Basic BES PWA procedure (Part A)

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2009.05.25

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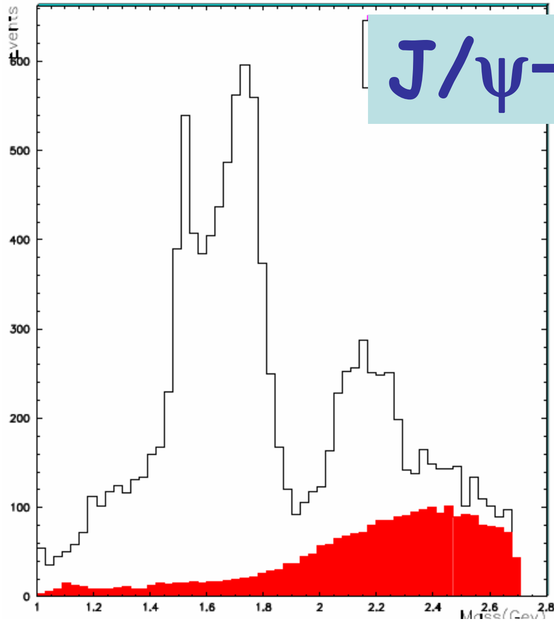
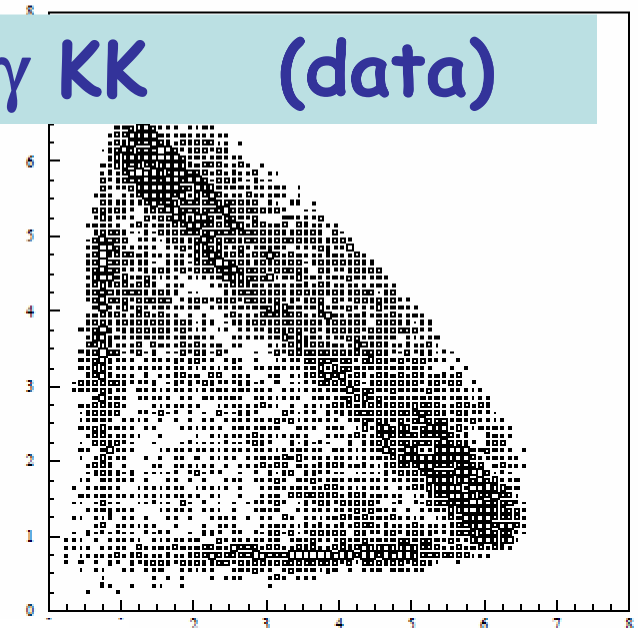
Why PWA



$J/\psi \rightarrow \gamma$ KK (data)



Left: M(KK), red is bkg estimated.

right: dalitz plot $M^2(\gamma K+)$ vs. $M^2(\gamma K-)$

Partial Wave Analysis is an important tool for determining resonance properties (like mass, width, branching fraction, spin and parity) in hadron spectroscopy.

PWA can deal with the interference of resonances.

Input

- **The four momentum of all final states.**
  - **Input files:**
    - **data.dat contains the four momentum of γ K+, K- from experimental data.**
    - **bg1.dat contains the four momentum of γ K+, K- from $\rho\ \pi$ background channel.**
    - **bg2.dat contains the four momentum of γ K+, K- from others background channel.**
    - **mc.dat contains the four momentum of γ K+, K- from phase space MC data.**

Construct likelihood function

ξ is the physical quantity measured by experiment

$\omega(\xi)$ is the probability density to produce it $\omega(\xi) = \frac{d\sigma}{d\Phi_i}$

$\varepsilon(\xi)$ is the efficiency， $P(\xi)$ is the probability to observe it ,

and $P(\xi)$ is defined as: $P(\xi) = \frac{\omega(\xi)\epsilon(\xi)}{\int d\xi \omega(\xi)\epsilon(\xi)}$

For n events, the probability density:

$$P(\xi_i, \xi_2, \cdots, \xi_n) = \prod_{i=1}^n P(\xi_i) = \prod_{i=1}^n \frac{\omega(\xi_i)\epsilon(\xi_i)}{\int d\xi \omega(\xi)\epsilon(\xi)}$$

$$\ln P(\xi_i, \xi_2, \cdots, \xi_n) = \sum_{i=1}^n \ln\left(\frac{\omega(\xi_i)}{\int d\xi \omega(\xi)\epsilon(\xi)}\right) + \underbrace{\sum_{i=1}^n \ln \epsilon(\xi_i)}_{constant}$$

Definition of likelihood function: $\mathcal{L} = P(\xi_i, \xi_2, \cdots, \xi_n)$

The logarithm of it: $\ln \mathcal{L} = \ln P(\xi_i, \xi_2, \cdots, \xi_n) = \sum_{i=1}^n \ln\left(\frac{d\sigma}{d\Phi_i}/\sigma\right)$

The total observed cross section: $\sigma = \int d\xi \omega(\xi)\epsilon(\xi)$

cross section

The total observed cross section is calculated by MC integration:

$$\sigma = \int d\xi \omega(\xi)\epsilon(\xi) = \sum_i \Delta\xi_i \omega(\xi_i)\epsilon(\xi_i)$$
$$= \frac{1}{N_{gen}} \sum_i N_{gen} \Delta\xi_i \omega(\xi_i)\epsilon(\xi_i)$$
$$= \frac{1}{N_{gen}} \sum_i N_{\xi_i} \omega(\xi_i) = \frac{1}{N_{gen}} \sum_{i=1}^{N_{MC}} \omega(\xi_i)$$

The total observed cross section:

$$\sigma = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \left(\frac{d\sigma}{d\Phi}\right) = \frac{1}{N_{MC}} \sum_{i=1}^{N_{MC}} \left(\sum_j A_j\right)^2$$

The differential cross section: $\frac{d\sigma}{d\Phi_i} = |\sum_j A_j|^2$

The amplitude: $A = \sum_j A_j = \psi_\mu(m) A^\mu = \psi_\mu(m) \sum_j (\Lambda_j U_j^\mu)$

$\psi_\mu(m)$ is the polarization vector; m=1,2
U_j^μ is the j-th partial wave amplitude constructed in the covariant tensor formalism, in which BW appears;

Λ_j =a $\cdot$ e^{ib} is the complex parameter for j-th partial wave.

Breit-Wigner propagator

According to the quantum number conservation, all the possible resonance can be listed.

The decay process is modeled by a phase space contribution plus several cascade two-body decays:

$$J/\psi \rightarrow \gamma f_0, f_0 \rightarrow K^+K^-;$$
$$J/\psi \rightarrow \gamma f_2, f_2 \rightarrow K^+K^-;$$
$$J/\psi \rightarrow \gamma f_4, f_4 \rightarrow K^+K^-;$$

Resonances are parameterized by BWs:

$$BW(R) = \frac{1}{s - M_R^2 + iM_R\Gamma_R}$$

By now, we notice that to describe one resonance, 4 parameters are needed (Magnitude, phase angle, mass, width)

$$\frac{d\sigma}{d\Phi_n} = \frac{(2\pi)^4}{2M_\psi} \cdot \frac{1}{2} \sum_{m=1}^{2} \psi_\mu(m) A^\mu \psi^*_{\mu'}(m) A^{*\mu'} \tag{3}$$

where M_ψ is the mass of ψ and $d\Phi_n$ is the standard element of n-body phase space given by

$$d\Phi_n(p_\psi; p_1, \cdots p_n) = \delta^4(p_\psi - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i}. \tag{4}$$

Note that

$$\sum_{m=1}^{2} \psi_\mu(m) \psi^*_{\mu'}(m) = \delta_{\mu\mu'}(\delta_{\mu 1} + \delta_{\mu 2}), \tag{5}$$

so we have

$$\frac{d\sigma}{d\Phi_n} = \frac{1}{2} \sum_{\mu=1}^{2} A^\mu A^{*\mu} = \frac{1}{2} \sum_{i,j} \Lambda_i \Lambda_j^* \sum_{\mu=1}^{2} U_i^\mu U_j^{*\mu} \equiv \sum_{i,j} P_{ij} \cdot F_{ij} \tag{6}$$

where

$$P_{ij} = P^*_{ji} = \Lambda_i \Lambda^*_j, \tag{7}$$

$$F_{ij} = F^*_{ji} = \frac{1}{2} \sum_{\mu=1}^{2} U_i^\mu U_j^{*\mu}. \tag{8}$$

Zou B.S. et al Eur.Phys.J.A16:537-547,2003.

The likelihood calculation

Most memory consuming

Coupling parameters (determined by fit)

Partial wave amplitudes (precalculated)

$$\ln L = \sum_{n=1}^{N_{data}} \ln\left(\frac{d\sigma}{d\Phi} / \sigma\right) = \sum_{n=1}^{N_{data}} \ln\left(\frac{\sum_{i,j} PA_{ij} \cdot FU_{ij}}{\sum_{i,j} PA_{ij} \left(\frac{1}{N_{gen}} \sum_{k=1}^{N_{MC}} FU_{ij}\right)}\right)$$

Sum over data events

Independent of fit parameters (precalculated)

Most time consuming

background subtraction

Since the log-likelihood is a sum over the number of data events, so background can be subtracted by subtracting log-likelihood value of background events.

$$
\begin{aligned}
N_{data} &= N_{signal} + N_{bkg} \\
-\ln \mathcal{L}_{data} &= -\ln \mathcal{L}_{signal} - \ln \mathcal{L}_{bkg} \\
-\ln \mathcal{L}_{signal} &= -\ln \mathcal{L}_{data} - (-\ln \mathcal{L}_{bkg}) \\
&= \sum_{i=1}^{N_{data}} \left(\frac{d\sigma}{d\Phi_i}/\sigma\right) - \sum_{i=1}^{N_{bkg}} \left(\frac{d\sigma}{d\Phi_i}/\sigma\right)
\end{aligned}
$$

Maximum likelihood PWA fit

- Likelihood has been constructed
- Instead of maximum likelihood fit, in fact, s=-ln L is used, and minimize s is used to get the parameters fitted.
- We use the minimization package provided by CERNLIB(D510): FUMILI .

- There are 4 parameters need to be determined for each resonance: magnitude, phase angle, mass and width.
- Fitting (magnitude, phase angle)
- Scanning (mass and width)

statistical significance

- 信号的统计显著性是物理学家对“观测到了信号事例”这一判断的定量化的表征。
- 零假设**H0:** 通常表示为观察到实验现象可以只用已知的现象或者本底函数圆满地描述。
- 备择假设**H1:** 表示观察到的实验现象需要用已知的现象或者本底函数，加上未知的、待寻找的新信号过程的贡献才能完整的描述。
- **H0(**似然函数**L(b))** 是对**H1(**似然函数**L(s+b))**中**k**个待定参数中**r**个参数加以固定，或者加上了**r**个约束条件。
- 似然比统计量：$\lambda = L_m(b)/L_m(s+b)$
- 当子样容量很大时，**$u=-2\ln\lambda$** 渐进地服从χ^2**(r)** 分布。
- 利用似然比统计量计算信号统计显著性**S**的表达式:

$$\int_{-S}^{+S} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 1 - P(u_{obs}) = \int_0^{u_{obs}} \chi^2(u;r) du$$

《实验物理中的概率与统计》**p429**，朱永生

statistical significance

For a resonance X,

$\mathcal{L}_0$ **is the likelihood when X is NOT included in the fit;**

$\mathcal{L}_1$ **is the likelihood when X is included in the fit;**

Δndf **is the number of changed degree of freedom which equals the free parameters of X;**

u **equals the twice difference of the log-likelihood.**

$$u = -2\ln\mathcal{L}_0 - (-2\ln\mathcal{L}_1) \quad obeying \quad \chi^2(u, \Delta ndf)$$

Using functions provided by CERNLIB(G100, G105), the statistical significance of X will be given by:

$$\boxed{prob(u, \Delta ndf)}$$

$$\boxed{\mathrm{S}=dgausn(1-0.5*prob(u,\Delta ndf))}$$

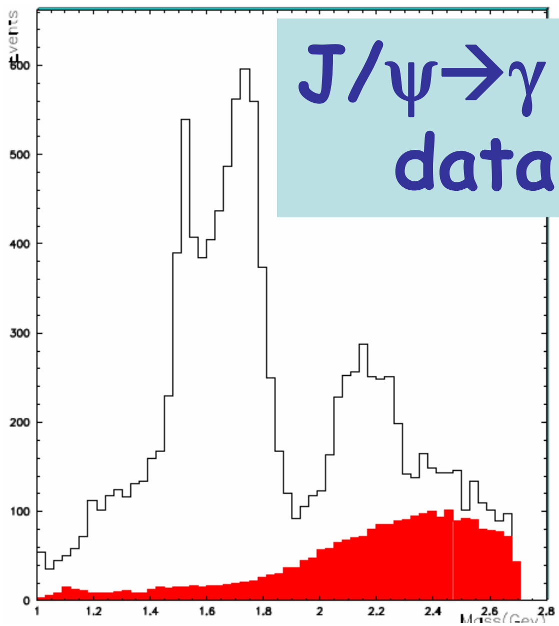
Input : u, Δndf, Output : S

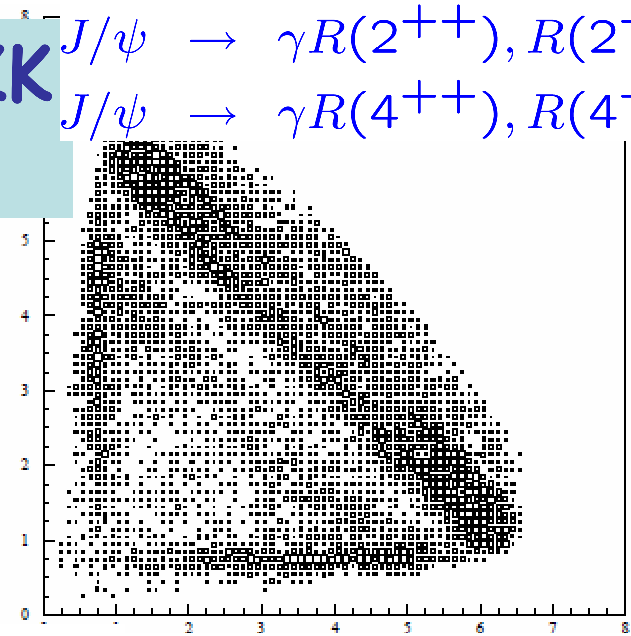
Output

- By the fitting, we can get the fitted parameters (magnitude, phase angle, mass, width).
- And according them, we can get the contribution (weight) of each resonance and the interference between all the intermediate states.
- By projecting the MC phase space 4-momentum to various plot, we can see the fitted result directly.
- We can calculate the branching ratio of each decay mode with consideration of the interference with other modes.

Example of implementation PWA $J/\psi \to \gamma K K$

$$J/\psi \rightarrow \gamma R(0^{++}), R(0^{++}) \rightarrow K^+K^-;$$
$$J/\psi \rightarrow \gamma R(2^{++}), R(2^{++}) \rightarrow K^+K^-;$$
$$J/\psi \rightarrow \gamma R(4^{++}), R(4^{++}) \rightarrow K^+K^-;$$





Left: M(KK),
red is bkg estimated.

right: dalitz plot
$M^2(\gamma K+)$ vs. $M^2(\gamma K-)$

The result of one combination ("best" solution)

In this combination these resonances are included.

$$
\begin{aligned}
J/\psi &\rightarrow \gamma f_2(1270) \\
&\rightarrow \gamma f_0(1500) \\
&\rightarrow \gamma f_2(1525) \\
&\rightarrow \gamma f_0(1710) \\
&\rightarrow \gamma f_0(2100) \\
&\rightarrow \gamma f_2(2150) \\
&\rightarrow \gamma f_2(2230) \\
&\rightarrow \gamma + broad0^{++}
\end{aligned}
$$

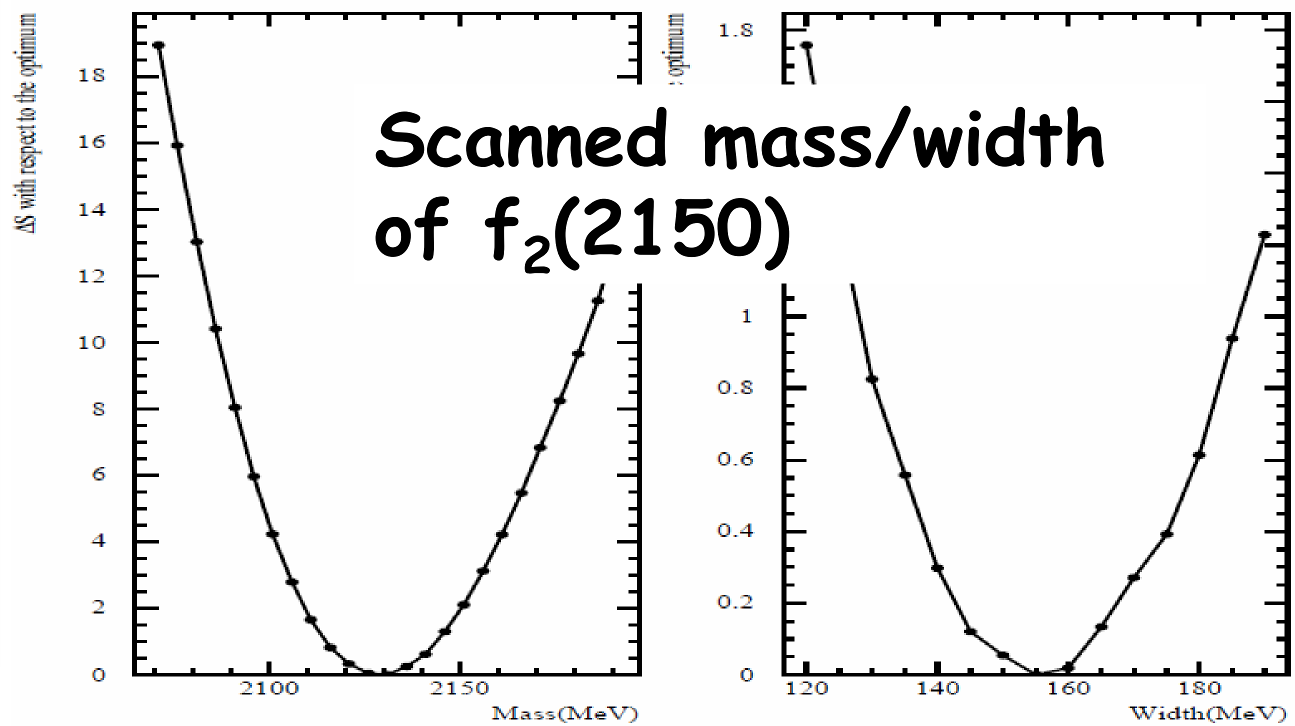
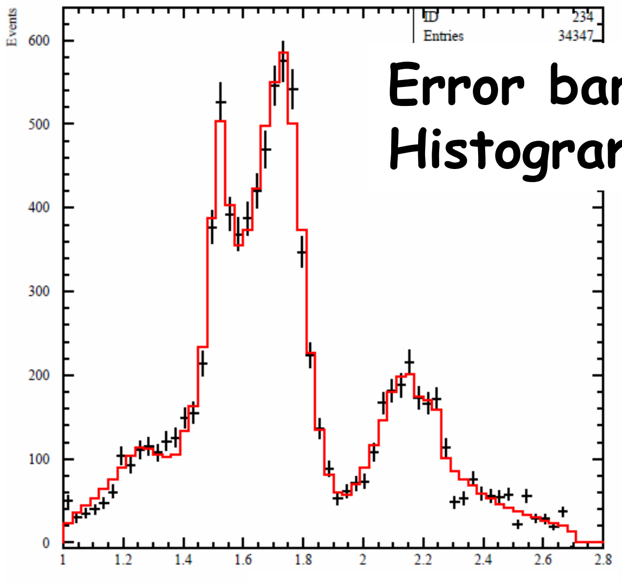


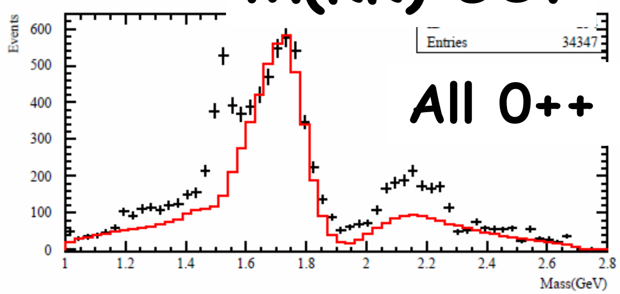
图 4.15 : f_2(2150) 的质量(左图)和宽度(右图)的扫描结果。

The results of official solution

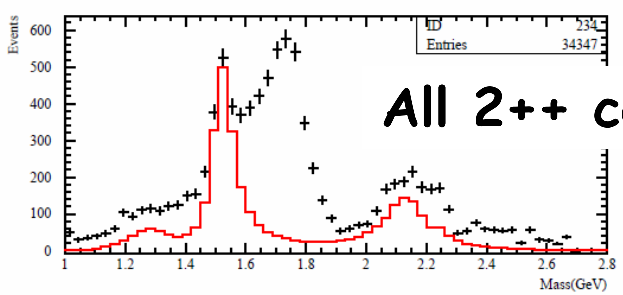


Error bar: data
Histogram: fitted result

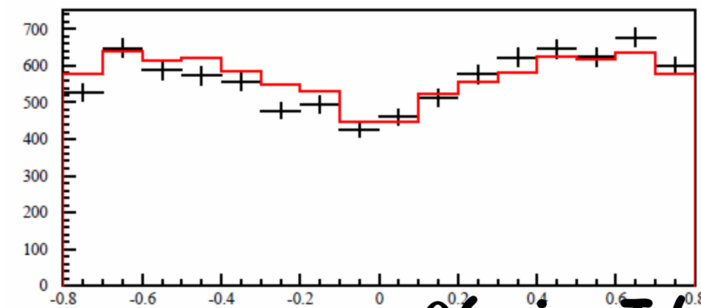
M(KK) GeV



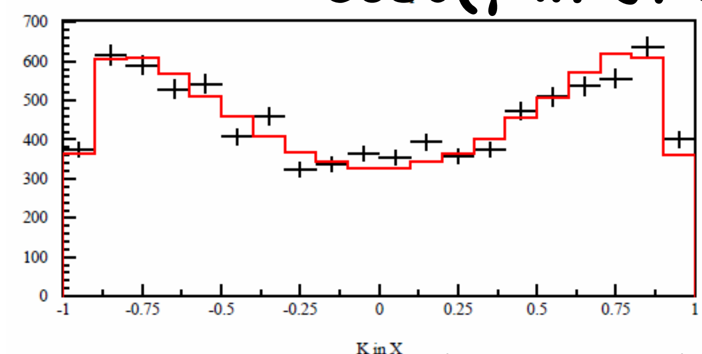
All 0++ component



All 2++ component



cosθ(γ in J/ψ)



cosθ(K in KK)

The results of official solution

After PWA, the mass, width, contribution/events of each resonance in this combination.

共振态	质量(MeV)	宽度(MeV)	事例数
$f_2(1270)$			321.
$f_0(1500)$			51.
$f_2(1525)$	1520^{+4}_{-3}	72^{+5}_{-5}	1412.
$f_0(1710)$	1755^{+7}_{-4}	206^{+10}_{-6}	6123.
$f_0(2100)$	2035^{+15}_{-25}	485^{+200}_{-40}	2132.
$f_2(2150)$	2131^{+9}_{-12}	155^{+22}_{-18}	981.
$f_2(2230)$	2238^{+5}_{-3}	32^{+10}_{-9}	282.
$f_0(wide)$			863.
常数项本底			598.

表 4.3：最佳解中各共振态或成分的质量、宽度、所占比例以及事例数。

To be continued …

Thank you!