Large Scale Lorentz Violation and Dark Energy

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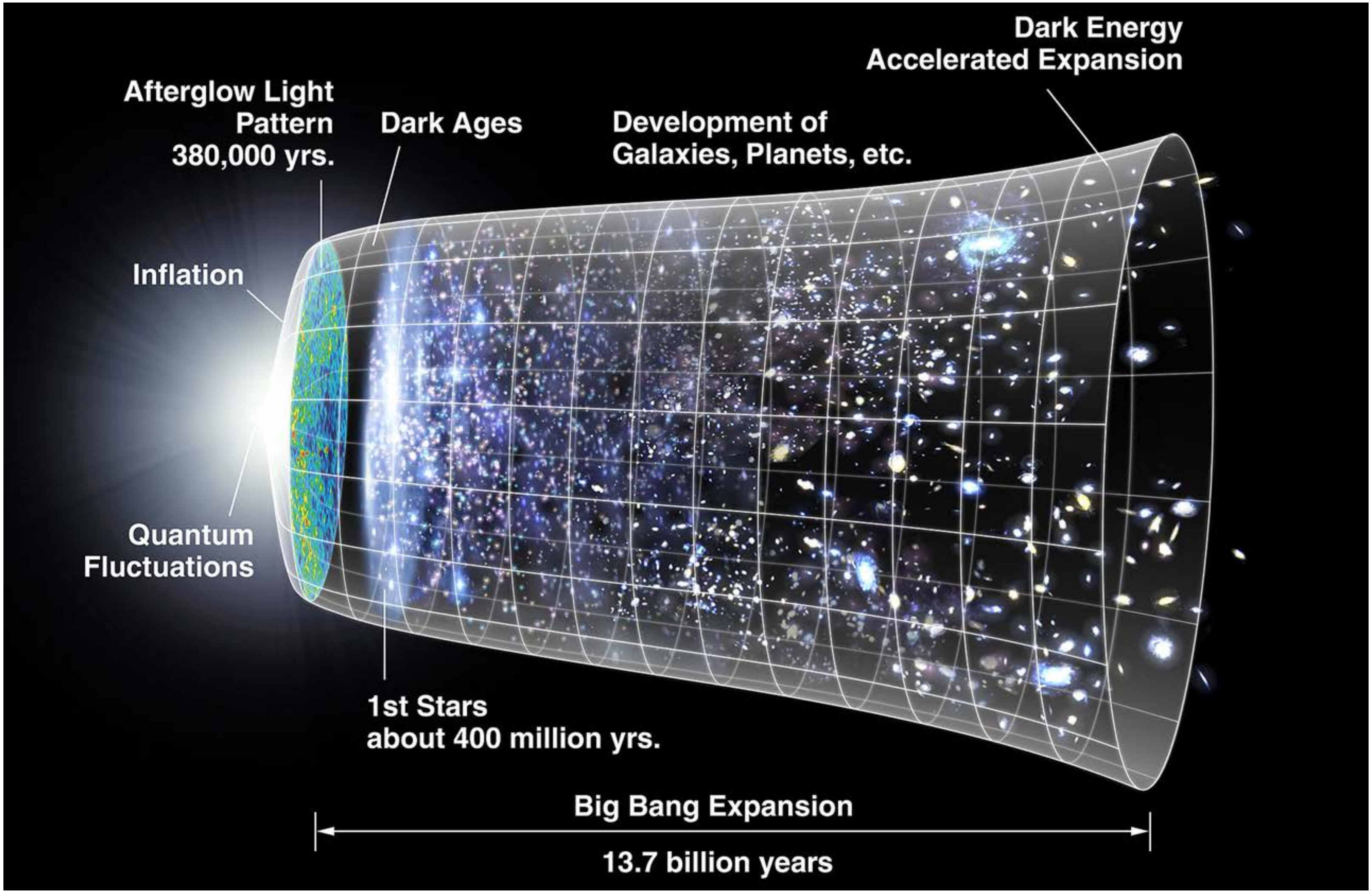
Aug. 13, SI2018, Panshan, Tianjin

Deviation from GR

- Galaxy rotation curve etc.
- Acceleratiing expansion of the universe



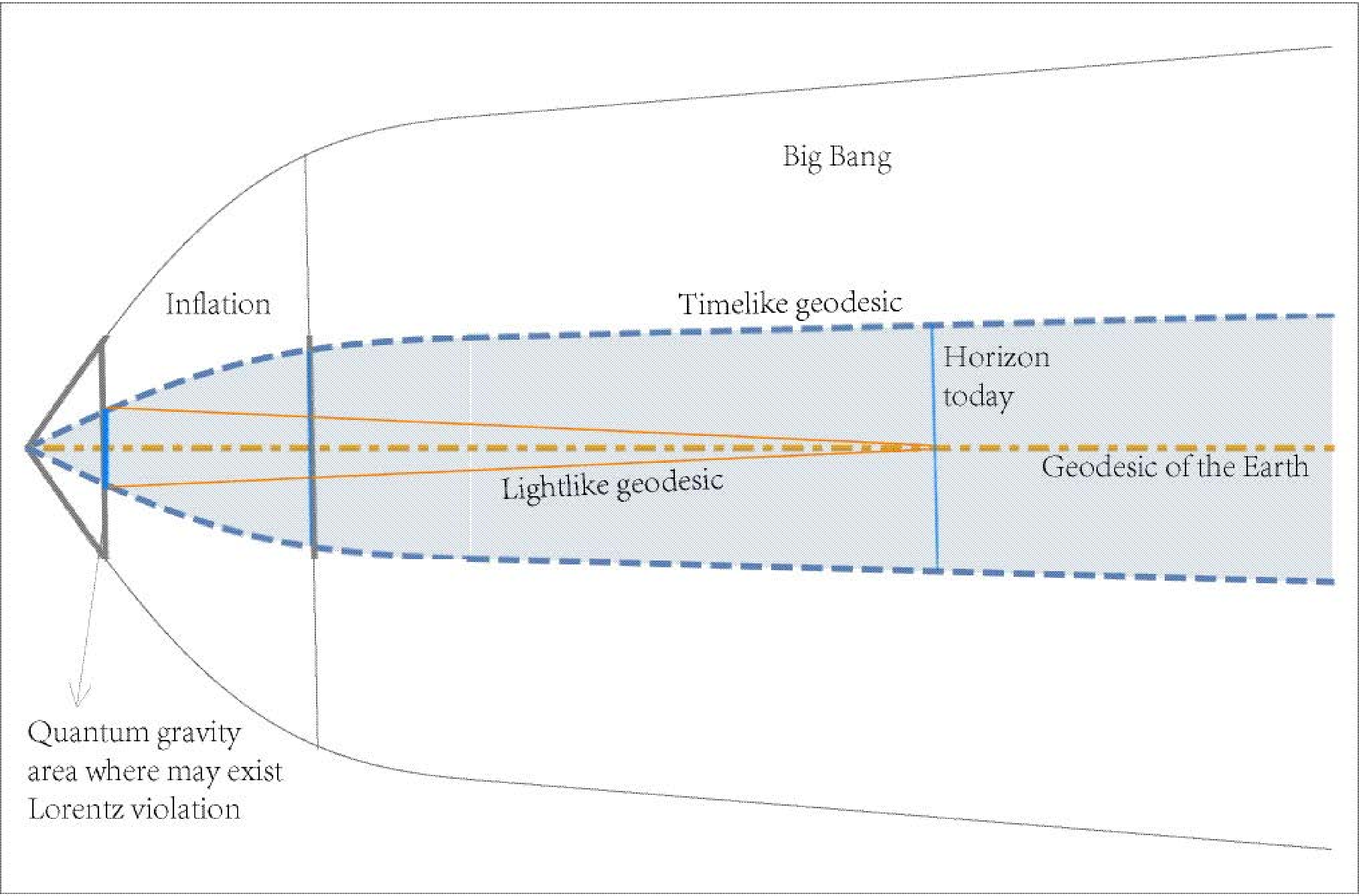
- Timeline of the universe

Quantum Gravity Leads to Local Lorentz Violation

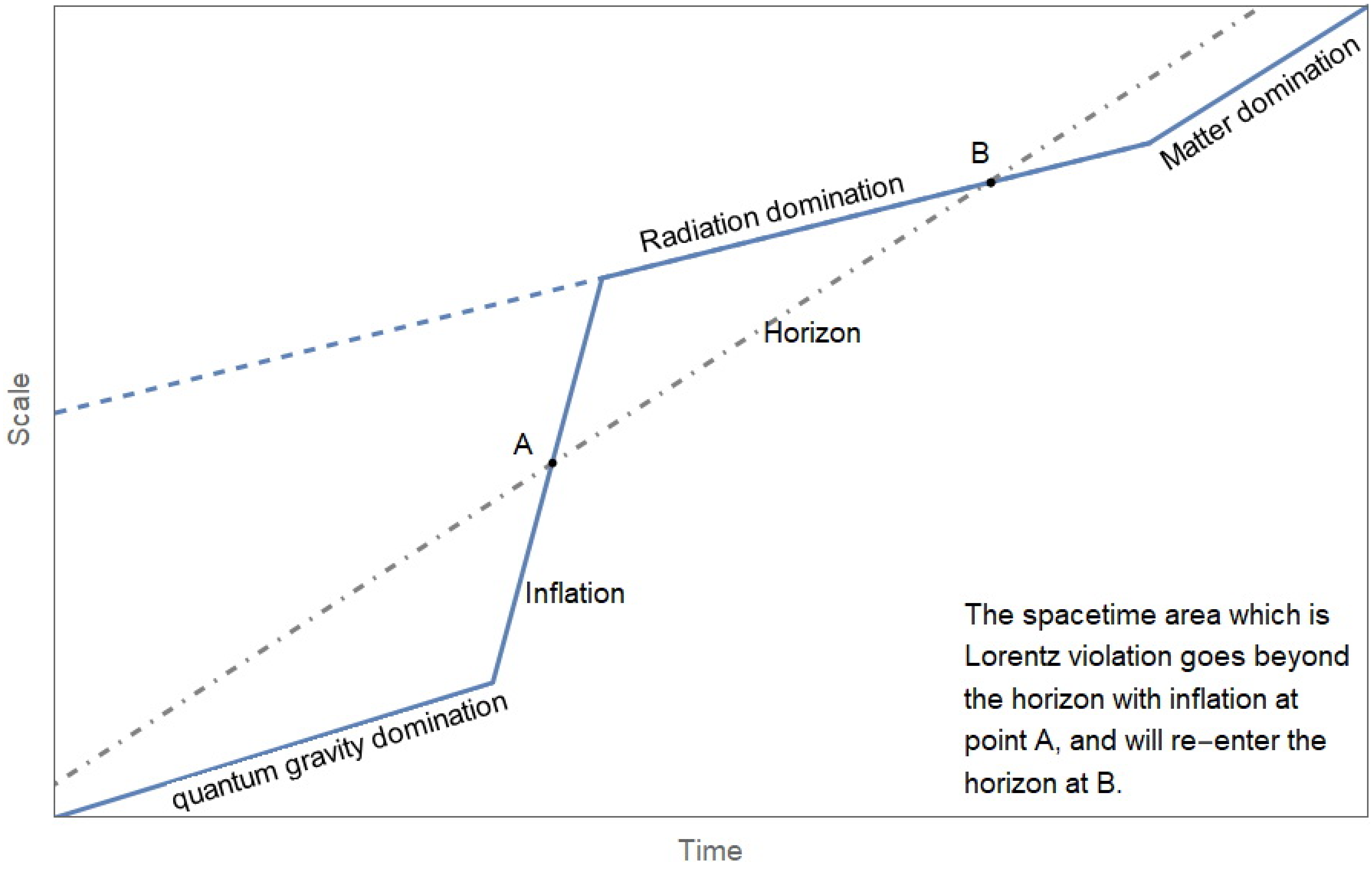
- **At the very beginning of the universe, it is believed that all the four basic interactions unified into the only quantum gravity.**
- **The existence of minimum length scale** L_p
and the maximum energy scale M_p
- **The universe experienced a period called cosmic inflation , by which the observable universe at now is expanded from a very small area at the beginning of the inflation.**
- **The local Lorentz violation effect at the moment may be transformed into the large scale one.**
- **Pre-inflation physics**

Large scale Local Lorentz Violation

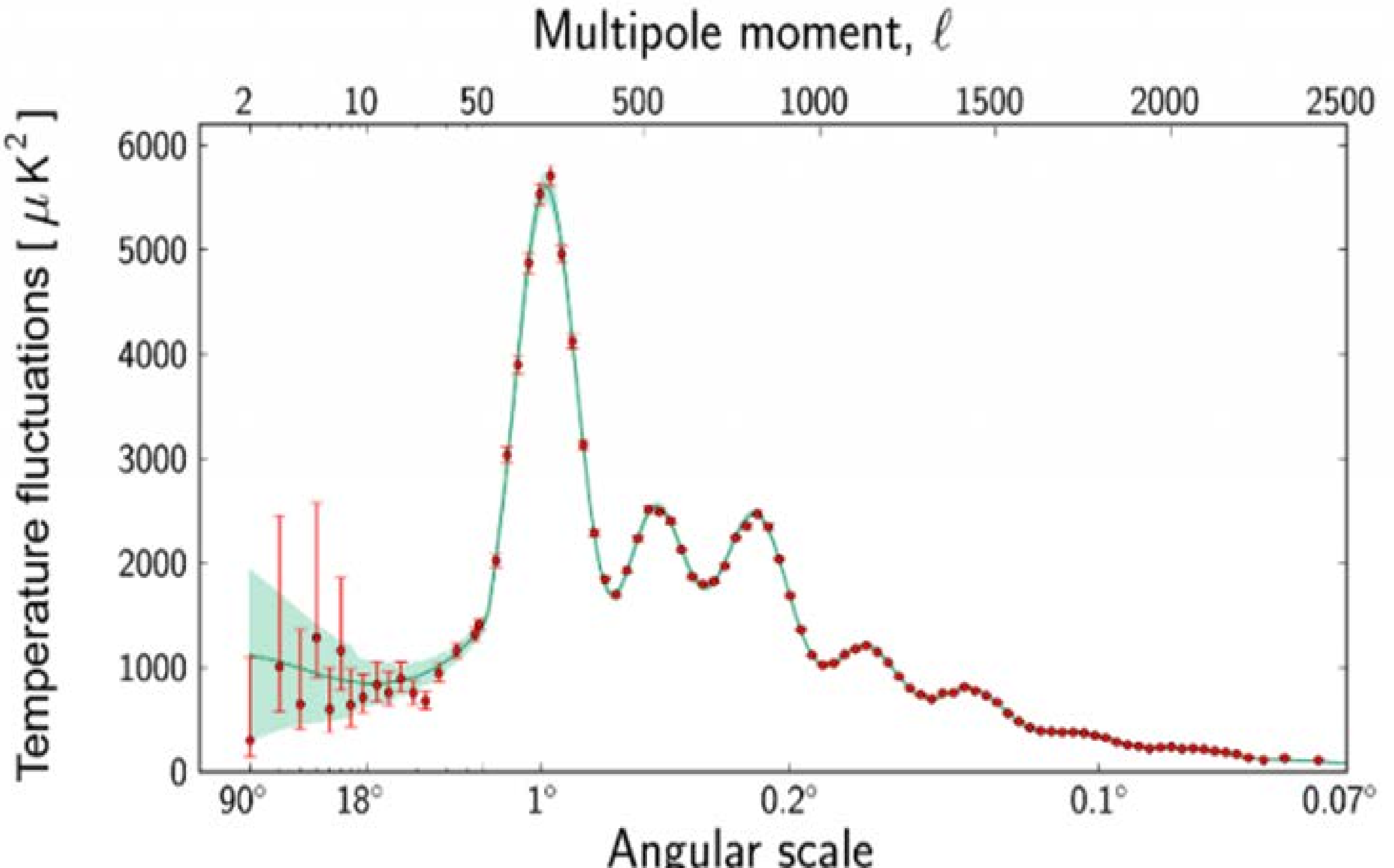
- **Lorentz violation in quantum gravity →local Lorentz symmetry in low energy gravity, GR, by interaction**
- **Local area ineracting via quantum gravity was separated to lose interaction by inflation so as to keep Lorentz violation at large scale.**



Universe time


Scale
Time
Matter domination
B
Radiation domination
Horizon
A
Inflation
quantum gravity domination
The spacetime area which is Lorentz violation goes beyond the horizon with inflation at point A, and will re−enter the horizon at B.

Anistropies of CMB



The ΛCDM model

- Einstein equation with cosmological constant

$$S_E = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R-2\Lambda\right)$$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\left(T_M\right)_{\mu\nu}$$

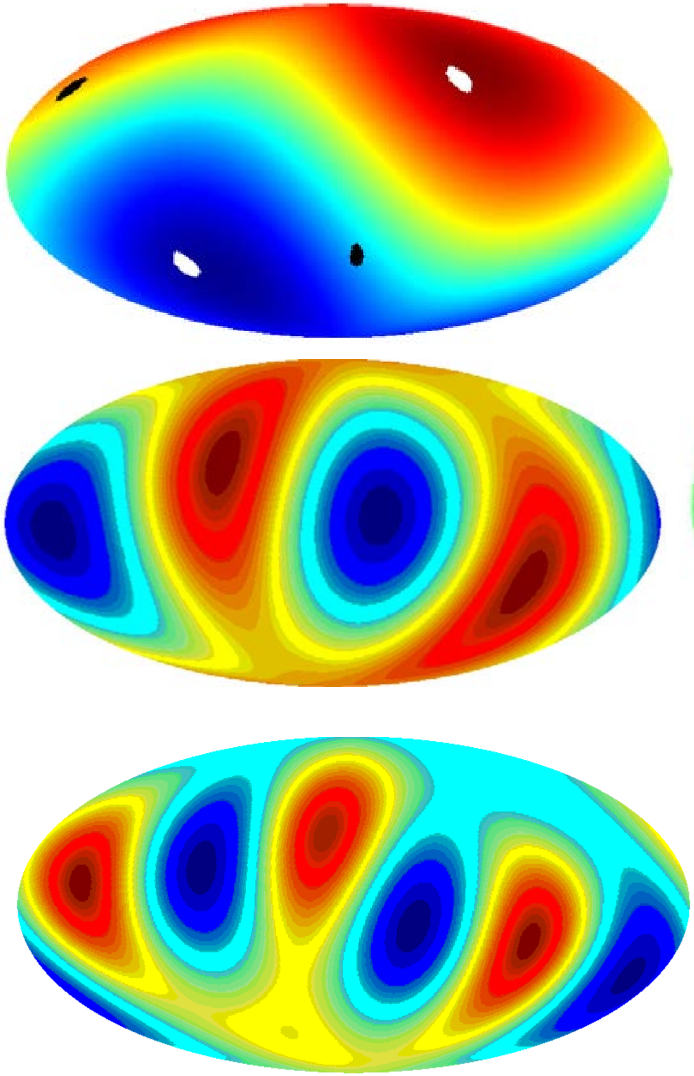
Large-scale Anomalies

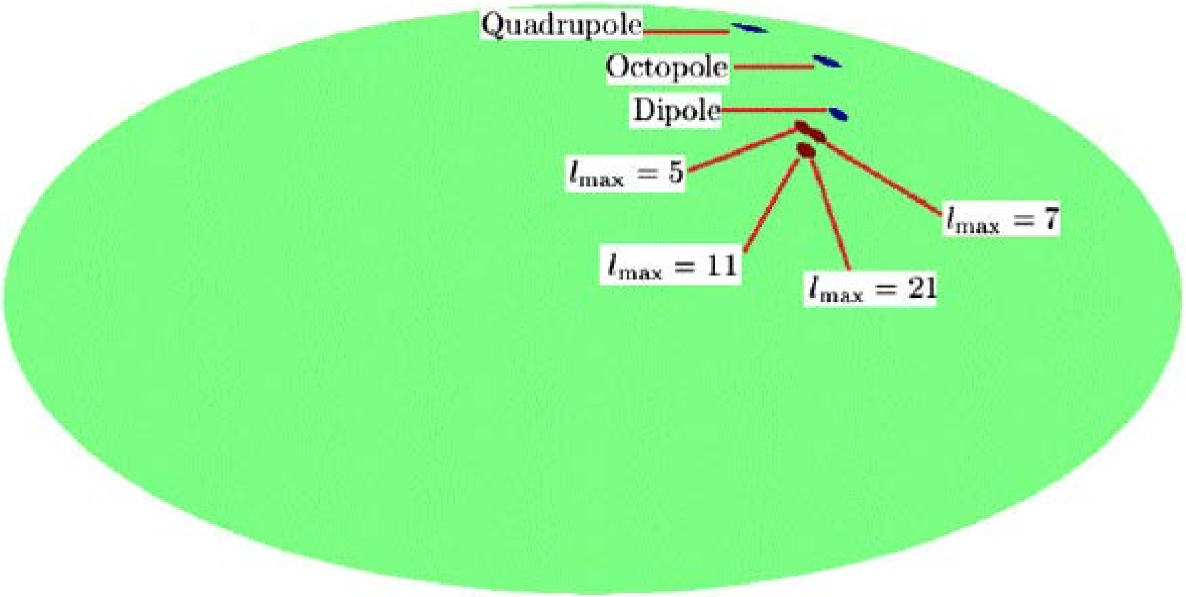
- Despite of the success of ΛCDM model , a number of large-scale “anomalies” have also been reported challenges of ΛCDM :

Alignment of CMB low multipoles:The normals to the octopole and quadrupole planes are aligned with the direction of the cosmological dipole at a level inconsistent with Gaussian random

L. Perivolaropoulos, Galaxies 2014, 2, 22-61

Comparing with preferred directions in CMB dipole, quadrupole and octopole





Large scale effective gravity

- In the frame relative to CMB static there holds cosmological principle. From such a frame to the peculiar motion frame, the transformation is not simply the Lorentz boost.
- within the solar system, local Lorentz invariance is verified at high accuracy
- Assuming the local Lorentz symmetry begins to break down from the scale of Galaxy scale to cosmic scale.
- It needs to take the local Lorentz violation into account when constructing the Large scale effective gravitation theory.

Lorentz violation, the theoretical investigations

- Considerable progress and lot of attentions on the theoretical investigation and experimental examination of Lorentz symmetry since the mid of 1990s.
- Coleman and Glashow, boost invariance violation in the rest frame of the cosmic background radiation
- Colladay and Kostelecky standard model extension incorporating Lorentz and CPT violation
- Cohen-Glashow's very special relativity model

Phys Rev D, 1998, 58, 116002

The identified VSR subgroups up to isomorphism

- T(2) (2-dimensional translations) with generators $T_1 = K_x + J_y$ and $T_2 = K_y - J_x$, where J and K are the generators of rotations and boosts respectively
- E(2) (3-parameter Euclidean motion) with generators T_1 ; T_2 and J_z ,
- HOM(2) (3-parameter orientation preserving transformations) with generators T_1; T_2 and K_Z
- SIM(2) (4-parameter similitude group)with generators T_1 ; T_2; J_z and K_Z

- We take very special relativity symmetry Sim(2),Hom(2) and E(2) gauge theories as an example of such motivation to illustrate the so called dark energy effect, the deviation of astronomical observation from Einstein's GR prediction such as the accelerating expansion of the universe etc., may be emerged from the Lorentz violation effect at the large scale.

arXiv:1510.00814, Chinese Science Bulletin 2016

Equivalence principle, local Lorentz symmetry

- There exists the free falling observer who does not feel gravity everywhere. One can always have Lorentz symmetry locally.

$$\psi \xrightarrow{x^{\mu} \to \Lambda^{\mu}{}_{\nu} x^{\nu}} U\left(\Lambda\left(x\right)\right)\psi$$

- Utiyama,Sciama, Kibble, Tautman, Heyl

- The localization of Lorentz symmetry can be realized by introducing Lorentz connection

$$\mathscr{L}\left(\partial_\mu \psi, \cdots\right) \longrightarrow \mathscr{L}\left(\mathscr{D}_\mu \psi, \cdots\right)$$

$$\mathscr{D}_\mu = \partial_\mu - \frac{i}{2} A^{ab}{}_\mu S_{ab}$$

- the Lorentzian group generator

$$S_{ab}$$

- the Lorenz gauge field or the Lorentz connection.

$$A_\mu = \frac{1}{2} A^{ab}{}_\mu S_{ab}$$

- Tetrad field $h^a{}_\mu$
- Relation with metric: $g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu$

$$\eta_{ab} = g_{\mu\nu} h_a{}^\mu h_b{}^\nu$$

- relation with linear connection

$$A^a{}_{b\mu} = h^a{}_\nu \partial_\mu h_b{}^\nu + h^a{}_\nu \Gamma^\nu{}_{\rho\mu} h_b{}^\rho \equiv h^a{}_\nu \nabla_\mu h_b{}^\nu$$

- tetrad basis $h_a = h_a{}^\mu \partial_\mu$

- commutation relation $\left[h_a, h_b\right] = f^c{}_{ab} h_c$
- the structure coefficients

$$f^c{}_{ab} = h_a{}^\mu h_b{}^\nu \left(h^c{}_{\mu,\nu} - h^c{}_{\nu,\mu}\right)$$

- spin connection in terms of the coefficients of anholonomy

$$A^a{}_{bc} = A^a{}_{b\mu} h_c{}^\mu$$

$$A^a{}_{bc} = \frac{1}{2}\left(f_b{}^a{}_c + f_c{}^a{}_b - f^a{}_{bc} + T_b{}^a{}_c + T_c{}^a{}_b - T^a{}_{bc} \right)$$

- The curvature in terms of spin connection

$$R^a{}_{b\nu\mu} = A^a{}_{b\mu,\nu} - A^a{}_{b\nu,\mu} + A^a{}_{e\nu} A^e{}_{b\mu} - A^a{}_{e\mu} A^e{}_{b\nu}$$

- the spacetime-indexed forms

$$R^\rho{}_{\lambda\nu\mu} = h_a{}^\rho h^b{}_\lambda R^a{}_{b\nu\mu} = \Gamma^\rho{}_{\lambda\mu,\nu} - \Gamma^\rho{}_{\lambda\nu,\mu} + \Gamma^\rho{}_{\eta\nu} \Gamma^\eta{}_{\lambda\mu} - \Gamma^\rho{}_{\eta\mu} \Gamma^\eta{}_{\lambda\nu}$$

Local Lorentz symmetry, the dynamics

- To describe the dynamics of gravity, the action must have the invariance group implied by the equivalence principle, i.e. Lorentz gauge invariance.
- field strength of Lorentz connection and vielbein fields.

$$D_a = h_a{}^\mu \mathscr{D}_\mu = h_a{}^\mu \left(\partial_\mu - \frac{i}{2} A^{cd}{}_\mu S_{cd} \right)$$

are curvature and torsion

$$\left[D_a , D_b \right] = T_{ab}{}^p D_p + \frac{i}{2} R_{ab}{}^{pq} S_{pq}$$

- The Yang-Mills type of action does not supply the space for gravitational coupling constant. A natural choice for the Lorentz gauge field is the Einstein action

$$S_E = \frac{1}{16\pi G}\int d^4 x h R_{ab}{}^{ab}, \; h = \det\left(h^a{}_\mu\right)$$

- EOM of Lorentz connection $\mathscr{D}_\nu\left(h\left(h_a{}^\nu h_b{}^\mu - h_a{}^\mu h_b{}^\nu\right)\right) = 0$

- Levi-Civita connection $\Gamma^\rho{}_{\nu\mu} = \frac{1}{2} g^{\rho\lambda}\left(g_{\mu\lambda,\nu} + g_{\nu\lambda,\mu} - g_{\mu\nu,\lambda}\right)$

- EOM of tetrad

$$R_c{}^a - \frac{1}{2}\delta_c{}^a R = 0$$

- In the presence of source matter field

$$\delta S_M = \int d^4 x h \left(\frac{1}{2} \delta h^c{}_a \left(T_M \right)_c{}^a + \delta A^{ab}{}_\mu \left(C_M \right)_{ab}{}^\mu \right)$$

- the full EOM for tetrad field :

$$\tilde{R}_c{}^a - \frac{1}{2} \delta_c{}^a \tilde{R} = 8\pi G \left(T_M \right)_c{}^a$$

- In the presence of matter field source, the theory is not torsion free in general. The scalar source field implies zero torsion while spinor does not.

$$\mathcal{D}_\nu \left(h \left(h_a{}^\nu h_b{}^\mu - h_a{}^\mu h_b{}^\nu \right) \right) = 16\pi G \left(C_M \right)_{ab}{}^\mu$$

Equivalence principle, local Sim(2) symmetry

- local Sim(2) symmetry invariant theory, gravity, the local transformation constrained on Sim(2).

$$\psi \xrightarrow{x^\mu \to \Lambda^\mu{}_\nu x^\nu} U\left(\Lambda(x)\right)\psi,\ \Lambda(x) \in Sim(2)$$

- The connection 1-form:

$$A_\mu = \frac{1}{2} A^{ab}{}_\mu S_{ab} = A^{10}{}_\mu S_{10} + A^{20}{}_\mu S_{20} + A^{30}{}_\mu S_{30} + A^{12}{}_\mu S_{12} + A^{23}{}_\mu S_{23} + A^{31}{}_\mu S_{31}$$

$$= \frac{1}{2}\left(A^{10}{}_\mu + A^{31}{}_\mu\right) T_1 + \frac{1}{2}\left(A^{20}{}_\mu - A^{23}{}_\mu\right) T_2 + A^{30}{}_\mu K_3 + A^{12}{}_\mu J_3$$

$$+ \frac{1}{2}\left(A^{10}{}_\mu - A^{31}{}_\mu\right)\left(S_{10} - S_{31}\right) + \frac{1}{2}\left(A^{20}{}_\mu + A^{23}{}_\mu\right)\left(S_{20} + S_{23}\right)$$

the dynamics with constrains

- the Sim(2) constrains

$$A^{10}{}_{\mu} - A^{31}{}_{\mu} = 0,\ \ A^{20}{}_{\mu} + A^{23}{}_{\mu} = 0$$

- The action：

$$S_E = \frac{1}{16\pi G}\int d^4xh\left(R^{ab}{}_{ab} + \lambda_1{}^{\mu}\left(A^{10}{}_{\mu} - A^{31}{}_{\mu}\right) + \lambda_2{}^{\mu}\left(A^{20}{}_{\mu} + A^{23}{}_{\mu}\right)\right)$$

- EOM of connections

$$\mathscr{D}_{\nu}\left(h\left(h_a{}^{\nu}h_b{}^{\mu} - h_a{}^{\mu}h_b{}^{\nu}\right)\right) = \lambda_1{}^{\mu}h\left(\delta_a{}^1\delta_b{}^0 - \delta_a{}^3\delta_b{}^1\right) + \lambda_2{}^{\mu}h\left(\delta_a{}^2\delta_b{}^0 - \delta_a{}^2\delta_b{}^3\right)$$

- The constrained EOM for connections

$$\lambda_1{}^{\mu} h = \mathcal{D}_{\nu}\left(h\left(h_1{}^{\nu} h_0{}^{\mu} - h_1{}^{\mu} h_0{}^{\nu}\right)\right) = -\mathcal{D}_{\nu}\left(h\left(h_3{}^{\nu} h_1{}^{\mu} - h_3{}^{\mu} h_1{}^{\nu}\right)\right)$$

$$\lambda_2{}^{\mu} h = \mathcal{D}_{\nu}\left(h\left(h_2{}^{\nu} h_0{}^{\mu} - h_2{}^{\mu} h_0{}^{\nu}\right)\right) = \mathcal{D}_{\nu}\left(h\left(h_2{}^{\nu} h_3{}^{\mu} - h_2{}^{\mu} h_3{}^{\nu}\right)\right)$$

- And $\mathcal{D}_{\nu}\left(h\left(h_3{}^{\nu} h_0{}^{\mu} - h_3{}^{\mu} h_0{}^{\nu}\right)\right) = 0$

$$\mathcal{D}_{\nu}\left(h\left(h_1{}^{\nu} h_2{}^{\mu} - h_1{}^{\mu} h_2{}^{\nu}\right)\right) = 0$$

- Independent number of Eqn: $24 - 2 \times 4 = 16$
- Free components of connections : 8
- The Lagrange-multipliers term canbe regarded as effective angule momentum distribution $C_{M\,eff}$

- the spin connection can be decomposed into

$$A^{a}{}_{bc} = \tilde{A}^{a}_{bc} + K^{a}{}_{bc}$$

- the torsion free connection in GR, $\tilde{A}^{a}_{bc}$

$$\tilde{A}^{a}_{bc} = \frac{1}{2}\left(f_{b}{}^{a}{}_{c} + f_{c}{}^{a}{}_{b} - f^{a}{}_{bc} \right)$$

- the contorsion

$$K^{a}{}_{bc} = \frac{1}{2}\left(T_{b}{}^{a}{}_{c} + T_{c}{}^{a}{}_{b} - T^{a}{}_{bc} \right)$$

- Recall the local Lorentz case, the EOM for connection reduced to constrain on connections and resulted in Levi-Civita ones.
- In local Sim(2) case, the contorsion has 8 independent components,

$$K^{10}{}_{0}, K^{10}{}_{1}, K^{10}{}_{2}, K^{20}{}_{0}, K^{30}{}_{0}, K^{30}{}_{1}, K^{30}{}_{2}, K^{12}{}_{0}$$

- The 8 constrain eqns reduced to:

$$f^{1}{}_{10} + f^{1}{}_{31} = 0$$

$$f^{2}{}_{23} - f^{2}{}_{20} = 0$$

$$f^{2}{}_{03} + f^{0}{}_{23} + 2f^{3}{}_{23} - f^{3}{}_{20} = 0$$

$$f^{1}{}_{30} + f^{0}{}_{31} + f^{3}{}_{10} + 2f^{3}{}_{31} = 0$$

$$2f^{0}{}_{20} - f^{2}{}_{30} + f^{3}{}_{20} - f^{0}{}_{23} = 0$$

$$2f^{0}{}_{10} + f^{0}{}_{31} - f^{1}{}_{30} + f^{3}{}_{10} = 0$$

$$f^{2}{}_{10} + f^{2}{}_{31} + f^{0}{}_{12} + f^{3}{}_{12} + f^{1}{}_{20} - f^{1}{}_{23} = 0$$

$$-f^{1}{}_{20} + f^{3}{}_{12} + f^{0}{}_{12} - f^{2}{}_{10} + f^{1}{}_{23} - f^{2}{}_{31} = 0$$

The local Sim(2) symmetry case

- The Einstein equation in GR

$$\tilde{R}_c{}^a - \frac{1}{2}\delta_c{}^a \tilde{R} = 8\pi G \left(T_M\right)_c{}^a$$

- In local Sim(2) case

$$\tilde{R}_c{}^a - \frac{1}{2}\delta_c{}^a \tilde{R} = 8\pi G \left(T_{Sim(2)} + T_M\right)_c{}^a$$

- And

$$\left(T_{Sim(2)}\right)_c{}^a = \frac{1}{8\pi G}\left(\frac{1}{2}\delta_c{}^a\left(R_K + R_{CK}\right) - \left(R_{K\,c}{}^a + R_{CK\,c}{}^a\right)\right)$$

Local Sim(2) , with presence of source matter field

- In the presence of source matter field

$$\delta S_M = \int d^4 x h \left(\frac{1}{2} \delta h^c{}_a \left(T_M \right)_c{}^a + \delta A^{ab}{}_\mu \left(C_M \right)_{ab}{}^\mu \right)$$

- the full EOM for tetrad field under constrain condition :

$$\tilde{R}_c{}^a - \frac{1}{2} \delta_c{}^a \tilde{R} = 8\pi G \left(T_{Sim(2)} + T_M \right)_c{}^a$$

$$A^{10}{}_\mu - A^{31}{}_\mu = 0, \;\; A^{20}{}_\mu + A^{23}{}_\mu = 0$$

The dynamics with Local Sim(2)

- It can be viewed as the non-Einstein gravity part $T_{Sim(2)}$ may contribute effectively as dark partener.
- It should be noted that the non-Einstein gravity contribution

$$T_{Sim(2)}$$

vanishes identically if the whole space is empty. The Minkowski space is still a solution of the equation.

The Self Consistency of Sim(2) Gauge Theory

- Employing the constrain 8 eqns

$$A^{10}{}_{\mu} - A^{31}{}_{\mu} = 0, \;\; A^{20}{}_{\mu} + A^{23}{}_{\mu} = 0$$

- obtain the Sim(2) invariant theory, we make the substitution

$$\partial_{\mu} \to \mathscr{D}_{\mu} = \partial_{\mu} - i\left(A^{10}{}_{\mu} T_1 + A^{20}{}_{\mu} T_2 + A^{12}{}_{\mu} J_3 + A^{20}{}_{\mu} K_3 \right)$$

- One need to verify the Maurer–Cartan eq. holds within Sim(2) algebra and the Bianchi Identity holds for the curvature and torsion of the Sim(2) connection.
- The curvature 2-form is indeed closed within the Sim(2) algebra

$$R^{pq}{}_{ab}S_{pq} = 2\left(h_a{}^{\nu}h_b{}^{\mu} - h_a{}^{\mu}h_b{}^{\nu}\right)$$

$$\Big[\left(\partial_\mu A^{10}{}_\nu + A^{12}{}_\mu A^{20}{}_\nu - A^{10}{}_\mu A^{30}{}_\nu\right)T_1$$

$$+\left(\partial_\mu A^{20}{}_\nu - A^{12}{}_\mu A^{10}{}_\nu - A^{20}{}_\mu A^{30}{}_\nu\right)T_2$$

$$+\partial_\mu A^{12}{}_\nu J_3 + \partial_\mu A^{30}{}_\nu K_3\Big]$$

- With the contribution from torsion, one gets the Maurer–Cartan eq. on sim(2) algebra

$$\left[\mathscr{D}_a, \mathscr{D}_b\right] = T^p{}_{ab}\mathscr{D}_p + \frac{i}{2} R^{pq}{}_{ab} S_{pq}$$

- By the Jacobi Identity

$$\left[\mathscr{D}_m, \left[\mathscr{D}_n, \mathscr{D}_p\right]\right] + \left[\mathscr{D}_p, \left[\mathscr{D}_m, \mathscr{D}_n\right]\right] + \left[\mathscr{D}_n, \left[\mathscr{D}_p, \mathscr{D}_m\right]\right] = 0$$

- and Maurer–Cartan eq. and the Sim(2) constrain, the first Bianchi Identity

$$\mathscr{D}_d T^a{}_{bc} + \mathscr{D}_c T^a{}_{db} + \mathscr{D}_b T^a{}_{cd}$$

$$= R^a{}_{bcd} + R^a{}_{dbc} + R^a{}_{cdb}$$

$$+ T^e{}_{bd} T^a{}_{ec} + T^e{}_{dc} T^a{}_{eb} + T^e{}_{cb} T^a{}_{ed}$$

- and the second Bianchi Identity still hold.

$$\mathscr{D}_\nu R^a{}_{b\rho\mu} + \mathscr{D}_\mu R^a{}_{b\nu\rho} + \mathscr{D}_\rho R^a{}_{b\mu\nu} = 0$$

- Similar analysis for other VSR subgroups.

Conclusion

- All VSR gauge theories are gravity theory with non-trivial torsion in general. Lorentz violation leads to non-trivial distribution of contortion
- The non-trivial effective concribution to the energy-momentum distribution by contortion is expected to be responsible for the dark partner of the matter.

$$\tilde{R}_c{}^a - \frac{1}{2}\delta_c{}^a \tilde{R} = 8\pi G\left(T_{eff} + T_M\right)_c{}^a$$

- The Bianchi Identities imply the conservation of T_{eff}

Modified Constrain for SO(3)

- In SO(3) case. The constrain condition

$A^{10}{}_{\mu}=0$, $A^{20}{}_{\mu}=0$, $A^{30}{}_{\mu}=0$

Are too strong so that it induces the degeneracy of the dynamics.

- In the Lorenzian case, the transformation law under local Lorentz transformation is

$$A'^{a}{}_{b\mu} = \Lambda^{a}{}_{c}(x) A^{c}{}_{d\mu} \Lambda_{b}{}^{d}(x) + \Lambda^{a}{}_{c}(x) \partial_{\mu} \Lambda_{b}{}^{d}(x)$$

arXiv:1802.03502

The Modified Constrain for SO(3)

- For SO(3) $\Lambda_0{}^j(x) = 0$

$$A'^i{}_{0\mu} = \Lambda^i{}_j(x) A^j{}_{0\mu} \Lambda_0{}^0(x) + \Lambda^i{}_j(x) \partial_\mu \Lambda_0{}^j(x)$$

$$= \Lambda^i{}_j(x) A^j{}_{0\mu}$$

- The Modified Constrain for SO(3) can be

$$S_E = \frac{1}{16\pi G} \int d^4 x h \left(R^{ab}{}_{ab} + \lambda^\mu \left(\left(A^{01}{}_\mu \right)^2 + \left(A^{02}{}_\mu \right)^2 + \left(A^{03}{}_\mu \right)^2 - f_\mu{}^2 \right) \right)$$

- Where f_μ can be regarded as a spacetime indexed field which must be invariant under local transformation among tetrad field.

Accelerating Expansion of the Universe

- To construct the FRW like solution of the model

$$ds^2 = \mathrm{d}t^2 - a(t)^2\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2\right)$$

- The naïve tetrad can be chosen as

$$h^0 = \mathrm{d}t,\ h^1 = \frac{a(t)}{\sqrt{1-kr^2}}\mathrm{d}r,\ h^2 = ra(t)\mathrm{d}\theta,\ h^3 = r\sin\theta a(t)\mathrm{d}\varphi$$

- And

$$h_0 = \frac{\partial}{\partial t},\ h_1 = \frac{\sqrt{1-kr^2}}{a(t)}\frac{\partial}{\partial r},\ h_2 = \frac{1}{ra(t)}\frac{\partial}{\partial\theta},\ h_3 = \frac{1}{r\sin\theta a(t)}\frac{\partial}{\partial\varphi}$$

Accelerating Expansion of the Universe

- The field eqn for the tetrad field by $\dfrac{\delta S}{\delta h^{a}{}_{\mu}}$

$$G^{a}{}_{b} \equiv R^{a}{}_{b} - \frac{1}{2} R \delta^{a}{}_{b} = \frac{8\pi G}{c^{4}} T^{a}{}_{b}$$

- The field eqn for the connections by $\frac{\delta S}{\delta A^{a}{}_{b\mu}}$ and the decomposition of connections $A^{a}{}_{b\mu} = \Gamma^{a}{}_{b\mu} + K^{a}{}_{b\mu}$

$$K^{0}{}_{12} = K^{0}{}_{21},\ K^{1}{}_{23} = 0,\ K^{2}{}_{12} = -K^{0}{}_{10},\ K^{3}{}_{13} = -K^{0}{}_{10}$$

$$K^{0}{}_{23} = K^{0}{}_{32},\ K^{2}{}_{31} = 0,\ K^{3}{}_{23} = -K^{0}{}_{20},\ K^{1}{}_{21} = -K^{0}{}_{20}$$

$$K^{0}{}_{31} = K^{0}{}_{13},\ K^{3}{}_{12} = 0,\ K^{1}{}_{31} = -K^{0}{}_{30},\ K^{2}{}_{32} = -K^{0}{}_{30}$$

$$2K^{0}{}_{10}h_{0}{}^{\mu} + \left(K^{0}{}_{22} + K^{0}{}_{33}\right)h_{1}{}^{\mu} - \left(K^{1}{}_{20} + K^{0}{}_{21}\right)h_{2}{}^{\mu} + \left(K^{3}{}_{10} - K^{0}{}_{31}\right)h_{3}{}^{\mu}$$

$$+\lambda^{\mu}\left(A^{0}{}_{10}h^{0}{}_{\mu} + A^{0}{}_{11}h^{1}{}_{\mu} + A^{0}{}_{12}h^{2}{}_{\mu} + A^{0}{}_{13}h^{3}{}_{\mu}\right) = 0$$

$$2K^{0}{}_{20}h_{0}{}^{\mu} + \left(K^{1}{}_{20} - K^{0}{}_{12}\right)h_{1}{}^{\mu} + \left(K^{0}{}_{11} + K^{0}{}_{33}\right)h_{2}{}^{\mu} - \left(K^{2}{}_{30} + K^{0}{}_{32}\right)h_{3}{}^{\mu}$$

$$+\lambda^{\mu}\left(A^{0}{}_{20}h^{0}{}_{\mu} + A^{0}{}_{21}h^{1}{}_{\mu} + A^{0}{}_{22}h^{2}{}_{\mu} + A^{0}{}_{23}h^{3}{}_{\mu}\right) = 0$$

$$2K^{0}{}_{30}h_{0}{}^{\mu} - \left(K^{3}{}_{10} + K^{0}{}_{13}\right)h_{1}{}^{\mu} + \left(K^{2}{}_{30} - K^{0}{}_{23}\right)h_{2}{}^{\mu} + \left(K^{0}{}_{11} + K^{0}{}_{22}\right)h_{3}{}^{\mu}$$

$$+\lambda^{\mu}\left(A^{0}{}_{30}h^{0}{}_{\mu} + A^{0}{}_{31}h^{1}{}_{\mu} + A^{0}{}_{32}h^{2}{}_{\mu} + A^{0}{}_{33}h^{3}{}_{\mu}\right) = 0$$

Cosmic solution of contortion

- The ideal fluid of cosmic media demands

$$G^1{}_1 = G^2{}_2 = G^3{}_3$$

- A simple solution can be chosen as

$$K^0{}_{11} = K^0{}_{22} = K^0{}_{33} = \mathcal{K}(t)$$

- With other contortion components vanish.
- And the relation with $f_\mu(x)$ is

$$\left(f_t, f_r, f_\theta, f_\varphi\right) = \left(a(t)\mathcal{K}(t) + \dot{a}(t)\right) \cdot \left(0, \frac{1}{\sqrt{1-kr^2}}, r, r\sin\theta\right)$$

- The degree of freedom of $f_\mu(x)$ is actually 4, which hide in the choice of frames by Lorentz boost.

- Denoting $\tilde{G}^a_c$ the Einstein tensor of Levi-Civita Connection

$$G^a{}_c = \tilde{G}^a_c + 2\left(\tilde{\nabla}_{[c} K^{ab}{}_{b]} + K^a{}_{e[c} K^{eb}{}_{b]} - \frac{1}{2}\left(\tilde{\nabla}_d K^{db}{}_b + K^d{}_{e[d} K^{eb}{}_{b]}\right)\delta^a{}_c\right)$$

- The gravity field equation

$$\tilde{R}^a{}_c - \frac{1}{2}\tilde{R}\delta^a{}_c = 8\pi G\left(T + T_\Lambda\right)^a{}_c, \quad T_\Lambda{}^a{}_c = \frac{1}{8\pi G}\Lambda^a{}_c = \frac{1}{8\pi G}\left(\tilde{G}^a_c - G^a{}_c\right)$$

- The gravity field equations for the naïve tetrad of RW metric of $k = 0$

$$3\left(\mathcal{K} + \frac{\dot{a}}{a}\right)^2 = 8\pi G\rho$$

$$\left(\mathcal{K} + \frac{\dot{a}}{a}\right)^2 + 2\left(\dot{\mathcal{K}} + \mathcal{K}\frac{\dot{a}}{a} + \frac{\ddot{a}}{a}\right) = -8\pi G p$$

- And

$$[T_\Lambda]^a{}_c = Diag(\rho_\Lambda, -p_\Lambda, -p_\Lambda, -p_\Lambda)$$

$$\rho_\Lambda = -\frac{1}{8\pi G}\left(3\mathcal{K}^2 + 6\mathcal{K}\frac{\dot{a}}{a}\right), \quad p_\Lambda = \frac{1}{8\pi G}\left(\mathcal{K}^2 + 4\mathcal{K}\frac{\dot{a}}{a} + 2\dot{\mathcal{K}}\right)$$

- the modified Friedmann Equation

$$\left(\frac{\dot{a}}{a}\right)^2 + 2\mathcal{K}\frac{\dot{a}}{a} + \mathcal{K}^2 = \frac{8\pi G}{3}\rho$$

$$\ddot{a} = -4\pi G \cdot a\left(p + \frac{\rho}{3}\right) - \frac{\mathrm{d}}{\mathrm{d}t}(a\mathcal{K})$$

- In ΛCDM

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{1}{3}\Lambda = \frac{8\pi G}{3}\rho$$

$$\ddot{a} = -4\pi G \cdot a\left(p + \frac{\rho}{3}\right) + \frac{1}{3}a\Lambda$$

- Two conditions.

$$2\mathcal{K}(t)\frac{\dot{a}(t)}{a(t)}+\mathcal{K}(t)^2=-\frac{1}{3}\Lambda$$

- And

$$-\frac{\mathrm{d}}{\mathrm{d}t}\left(a\mathcal{K}\right)=\frac{1}{3}a\Lambda$$

- Choose one as the initial value condition at present $t_0=H_0^{-1}$

$$2\mathcal{K}(t_0)\frac{\dot{a}(t_0)}{a(t_0)}+\mathcal{K}(t_0)^2=-\frac{1}{3}\Lambda$$

- or $\mathcal{K}(t_0)=H_0\left(\pm\sqrt{1-\Omega_{\Lambda 0}}-1\right)\approx -0.465H_0, -1.535H_0$

- The condition for accelerating expansion

$$4\pi G\cdot a\left(p+\frac{\rho}{3}\right)+\frac{\mathrm{d}}{\mathrm{d}t}\left(a\mathcal{K}\right)<0$$

- Case A: $-\frac{\mathrm{d}}{\mathrm{d}t}\left(a\mathcal{K}\right)=\frac{1}{3}a\Lambda$

- A1: $\mathcal{K}(t_0)=-0.465H_0$ A2: $\mathcal{K}(t_0)=-1.535H_0$
- Case B:

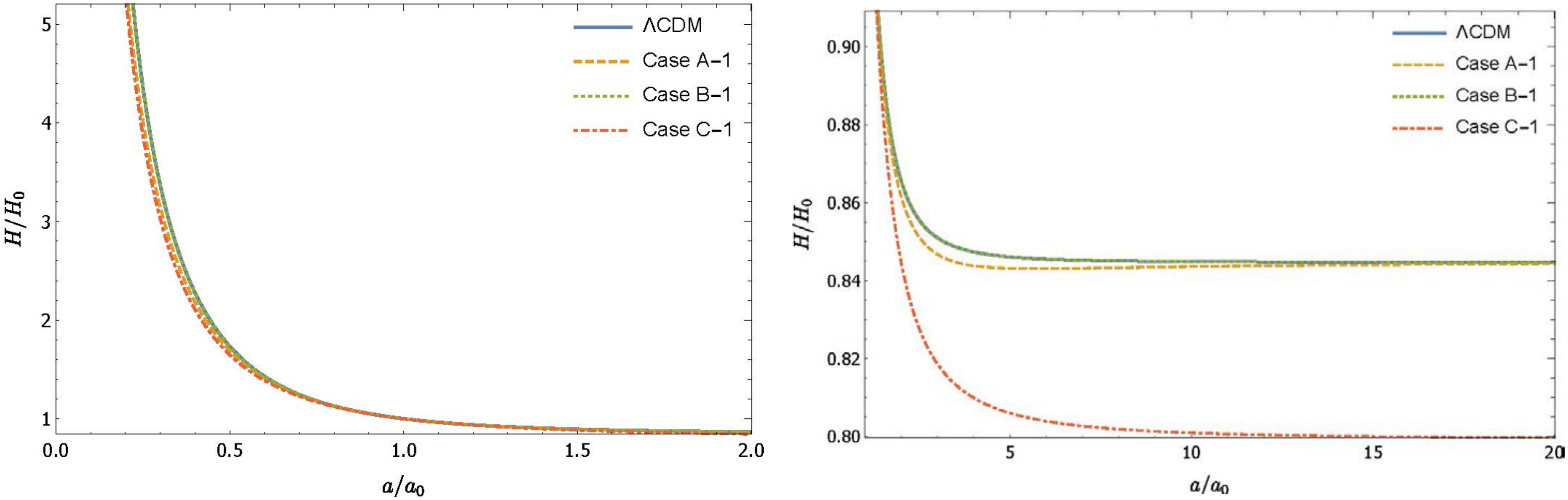
$$\frac{\ddot{a}}{a}+\frac{3w+1}{2}\left(\frac{\dot{a}}{a}\right)^2=\frac{w+1}{2}\Lambda$$

$$\frac{\ddot{a}}{a}+\frac{3w+1}{2}\left(\frac{\dot{a}}{a}\right)^2=-\dot{\mathcal{K}}-\frac{3w+1}{2}\mathcal{K}^2-(3w+2)\frac{\dot{a}}{a}\mathcal{K}$$

$$\dot{\mathcal{K}}+\frac{3w+1}{2}\mathcal{K}^2+(3w+2)\frac{\dot{a}}{a}\mathcal{K}=-\frac{w+1}{2}\Lambda$$

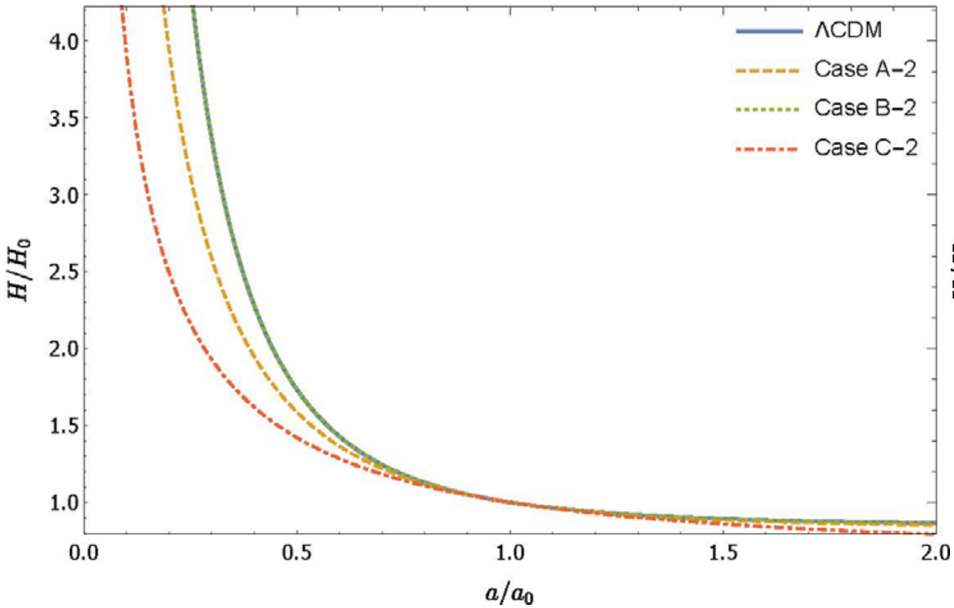
- B1: $\mathcal{K}(t_0)=-0.465H_0$ B2: $\mathcal{K}(t_0)=-1.535H_0$

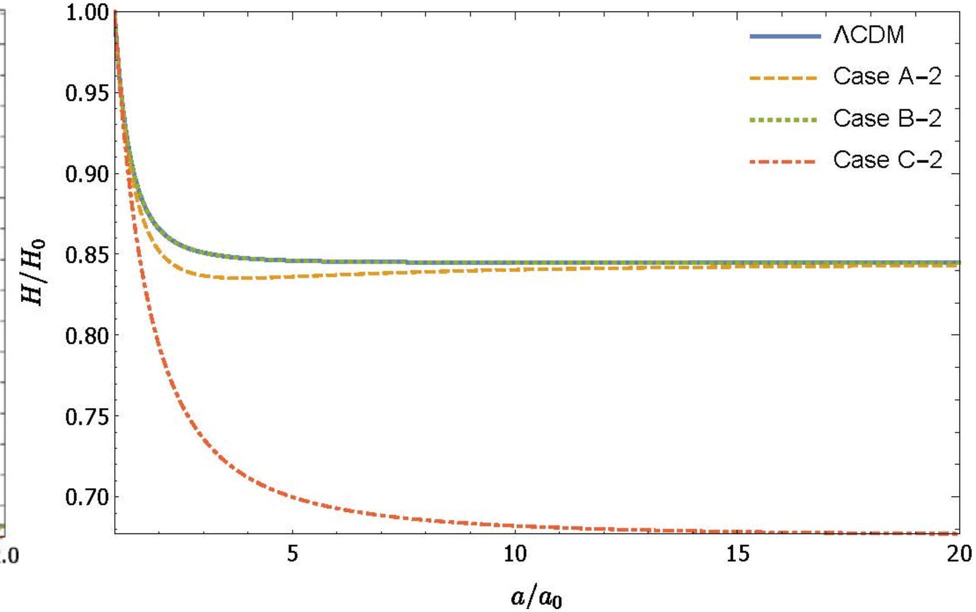
- To mimic the contribution by cosmological constant to cosmology with one by contortion, we make an alternative proposition by requiring
- $T_{\Lambda}{}^{a}{}_{b} \propto \delta^{a}{}_{b}$ or $T_{\Lambda}{}^{0}{}_{0} = T_{\Lambda}{}^{1}{}_{1} = T_{\Lambda}{}^{2}{}_{2} = T_{\Lambda}{}^{3}{}_{3}$
- which leads to the equation satisfied by contortion
- Case C: $\dot{\mathcal{K}} = \mathcal{K}^2 + \mathcal{K}\frac{\dot{a}}{a}$
- C1: $\mathcal{K}(t_0) = -0.465 H_0$ C2: $\mathcal{K}(t_0) = -1.535 H_0$



- H's evolution from a□0 to $a=2a_0$ and from a□0 to $a=20a_0$

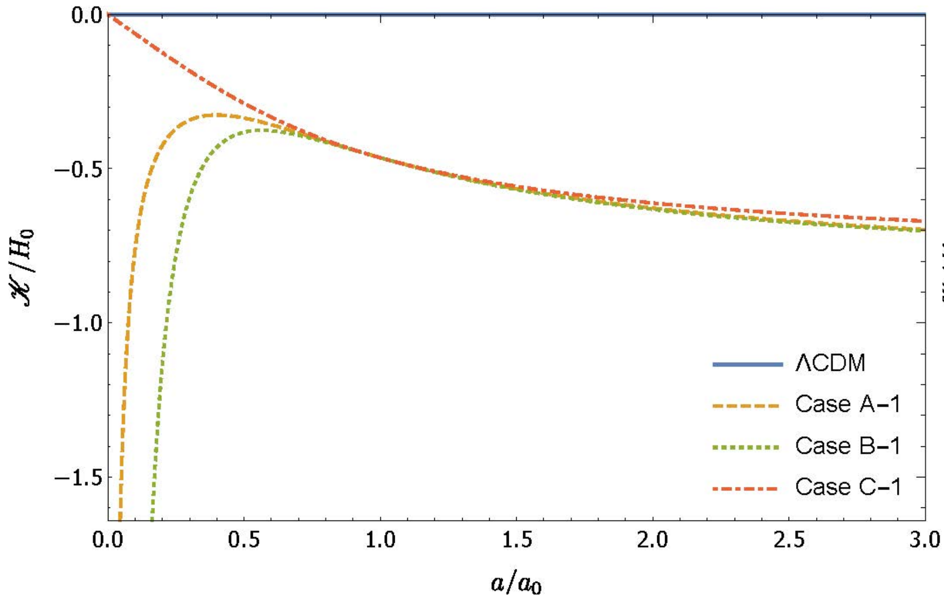
- Evolutions of H between LSLV model and CDM model, with initial condition $\mathcal{K}(t_0)=-0.465H_0$

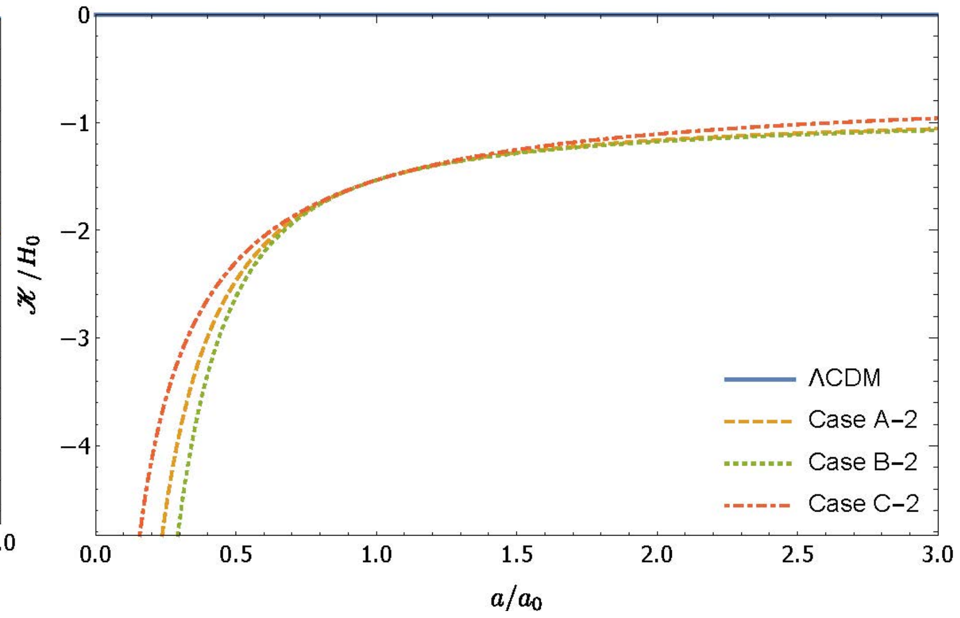




- H's evolution from a □ 0 to $a = 2a_0$ and from a □ 0 to $a = 20a_0$

- Evolutions of H between LSLV model and CDM model, with initial condition $\mathcal{K}(t_0) = -1.535H_0$



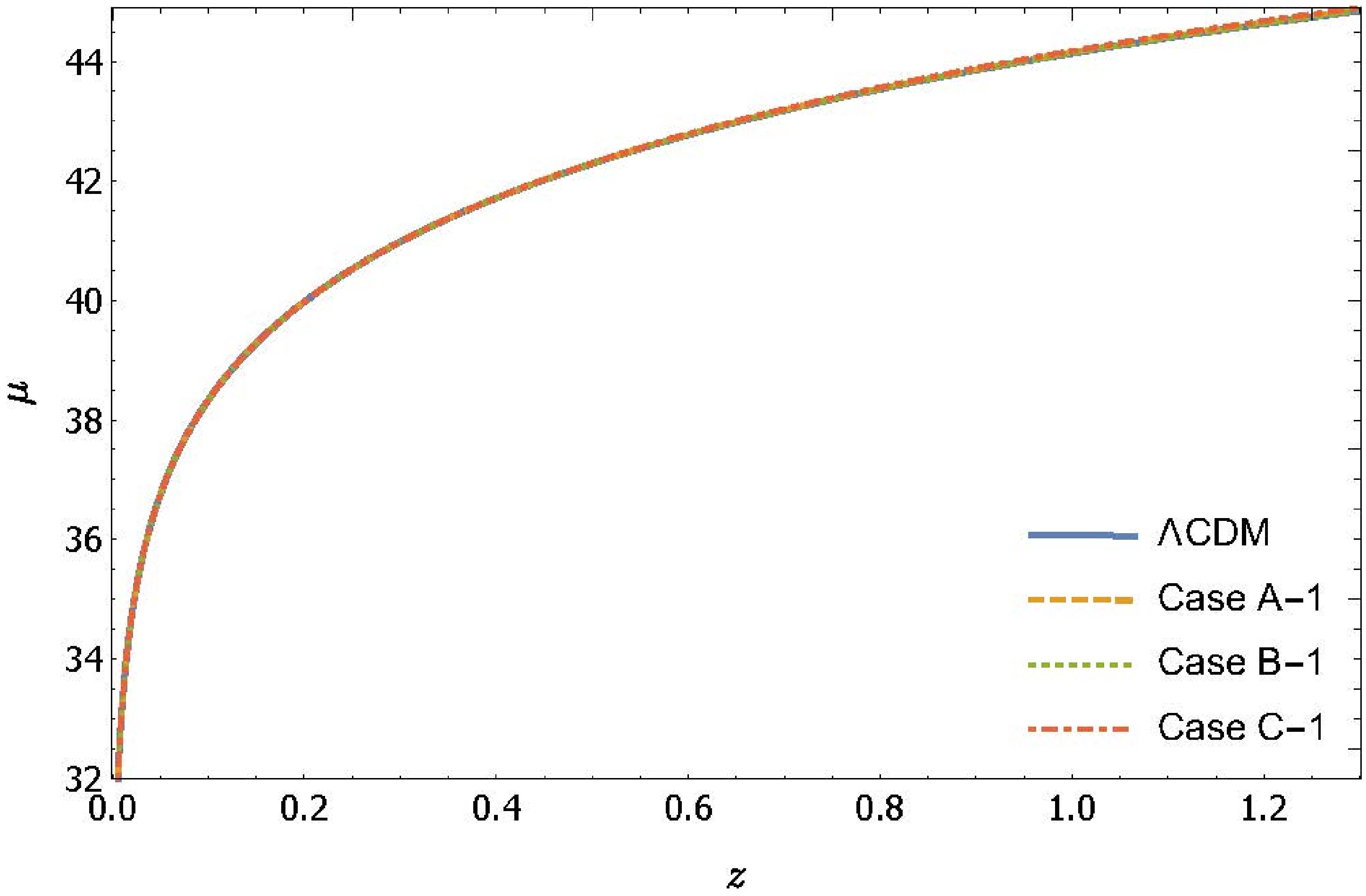


- Evolutions of $\mathcal{K}$ with $\mathcal{K}(t_0) = -0.465H_0$ and $\mathcal{K}(t_0) = -1.535H_0$

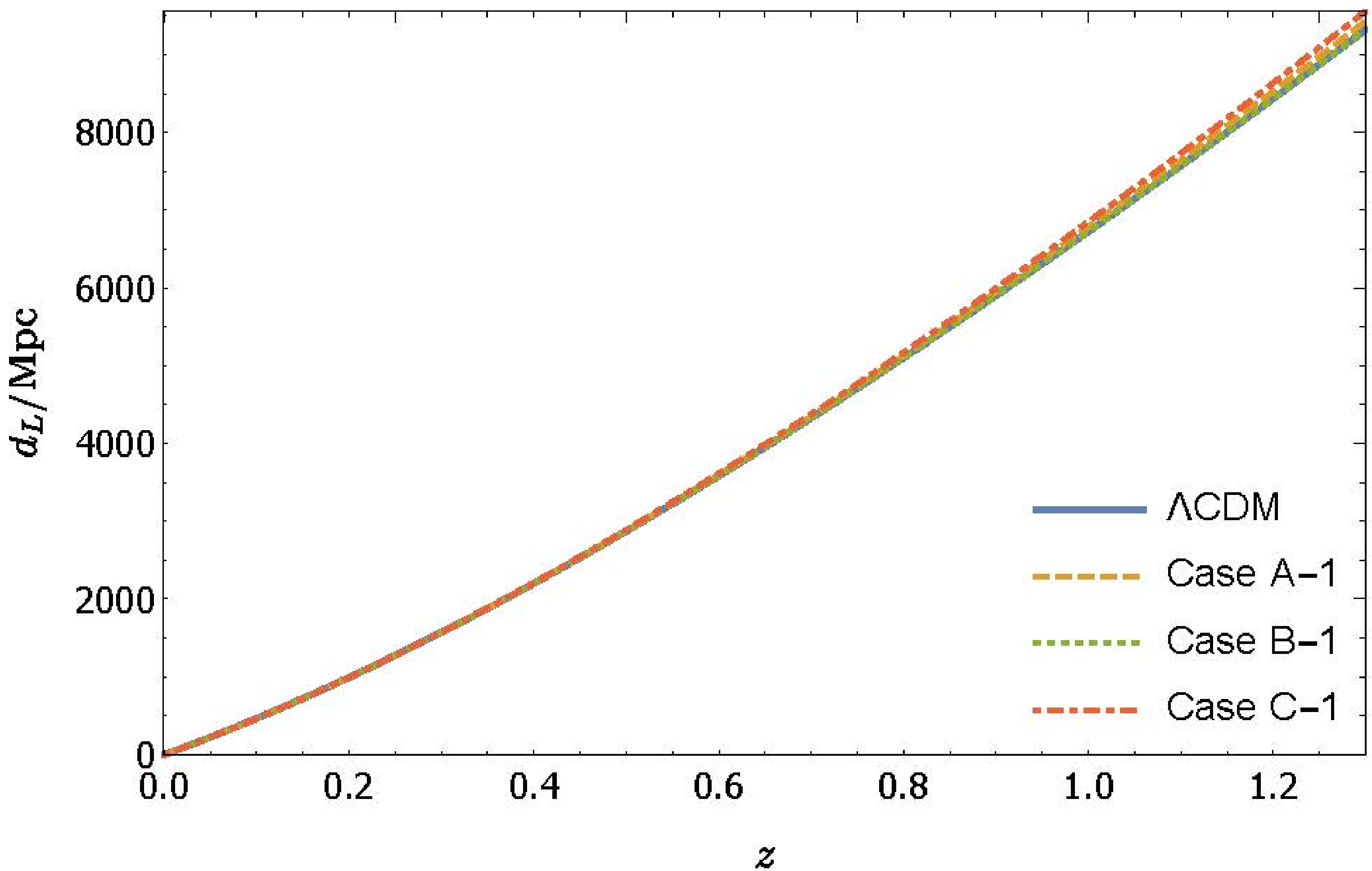
- Evolutions of $\mathcal{K}$ from $a \to 0$ to $a = 3a_0$

- Under the condition that contortion don't vanish, curve $x = x(\beta)$ that satisfies $\nabla_{\partial_\beta}\partial_\beta = 0$ is the autoparallel curve, while one satisfies $\tilde{\nabla}_{\partial_\beta}\partial_\beta = 0$ with the Levi-Civita connection is the geodesic.
- Due to Hamiltonian Principle, particle moves along the geodesic rather than autoparallel curve.
- The formula for the redshift remains unchanged as in the Lorentzian invariant case. $1 + z = \frac{a_0}{a}$
- The dependence of luminosity distance d_L with redshift and Hubble constant.

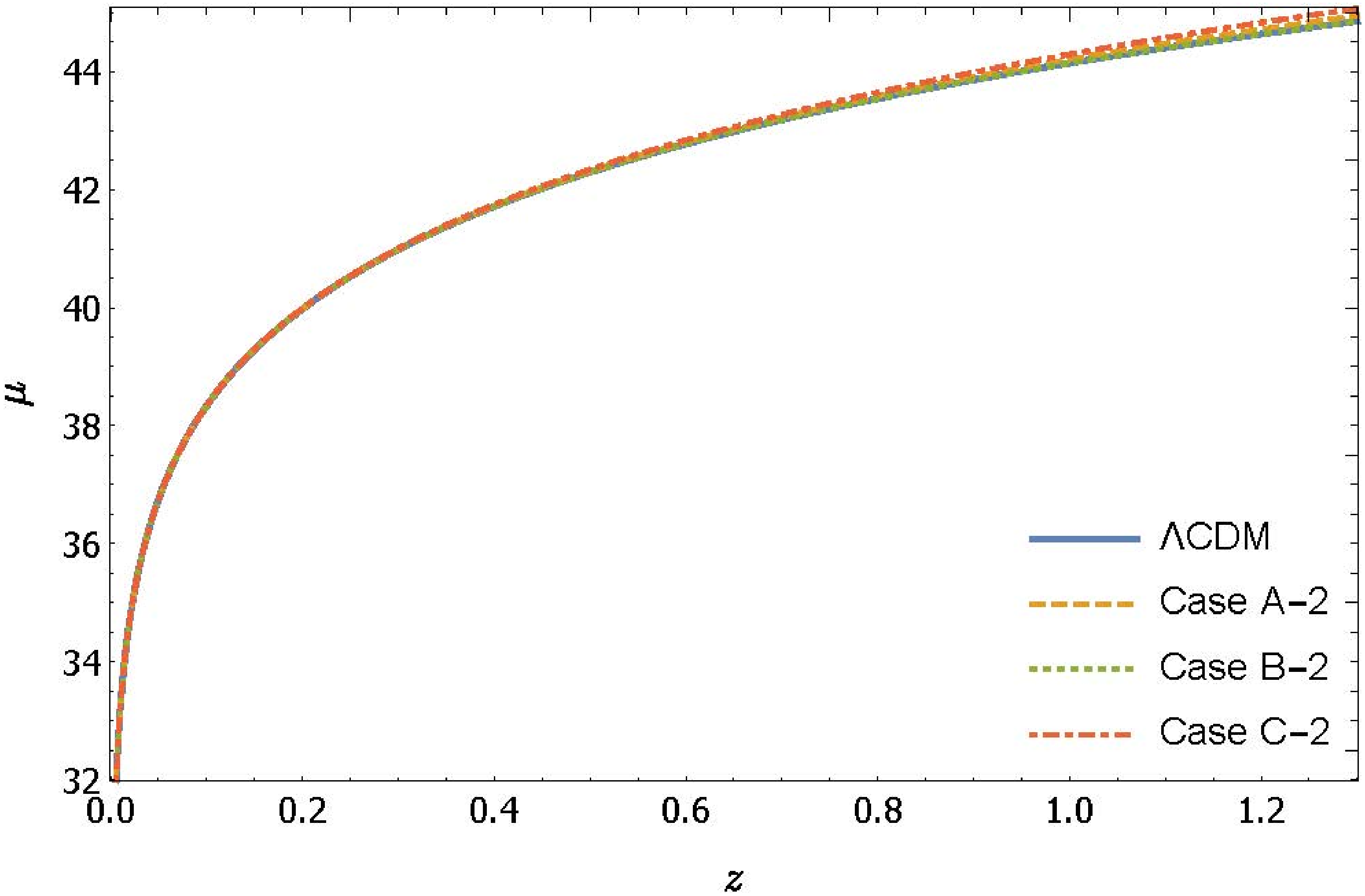
$$H(z) = \left(\frac{\mathrm{d}}{\mathrm{dz}} \frac{d_L}{1+z} \right)^{-1}, \qquad \frac{\mathrm{d}t}{\mathrm{dz}} = -\frac{1}{1+z} \frac{\mathrm{d}}{\mathrm{dz}} \left(\frac{d_L}{1+z} \right)$$



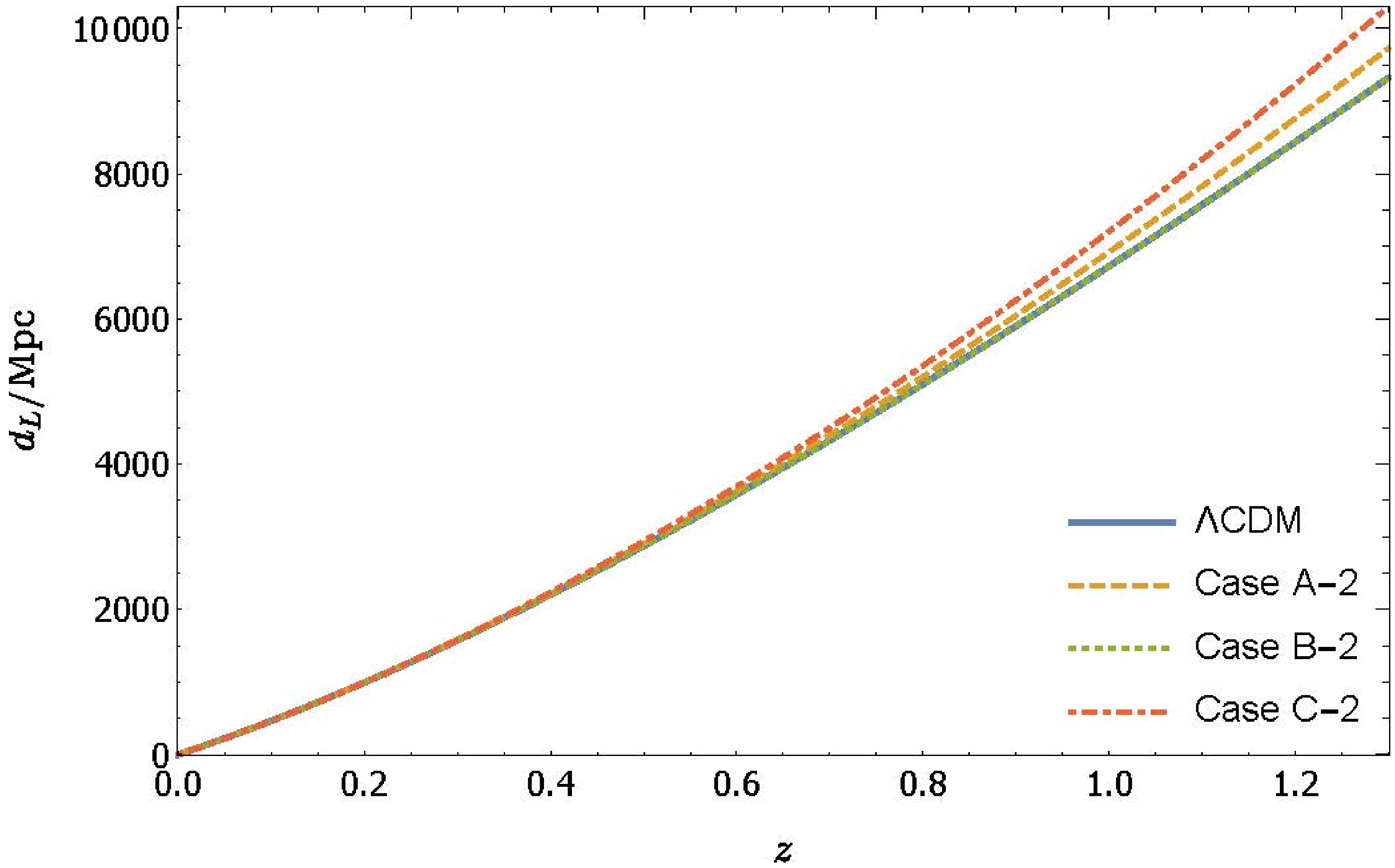
- Comparison of distance magnitudes. Curves of different models are hard to distinguish
- $\mathcal{K}(t_0) = -0.465 H_0$ case



- Comparison of luminosity distance of these models
- $\mathcal{K}(t_0) = -0.465 H_0$ case



- Comparison of distance magnitudes. Divergences of different models are bigger than $\mathcal{K}(t_0) = -0.465H_0$ case
- $\mathcal{K}(t_0) = -1.535H_0$ case



- Divergences of luminosity distance are obvious bigger than $\mathcal{K}(t_0) = -0.465H_0$ case
- $\mathcal{K}(t_0) = -1.535H_0$ case

Summary

- The very special relativity symmetry, is illustrated as an example of large scale local symmetry and its corresponding gravity theory is constructed.
- All VSR gauge theories are gravity theory with non-trivial torsion in general. The contribution by contortion is expected responsible to the dark effect.
- For the modified local SO(3) case，dark energy effect is shown to be an emerge effect from large scale Lorentz violation.
- The Large Scale Lorentz Violation may originate from quantum gravity and hence dark energy or the accelerating expansion can be traced to quantum gravity effect.
- The quantum gravity remneant in the late universe

Outlook

- 1) The field theory realization of our guess on quantum gravity inducing large scale Lorentz violation.
- 2)The model independent relation between large scale Lorentz violation and the dark energy
- 3) The verification with other framework such as Teleparallele gravity.

THANKS!

Teleparallel Gravity Framework

Sim(2)case:

$$L = \frac{c^4 h}{16\pi G}\left(\frac{1}{4}T^a{}_{bc}T_a^{bc} + \frac{1}{2}T^a{}_{bc}T^{cb}{}_a - T^a{}_{ba}T^{cb}{}_c\right)$$

The equations of motion for connections $A^a{}_{bc}$ give

$$K^2_{10} = \frac{1}{2}(f^2{}_{10} - f^0{}_{12} - f^1{}_{20}); K^2_{11} = \frac{1}{2}(f^1{}_{12} - f^1{}_{21}); K^2_{12} = f^2{}_{12}; K^2_{13} = \frac{1}{2}(f^2{}_{13} + f^3{}_{12} - f^1{}_{23})$$

$$K^3_{00} = f^0{}_{30}; K^3_{01} = \frac{1}{2}(f^0{}_{31} - f^1{}_{30} + f^3{}_{01}); K^3_{02} = \frac{1}{2}(f^0{}_{32} - f^2{}_{30} + f^3{}_{02}); K^3_{03} = f^3{}_{03}$$

And

$$T^2{}_{12} + T^3{}_{13} + T^0{}_{31} = 0; T^0{}_{30} + T^2{}_{32} + T^2{}_{02} + T^3{}_{03} = 0; T^2{}_{01} + T^2{}_{31} = 0; T^3{}_{01} + T^0{}_{10} + T^2{}_{12} = 0$$

$$T^1{}_{21} + T^3{}_{23} + T^0{}_{32} = 0; T^1{}_{02} + T^1{}_{32} = 0; T^0{}_{30} + T^1{}_{31} + T^1{}_{01} + T^3{}_{03} = 0; T^3{}_{02} + T^0{}_{20} + T^1{}_{21} = 0$$

Where $T^a{}_{bc}$ are the torsion tensor.

- These equations can be finally solved the contortion with the expression of tetrad

$$K^2_{00}=\frac{1}{2}(f^0{}_{20}-f^1{}_{21}+f^3{}_{20});K^2_{01}=\frac{1}{2}(f^0{}_{21}-f^1{}_{20}-f^2{}_{31});K^2_{02}=-\frac{1}{2}(f^3{}_{03}+f^0{}_{30}+f^2{}_{20}+f^2{}_{32})$$

$$K^2_{03}=\frac{1}{2}(f^1{}_{21}-f^3{}_{32}-f^2{}_{30}+f^3{}_{02});K^1_{00}=\frac{1}{2}(f^0{}_{10}+f^3{}_{10}-f^2{}_{12})K^1_{01}=-\frac{1}{2}(f^1{}_{10}+f^1{}_{31}+f^0{}_{30}+f^3{}_{03})$$

$$K^1_{02}=-\frac{1}{2}(f^2{}_{10}+f^0{}_{21}+f^1{}_{32});K^1_{03}=\frac{1}{2}(f^2{}_{12}-f^3{}_{31}-f^1{}_{30}+f^3{}_{01});K^3_{10}=\frac{1}{2}(f^0{}_{10}-f^1{}_{30}+f^0{}_{31}+f^2{}_{12})$$

$$K^3_{11}=\frac{1}{2}(f^0{}_{30}+f^3{}_{03}-f^1{}_{10}-f^1{}_{31});K^3_{12}=\frac{1}{2}(f^2{}_{13}-f^1{}_{20}+f^3{}_{12});K^3_{13}=-\frac{1}{2}(f^2{}_{12}+f^0{}_{31}+f^3{}_{31})$$

$$K^3_{20}=\frac{1}{2}(f^0{}_{20}+f^0{}_{32}+f^1{}_{21}-f^2{}_{30});K^3_{21}=\frac{1}{2}(f^1{}_{23}-f^2{}_{10}-f^3{}_{12});K^3_{22}=\frac{1}{2}(f^0{}_{30}-f^2{}_{20}-f^2{}_{32}+f^3{}_{03})$$

$$K^3_{23}=-\frac{1}{2}(f^1{}_{21}+f^0{}_{32}+f^3{}_{32})$$

- The equation of motion for the tetrad

$$\partial_\sigma(hS_a{}^{\rho\sigma})-khJ_a{}^\rho=0$$

- Where

$$S_a{}^{\rho\sigma}=K^{\rho\sigma}{}_a-h_a{}^\sigma T^{\theta\rho}{}_\theta+h_a{}^\rho T^{\theta\sigma}{}_\theta;J_a{}^\rho=\frac{1}{k}h_a{}^\mu S_c{}^{\nu\rho}T^c{}_{\nu\mu}-\frac{h_a{}^\rho}{h}L+\frac{1}{k}A^c{}_{a\sigma}S_c{}^{\rho\sigma}$$

- To compare with the GR gravity equation

$$\partial_\sigma (hS_a{}^{\rho\sigma}) - khJ_a{}^\rho = \overset{\circ}{R}_a{}^\rho - \frac{1}{2} h_a^\rho \overset{\circ}{R} + M_a{}^\rho + N_a{}^\rho$$

- Where

$$M_a{}^\rho = \partial_\sigma \{\frac{h}{2}[D^\sigma{}_a{}^\rho - D^\rho{}_a{}^\sigma - D^{\sigma\rho}{}_a + D^{\rho\sigma}{}_a + \eta_{ba} D^{b\sigma\rho} - \eta_{ba} D^{b\rho\sigma}$$

$$+2(h_a{}^\rho D^b{}_b{}^\sigma + h_a^\sigma D^b{}_b{}^\rho - 2h_a{}^\rho D^{b\sigma}{}_b - 2h_a{}^\sigma D^{b\rho}{}_b)]\}$$

$$N_a{}^\rho = -hh_a{}^\mu S_c{}^{\rho\nu} h_\nu^b D^c{}_{b\mu} + \frac{1}{2} h(h_a{}^\mu T^c{}_{\sigma\mu} - A^c{}_{a\sigma})$$

$$[D^\sigma{}_a{}^\rho - D^\rho{}_a{}^\sigma - D^{\sigma\rho}{}_a + D^{\rho\sigma}{}_a + \eta_{ba} D^{b\sigma\rho} - \eta_{ba} D^{b\rho\sigma} + 2(h_a{}^\rho D^b{}_b{}^\sigma + h_a^\sigma D^b{}_b{}^\rho - 2h_a{}^\rho D^{b\sigma}{}_b - 2h_a{}^\sigma D^{b\rho}{}_b)]$$

$$+\frac{h}{8} h_a{}^\rho \{[(\eta_{cb} T^{c\tau}{}_\mu (D^{b\mu}{}_\tau - D^b{}_\tau{}^\mu) + (T_c{}^{\mu b} - T_c{}^{b\mu}) D^c{}_{b\mu}] + 2[T^c{}_{\mu\nu}(D^\nu{}_c{}^\mu - D^{\nu\mu}{}_c) + (T^{b\mu}{}_c - T^{\mu b}{}_c) D^c{}_{b\mu}]\}$$

- $\overset{\circ}{R}{}^a{}_b$ is the Ricci curvature with Levi-Civita connection and $D^a{}_{b\mu}$ is the deviation of connection from the Weitzenbock ones $\dot{A}^a_{b\mu}$

$$D^a{}_{b\mu} = A^a{}_{b\mu} - \dot{A}^a{}_{b\mu}$$

- The SO(3) case, take the constrain condition

$$(A_{0\mu}^1)^2 + (A_{0\mu}^2)^2 + (A_{0\mu}^3)^2 = f_\mu{}^2$$

- The constrain condition and equations of motion for the connection reduce to

$$-\lambda^1 = \frac{K_{02}^2 + K_{03}^3}{A_{01}^1} = \frac{K_{20}^1}{K_{01}^2} - 1 = \frac{K_{30}^1}{K_{01}^3} - 1$$

$$-\lambda^2 = \frac{K_{01}^1 + K_{03}^3}{A_{02}^2} = \frac{K_{10}^2}{K_{02}^1} - 1 = \frac{K_{30}^2}{K_{02}^3} - 1$$

$$-\lambda^3 = \frac{K_{01}^1 + K_{02}^2}{A_{03}^3} = \frac{K_{20}^3}{K_{03}^2} - 1 = \frac{K_{10}^3}{K_{03}^1} - 1$$

$$K_{12}^2 + K_{13}^3 = 0$$

$$K_{23}^3 - K_{11}^2 = 0$$

$$K_{11}^3 + K_{22}^3 = 0$$

$$K_{01}^2 - K_{02}^1 = 0; K_{21}^3 - K_{12}^3 = 0; K_{01}^3 - K_{03}^1 = 0; K_{02}^3 - K_{03}^2 = 0$$

$$K_{12}^3 - K_{13}^2 = 0; K_{00}^3 - K_{11}^3 = 0; K_{00}^2 - K_{11}^2 = 0; K_{00}^3 - K_{22}^3 = 0$$

$$K_{23}^3 + K_{00}^2 = 0; K_{00}^1 + K_{13}^3 = 0; K_{00}^1 + K_{12}^2 = 0; K_{21}^3 + K_{13}^2 = 0$$

- Unlike the general relativity, the Teleparallel Gravity framework approach is generally inequivalent to the Hilbert-Einstein framework approach in the Lorentz violation case .