SI2018, 16 August, Tianjin, China

Inflationary Cosmology
in Composite Scalar Model

Tomohiro Inagaki (Hiroshima University)
with Sergei D. Odintsov and Hiroki Sakamoto

Role of fermion in cosmology

Can a fermion dominate the energy density of the early universe?

Can the effective potential $V(\bar{\psi}\psi)$ accelerate the expansion of the universe?

- A free fermion gives a negative contribution.

Lecture by Prof. Kugo

- A fermion mass contributes as an ordinary matter.

$$a(t) \propto t^{2/3}$$

Outline

- Cosmological inflation
- Gauged Nambu-Jona-Lasinio (gNJL) model
- Inflationary Cosmology in the gNJL Model
- Concluding remarks

T. I., S. D. Odintsov and H. Sakamoto, Astr. Space Sci. 360 (2015) 67,
T. I., S. D. Odintsov and H. Sakamoto, Nucl. Phys. B (2017),
T. I., S. D. Odintsov and H. Sakamoto, Europhys. Lett. 118 (2017) 29001.

Cosmological inflation

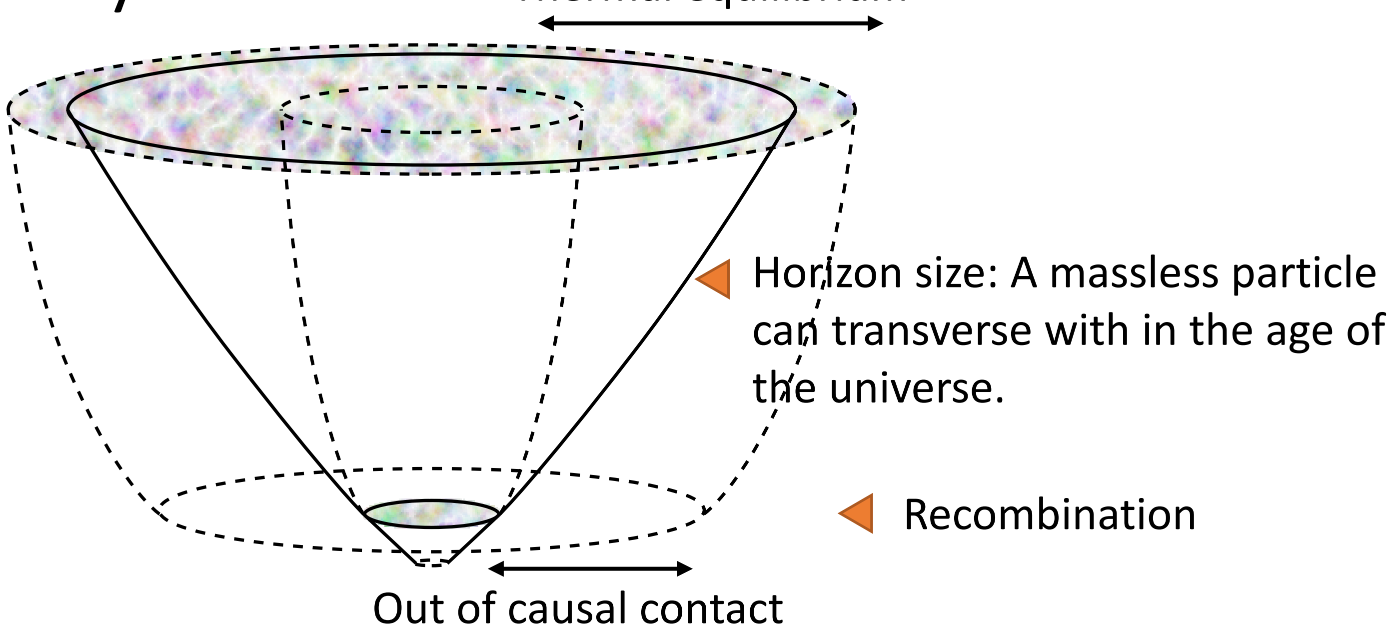
Cosmological problems

- Horizon problem
- Flatness problem
- Monopole problem
- Singularity problem

C. W. Misner, K. S. Thorne, J. A. Wheeler , Gravitation (1973)
A. D. Linde, Contemp. Concepts Phys. 5, 1 (1990).

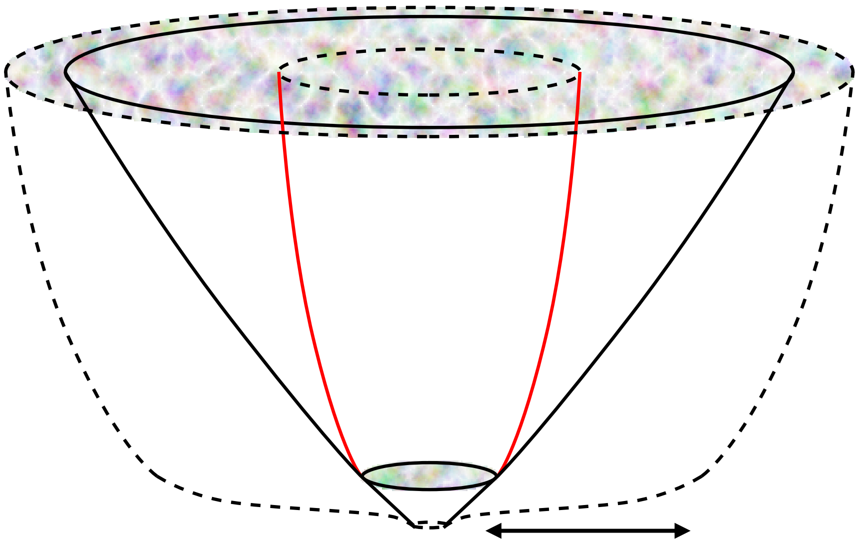
Cosmological problems

- Horizon problem: Horizon size at the time of recombination when the cosmic microwave background radiated is much smaller than that of today.



Inflationary expansion

- If we assume inflationary expansion of the early universe, the current horizon size can be in causal contact at very early universe.



A. Guth and K. Sato, 1981

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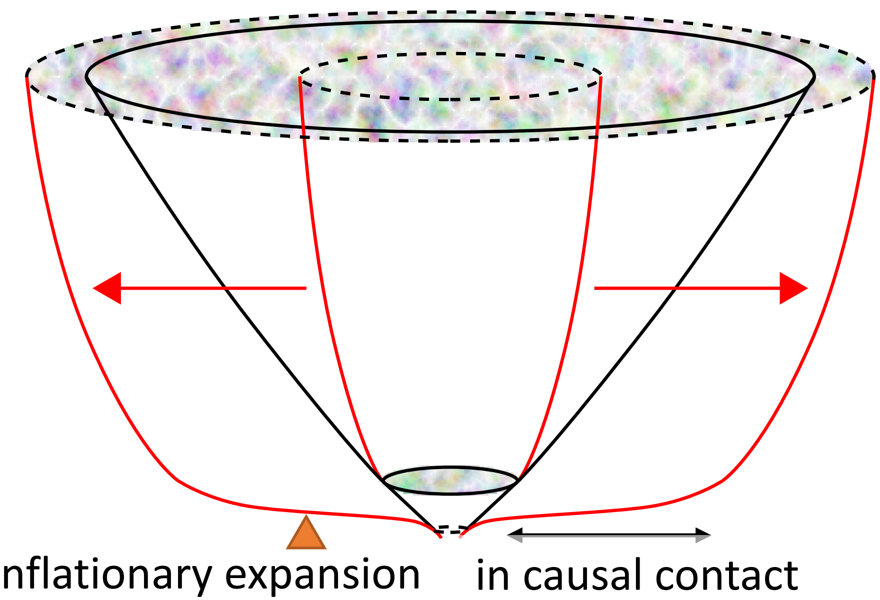

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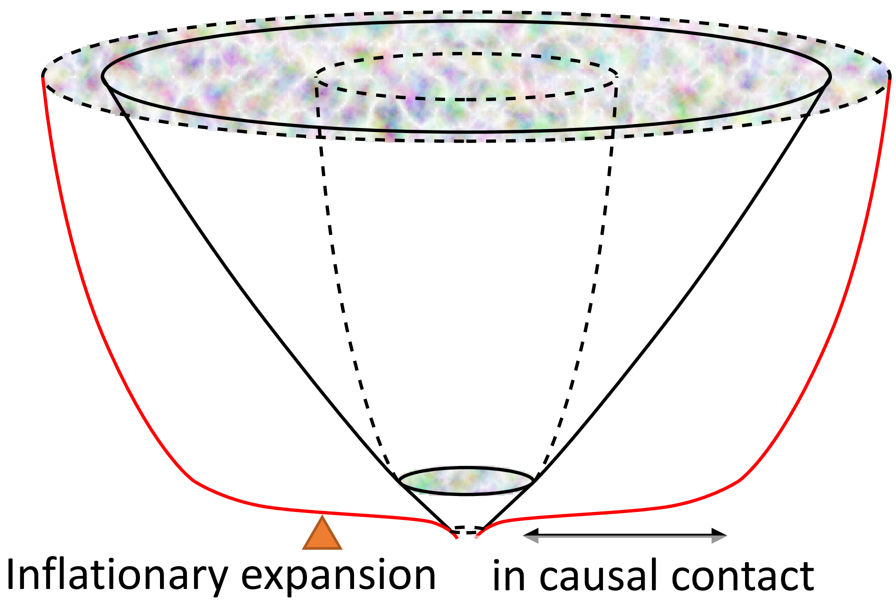

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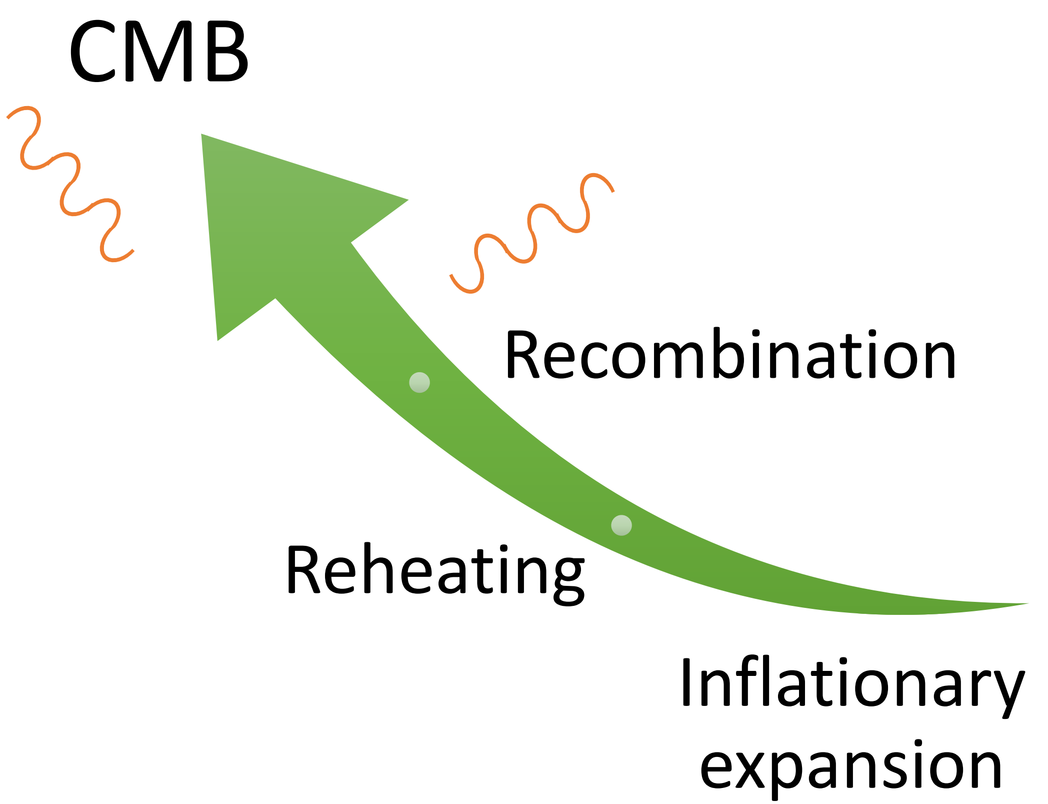
Evidence for Inflation



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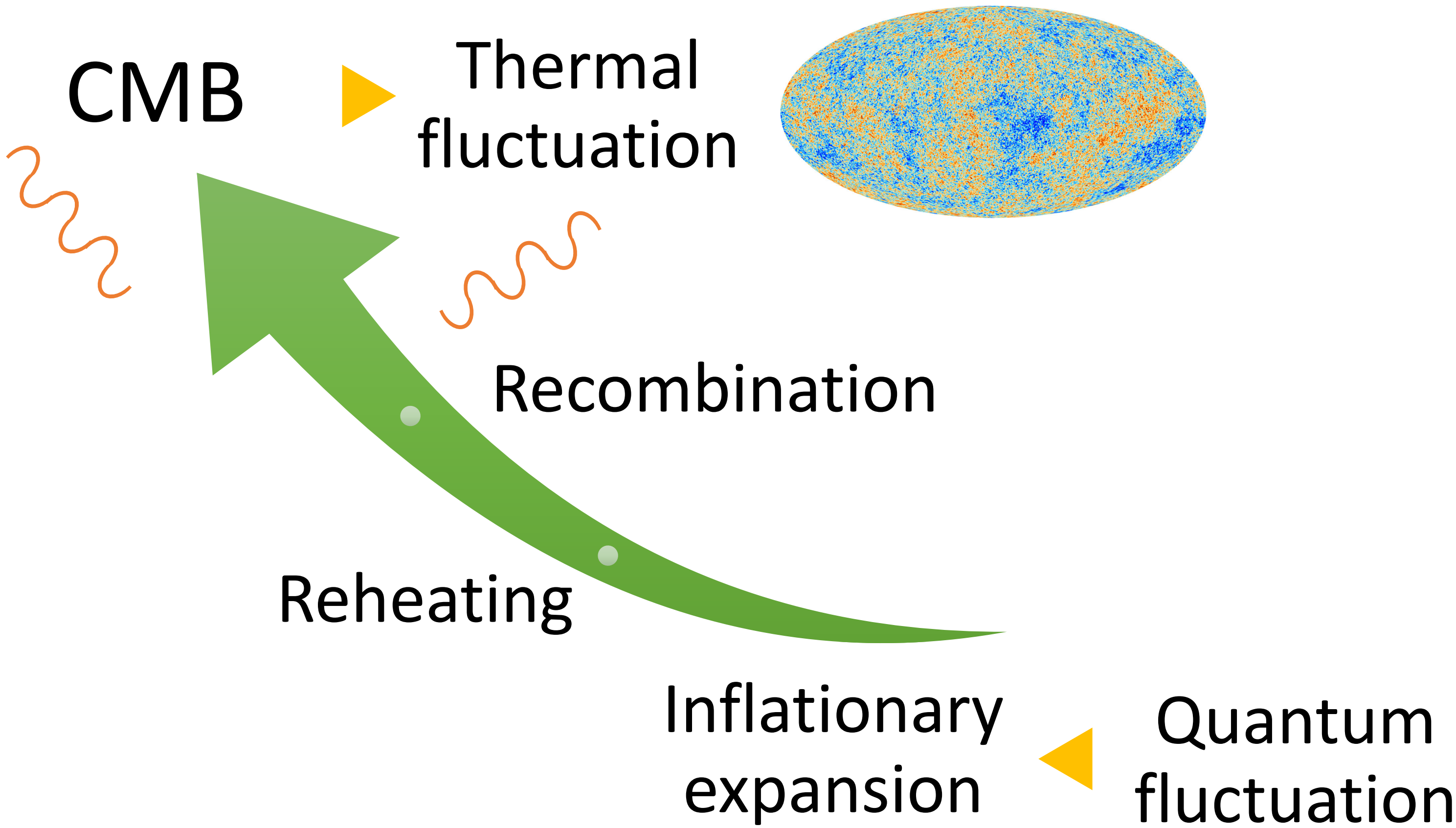
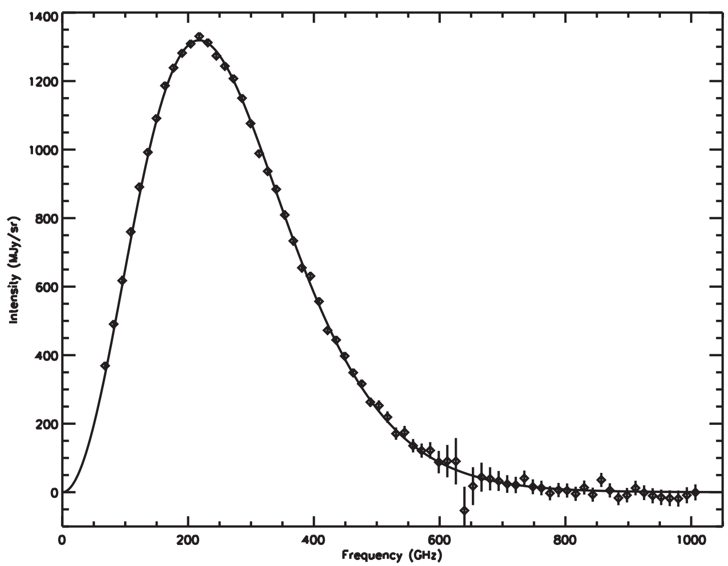


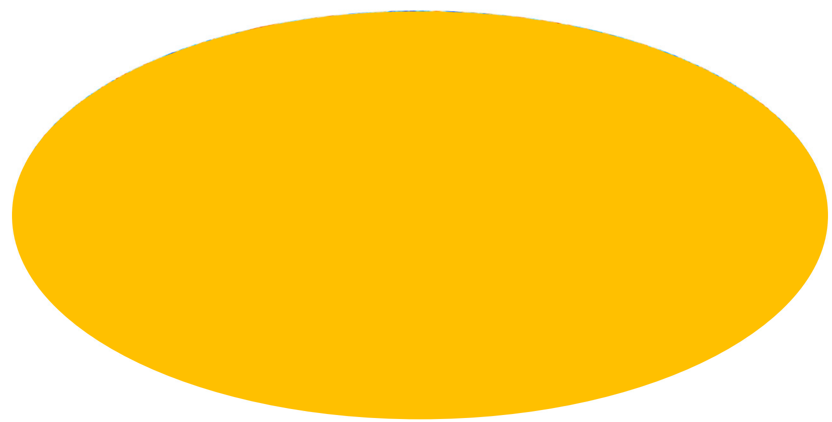
Observed CMB

- Black-body radiation at $T=2.72548 \pm 0.00057$ K



D. J. Fixsen, 2009

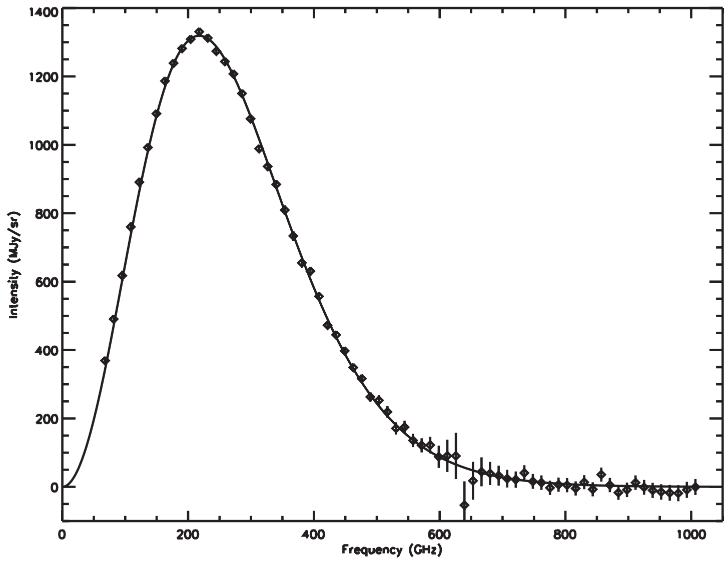
- CMB intensity



If there is no CMB fluctuation,

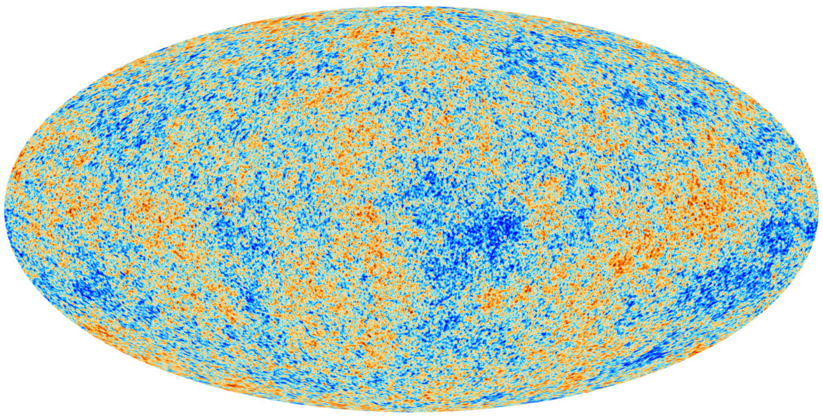
Observed CMB

- Black-body radiation at $T=2.72548 \pm 0.00057$ K



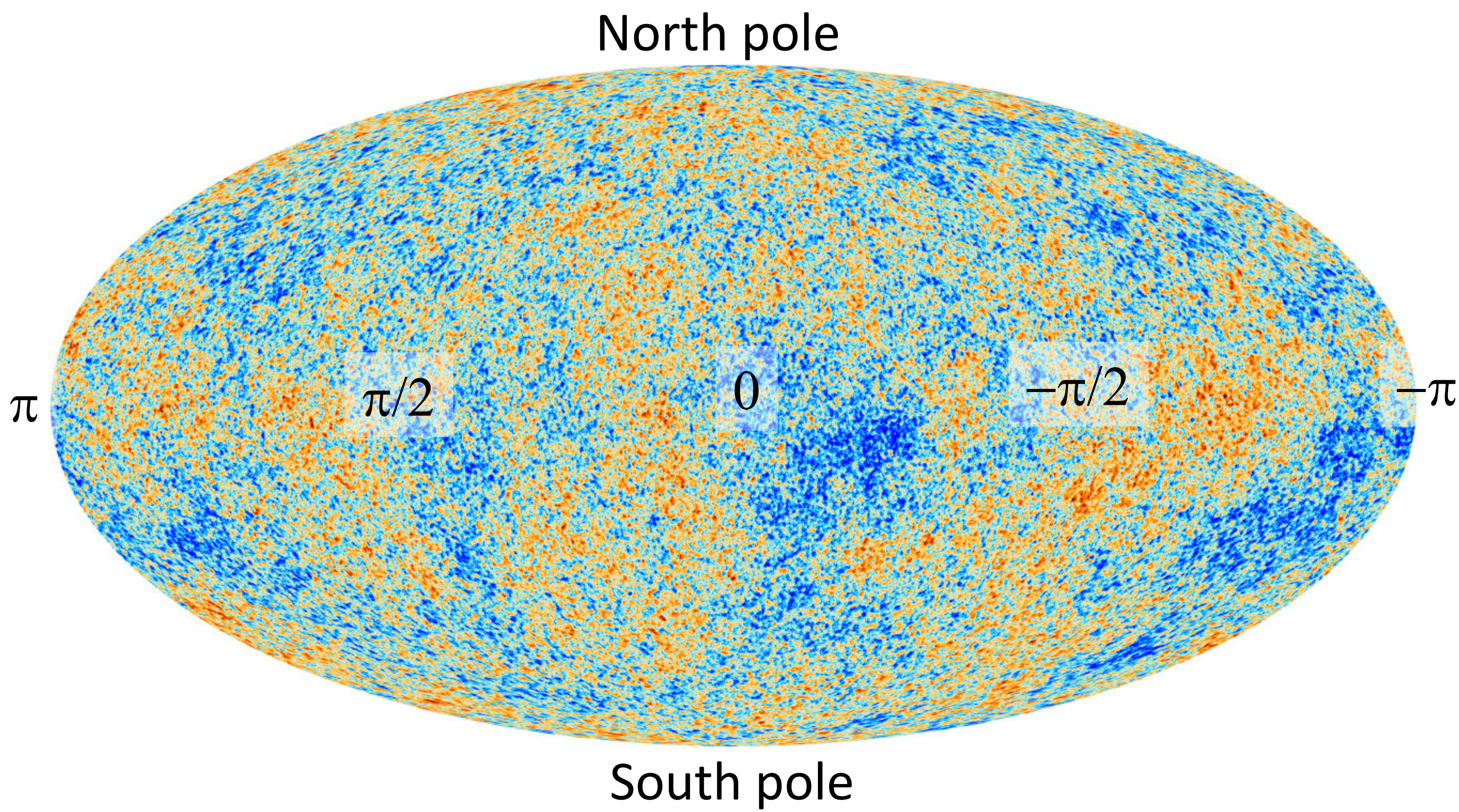
D. J. Fixsen, 2009

- CMB intensity



Planck, 2015

Observed CMB fluctuations

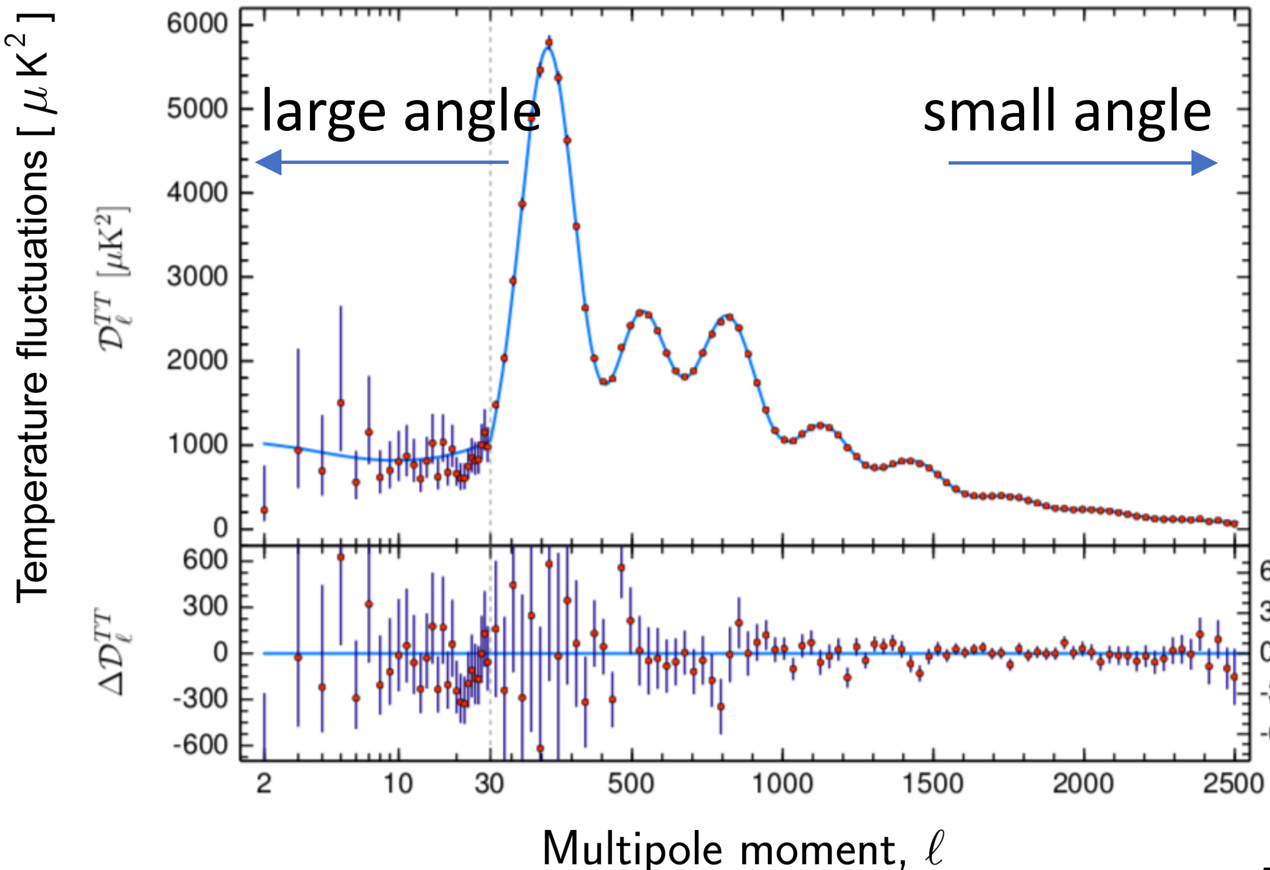


Mollweide projection of the elestial sphere

Angular power spectrum

Talk by Prof. Xun

$$\text{Angle} = \pi / l$$



Planck, 2018

Quantum fluctuations

$$\boxed{\begin{aligned}&\varphi + \delta\varphi \\ &\quad \to \mathcal{P}_s(k)\end{aligned}}$$

Scalar type fluctuation
Origin: quantum fluctuation of scalar field

Tensor type fluctuation
Origin: quantum fluctuation of space-time

$$\boxed{\begin{aligned}&g^{\mu\nu} + \delta h^{\mu\nu} \\ &\quad \to \mathcal{P}_t(k)\end{aligned}}$$

Observed CMB fluctuations

- Rescaled scalar type fluctuation

$$\mathcal{P}_s(k) \equiv A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

- Rescaled tensor type fluctuation

$$\mathcal{P}_t(k) \equiv A_t \left(\frac{k}{k_0}\right)^{n_t}$$

- Tensor to scalar ratio

$$r \equiv \frac{\mathcal{P}_t(k)}{\mathcal{P}_s(k)}$$

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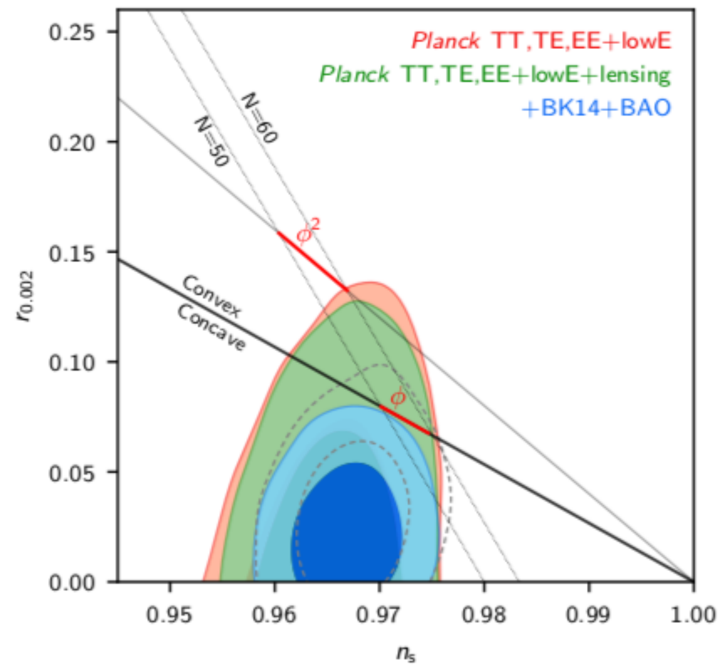
Planck, 2018

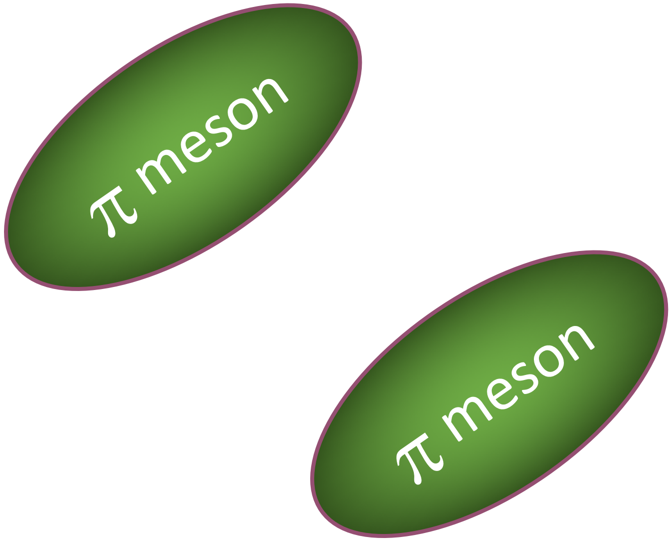


Gauged Nambu-Jona-Lasinio (gNJL) model

Original gNJL model

Lecture by Prof. Craig

- Low energy effective theory of light scalar mesons



Y. Nambu and G. Jona-Lasinio (1961).

V. A. Miransky, Dynamical Symmetry Breaking in Quantum Field Theories (1993).

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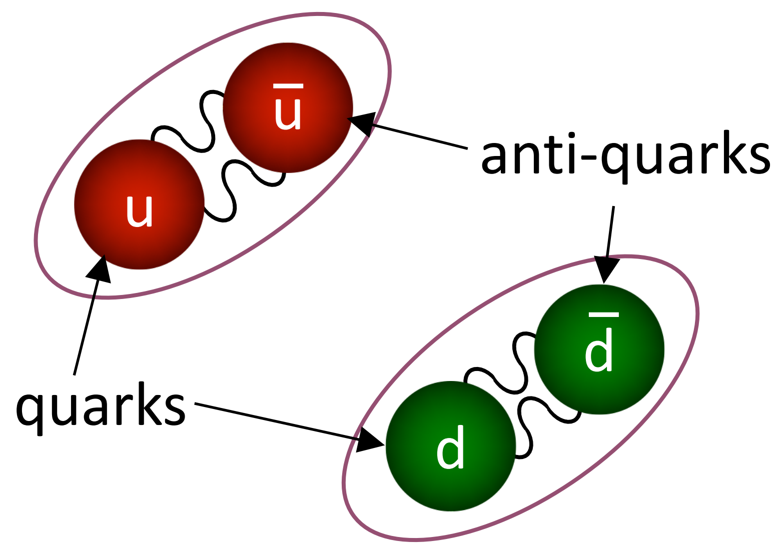

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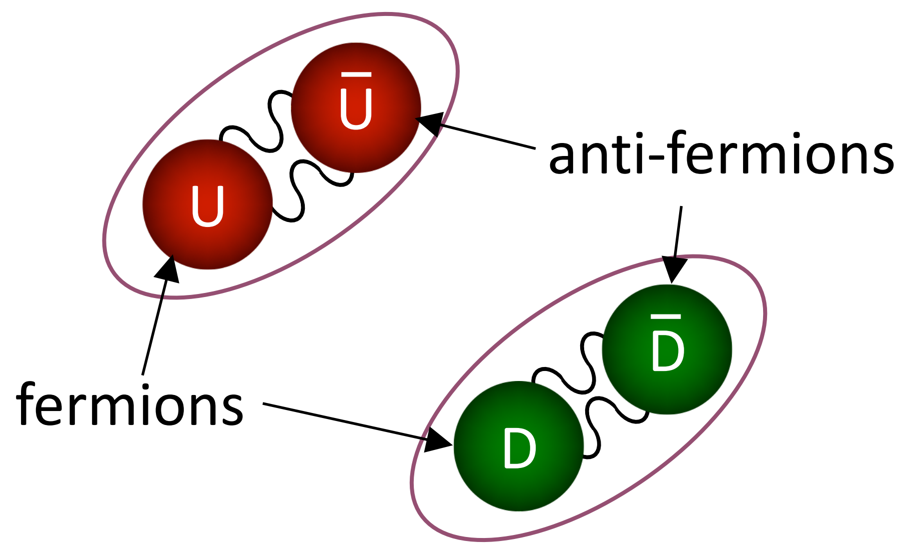
Original gNJL model

Lecture by Prof. Craig

- Low energy effective theory of light scalar mesons constructed by a quark and an anti-quark.
- Here we scale up the model from the QCD scale to the inflation scale



Y. Nambu and G. Jona-Lasinio (1961).

V. A. Miransky, Dynamical Symmetry Breaking in Quantum Field Theories (1993).

Scale up version of gNJL model

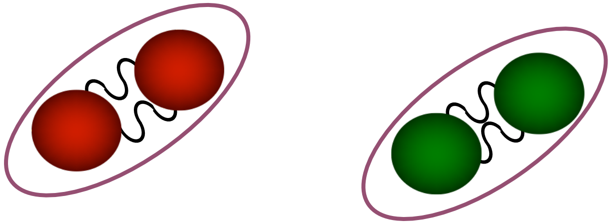
$SU(N_c) \otimes \mathcal{G}$ gauge theory with N_f fermion flavors

$\downarrow$ Strong enough

Four-fermion interaction

- Lagrangian density

$$\mathcal{L}_{gNJL} = \mathcal{L}_{SU(N_c)gauge} + \bar{\psi} i \hat{\not{D}} \psi + \frac{16\pi^2 g_4}{8 N_f N_c \Lambda^2} \left[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \tau^a \psi)^2 \right]$$



Auxiliary field method

- Equivalent Lagrangian density

$$\mathcal{L}_{aux} = \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - \sigma - i\gamma_5\tau^a\pi^a\right)\psi - \frac{2N_f N_c \Lambda^2}{16\pi^2 g_4}\left(\sigma^2 + \pi^{a2}\right)$$

with

$$\sigma = -\frac{16\pi^2 g_4}{4N_f N_c \Lambda^2}\bar{\psi}\psi, \quad \pi^a = -\frac{16\pi^2 g_4}{4N_f N_c \Lambda^2}\bar{\psi}i\gamma_5\tau^a\psi$$

Auxiliary field method

- Equivalent Lagrangian density

$$\mathcal{L}_{aux} = \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - \sigma - i\gamma_5\tau^a\pi^a\right)\psi$$
$$-\frac{2N_f N_c \Lambda^2}{16\pi^2 g_4}\left(\sigma^2 + \pi^{a\,2}\right)$$

- Gauged Higgs-Yukawa Lagrangian

$$\mathcal{L}_{gHY} = \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - y\sigma - yi\gamma_5\tau^a\pi^a\right)\psi$$
$$-\frac{1}{2}m^2(\sigma^2 + \pi^a\pi^a) + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a$$
$$-\frac{1}{2}\xi R(\sigma^2 + \pi^a\pi^a) - \frac{\lambda}{4}(\sigma^2 + \pi^a\pi^a)^2$$

Auxiliary field method

- Equivalent Lagrangian density

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- Gauged Higgs-Yukawa Lagrangian

$$\mathcal{L}_{gHY} = \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - y\sigma - yi\gamma_5\tau^a\pi^a\right)\psi - \frac{1}{2}m^2(\sigma^2 + \pi^a\pi^a) + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a - \frac{1}{2}\xi R(\sigma^2 + \pi^a\pi^a) - \frac{\lambda}{4}(\sigma^2 + \pi^a\pi^a)^2$$

Conventional normalization

- Transforming the fields in the gauged Higgs-Yukawa Lagrangian

  $\sigma \to \sigma/y,\ \pi^a \to \pi^a/y$

  we get

$$
\begin{aligned}
\mathcal{L}_{gHY} \quad = \quad & \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - \sigma - i\gamma_5\tau^a\pi^a\right)\psi \\
& -\frac{1}{2}\frac{m^2}{y^2}(\sigma^2 + \pi^a\pi^a) + \frac{1}{2y^2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2y^2}\partial_\mu\pi^a\partial^\mu\pi^a \\
& -\frac{\xi}{2y^2}R(\sigma^2 + \pi^a\pi^a) - \frac{\lambda}{4y^4}(\sigma^2 + \pi^a\pi^a)^2
\end{aligned}
$$

W. A. Bardeen, C. Hill & M. Lindner, Phys. Rev. D41 (1990) 1647
C. T. Hill & D. S. Salopek, Annals Phys. 213 (1992) 21

Compositeness condition

- We set the following conditions at the composite scale Λ

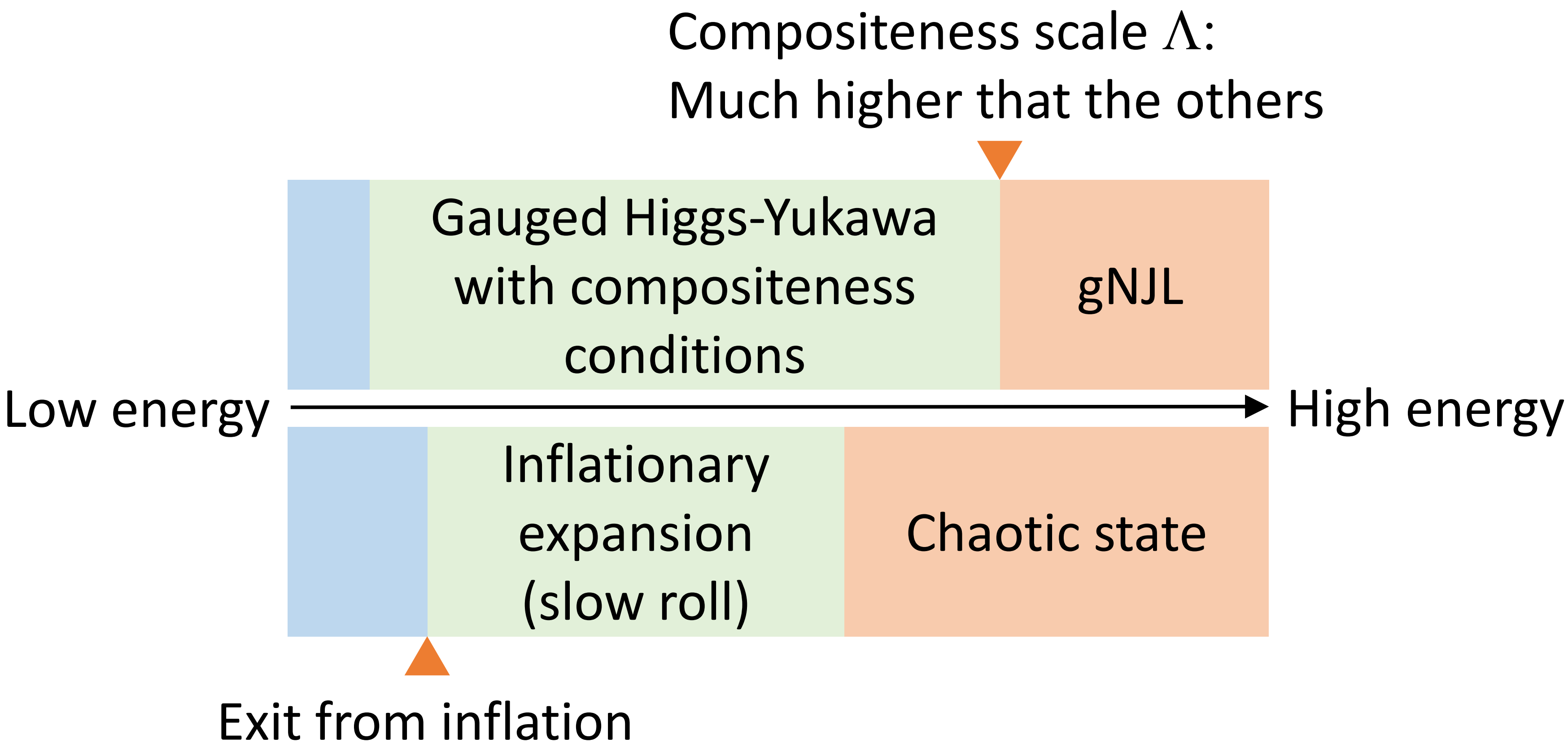
$$\frac{1}{y^2(\Lambda)} = 0,\ \frac{\lambda(\Lambda)}{y^4(\Lambda)} = 0,\ \xi(\Lambda) = \frac{1}{6},\ \frac{m^2(\Lambda)}{y^2(\Lambda)} = \frac{2a}{16\pi^2}\Lambda^2\left(\frac{1}{g_4} - \frac{1}{\Omega(\Lambda)}\right)$$

$$\begin{aligned}\mathcal{L}_{gHY} \quad = \quad & \mathcal{L}_{SU(N_c)gauge} + \bar{\psi}\left(i\hat{\not{D}} - \sigma - i\gamma_5\tau^a\pi^a\right)\psi \\ & -\frac{1}{2}\frac{m^2}{y^2}(\sigma^2 + \pi^a\pi^a) + \cancel{\frac{1}{2y^2}}\partial_\mu\sigma\partial^\mu\sigma + \cancel{\frac{1}{2y^2}}\partial_\mu\pi^a\partial^\mu\pi^a \\ & -\cancel{\frac{\xi}{2y^2}}R(\sigma^2 + \pi^a\pi^a) - \cancel{\frac{\lambda}{4y^4}}(\sigma^2 + \pi^a\pi^a)^2\end{aligned}$$

σ, π^a: composite scalar fields

W. A. Bardeen, C. Hill & M. Lindner, Phys. Rev. D41 (1990) 1647
C. T. Hill & D. S. Salopek, Annals Phys. 213 (1992) 21

Assumptions of our analysis



Assumptions of our analysis

- We neglect the running of the SU(Nc) gauge coupling, α.
- We omit higher order terms in R.
- Only the field, σ, contributes the inflationary expansion.

M. Harada, Y. Kikukawa, T. Kugo, H. Nakano, Prog. Theor. Phys. 92 (1994) 1161
B. Geyer and S. D. Odintsov, Phys. Lett. B376 (1996a) 260

Composite scalar field theory

- Composite scalar field

$$\bar{\psi}\psi \to \sigma$$

- Renormalization group improvement

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma) + \mathcal{L}_{int}\right]$$

$$V(\sigma) = \frac{B}{2}\sigma^2 + \frac{C_1}{4}\sigma^{4/(1+A\alpha)} - \frac{C_2}{4}\sigma^4 + \frac{R}{2}\frac{D_1}{6}\sigma^{2/(1+A\alpha)} - \frac{R}{2}\frac{D_2}{6}\sigma^2$$

We assume that only the composite scalar field, σ, contributes the inflation.

C. N. Leung, S. T. Love and W. A. Bardeen, Nucl. Phys. B273, 649 (1986) 649.
B. Geyer and S. D. Odintsov, Phys. Lett. B376 (1996a) 260.

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Einstein frame

- Weyl transformation and field redefinition

$$g_{\mu\nu} \to \Omega^2(\sigma) g_{\mu\nu} \qquad\qquad \sigma \to \varphi$$

- Effective action in the Einstein frame

$$S = \int d^4x \sqrt{-\tilde{g}} \left[-\frac{1}{2}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V_E(\varphi) + \mathcal{L}_{int} \right]$$

$$V_E = \Omega^{-4}(\sigma) \left(\frac{B}{2}\sigma^2 + \frac{C_1}{4}\sigma^{4/(1+A\alpha)} - \frac{C_2}{4}\sigma^4 \right)$$

T. I., S. D. Odintsov and H. Sakamoto, Astrophys. Space Sci. 360 (2015) 67,
T. I., S. D. Odintsov and H. Sakamoto, Nucl. Phys. B919 (2017) 297.

Inflationary Cosmology in the gNJL Model

Origin of inflationary expansion

- Sources of energy density

Radiation	$a(t) \propto t^{1/2}$
Matter	$a(t) \propto t^{2/3}$
Potential energy	$a(t) \propto \exp(\alpha t)$
Cosmological constant	

- Another possibility

Modified gravity

Origin of inflationary expansion

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Radiation	$a(t) \propto t^{1/2}$
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Potential energy	$a(t) \propto \exp(\alpha t)$
Cosmological constant	

- Another possibility

Modified gravity

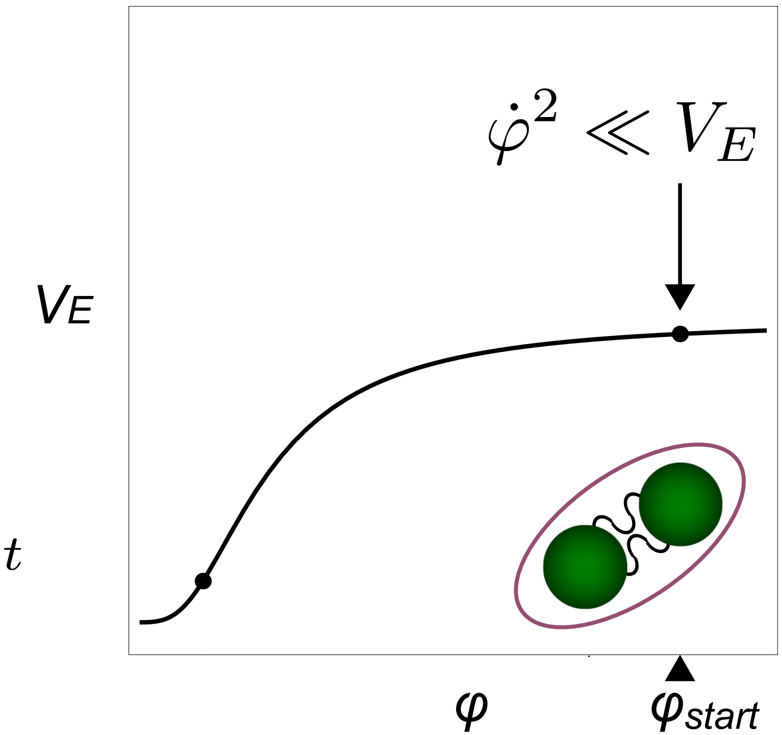
Quasi de-Sitter expansion

- Friedman equation

$$3\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{2}\dot{\varphi}^2 + V_E$$

- Assumption $\dot{\varphi}^2 \ll V_E$

$$a(t+\Delta t) \sim a(t)e^{\sqrt{\frac{V_E}{3}}\Delta t}$$



A. D. Linde, Contemp. Concepts Phys. 5, 1 (1990).

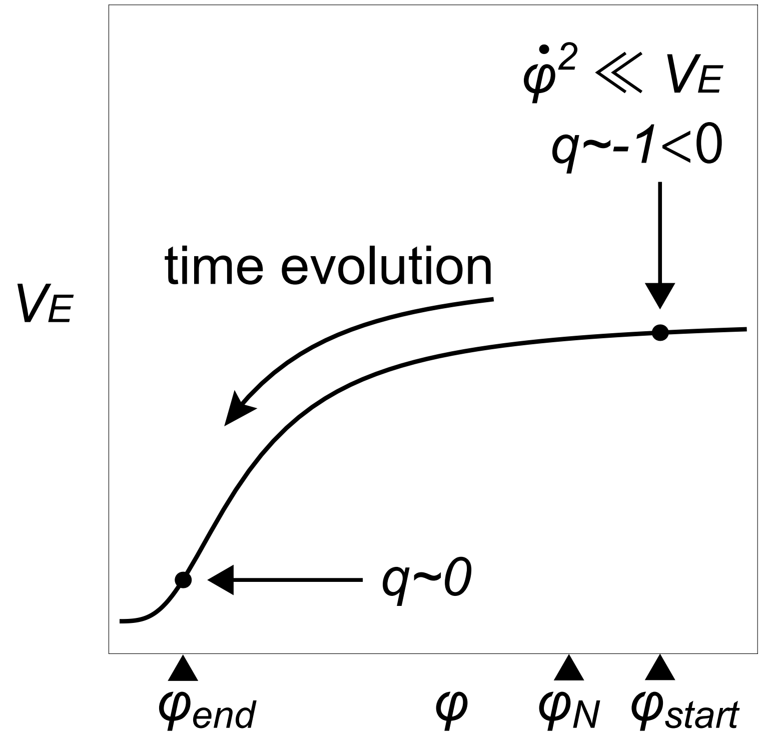
Exit from Inflation

- Equation of motion

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} = -\frac{\partial V_E}{\partial \varphi}$$

- Deceleration parameter

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2} \to 0$$



A. D. Linde, Contemp. Concepts Phys. 5, 1 (1990).

To solve the horizon problem

- The horizon problem can be solved, if

$$\frac{1}{\dot{a}(t_{today})} < \frac{1}{\dot{a}(t_{start})}$$

- Time derivative of the scale factor

$$\frac{\dot{a}(t_{today})}{\dot{a}(t_{end})} \sim \frac{T_0}{T_{end}} \sim 10^{-27}$$

- E-folding number (We assume that $\frac{\dot{a}}{a}$ is constant.)

$$\frac{a(t_{end})}{a(t_{start})} > 10^{27} \qquad N \equiv \log \frac{a(t_{end})}{a(t_{start})} > 50 \sim 60$$

CMB fluctuations

The exit from the inflation is found by evaluating the deceleration parameter $q=0$.

The value of φ at the start point (horizon crossing) is fixed to generate a suitable e-folding number, $N=50\text{-}60$.

We evaluate time evolution of the scalar and tensor fluctuations and find a constraint from CMB fluctuations.

Slow roll parameters

- Here we introduce two parameters,

$$\varepsilon = \frac{1}{2}\left(\frac{1}{V}\frac{\partial V}{\partial \phi}\right), \eta = \frac{1}{V}\frac{\partial^2 V}{\partial \phi^2}$$

- Then we caluculate

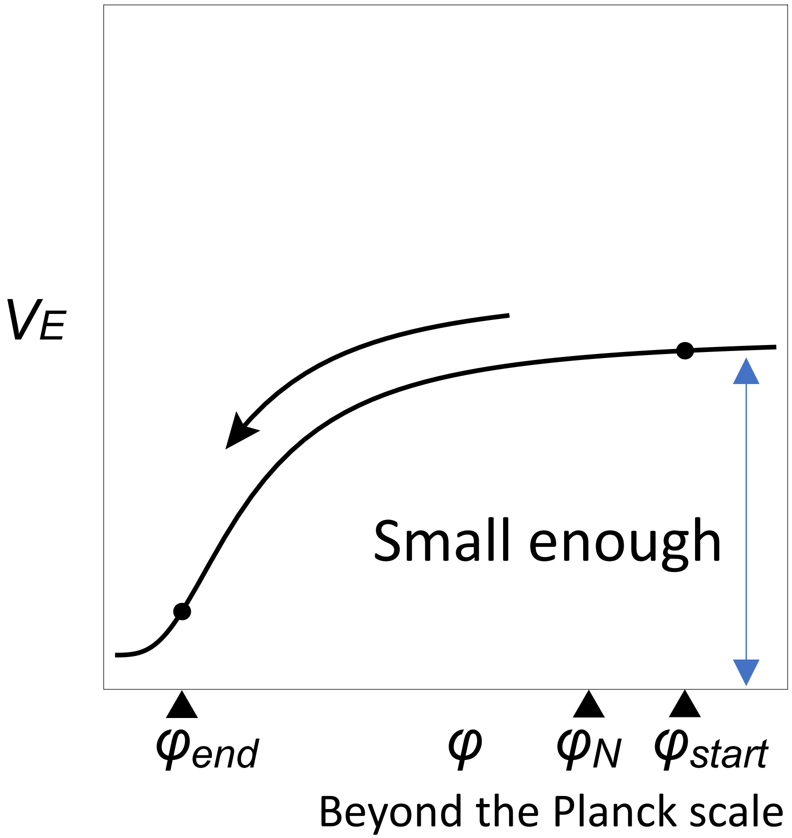
$$\phi_{end} : \varepsilon = 1 \text{ or } \eta = 1$$

$$\phi_N : \quad N = \int_{\phi_{end}}^{\phi_N} \frac{V}{\partial V/\partial \phi} d\phi \sim 50 \sim 60$$

$$n_s - 1 = (2\eta - 6\varepsilon)|_{\phi=\phi_N}$$

$$r = 16\varepsilon|_{\phi=\phi_N}$$

Observed small amplitude, As



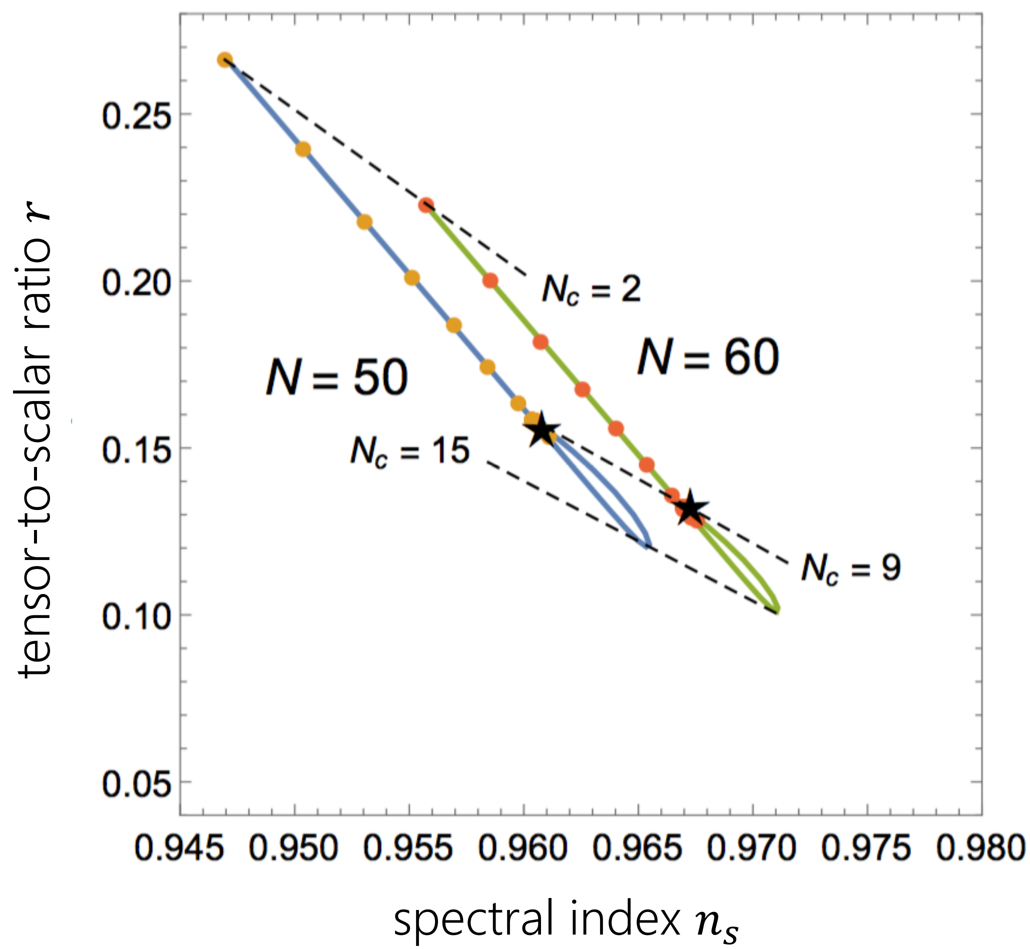
- Tune the gauge coupling, α, the renormalization scale, μ, and the compositeness scale, Λ.

  T. I., S. D. Odintsov and H. Sakamoto, Nucl. Phys. B (2017),

- Introduce a huge curvature coupling

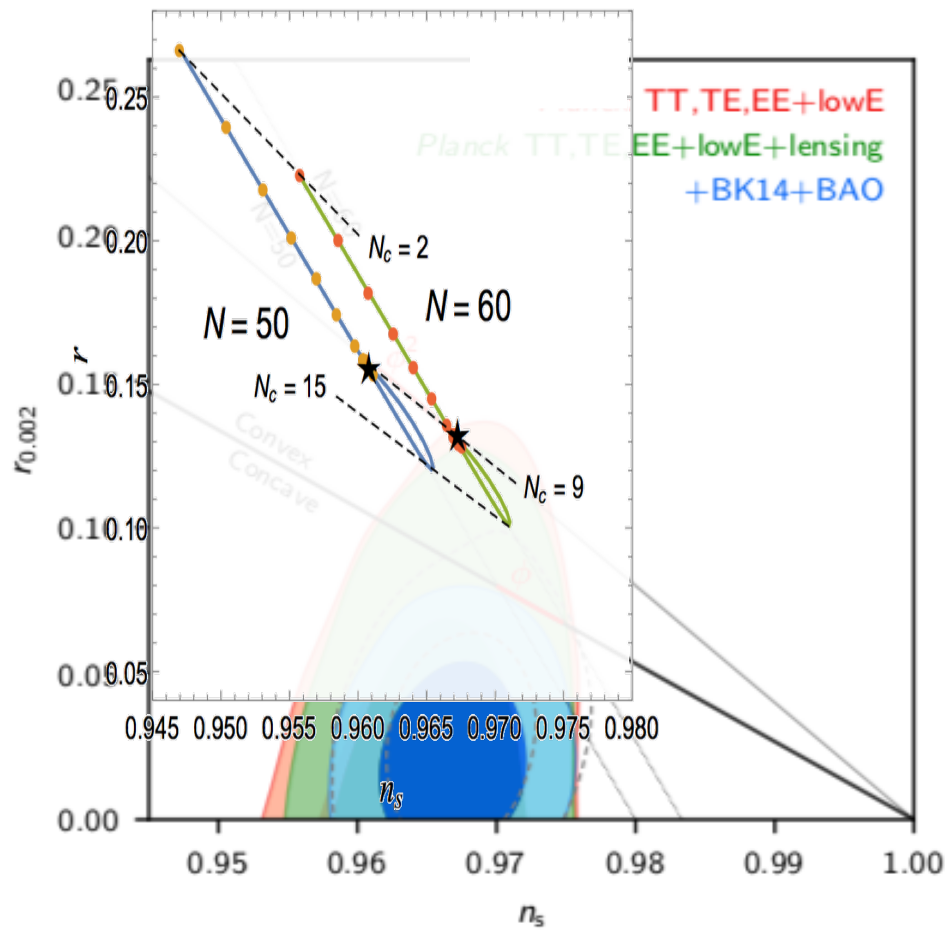
  P. Channuie and C. Xiong, Phys. Rev. D 94, 043521 (2017)

Numerical results



$$G_{4r} = 10^4,\ \alpha = 0.5,\ N_f = 1,\ \Lambda = 20M_p$$

Numerical results



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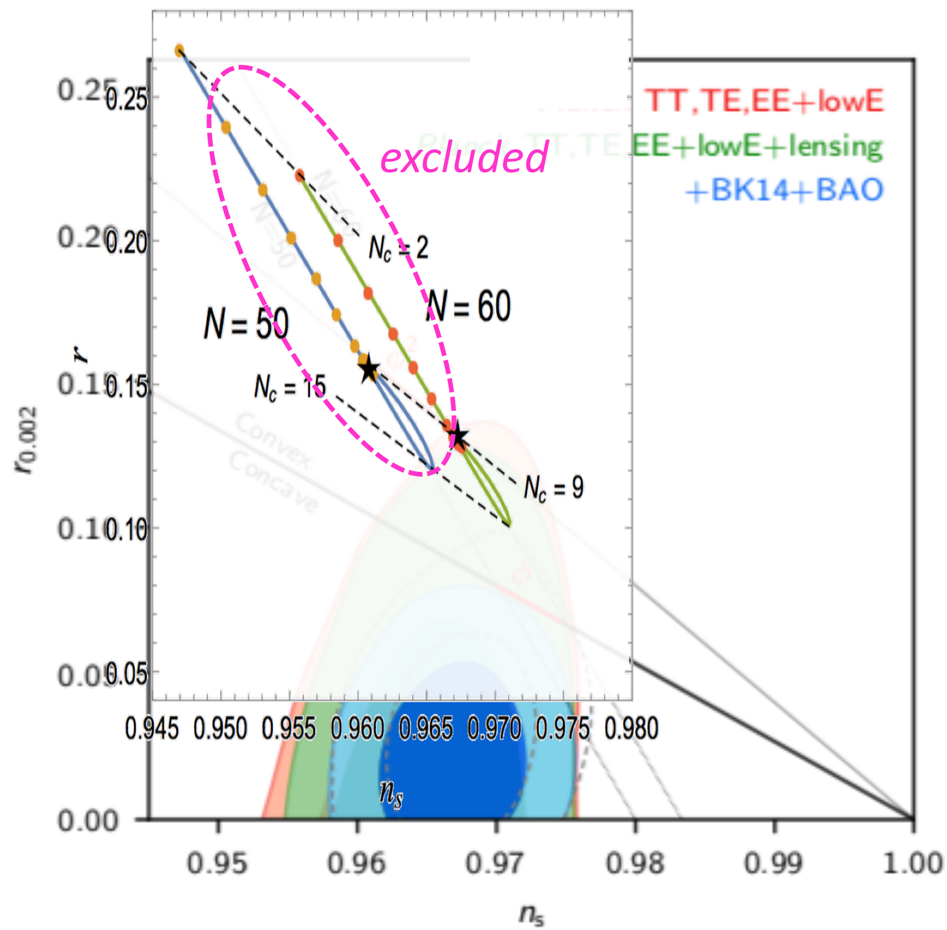

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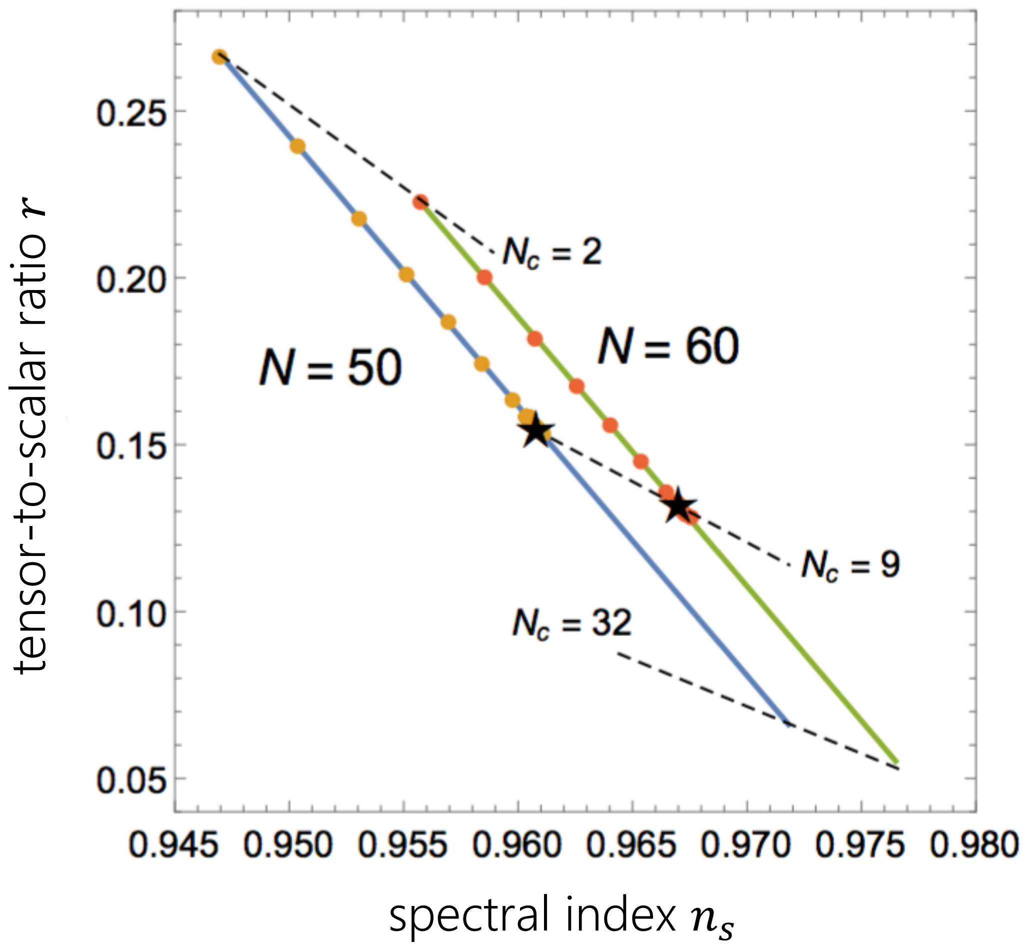

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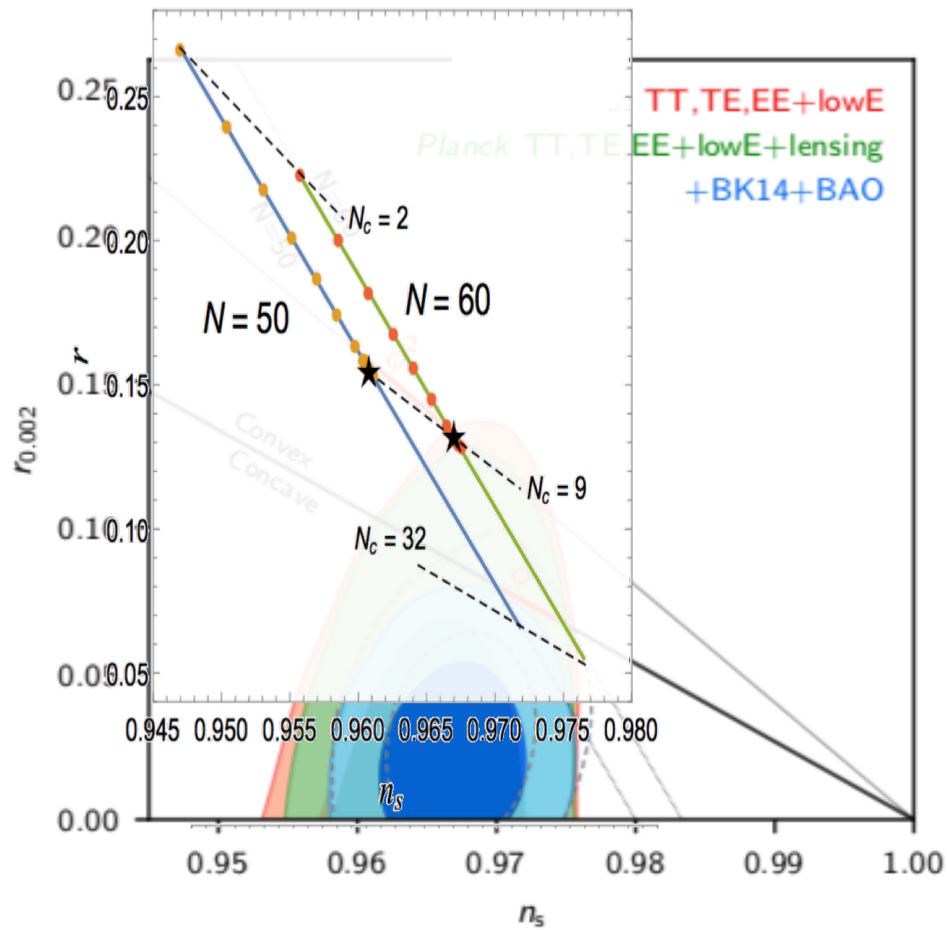

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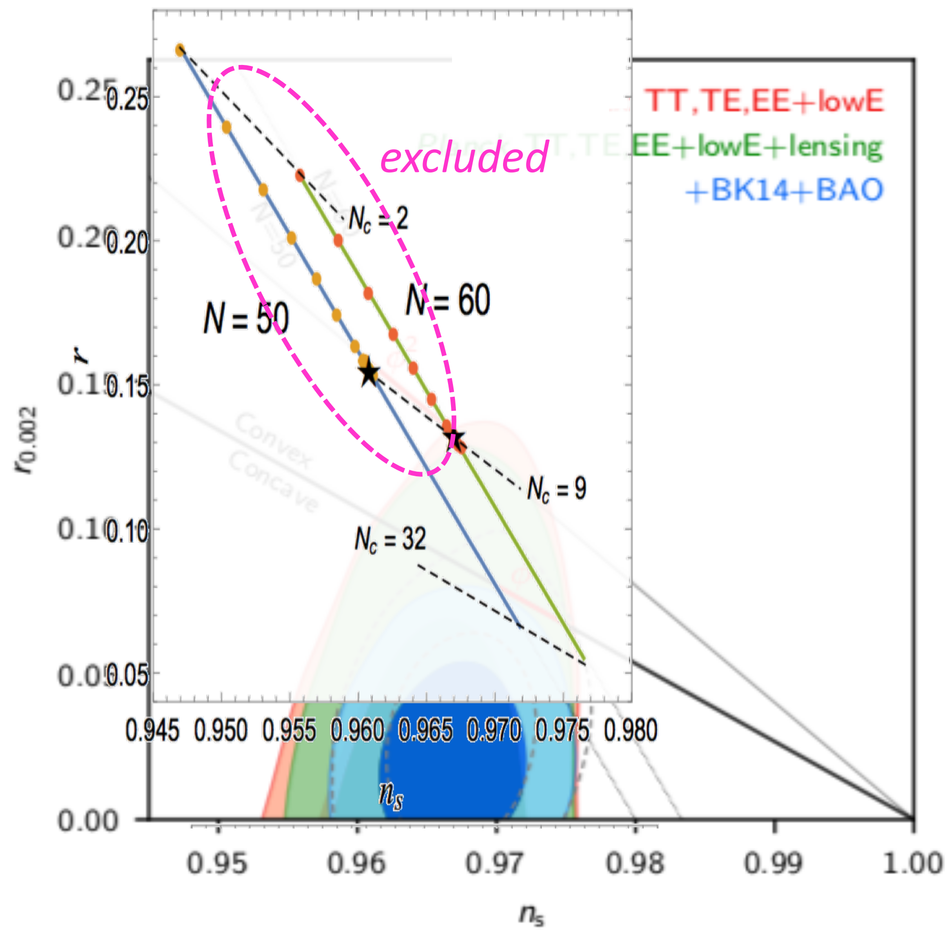
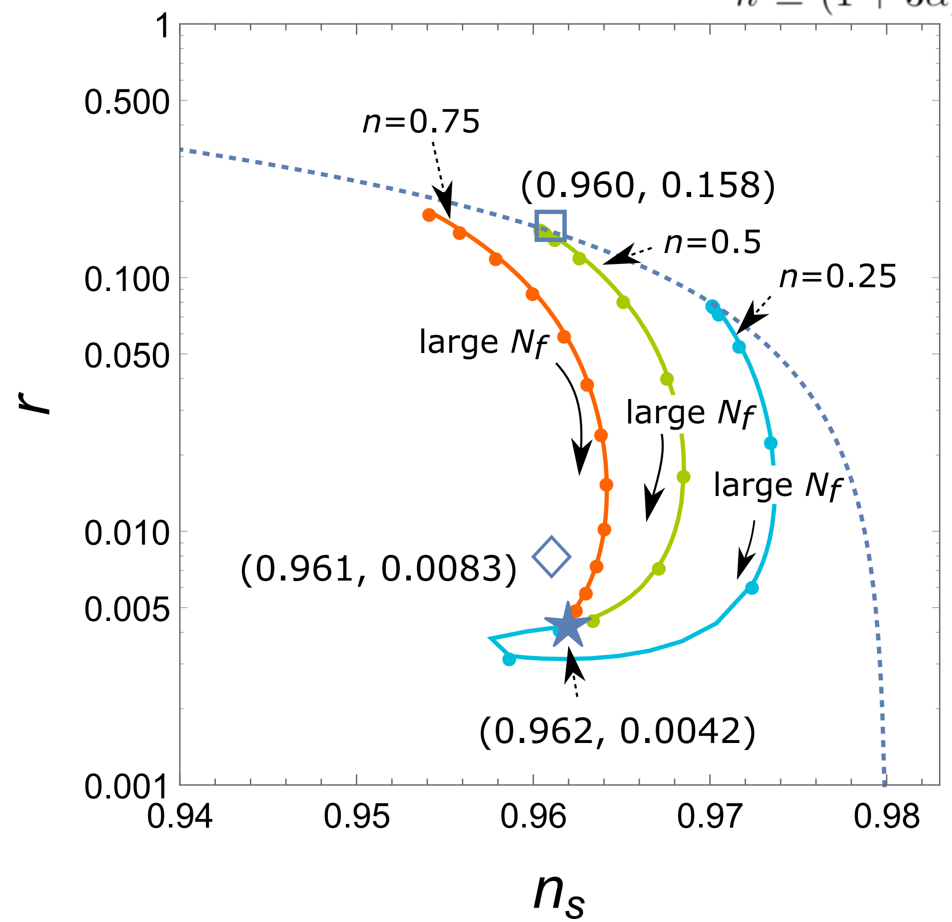


$$G_{4r} = 10^{10},\ \alpha = 0.5,\ N_f = 1,\ \Lambda = 20M_p$$

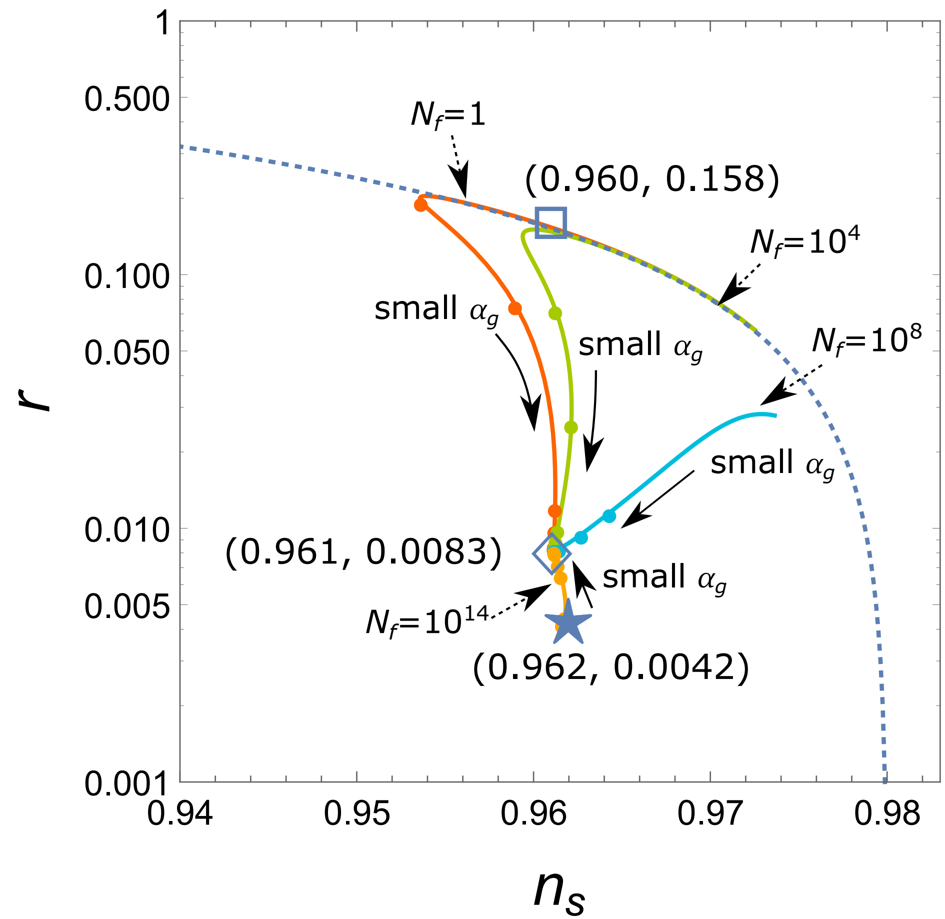
Numerical results

$n \equiv (1 + 3\alpha\ (N_c^2 - 1)/(4\pi N_c))^{-1}$



$$G_{4r} = 10^{10},\ \alpha = 0.5,\ \Lambda/\mu = 10^3$$

Numerical results



$$G_{4r} = 10^{10},\ N_c = 2,\ \Lambda/\mu = 10^3$$

Analytical expressions

- Flat limit (chaotic inflation)

$$n_s = 1 - \frac{m+1}{N} \qquad r = \frac{8m}{N}$$

- Steep limit (Starobinsky model, $N_f N_c \sim \mathrm{O}(10^{10})$)

$$n_s = 1 - \frac{2}{N} \qquad r = \frac{12}{N^2}$$

Universal attractor, α=1

R. Kallosh, A. Linde and D. Roest, Phys. Rev. Lett. 112 (2014) 011303

- Weak coupling limit $\alpha \to +0, M_P \ll \Lambda$

$$n_s = 1 - \frac{2}{N} \qquad r = \frac{24}{N^2}$$

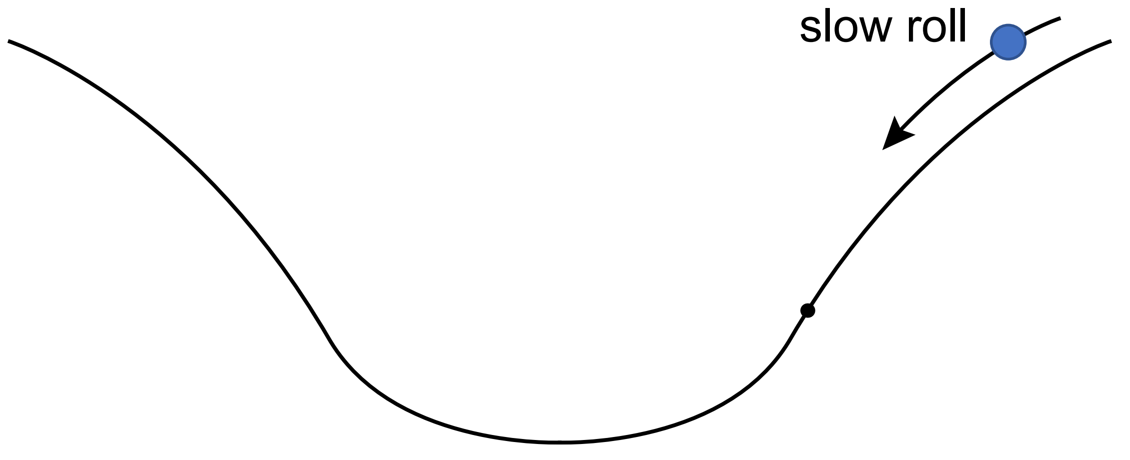
α=2, α–attractor model

T. I., S. D. Odintsov and H. Sakamoto, Nucl. Phys. B (2017).

T. I., S. D. Odintsov and H. Sakamoto, Europhys. Lett. 118 (2017) 29001.

Reheating process

- The potential energy of the inflaton is released through the decay process of the inflaton into light particles.



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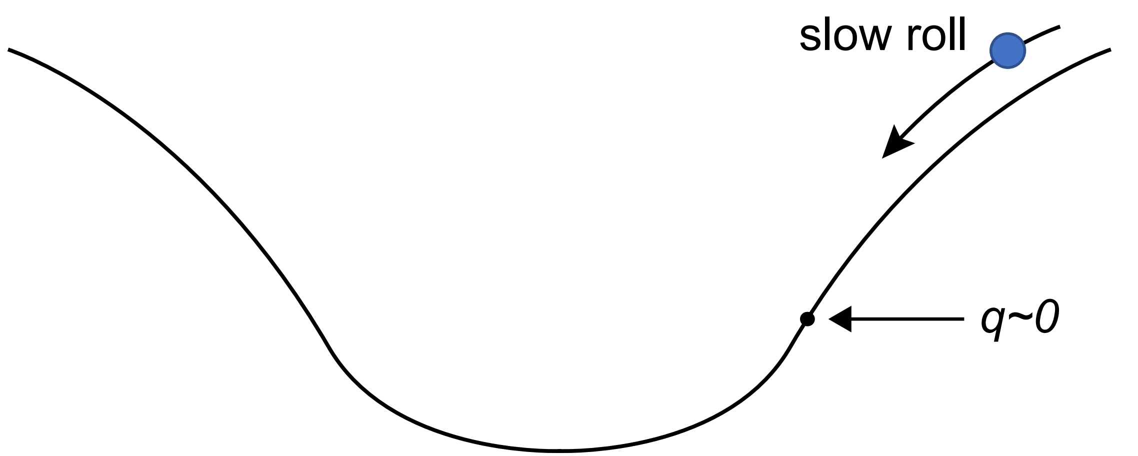

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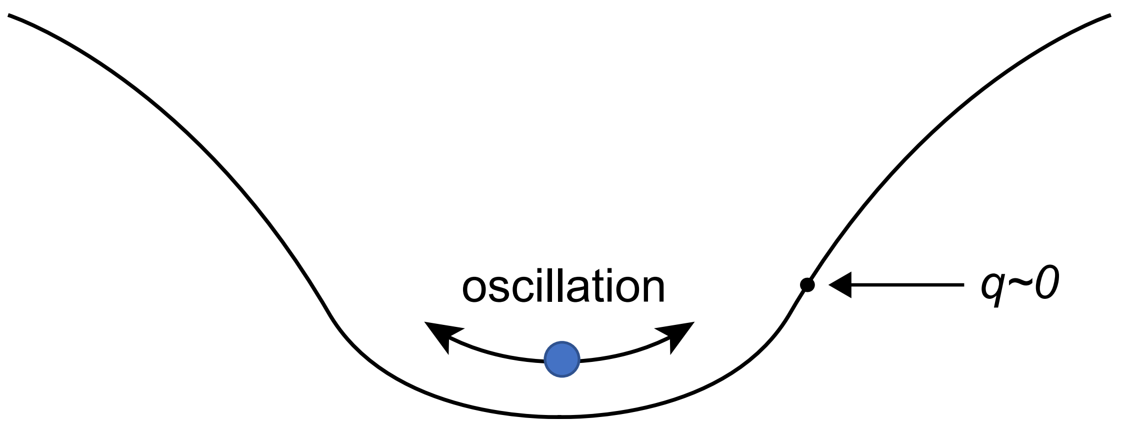

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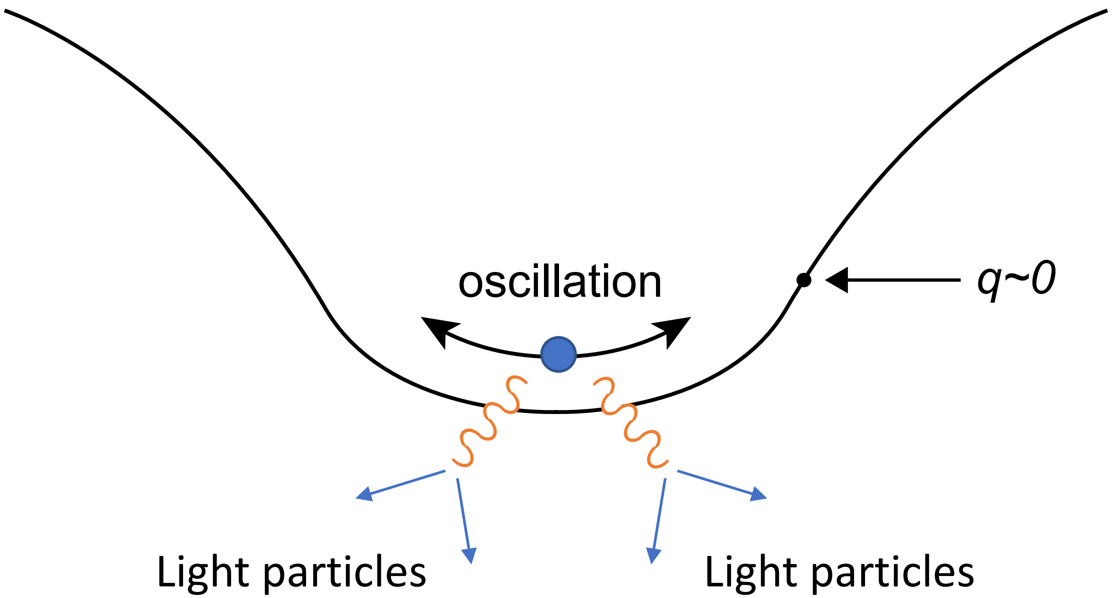
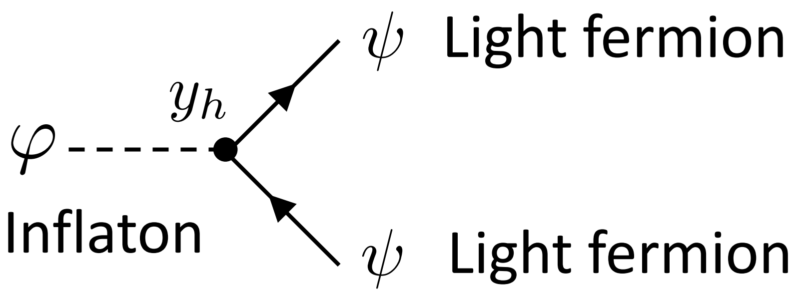


Inflaton decay

- Decay process (example)



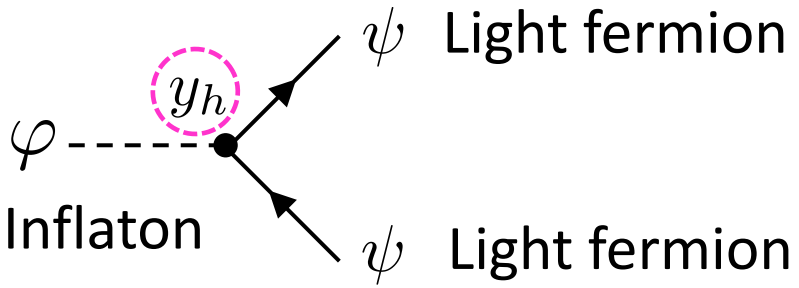
- Reheating temperature

$$T_R \sim 0.2\sqrt{\frac{y_h^2}{8\pi} M M_p}$$

M: Inflaton mass, y_h: Yukawa coupling

Inflaton decay

- Decay process (example)



- Reheating temperature

$$T_R \sim 0.2\sqrt{\frac{y_h^2}{8\pi} M M_p} \quad \xrightarrow[y_h < 1]{} \quad \begin{array}{l} T_R < 10^{15}\mathrm{GeV} \\ \alpha = 0.5, G_{4r} = 10^{10} \end{array}$$

M: Inflaton mass, y_h: Yukawa coupling

Dark matter candidate

- If there is no coupling with the SM particles, the composite scalar can be a dark matter candidate.

M. Holthausen, J. Kubo, K. S. Lim, M. Lindner. JHEP 1312 (2013) 076,
P. Channuie and C. Xiong, Phys. Rev. D 94, 043521 (2017)

Concluding remarks

Summary

- Inflationary expanding universe has been investigated in a composite model, the gauged NJL model.
- CMB fluctuations are calculated under the slow roll approximation.
- At flat, steep and weak coupling limits we obtain the explicit expressions of the CMB fluctuations.
- We obtain a consistent spectral index, tensor-to-scalar ratio with the Planck 2018 data.