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Tomography by neutrino pair beam

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arXiv:1805.10793[hep-ph]

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Introduction

Development of the neutrino physics

Our understanding of neutrino has been improved greatly since the end of the last century.

Especially, the observation of flavor oscillations of neutrino has shown the presence of new physics beyond the standard model.

Takaaki Kajita

It's me !

The Nobel Prize in Physics 2015

- **Takaaki Kajita (SK)**
- **Arthur B. McDonald (SNO)**

For the discovery of neutrino oscillation, which shows the neutrinos have mass.

- **This is inconsistent with the prediction of the Standard Model that predicts massless neutrinos.**
- **This is a clear signature of new physics beyond the Standard Model.**

Neutrino Oscillation Parameter

From neutrino oscillation experiments

$s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij}$ $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$

Majorana phase

PMNS matrix

CP phase

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

Atmospheric neutrino   **Reactor neutrino / Accelerator neutrino**   **Solar neutrino**

NuFIT 3.0 (2016)

Normal Ordering

$m_1 < m_2 < m_3$

Inverted Ordering

$m_3 < m_2 < m_1$

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 0.83$)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.345$	$0.306^{+0.012}_{-0.012}$	$0.271 \to 0.345$	$0.271 \to 0.345$
$\theta_{12}/^\circ$	$33.56^{+0.77}_{-0.75}$	$31.38 \to 35.99$	$33.56^{+0.77}_{-0.75}$	$31.38 \to 35.99$	$31.38 \to 35.99$
$\sin^2\theta_{23}$	$0.441^{+0.027}_{-0.021}$	$0.385 \to 0.635$	$0.587^{+0.020}_{-0.024}$	$0.393 \to 0.640$	$0.385 \to 0.638$
$\theta_{23}/^\circ$	$41.6^{+1.5}_{-1.2}$	$38.4 \to 52.8$	$50.0^{+1.1}_{-1.4}$	$38.8 \to 53.1$	$38.4 \to 53.0$
$\sin^2\theta_{13}$	$0.02166^{+0.00075}_{-0.00075}$	$0.01934 \to 0.02392$	$0.02179^{+0.00076}_{-0.00076}$	$0.01953 \to 0.02408$	$0.01934 \to 0.02397$
$\theta_{13}/^\circ$	$8.46^{+0.15}_{-0.15}$	$7.99 \to 8.90$	$8.49^{+0.15}_{-0.15}$	$8.03 \to 8.93$	$7.99 \to 8.91$
$\delta_{CP}/^\circ$	261^{+51}_{-59}	$0 \to 360$	277^{+40}_{-46}	$145 \to 391$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.03 \to 8.09$	$7.03 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.524^{+0.039}_{-0.040}$	$+2.407 \to +2.643$	$-2.514^{+0.038}_{-0.041}$	$-2.635 \to -2.399$	$\begin{bmatrix} +2.407 \to +2.643 \\ -2.629 \to -2.405 \end{bmatrix}$

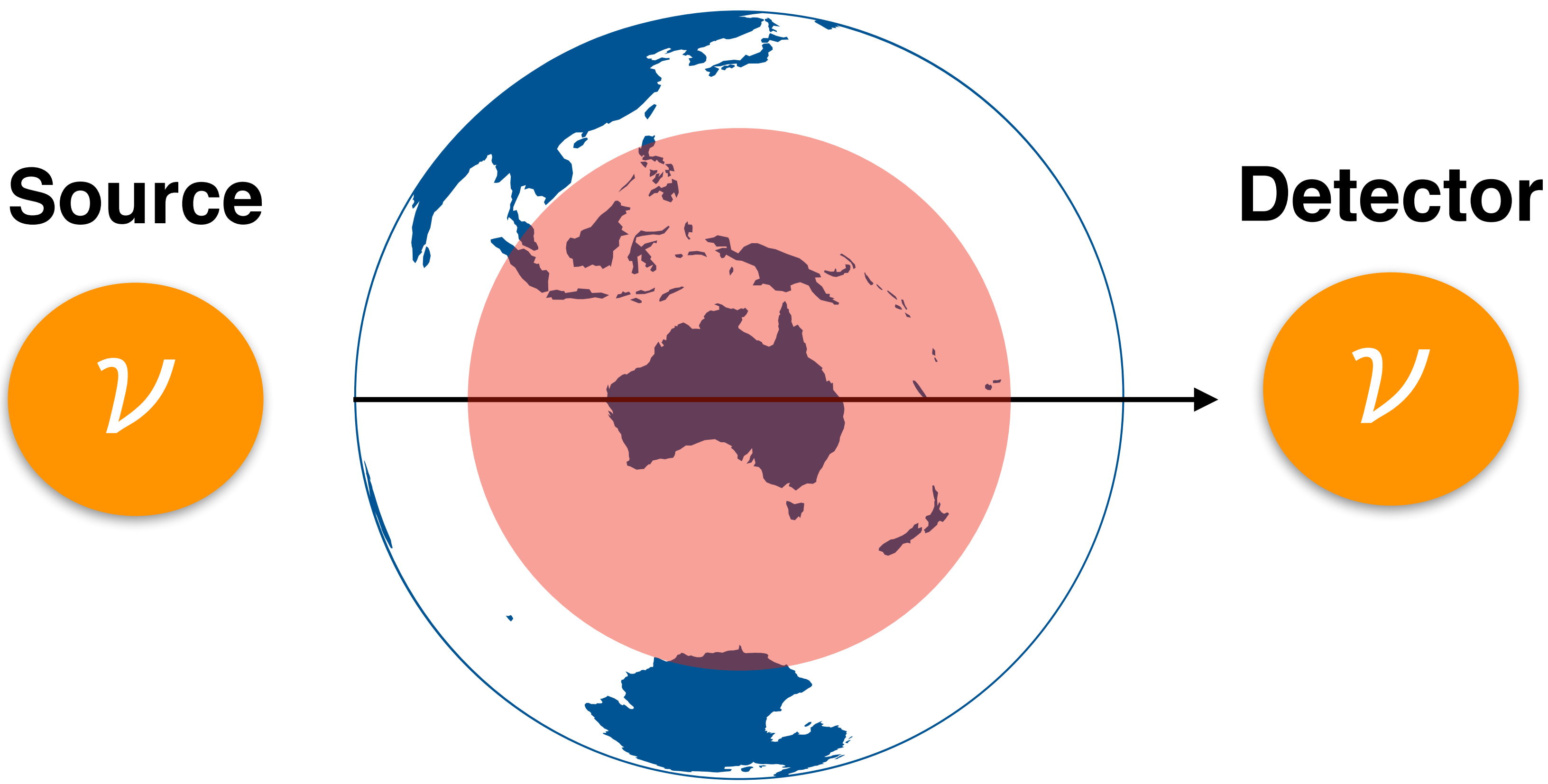
Thanks to the remarkable efforts of various experiments θ_{ij} Δm_{ij} has been measured accurately.

So, we consider seriously the application of neutrino physics to various fields of basic science.

* **Absolute value of neutrino mass, CP phase, Majorana phase, mass ordering have not yet determined.**

The idea of Neutrino Tomography

Imaging of the Earth's interior structure using the neutrino.



Neutrino can easily transmit the Earth due to the weakness of its interaction.

Neutrino Tomography

3 different methods of Neutrino Tomography

1. Neutrino Absorption Tomography

- Using the absorption of neutrino by matter.
- Same mechanism to the X-ray computed tomography.
- This method needs the high energy neutrinos (E_ν > 10 TeV).

  - L. V. Volkova and G. T. Zatsepin, Bull. Acad. Sci. USSR, Phys. Ser. 38 (1974) 151.

  And more …

2. Neutrino Oscillation Tomography

- Using the matter effect of neutrino oscillation.

  - T. Ohlsson andW.Winter, Europhys. Lett. 60 (2002) 34
  - E. K. Akhmedov, M. A. Tortola and J.W. F. Valle, JHEP 0506, 053 (2005)
  - W.Winter, Nucl. Phys. B 908 (2016) 250
  - A.N. Ioannisian and A. Y. Smirnov, Phys. Rev. D 96 (2017) no.8, 083009

  And more …

In this talk, we discuss about this type !

(3. Neutrino Diffraction Tomography)

- Measure the diffraction pattern of crystalline matter in the deep interior of the Earth.
- Not realistic yet.

  - A.D. Fortes, I. G.Wood, and L. Oberauer, Astron. Geophys. 47(2006) 5.31–5.33.
  - R. Lauter, Astron. Nachr. 338 (2017) no.1, 111.

There is no precise tomography method.
- There is no powerful source.
- There is no established reconstruction method.

Neutrino Oscillation

Neutrino oscillation is phenomenon that the neutrino flavor will vary with distance. It is caused by the quantum mechanical superposition.

Neutrino flavor eigenstates is written by superposition of the mass eigenstates.

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = U_{PMNS}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

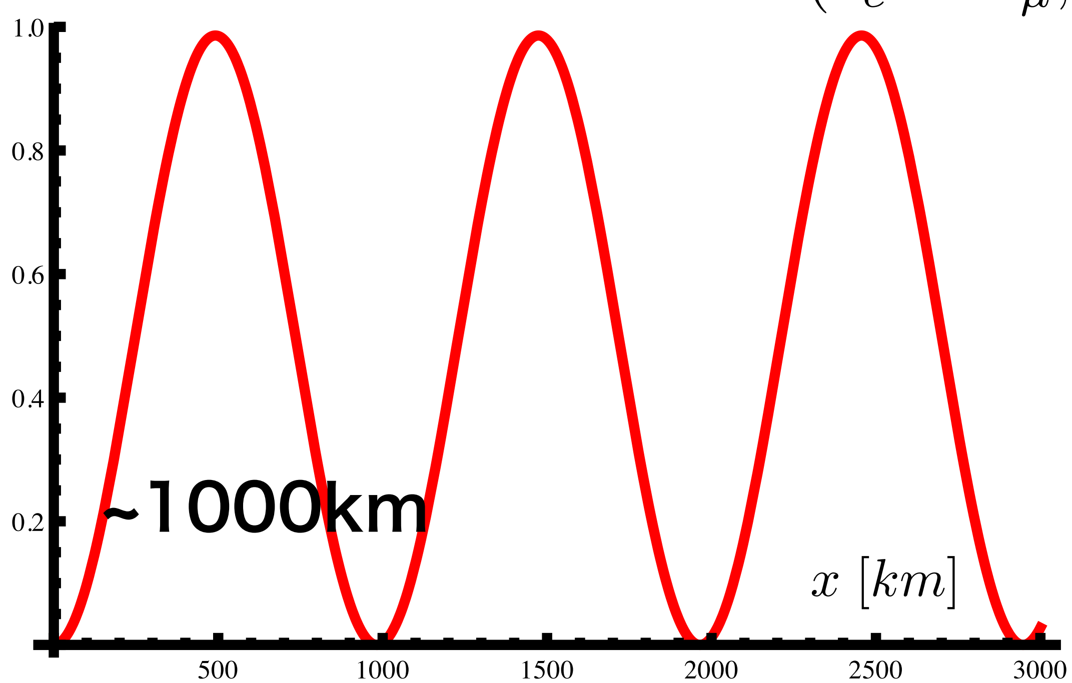
flavor eigenstate — Mass eigenstate

U_{PMNS} **Pontecorvo-Maki-Nakagawa-Sakata matrix**

Mass eigenstates evolve respectively in time.
Then, because of the interference between the mass state, the flavor transition probability behaves oscillatory.

Ex) 2 flavor case

$$P(\nu_e \to \nu_\mu; E, t) = |\langle\nu_\mu|\nu_e(t)\rangle|^2 = |\sin\theta\cos\theta(1 - e^{-i(E_2-E_1)t})|^2$$
$$= \sin^2(2\theta)\sin^2(\frac{\Delta m^2}{4E}t)$$



$$E = 1\ [GeV]$$
$$\Delta m^2 = 2.524 \times 10^{-3}\ [eV]$$
$$\theta = \frac{41.6}{180}\pi$$

Neutrino Oscillation in Matter

Evolution equation of transition amplitudes of neutrino flavors is written as follow. In matter, additional effective potential is added to the vacuum Hamiltonian.

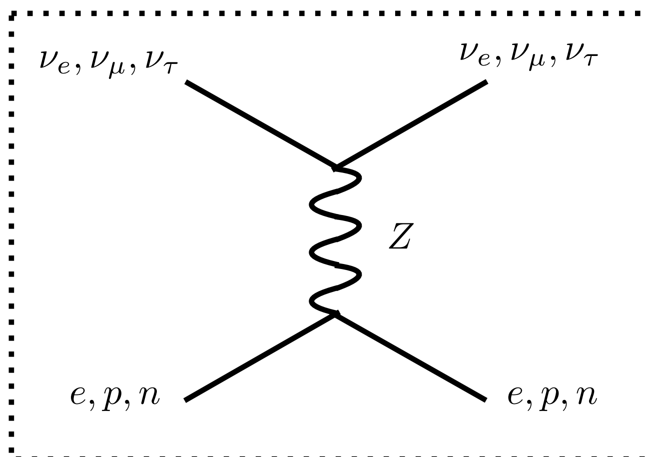
$$i\frac{d}{dx}\vec{A}(x) = [H_0^F + V^F]\vec{A}(x)$$

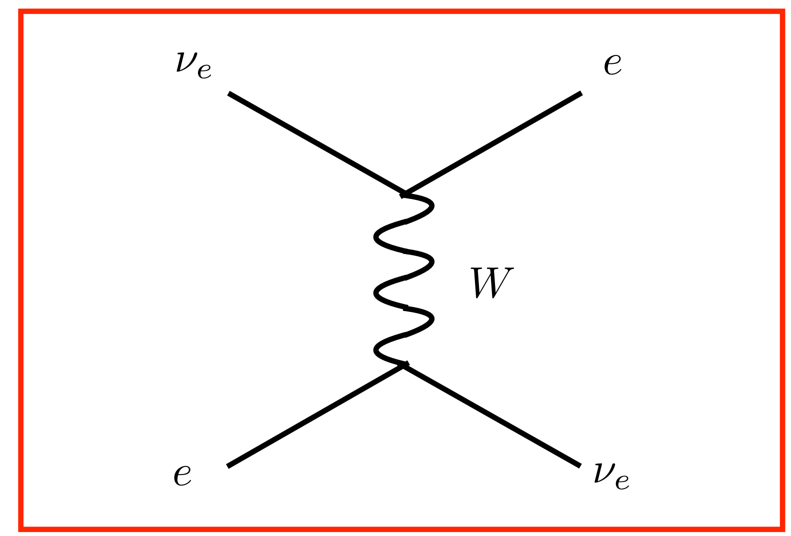
Vacuum contribution (H_0^F)   Additional effective potential (V^F)

In 2 flavor case

$$\vec{A}(x) = (A_{\nu_e\to\nu_e}(x), A_{\nu_e\to\nu_\mu}(x))^T$$

$$A_{\beta\alpha}(x) = \langle\nu_\beta|\nu_\alpha(x)\rangle$$





- **Neutrino interacts with the electron, proton, neutron in matter, through the CC and NC interaction.**

- **The contribution of the NC interaction is common to all flavors, and eliminated by the common phase shift.**

- **Therefore, the main contribution to the potential is the CC interaction and effective potential depend on the electron number density.**

Neutrino Oscillation in Matter

For simplicity, we consider the 2 flavor neutrino oscillation

$$i\frac{d}{dx}\begin{pmatrix} A_{\nu_e\to\nu_e} \\ A_{\nu_e\to\nu_\mu} \end{pmatrix} = \left[U \begin{pmatrix} 0 & 0 \\ 0 & \frac{\Delta m^2}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC}(x) & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} A_{\nu_e\to\nu_e} \\ A_{\nu_e\to\nu_\mu} \end{pmatrix}$$

Effective potential is written as

$$V_{CC}(x) = \sqrt{2} G_F n_e(x)$$

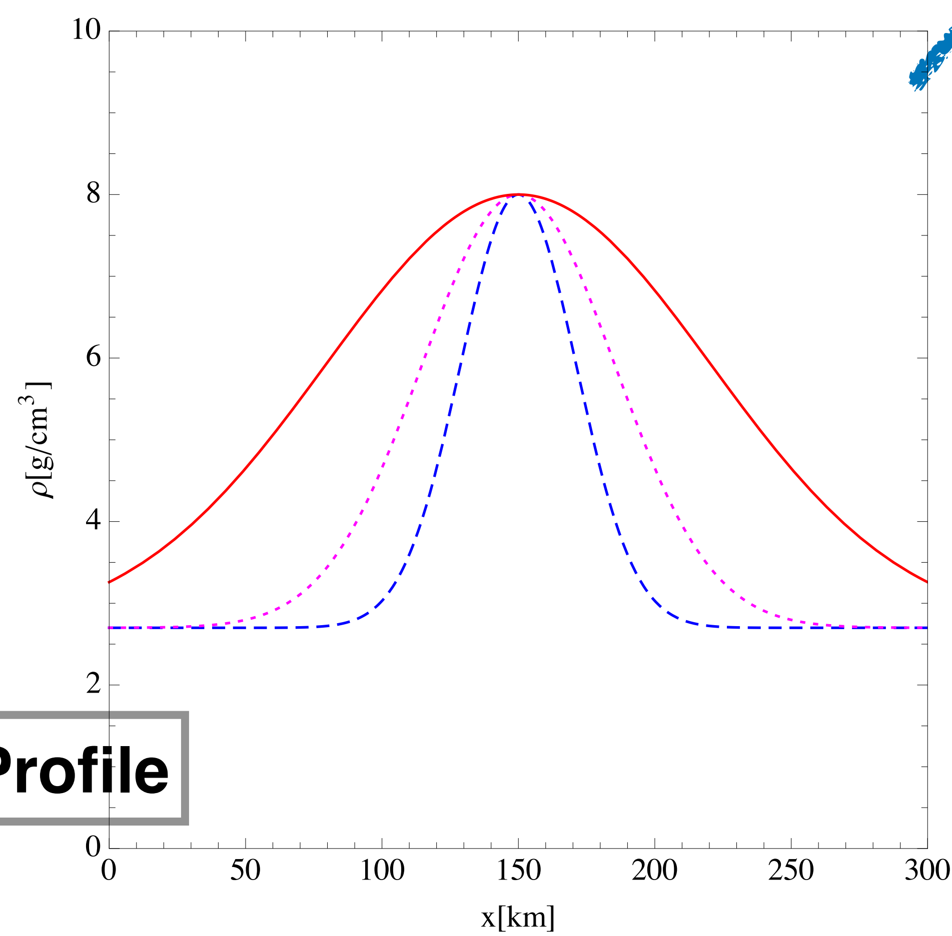
The electron number density is translated into the matter density.

$$n_e(x) \simeq \frac{\rho(x)}{2m_p}$$

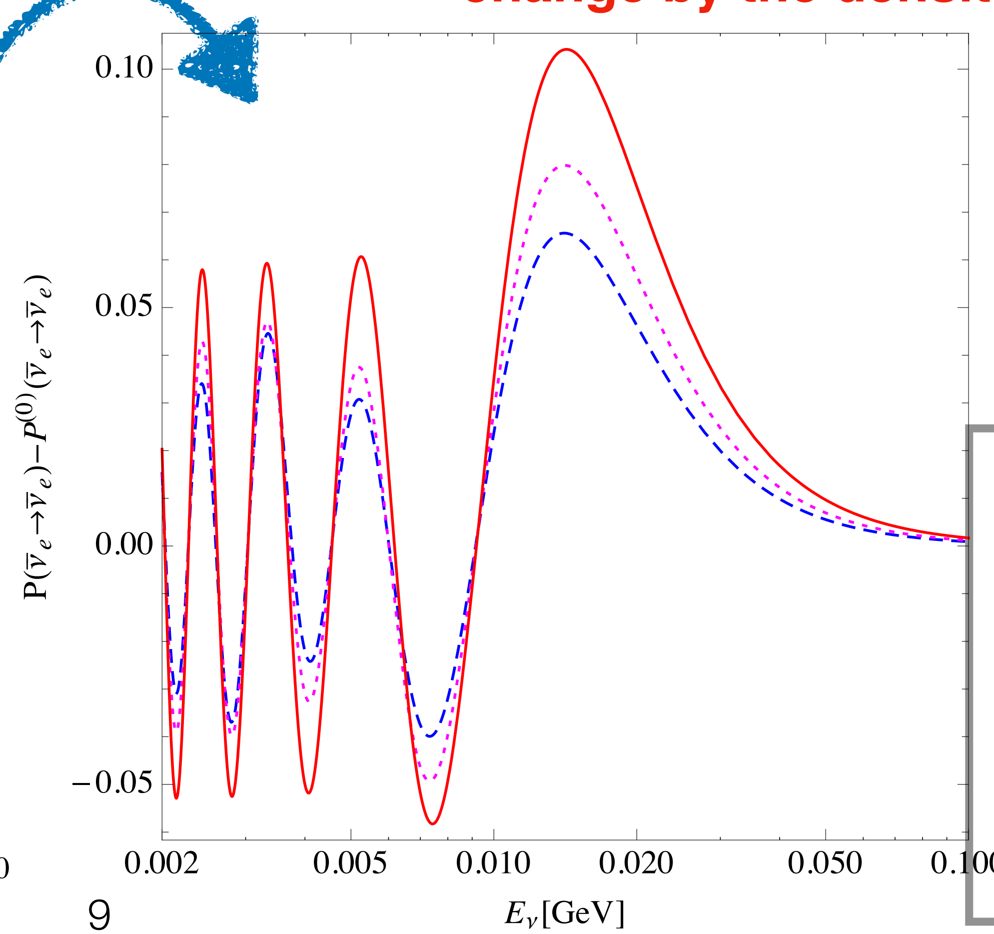
Probability is calculated as follow

$$P_{\nu_\alpha\to\nu_\beta}(E_\nu, x) = |A_{\nu_\alpha\to\nu_\beta}(E_\nu, x)|^2$$

Energy Spectrum of the Oscillation Probability change by the density profile.



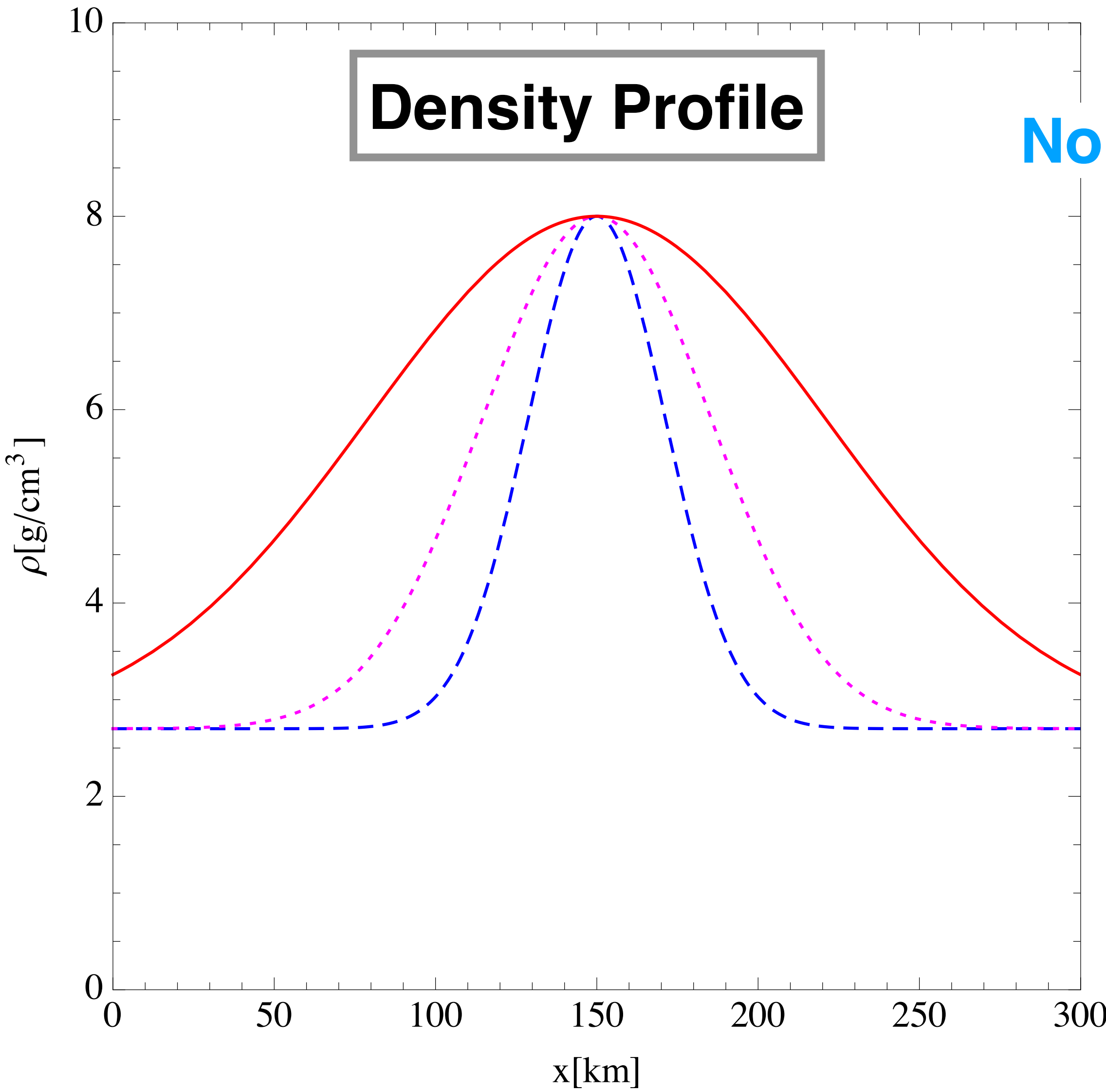
Density Profile



Oscillation Probability

(subtracted the vacuum contribution)

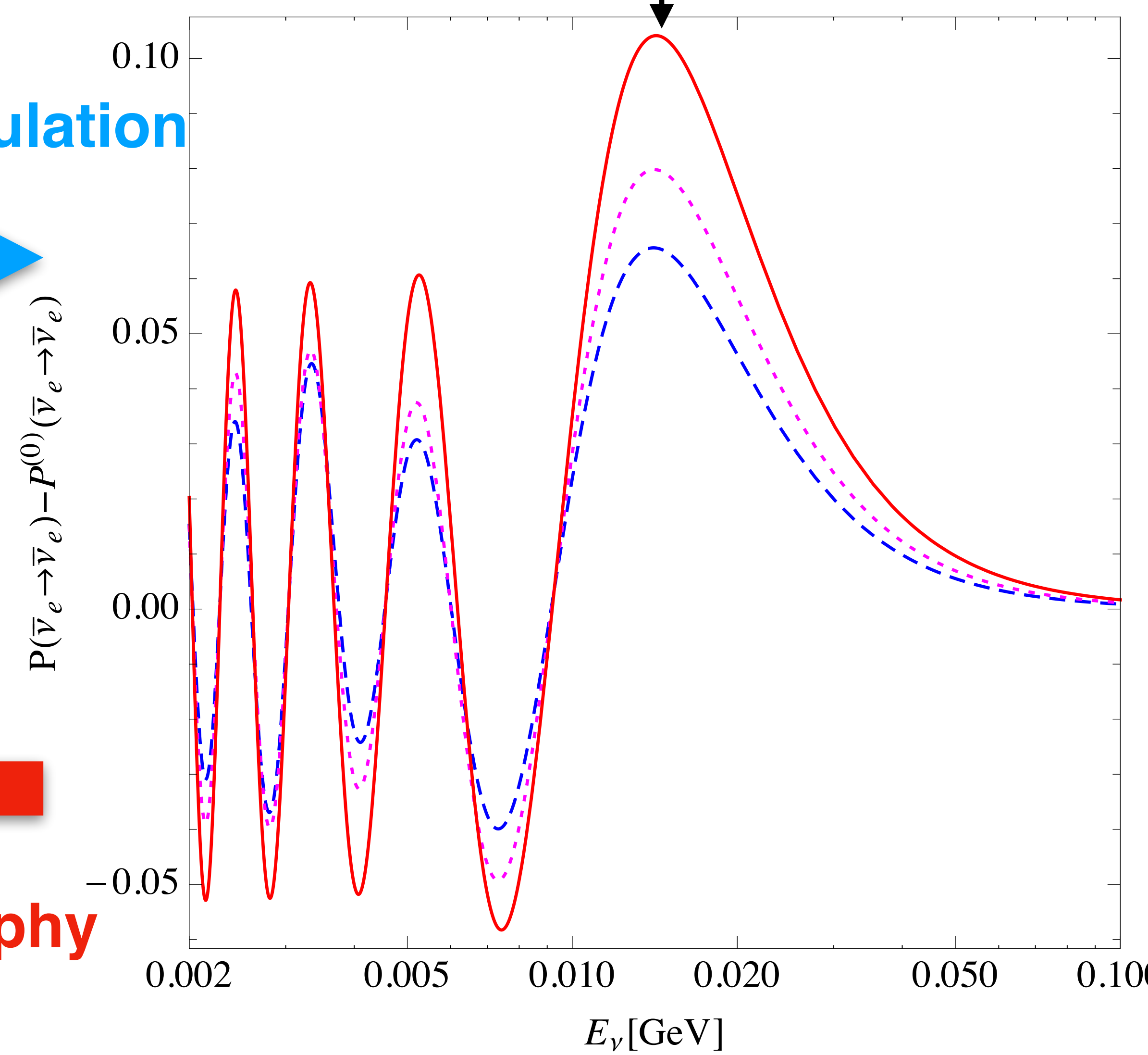
Neutrino Oscillation Tomography



Normal calculation

Tomography

There include the information of the density profile



OscillationProbability (subtracted the vacuum contribution)

It is required the precise measurement of the energy spectrum. So, powerful neutrino source is required.

Method

Neutrino Pair Beam

The pair beam, which has been proposed recently, can produced a large amount of neutrino pairs from the circulating partially stripped ions.

[Yoshimura, Sasao, Phys. Rev. D 92, 073015 (2015)]

Characteristics of the Neutrino Pair Beam

- **It generates the all flavor neutrino pairs** $(\nu_e, \overline{\nu_e}), (\nu_\mu, \overline{\nu_\mu}), (\nu_\tau, \overline{\nu_\tau})$
- **Very high intensity flux of neutrino beam**
- **High beam directivity**

Forward production spectrum

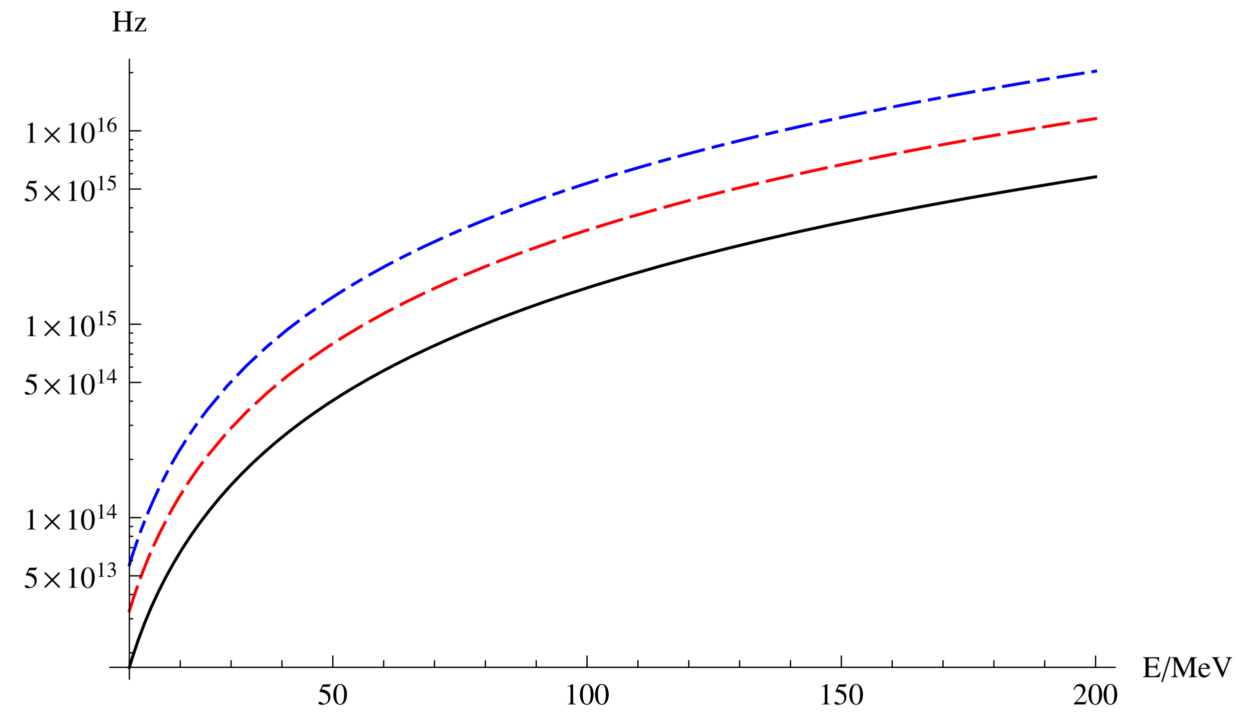


Fig. 7. Neutrino energy spectrum rate at the forward direction of solid angle area π/γ^2. Assumed parameters are $\rho\epsilon_{eg} = 10^{14}$, $N = 10^8$ and $\epsilon_{eg} = 50$ keV, $\gamma = 4000$ in solid black, 5000 in dashed red, 6000 in dash-dotted blue. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

[Asaka,Tanaka,Yoshimura, Phys.Lett. B760 (2016) 359-364]

Neutrino Tomography requires the precise measurement of the energy spectrum for the precise reconstruction of the density profile.

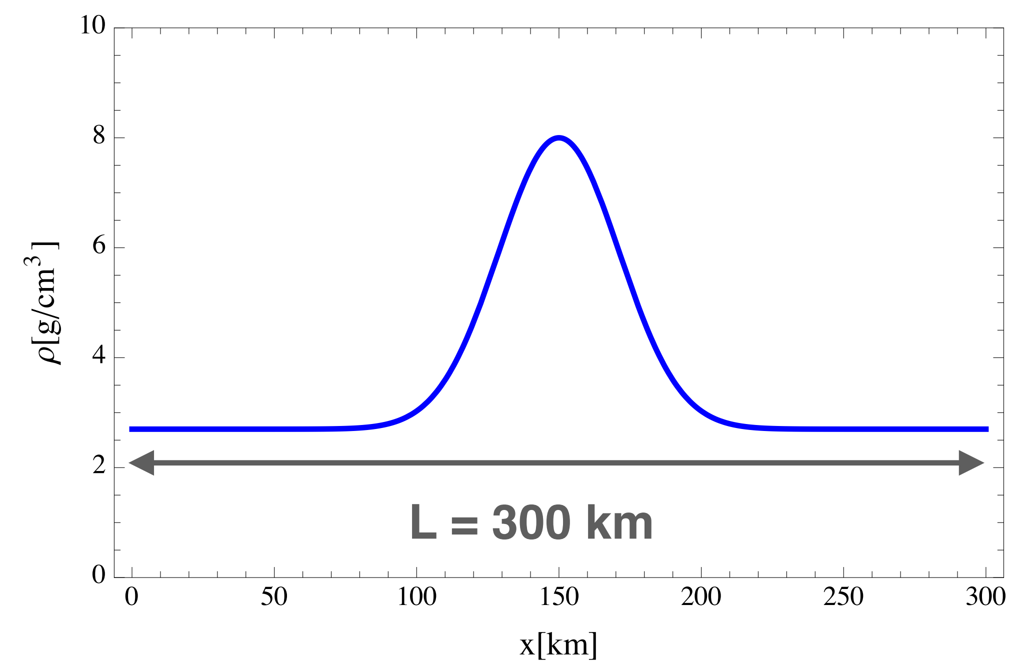
This high event rate (high flux) is essential.

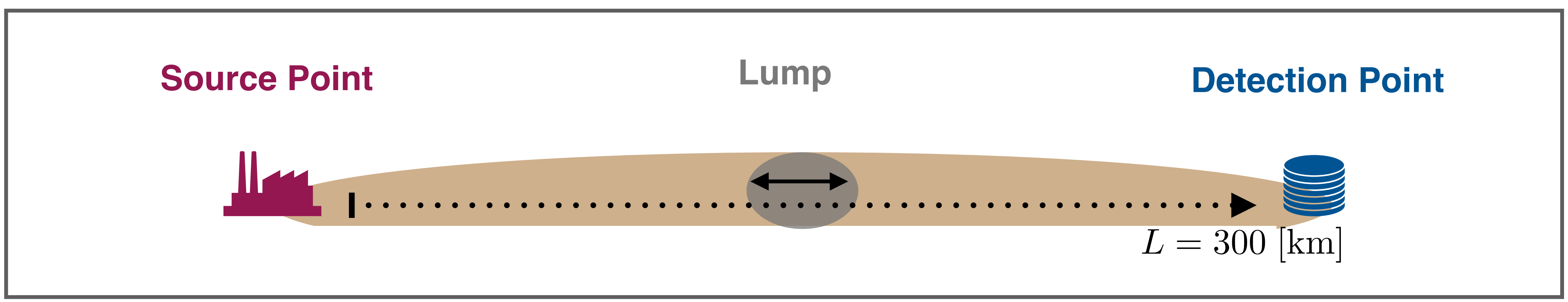
Toy model

- **We consider the symmetric exponential type of the density profile.**

$$\rho(x) = \bar{\rho} + (\rho_l - \bar{\rho}) \exp\left[-\frac{\left(x-\frac{L}{2}\right)^2}{D_l^2}\right]$$

L : length of the baseline
D_l : width of the lump





- **We consider the low energy $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillation.**

$$E_\nu : 2 \sim 100 \ [\mathrm{MeV}]$$

- **We assume the huge liquid Argon as the neutrino detector.**

Fiducial volume 10^5 m^3

Statistical Test

We estimate how precisely the width(D_*) and density(ρ_*) of the lump can be reconstructed under this set up.

$$\rho(x) = \bar{\rho} + (\rho_l - \bar{\rho}) \exp\left[-\frac{\left(x - \frac{L}{2}\right)^2}{D_l^2}\right] \qquad \bar{\rho} = 2.7[\mathrm{g/cm^3}]$$

We perform the χ^2 analysis.

$$\Delta\chi^2 = \sum_{i=1}^{N_b} \frac{\left[N(E_i)|_{D_*,\rho_*} - N(E_i)|_{D_l,\rho_l}\right]^2}{\sigma^2(E_i)}$$

$N_b = 100$: the number of energy bin

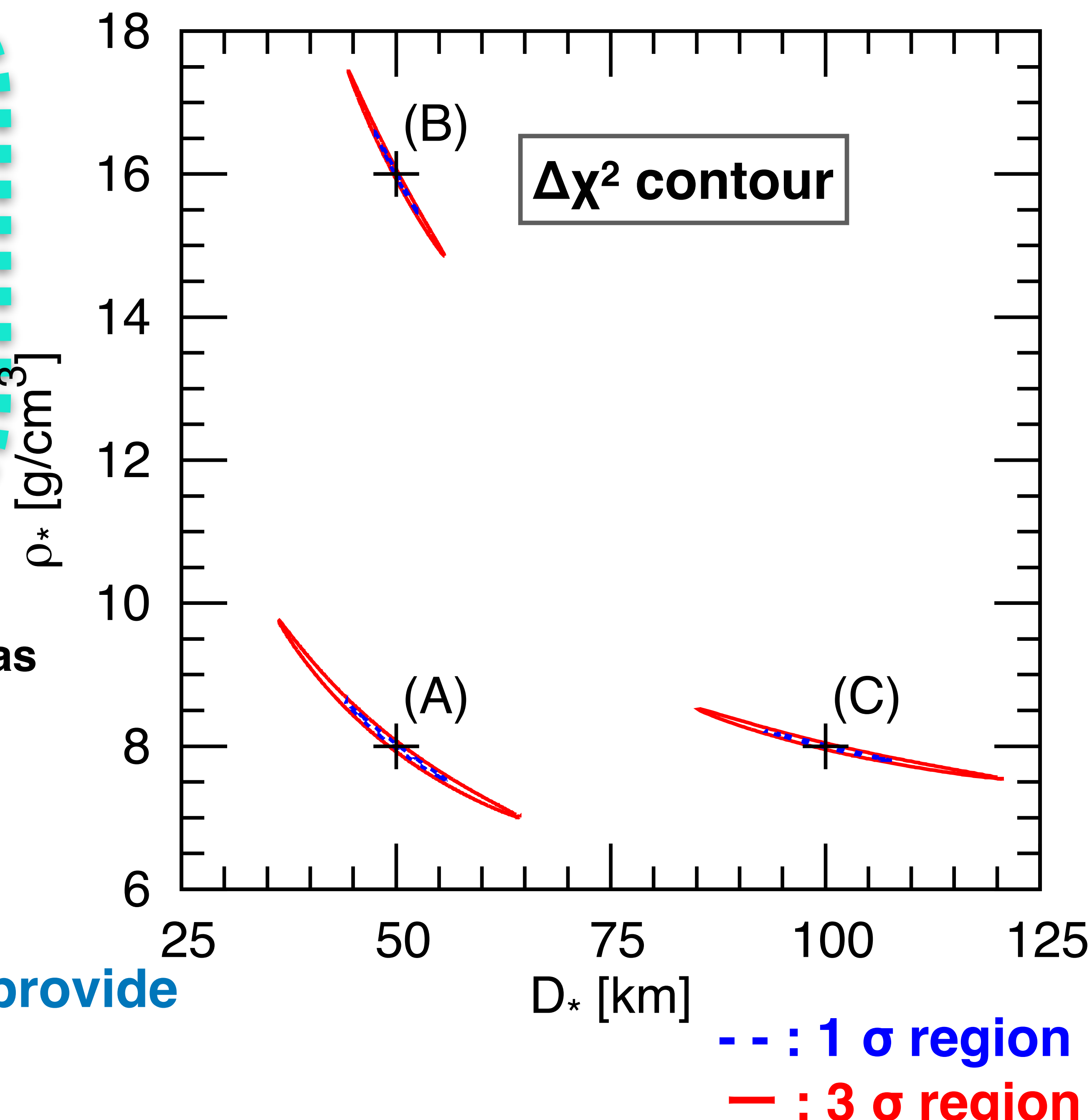
N = flux × oscillation probability × detection rate

We assume 1 year as experimental running time.

We assume the 3 density profile.

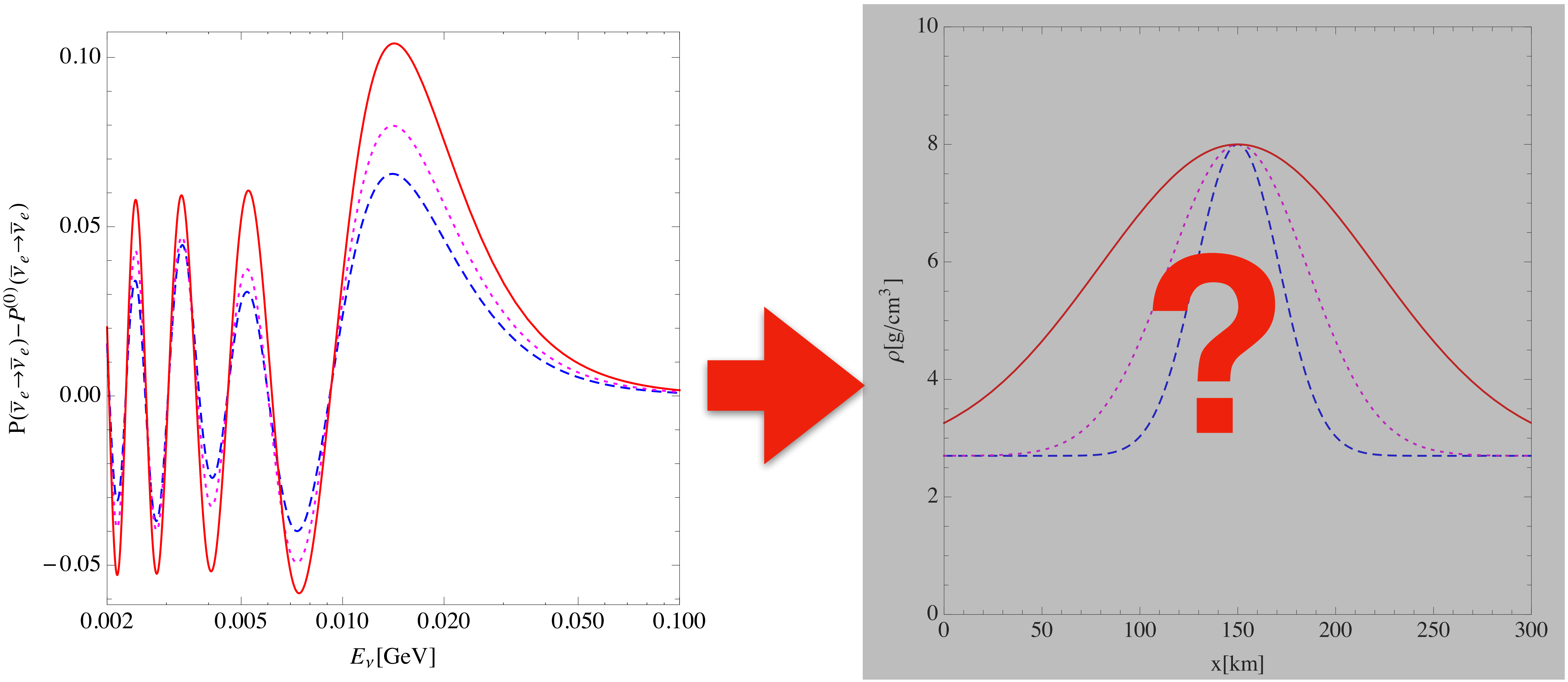
The pair beam can probe the lump at the 1 σ level as

$$\begin{aligned}
&(A) \quad D_* = 50^{+5.9}_{-5.9}\,\mathrm{km} \quad \text{and} \quad \rho_* = 8.0^{+0.62}_{-0.48}\,\mathrm{g\,cm^{-3}},\\
&(B) \quad D_* = 50^{+2.5}_{-2.4}\,\mathrm{km} \quad \text{and} \quad \rho_* = 16^{+0.58}_{-0.53}\,\mathrm{g\,cm^{-3}},\\
&(C) \quad D_* = 100^{+8.2}_{-7.1}\,\mathrm{km} \quad \text{and} \quad \rho_* = 8.0^{+0.22}_{-0.21}\,\mathrm{g\,cm^{-3}},
\end{aligned}$$



It is seen that the neutrino pair beam can provide the measurement of the density profile.

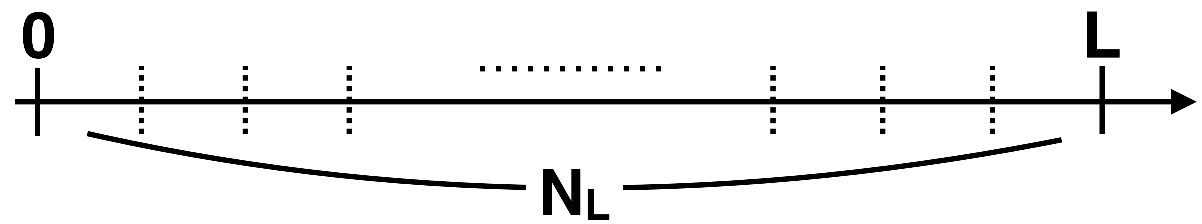
Neutrino Oscillation Tomography



How reconstruct the density profile from the energy spectrum of the neutrino oscillation?

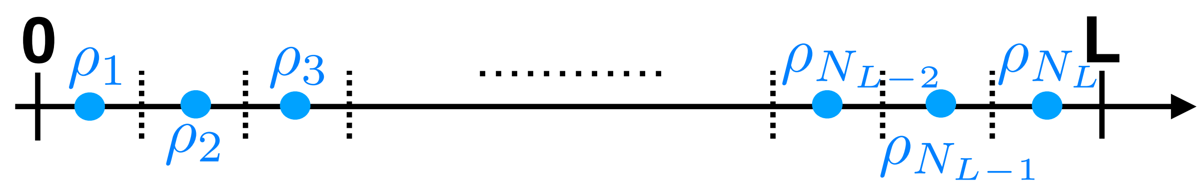
Density Profile Reconstruction Method

1. We discretize the neutrino baseline into the N_L segments.



2. We consider the matter densities for these segments as free parameters ρ_j .

$j = 1, \cdots, N_L$

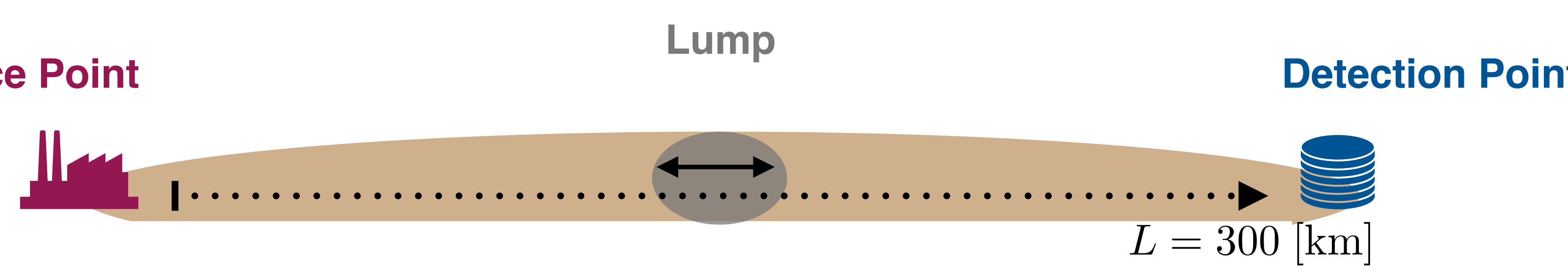


We assume that the each density is constant within each segment.

3. We also divide the energy range into the N_E parts, and define the χ^2 function

$$\chi^2 = \sum_{i=1,N_E} \frac{\left[N^{\rm obs}(E_i) - N^{\rm th}(E_i)\right]^2}{\sigma^2(E_i)} \qquad \sigma(E_i) = \sqrt{N^{\rm obs}(E_i)}.$$

4. We determine those density by minimizing the χ^2 function by comparing the experimental data $N^{\rm obs}(E_i)$ for a given original profile ρ(x) with the theoretical prediction $N^{\rm th}(E_i)$ from unknown parameters ρ_j .



Perturbation Formula

We introduce the perturbation formula of the neutrino oscillation probability which is used for the theoretical prediction $N^{\mathrm{th}}(E_i)$ from unknown parameters ρ_j .

$$N^{\mathrm{th}}(E_i) = \mathrm{flux} \times P_{\bar{\nu}_e \to \bar{\nu}_e}(E, L) \times \mathrm{detection\ rate}$$

Neutrino Oscillation Probability

Neutrino Oscillation Probability is calculated from the this evolution equation.

$$i\frac{d}{dx}\vec{A}(x) = [H_0^F + V^F]\vec{A}(x)$$

Then we assume the relation $H_0^F > V^F$

And calculate the oscillation probability by perturbation.

$$P_{\alpha\beta} = |A_{\beta\alpha}^{(0)} + A_{\beta\alpha}^{(1)} + A_{\beta\alpha}^{(2)} + \cdots|^2$$
$$= |A_{\beta\alpha}^{(0)}|^2 + A_{\beta\alpha}^{(0)*}A_{\beta\alpha}^{(1)} + A_{\beta\alpha}^{(0)}A_{\beta\alpha}^{(1)*} + |A_{\beta\alpha}^{(1)}|^2 + A_{\beta\alpha}^{(0)*}A_{\beta\alpha}^{(2)} + A_{\beta\alpha}^{(0)}A_{\beta\alpha}^{(2)*} + \cdots$$

0th 1st 2nd

Ex) perturbation formula at 1st order is written as

$$P^{(1)}(E_i) \propto \sum \rho(x_j)\left[\sin\left\{\frac{\Delta m^2}{2E_I}L\right\} - \sin\left\{\frac{\Delta m^2}{2E_I}x_j\right\} - \sin\left\{\frac{\Delta m^2}{2E_\nu}(L - x_j)\right\}\right]$$

We find the 2nd order perturbation is important for the successful reconstruction.

Assumption

★ **Minimize** $\chi^2 = \sum_{i=1,N_E} \frac{\left[N^{\text{obs}}(E_i) - N^{\text{th}}(E_i)\right]^2}{\sigma^2(E_i)}$

And reconstruct the density profile.

N = flux × oscillation probability × detection rate

★ **We assume about the fitting parameter (matter density in the each segment)**

$$\rho_j \geq 0 \qquad\qquad N^{th}(E_i, \rho_j)$$

★ **We assume $N^{\text{obs}}(E_i)$ as event rate by the calculation from evolution equation with original matter density profile.**

$$i\frac{d}{dx}\begin{pmatrix} A_{\nu_e\to\nu_e} \\ A_{\nu_e\to\nu_\mu} \end{pmatrix} = \left[U\begin{pmatrix} 0 & 0 \\ 0 & \frac{\Delta m^2}{2E} \end{pmatrix}U^\dagger + \begin{pmatrix} V_{CC}(x) & 0 \\ 0 & 0 \end{pmatrix}\right]\begin{pmatrix} A_{\nu_e\to\nu_e} \\ A_{\nu_e\to\nu_\mu} \end{pmatrix}$$

$$V_{CC}(x) = \sqrt{2}G_F n_e(x)$$

$$n_e(x) \simeq \frac{\rho(x)}{2m_p}$$

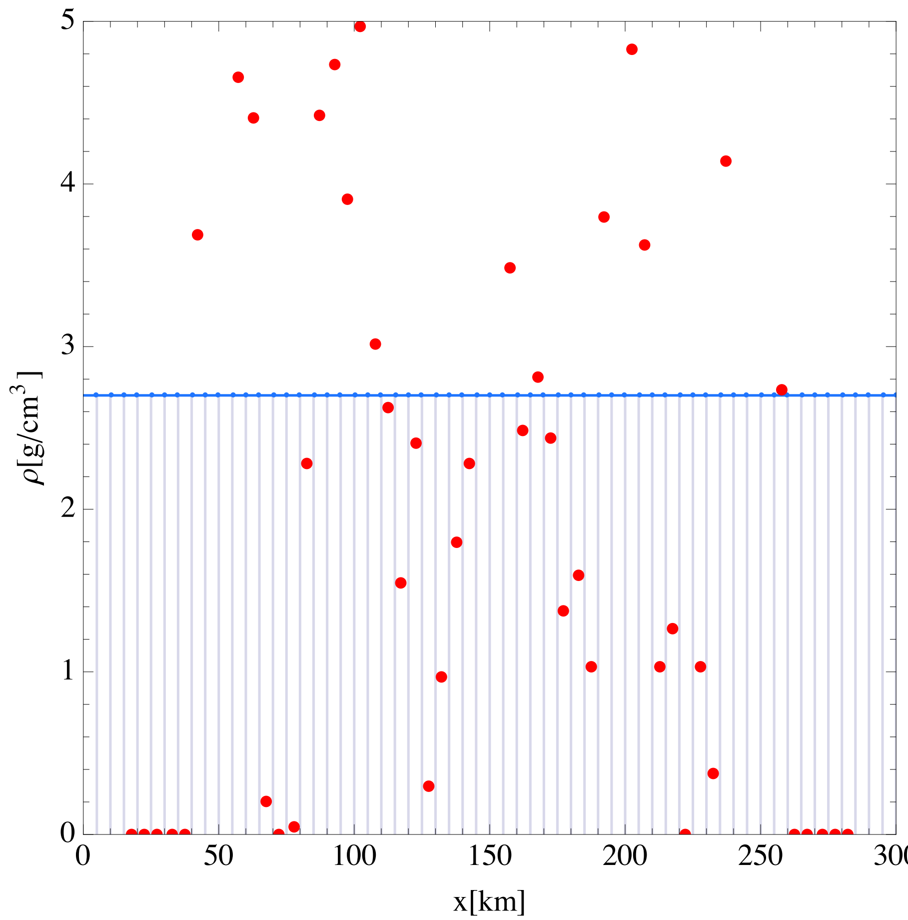
Results of reconstruction

Result of the flat density

$$\bar{\rho} = 2.7\ [\mathrm{g/cm^3}]$$

Result with using the 1st order formula

Result with using the 2nd order formula



The 2nd order matter effect is important for the reconstruction

- : Original density profile

• : reconstructed density profile

Reconstruction of 60 points

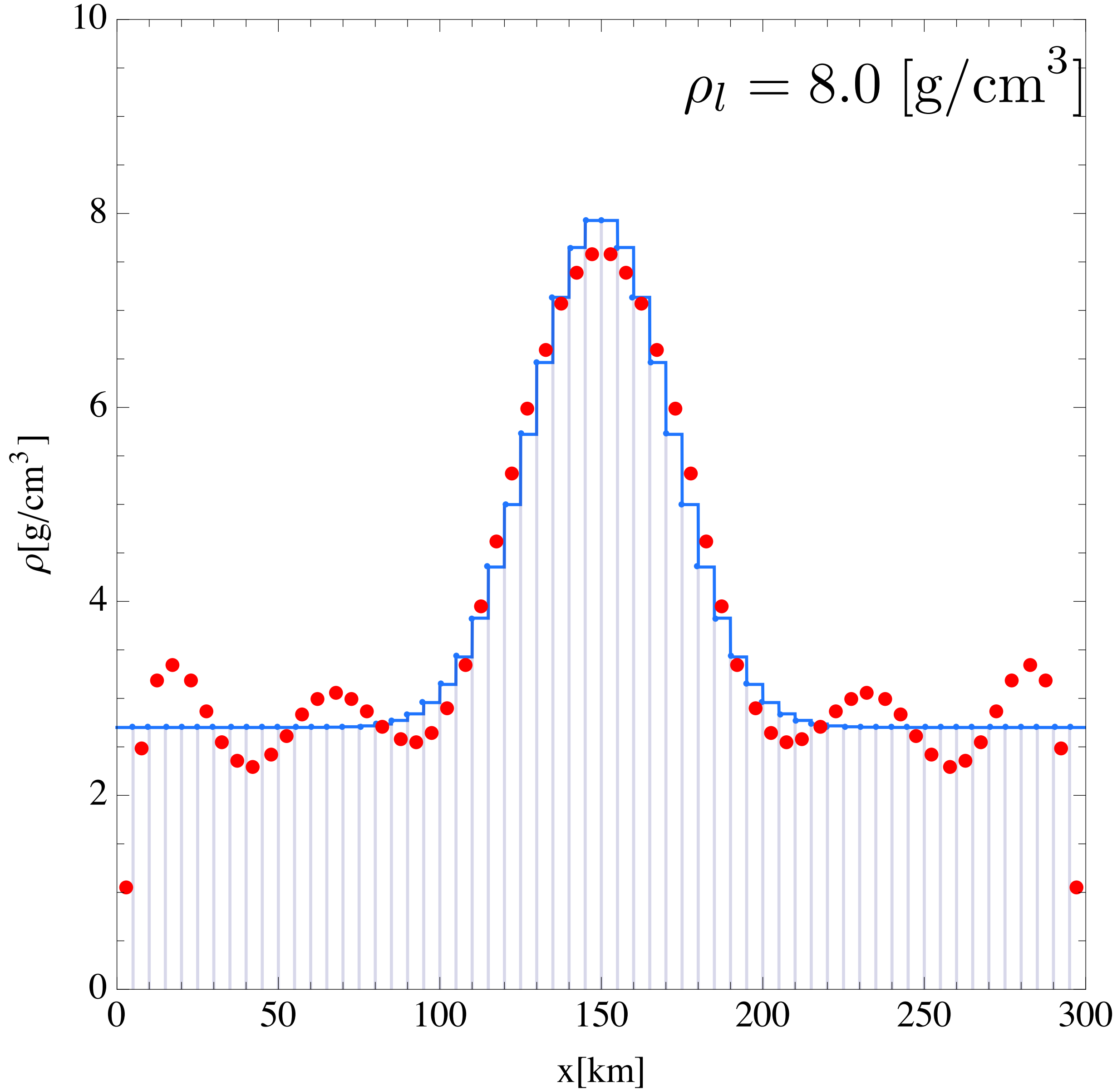
100 Energy bin

Result with using the 2nd order formula

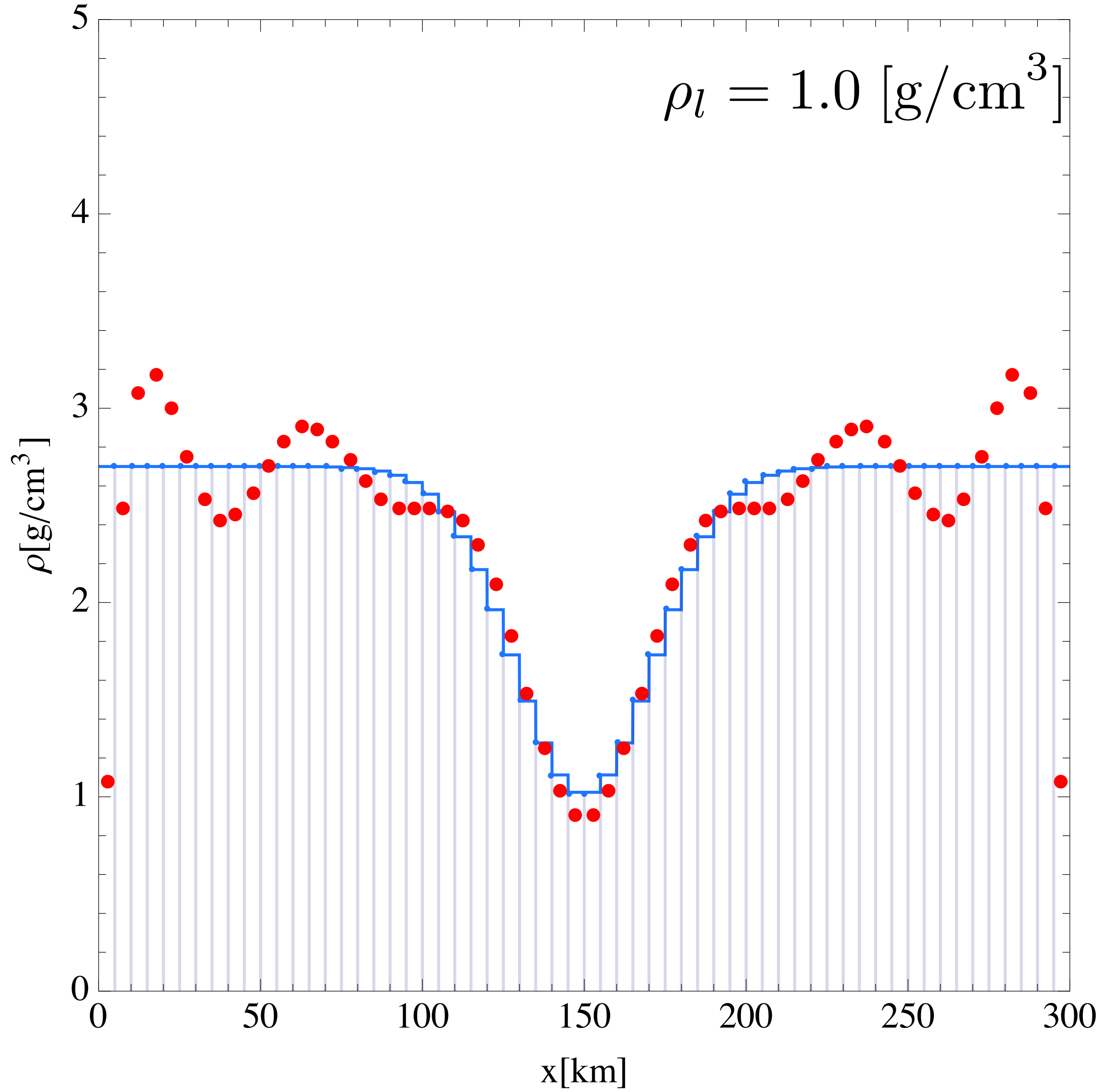
- : Original density profile
• : reconstructed density profile

$\bar{\rho} = 2.7\ [\mathrm{g/cm^3}]$

The lump of iron



The lump of water



Original density profile

$$\rho(x) = \bar{\rho} + (\rho_l - \bar{\rho}) \exp\left[-\frac{\left(x-\frac{L}{2}\right)^2}{D_l^2}\right]$$

We could reconstruct the density profile of lump.

Reconstruction of 60 points 100 Energy bin

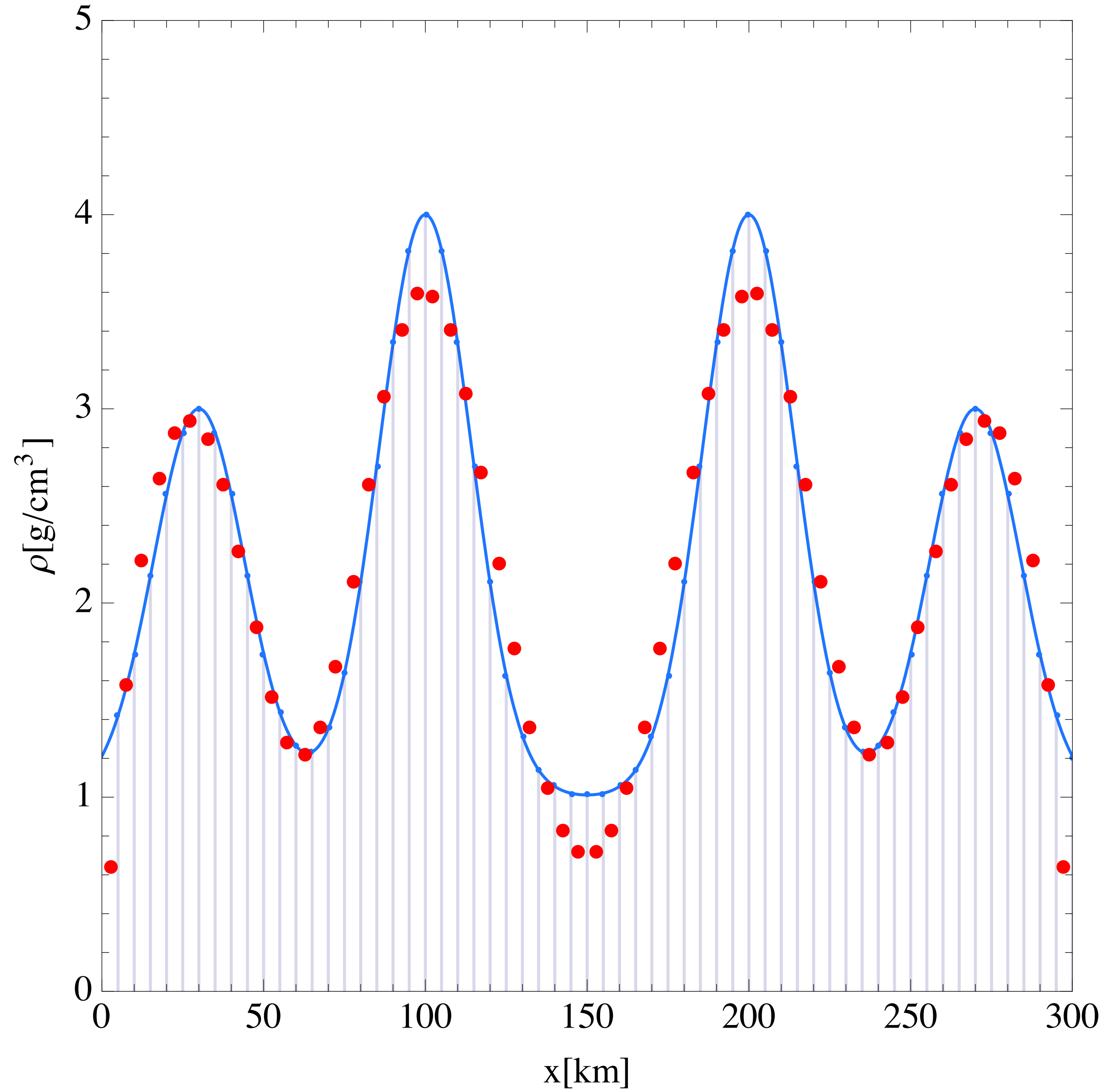
Result of the exotic density profile

- : Original density profile

• : reconstructed density profile

using the 2nd order formula

4 lump



Reconstruction of 60 points

100 Energy bin

We could reconstruct the exotic density profile.

Summary

Summary

We have investigated the oscillation tomography by the neutrino pair beam.

This talk

- **The neutrino pair beam is powerful source to the probe of the Earth's interior.**
- **The reconstruction method with the 2nd order perturbation formula is powerful tool.**
- **It has been demonstrated that the profile can be reconstructed well by including the 2nd order correction. We believe that these two ingredients give considerable progress toward the realization of the neutrino tomography.**

Toward to the realization of the neutrino tomography

- **the realistic 3 flavor oscillation.**
- **the method with the reconstruction of the asymmetric density profile.**
- **the more realistic set up.**
  (- uncertainty of the real experiment)
  (- realistic target of the neutrino tomography)
  ex.) Earth's core and mantle, mineral, oil, etc…

Back Up

Degeneracy of the 2 flavor Neutrino Oscillation

If we consider the 2 flavor oscillation, probability degenerate by the Unitarity.

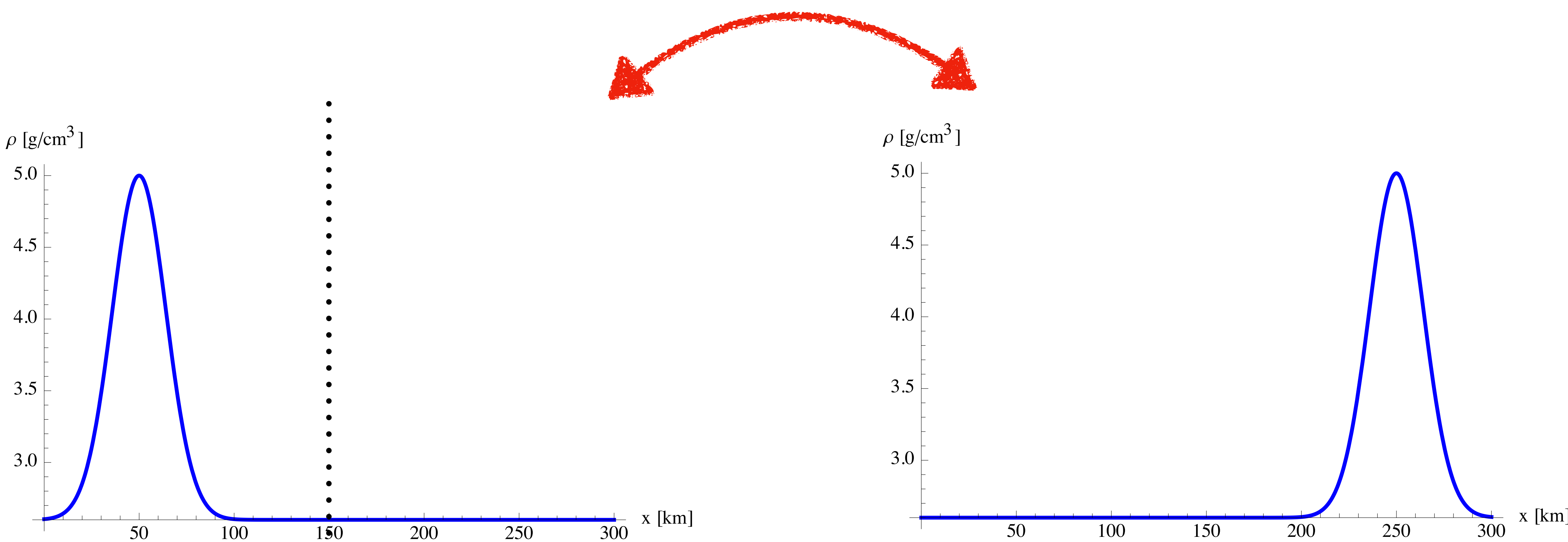
$$P(\nu_e \to \nu_e) + P(\nu_e \to \nu_\mu) = 1$$

$$P(\nu_e \to \nu_e) + P(\nu_\mu \to \nu_e) = 1$$

$$\therefore P(\nu_\mu \to \nu_e) = P(\nu_e \to \nu_\mu)$$

So, Oscillation probability with asymmetric density profile coincide with the another one.

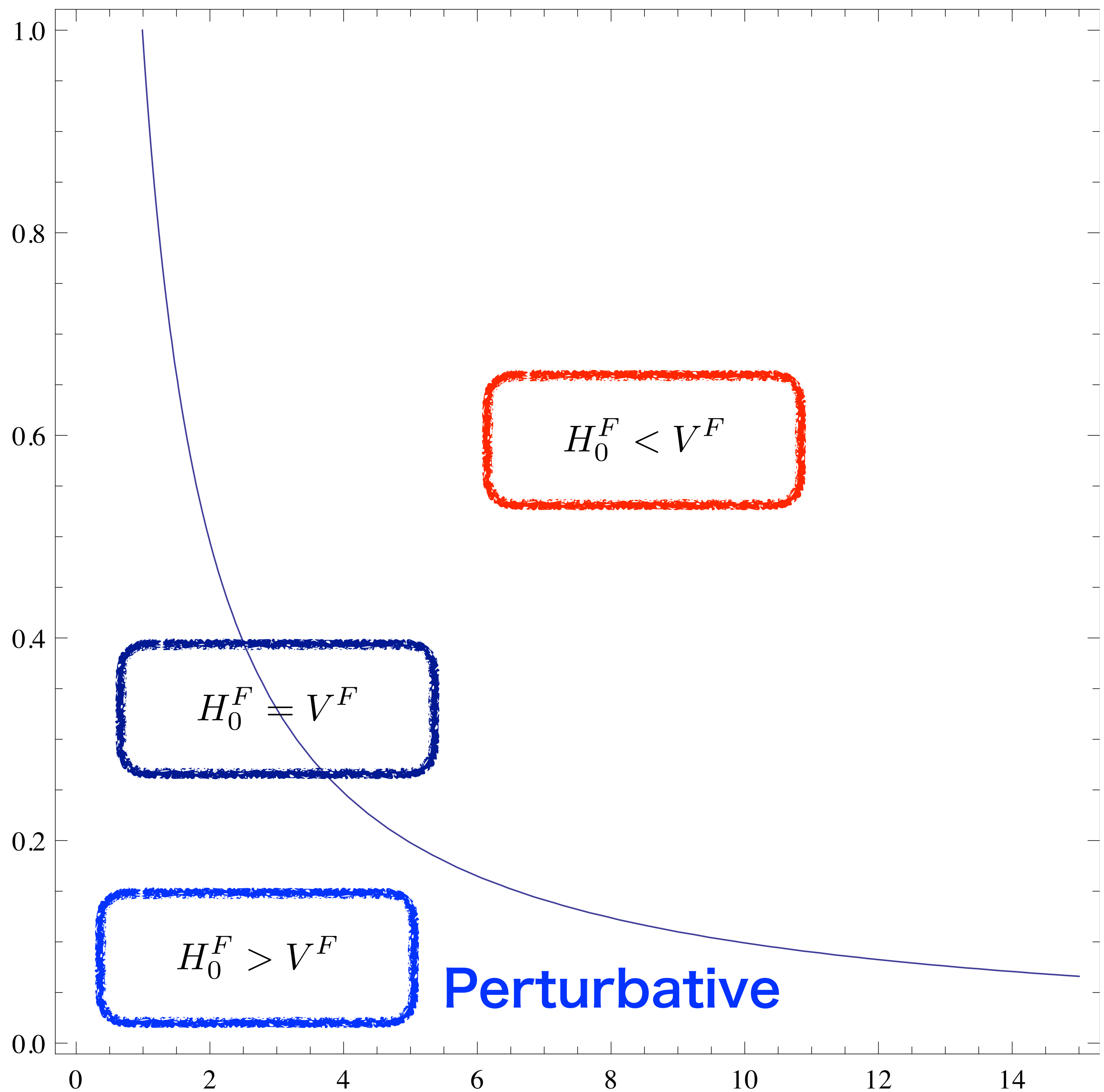
Same Oscillation Probability



Density profile with left side lump

Density profile with right side lump

Condition of perturbation



Neutrino Energy
$E_\nu \ [GeV]$

Density
$\rho \ [g/cm^3]$

Evolution equation

$$i\frac{d}{dx}\vec{A}(x) = [H_0^F + V^F]\vec{A}(x)$$

Condition of perturbation

$$H_0^F > V^F$$

$$\Delta m^2_{SOL} = 7.5 \times 10^{-5}[eV]$$

Perturbation formulae

1st order

$$P^{(1)}(\nu_e \to \nu_e) = \frac{G_F}{2\sqrt{2}m_p} \sin^2 2\theta \cos 2\theta \int_0^L dx\ \rho(x)[\sin\{\frac{\Delta m^2}{2E} L\} - \sin\{\frac{\Delta m^2}{2E} x\} - \sin\{\frac{\Delta m^2}{2E}(L-x)\}]$$

2nd order

$$P^{(2)}(\nu_e \to \nu_e; t) = P^{(2a)}(\nu_e \to \nu_e; t) + P^{(2b)}(\nu_e \to \nu_e; t)$$

$$\begin{aligned} P^{(2a)}(\nu_e \to \nu_e; t) = & [\cos^8\theta + \sin^8\theta + 2\cos^4\theta \sin^4\theta \cos(\Phi t)] G_1(t)^2 \\ & + \cos^4\theta \sin^4\theta [G_2(t)^2 + G_3(t)^2] \\ & + 2(\cos^4\theta + \sin^4\theta)\cos^2\theta \sin^2\theta G_1(t) G_2(t) \end{aligned}$$

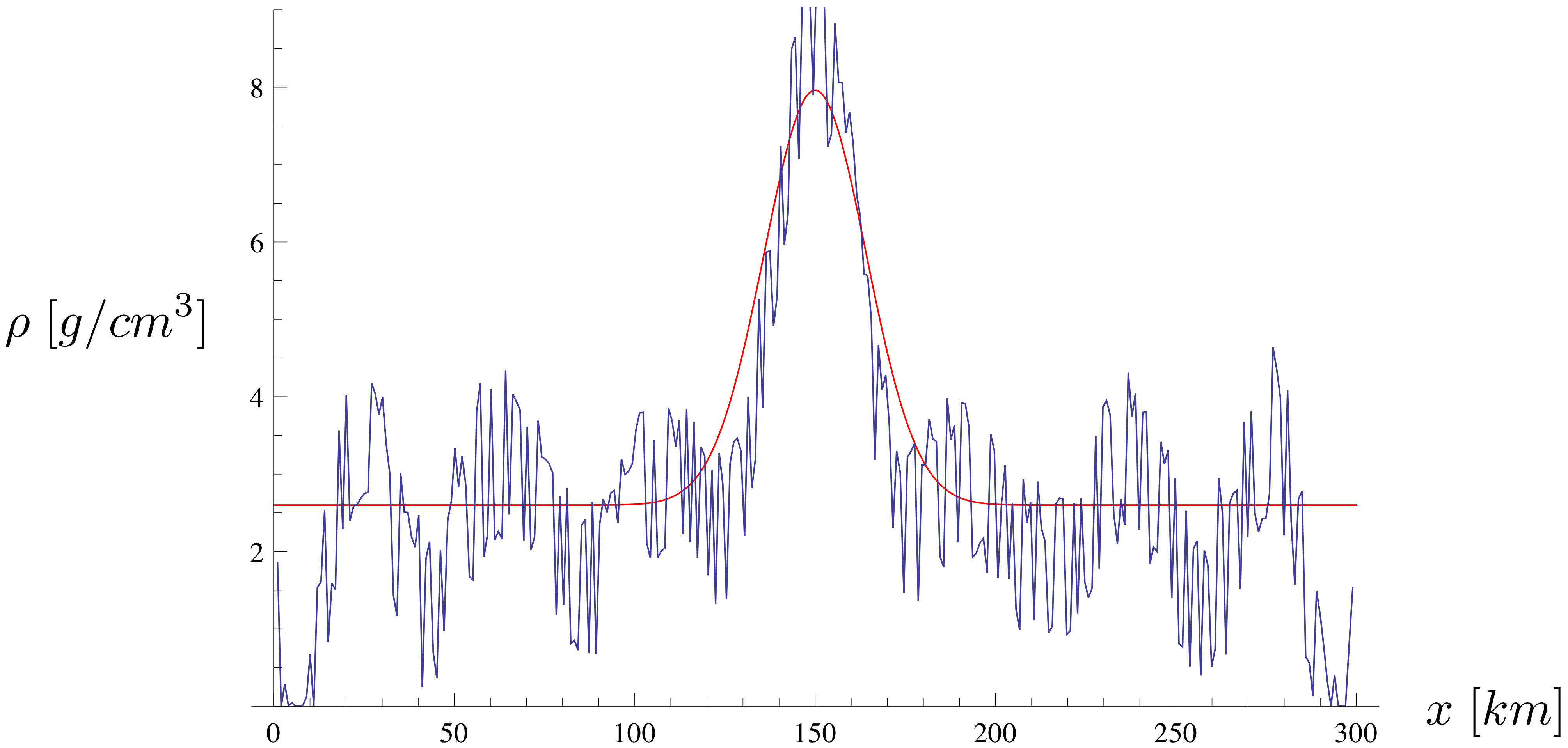
$$\begin{aligned} P^{(2b)}(\nu_e \to \nu_e; t) = & -2\int_0^t dt_1 \int_0^{t_1} dt_2 V_{CC}(t_1) V_{CC}(t_2) \\ & \times \{+\cos^8\theta + \sin^8\theta \\ & + \cos^2\theta \sin^2\theta(\cos^4\theta + \sin^4\theta)[\cos(\Phi t) + \cos(\Phi t_2) + \cos(\Phi(t_2 - t_1)) + \cos(\Phi(t_1 - t))] \\ & + 2\cos^4\theta \sin^4\theta[\cos(\Phi(t_2 - t)) + \cos(\Phi t_1) + \cos(\Phi(t_2 - t_1 + t))]\} \end{aligned}$$

$$\begin{aligned} G_1(t) &= \int_0^t dt_1 V_{CC}(t_1) \\ G_2(t) &= \int_0^t dt_1 V_{CC}(t_1)[\cos(\Phi t_1) + \cos(\Phi(t - t_1))] \\ G_3(t) &= \int_0^t dt_1 V_{CC}(t_1)[\sin(\Phi t_1) + \sin(\Phi(t - t_1))] \end{aligned}$$

Reconstruction with 1st order perturbation

The result when the number of devisors is increased.

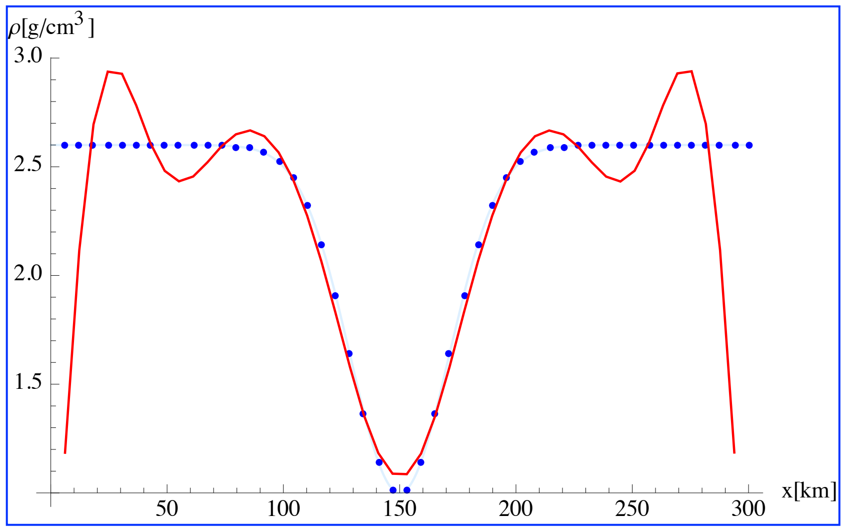
N_E = 300
N_L = 300

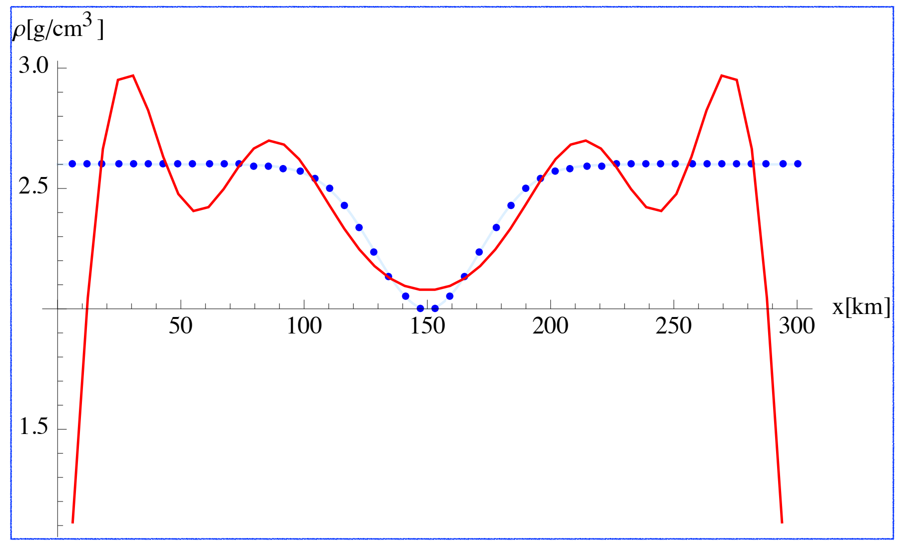


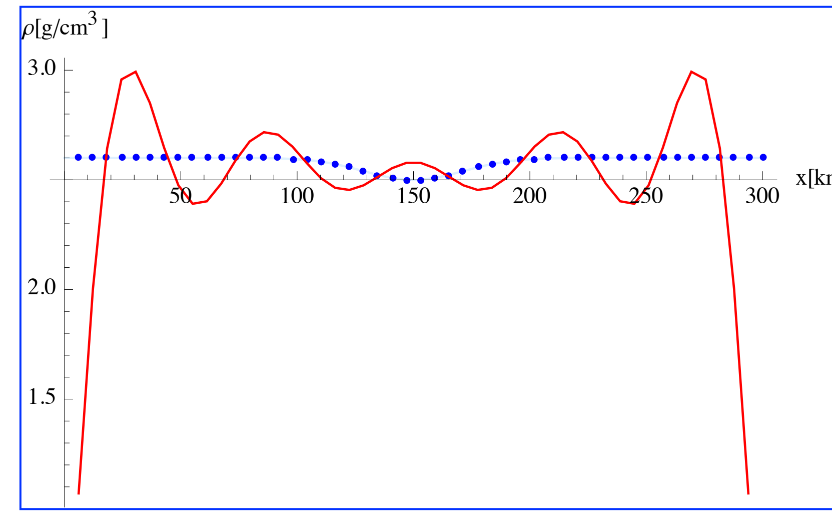
- : Original density profile
• : reconstructed density profile

Reconstruction with 2nd order perturbation

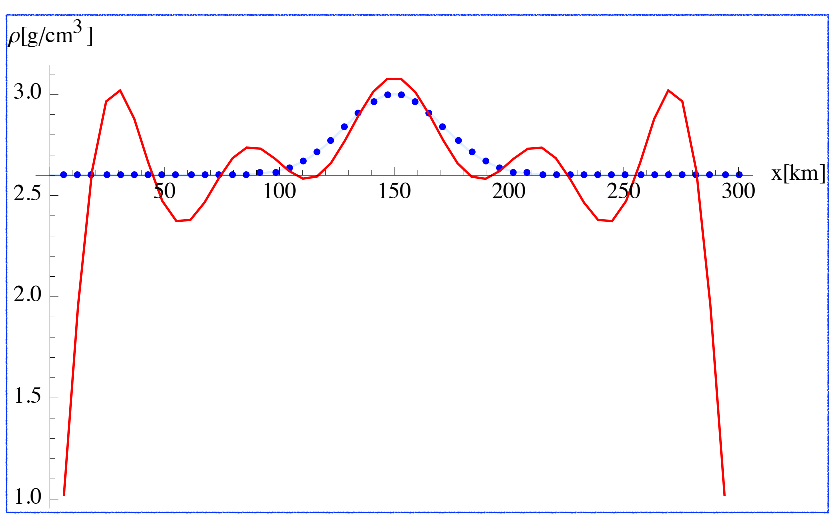
We see the dependence of density.

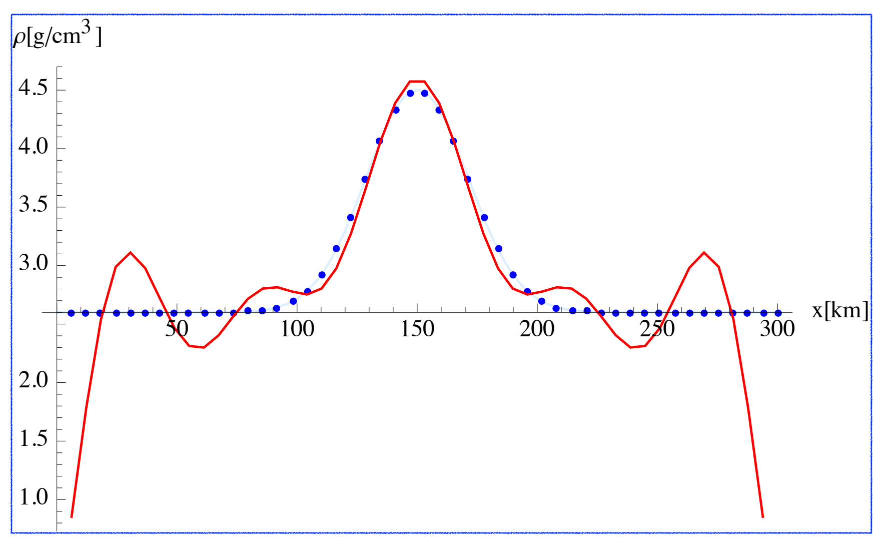


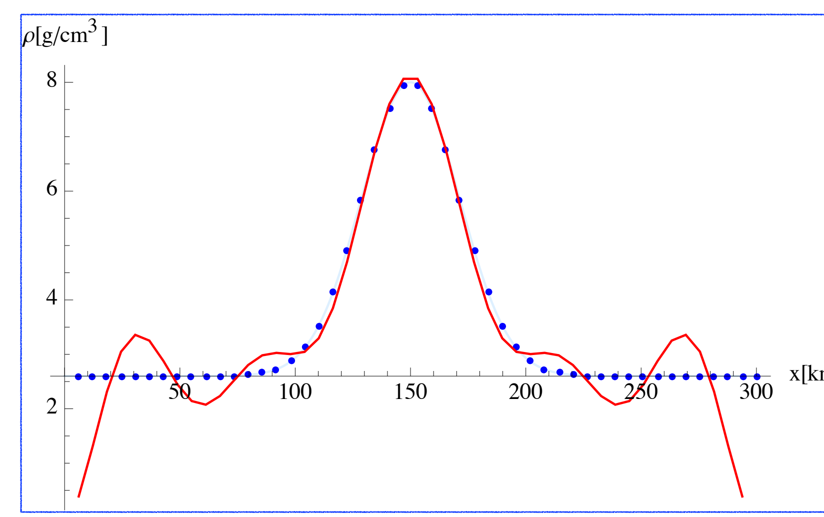




We can't reconstruct if the density becomes too small in this method.





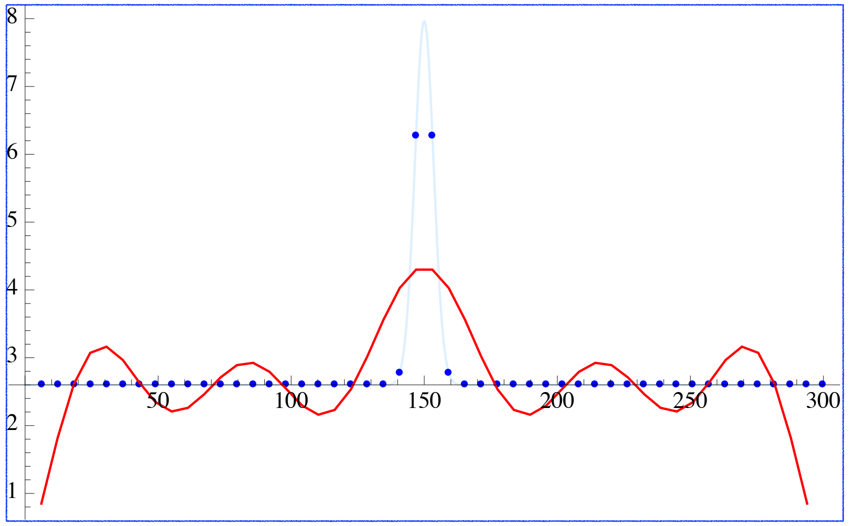


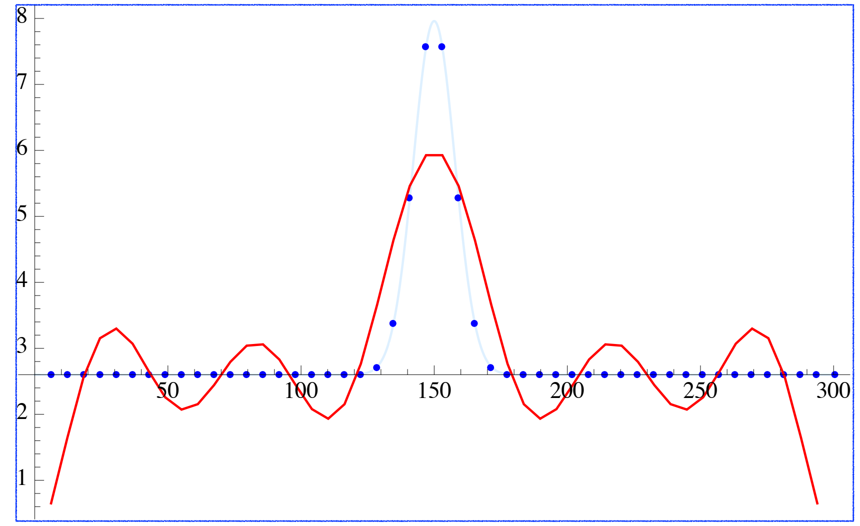
- : reconstructed density profile

• : Original density profile

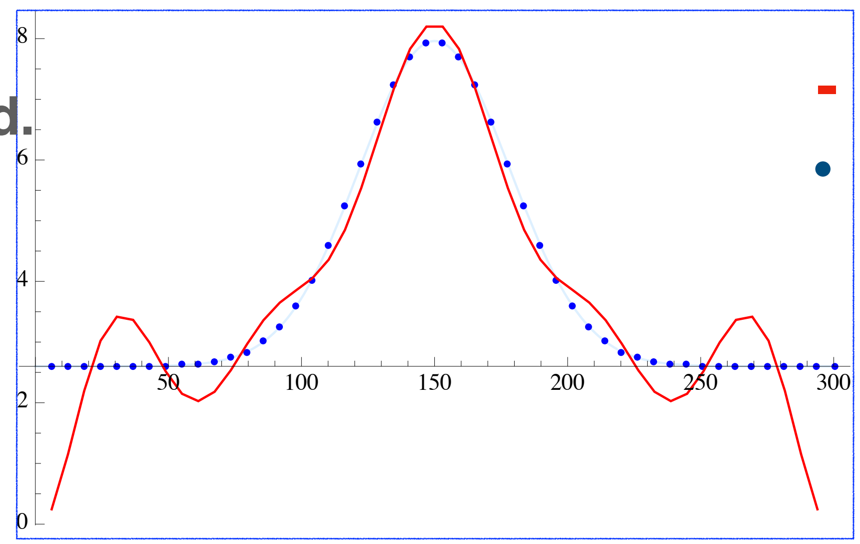
Reconstruction with 2nd order perturbation

We see the dependence of width of lump.





We can't reconstruct if the density becomes too narrow in this method.



- : reconstructed density profile

• : Original density profile