

# Welcome to the 24<sup>th</sup> International Summer Institute on Phenomenology of Elementary Particle Physics and Cosmology (SI 2018)!

- SI 1995 – 2005: Fuji-Yoshida, Japan
- SI 2006: Pohang, Korea
- SI 2008: Chi-Tou, Taiwan
- SI 2015: Huarou, Beijing, China
- 2018, the 24<sup>th</sup> SI: PanShan, Tianjin, China  
by **Nankai University**

Thanks to the local organizing committee:

Profs. Xueqian Li, Lei Chang, Yuming Wang + ...

Thanks to the International Advisory Committee!

**Thank you all for coming!**

**Enjoy the conference and the scenery!**



# EW SECTOR @ HIGH ENERGIES

Univ. of Pittsburgh & Tsinghua University

Tao Han 韩涛

24<sup>th</sup> International Summer Institute

PanShan, Aug. 13, 2018



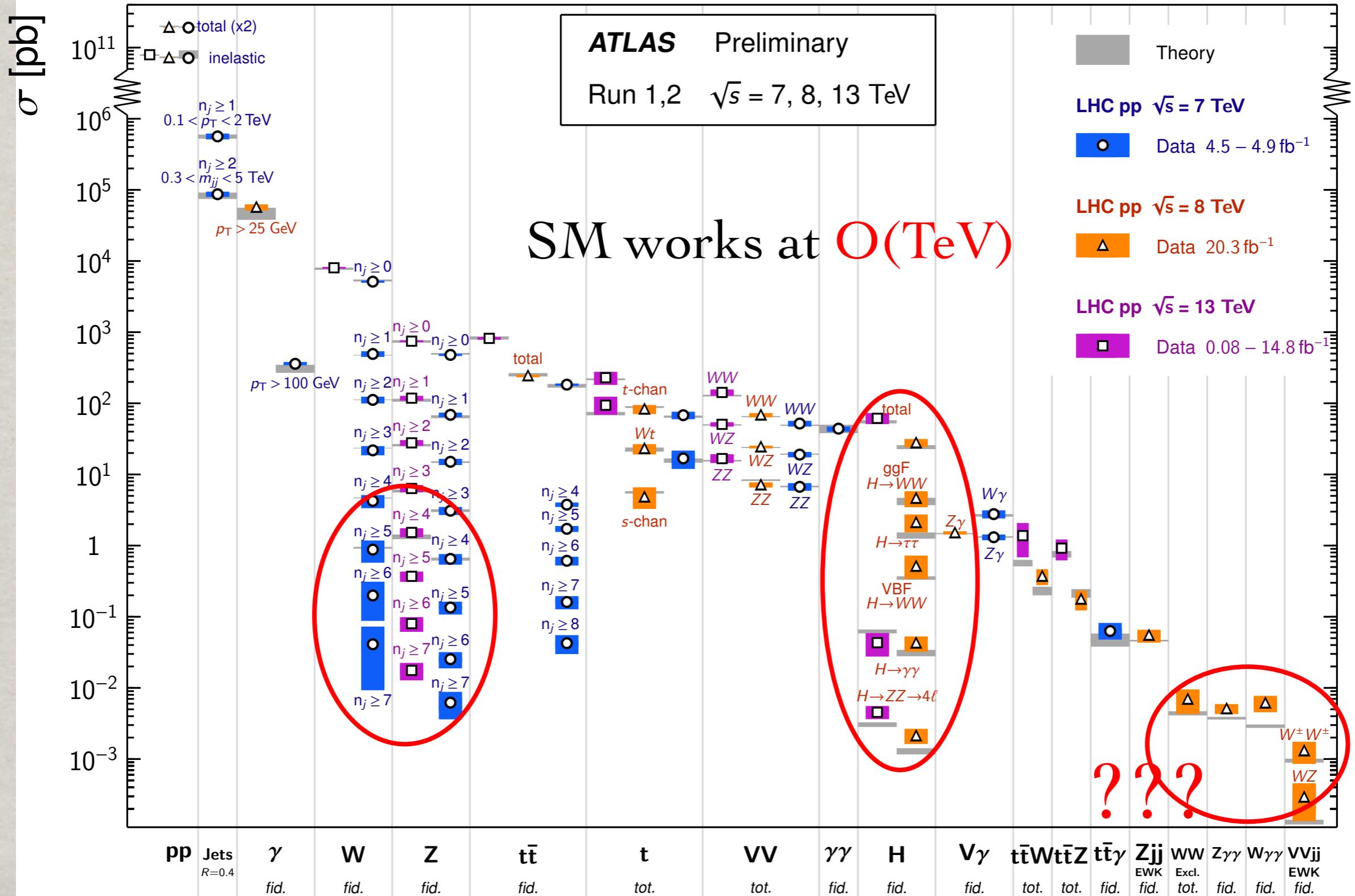
With J.M. Chen & B. Tweedie, arXiv:1611.00788; arXiv:18xx



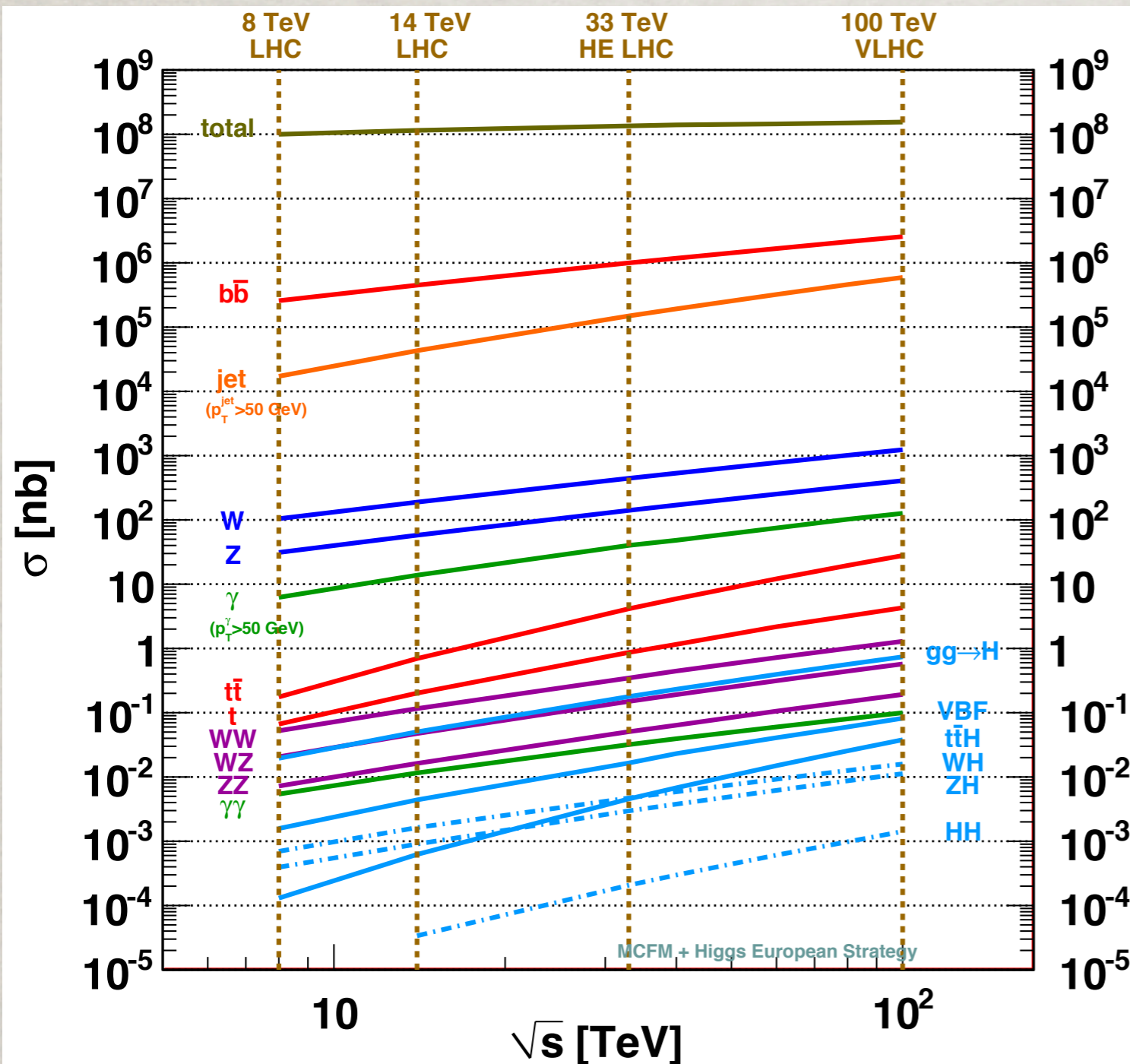
# LHC ROCKS!

## Standard Model Production Cross Section Measurements

Status: August 2016



# Future High Energy Frontier: FCC<sub>hh</sub>/SPPC



Process	$\sigma (100 \text{ TeV})/\sigma (14 \text{ TeV})$
Total pp	1.25
W	$\sim 7$
Z	$\sim 7$
WW	$\sim 10$
ZZ	$\sim 10$
tt	$\sim 30$
H	$\sim 15$ (ttH $\sim 60$ )
HH	$\sim 40$
stop (m=1 TeV)	$\sim 10^3$

Snowmass QCD Working Group: arXv:1310.5189;  
 N. Arkani-Hamed, TH, M. Mangano, L.-T. Wang, 1511.06495;  
 CERN Yellow books, + many others ...



# EW AT HIGHER ENERGIES

## Some numerology:

$$(1). \frac{E}{v} : G_F E_\beta^2 \sim \left( \frac{\text{MeV}}{M_W} \right)^2 \sim 10^{-8}, \quad \left( \frac{10 \text{ TeV}}{M_W} \right)^2 \sim 10^4 !$$

$$\epsilon_L^\mu(p) \sim \frac{p^\mu}{M_W} \rightarrow \text{need a proper treatment.}$$

$$(2). \frac{v}{E} : \frac{v}{100 \text{ TeV}} \sim \frac{\Lambda_{QCD}}{100 \text{ GeV}}$$

- $v/E$  power counting  $\rightarrow$  Higher twist effects.

$$v/E, m_t/E, M_W/E \rightarrow 0!$$

massless theory; EW symmetry restored !



# Some numerology:

$$(3). \quad \frac{m_t}{100 \text{ TeV}} \sim \frac{m_b}{2 \text{ TeV}}$$

The top quark at the FCC/SppC would be as “massless” as b-quark was at the Tevatron.

→ Top quark PDF? 6-flavors?

Daswon, Ismail, I. Low (2014);  
TH, Sayre, Westhoff (2015).

$$\text{At scale } Q: \quad \frac{\alpha_s}{\pi} C_F \ln \frac{Q^2}{m_t^2} \sim \delta$$

$$Q \approx m_t \cdot \exp\left(\frac{\pi\delta}{2\alpha_s C_F}\right)$$

For  $\delta = 20\% - 30\%$ ,  $\alpha_s \sim 0.08$ ,

$$Q = (25 - 110)m_t \Rightarrow (4 - 20) \text{ TeV.}$$



# Some numerology:

## (4). EW logarithms

$$\text{At scale } Q: \frac{\alpha_2}{\pi} C_w \ln^2 \frac{Q^2}{M_W^2} \sim \delta$$
$$Q \approx M_W \cdot \exp\left(\frac{\pi\delta}{4\alpha_2 C_w}\right)^{\frac{1}{2}}$$

J. Chiu, A. Manohar et al., 2005;  
Manohar, Bauer et al. (SCET);  
M. Chiesa et al., PRL (2013);  
T. Becher et al., 1305.4202;  
Bauer, Ferland, 1601.07190;

For  $\delta = 50\%$ ,  $\alpha_2 \sim 0.035$ ,

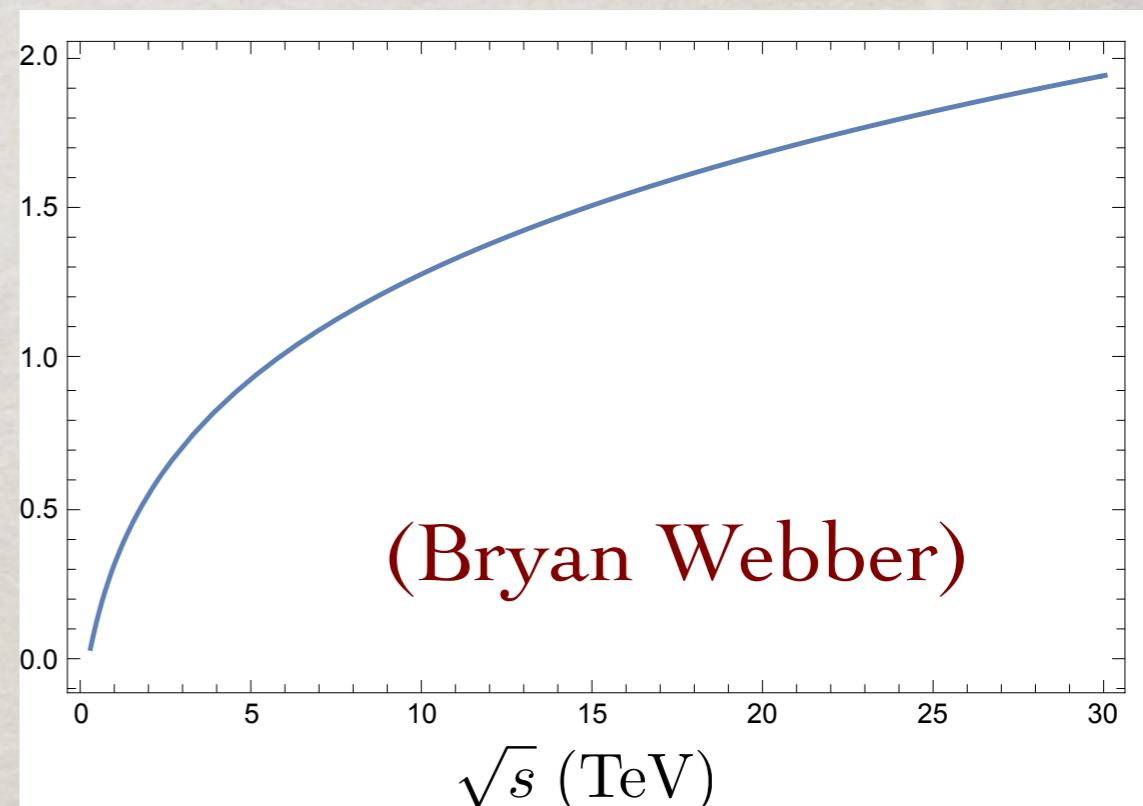
$Q \approx 30M_W \Rightarrow 2.5 \text{ TeV}$ .

- Virtual Sudakov suppression;
- Real emission enhancement.

SU(2) versus SU(3):

Gauge boson splitting

$$\text{“Color factors”} : \frac{C_A}{C_F} = \frac{2N^2}{N^2 - 1} \Rightarrow \left(\frac{9}{4}\right)_{N=3} \text{ and } \left(\frac{8}{3}\right)_{N=2}.$$



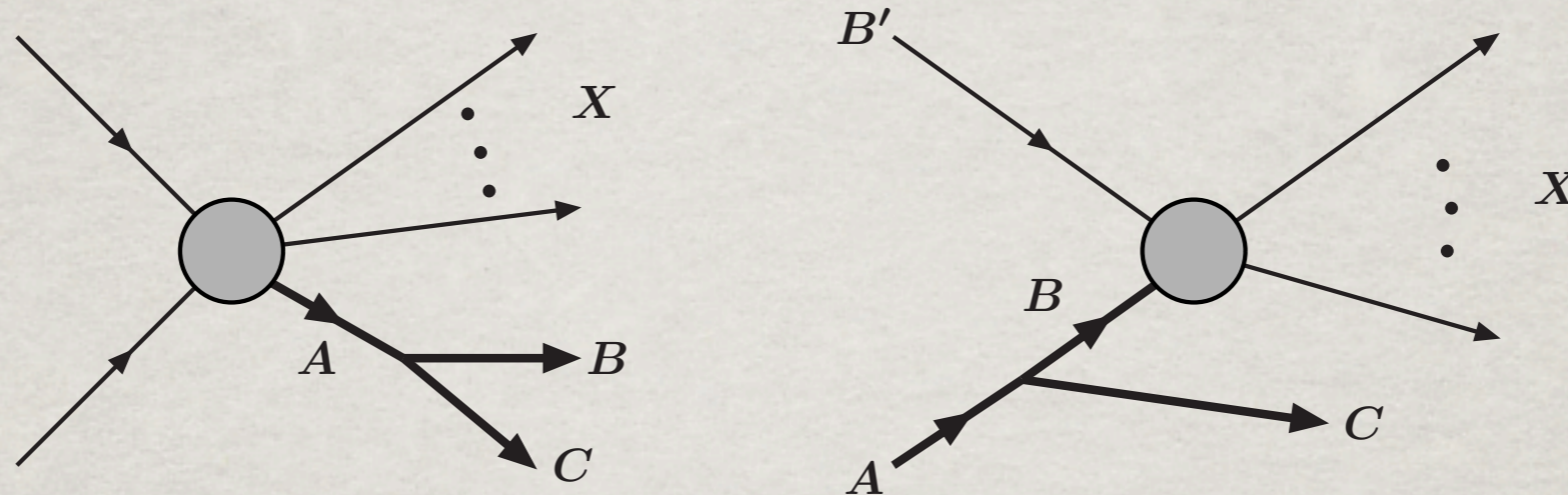


# TODAY:

1. EW SPLITTING FUNCTIONS
2. EW SHOWERING
3. EW PDF: FACTORIZATION,  
RESUMMATION (on going efforts)



# FORMALISM:



$$d\sigma_{X,BC} \simeq d\sigma_{X,A} \times d\mathcal{P}_{A \rightarrow B+C}$$

$$E_B \approx zE_A, \quad E_C \approx \bar{z}E_A, \quad k_T \approx z\bar{z}E_A\theta_{BC}$$

$$\frac{d\mathcal{P}_{A \rightarrow B+C}}{dz dk_T^2} \simeq \frac{1}{16\pi^2} \frac{z\bar{z} |\mathcal{M}^{(\text{split})}|^2}{(k_T^2 + \bar{z}m_B^2 + zm_C^2 - z\bar{z}m_A^2)^2}$$

On the dimensional ground:  $|\mathcal{M}_{split}|^2 \sim k_T^2$  or  $m^2$

In general, the splitting formalism must be

- infra-red safe
- leading behavior

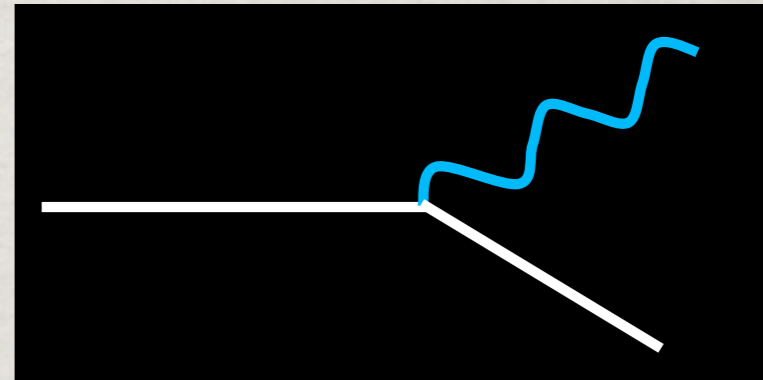


# SPLITTING FUNCTIONS: QED

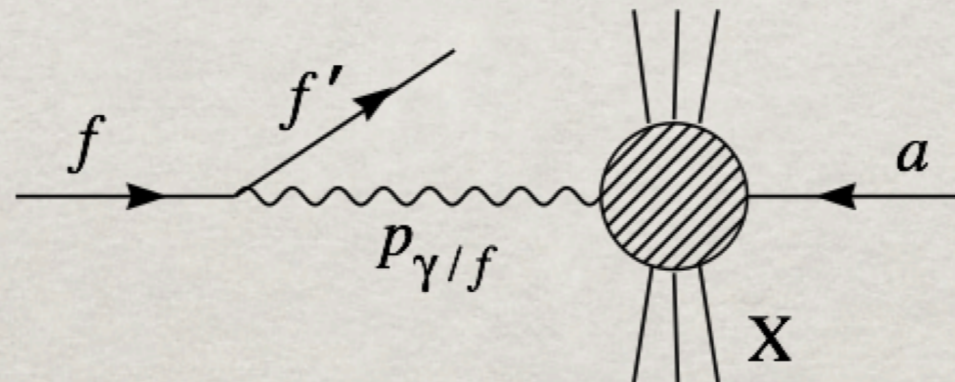
Most familiar example in QED:  $f \rightarrow f \gamma$

$$p_{\gamma/f}(z) = \frac{1 + \bar{z}}{z}, \quad \bar{z} = 1 - z.$$

$$P_{\gamma/f}(z) = \frac{\alpha}{2\pi} \frac{1 + \bar{z}}{z} \ln \frac{Q^2}{m_f^2}.$$



The familiar Weizsäcker-Williams approximation



$$\sigma(fa \rightarrow f'X) \approx \int dx dp_T^2 P_{\gamma/f}(x, p_T^2) \sigma(\gamma a \rightarrow X),$$

$$P_{\gamma/e}(x, p_T^2) = \frac{\alpha}{2\pi} \frac{1 + (1-x)^2}{x} \left(\frac{1}{p_T^2}\right) \Big|_{m_e}^E.$$

Note the infrared & collinear behavior.



# SPLITTING FUNCTIONS: QCD

Most common in hadronic collisions:  $q, g$

$$P_{gq}(z) = \frac{1 + \bar{z}^2}{z}, \quad P_{gg}(z) = \frac{(1 - z\bar{z})^2}{z\bar{z}}, \quad P_{qq}(z) = \frac{z^2 + \bar{z}^2}{2}.$$

ISR, PDF (DGLAP):

$$f_B(z, \mu^2) = \sum_A \int_z^1 \frac{d\xi}{\xi} f_A(\xi) \int_{m^2}^{\mu^2} d\mathcal{P}_{A \rightarrow B+C}(z/\xi, k_T^2).$$

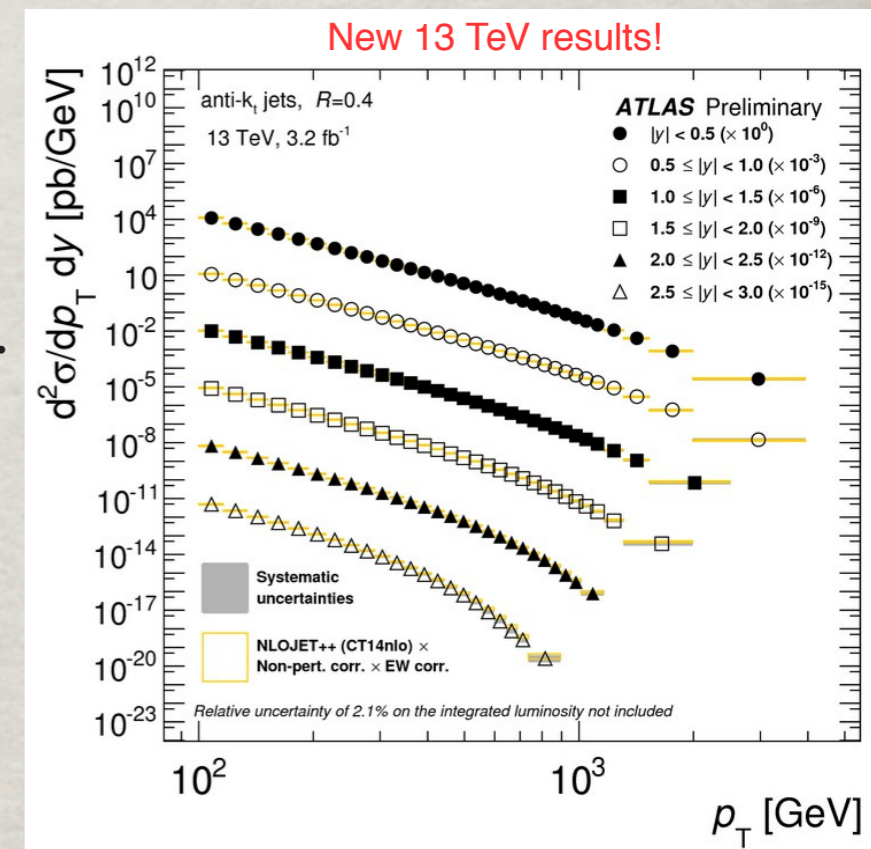
$$\frac{\partial f_B(z, \mu^2)}{\partial \mu^2} = \sum_A \int_z^1 \frac{d\xi}{\xi} \frac{d\mathcal{P}_{A \rightarrow B+C}(z/\xi, \mu^2)}{dz dk_T^2} f_A(\xi, \mu^2).$$

FSR, parton showers:

$$\Delta_A(t) = \exp\left[-\sum_B \int_{t_0}^t \int dz P_{A \rightarrow BC}(z)\right],$$

$$f_A(x, t) = \Delta_A(t) f_A(x, t_0) + \int_{t_0}^t \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} P_{A \rightarrow BC}(z) f_A(x/z, t')$$

Very important formulation for LHC physics!





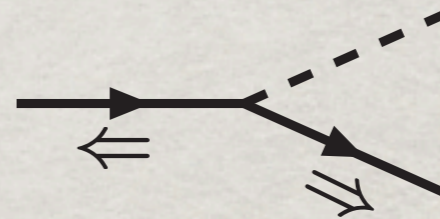
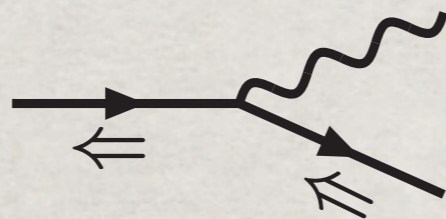
# SPLITTING FUNCTIONS: EW

Start from the unbroken phase – all massless.

$$\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{Yuk}$$

Chiral fermions:  $f_s$ , gauge bosons:  $B, W^0, W^\pm$ ;  $H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(h - i\phi^0) \end{pmatrix}$

## Fermion splitting:



	$\frac{1}{8\pi^2} \frac{1}{k_T^2} \left( \frac{1 + \bar{z}^2}{z} \right)$	$\frac{1}{8\pi^2} \frac{1}{k_T^2} \left( \frac{z}{2} \right)$	Ciafaloni et al., Hep-ph/0505047.
	$\rightarrow V_T f_s^{(\prime)}$ $[BW]_T^0 f_s$	$H^{0(*)} f_{-s}$ or $\phi^\pm f'_{-s}$	
$f_{s=L,R}$	$g_V^2 (Q_{f_s}^V)^2$	$g_1 g_2 Y_{f_s} T_{f_s}^3$	$y_{f_R}^2$

Infrared & collinear singularities ( $P_{gq}$ )

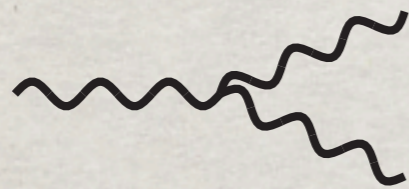
Collinear singularity,  
Chirality-flip, Yukawa (new)



# SPLITTING FUNCTIONS: EW

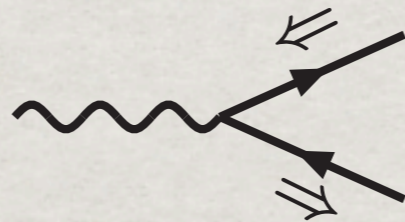
SM in the unbroken phase

Gauge boson splitting:



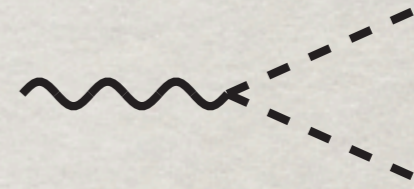
$$\frac{1}{8\pi^2} \frac{1}{k_T^2} \left( \frac{(1 - z\bar{z})^2}{z\bar{z}} \right)$$

$\rightarrow W_T W_T$



$$\frac{1}{8\pi^2} \frac{1}{k_T^2} \left( \frac{z^2 + \bar{z}^2}{2} \right)$$

$f_s \bar{f}_{-s}^{(f)}$



$$\frac{1}{8\pi^2} \frac{1}{k_T^2} (z\bar{z})$$

$\phi^+ \phi^-$  or  $H^0 H^{0*}$      $\phi^+ H^{0*}$  or  $\phi^- H^0$

$V_T$

$$2g_2^2 (V = W^{0,\pm})$$

$$N_f g_V^2 (Q_{f_s}^V)^2$$

$$\frac{1}{4} g_V^2$$

$$\frac{1}{2} g_2^2$$

$[BW]_T^0$

$$0$$

$$N_f g_1 g_2 Y_{f_s} T_{f_s}^3$$

$$\frac{1}{2} g_1 g_2 T_{\phi^+, H^0}^3$$

$$0$$

Infrared &  
collinear ( $P_{gg}$ )

Collinear ( $P_{qg}$ )  
Interference ( $BW^0$ )  
must be included!

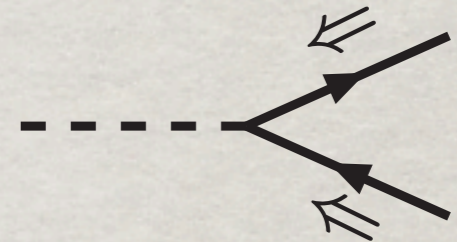
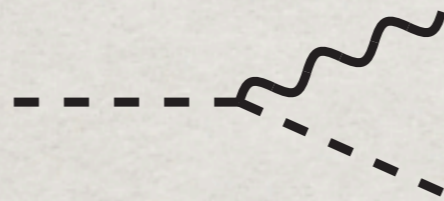
Collinear (new)



# SPLITTING FUNCTIONS: EW

SM in the unbroken phase

Scalar splitting (new):



	$\frac{1}{8\pi^2} \frac{1}{k_T^2} \begin{pmatrix} 2\bar{z} \\ z \end{pmatrix}$	$\frac{1}{8\pi^2} \frac{1}{k_T^2} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
$\rightarrow V_T^0 H$	$[BW]_T^0 H$	$W_T^\pm H'$
$H = \phi^+, H^0$	$\frac{1}{4}g_V^2$	$\frac{1}{2}g_1g_2T_{\phi^+,H^0}^3$
	$\frac{1}{2}g_2^2$	$3y_u^2$
		$N_{d,e}y_{d,e}^2$

Infrared & collinear singularities  
(a charge source, similar to  $P_{gq}$ )

Collinear,  
similar to  $(P_{qg})$



# EW Symmetry breaking & Goldstone-boson Equivalence Theorem (GET):

Lee, Quigg, Thacker (1977); Chanowitz & Gailard (1984)

At high energies  $E \gg M_W$ , the longitudinally polarized gauge bosons behave like the corresponding Goldstone bosons. (They remember their origin!)

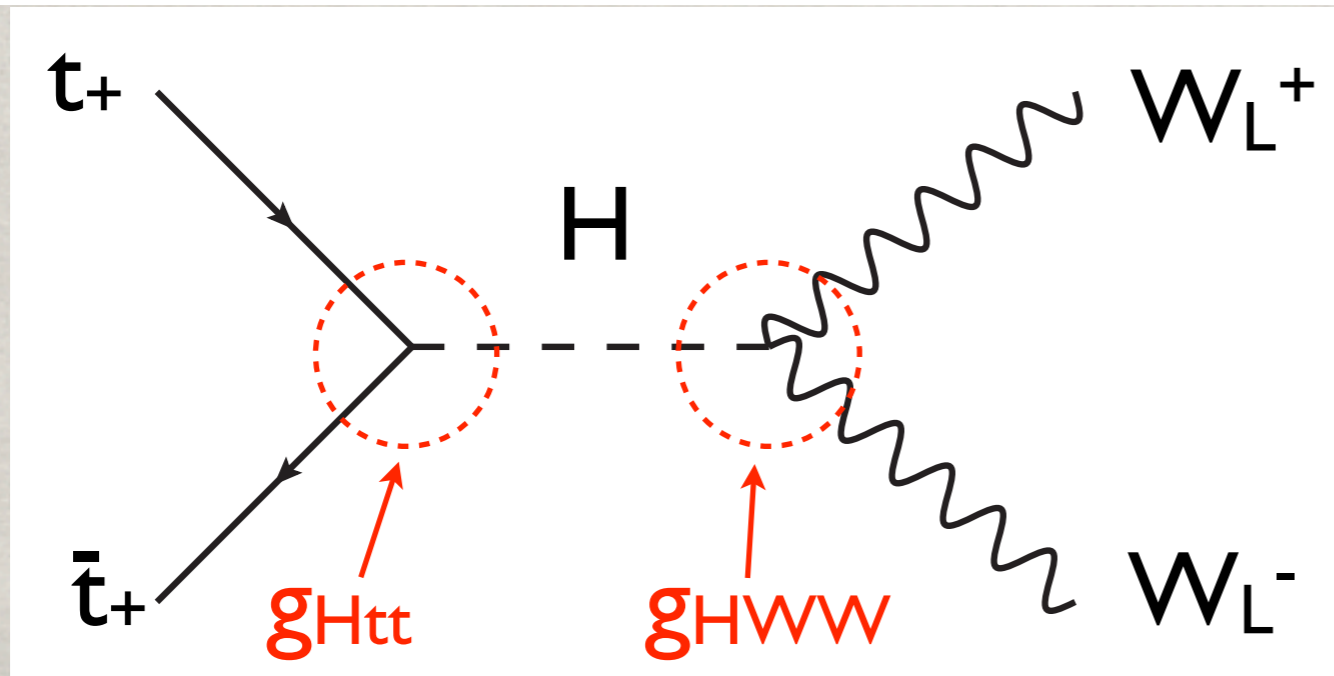
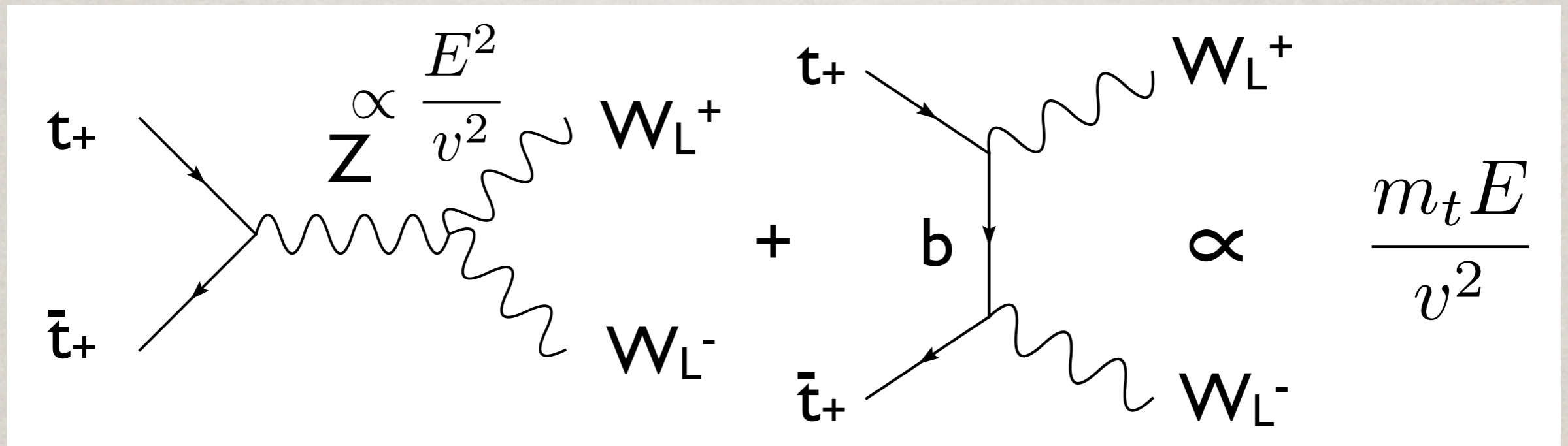
“Scalarization” to implement the Goldstone-boson Equivalence Theorem (GET):

$$\epsilon(k)_L^\mu = \frac{E}{m_W} (\beta_W, \hat{k}) \approx \frac{k^\mu}{m_W} + \mathcal{O}(M_W/E)$$



# (a). Unitarity at higher energies:

$$\epsilon(k)_L^\mu = \frac{E}{m_W} (\beta_W, \hat{k}) \approx \frac{k^\mu}{m_W} \quad \text{bad high-energy behavior!}$$



A “light Higgs” fixes it:

$$\propto \frac{m_t m_H}{v^2}$$

D. Dicus & V. Mathur (1973);  
Lee, Quigg, Thacker (1977).



## (b). Puzzle of massless fermion radiation

$V_L$  contributions dominant at high energies:

$$\epsilon(k)_L^\mu = \frac{E}{m_W} (\beta_W, \hat{k}) \approx \frac{k^\mu}{m_W}$$

Then, massless fermion splitting

$$f \rightarrow f V_L$$

would be zero, in accordance with GET for

$$f \rightarrow f \phi \quad (y_f \rightarrow 0).$$

GET ignored the EWSB effects at the order  $M_W/E$   
(higher twist effects)



# Corrections to GET

## 1<sup>st</sup> example: “Effective $W$ -Approximation”

S. Dawson (1985); G. Kane et al. (1984);  
Chanowitz & Gailard (1984)

At colliding energies  $E \gg M_W$ ,

$$P_{q \rightarrow q V_T} = (g_V^2 + g_A^2) \frac{\alpha_2}{2\pi} \frac{1 + (1-x)^2}{x} \ln \frac{Q^2}{\Lambda^2}$$

$$P_{q \rightarrow q V_L} = (g_V^2 + g_A^2) \frac{\alpha_2}{\pi} \frac{1-x}{x}$$

- Vector boson fusion observed at the LHC  
 $WW, ZZ \rightarrow h$  &  $W^+W^+$  scattering
- $f \rightarrow f W_L, f Z_L$  do not vanish; no collinear-log!

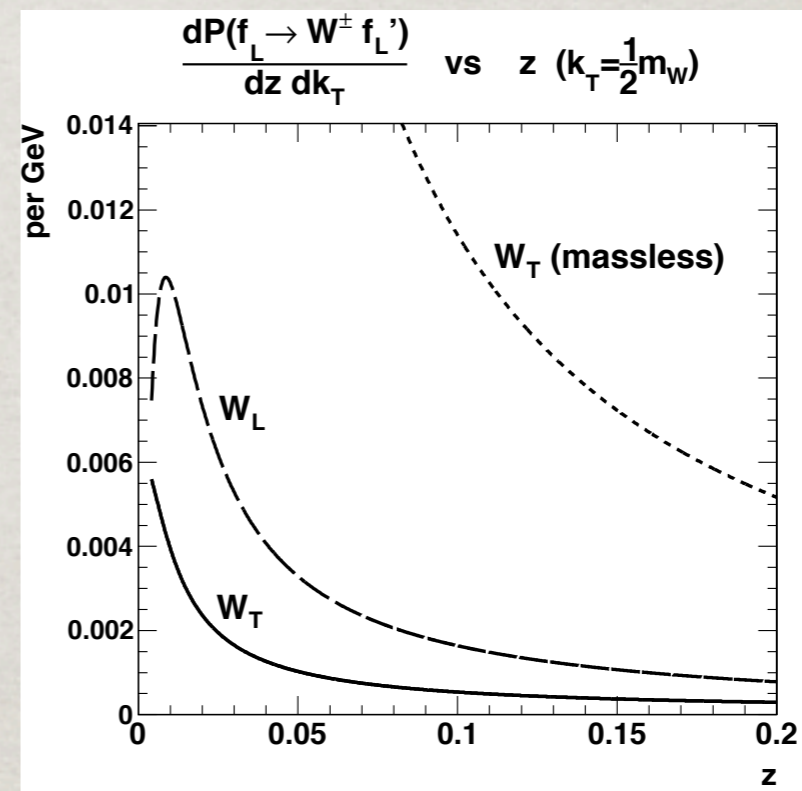
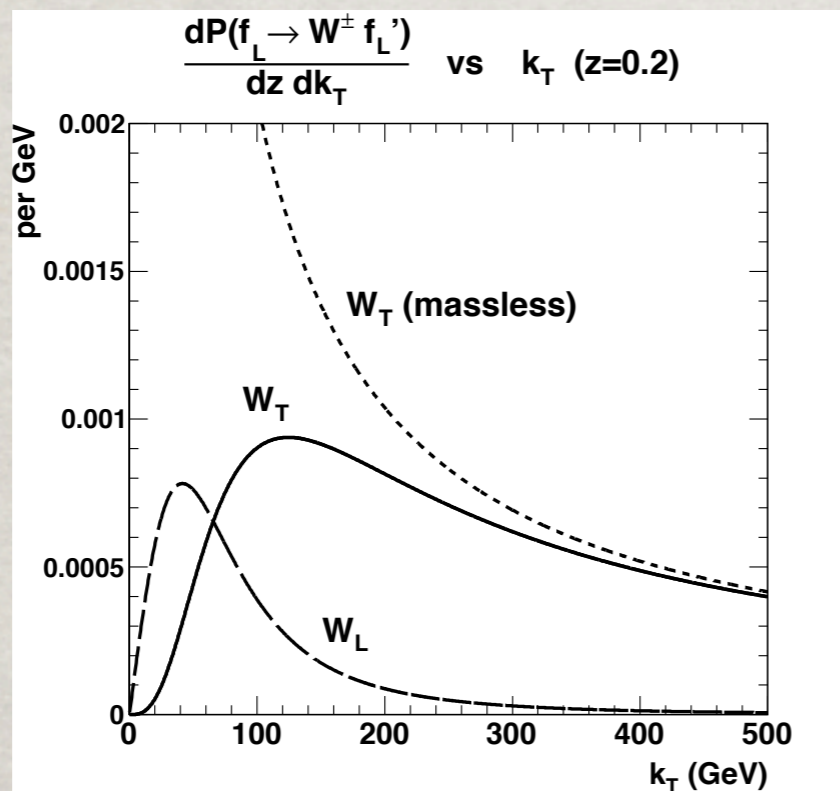


# “Ultra collinear behavior”

New characteristics with the mass:

$$k_T^2 > m_W^2, \text{ it shuts off; } \frac{v^2}{k_T^2} \frac{dk_T^2}{k_T^2} \sim \left(1 - \frac{v^2}{Q^2}\right)$$

$$k_T^2 < m_W^2, \text{ flattens out!}$$



- Kinematic basis for “forward jet-tagging, central jet-vetoing” ! Barger, Cheung, TH, Phillips (1989).
- The DPFs for  $W_L$  thus don't run at leading log: “Bjorken scaling” restored (higher-twist effects)!



# “GOLDSTONE EQUIVALENCE GAUGE” (GEG)

$$\epsilon(k)_L^\mu = \frac{E}{m_W} (\beta_W, \hat{k}) = \frac{k^\mu}{m_W} - \frac{m_W}{E + |\vec{k}|} n^\mu, \quad n^\mu = (1, -\hat{k}).$$

1<sup>st</sup> term leads to GET  $\sim \phi$ , well behaved;

2<sup>nd</sup> term captures EWSB  $\sim A_n^\mu$ , well behaved

Separate them out by a special gauge choice:

(hybrid of Coulomb & light-cone gauge)

$$\mathcal{L}_{\text{fix}} = -\frac{1}{2\xi} [n(k) \cdot W(k)] [n(k) \cdot W(-k)] \quad (\xi \rightarrow 0)$$

$$n^0(k) \equiv 1, \quad \vec{n}(k) \equiv -\frac{k^0}{|k^0|} \frac{\vec{k}}{|\vec{k}|},$$

$$\epsilon_{\text{long}}^\mu(k) \rightarrow \frac{\sqrt{|k^2|}}{n(k) \cdot k} n^\mu(k) \xrightarrow{\text{on-shell}} \frac{m_W}{E + |\vec{k}|} (-1, \hat{k}).$$

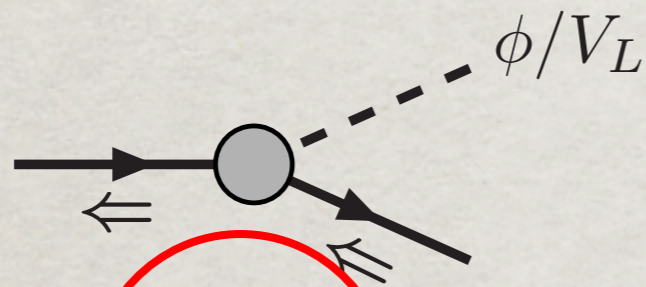
A similar work by A. Wulzer, arXiv:1703.08562.



# SPLITTING IN THE BROKEN GAUGE CORRECTIONS TO GET

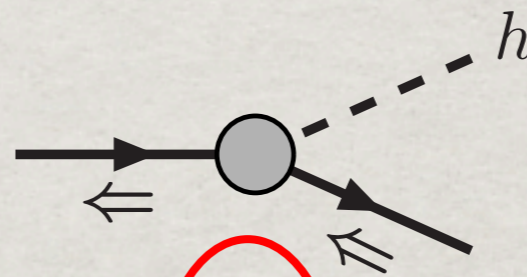
New fermion splitting:  $\frac{v^2}{k_T^2} \frac{dk_T^2}{k_T^2} \sim \left(1 - \frac{v^2}{Q^2}\right)$

$V_L$  is of IR, h no IR



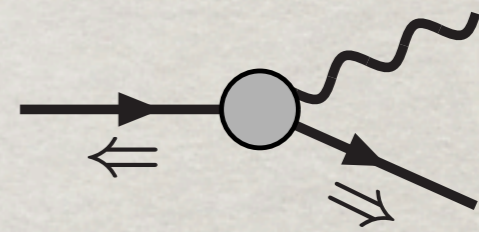
$$\frac{1}{16\pi^2} \frac{v^2}{\tilde{k}_T^4} \begin{pmatrix} 1 \\ z \end{pmatrix}$$

$$\rightarrow V_L f_s^{(\prime)} \quad (V \neq \gamma)$$



$$\frac{1}{16\pi^2} \frac{v^2}{\tilde{k}_T^4}$$

$$h f_s$$



$$\frac{1}{16\pi^2} \frac{v^2}{\tilde{k}_T^4}$$

$$V_T f_{-s}^{(\prime)}$$

$f_{s=L}$	$(I_f^V (y_f^2 \bar{z} - y_{f^{(\prime)}}^2) z - \underline{Q_{f_L}^V g_V^2 \bar{z}})^2$	$\frac{1}{4} y_f^4 z (1 + \bar{z})^2$	$g_V^2 z (Q_{f_R}^V y_f \bar{z} - Q_{f_L}^V y_{f^{(\prime)}})^2$
$f_{s=R}$	$(I_f^V y_f y_{f^{(\prime)}} z^2 - \underline{Q_{f_R}^V g_V^2 \bar{z}})^2$	$\frac{1}{4} y_f^4 z (1 + \bar{z})^2$	$g_V^2 z (Q_{f_L}^V y_f \bar{z} - Q_{f_R}^V y_{f^{(\prime)}})^2$

Chirality conserving:  
Non-zero for massless f

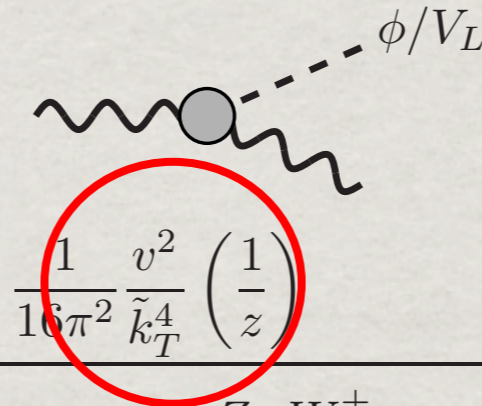
Chirality flipping:  $\sim m_f$



# SPLITTING IN THE BROKEN GAUGE

## New gauge boson splitting to $W_L W_T$

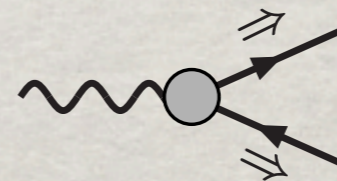
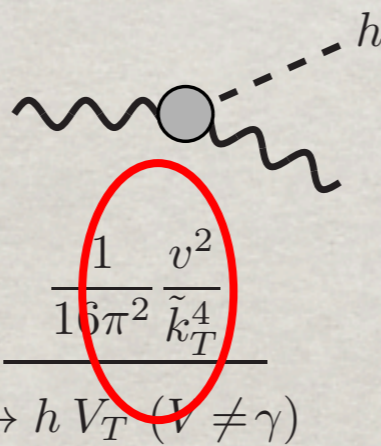
Vector boson  $V_L$  is of IR.



$$\frac{v^2}{k_T^2} \frac{dk_T^2}{k_T^2} \sim \left(1 - \frac{v^2}{Q^2}\right)$$

	$\rightarrow W_L^\pm \gamma_T$	$W_L^\pm Z_T$	$Z_L W_T^\pm$	$W_L^+ W_T^-$ or $W_L^- W_T^+$
$W_T^\pm$	$e^2 g_2^2 \bar{z}^3$	$\frac{1}{4} c_W^2 g_2^4 \bar{z} \left( (1 + \bar{z}) + t_W^2 z \right)^2$	$\frac{1}{4} g_2^4 \bar{z} (1 + \bar{z})^2$	0
$\gamma_T$	0	0	0	$e^2 g_2^2 \bar{z}$
$Z_T$	0	0	0	$\frac{1}{4} c_W^2 g_2^4 \bar{z} \left( (1 + \bar{z}) - t_W^2 z \right)^2$
$[\gamma Z]_T$	0	0	0	$\frac{1}{2} c_W e g_2^3 \bar{z} \left( (1 + \bar{z}) - t_W^2 z \right)$

$h$  &  $f$  have no IR.



	$\rightarrow h V_T \ (V \neq \gamma)$	$f_s \bar{f}_s^{(\prime)}$
$V_T$	$\frac{1}{4} z \bar{z} g_V^4$	$\frac{1}{2} g_V^2 \left( Q_{f_s}^V y_{f^{(\prime)}} z + Q_{f-s}^V y_f \bar{z} \right)^2$
$[\gamma Z]_T$	0	$\frac{1}{2} e g_Z y_f^2 Q_f^\gamma \left( Q_{f_s}^Z z + Q_{f-s}^Z \bar{z} \right)$



# SPLITTING IN THE BROKEN GAUGE

## New gauge boson splitting in $3-W_L$

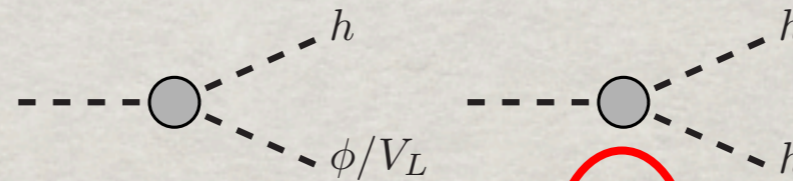
Vector boson  $V_L$  is of IR.

$$\frac{1}{16\pi^2} \frac{v^2}{\tilde{k}_T^4} \left( \frac{1}{z\bar{z}} \right)$$

$$\frac{v^2}{k_T^2} \frac{dk_T^2}{k_T^2} \sim \left( 1 - \frac{v^2}{Q^2} \right)$$

	$\rightarrow W_L^+ W_L^-$	$Z_L W_L^\pm / Z_L$
$W_L^\pm$	0	$\frac{1}{16} g_2^4 ((\bar{z} - z)(2 + z\bar{z}) - t_W^2 \bar{z}(1 + \bar{z}))^2$
$h$	$\frac{1}{4} (g_2^2(1 - z\bar{z}) - \lambda_h z\bar{z})^2$	$\frac{1}{8} (g_Z^2(1 - z\bar{z}) - \lambda_h z\bar{z})^2$
$Z_L$	$\frac{1}{16} g_2^4 ((\bar{z} - z)(2 + z\bar{z} - t_W^2 z\bar{z}))^2$	0
$[hZ_L]$	$\frac{i}{8} g_2^2 (g_2^2(1 - z\bar{z}) - \lambda_h z\bar{z}) (\bar{z} - z) (2 + z\bar{z} - t_W^2 z\bar{z})$	0

$h$  has no IR.



	$\rightarrow h W_L^\pm / Z_L$	$h h$
$W_L^\pm$	$\frac{1}{4} z (g_2^2(1 - z\bar{z}) + \lambda_h \bar{z})^2$	0
$h$	0	$\frac{9}{8} \lambda_h^2 z\bar{z}$
$Z_L$	$\frac{1}{4} z (g_Z^2(1 - z\bar{z}) + \lambda_h \bar{z})^2$	0
$[hZ_L]$	0	0



# SPLITTING PROBABILITIES:

Process	gauge couplings $\approx \mathcal{P}(E)$	$\mathcal{P}(1 \text{ TeV})$	$\mathcal{P}(10 \text{ TeV})$
$q \rightarrow V_T q^{(\prime)}$ (CL+IR)	$(3 \times 10^{-3}) \left[ \log \frac{E}{m_W} \right]^2$	3%	7%
$q \rightarrow V_L q^{(\prime)}$ (UC+IR)	$(2 \times 10^{-3}) \log \frac{E}{m_W}$	0.8%	1.1%
$t_R \rightarrow W_L^+ b_L$ (CL)	$(8 \times 10^{-3}) \log \frac{E}{m_W}$	2%	4%
$t_R \rightarrow W_T^+ b_L$ (UC)	$(6 \times 10^{-3})$	0.6%	0.6%
$V_T \rightarrow V_T V_T$ (CL+IR)	$(0.015) \left[ \log \frac{E}{m_W} \right]^2$	8%	36%
$V_T \rightarrow V_L V_T$ (UC+IR)	$(0.014) \log \frac{E}{m_W}$	3%	7%
$V_T \rightarrow f \bar{f}$ (CL)	$(0.02) \log \frac{E}{m_W}$	5%	10%
$V_L \rightarrow V_T h$ (CL+IR)	$(2 \times 10^{-3}) \left[ \log \frac{E}{m_W} \right]^2$	1%	4%
$V_L \rightarrow V_L h$ (UC+IR)	$(2 \times 10^{-3}) \log \frac{E}{m_W}$	0.4%	1%

- Non-Abelian gauge splitting larger than fermion splitting!
- Collinear splittings larger than perturbative radiation!

**NOW SOME RESULTS AT 100 TEV →**



# MULTI GAUGE-BOSON PRODUCTION

W radiation costs  $\sim 1/10$

At 100 TeV: M. Mangano's talk  
Diagrammatic calculations

W W  $\sigma = 770$  pb

W W W  $\sigma = 2$  pb

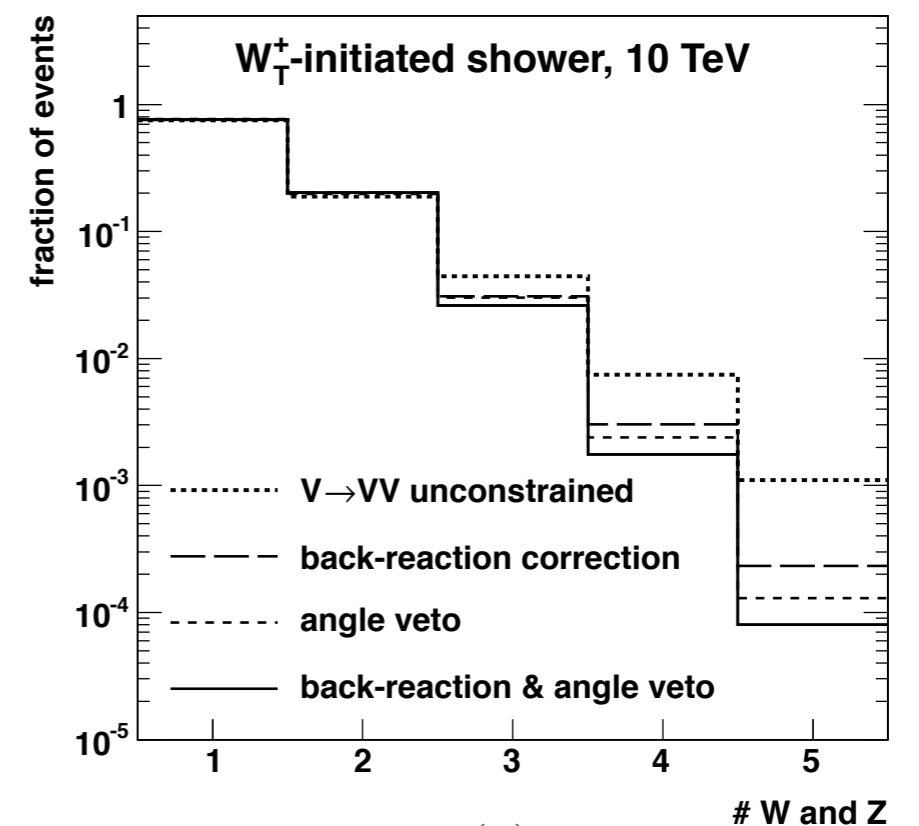
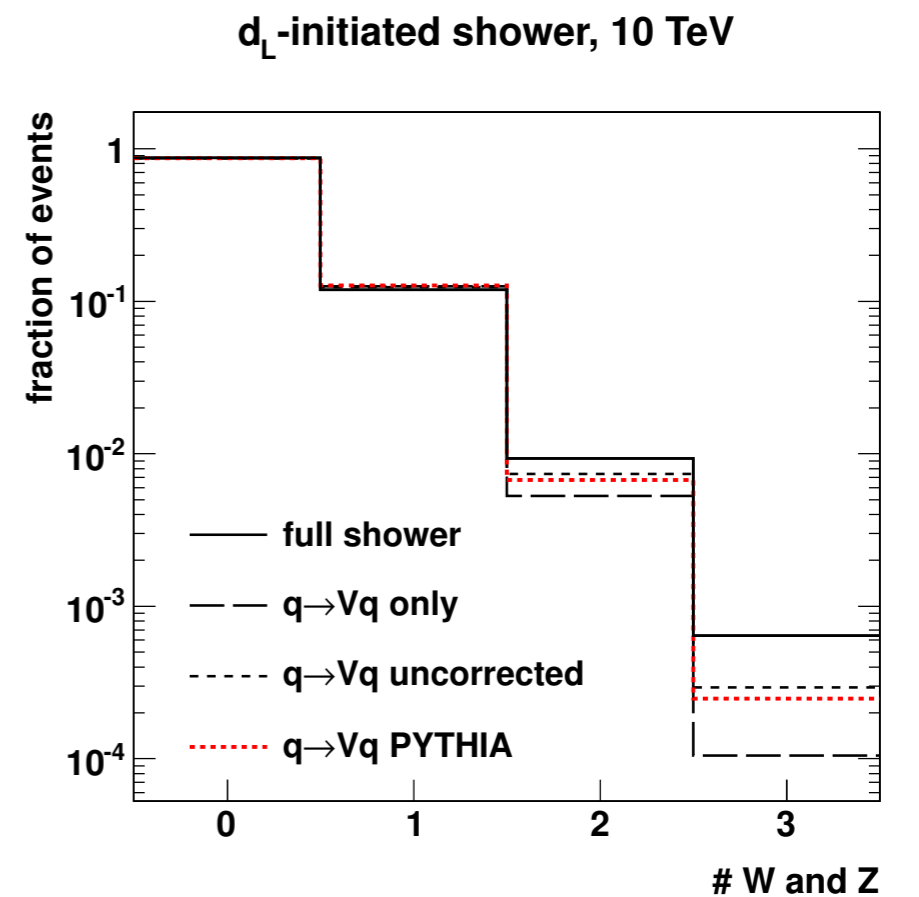
W W Z  $\sigma = 1.6$  pb

W W W W  $\sigma = 15$  fb

W W W Z  $\sigma = 20$  fb

....

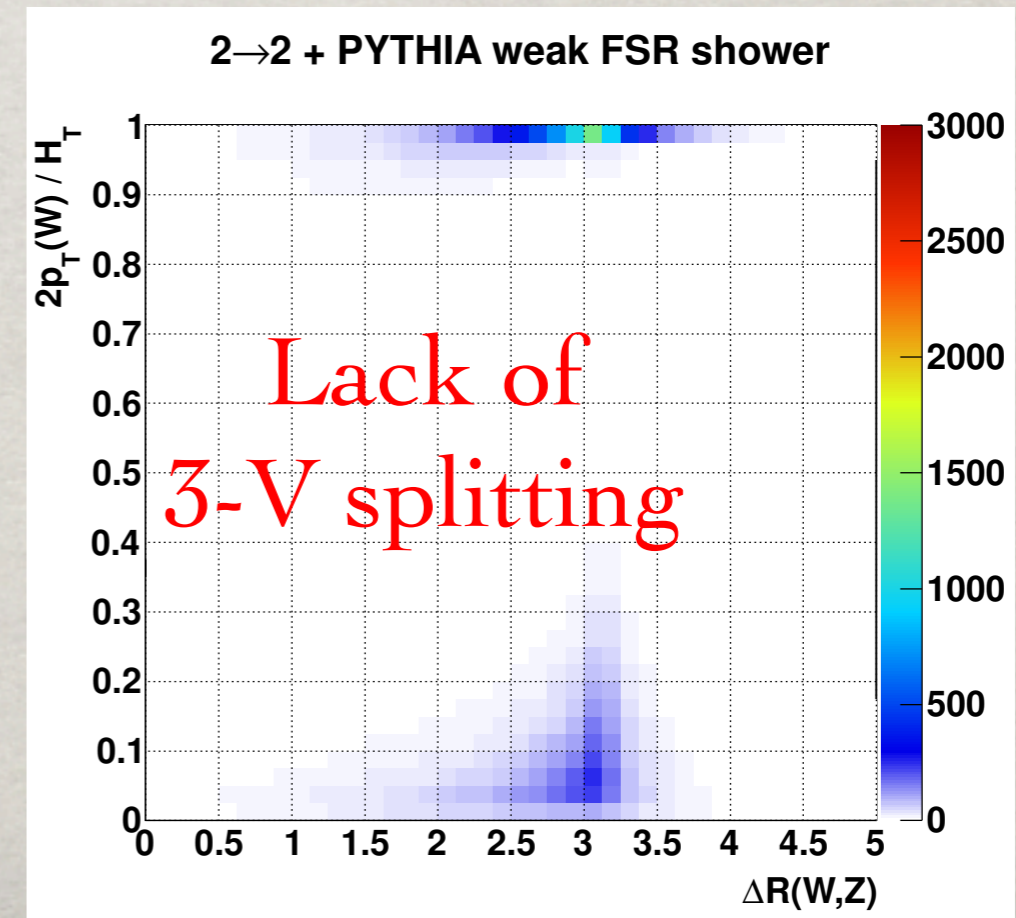
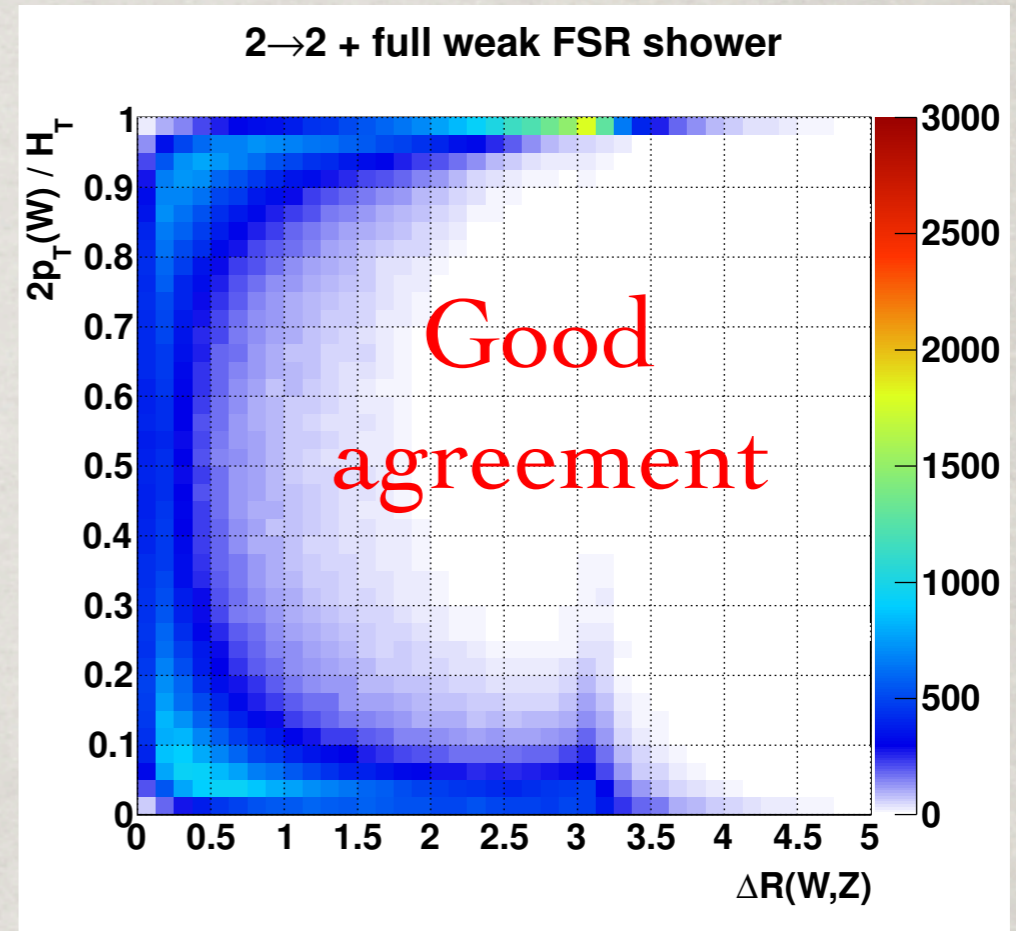
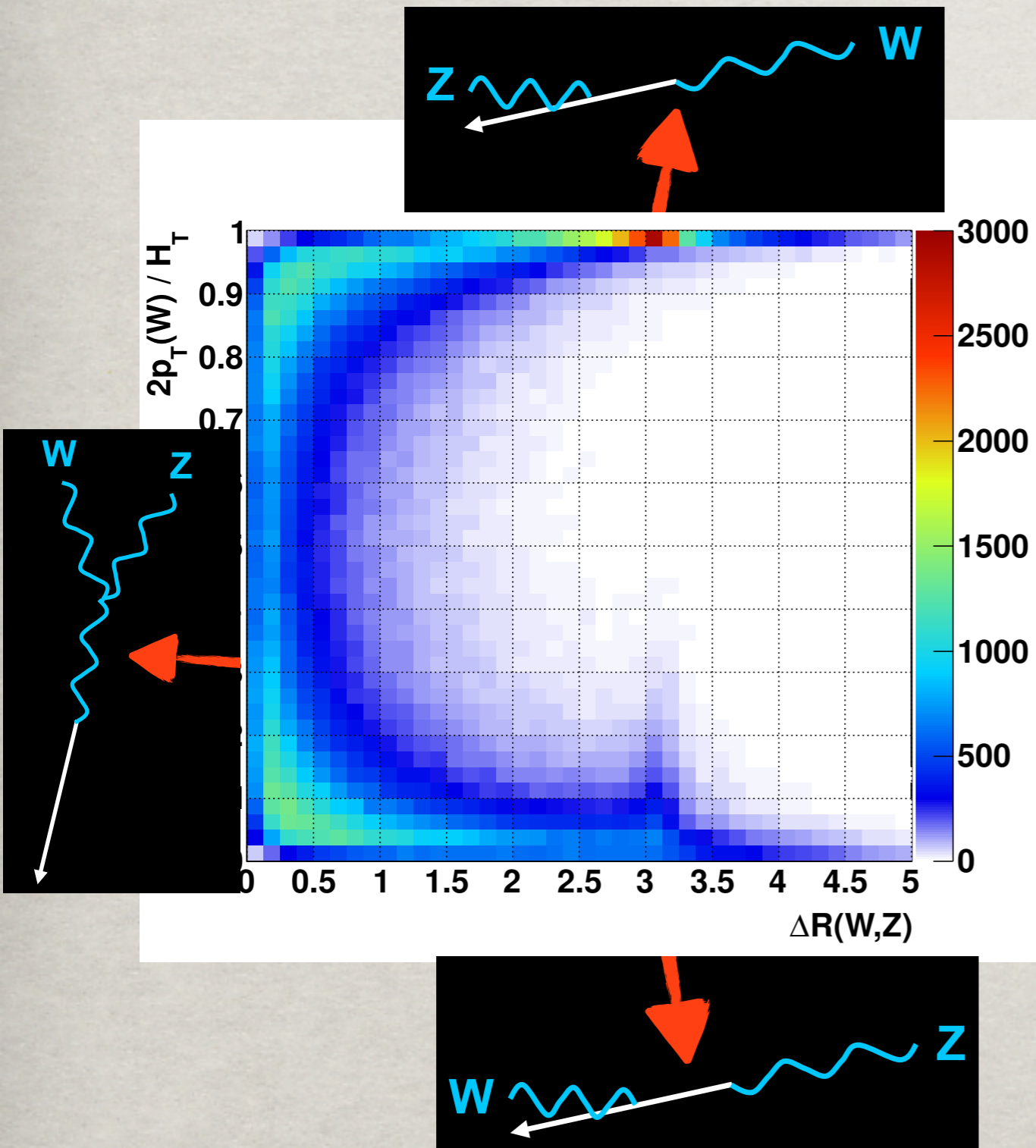
Each  $W$  costs you a factor of  $\sim 1/100$  (EW coupling)





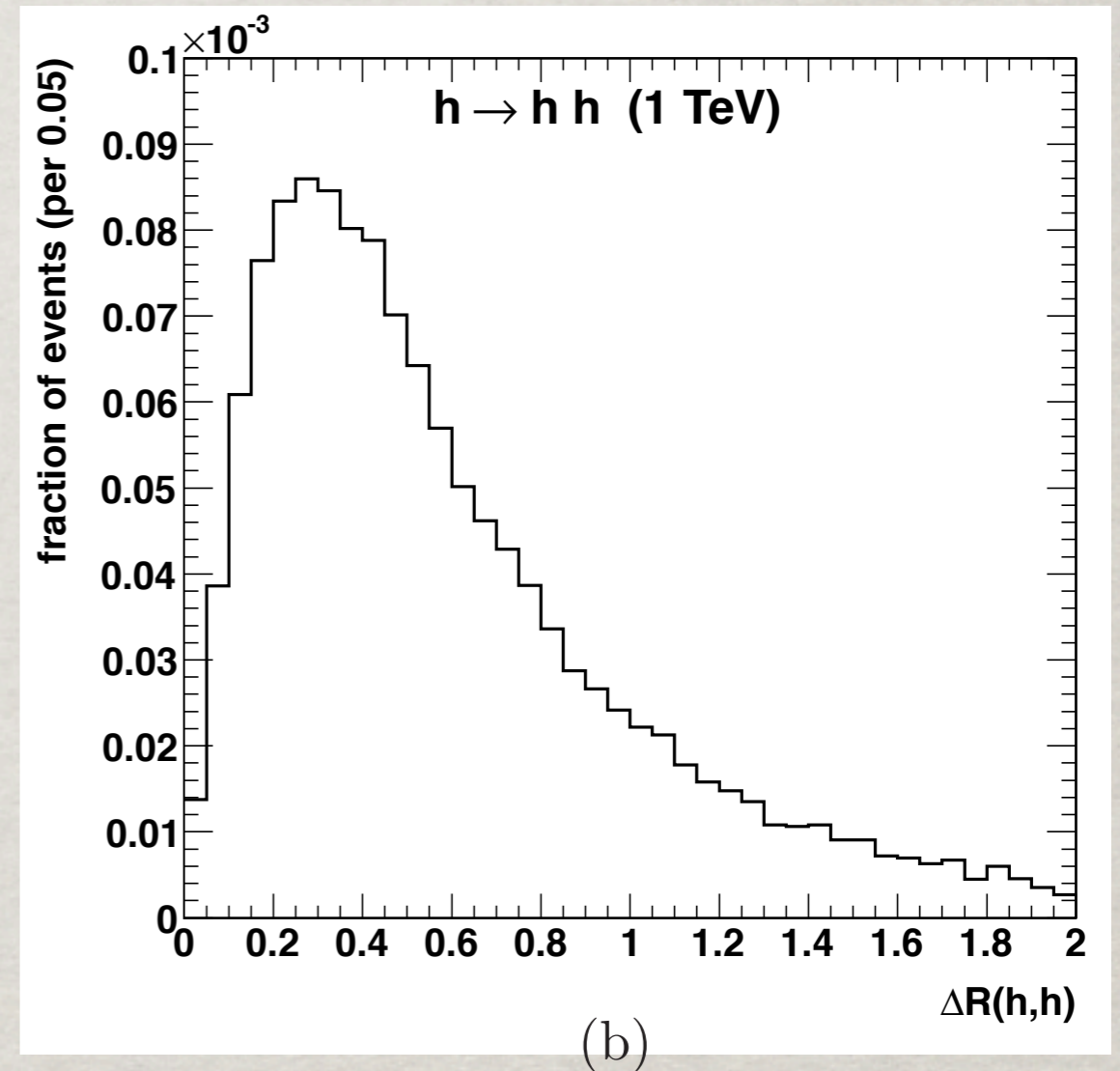
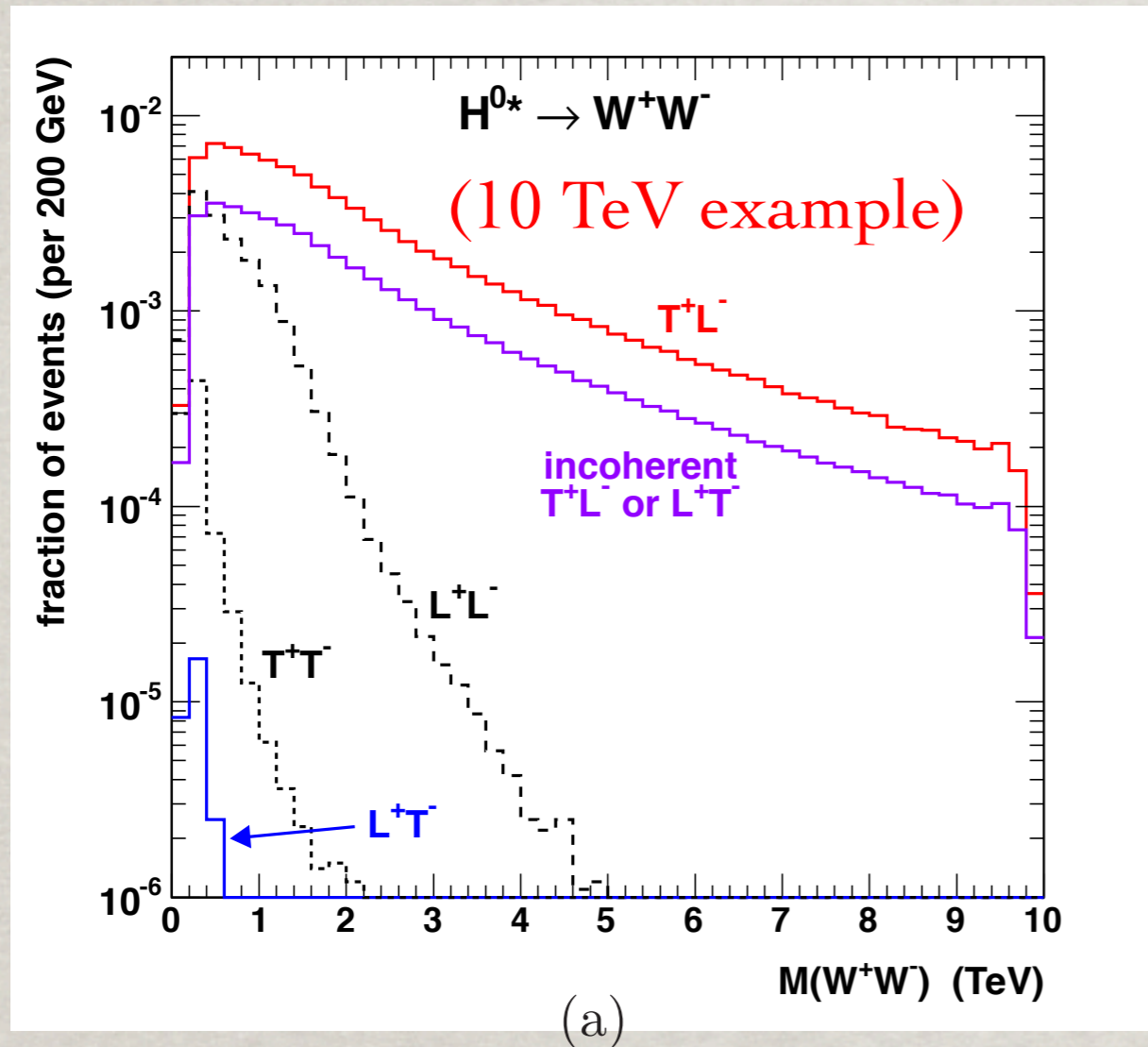
# AN EXAMPLE: $WZ+J$ @ 100 TEV

MadGraph  $2 \rightarrow 3$  fixed order





# Higgs boson showering:



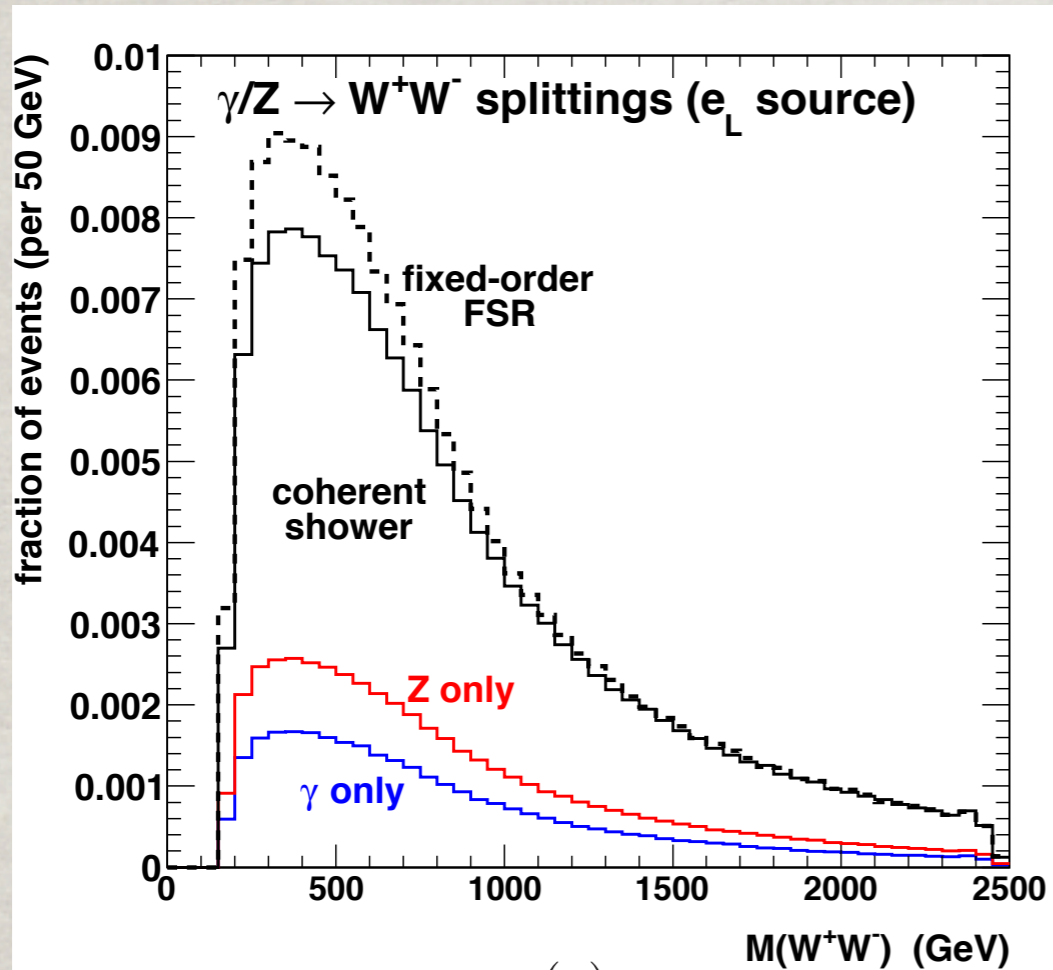
- $H^{0*} = h^0 - i\phi^0$  coherent:  
 $W_T^+ h^- \gg W_T^- h^+$
- $h/Z_L$  separate wrong!

Ultra-collinear behavior:  
 Some guidance for  $h^3$  search.

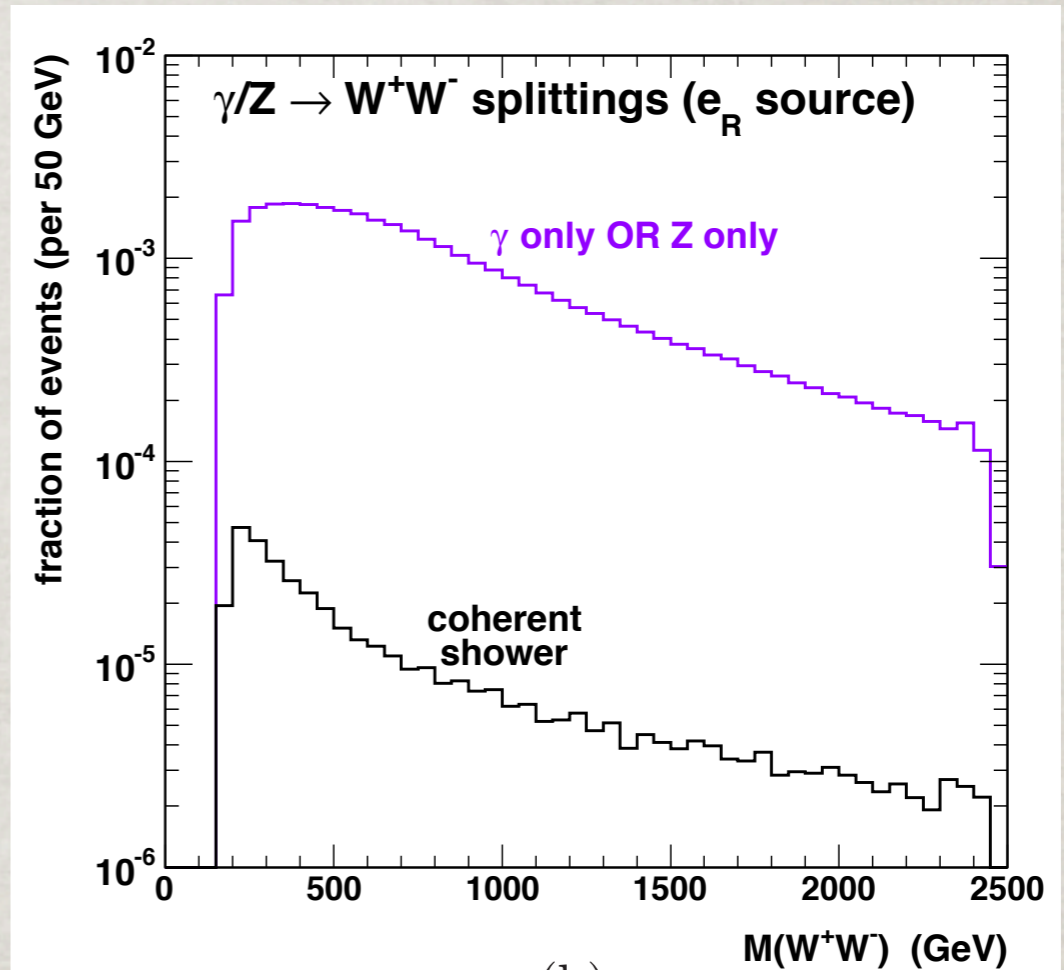


$$e^-_L e^+_R \text{ \& } e^-_R e^+_L \rightarrow W^+ W^- + \text{shower}$$

$$(e^+ e^- E_{\text{cm}} = 5 \text{ TeV})$$



(a)



(b)

$W/Z$  shower important;  
Coherent treatment important.

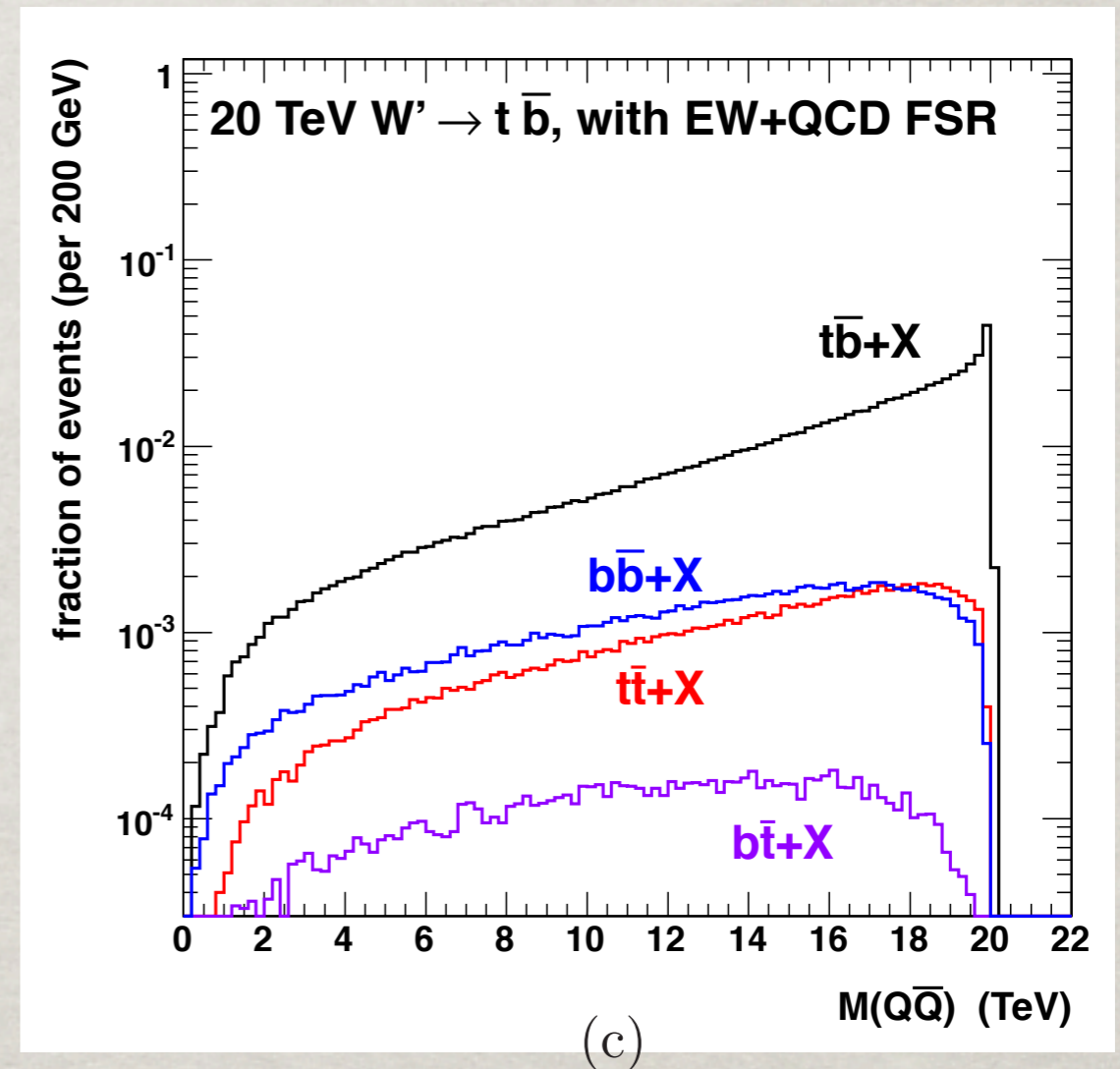
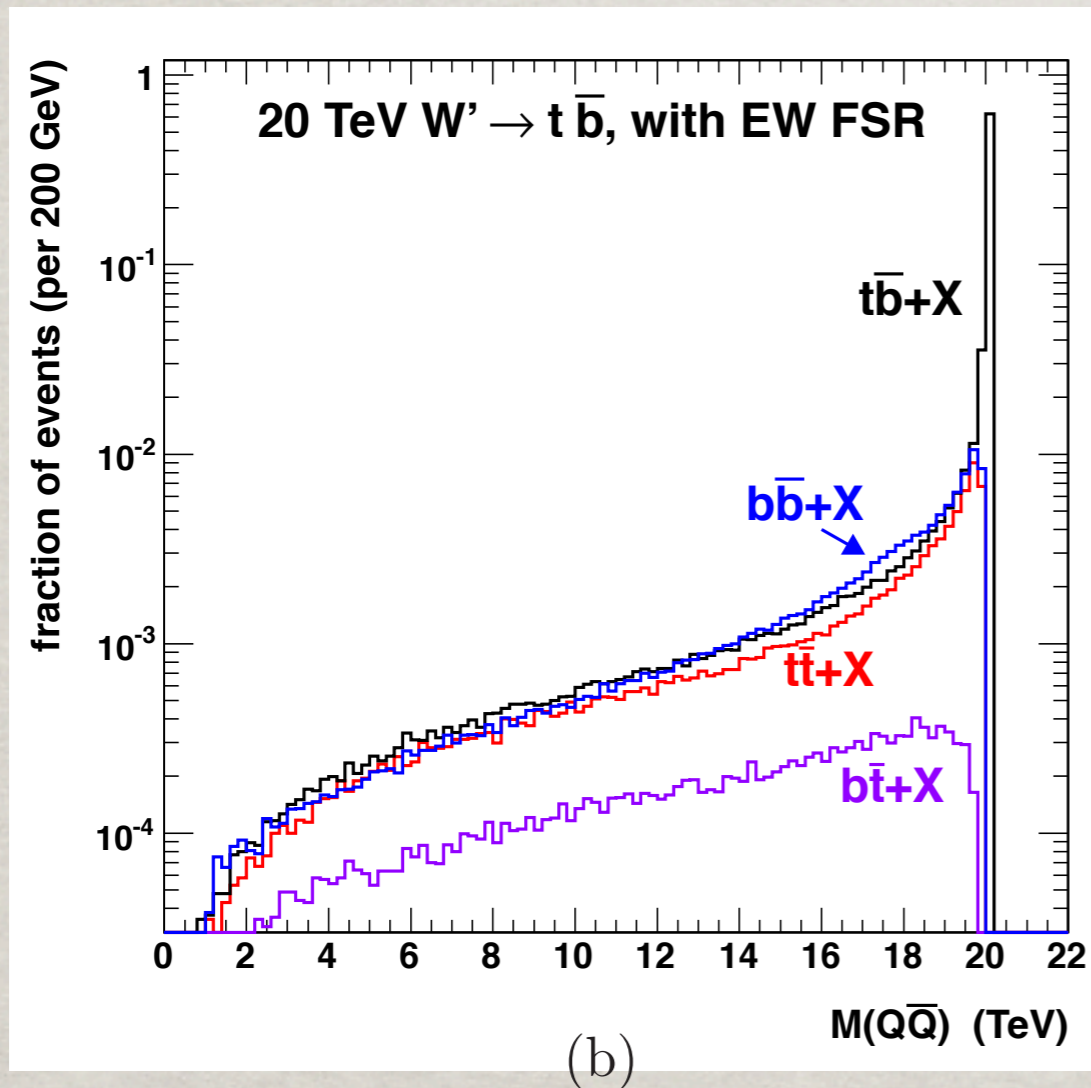
Pure  $B^0$  exchange:  
Small.

$SU(2)_L \times U(1)_Y$  interactions restored!



# $W'^+$ Shower examples:

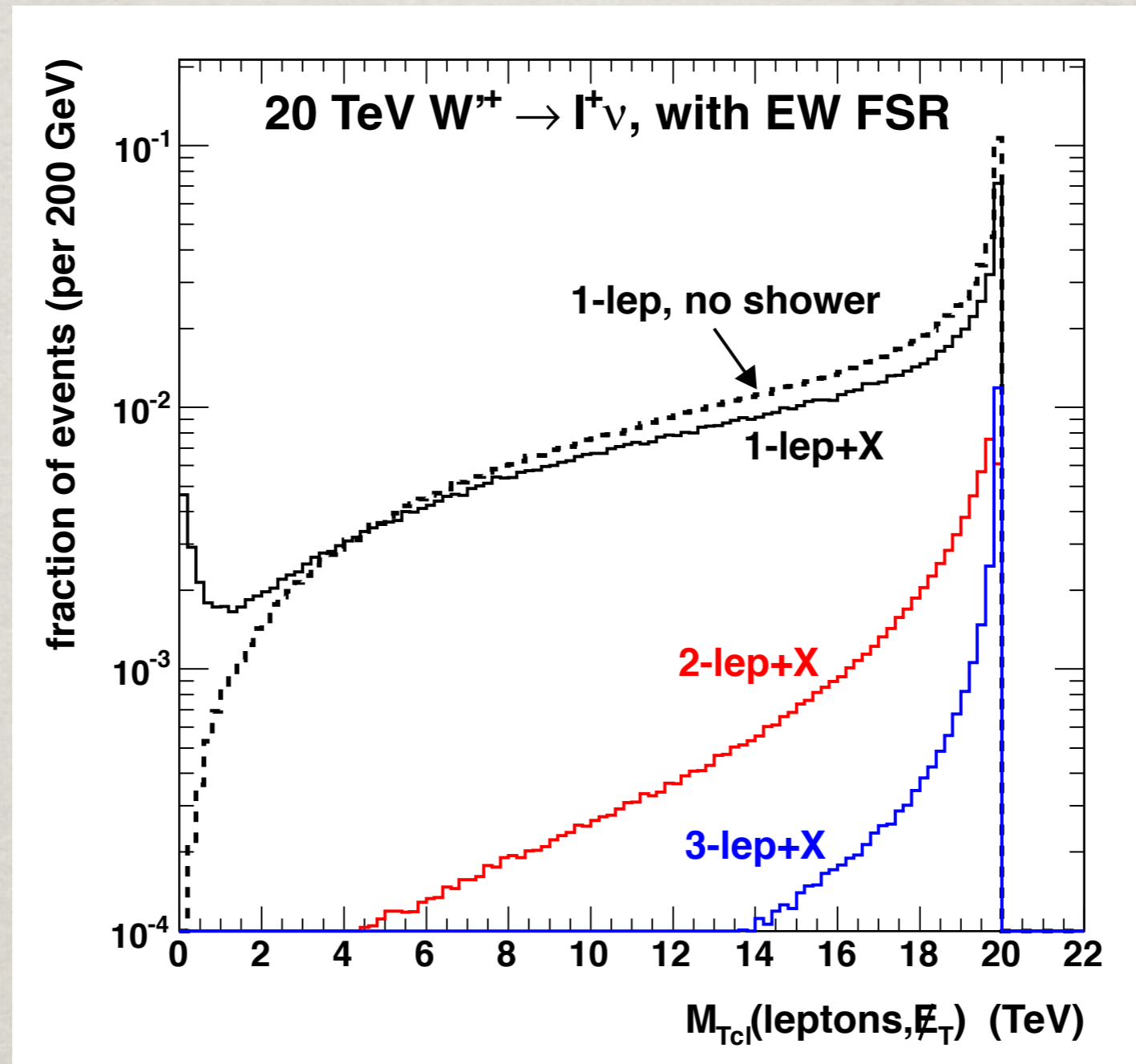
$$W_L^{+'} \rightarrow t\bar{b}, t\bar{t}(W^-), b\bar{b}(W^+), b\bar{t}(W^+W^+).$$



With  $W/Z$  showers, ALL  $t/b$  iso-spin components exist.



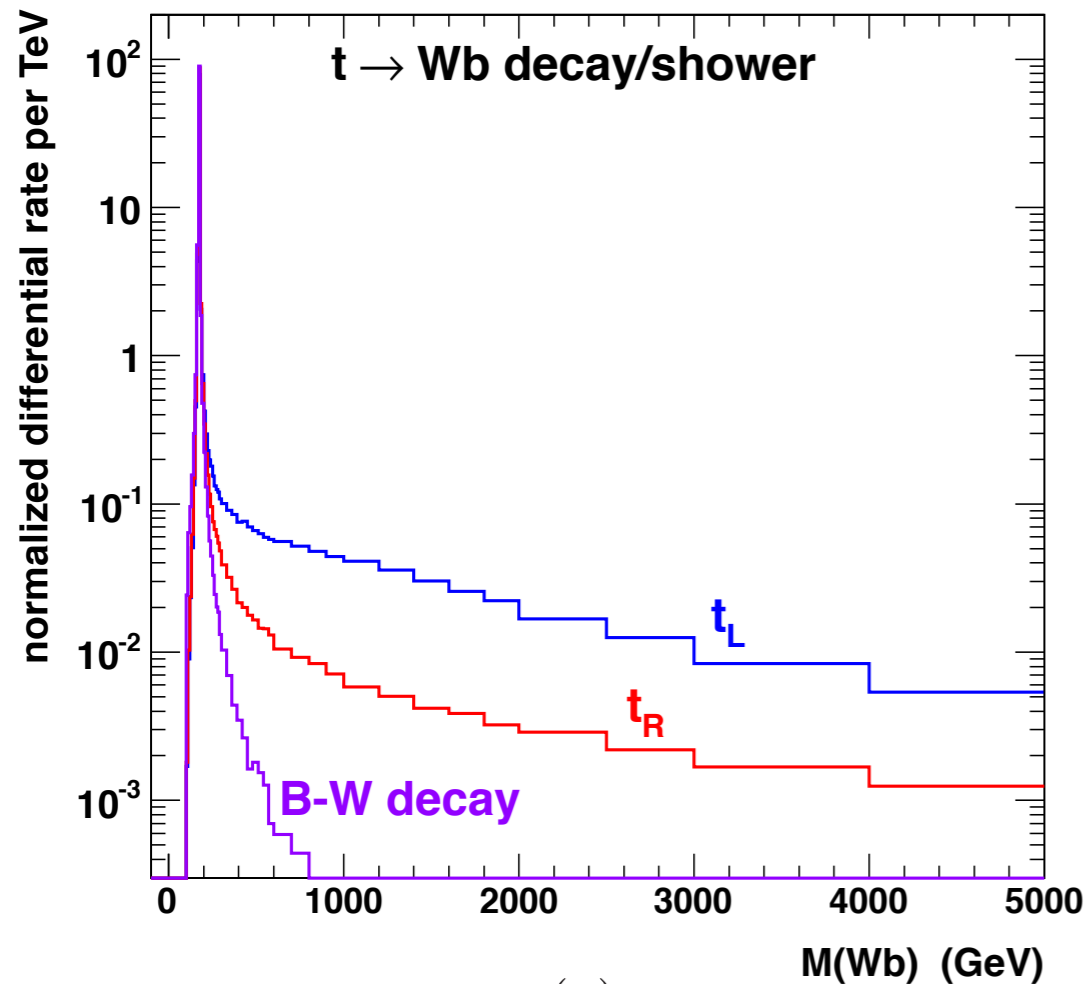
# $W^+$ Shower example: Lepton final states



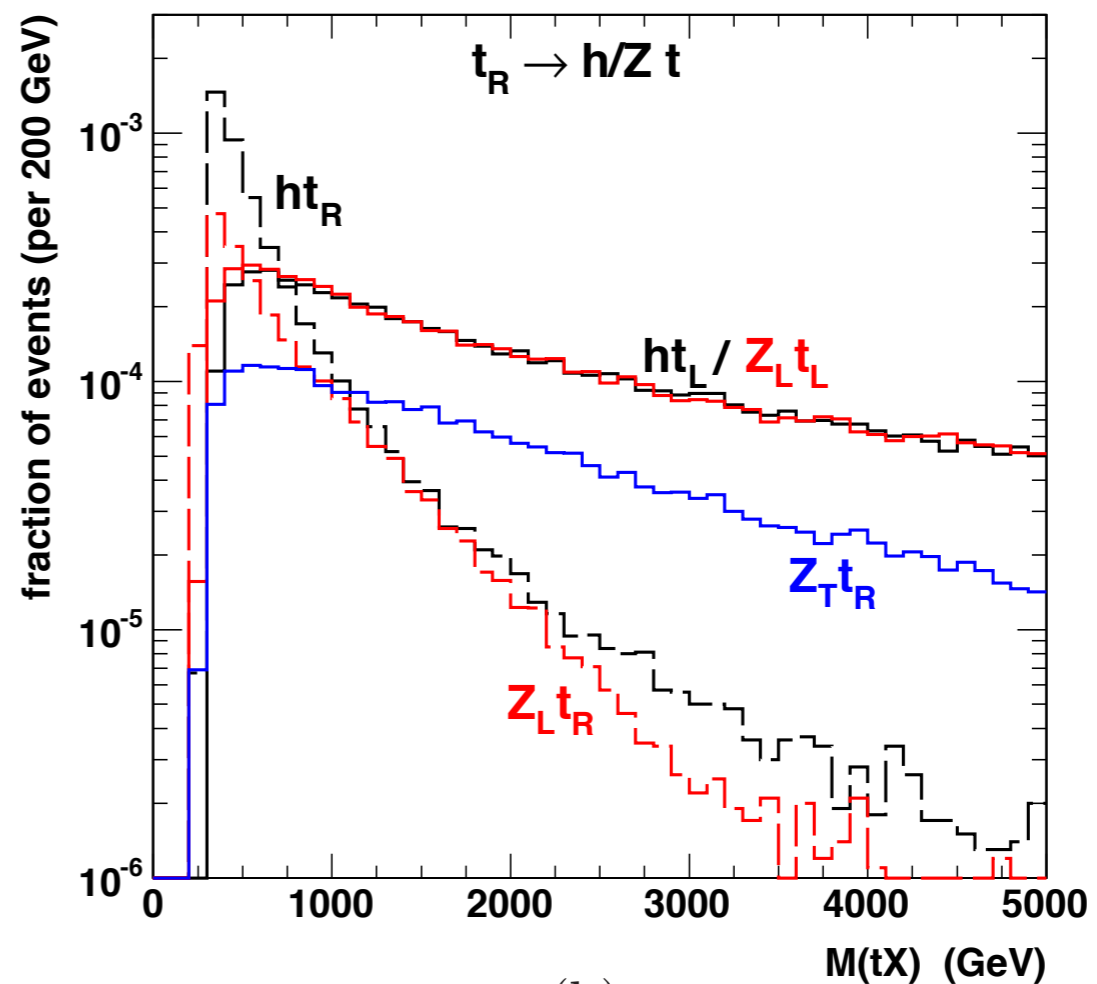
With  $W/Z$  showers, all **leptons/neutrino** components exist.



# Top decay/showering (10 TeV):



(a)



(b)

Yukawa:  $\mathcal{P}(t_R \rightarrow ht_L) \simeq \mathcal{P}(t_R \rightarrow Z_L t_L) \approx 7.2 \times 10^{-3}$

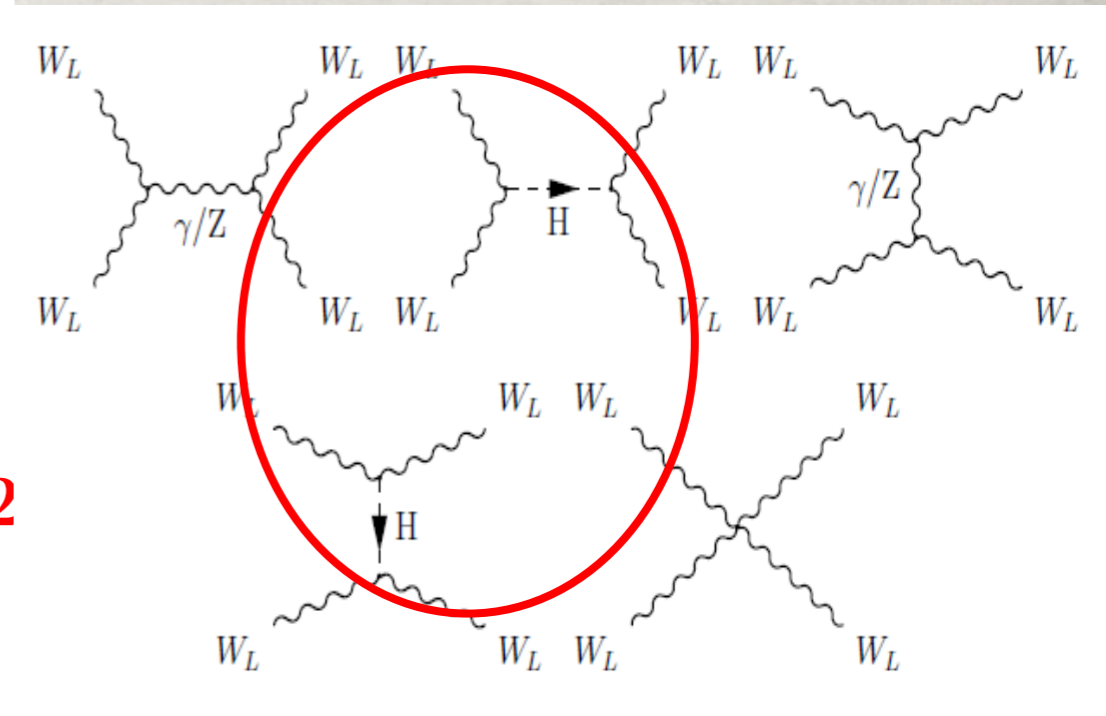
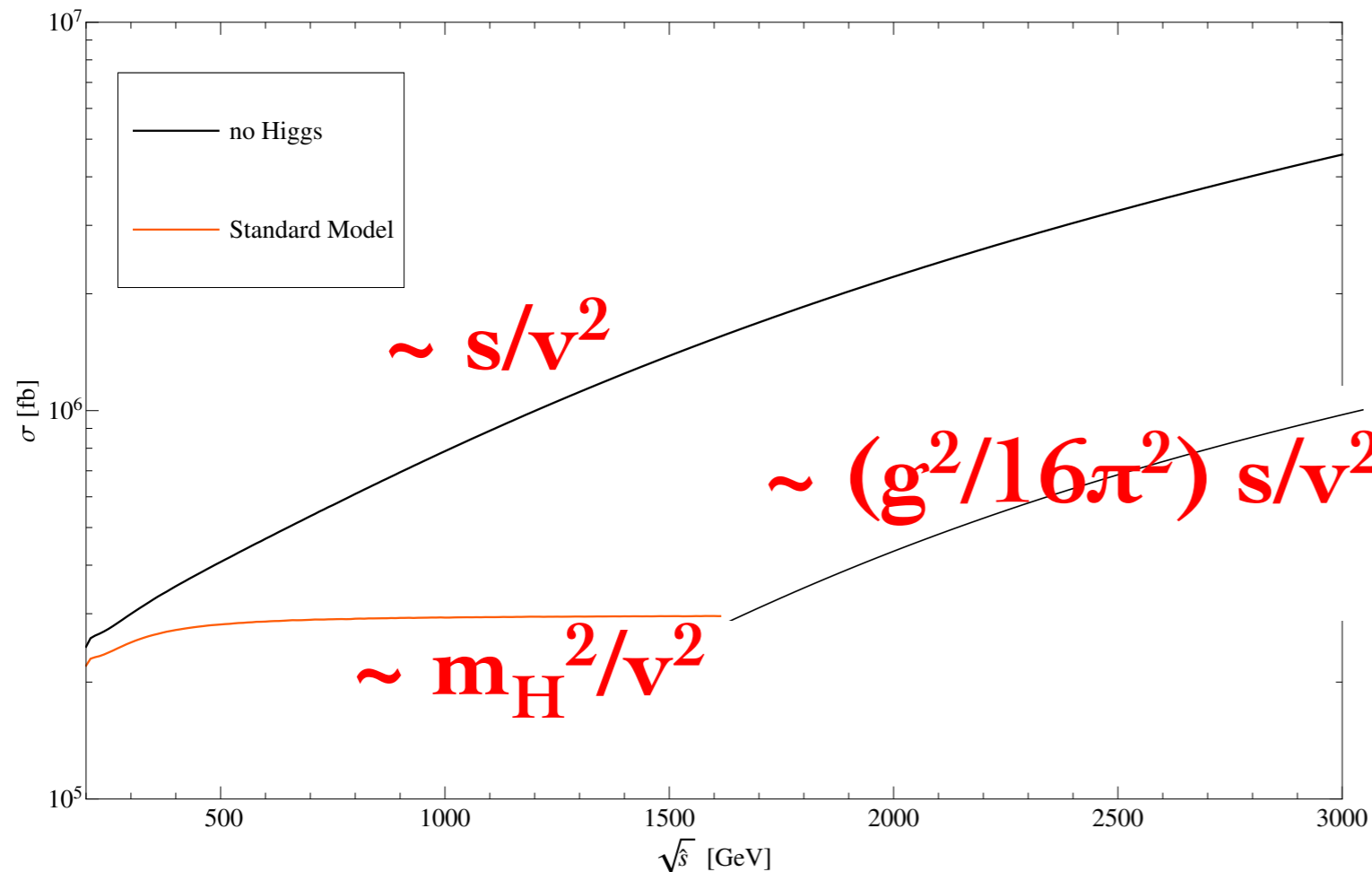
U(1) gauge:  $\mathcal{P}(t_R \rightarrow Z_T t_R) \approx 4.5 \times 10^{-3}$

Ultra-collinear:  $t_R \rightarrow ht_R, Z_L t_R$



# $W_L W_L$ Scattering:

- The existence of a light, weakly coupled Higgs boson unitarize the WW amplitude:



- Consistent perturbative theory up to  $\Lambda$  (?)
- New strong dynamics effects may still exist, but “delayed” to  $v^2/\Lambda^2$ .



# EW PDF's

QCD factorization: **Colins, Soper, Sterman (1985).**

$$\sigma(pp \rightarrow X + \text{anything}) = \int_{\tau_0}^1 d\tau \sum_{ij} \frac{d\mathcal{L}_{ij}}{d\tau} \hat{\sigma}(ij \rightarrow X),$$

$$\frac{d\mathcal{L}_{ij}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_{\tau}^1 \frac{d\xi}{\xi} \left[ f_{i/p}(\xi, Q_f^2) f_{j/p} \left( \frac{\tau}{\xi}, Q_f^2 \right) + (i \leftrightarrow j) \right]$$

EW partons:

$$\begin{aligned} \Phi_{VV'}(\tau) &= \frac{1}{(\delta_{VV'} + 1)} \int_{\tau}^1 \frac{d\xi}{\xi} \int_{\tau/\xi}^1 \frac{dz_1}{z_1} \int_{\tau/\xi/z_1}^1 \frac{dz_2}{z_2} \sum_{q,q'} \quad (7) \\ &\times \left[ f_{V/q}(z_2) f_{V'/q'}(z_1) f_{q/p}(\xi) f_{q'/p} \left( \frac{\tau}{\xi z_1 z_2} \right) + f_{V/q}(z_2) f_{V'/q'}(z_1) f_{q/p} \left( \frac{\tau}{\xi z_1 z_2} \right) f_{q'/p}(\xi) \right] \end{aligned}$$

**Chen, TH, Tweedie, arXiv:1611.00788;**

**Bauer, Ferland, Webber, arXiv:1703.08562, 1712.07147.**



# EW Evolution @ Leading Double Log

Bauer, Ferland, Webber, arXiv:1703.08562, 1712.07147;  
Chen, TH, Tweedie, arXiv:18xx.

⇒ Sudakov factor :  $\Delta_i \sim \exp\left[-C_i \frac{\alpha_2}{\pi} \ln^2\left(\frac{Q^2}{M_W^2}\right)\right]$

Following SU(2) $\times$ U(1) DGLAP equations.

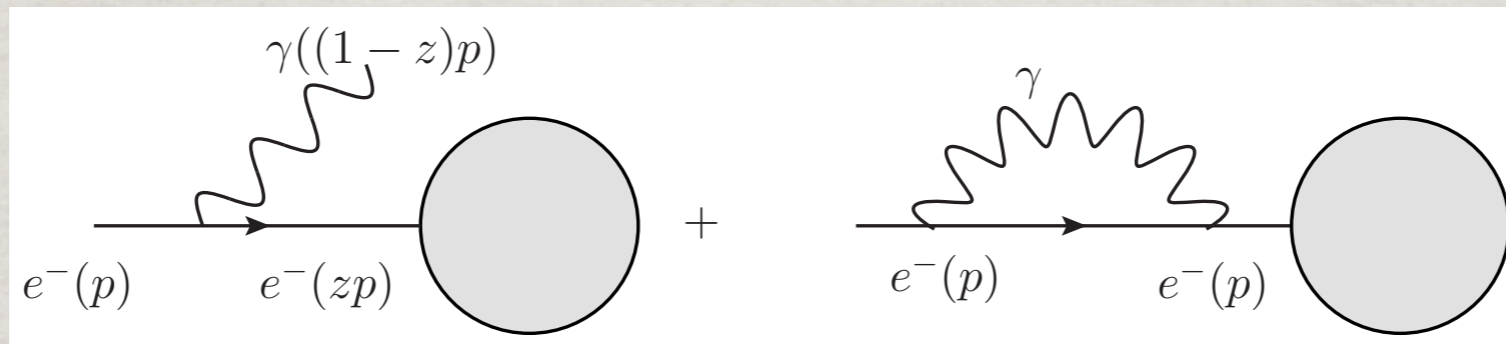
e.g., iso-spin state evolution @ leading log:

$$f_e(x, \mu) \simeq f_e(x, 0) \frac{1 + e^{-(\alpha_2/\pi) \log^2(\mu/m_W)}}{2} + f_\nu(x, 0) \frac{1 - e^{-(\alpha_2/\pi) \log^2(\mu/m_W)}}{2}$$

with **W/Z** showers, **leptons/neutrinos** redistributed.



# EW Evolution beyond Leading Log



Incomplete cancellation for non-inclusive process in SU(2)

→ Bloch-Nordsieck theorem violation

$$d\mathcal{P}_{\nu \leftarrow e} = d\mathcal{P}_{e \leftarrow \nu} \sim \frac{(T^\pm)^2}{1-z} = \frac{(1/\sqrt{2})^2}{1-z}$$

$$d\mathcal{P}_{e \leftarrow e} = d\mathcal{P}_{\nu \leftarrow \nu} \sim \frac{(T^3)^2}{1-z} = \frac{(1/2)^2}{1-z}$$

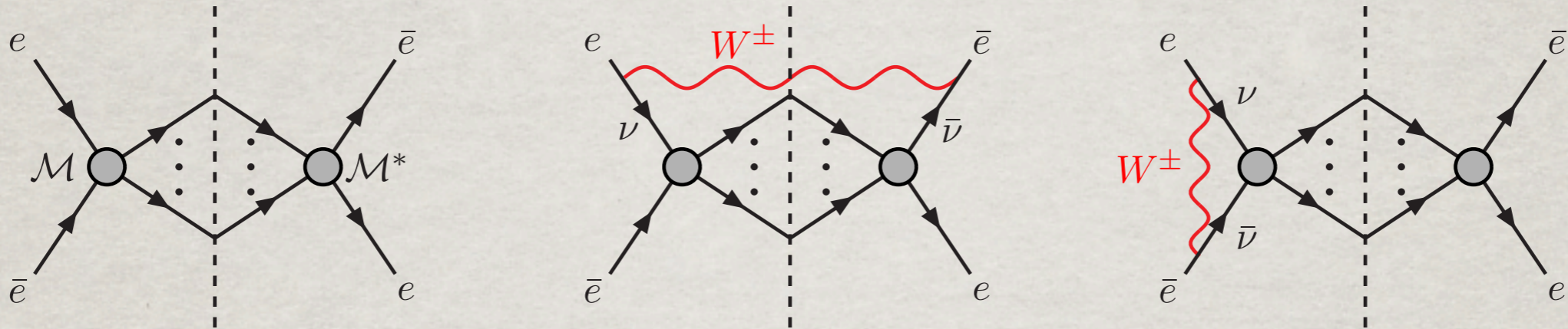
$$dV_{\nu \leftarrow e} = dV_{e \leftarrow \nu} = 0$$

$$dV_{e \leftarrow e} = dV_{\nu \leftarrow \nu} \sim - \int dz \frac{C_2(\mathbf{2})}{1-z} = - \int dz \frac{3/4}{1-z}$$

→ non-cancelled sub-leading  $\log(Q^2/M_w^2)$



Consider  $e^+e^- \rightarrow X$



$$\Delta\sigma_{e\bar{e}\rightarrow X} \simeq \left[ \frac{\alpha_2}{\pi} \log\left(\frac{Q}{m_W}\right) \mathcal{A}(e, e) \left(\frac{1}{\sqrt{2}}\right)^2 \right] \times \sigma_{\nu\bar{e}\rightarrow X}$$

$$\Delta\sigma_{e\bar{e}\rightarrow X} \simeq \left[ \frac{\alpha_2}{\pi} \log\left(\frac{Q}{m_W}\right) \mathcal{A}(e, \bar{e}) \left(\frac{1}{\sqrt{2}}\right)^2 \right] \times \sigma_{\nu\bar{\nu}\rightarrow X, \bar{e}e\rightarrow \bar{X}}$$

Ciafaloni et al., hep=ph/0007096.

$$\begin{aligned} \sigma = & F_{e\bar{e}, \bar{e}e} \hat{\sigma}_{e\bar{e}\rightarrow X, \bar{e}e\rightarrow \bar{X}} + F_{\nu\bar{\nu}, \bar{\nu}\nu} \hat{\sigma}_{\nu\bar{\nu}\rightarrow X, \bar{\nu}\nu\rightarrow \bar{X}} + \\ & F_{e\bar{\nu}, \bar{e}\nu} \hat{\sigma}_{e\bar{\nu}\rightarrow X, \bar{e}\nu\rightarrow \bar{X}} + F_{\nu\bar{e}, \bar{\nu}e} \hat{\sigma}_{\nu\bar{e}\rightarrow X, \bar{\nu}e\rightarrow \bar{X}} + \\ & F_{e\bar{e}, \bar{\nu}\nu} \hat{\sigma}_{e\bar{e}\rightarrow X, \bar{\nu}\nu\rightarrow \bar{X}} + F_{\nu\bar{\nu}, \bar{e}e} \hat{\sigma}_{\nu\bar{\nu}\rightarrow X, \bar{e}e\rightarrow \bar{X}} + \dots \end{aligned}$$

→ State ensembles:

Parton-Luminosity Ensembles (PLE).



# Our Approach

- Decompose an incoming state into gauge multi-plets:  $SU(2) \ f_L \rightarrow 1 + 3$ .
  - $1 : (e\nu - \nu e)/\sqrt{2}$
  - $3 : ee, \nu\nu, (e\nu + \nu e)/\sqrt{2}$
- Gauge eigenstates properly evolve with  $Q^2$ ; and the off-diagonal terms never develop:  
 $1_{in}$  or  $3_{in}$  would not fix.
- At which level would it break down?

More to come ...



# CONCLUSIONS

- With the discovery of the Higgs boson, we have a consistent QM, relativistic, unitary theory up to (possibly exponentially) high scales, but where is it? We wish  $\Lambda \sim 4 \pi v$  (?)
- EW sector @ high scale holds the hope for the probe!
- First, bread & butter rich physics:
  - Perturbative cutoff via SSB
  - Longitudinals/scalars
  - Chirality
  - Yukawa showers
  - Neutral boson interference
  - Weak isospin self-averaging

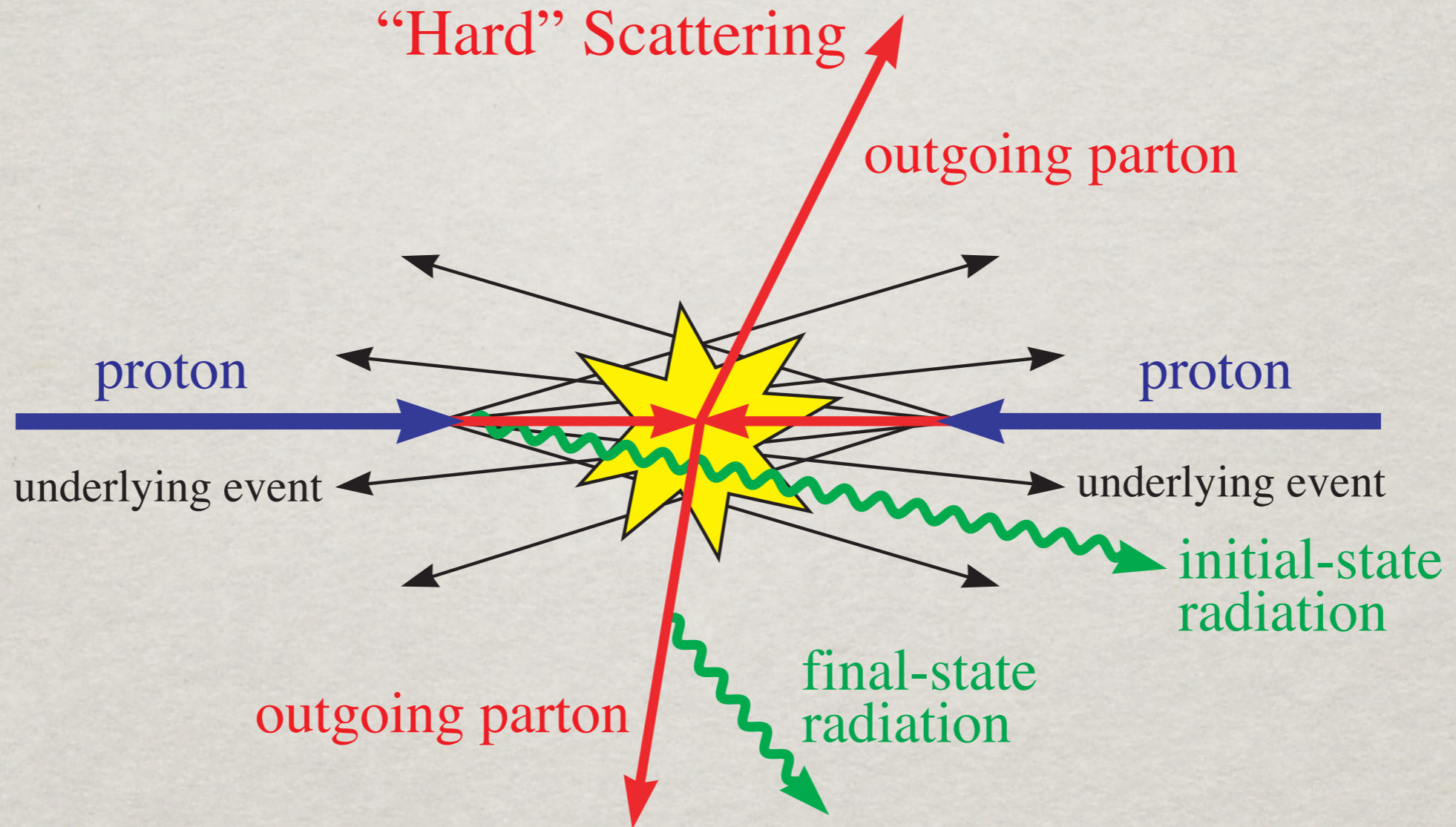


# CONCLUSIONS

- EW splitting/showering will become an increasingly important part at higher energies.
- It still has technical & conceptual challenges at higher energies.
- Be prepared:  
Very high-energy  $W, Z, h, t$  may serve as tools for the next discovery !



# ***REALITY IN HADRONIC COLLISIONS***



**Collinear splitting, ISR & FSR,  
is one of the dominant phenomena.**



# EW SPLITTING FUNCTIONS

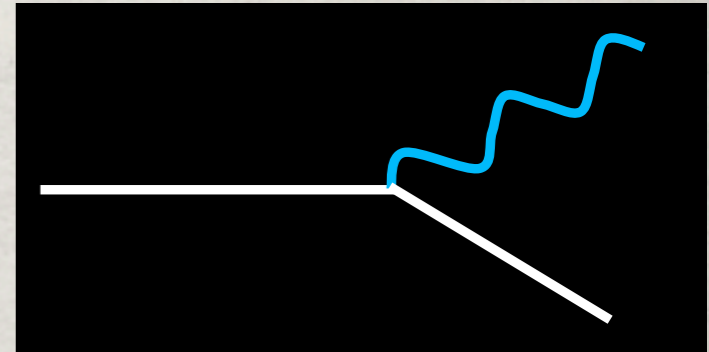
## Motivations:

- We have marched into the territory where  $E \gg M_W$  where EW symmetry can be restored.
- Conceptually different from QCD:  $\Lambda_{\text{QCD}}$  vs  $v_{\text{ev}}$ : EW sector remains perturbative.
- New degrees of freedom:  
*the Higgs sector / Longitudinal vector bosons*
- Clear understanding of the “Equivalence theorem”.
- Most sensitive to new physics above the EW scale.



# GAUGE-BOSON INITIATED PROCESSES

At colliding energies  $E \gg M_W$ ,  
EW gauge bosons are new “gluons”!

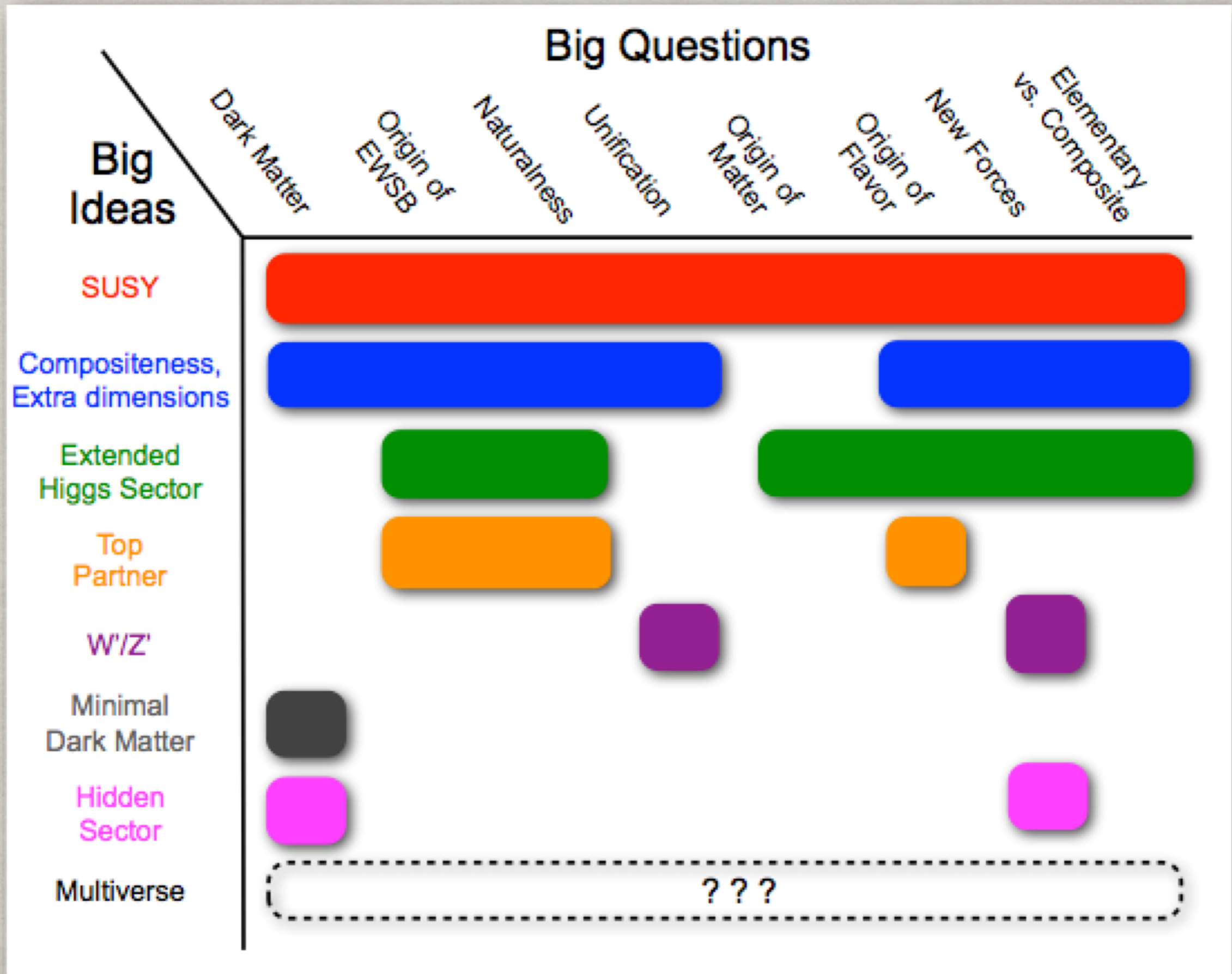


In the EW theory:

$$P_{q \rightarrow q V_T} = (g_V^2 + g_A^2) \frac{\alpha_2}{2\pi} \frac{1 + (1-x)^2}{x} \ln \frac{Q^2}{\Lambda^2}$$
$$P_{q \rightarrow q V_L} = (g_V^2 + g_A^2) \frac{\alpha_2}{\pi} \frac{1-x}{x}$$

- $V_T$  radiation the same as  $g, \gamma$ :  $|M|^2 \sim p_T^2$ :
  - “dead cone” at  $k_T \rightarrow 0$ :  $\sim k_T dk_T / m_W^2$
  - log-enhancement at high  $p_T$  & soft  $x$
- $V_L$  radiation no collinear enhancement/suppression, no log-running at leading order.







# NEW PHYSICS WITH ENERGETIC MULTI TOPS/GAUGE-BOSONS

SUSY examples:  $\tilde{b}\tilde{b}^* \rightarrow t\chi^- \bar{t}\chi^+, \tilde{t}W^- \tilde{t}^*W^+ \rightarrow 4W^\pm b\bar{b}.$

Heavy quark examples:  $TT', BB', \dots$

Heavy  $W', Z'$  decays.

Heavy DM annihilation in indirect searches

... ..

Ciafaloni, Riotto, Strumia, et al., 1009.0224;

Hook, Katz, 1407.2607;

M. Bauer, T. Cohen, et al., 1409.7492;

Baumgart, Rothstein, Vaidya (2014 - 2015)

... ..

→ Energetic  $W^\pm, Z, H, t$  from new radiation sources and decays.