Wavefunction Localization on Sphere with Magnetic Fluxes and Branes

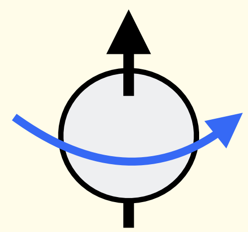
Sosuke Imai (Waseda Univ.)

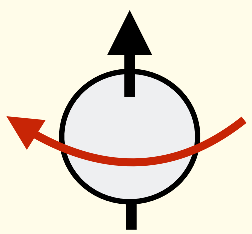
arXiv : 18xx.xxxxx

Collaboration with
Yoshiyuki Tatsuta (DESY)

Mysteries of the Standard Model

* Chiral asymmetry

Right  $(\mathbf{3},\mathbf{1})_{\frac{4}{3}} \oplus (\mathbf{3},\mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1},\mathbf{1})_{-2} \oplus (\mathbf{1},\mathbf{1})_{0}$

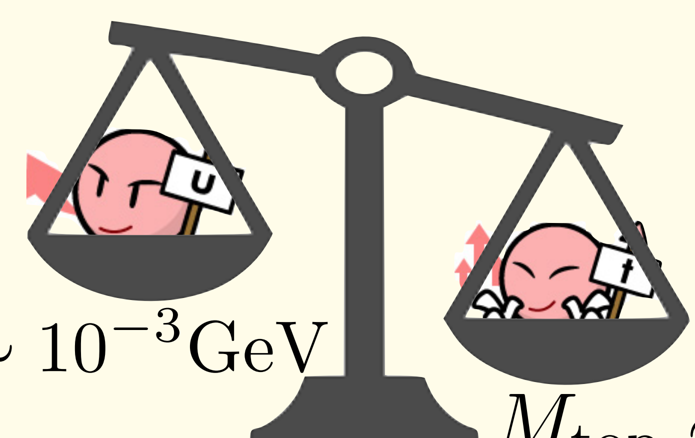
Left  $(\mathbf{3},\mathbf{2})_{\frac{1}{3}} \oplus (\mathbf{1},\mathbf{2})_{-1}$

* Three generation

same charges

* Yukawa hierarchy

$$M_{\rm top}/M_{\rm up} \sim 10^5$$



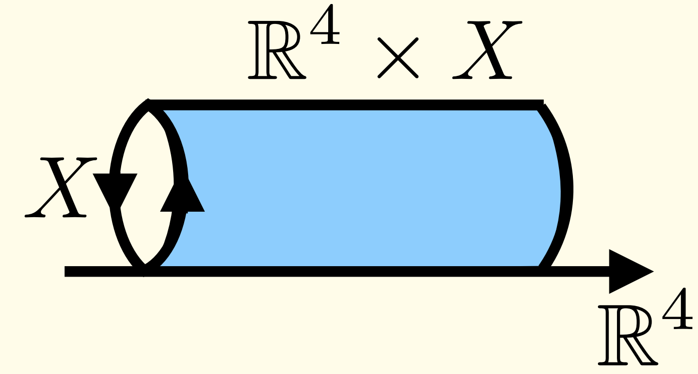
$M_{\rm up} \sim 10^{-3}\,{\rm GeV}$

$M_{\rm top} \sim 10^{2}\,{\rm GeV}$

Extra Dimensional Model

* Action (fermion)

$$S_{\rm fer} = \int_{\mathbb{R}^4} d^4x \left[\int_X \sqrt{|g|} d^{2n}y\, \bar{\psi}(i\not{D}_4 + i\not{D}_{2n})\psi \right]$$



* KK-mode expansion

$$\psi(x,y) = \sum_{k=0}^{\infty} \sum_{i=1}^{n_k} \psi_{k,i}(x) \otimes \chi_{k,i}(y)$$

$$i\not{D}_{2n}\chi_{k,i} = m_k \chi_{k,i}$$

$$m_k \propto \frac{1}{\sqrt[2n]{\mathrm{vol}(X)}}$$

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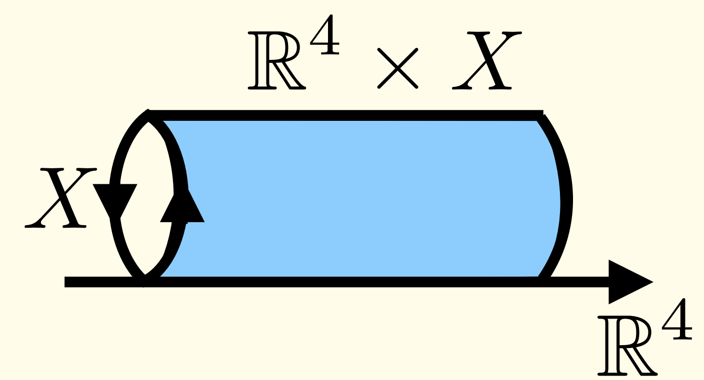

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$\int_X$

$$S_{\text{fer}} = \int_{\mathbb{R}^4} d^4x \sum_{k=0}^{\infty} \sum_{i=1}^{n_k} \bar{\psi}_{k,i}(i\not{D}_4 + m_k)\psi_{k,i}$$

Extra Dimensional Model

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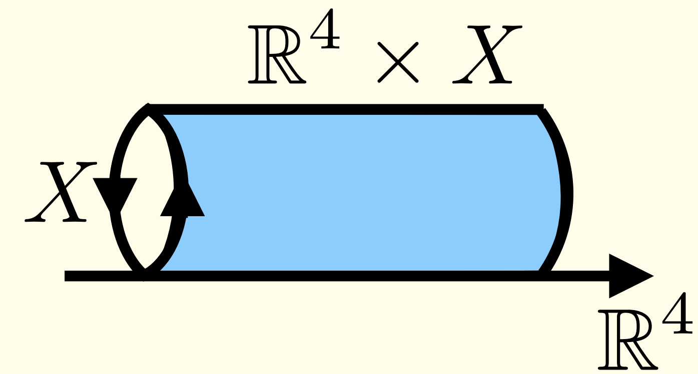

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$\mathrm{vol}(X)$:small

Only massless modes $m_0 = 0$ **appear**

$$\sim \int_{\mathbb{R}^4} d^4x \sum_{i=0}^{n_0} \bar{\psi}_{0,i} i\not{D}_4 \psi_{0,i}$$

Effective theory

Extra Dimensional Model

Zero-mode wavefunction

$\psi(x,y) \sim \sum_{i=1}^{n_0} \psi_{0,i}(x) \otimes \chi_{0,i}(y)$ at low energy, where $\chi_{0,i}$ satisfies

$$\not{D}_{2n}\chi_{0,i} = \begin{pmatrix} 0 & \not{D}_+ \\ \not{D}_- & 0 \end{pmatrix} \begin{pmatrix} \chi^-_{0,i} \\ \chi^+_{0,i} \end{pmatrix} = 0$$

Mysteries of the SM		Ans. from Extra dim. model
Chiral asymmetry	←	Asymmetry of $\ker \not{D}_\pm$
Three generation	←	Index of $\not{D}_{2n}$. $\dim\ker \not{D}_+ - \dim\ker \not{D}_- = 3$
Yukawa hierarchy	←	Overlap integral of wavefunctions $y_{ijk} = \int_X \sqrt{\lvert g \rvert} d^{2n}y\, \phi_{0,i}\chi^\dagger_{0,j}\chi_{0,k}$

J.P. Conlon, A. Maharana and F. Quevedo, JHEP**09** (2008) 104

Review : Sphere with Magnetic Fluxes

There are no zero-modes

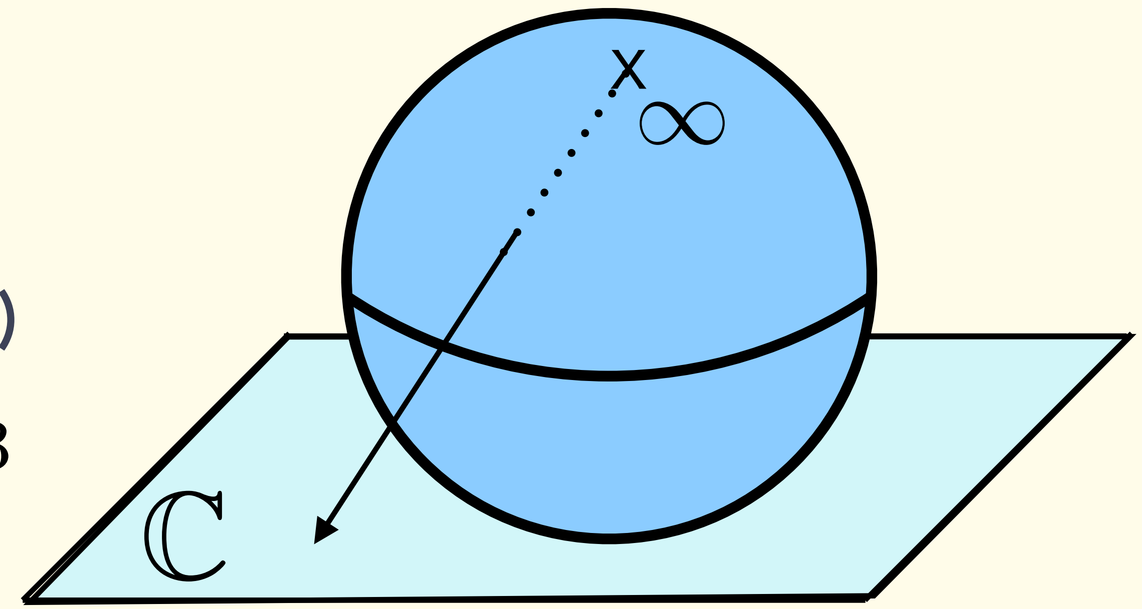
∵ Positive curvature of the sphere

↓

We need magnetic fluxes (gauge field background)

$$\langle F_{\mu\nu} \rangle = 0 \qquad \mu, \nu = 0, \ldots, 3$$

$$\langle F_{45} \rangle = \frac{2M}{(1+|z|^2)^2} \qquad M \in \mathbb{Z}$$



* Zero-mode wavefunctions

$$M > 0 \quad \Rightarrow \quad \chi_{0,i} = \begin{pmatrix} |g|^{\frac{M-1}{8}} z^i \\ 0 \end{pmatrix}$$

chiral asymmetry

$$i = 0, \ldots, |M| - 1$$

# of generations ($|M|$)

$$M < 0 \quad \Rightarrow \quad \chi_{0,i} = \begin{pmatrix} 0 \\ |g|^{\frac{-M-1}{8}} z^i \end{pmatrix}$$

Magnetic fluxes controls chiral asymm. & # of generations

*J.P. Conlon, A. Maharana and F. Quevedo, JHEP**09** (2008) 104*

Review : Sphere with Magnetic Fluxes

- ✔ Chiral asymmetry
- ✔ Three generation

← Magnetic fluxes

<u>How about Yukawa hierarchy ?</u>

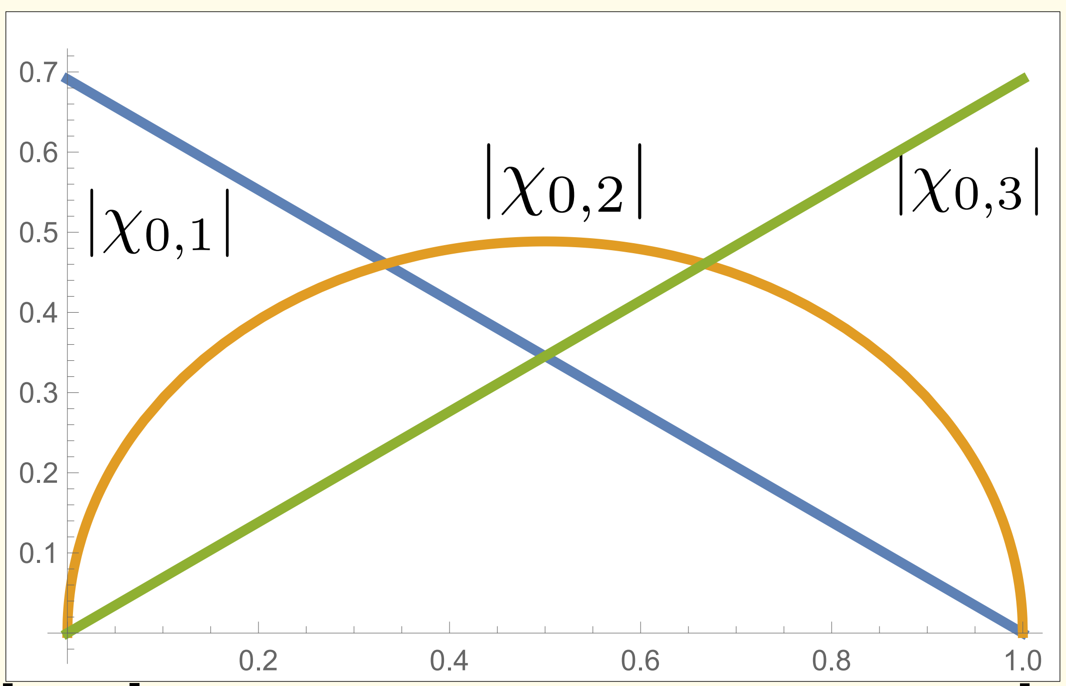
J.P. Conlon, A. Maharana and F. Quevedo, JHEP ***09*** *(2008) 104*

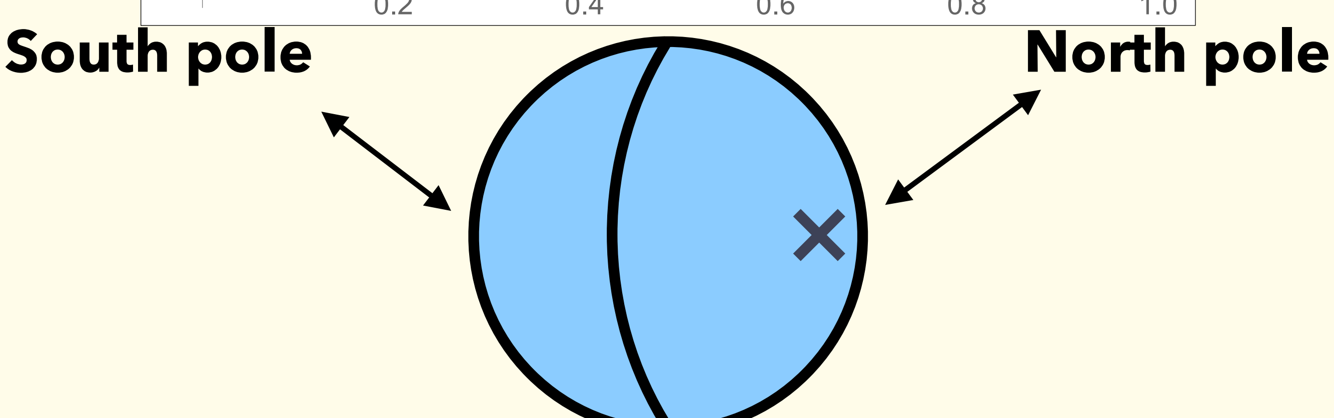
Review : Sphere with Magnetic Fluxes

- ✓ Chiral asymmetry
- ✓ Three generation

← Magnetic fluxes

How about Yukawa hierarchy ?





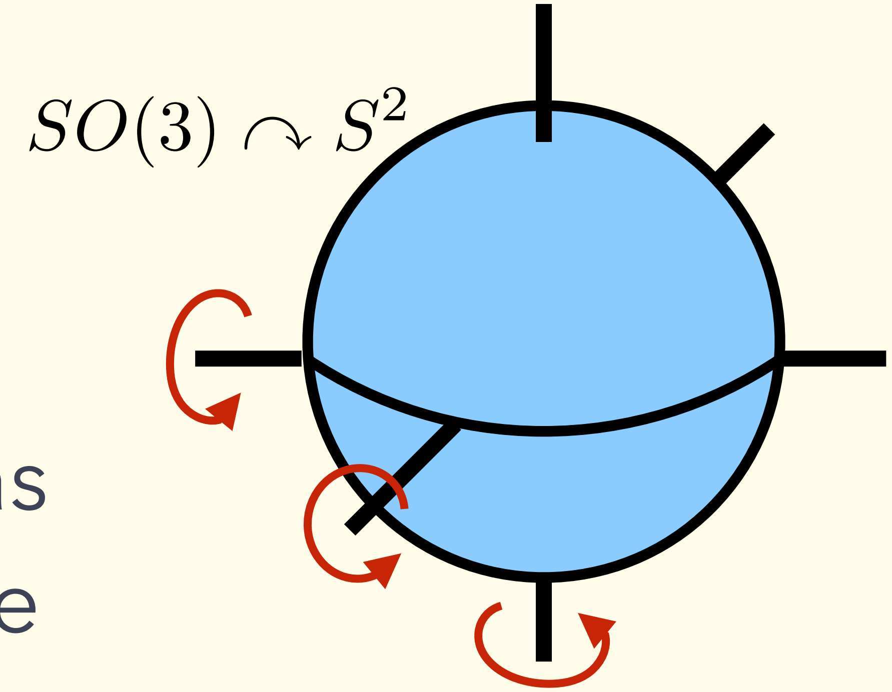
Large overlapping

➡ Less hierarchical Yukawa

$$y_{ijk} = \int_X \sqrt{|g|} d^{2n}y \, \phi_{0,i} \chi_{0,j}^{\dagger} \chi_{0,k}$$

✗ Yukawa hierarchy

Our idea

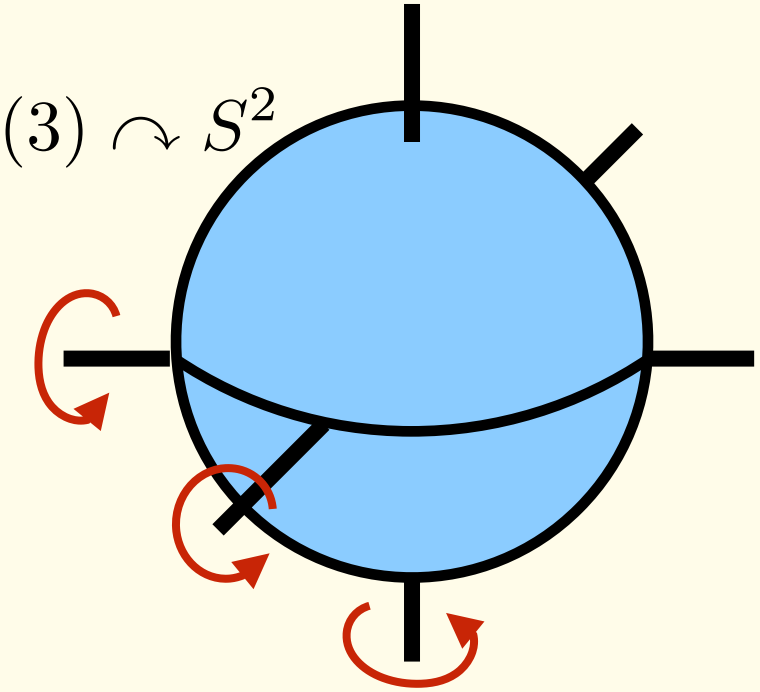


* Why is not Yukawa hierarchical ?

We thought that one of the reasons was the **Large isometry group** of the sphere

What happens if the isometry is broken ?

Our idea



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What happens if the isometry is broken ?

* Our idea

M_6 : 6d Plank mass

α_k : dimensionless tension

$$S \to S - 2\pi M_6^4 \sum_{k=1}^{N} \alpha_k \int_{\mathbb{R}^4 \times S^2} \sqrt{|g|} d^4x dz d\bar{z}\, \delta^2(z - z_k)$$

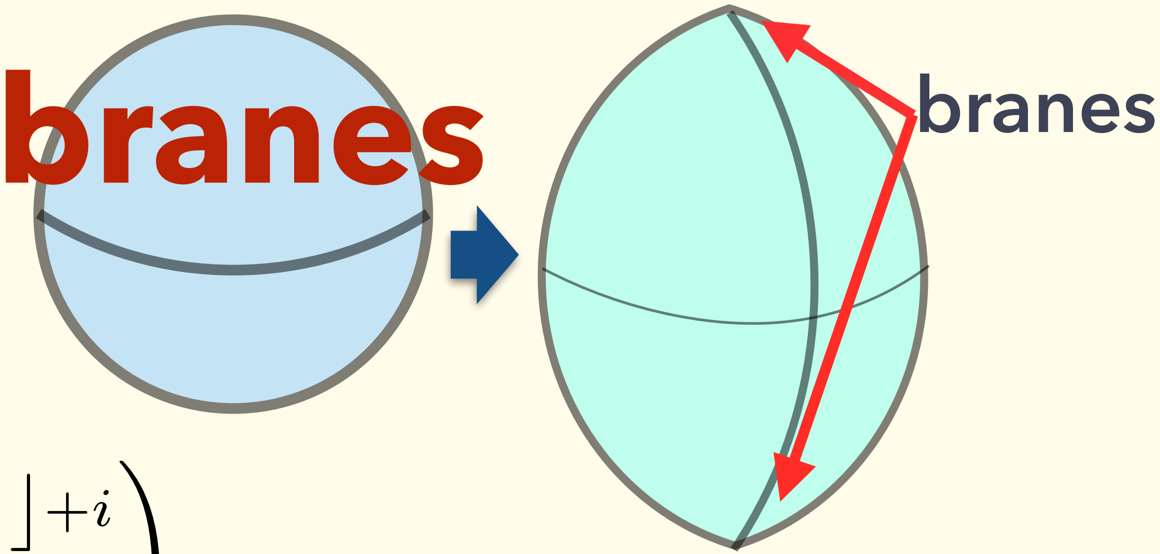
D3-branes

* The isometry is broken (explicitly)

* Solutions of the Einstein eq. for two and three branes are known

*M. Redi, Phys.Rev. D**71** (2005) 044006*

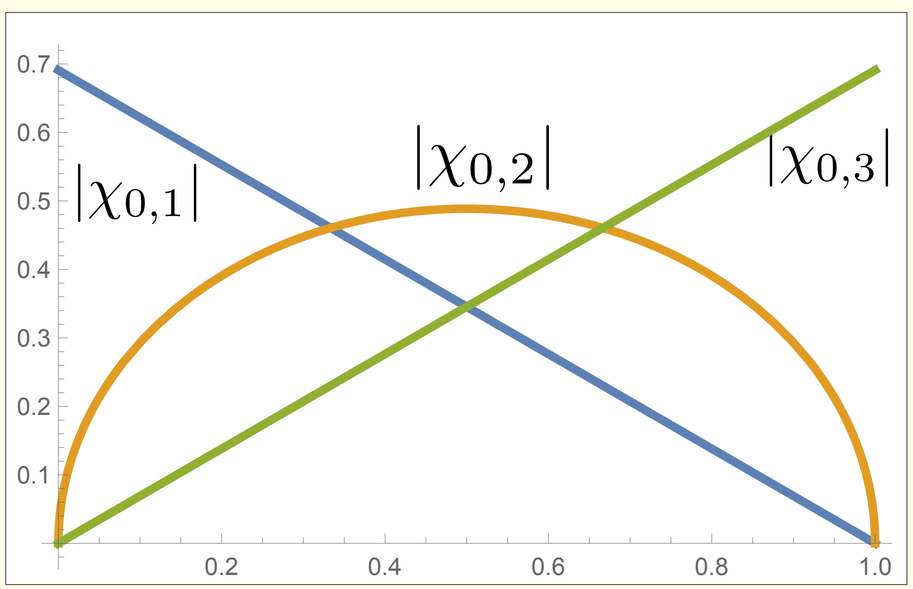
Result : two branes



* Zero-mode wavefunction

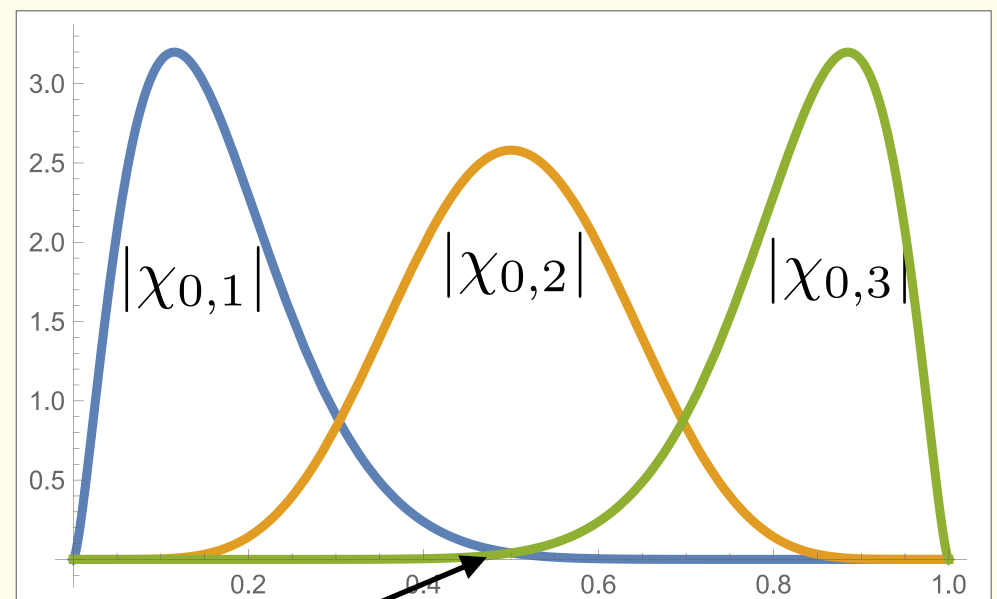
$$M > 2\left\lfloor \alpha \frac{M+1}{2} \right\rfloor \;\Rightarrow\; \chi_{0,i} = \begin{pmatrix} |g|^{\frac{M-1}{2}} z^{\lfloor \alpha \frac{M+1}{2} \rfloor + i} \\ 0 \end{pmatrix} \qquad i = 0, \ldots, M - 2\left\lfloor \alpha \frac{M+1}{2} \right\rfloor - 1$$

$$M < -2\left\lfloor \alpha \frac{-M+1}{2} \right\rfloor \;\Rightarrow\; \chi_{0,i} = \begin{pmatrix} 0 \\ |g|^{\frac{-M-1}{2}} z^{\lfloor \alpha \frac{-M+1}{2} \rfloor + i} \end{pmatrix} \qquad i = 0, \ldots, -M - 2\left\lfloor \alpha \frac{-M+1}{2} \right\rfloor - 1$$



introduce

two branes



$\alpha = 0.9$
$M = 27$

small overlapping

$$\left| \int_{S^2} \phi_{0,i} \chi_{0,1}^\dagger \chi_{0,3} \right| << \left| \int_{S^2} \phi_{0,i} \chi_{0,1}^\dagger \chi_{0,2} \right|$$

Wavefunctions are localized because of branes !

Result : two branes

<u>* Unsatisfactory points</u>

Yukawa is diagonal ➡ No mixing between generations

Yukawa is real ➡ No CP-violation

Q : What is an origin of these problems ?

Result : two branes

<u>* Unsatisfactory points</u>

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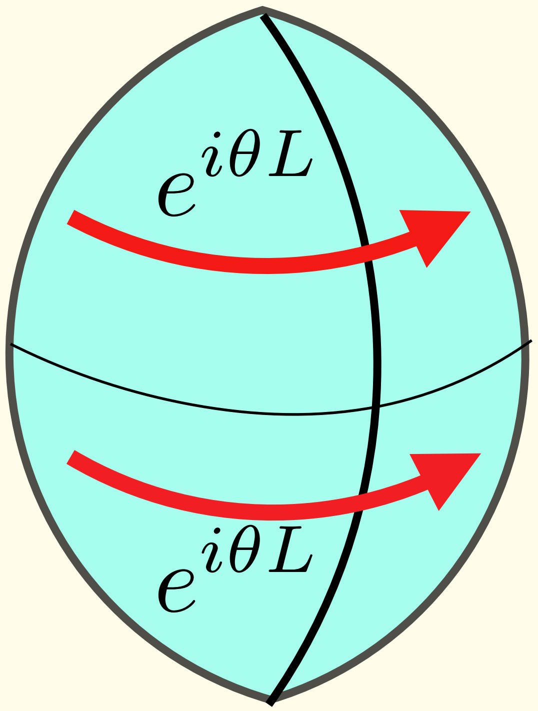
Q : What is an origin of these problems ?

A : U(1)-isometry of the background

$$[\not{D}, L] = 0$$



Result : two branes

<u>* Unsatisfactory points</u>

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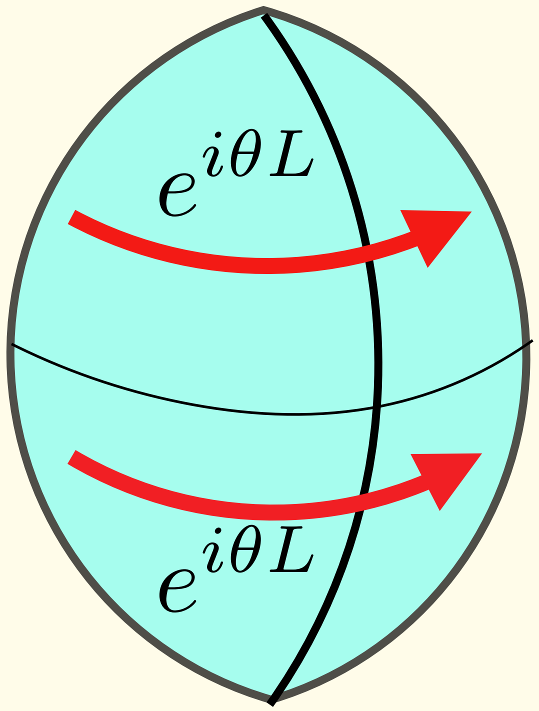


Q : What is an origin of these problems ?

A : U(1)-isometry of the background

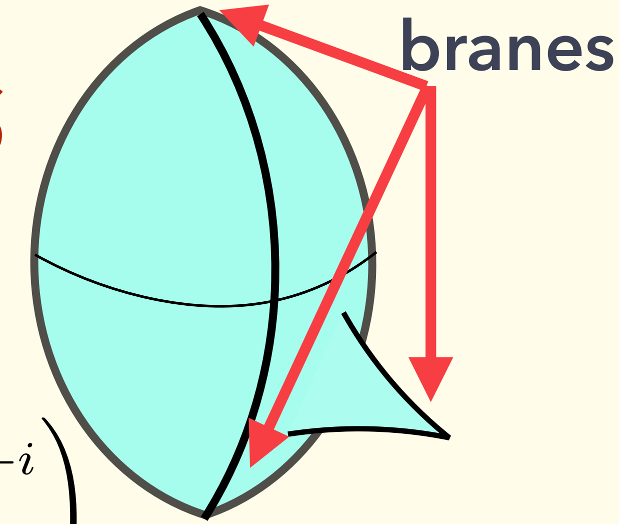
$$[\not{D}, L] = 0$$

<u>* How to overcome these problems ?</u>

Break the U(1)-isometry

by introducing one more brane

Result : three branes



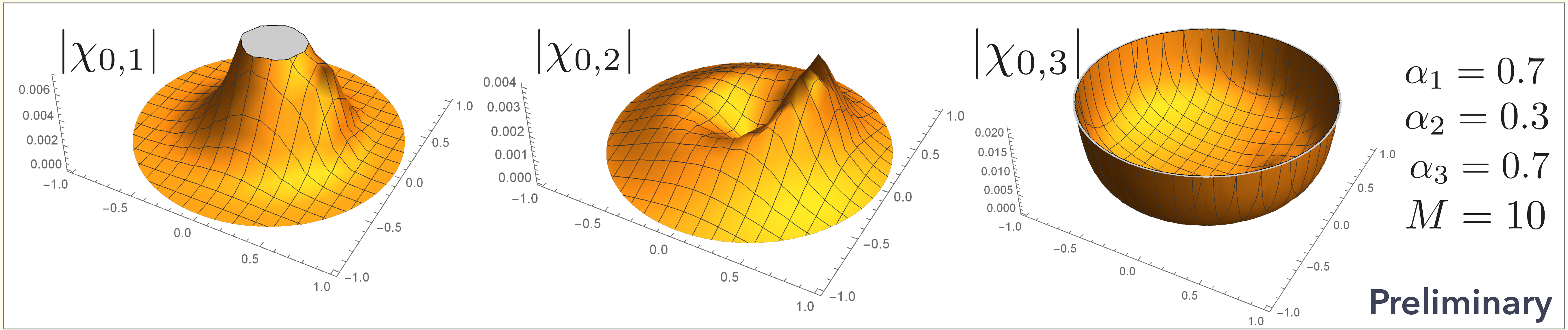
* Zero-mode wavefunction

$$M > \sum_{k=1}^{3} \left\lfloor \alpha_k \frac{M+1}{2} \right\rfloor \;\Rightarrow\; \chi_{0,i} = \begin{pmatrix} |g|^{\frac{M-1}{2}} (z-1)^{\lfloor \alpha_2 \frac{M+1}{2} \rfloor} z^{\lfloor \alpha_1 \frac{M+1}{2} \rfloor + i} \\ 0 \end{pmatrix}$$

$$i = 0, \ldots, M - \sum_{k=1}^{3} \left\lfloor \alpha_k \frac{M+1}{2} \right\rfloor - 1$$

$$M < -\sum_{k=1}^{3} \left\lfloor \alpha_k \frac{-M+1}{2} \right\rfloor \;\Rightarrow\; \chi_{0,i} = \begin{pmatrix} 0 \\ |g|^{\frac{-M-1}{2}} (z-1)^{\lfloor \alpha_2 \frac{-M+1}{2} \rfloor} z^{\lfloor \alpha_1 \frac{-M+1}{2} \rfloor + i} \end{pmatrix}$$

$$i = 0, \ldots, -M - \sum_{k=1}^{3} \left\lfloor \alpha_k \frac{-M+1}{2} \right\rfloor - 1$$



Again, we can observe the localization of wavefunctions.

U(1)-isometry is broken by the third brane

Summary & Future work

* Summary

S^2 with magnetic fluxes → ✓ Chiral asymmetry
✓ Three generation
✗ Yukawa hierarchy

S^2 with magnetic fluxes **and branes** → Wavefunction localization
→ Possibilities for hierarchical Yukawa

* Future work

Construct realistic models.

Survey on relation between brane positions and flavor symmetry.

Thank you !

Back up slides

Background solution

M. Redi, Phys.Rev. D**71** (2005) 044006

Gaussian curvature of $ds^2 = R^2 dz d\bar{z}$ is

$$K = -\frac{4}{R^2}\frac{\partial^2}{\partial z \partial \bar{z}} \log R$$

<u>* Einstein equation</u>

$$K = k + \frac{4\pi}{R^2}\sum_{k=1}^{N} \alpha_k \delta^2(z - z_k)$$

<u>Fact</u>

$\varphi : X \to (Y, g)$ holomorphic and $d\varphi \neq 0$. Then, $K_X = K_Y \circ \varphi$

$Y = S^2$

with

$K_Y = k > 0$

Induced metric on X satisfies $K_X = k$.

Moreover, if $\varphi \sim (z - z_k)^{1-\alpha}$ around $z_k \in X$

$$K_X = k + \frac{4\pi}{R^2}\alpha\delta^2(z - z_k)$$

Background solution

holomorphicity
$\varphi : X \to S^2$

M. Redi, Phys.Rev. D**71** (2005) 044006

$$K = k + \frac{4\pi}{R^2} \sum_{k=1}^{N} \alpha_k \delta^2(z - z_k)$$

multi-valuedness
$\varphi \sim (z - z_k)^{1-\alpha}$

* Monodromy

Although $\varphi : X \to S^2$ is multi-valued, the induced metric is well-defined when monodromies give isometry of S^2

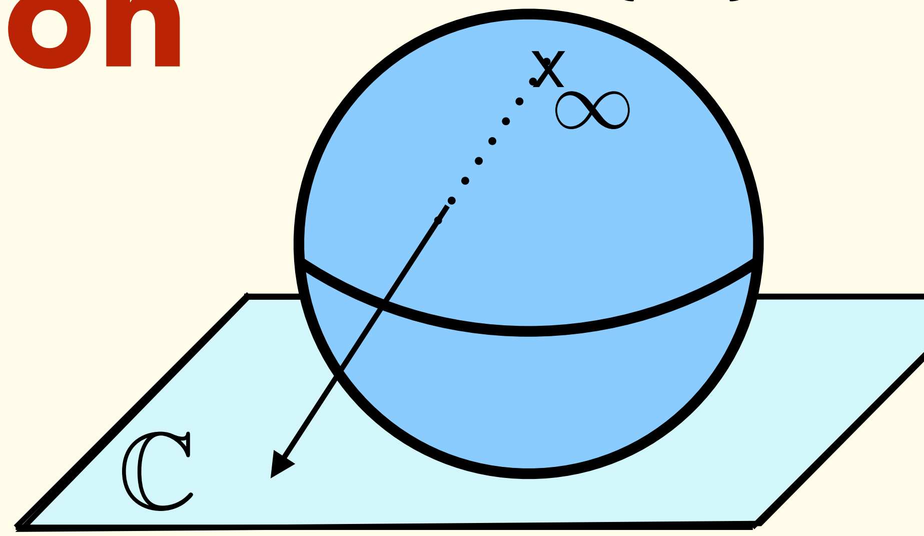
* Two branes

$\varphi(z) = z^{1-\alpha}$ (multi-valued)
$\varphi : S^2 \to S^2$

$$\Rightarrow \quad ds^2 = \frac{4(1-\alpha)}{k} \frac{|z|^{-2\alpha}}{(1+|z|^{2-2\alpha})^2} dz d\bar{z}$$

Flux quantization

$S^2 = \mathbb{C} \cup \{\infty\}$



the South part

* Sol. of Maxwell-eq. on the South part

$$A_4 = \frac{M}{1+|z|^2}\mathrm{Im}\, z \qquad A_5 = -\frac{M}{1+|z|^2}\mathrm{Re}\, z$$

* Sol. of Maxwell-eq. on the North part

$$A_4 = -\frac{M}{|w|^2(1+|w|^2)}\mathrm{Im}\, w \qquad A_5 = \frac{M}{|w|^2(1+|w|^2)}\mathrm{Re}\, w$$

where $w = \dfrac{1}{z}$

singular at the North pole $w = 0$!

We need gauge transformation to remove the singularity

* Charged matter

$$\chi \to \left(\frac{\bar{w}}{w}\right)^{\frac{M}{2}} \chi$$

M must be an integer

$\because$ single valuedness

Zero-mode counting

$\Sigma_{g,N}$: Riemann surface with N branes and non-flat metric
$(g \neq 1)$

Zero-mode counting formula

$$\dim \ker \not{D}_{\pm} = \pm M - \sum_{k=1}^{N} \left\lfloor \alpha_k \frac{\pm M + 1 - g}{2 - 2g} \right\rfloor + l(D_S \mp D_M + D_{\mp}^{\text{sing}})$$

"correction"

D_S : divisor for weyl spinors

D_M : divisor for magnetic fluxes

$D_{\mp}^{\text{sing}}$: divisor induced by branes

L-coordinate on S^2 with two branes

$$\int_{S^2} f\sqrt{|g|}dy_1 dy_2 = \int_0^\infty r dr \int_0^{2\pi} d\theta f R^2$$

where

$$ds^2 = R^2(dr^2 + r^2 d\theta)$$

$$R = \frac{2|1-\alpha|}{\sqrt{k}} \frac{r^{-\alpha}}{1+r^{2-2\alpha}}$$

We define

$$l = 1 - \frac{1}{1+r^{2-2\alpha}}$$ **then,** $$dl = \frac{k}{2-2\alpha} R^2 r dr$$

and

$$\int_{S^2} f\sqrt{|g|}dy_1 dy_2 = \frac{2-2\alpha}{k}\int_0^1 dl \int_0^{2\pi} f$$

Example : Magnetized SYM

*** 6d $\mathcal{N}=1,\ U(n_1+n_2)$ super Yang-Mills**

$$\mathcal{L} = \mathrm{tr}\left[-\frac{1}{4}F^{MN}F_{MN} + i\bar{\psi}\Gamma^M D_M \psi\right]$$

*** Magnetic fluxes**

$$\langle F_{45}\rangle \propto \begin{pmatrix} M_1 \times 1_{n_1} & \\ & M_2 \times 1_{n_2} \end{pmatrix} \quad (M_1 - M_2 > 0)$$

$$\Rightarrow \quad U(n_1+n_2) \to SU(n_1)\times SU(n_2)\times U(1)$$

*** Fermion in effective theory**

$$\psi \sim 1\times\begin{pmatrix} \psi_{11}^+ & \\ & \end{pmatrix} + 1\times\begin{pmatrix} & \\ & \psi_{22}^+ \end{pmatrix} + 1\times\psi_{\mathrm{tr}}^+$$ **: gaugino**

$$+\, n(M_1 - M_2)\times\begin{pmatrix} & \psi_{12}^+ \\ & \end{pmatrix} + n(M_2 - M_1)\times\begin{pmatrix} & \\ \psi_{21}^- & \end{pmatrix}$$ **: matter**

# of generation