plasma



y_t

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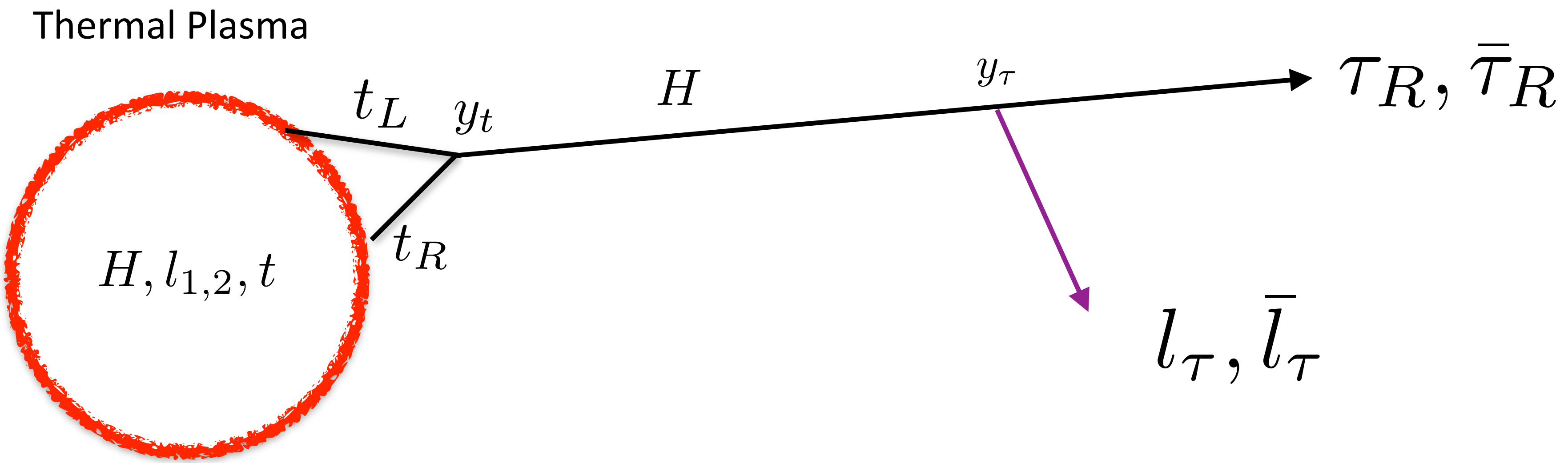

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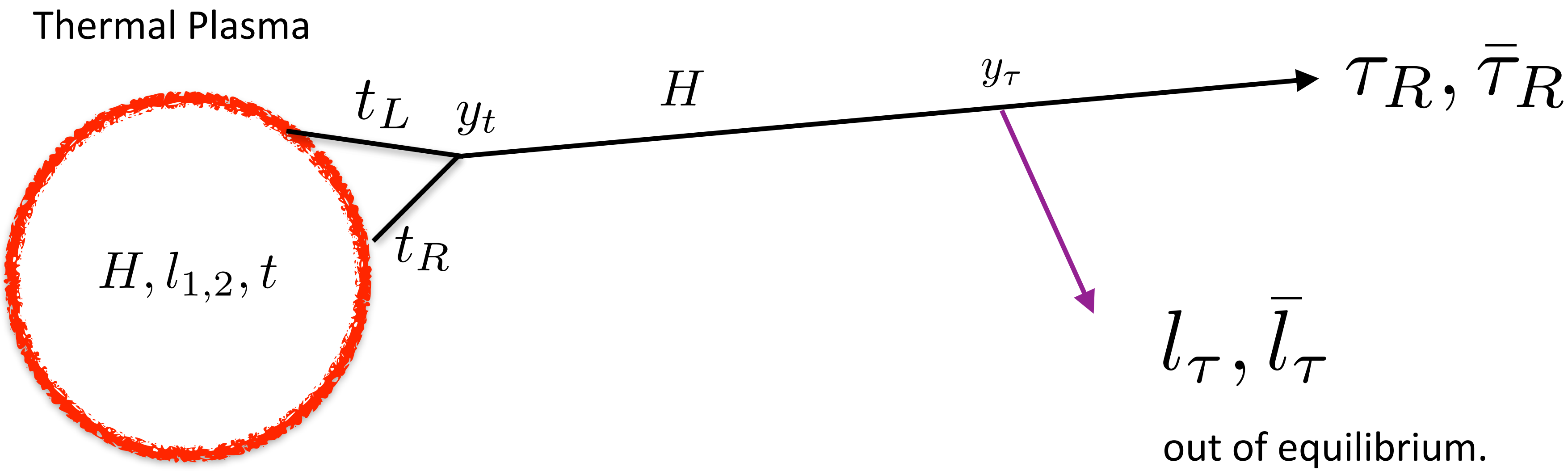

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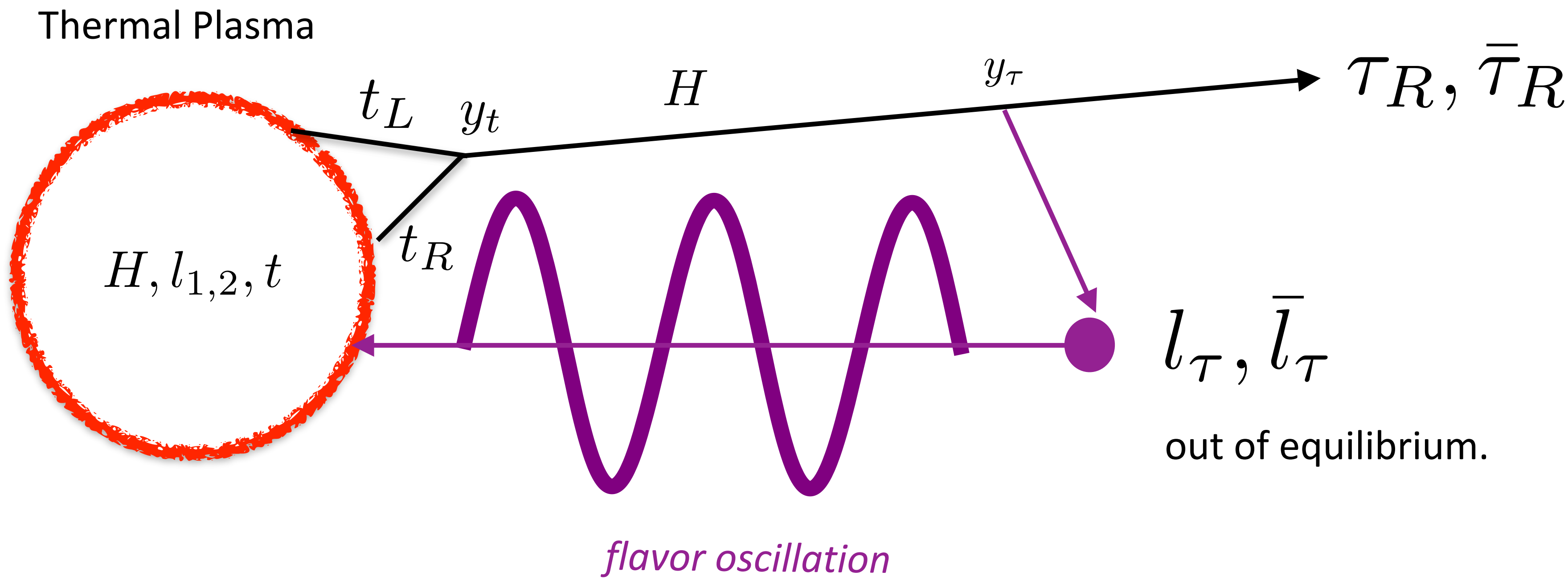
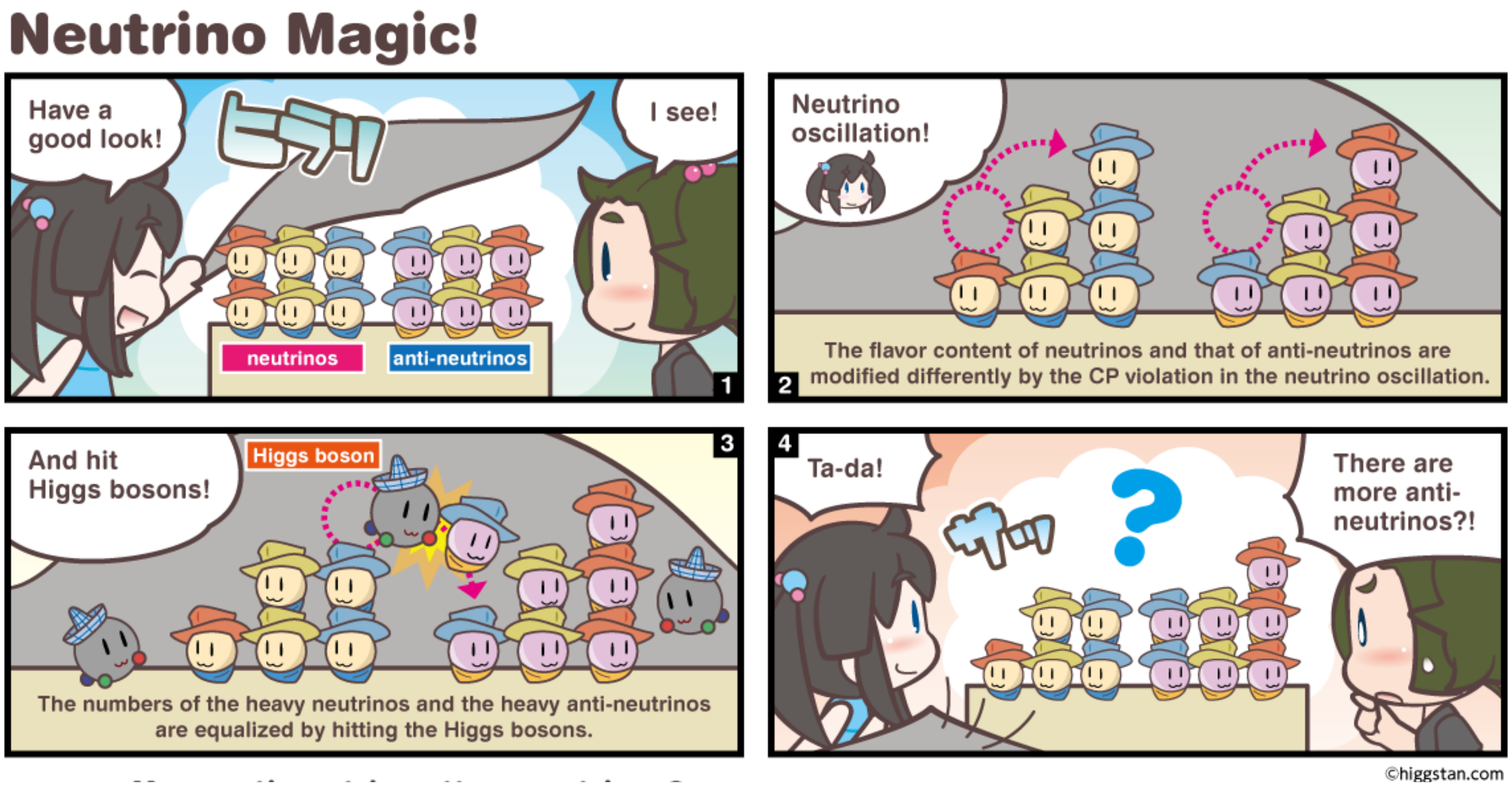


Neutrino Magic



Frame 3: Flavor dependent washout effect through llHH,
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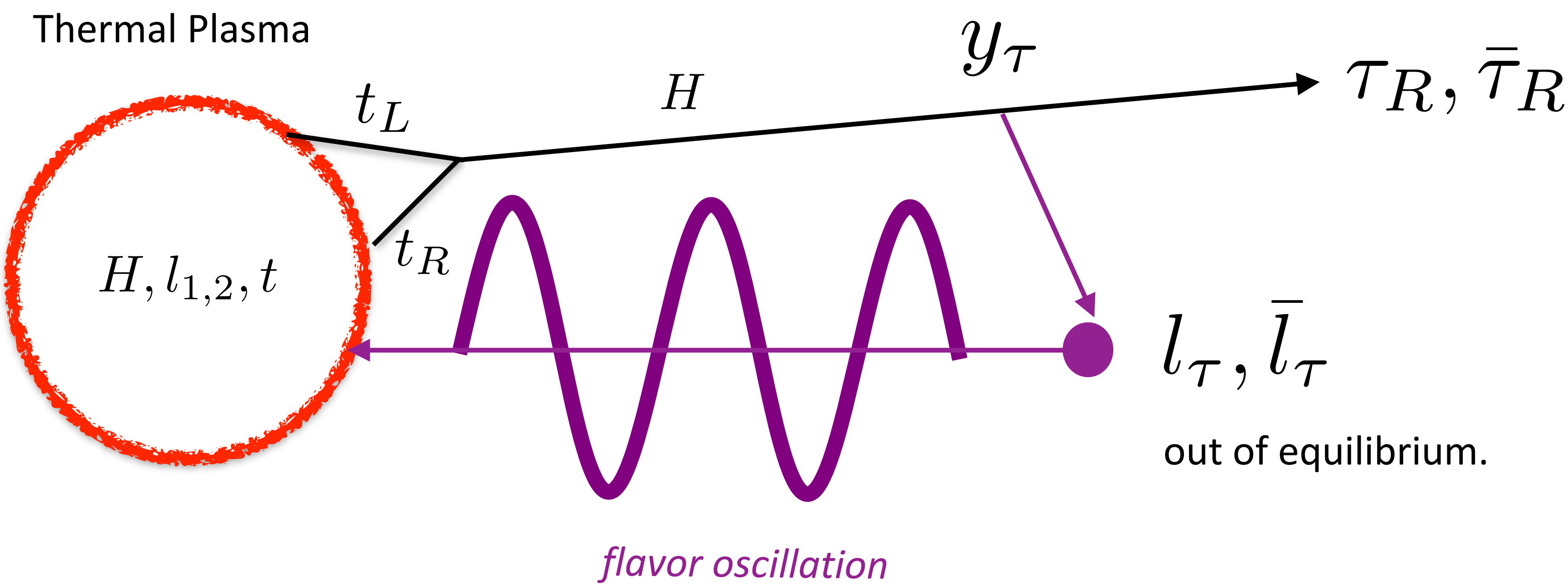
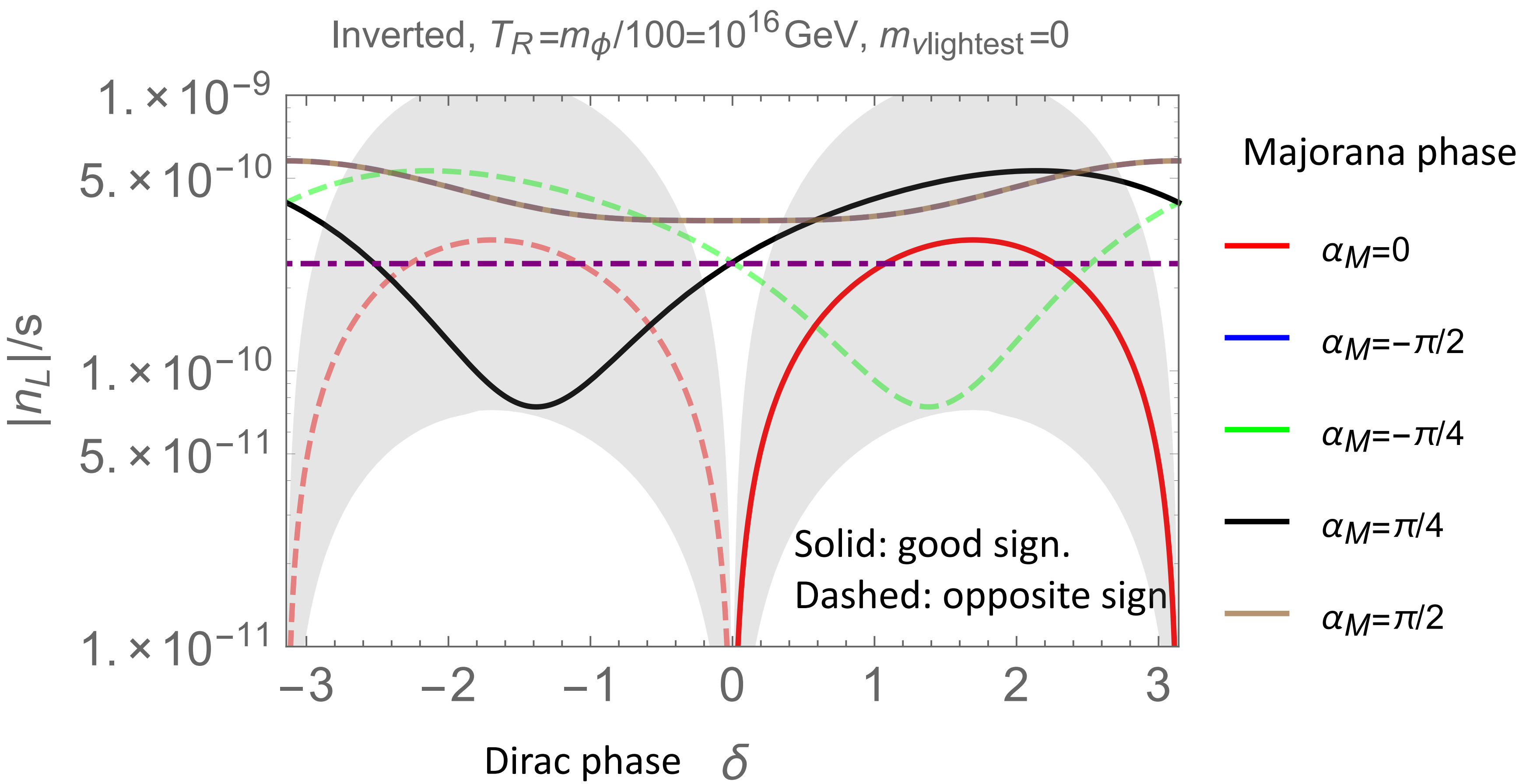


$$\frac{n_L}{s} \sim \left(-2 \times 10^{-10} \sin \alpha_M + 10^{-10} \sin(\alpha_M - \delta) + 4 \times 10^{-11} \sin(\alpha_M + \delta)\right) \cdot \left(\frac{30.25}{g_{*s}}\right).$$

Independent of T_R, m_{ϕ}***, but around the required value ~-10^(-10)!***

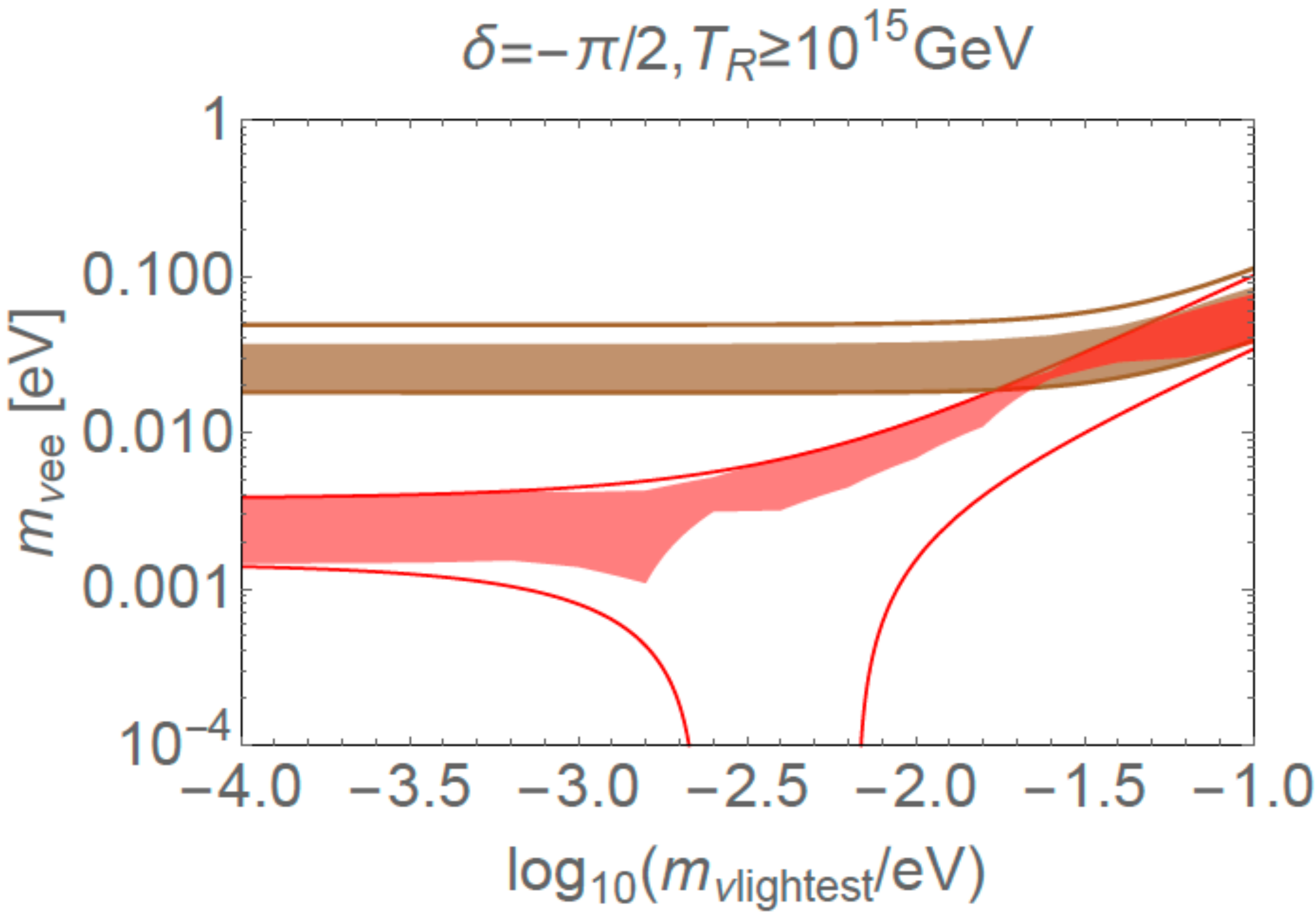
Even does not depend on $m_{\nu \mathrm{lightest}}$ (Degenerated case has almost same result.)

Phase Dep. at very High temp



Neutrinoless double beta decay

The CP phase and neutrino mass can be tested from neutrino exps.



Summary

Kitano, Hamada, WY,1807.06582

Baryon asymmetry explained within SM with

$$-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH + \text{h.c.}$$

- The observed asymmetry can be explained at reheat temp > $10^8\ GeV$

  A low scale inflation around this scale may open the axion window.
  Takahashi, WY, Guth, Phys.Rev. D98 (2018) 015042; Graham, Scherlis, 1805.07362

- For the Higgs case, the CP phase needed is within the PMNS mat.
  The scenario can be tested in future neutrino exps.

Backups

Not complete washout at very High Temp

$$-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j) H H + \text{h.c.}$$

Inverted Hierarchy

$$\kappa \simeq 1/v^2 \text{diag}[m_\nu, m_\nu, 0]$$

$SO(2) \times U(1)$ sym.

Conserved current for lepton (flavor) number!

Even with very high Temp, conserved current protects lepton asymmetry!

Not complete washout with degenerate case

$$-\frac{\kappa_{ij}}{2}(\bar{l}_i^c P_L l_j)HH + \text{h.c.}$$

$$\kappa \simeq 1/v^2 \text{diag}[m_\nu, m_\nu, m_\nu]$$

$SO(3)$ sym.

Conserved current for lepton flavor sym.
Certain flavor dependent asymmetry remains.

Even with very high Temp, conserved current protects lepton asymmetry!

Oscillation phase

$$t_{osc}^{-1} \sim \frac{y_\tau^2}{16} T \sim 10^7 \left(\frac{T}{10^{12} GeV} \right) GeV$$

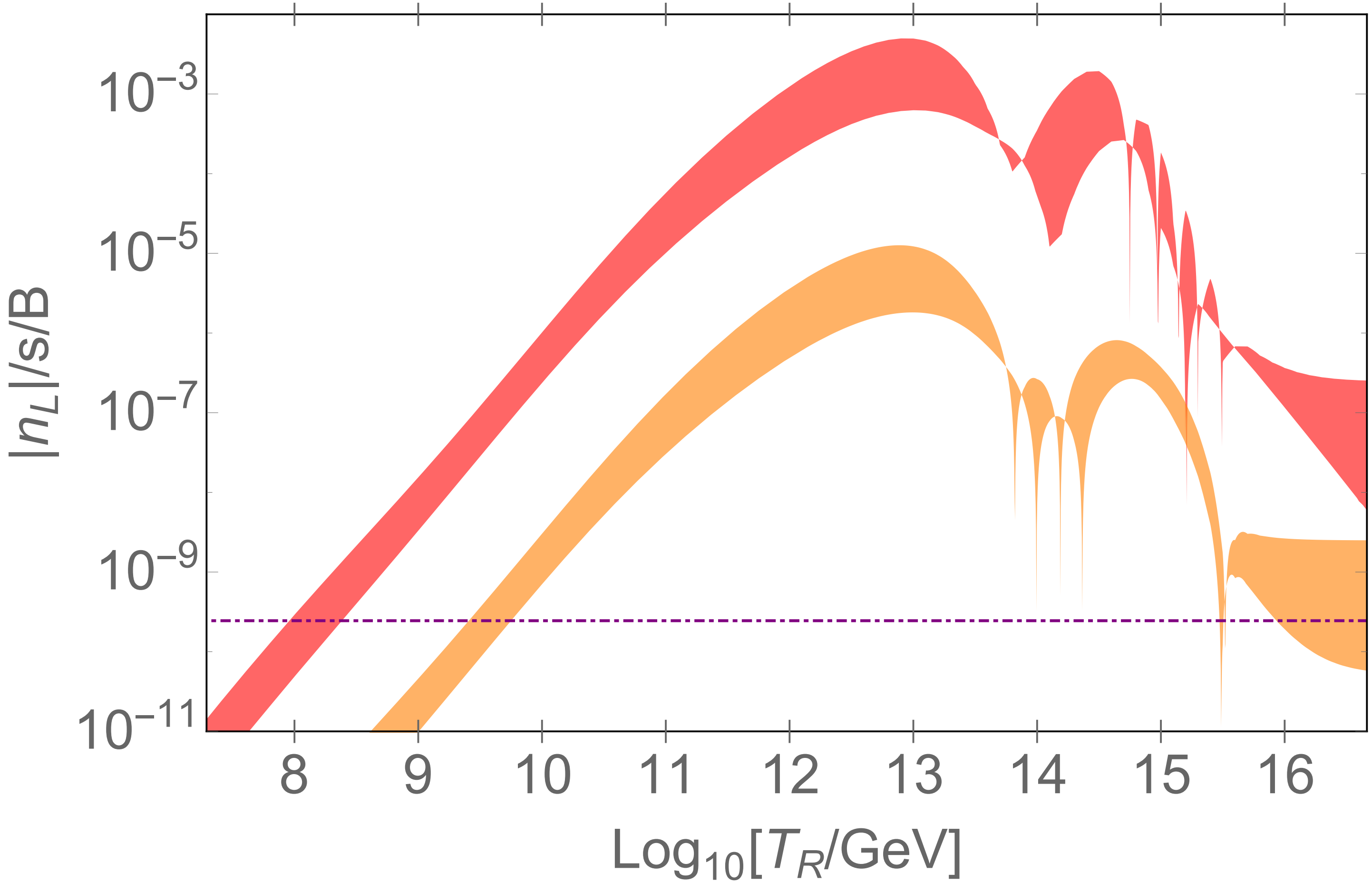
$$t_{scat}^{-1} \sim \alpha_2^2 T \sim 10^9 \left(\frac{T}{10^{12} GeV} \right) GeV$$

$$\text{Phase} \sim t_{osc}^{-1} t_{scat} \ll 1$$

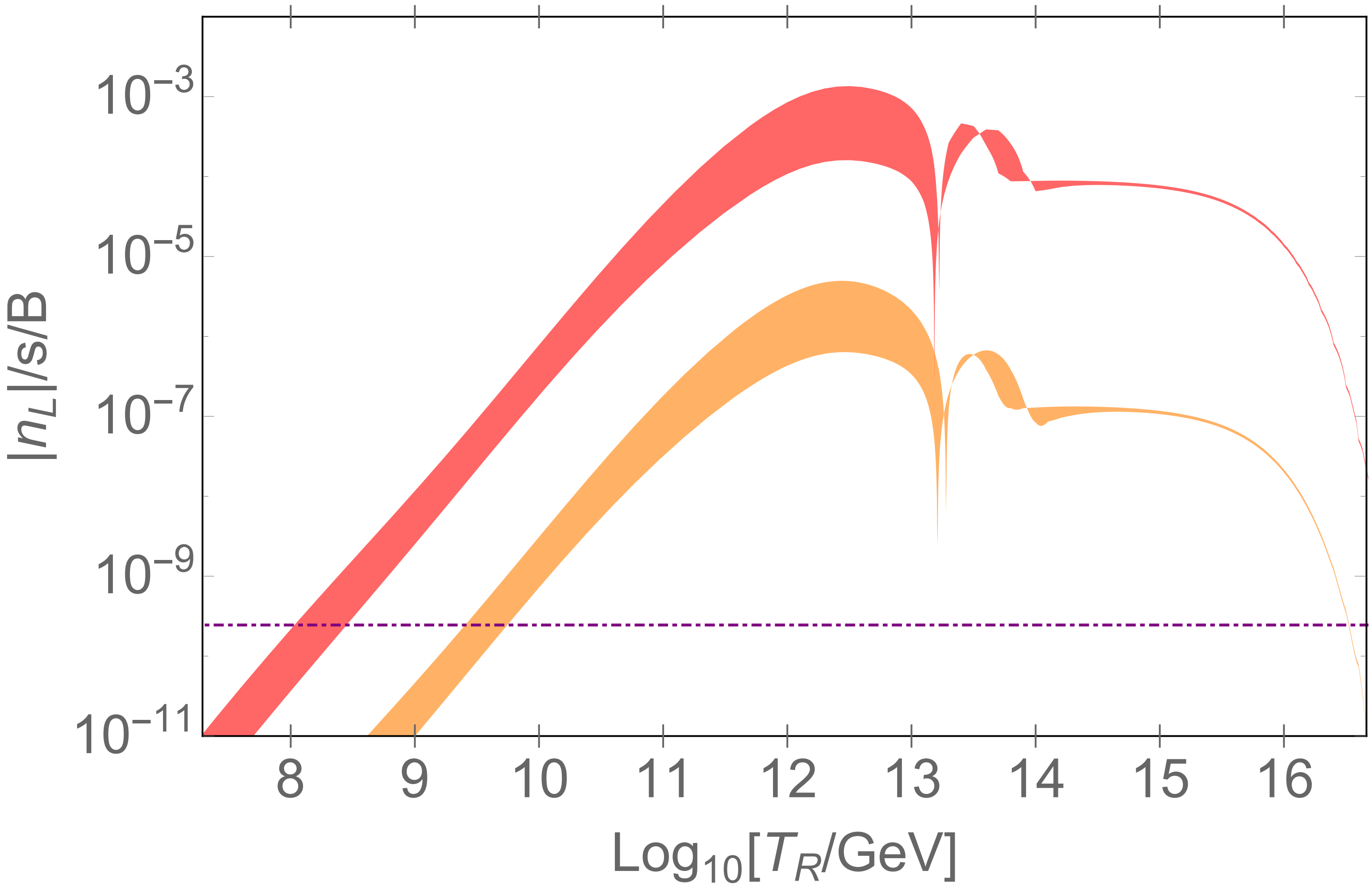
Coherently scattering (LPM effect) becomes important and it is included in the analysis.

FIGURES FOR INFLATON->LEPTON

normal hierarchy one massless neutrino.
Other parameters are same as the main part one.



normal order degenerate mass. mnulightest=0.07eV
Other parameters are same as the main part one.



inverted order degenerate mass. mnulightest=0.07eV
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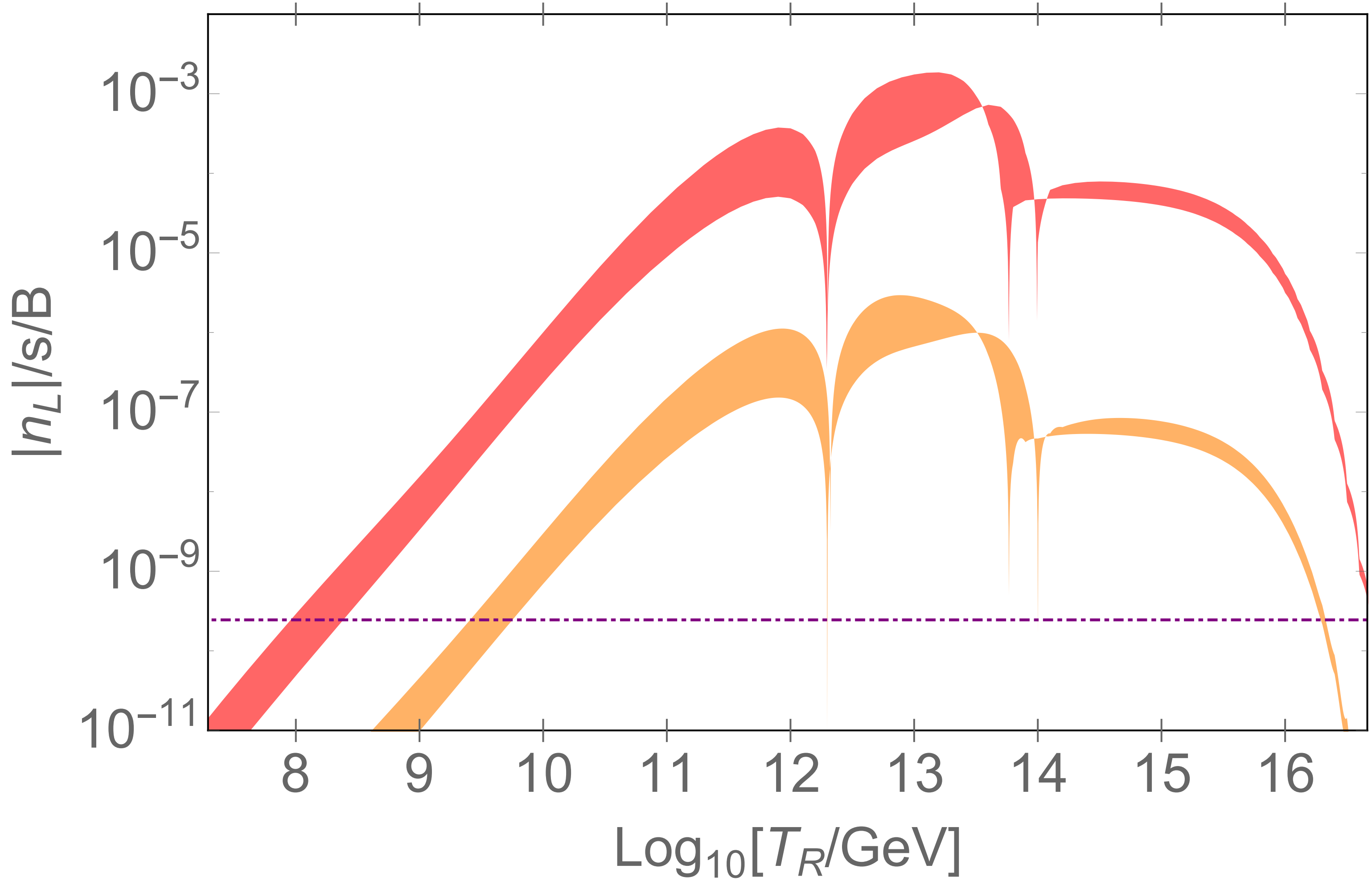
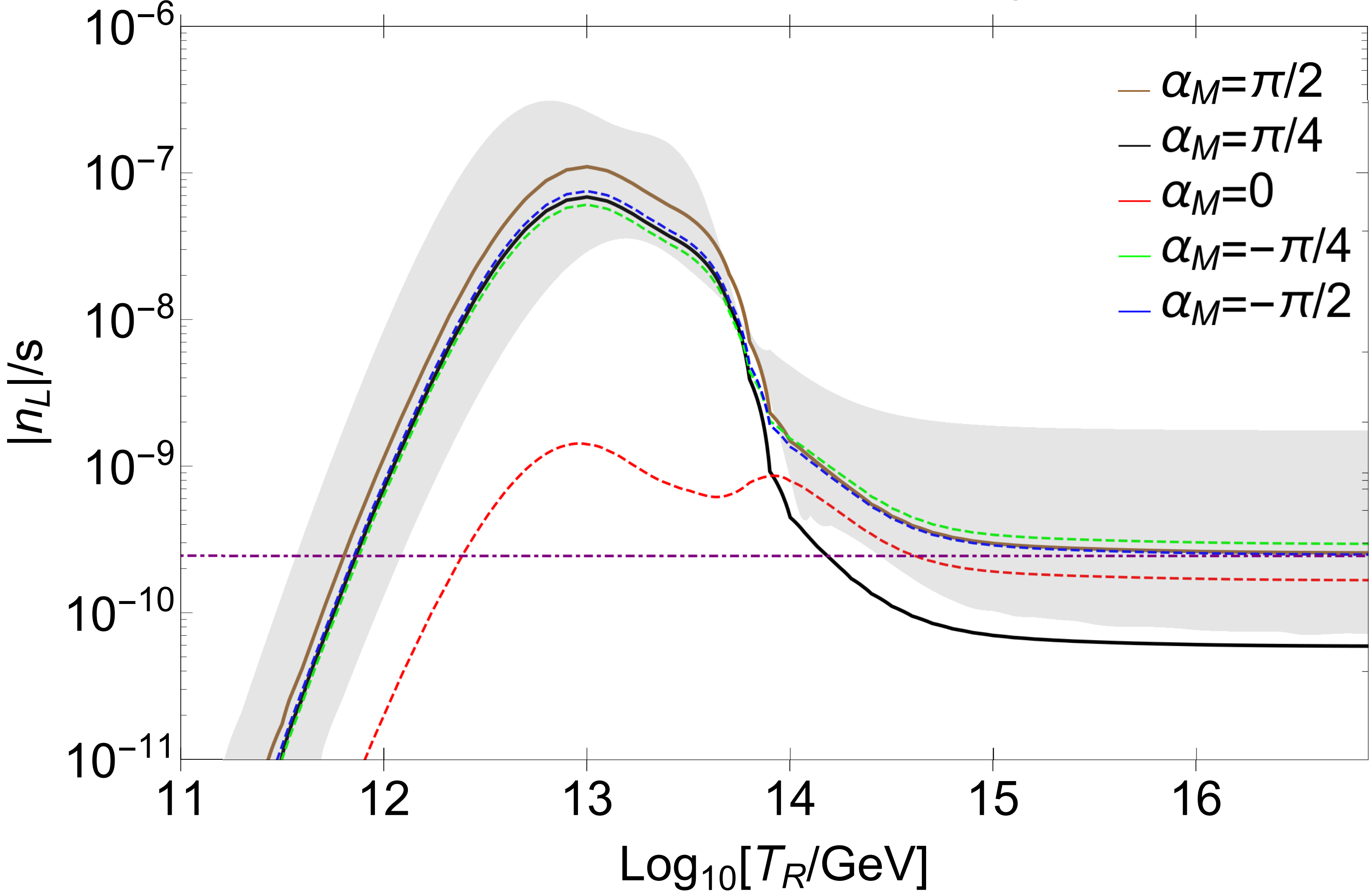
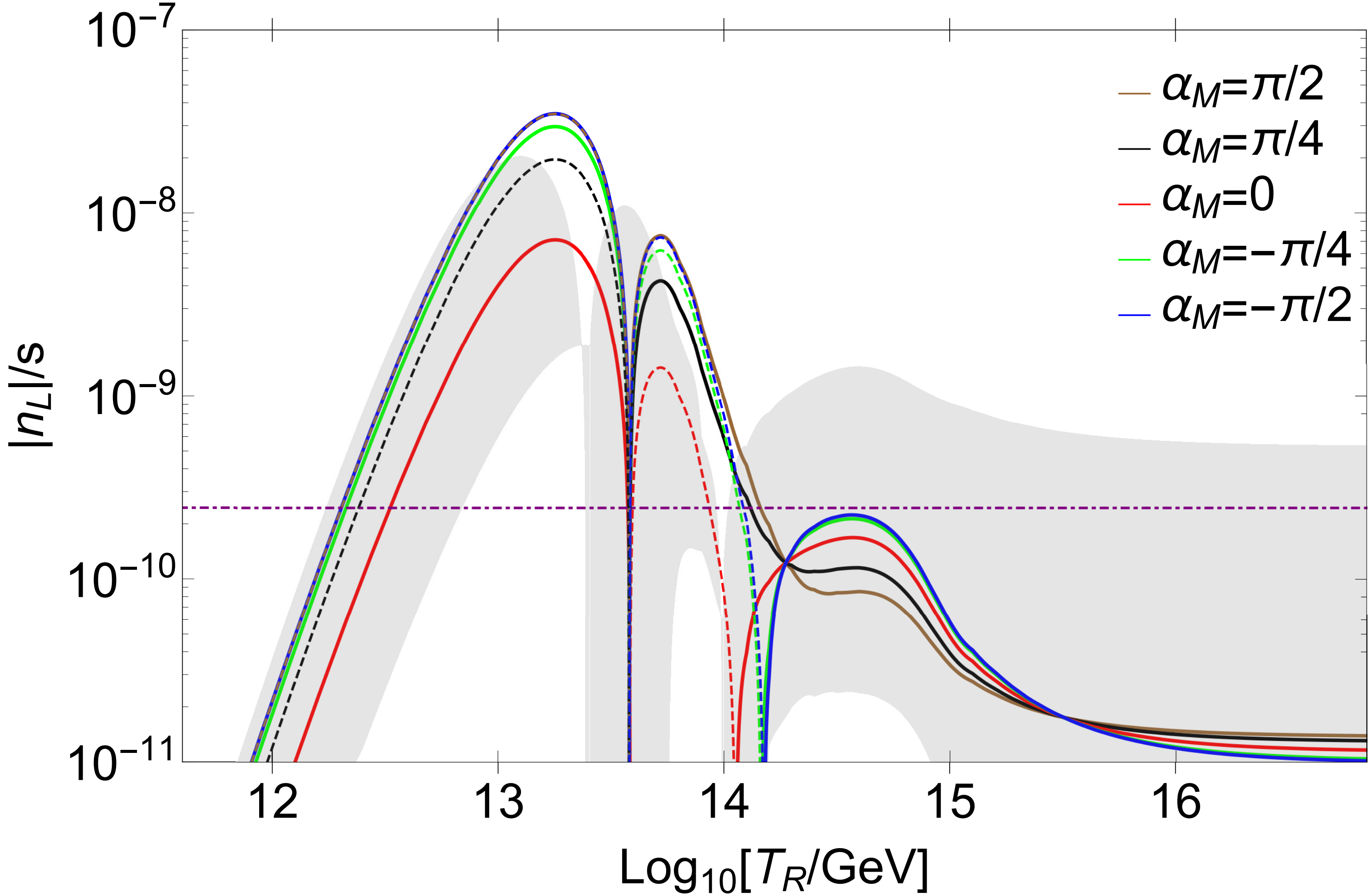


FIGURES FOR INFLATON->HIGGS

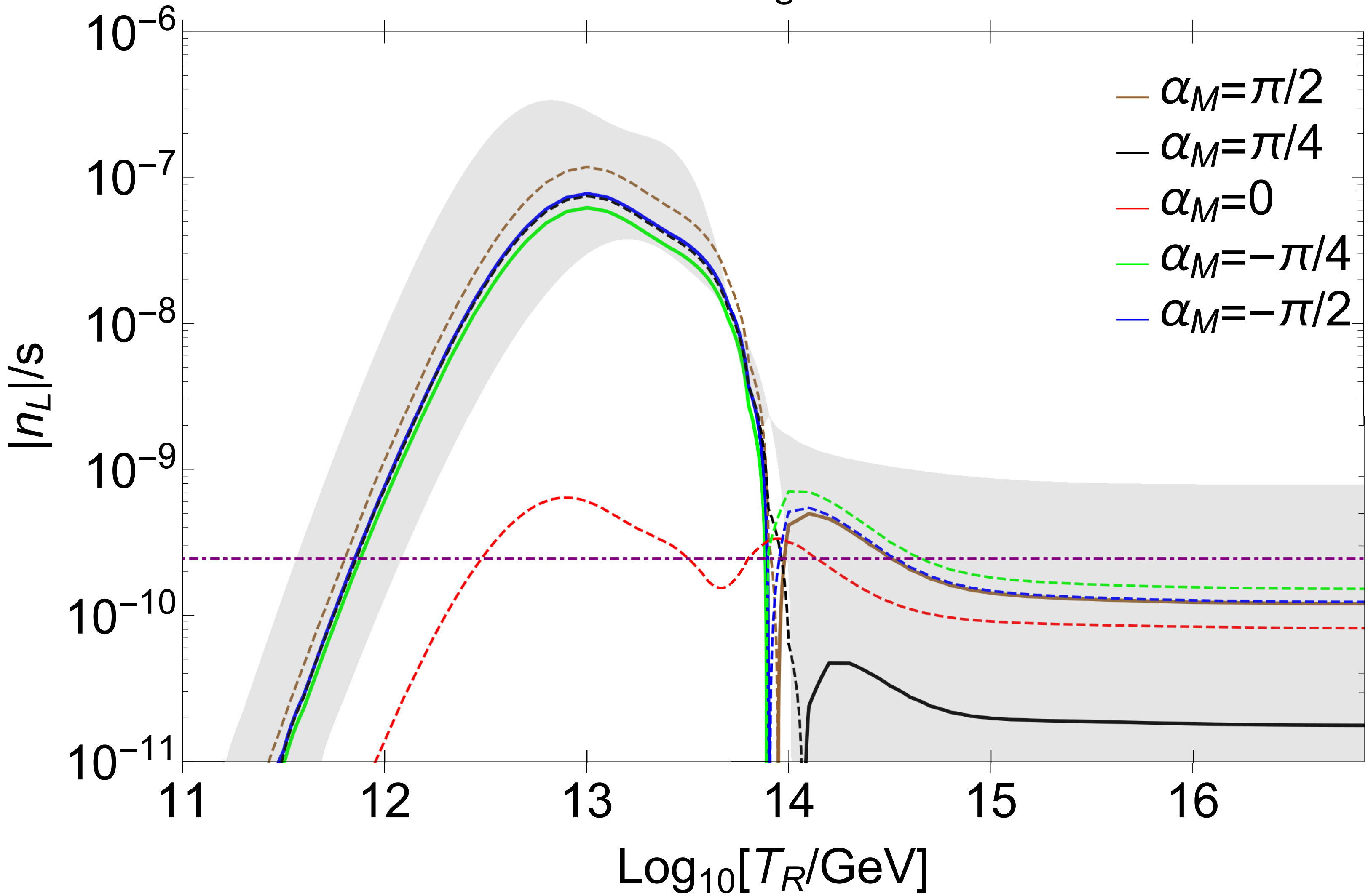
Inverted degenerate case.
mnulightest=0.07eV



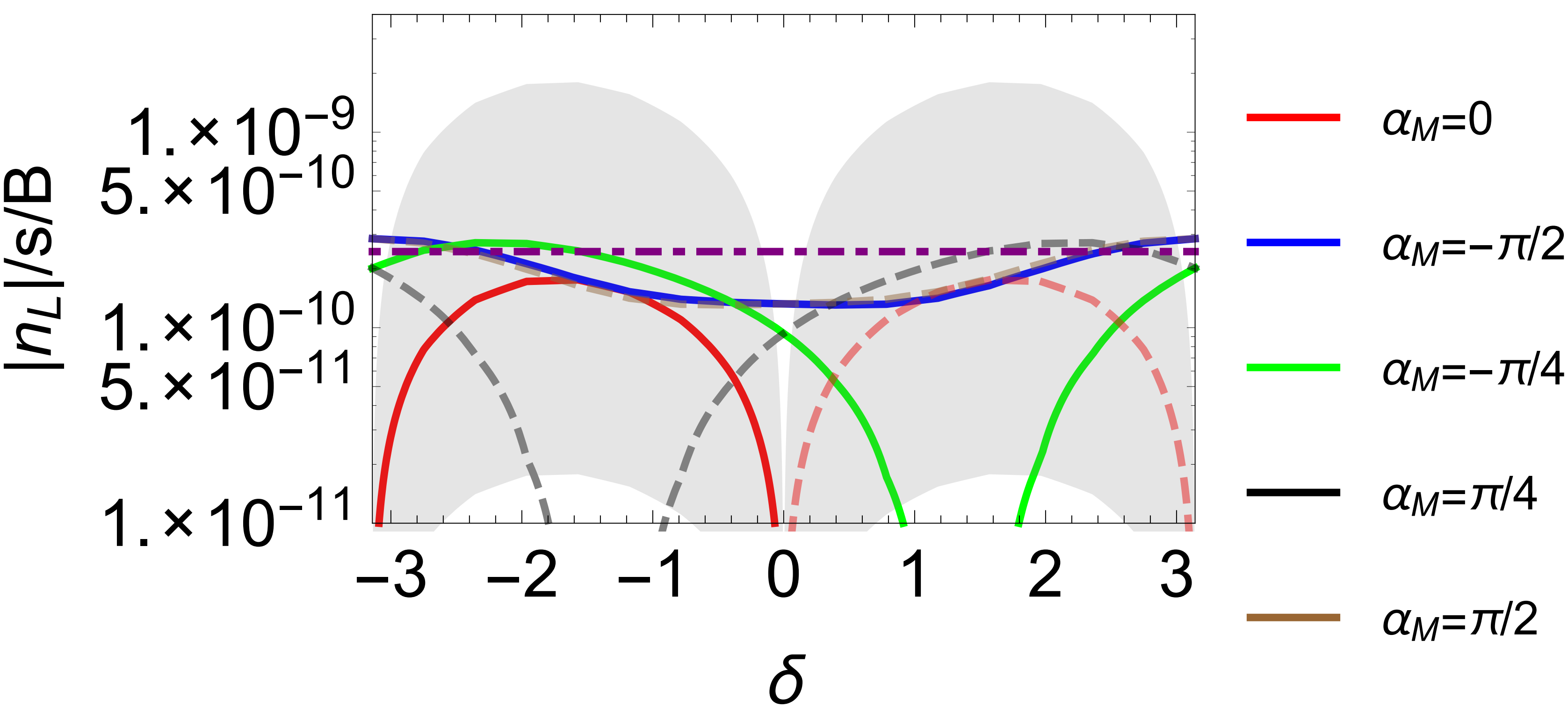
Normal hierarchy one massless neutrino.

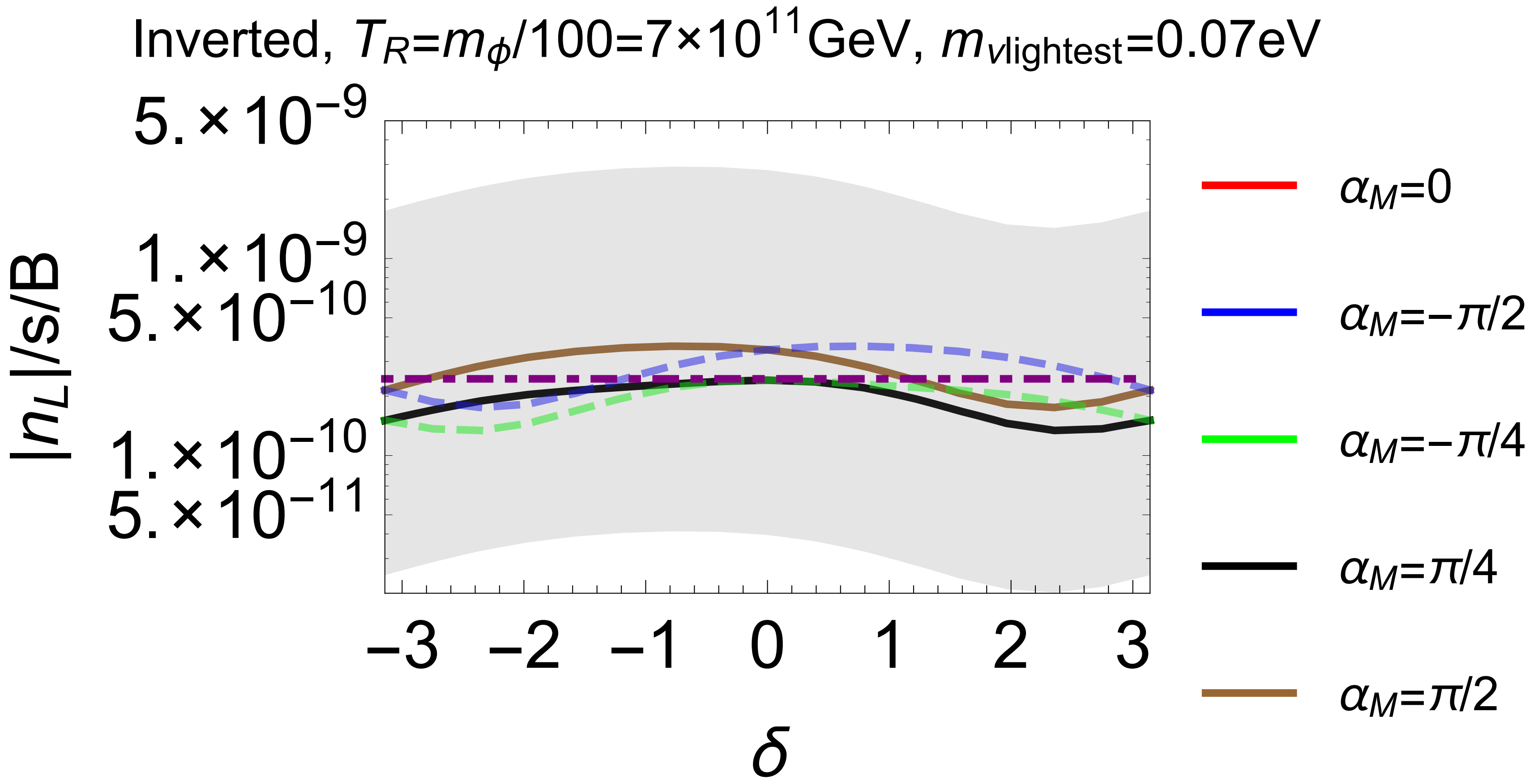


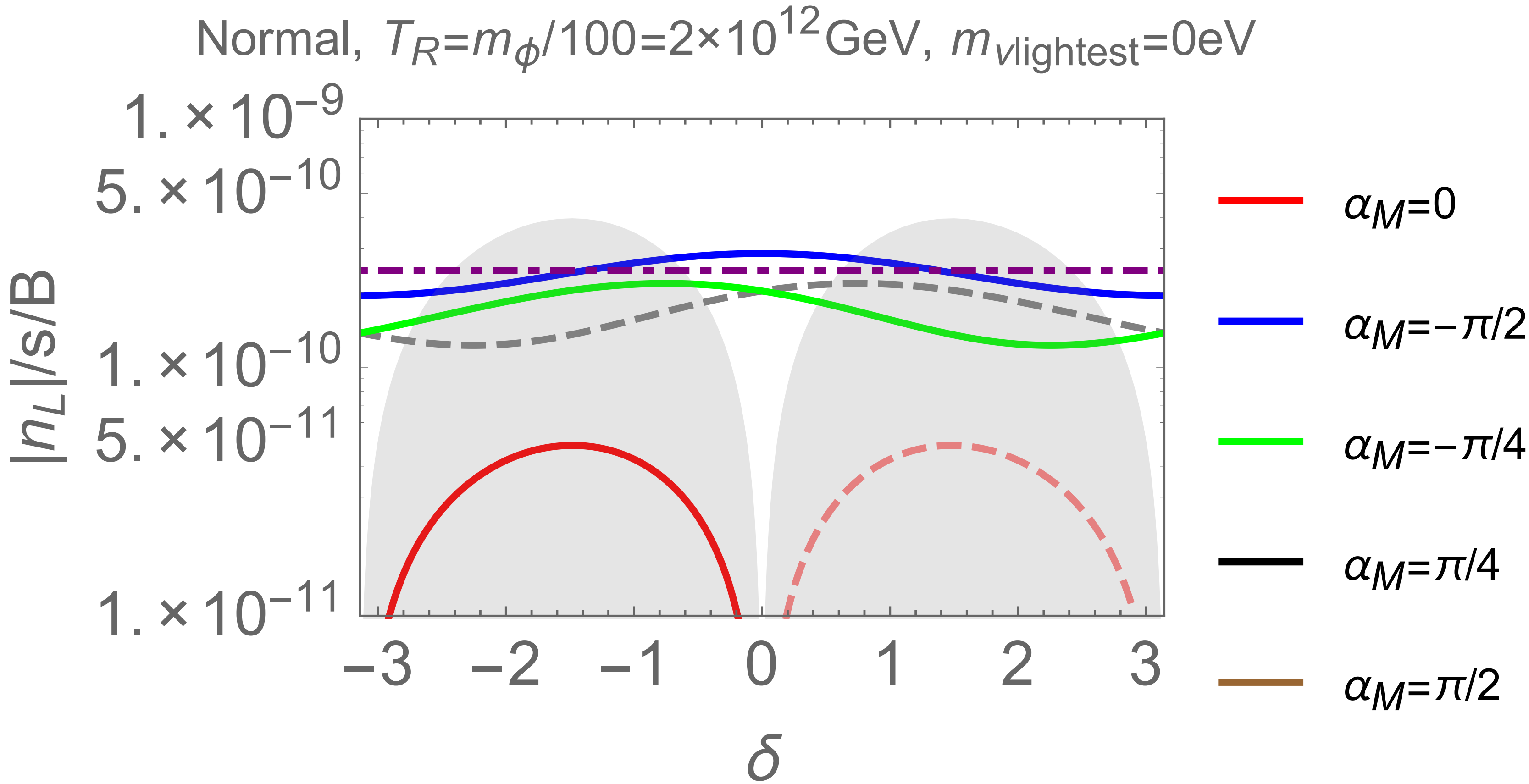
Normal hierarchy degenerate case.
mnulightest=0.07eV



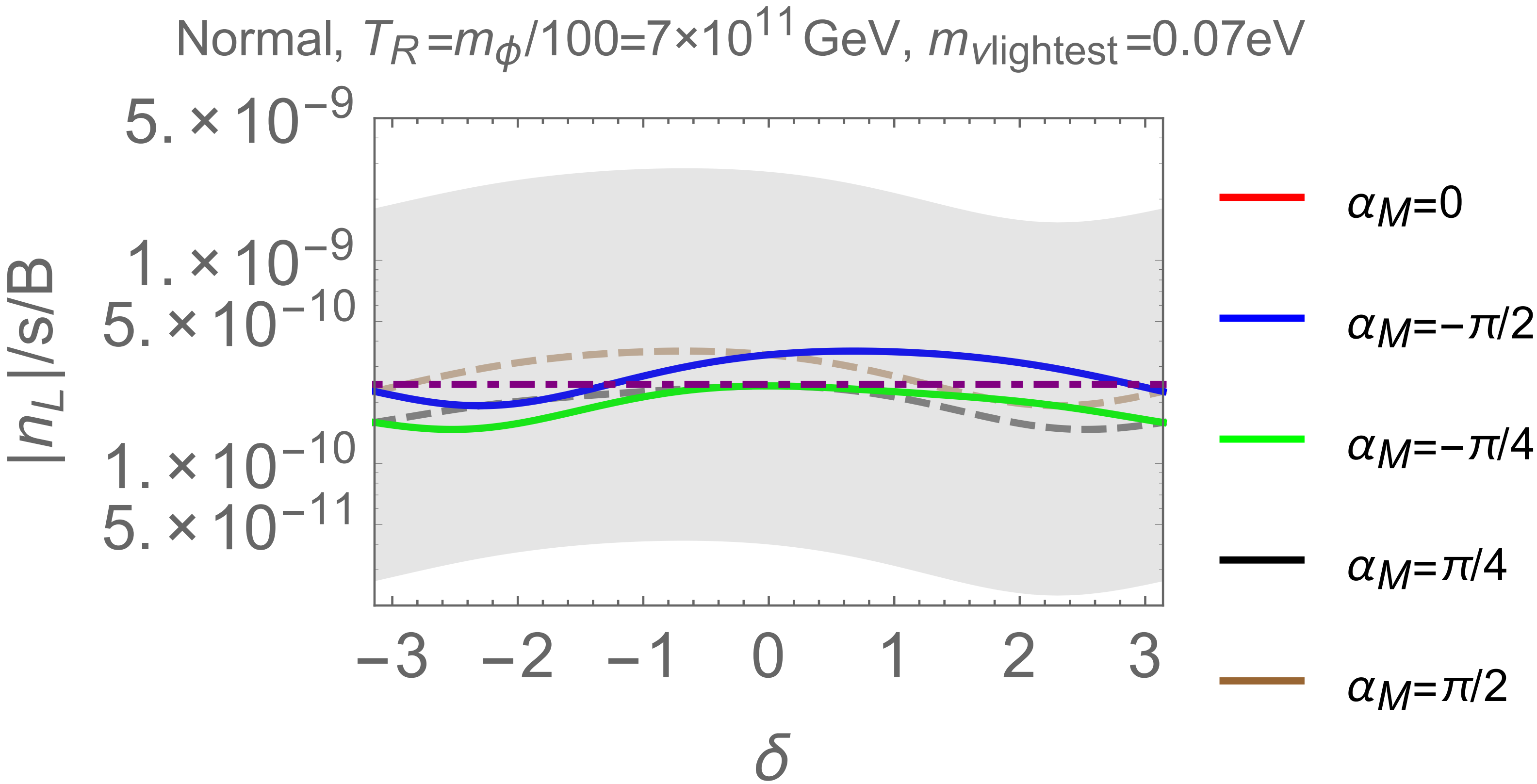
Inverted, $T_R = m_\phi/100 = 4\times10^{12}$GeV, $m_{\nu\mathrm{lightest}}$=0eV



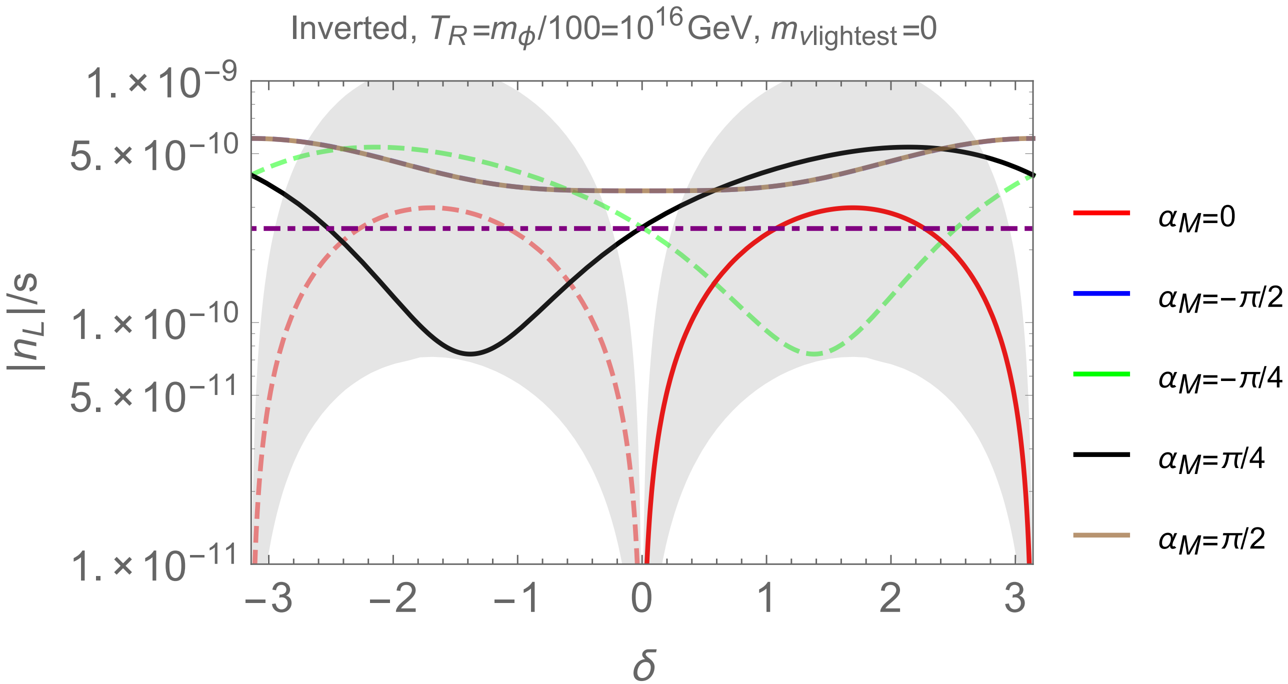

Inverted, $T_R = m_\phi/100 = 7\times10^{11}$GeV, $m_{\nu\mathrm{lightest}}$=0.07eV
$5.\times10^{-9}$
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$|n_L|/s/B$
−3
−2
−1
0
1
2
3
δ
$\alpha_M=0$
$\alpha_M=-\pi/2$
$\alpha_M=-\pi/4$
$\alpha_M=\pi/4$
$\alpha_M=\pi/2$



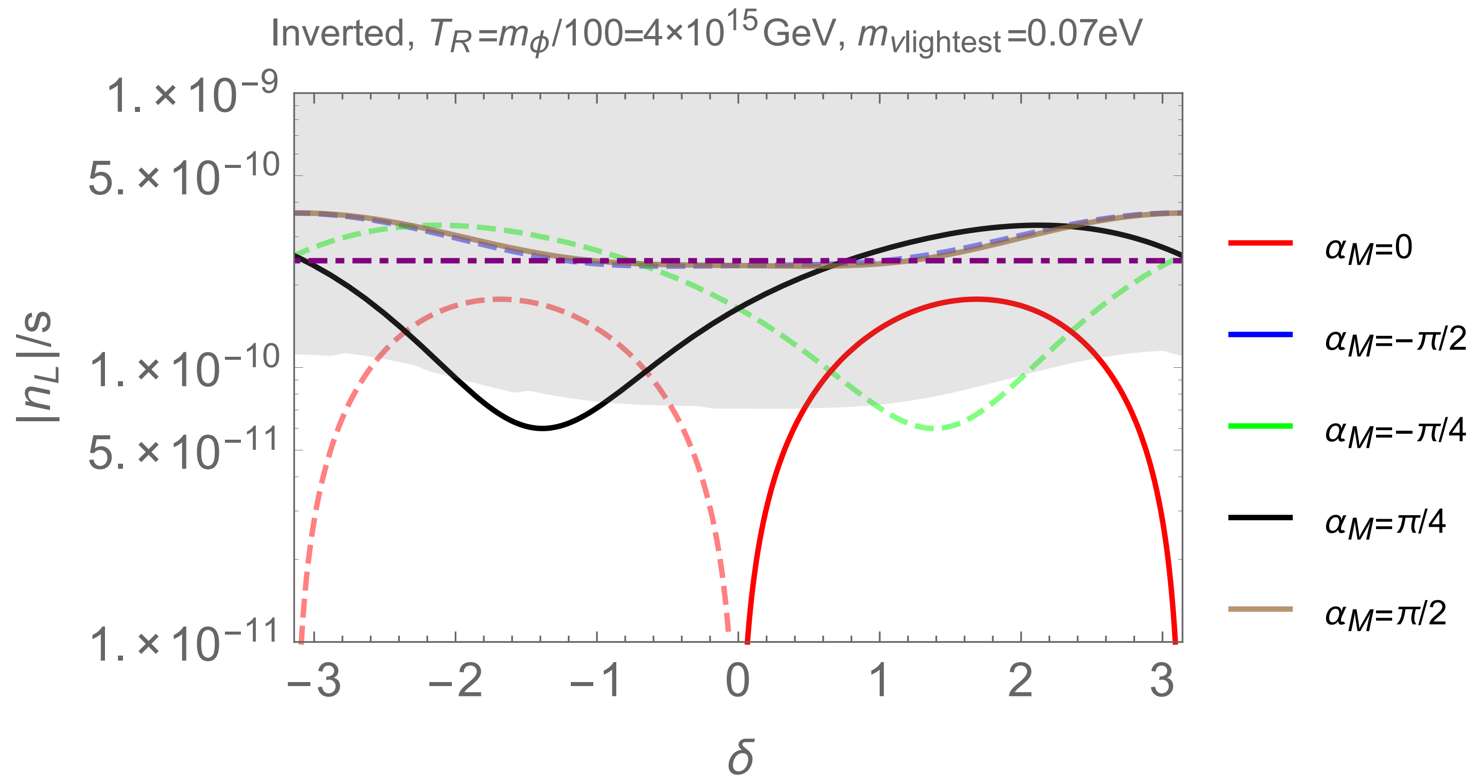
Normal, $T_R = m_\phi/100 = 2\times10^{12}$GeV, $m_{\nu\text{lightest}}$=0eV
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$1.\times10^{-11}$
$|n_L|$/s/B
−3
−2
−1
0
1
2
3
δ
α_M=0
$\alpha_M=-\pi/2$
$\alpha_M=-\pi/4$
$\alpha_M=\pi/4$
$\alpha_M=\pi/2$



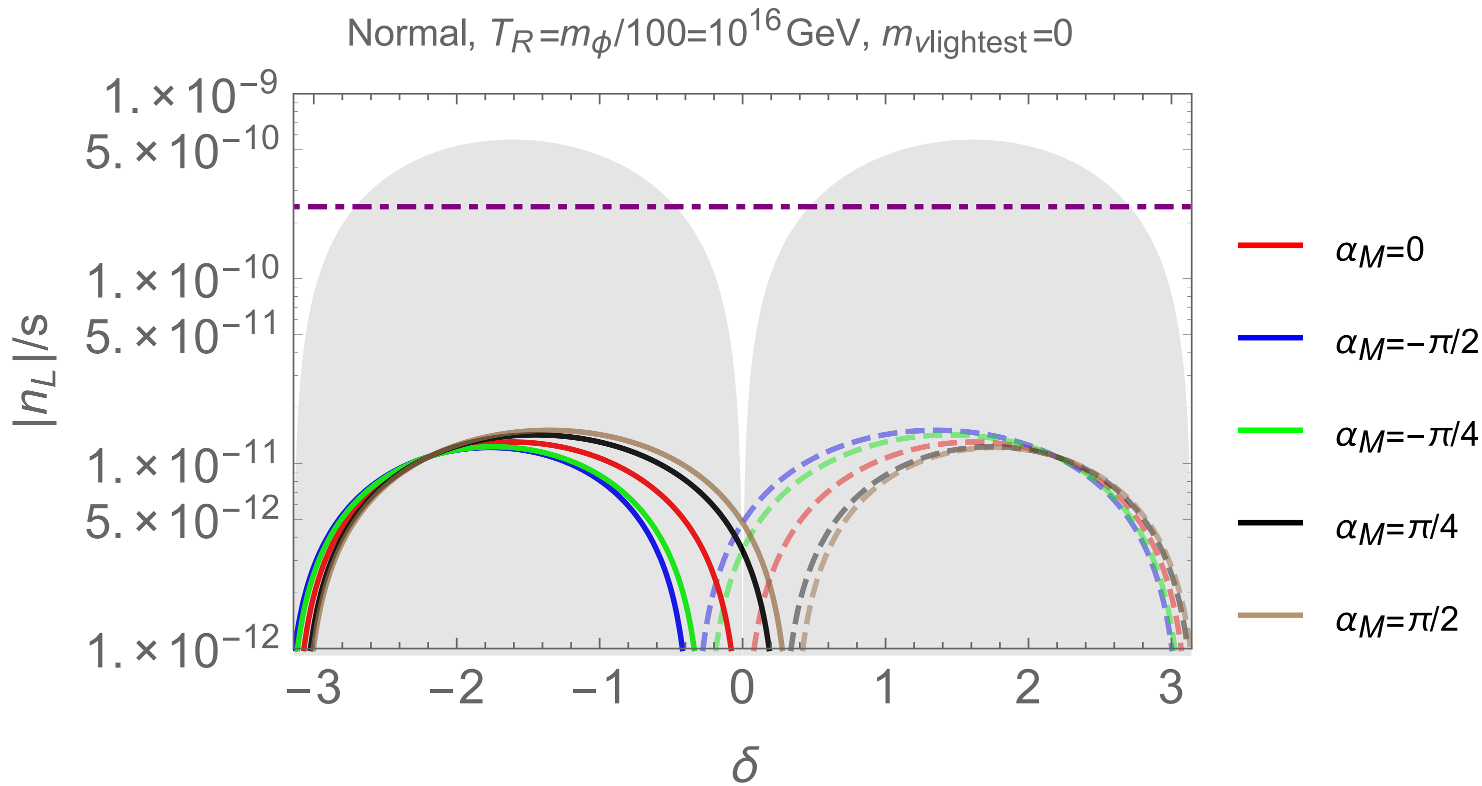
Normal, $T_R = m_\phi/100 = 7\times10^{11}$ GeV, $m_{\nu\text{lightest}} = 0.07$eV
$5.\times10^{-9}$
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$|n_L|/s/B$
−3 −2 −1 0 1 2 3
δ
$\alpha_M=0$
$\alpha_M=-\pi/2$
$\alpha_M=-\pi/4$
$\alpha_M=\pi/4$
$\alpha_M=\pi/2$



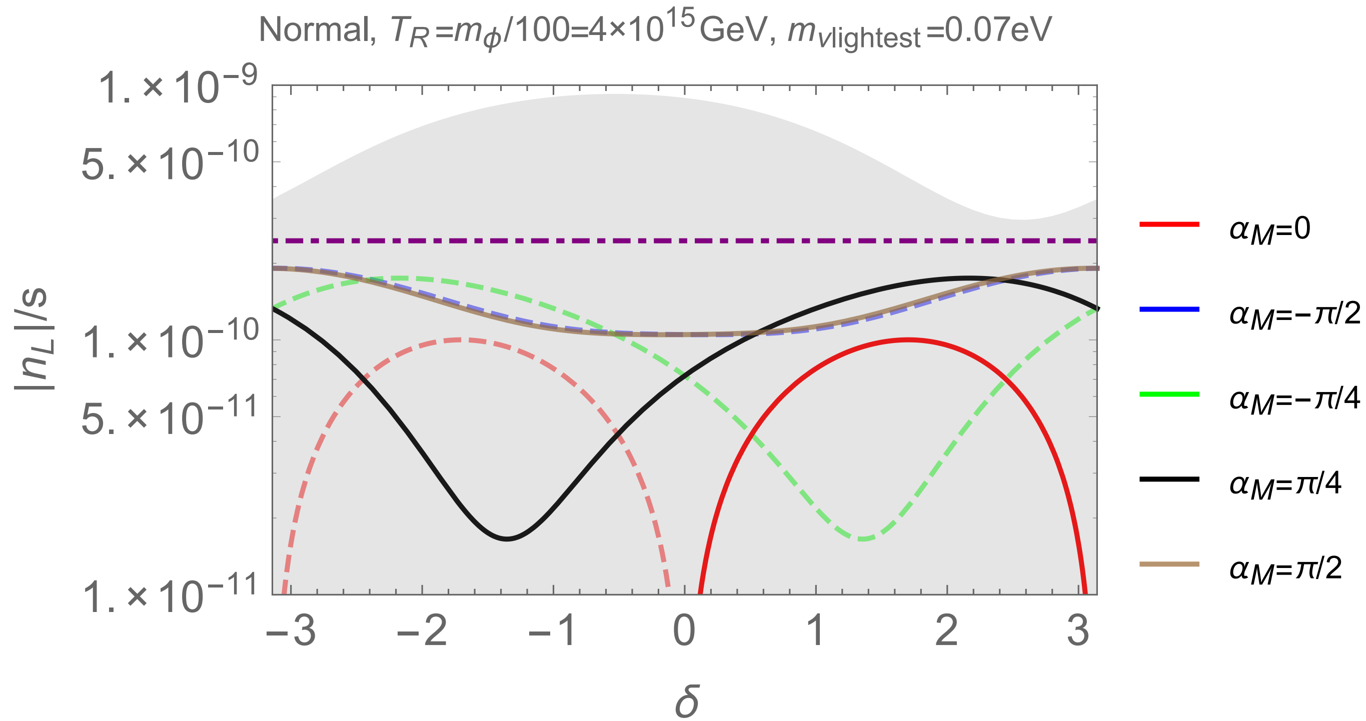
Inverted, $T_R = m_\phi/100 = 10^{16}$ GeV, $m_{\nu\text{lightest}} = 0$
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$1.\times10^{-11}$
$|n_L|/s$
−3
−2
−1
0
1
2
3
δ
$\alpha_M=0$
$\alpha_M=-\pi/2$
$\alpha_M=-\pi/4$
$\alpha_M=\pi/4$
$\alpha_M=\pi/2$



Inverted, $T_R = m_\phi/100 = 4\times10^{15}$ GeV, $m_{\nu\text{lightest}} = 0.07$eV
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$1.\times10^{-11}$
$|n_L|/s$
−3
−2
−1
0
1
2
3
δ
$\alpha_M=0$
$\alpha_M=-\pi/2$
$\alpha_M=-\pi/4$
$\alpha_M=\pi/4$
$\alpha_M=\pi/2$



Normal, $T_R=m_\phi/100=10^{16}$GeV, $m_{\nu\text{lightest}}=0$
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$1.\times10^{-11}$
$5.\times10^{-12}$
$1.\times10^{-12}$
$|n_L|/s$
−3
−2
−1
0
1
2
3
δ
α_M=0
α_M=−π/2
α_M=−π/4
α_M=π/4
α_M=π/2


Normal, $T_R = m_\phi/100 = 4\times10^{15}$GeV, $m_{\nu\text{lightest}} = 0.07$eV
$1.\times10^{-9}$
$5.\times10^{-10}$
$1.\times10^{-10}$
$5.\times10^{-11}$
$1.\times10^{-11}$
$|n_L|/s$
−3
−2
−1
0
1
2
3
δ
$\alpha_M=0$
$\alpha_M=-\pi/2$
$\alpha_M=-\pi/4$
$\alpha_M=\pi/4$
$\alpha_M=\pi/2$

$\delta = -3\pi/4,\ T_R \leq 10^{13}$GeV