PWA with Covariant Tensor Formalism

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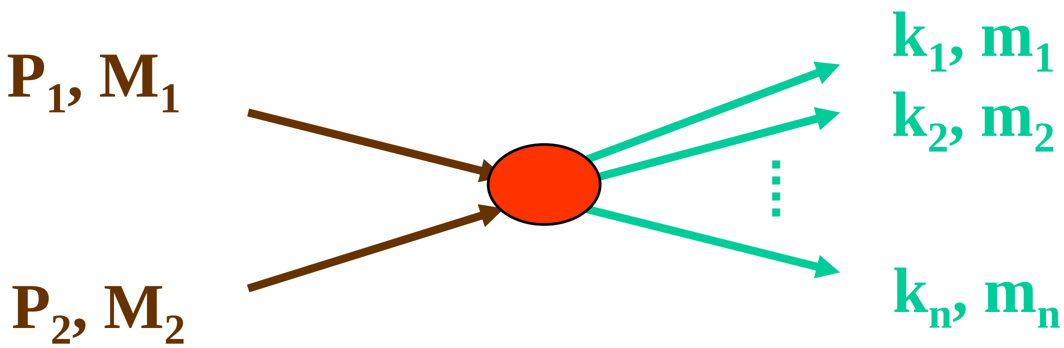
一. Experimental observables – Starting point for PWA



Experimental distribution probability:

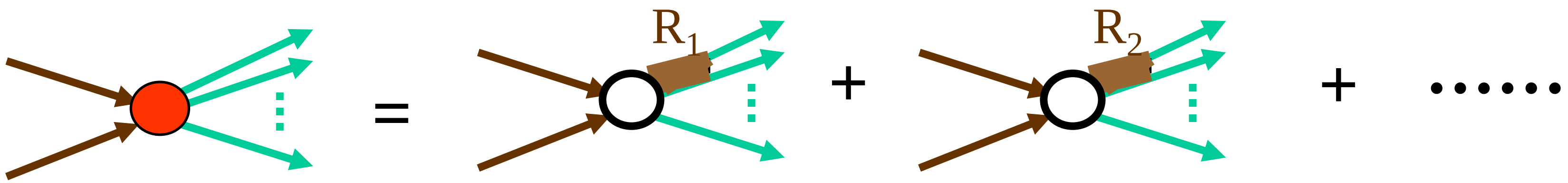
$$W_{exp}(k_1, \ldots, k_n) \sim |\mathcal{M}(k_1, \ldots, k_n)|^2 \, \varepsilon(k_1, \ldots, k_n) \, d\Phi_n(k_1, \ldots, k_n)$$

amplitude efficiency phase space

$$d\Phi_n(P; p_1, \ldots, p_n) = \delta^4 \left(P - \sum_{i=1}^{n} p_i\right) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

various projections

二. PWA framework



$\mathcal{M}(k_1, \ldots, k_n) = C_1 A_{R1}(k_1, \ldots, k_n) + C_2 A_{R2}(k_1, \ldots, k_n) + \ldots$

⇒ $W_{th}(k_1, \ldots, k_n; C_1, C_2, \ldots)$

PWA: fitting experimental data to get $C_1, C_2, \ldots$

Old fashion : χ^2-fit to various projections

Modern technique: multi-dimensional Maximum Likelihood fit to $W_{exp}(k_1, \ldots, k_n)$

CERNLIB programs : FUMILI, MINUIT

三. Construction of PWA covariant tensor amplitudes

Some references :

W.Rarita, J.Schwinger, Phys. Rev. 60, 61 (1941)
E.Behtends, C.Fronsdal, Phys. Rev. 106, 345 (1957)
S.U.Chung, PRD48, 1225 (1993); PRD57, 431 (1998)
B.S.Zou, D.V.Bugg, Eur. Phys. J. A16, 537 (2003)   for mesons
B.S.Zou, F.Hussian, PRC67, 015204 (2003)   for baryons

Basic ingredients :

(1) spin wave-function for single particle
(2) orbital wave-function for two-particle system
(3) $g_{\mu\nu}$, $\varepsilon_{\mu\nu\lambda\sigma}$
(4) Breit-Wigner propagator, form factor

(1) spin wave-function for single particle

S=1 : $\phi^{\alpha}(m)$, m=+1, 0, -1

at rest frame :

$$\phi^{\alpha}(\pm) = \mp\frac{1}{\sqrt{2}} \quad (\quad 0;\ 1,\ \pm i,\ 0 \quad)$$
$$\phi^{\alpha}(0) = \quad (\quad 0;\ 0,\ 0,\ 1 \quad)$$

S=2 : $\phi^{\alpha\beta}(m)$, m=+2, +1, 0, -1, -2

$$\phi^{\alpha\beta}(m) = \sum_{m_1 m_2} (1m_1 1m_2|2m)\phi^{\alpha}(m_1)\phi^{\beta}(m_2)$$

S=3 : $\phi^{\alpha\beta\gamma}(m)$, S=4 : $\phi^{\alpha\beta\gamma\delta}(m)$, ……

Some useful properties :

$p_{\mu}\,\phi^{\cdots\mu\cdots}(p,m)$, $\phi^{\cdots\mu\cdots\nu\cdots} = \phi^{\cdots\nu\cdots\mu\cdots}$, $g_{\mu\nu}\,\phi^{\cdots\mu\cdots\nu\cdots} = 0$

Spin projection operators :

$$
\begin{aligned}
P^{(1)}_{\mu\mu'}(p_a) &= \sum_m \phi_\mu(p_a,m)\phi^*_{\mu'}(p_a,m) = -g_{\mu\mu'} + \frac{p_{a\mu}p_{a\mu'}}{p_a^2} \equiv -\tilde{g}_{\mu\mu'}(p_a), \\
P^{(2)}_{\mu\nu\mu'\nu'}(p_a) &= \sum_m \phi_{\mu\nu}(p_a,m)\phi^*_{\mu'\nu'}(p_a,m) = \frac{1}{2}(\tilde{g}_{\mu\mu'}\tilde{g}_{\nu\nu'} + \tilde{g}_{\mu\nu'}\tilde{g}_{\nu\mu'}) - \frac{1}{3}\tilde{g}_{\mu\nu}\tilde{g}_{\mu'\nu'}, \\
P^{(3)}_{\mu\nu\lambda\mu'\nu'\lambda'}(p_a) &= \sum_m \phi_{\mu\nu\lambda}(p_a,m)\phi^*_{\mu'\nu'\lambda'}(p_a,m) \\
&= -\frac{1}{6}(\tilde{g}_{\mu\mu'}\tilde{g}_{\nu\nu'}\tilde{g}_{\lambda\lambda'} + \tilde{g}_{\mu\mu'}\tilde{g}_{\nu\lambda'}\tilde{g}_{\lambda\nu'} + \tilde{g}_{\mu\nu'}\tilde{g}_{\nu\mu'}\tilde{g}_{\lambda\lambda'} \\
&\qquad + \tilde{g}_{\mu\nu'}\tilde{g}_{\nu\lambda'}\tilde{g}_{\lambda\mu'} + \tilde{g}_{\mu\lambda'}\tilde{g}_{\nu\nu'}\tilde{g}_{\lambda\mu'} + \tilde{g}_{\mu\lambda'}\tilde{g}_{\nu\mu'}\tilde{g}_{\lambda\nu'}) \\
&\quad + \frac{1}{15}(\tilde{g}_{\mu\nu}\tilde{g}_{\mu'\nu'}\tilde{g}_{\lambda\lambda'} + \tilde{g}_{\mu\nu}\tilde{g}_{\nu'\lambda'}\tilde{g}_{\lambda\mu'} + \tilde{g}_{\mu\nu}\tilde{g}_{\mu'\lambda'}\tilde{g}_{\lambda\nu'} \\
&\qquad + \tilde{g}_{\mu\lambda}\tilde{g}_{\mu'\lambda'}\tilde{g}_{\nu\nu'} + \tilde{g}_{\mu\lambda}\tilde{g}_{\mu'\nu'}\tilde{g}_{\nu\lambda'} + \tilde{g}_{\mu\lambda}\tilde{g}_{\nu'\lambda'}\tilde{g}_{\nu\mu'} \\
&\qquad + \tilde{g}_{\nu\lambda}\tilde{g}_{\nu'\lambda'}\tilde{g}_{\mu\mu'} + \tilde{g}_{\nu\lambda}\tilde{g}_{\mu'\nu'}\tilde{g}_{\mu\lambda'} + \tilde{g}_{\nu\lambda}\tilde{g}_{\mu'\lambda'}\tilde{g}_{\mu\nu'}), \\
P^{(4)}_{\mu\nu\lambda\sigma\mu'\nu'\lambda'\sigma'}(p_a) &= \sum_m \phi_{\mu\nu\lambda\sigma}(p_a,m)\phi^*_{\mu'\nu'\lambda'\sigma'}(p_a,m) \\
&= \frac{1}{24}[\tilde{g}_{\mu\mu'}\tilde{g}_{\nu\nu'}\tilde{g}_{\lambda\lambda'}\tilde{g}_{\sigma\sigma'} + \cdots (\mu', \nu', \lambda', \sigma' \ permutation,\ 24\ terms)] \\
&\quad - \frac{1}{84}[\tilde{g}_{\mu\nu}\tilde{g}_{\mu'\nu'}\tilde{g}_{\lambda\lambda'}\tilde{g}_{\sigma\sigma'} + \cdots (\mu, \nu, \lambda, \sigma \ permutation, \\
&\qquad\qquad \mu', \nu', \lambda', \sigma' \ permutation,\ 72\ terms)] \\
&\quad + \frac{1}{105}(\tilde{g}_{\mu\nu}\tilde{g}_{\lambda\sigma} + \tilde{g}_{\mu\lambda}\tilde{g}_{\nu\sigma} + \tilde{g}_{\mu\sigma}\tilde{g}_{\nu\lambda})(\tilde{g}_{\mu'\nu'}\tilde{g}_{\lambda'\sigma'} + \tilde{g}_{\mu'\lambda'}\tilde{g}_{\nu'\sigma'} + \tilde{g}_{\mu'\sigma'}\tilde{g}_{\nu'\lambda'}).
\end{aligned}
$$

(2) orbital wave-function for two-particle system

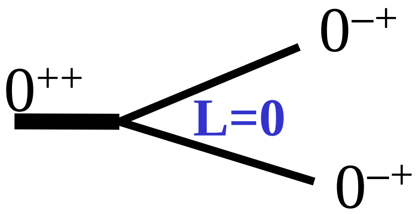
a → b + c ,   r = p_b - p_c

$$\tilde{t}^{(L)}_{\mu_1\cdots\mu_L} = (-1)^L P^{(L)}_{\mu_1\cdots\mu_L\mu'_1\cdots\mu'_L} r^{\mu'_1}\cdots r^{\mu'_L} B_L(Q_{abc})$$

$$\begin{aligned}
B_1(Q_{abc}) &= \sqrt{\frac{2}{Q^2_{abc}+Q^2_0}},\\
B_2(Q_{abc}) &= \sqrt{\frac{13}{Q^4_{abc}+3Q^2_{abc}Q^2_0+9Q^4_0}},\\
B_3(Q_{abc}) &= \sqrt{\frac{277}{Q^6_{abc}+6Q^4_{abc}Q^2_0+45Q^2_{abc}Q^4_0+225Q^6_0}},\\
B_4(Q_{abc}) &= \sqrt{\frac{12746}{Q^8_{abc}+10Q^6_{abc}Q^2_0+135Q^4_{abc}Q^4_0+1575Q^2_{abc}Q^6_0+11025Q^8_0}}
\end{aligned}$$

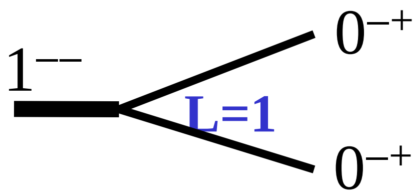
Examples:

1. $f_0 \rightarrow \pi\pi$



$\mathcal{M} = C$ => isotropic

2. $e^+e^- \rightarrow \phi \rightarrow K^+K^-$



$\mathcal{M} = C\ \phi^{\mu}(m)\tilde{t}^{(1)}_{\mu}$, $\quad \tilde{t}^{(1)}_{\mu} = (-g_{\mu\nu} + \frac{P_\mu P_\nu}{M_\phi^2})(p_1^\nu - p_2^\nu) = p_{2\mu} - p_{1\mu}$

in ϕ at-rest frame:

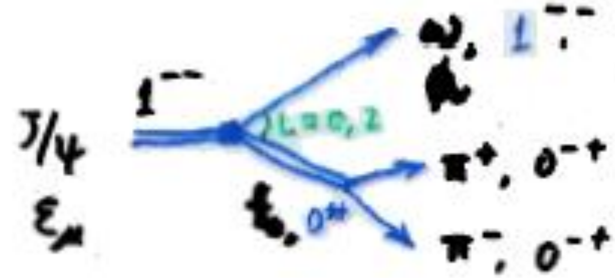
$$\tilde{t}^{(1)}_{\mu} = (0, q_x, q_y, q_z)\ , \quad \vec{q} = \vec{p}_2 - \vec{p}_1$$

$$\phi^{\mu}(\pm) = (0, \mp\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0), \quad \phi^{\mu}(0) = (0, 0, 0, 1)$$

$$|\mathcal{M}(+)|^2 + |\mathcal{M}(-)|^2 = |C|^2(q_x^2 + q_y^2) = |C|^2(|\vec{q}|^2 - q_z^2) = |C|^2|\vec{q}|^2(1 - cos^2\theta)$$

$$|\mathcal{M}(0)|^2 = |C|^2 q_z^2 = |C|^2|\vec{q}|^2 cos^2\theta$$

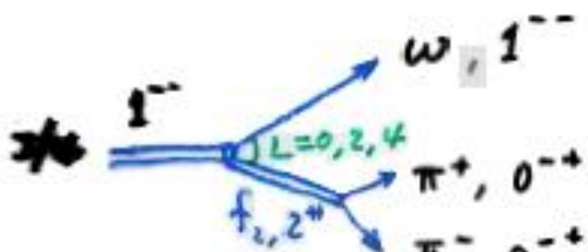
3 $J/\psi \to \omega \pi^+ \pi^-$



$L=0$, $A_1 = \varepsilon^*_\mu(m_\psi)\, \phi^\mu(m_\omega)\, BW(f_0)$

$L=2$, $A_2 = \varepsilon^*_\mu \phi_\nu \tilde{t}^{(2)\mu\nu}(g_{\omega f_0})\, BW(f_0)$

$= \varepsilon^*_\mu \phi_\nu P^\mu_\omega P^\nu_{f_0} BW(f_0) - \frac{1}{3}|\vec{g}_{\omega f_0}|^2 A_1$



$L=0$, $A_3 = \varepsilon^*_\mu \phi_\nu \phi^{\mu\nu} BW(f_2)\, \phi^{*\alpha\beta} \tilde{T}^{(2)}_{\alpha\beta}$

$= \varepsilon^*_\mu \phi_\nu \tilde{T}^{(2)\mu\nu} BW(f_2)$

$L=2$, $A_4 = \varepsilon^*_\mu \phi^\mu \tilde{t}^{(2)}_{\alpha\beta} \tilde{T}^{(2)\alpha\beta} BW(f_2)$

$\sum_m \phi^{\mu\nu}(m)\, \phi^{*\alpha\beta}(m)\, \tilde{T}^{(2)}_{\alpha\beta}$

$= \tilde{T}^{(2)\mu\nu}$

$A_5 = \varepsilon^*_\mu \phi_\nu P_\alpha \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{\beta\gamma\delta\sigma} P^\gamma \tilde{t}^{\delta\lambda} \tilde{T}^\sigma_\lambda\, BW(f_2)$

$A_6 = \varepsilon^*_\mu \phi_\nu \tilde{t}^{\mu\lambda} \tilde{T}^\nu_\lambda\, BW(f_2)$

$L=4$, $A_7 = \varepsilon^*_\mu \phi_\nu \tilde{t}^{\mu\nu\lambda\sigma} \tilde{T}_{\lambda\sigma} BW(f_2)$

$$\mathcal{M} = \sum_i \Lambda_i A_i = \varepsilon^*_\mu \phi_\nu \sum_i \Lambda_i T_i^{\mu\nu}$$

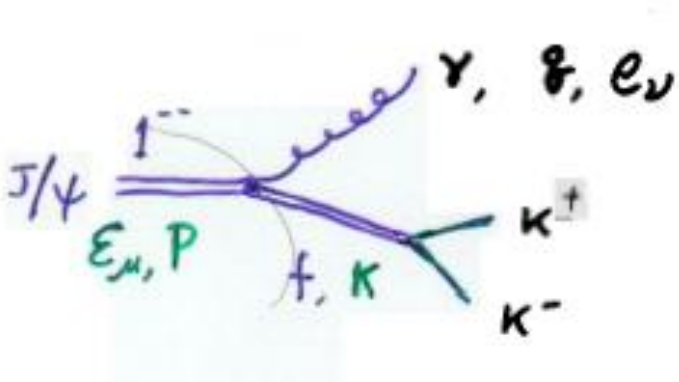
$$\frac{d\sigma}{d\Phi} = \frac{1}{2} \sum_{m_1=1}^{2} \sum_{m_2=1}^{3} \varepsilon^*_\mu(m_1) \phi_\nu(m_2) \sum_i \Lambda_i T_i^{\mu\nu} \, \varepsilon_{\mu'}(m_1) \phi^*_{\nu'}(m_2) \sum_j \Lambda^*_j T_j^{*\mu'\nu'}$$

$$\sum_{m_2=1}^{3} \phi_\nu(m_2) \phi^*_{\nu'}(m_2) = -g_{\nu\nu'} + \frac{p_\nu(\omega) p_{\nu'}(\omega)}{m_\omega^2} =: \tilde{g}$$

$$\sum_{m_1=1}^{2} \varepsilon^*_\mu(m_1) \varepsilon_{\mu'}(m_1) = \delta_{\mu\mu'} (\delta_{\mu 1} + \delta_{\mu 2})$$

$$= \frac{1}{2} \sum_{i,j} \Lambda_i \Lambda_j^* \sum_{\mu=1}^{2} T_i^{\mu\nu} \tilde{g}_{\nu\nu'} T_j^{*\mu\nu'}$$

4. J/ψ→ γ K^+K^-



Compare with J/ψ→ ω K^+K^- :

ϕ_ν→ e_ν, $e_\nu q^\nu=0$, $q^2=0$

Coulomb gauge : $P^\nu e_\nu= 0$ → $K^\nu e_\nu= (P^\nu-q^\nu)e_\nu= 0$

$$\sum_m e^*_\mu(m)e_\nu(m) = -g_{\mu\nu} + \frac{q_\mu K_\nu + K_\mu q_\nu}{q\cdot K} - \frac{K\cdot K}{(q\cdot K)^2}q_\mu q_\nu \equiv -g^{(\perp\perp)}_{\mu\nu}$$

c.f. Greiner & Schafer, Quantum Chromodynamics, p.185

The method to get the equation :

$$g^{(LL)}_{\mu\nu} = A\, g_{\mu\nu} + B\, q_\mu K_\nu + C\, q_\nu K_\mu + D\, q_\mu q_\nu + E\, K_\mu K_\nu$$

$$q^\mu g^{(LL)}_{\mu\nu} = 0, \quad q^\nu g^{(LL)}_{\mu\nu} = 0, \quad K^\mu g^{(LL)}_{\mu\nu} = 0, \quad K^\nu g^{(LL)}_{\mu\nu} = 0,$$

$$g^{\mu\nu} g^{(LL)}_{\mu\nu} = -2 \quad \Rightarrow \quad A = -1, \quad B = C = \frac{1}{q\cdot K}$$

$$D = -\frac{K\cdot K}{(q\cdot K)^2}, \quad E = 0$$

$$A_1 = \varepsilon^{*}_{\mu}(m_\psi)\, e^{\mu}(m_\gamma)\, BW(f_0)$$

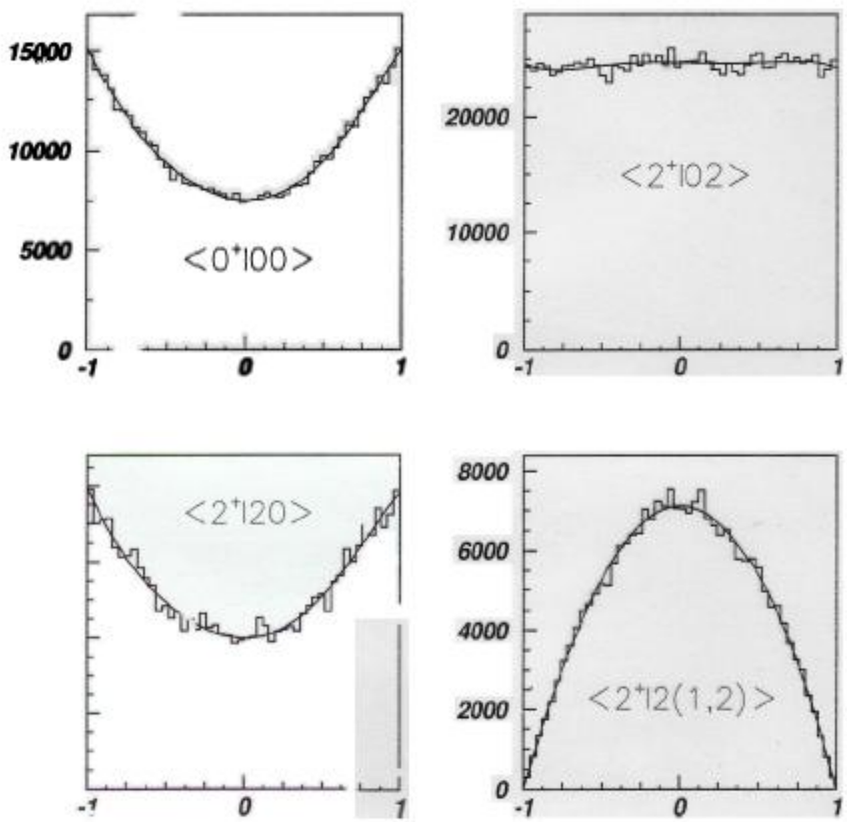
$$A_2 = \varepsilon^{*}_{\mu}\, e_\nu\, g^{\mu} K^{\nu} BW(f_0) - \frac{1}{3}|\vec{g}|^2 A_1$$

$$= \frac{1}{3}|\vec{g}|^2 A_1 \rightarrow \frac{1}{3} B_2(g)\, A_1 \sim A_1$$

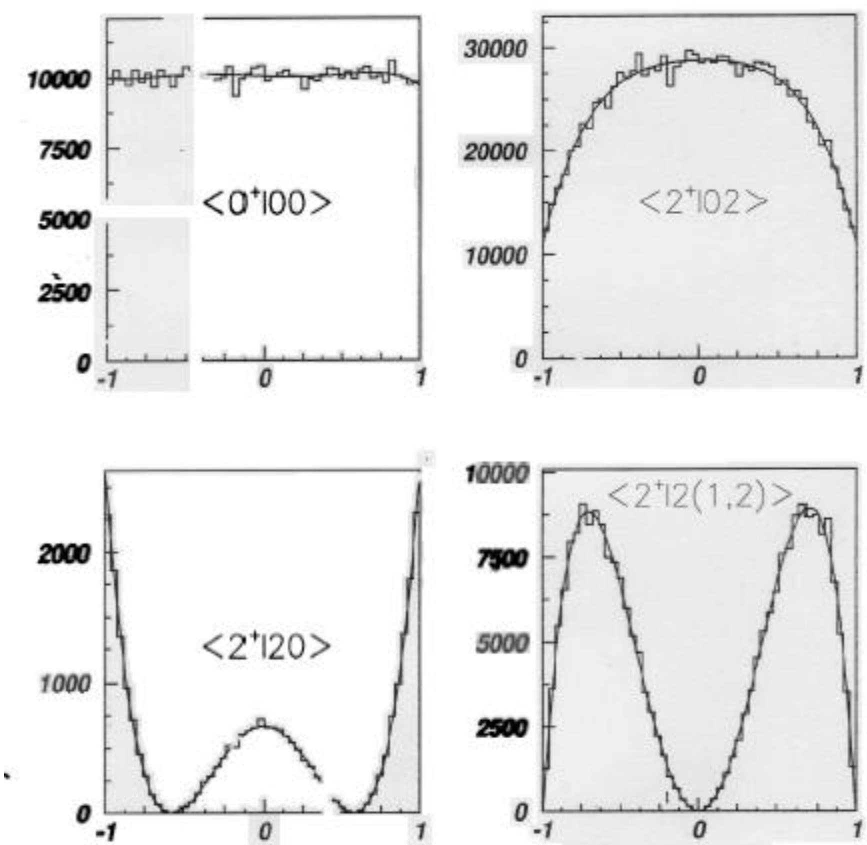
So A_1 and A_2 have the same angular distribution:

$$\sum_{m_1, m_2} \left| \varepsilon^{*}_{\mu}(m_1)\, e^{\mu}(m_2) \right|^2 = 1 + \cos^2\theta$$

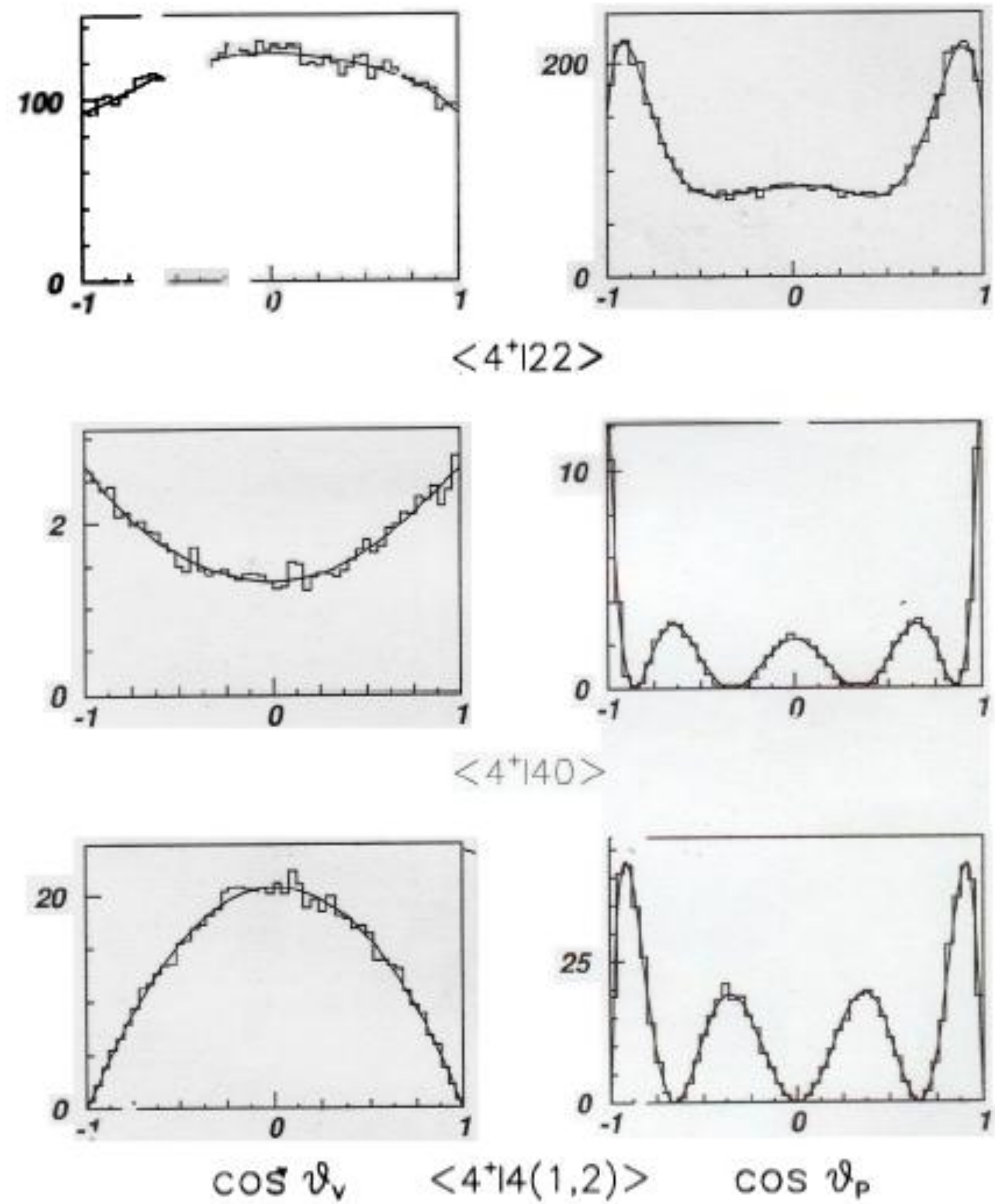
Similarly, there are only 3 independent angular distribution for A_3, A_4, A_5, A_6, A_7



Projection of cos ϑ_V distribution



Projection of cos ϑ_P distribution


100
0
-1
0
1
200
0
-1
0
1
<4⁺122>
2
0
-1
0
1
10
0
-1
0
1
<4⁺140>
20
0
-1
0
1
25
0
-1
0
1
cos ϑv
<4⁺14(1,2)>
cos ϑp

Crystal Barrel $p\bar{p} \rightarrow \eta\eta\eta$

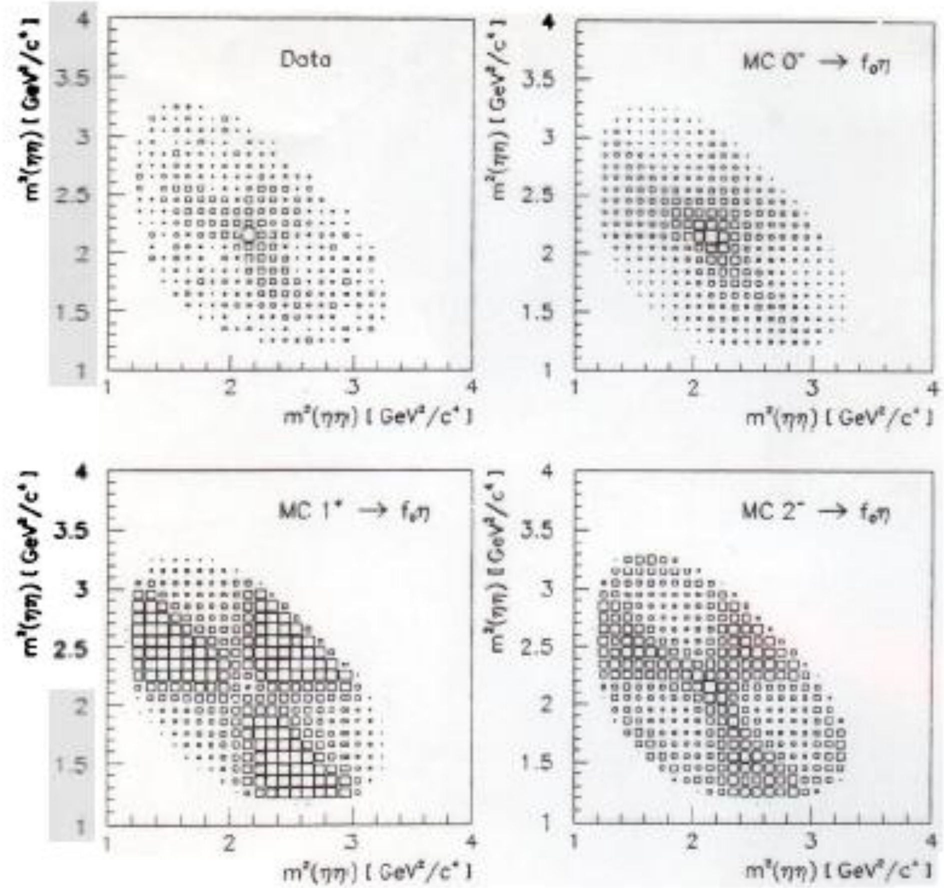


Figure 2. Real data and **Monte** Carlo Dalitz plots for $p\bar{p} \rightarrow \eta\eta\eta$ **at 1.8 GeV/c.**

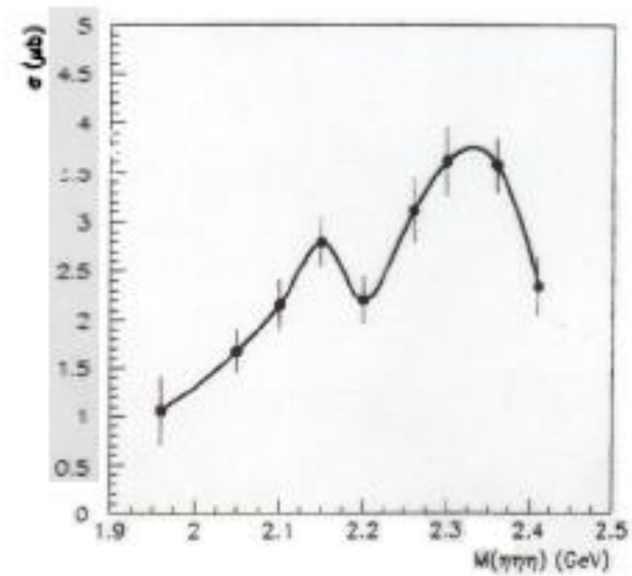


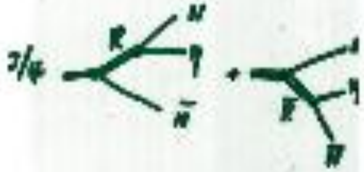
Figure 3. preliminary results of cross section for $\bar{p}p \rightarrow \eta\eta\eta$.

For ψ decay to baryons:

- Construction of PWA amplitudes.

Rarita-Schwinger covariant tensor formalism.

(1) $\frac{1}{2}^{\pm}$ N^* .



$$\mathcal{L}_{\eta N R} = -i g_{\eta N R} \bar{N} \Gamma R \eta + h.c.$$

$$\mathcal{L}_{\psi N R} = -g_{\psi_1} \bar{R} \Gamma_\mu N \psi^\mu + \frac{i g_{\psi_2}}{M_R + M_N} \bar{R} \Gamma_{\mu\nu} \xi^\nu_\psi N \psi^\mu + h.c.$$

where

$\Gamma = 1,\ \Gamma_\mu = \gamma_5 \gamma_\mu,\ \Gamma_{\mu\nu} = \gamma_5 \sigma_{\mu\nu}$ for $\frac{1}{2}^-$ N^*,

$\Gamma = \gamma_5,\ \Gamma_\mu = \gamma_\mu,\ \Gamma_{\mu\nu} = \sigma_{\mu\nu}$ for $\frac{1}{2}^+$ N^*.

$$A_{\frac{1}{2}^-} = G_1 \bar{u} \left[\frac{\not{k}_1 + \not{k}_2 + M_{N^*}}{M_{N^*}^2 - s_{12} - i M_{N^*}\Gamma_{N^*}} \gamma_5 \gamma_\mu \varepsilon^\mu + \gamma_5 \not{\varepsilon} \frac{-\not{k}_2 - \not{k}_3 + M_{N^*}}{M_{N^*}^2 - s_{23} - i M_{N^*}\Gamma_{N^*}} \right] v$$

$$+ G_2 \bar{u} \left[\frac{\not{k}_1 + \not{k}_2 + M_{N^*}}{M_{N^*}^2 - s_{12} - i M_{N^*}\Gamma_{N^*}} \gamma_5 \sigma_{\mu\nu} \xi^\mu_\psi \varepsilon^\nu + \gamma_5 \sigma_{\mu\nu} \xi^\mu_\psi \varepsilon^\nu \frac{-\not{k}_2 - \not{k}_3 + M_{N^*}}{M_{N^*}^2 - s_{23} - i M_{N^*}\Gamma_{N^*}} \right] v$$

$A_{\frac{1}{2}^+} = \cdots$

Three basic elements for constructing amplitudes:

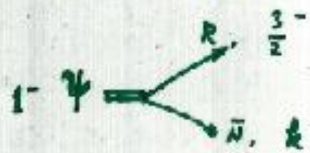
Wave functions, propagators, effective couplings.

(2) $\frac{3}{2}$ N^*

Wave function: $u_\mu(p, s_z) = \sum_{\lambda, s} (1\lambda \tfrac{1}{2} s | \tfrac{3}{2} s_z)\, e_\mu(p,\lambda)\, u(p,s)$

propagators: $P_{\mu\nu} = \frac{\not{p} + M_0}{p^2 - M_0^2 + i M_0 \Gamma_0} \left[g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{2 p_\mu p_\nu}{3 M_0^2} + \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3 M_0} \right]$

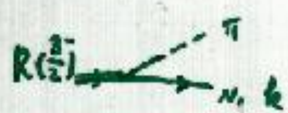
effective couplings:



(1) $\bar{R}^\mu \psi^\nu g_{\mu\nu} N$ $\qquad$ $\bar{R}^\mu \psi^\nu g_{\mu\nu} \gamma_5 N$

(2) $\bar{R}^\mu \psi^\nu \gamma_\nu k_\mu N$ $\qquad$ $\bar{R}^\mu \psi^\nu \gamma_\nu k_\mu \gamma_5 N$

(3) $\bar{R}^\mu \psi^\nu k_\mu k_\nu N$ $\qquad$ $\bar{R}^\mu \psi^\nu k_\mu k_\nu \gamma_5 N$



$i \bar{N} \phi \gamma_5 k_\mu R^\mu$ $\qquad$ $i \bar{N} \phi k_\mu R^\mu$

S. U. Chung, J. J. Zhu

(3) N^* with spin $J^* = n + \frac{1}{2}$,

wave function: $u_{\mu_1\mu_2\cdots\mu_n}(P, s^*) = \sum_{\lambda, s} (n\lambda \frac{1}{2} s | J^* s^*) e^{(n)}_{\mu_1\cdots\mu_n} u(P, s)$

propagator: $P_{\mu_1\cdots\mu_n\nu_1\cdots\nu_n}(P) = \frac{\gamma_\mu (\not{p} + M_{N^*}) \gamma_\nu}{p^2 - M^2_{N^*} + iM_{N^*}\Gamma_{N^*}} G^{(n+1)}_{\mu\mu_1\cdots\mu_n\nu\nu_1\cdots\nu_n}$

effective couplings: ...

↓

FDC (Automatic Feynman Diagram Calculation) – J. X. Wang

↓

Fortran Programs for amplitudes

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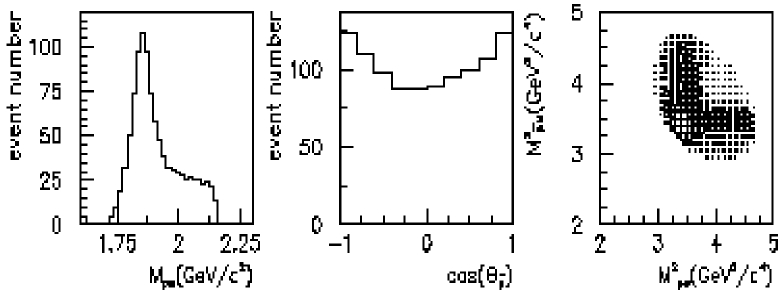
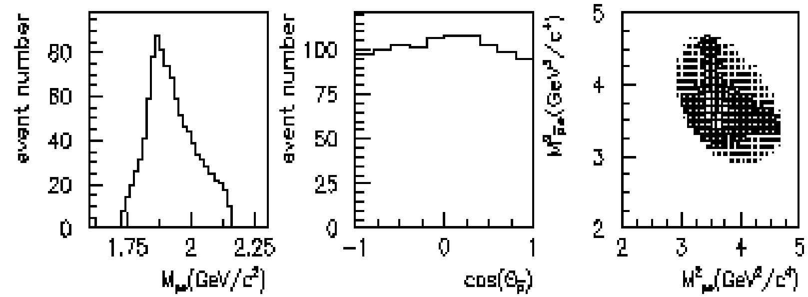
Fit to the data

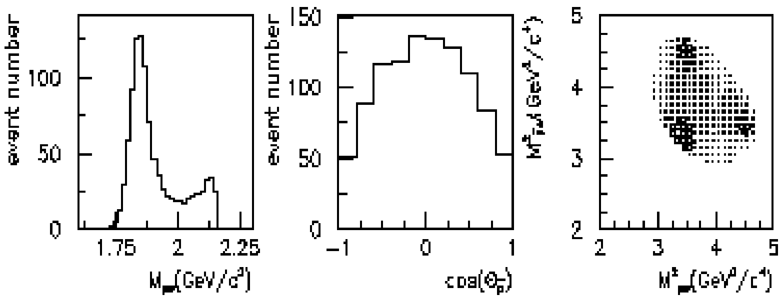
Monte Carlo Simulation for $J/\Psi \rightarrow p + \bar{p} + \omega$



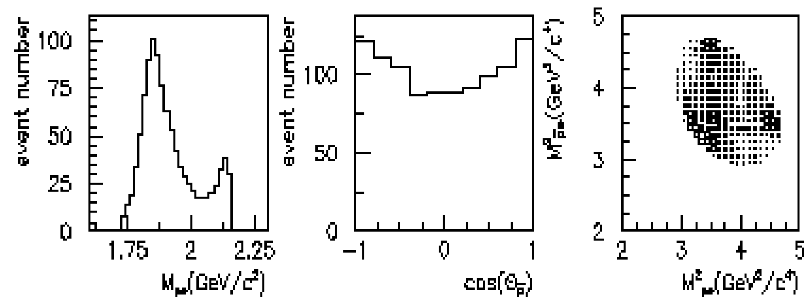
Plots for N^* resonance with $J^P = \frac{1}{2}^-$ and for decay mode 1.



Plots for N^* resonance with $J^P = \frac{3}{2}^+$ and for decay mode 3.



Plots for N^* resonance with $J^P = \frac{1}{2}^+$ and for decay mode 2.



Plots for N^* resonance with $J^P = \frac{5}{2}^-$ and for decay mode 4.

W.H.Liang, P.N.Shen, J.X.Wang, B.S.Zou, J. Phys. G28 (2002) 333

四. "Exotic" hadron-hadron S-wave Interaction

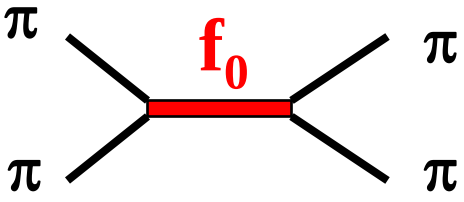
1. "Exotic" ππ S-wave Interaction

Why interested ?

1) a fundamental strong interaction process and a necessary input for many reactions involving multi-pions

2) I=0 ππ S-wave has the same quantum number of f_0 resonances which include :

σ/f_0(600) (σ-model, σ–exchange for NN interaction) and the lightest glueball candidate f_0(1500)/f_0(1710)



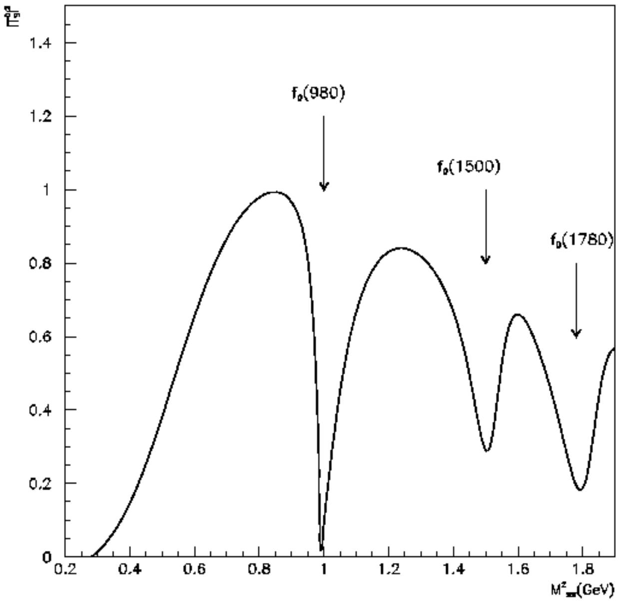
Time dependent well established f_0 resonances

PDG1992

$f_0(980)$
Γ=46 MeV

$f_0(1300)$
BR($\pi\pi$)>90%

$f_0(1590)$



PDG1996

$f_0(980)$
Γ= 46-400 MeV

$f_0(1370)$
BR($\pi\pi$) very small

$f_0(1500)$

$f_0(400\text{-}1200)$

References:

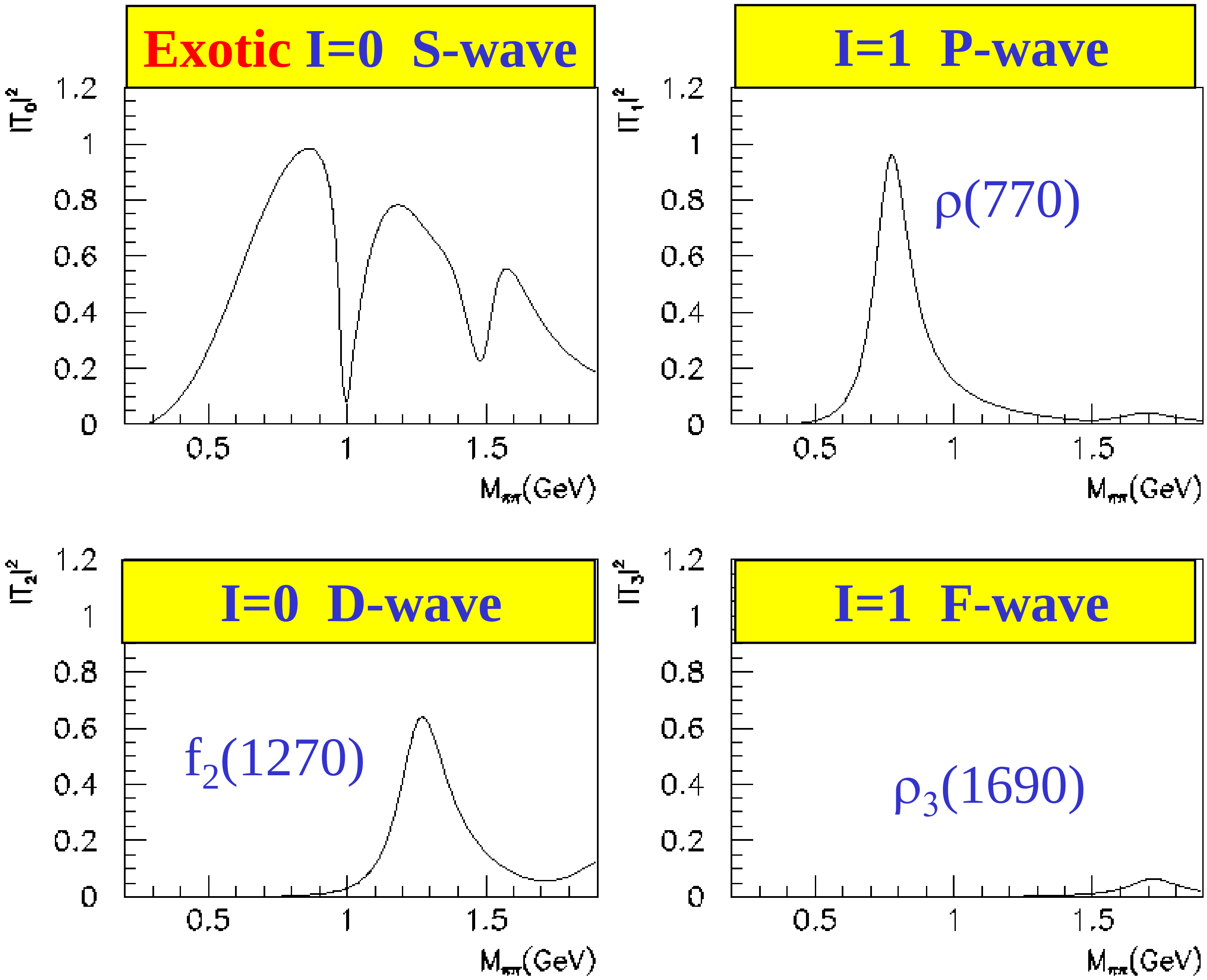
B.S.Zou, D.V.Bugg, Phys. Rev. D48, R3948 (1993)
"Is $f_0(975)$ a narrow resonance?"

B.S.Zou, D.V.Bugg, Phys. Rev. D50, 591 (1994)
"Remarks on I=0 JPC=0++ States:σ/ε and $f_0(975)$"

Cbar Coll., B.S.Zou, Phys. Lett. B323, 233 (1994)
"Observation of two $J^{PC}=0^{++}$ resonances at 1365 and 1520 MeV"

D.V.Bugg,I.Scott, B.S.Zou et al, Phys. Lett. B353, 378 (1995)
"Further amplitude analysis of $J/\psi \rightarrow \gamma(\pi\pi\pi\pi)$"

D.V.Bugg, A.Sarantsev, B.S.Zou, Nucl. Phys. B471, 59 (1996)
"New results on ππ phase shifts between 600 and 1900 MeV"

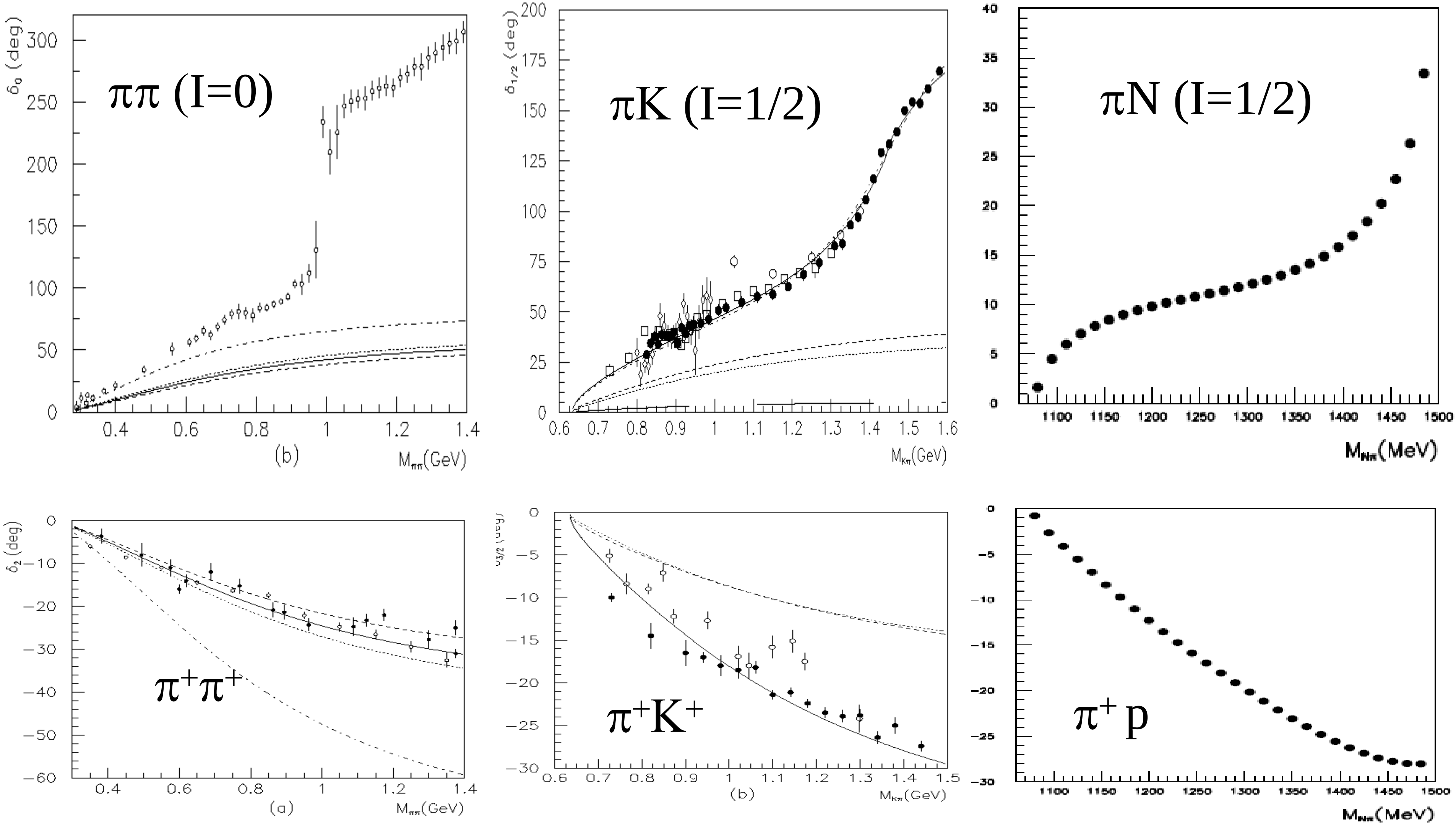


D.V.Bugg, A.Sarantsev,B.S.Zou, Nucl. Phys. B471 (1996) 59

“Exotic” $\pi\pi$ S-wave interaction :

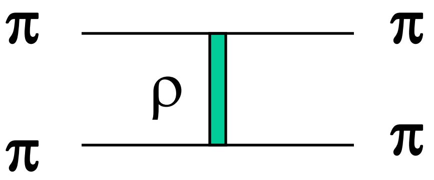
broad σ–background with narrow resonances as dips instead of peaks

Similarity for ππ, πK and πN s-wave scattering



What's the nature of the broad σ ?

Important role by t-channel ρ exchange for all these processes



ππ **πK & πN**

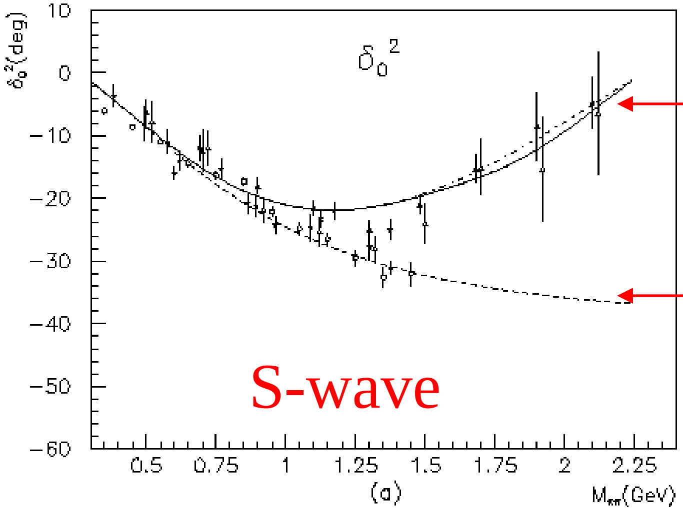
$K_\rho^{I=0} = -2\,K_\rho^{I=2}$, $K_\rho^{I=1/2} = -2\,K_\rho^{I=3/2}$

D. Lohse, J.W. Durso, K. Holinde, J. Speth, Nucl.Phys.A516, 513 (1990)
B.S.Zou, D.V.Bugg, Phys. Rev. D50, 591 (1994)

An interesting paper by T.Hyodo, D.Jido, A.Hosaka, PRL 97 (2006) 192002
"Exotic hadrons in s-wave chiral dynamics"

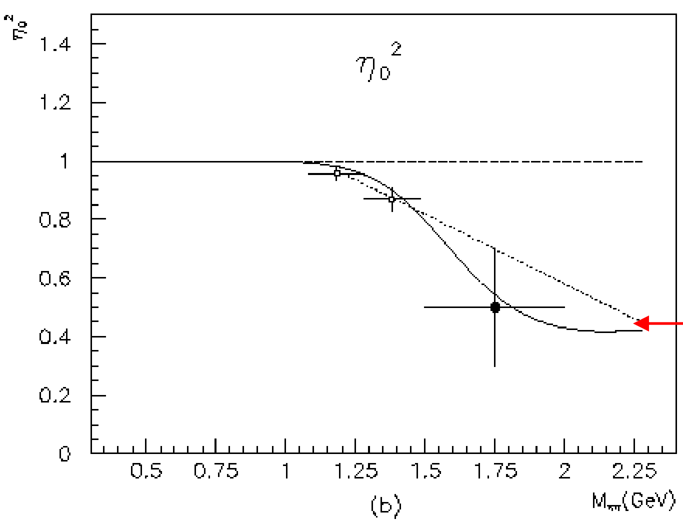
Basic features of I=2 $\pi\pi$ Interaction

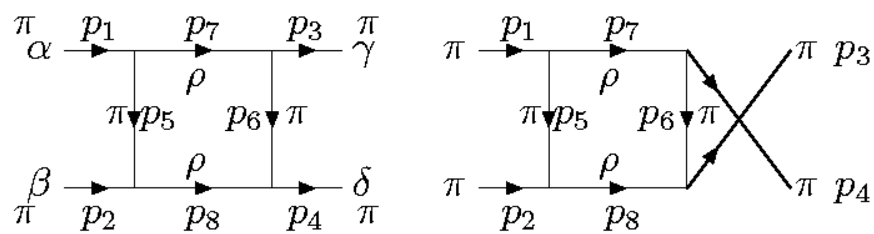
F.Q.Wu, B.S.Zou et al., Nucl. Phys.A735 (2004) 111



attractive force by t-channel f_2

repulsive force by t-channel ρ





Inelasticity by $\pi\pi \leftrightarrow \rho\rho$

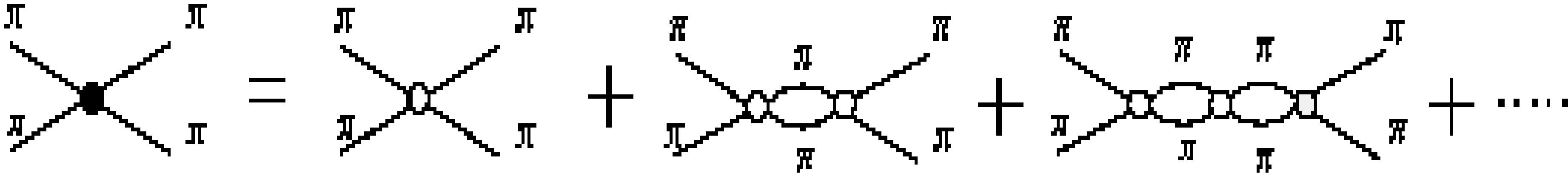
An important cause for hadron-hadron S-wave interactions appearing “exotic” is

t-channel meson-exchange amplitude has a comparable strength as s-channel resonance contribution for S-waves.

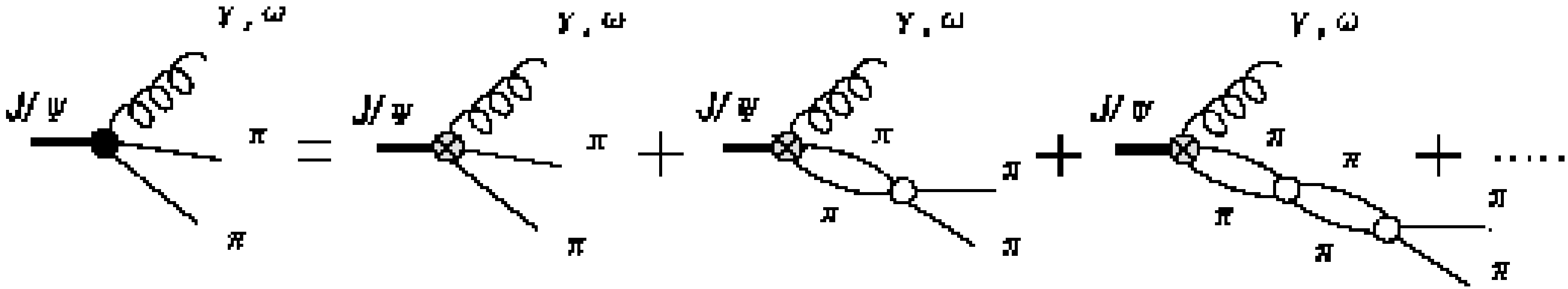
For higher partial waves, s-channel resonance contribution dominates.

Why broad σ appears narrower in production processes than in ππ elastic scattering?

$$T_{el} = K / (1 - i \rho K) = K + K i \rho K + K i \rho K i \rho K + \ldots$$



$$T_{prod} = P / (1 - i \rho K) = P + P i \rho K + P i \rho K i \rho K + \ldots$$

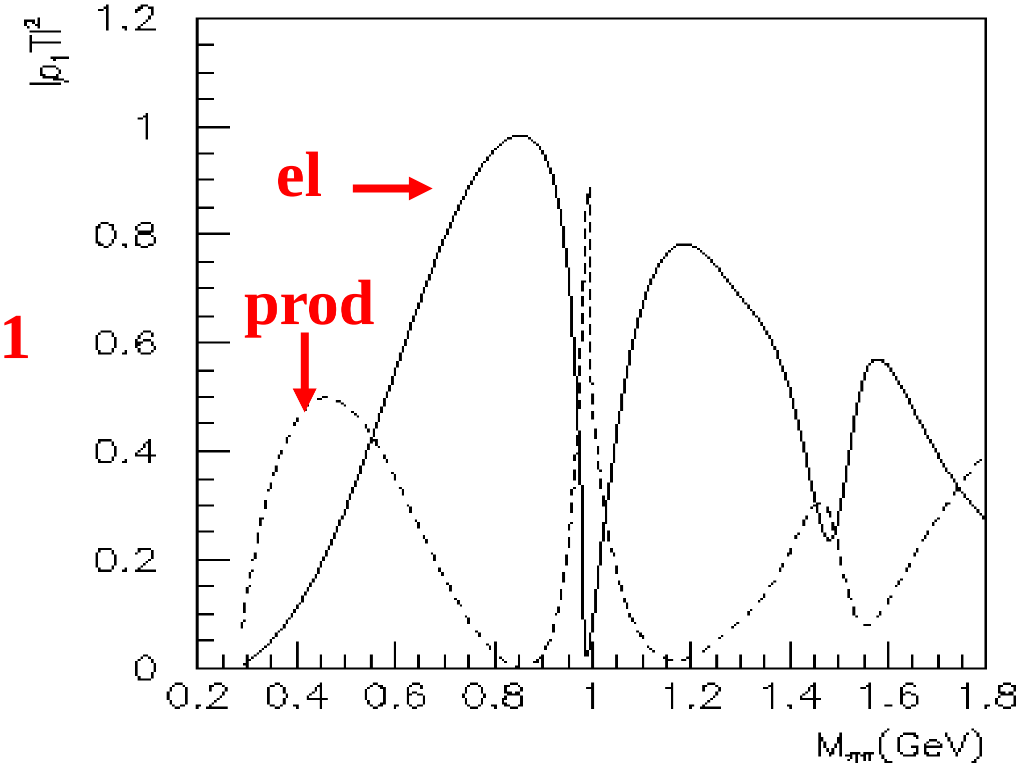


T_{el} from Bugg,Sarantsev,Zou Nucl. Phys. B471 (1996) 59

with σ pole at (0.571 – i 0.420) GeV

$$T_{prod} = T_{el} * P / K , \quad K = \tan\delta / \rho$$

assuming P=1



How about production vertex P ?

$$\mathbf{P(V'\rightarrow V\pi^+\pi^-)} = -\frac{4}{F_0^2}\left[\frac{g}{2}(m_{\pi\pi}^2 - 2M_\pi^2) + g_1 E_{\pi^+} E_{\pi^-}\right]\epsilon_\Psi^* \cdot \epsilon_{\Psi'}$$

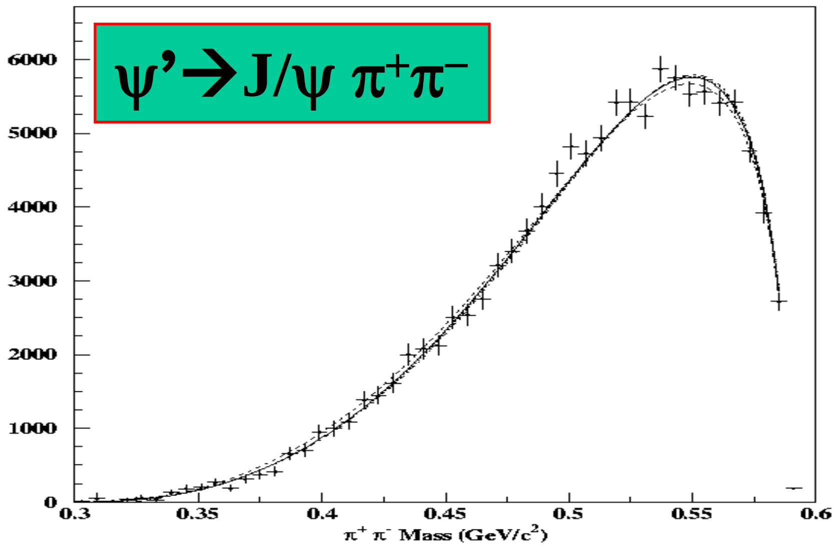
T. Mannel, R. Urech, Z. Phys.C73, 541 (1997);
Ulf-G. Meißner, J.Oller, Nucl.Phys. A679 (2001) 671;
M.Ishida et al., Phys. Lett. B518 (2001) 47;
L. Roca, J. Palomar, E. Oset, H.C.Chiang, Nucl. Phys. A744 (2004) 127
F.K.Guo,P.N.Shen, H.C.Chiang, R.G.Ping, Nucl.Phys.A761 (2005) 269

For $\psi'\rightarrow J/\psi\ \pi^+\pi^-$, E_π small, 1st term dominates
$\rightarrow$ higher σ peak

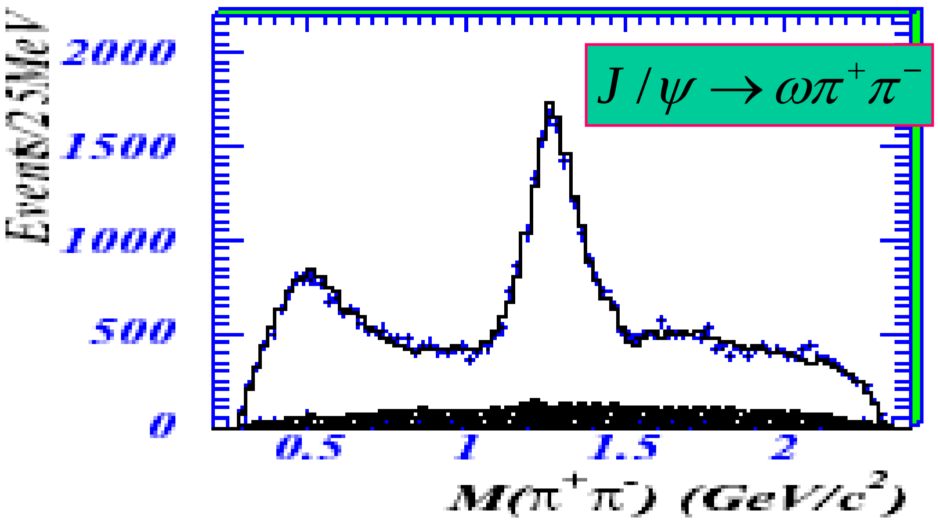
For $\psi \rightarrow \omega\ \pi^+\pi^-$, E_π large, 2nd term dominates
$\rightarrow$ lower σ peak

σ peak position is process dependent !

$$P \sim c_1 + c_2\, s$$



BES, Phys.Rev. D62 (2000) 032002



BES, Phys.Lett. B598 (2004) 149

Why $f_0(980)$'s peak width is so narrow ?

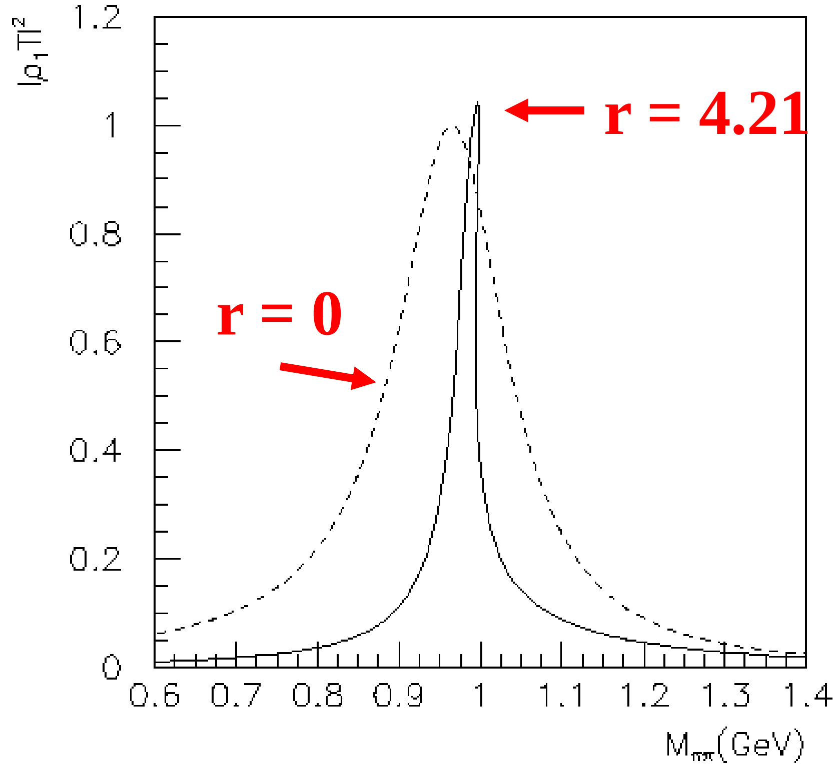
BES, PLB 607 (2005) 243

$$f = \frac{1}{M^2 - s - i(g_1\rho_{\pi\pi} + g_2\rho_{K\bar{K}})}.$$

M = 965 MeV

g_1 = 165 MeV^2

r = g_2/g_1 = 4.21

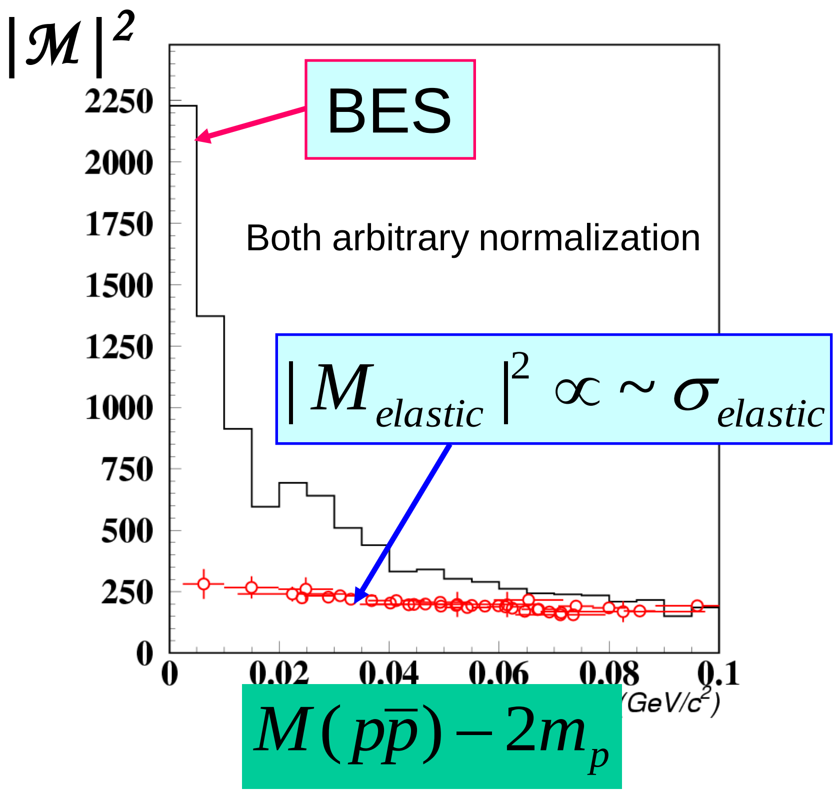


$$\rho_{K\bar{K}} = (1 - 4m_K^2/s)^{1/2}$$

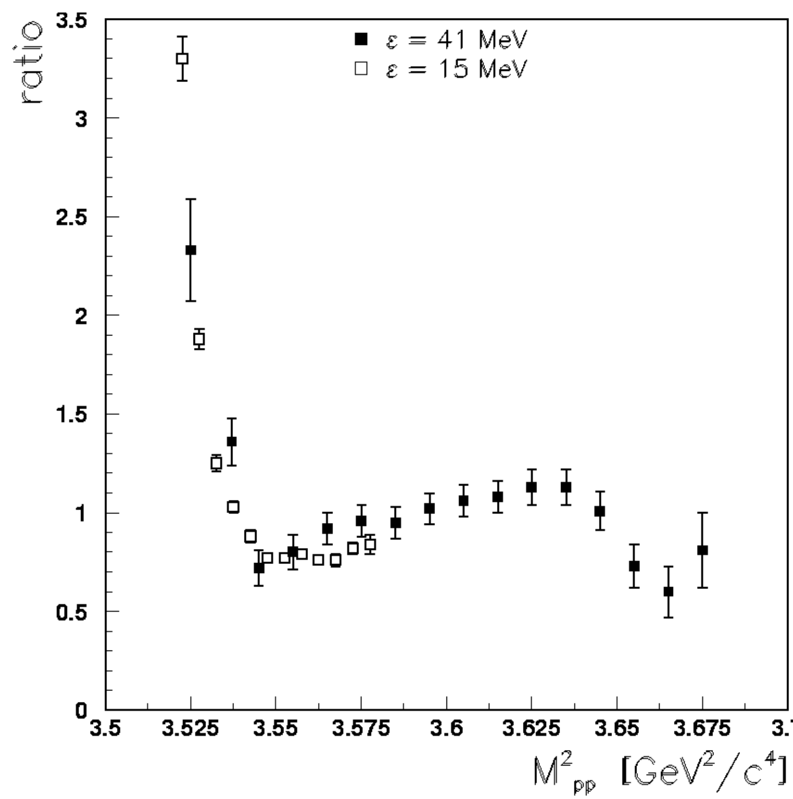
Strong coupling to $\bar{K}K$ strongly reduce the peak width of $f_0(980)$

2. I=0 $^1S_0$ $\bar{p}p$ & I=1 $^1S_0$ pp near threshold enhancement

$J/\psi \rightarrow \gamma\ \bar{p}p$



$p\,p \rightarrow p\,p\,\eta$



BES, Phys. Rev. Lett. 91, 022001 (2003)

COSY-TOF, Eur.Phys.J.A16, 127 (2003)

What should be the largest decay mode of I=0 $^1S_0$ $\bar{p}p$ state ?

I=0 $^1S_0$ $\bar{p}p$ atom : $\pi^0\pi^0\eta$ / $\pi^0\pi^0\eta'$ ~ 2

C.Amsler et al., B.S.Zou, Nucl. Phys. A720 (2003) 357

$J/\psi \to \gamma\,\eta\,\pi^+\pi^-$: BES, Phys. Lett. B446 (1999) 356

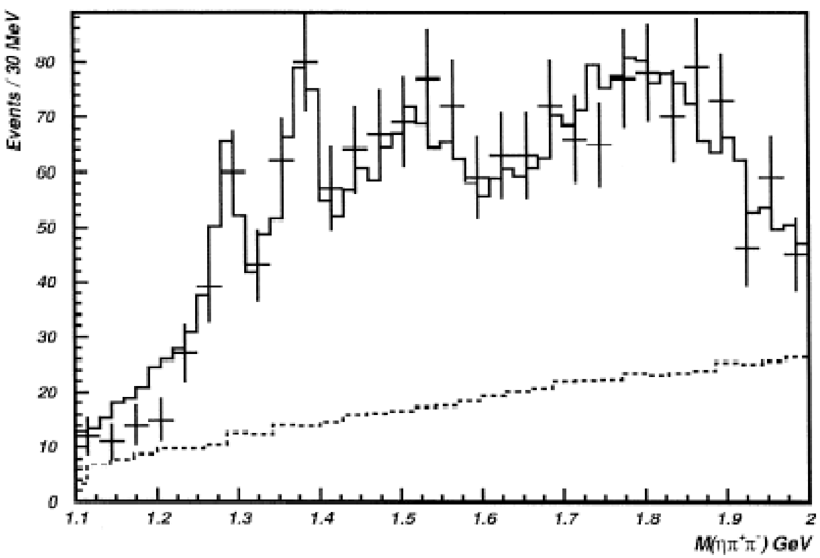
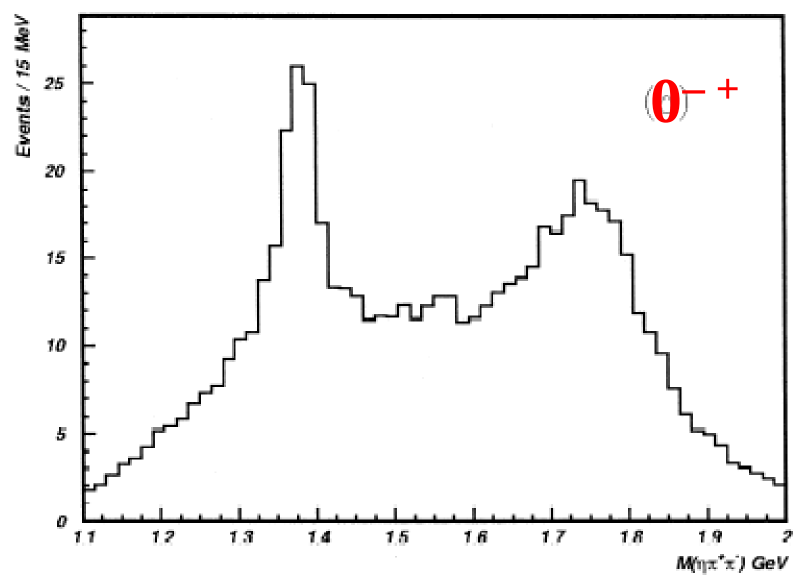


Fig. 2. The $\eta\pi^+\pi^-$ mass spectrum.

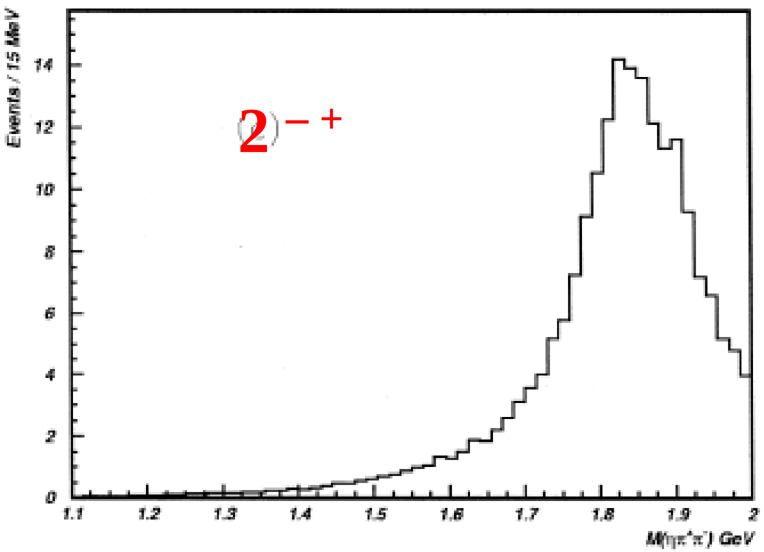


$\bar{p}p$ near threshold enhancement

= some broad sub-threshold 0^{-+} resonance(s) + FSI

Zou B.S., Chiang H.C. Phys.Rev.D69 (2004) 034004

$J/\psi \to \gamma\,\eta\,\pi^+\pi^-$: BES, Phys. Lett. B446 (1999) 356



$$M = 1840 \pm 15 \text{ MeV}$$

$$\Gamma = 170 \pm 40 \text{ MeV}$$

What's its relation with X(1835) observed in $J/\psi \to \gamma\,\eta'\,\pi^+\pi^-$

One-Pion-Exchange and BES p$\bar{p}$ ($^1S_0$) near threshold enhancement

Zou B.S., Chiang H.C. Phys.Rev.D69 (2004) 034004

NN interaction : $$V_\pi^{NN}(t) = \frac{f_\pi^2}{m_\pi^2 - t}\,\frac{1}{3}\,\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\tau}_1\cdot\vec{\tau}_2$$

$$\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{\tau}_1\cdot\vec{\tau}_2 = \begin{cases} 9 & (S, I) = (0,0) \\ 1 & (S, I) = (1,1) \\ -3 & (S, I) = (1,0) \text{ or } (0,1) \end{cases}$$

deuteron

$\bar{N}N$ interaction : $$V_\pi^{N\bar{N}}(t) = -V_\pi^{NN}(t)$$

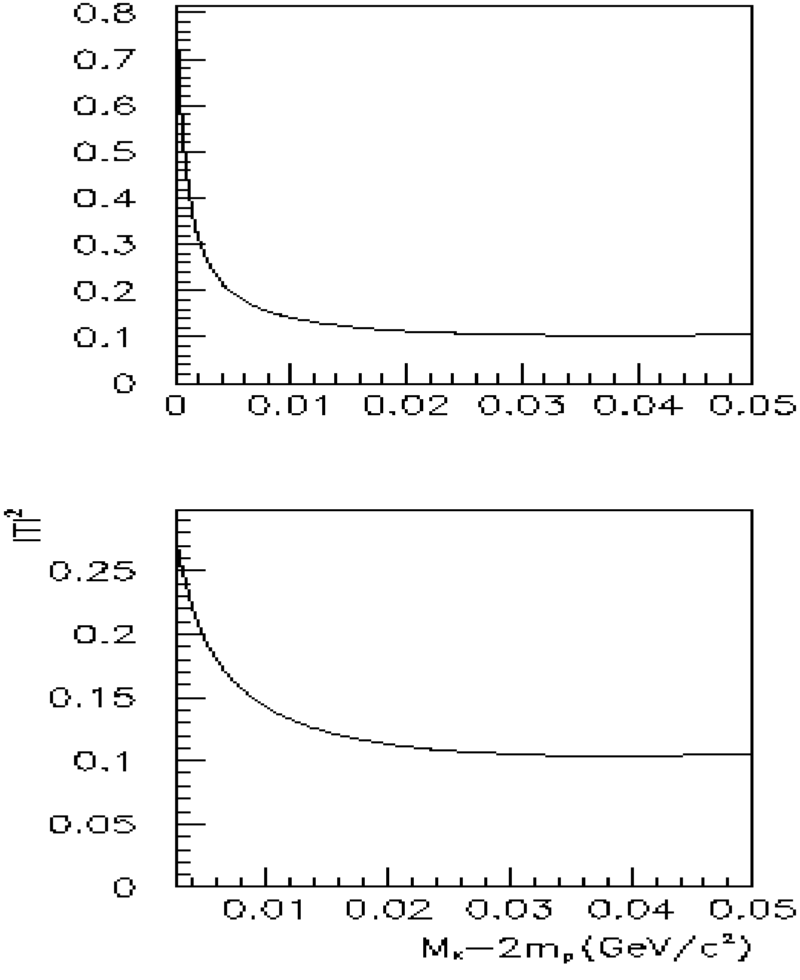
I=0, $\bar{p}$p ($^1S_0$) gets the biggest attractive force !

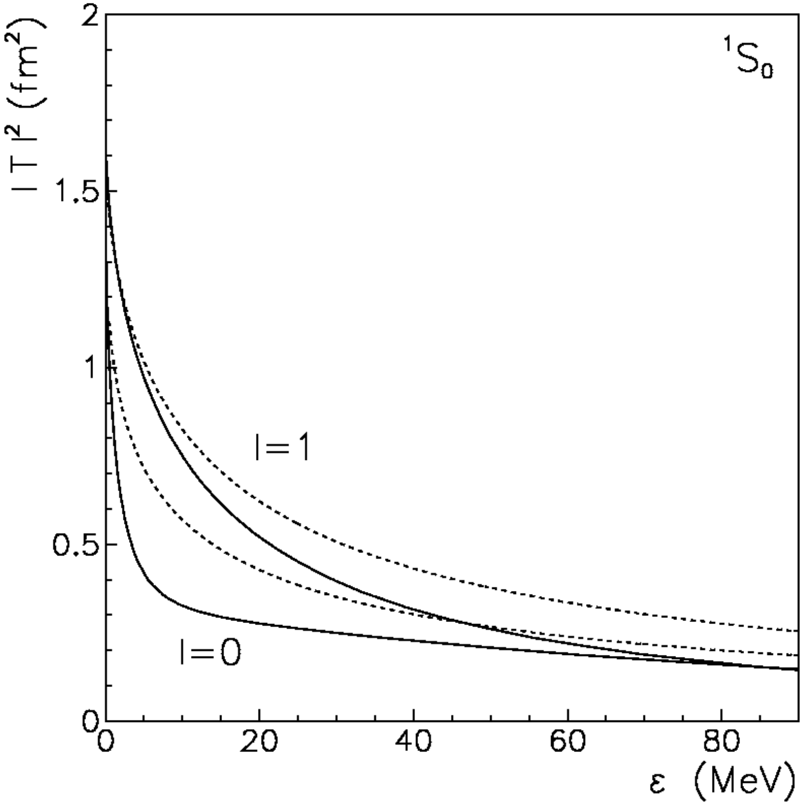
$$K_s = \frac{1}{4k^2}\int_{-4k^2}^{0} dt V_{p\bar{p}}^{\pi}(t) = -\frac{3f_\pi^2}{4k^2} ln(1+\frac{4k^2}{m_\pi^2})$$

$$T_{J/\psi\to\gamma p\bar{p}} = \frac{T^{(0)}_{J/\psi\to\gamma p\bar{p}}}{1 - i\rho_{p\bar{p}}K_s} = \frac{CK_\gamma}{1 + i\frac{3M_p^2}{k\sqrt{s}}\frac{f_\pi^2}{4\pi} ln(1+\frac{4k^2}{m_\pi^2})}$$

$\pi+\sigma+\rho+\omega$ exchange FSI & full FSI by A.Sibirtsev et al.

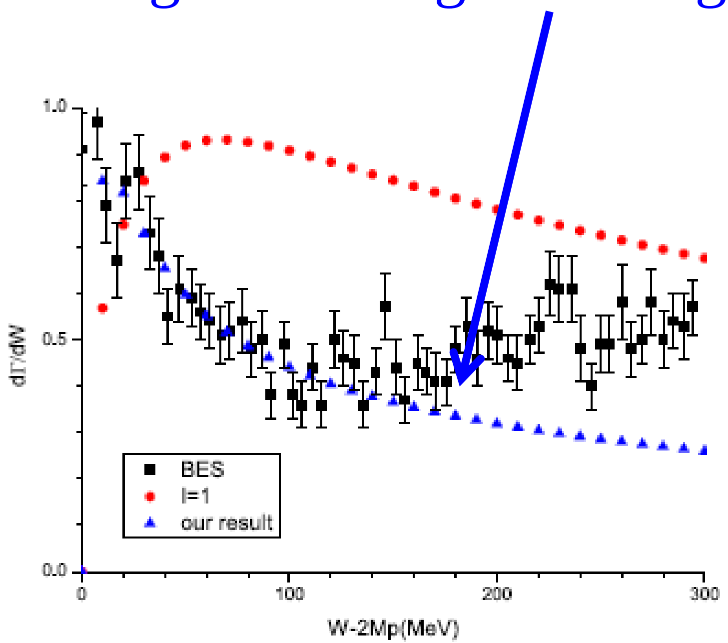
Phys.Rev. D71 (2005) 054010





G.Y. Chen, H.R. Dong, J.P. Ma, **arXiv:0807.0296** [hep-ph]

One-pion-exchange including zero-range repulsive force



In summary, $\bar{p}p$ near threshold enhancement is very likely due to some broad sub-threshold 0^{-+} resonance(s) plus FSI.

3. KΛ s-wave near threshold enhancement

complimentary BES and COSY experiments

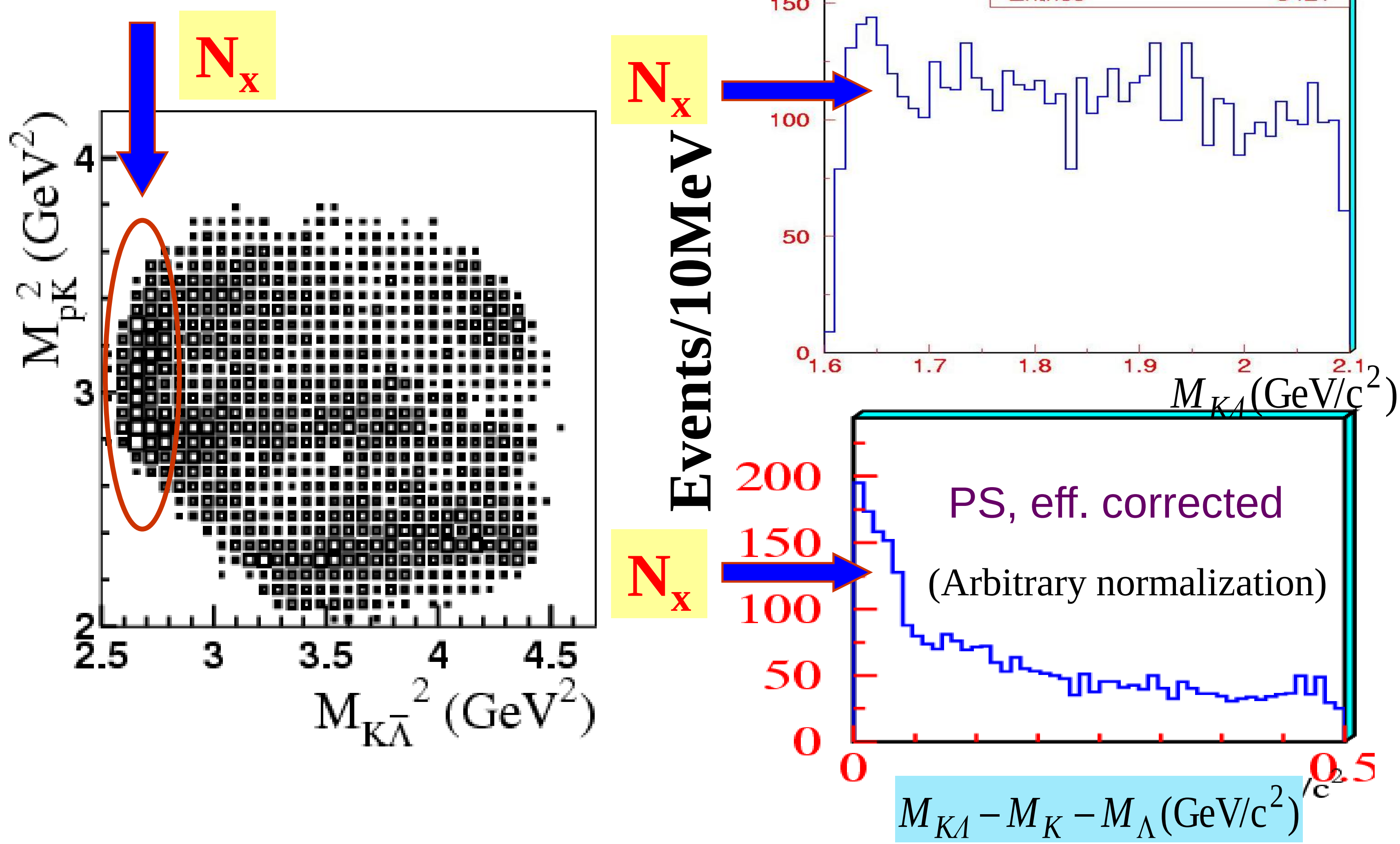
$J/\psi \rightarrow P K^- \bar{\Lambda}$ vs $P P \rightarrow P K^+ \Lambda$

$P \bar{\Lambda}$ & $P \Lambda$ — **the same t-channel interaction**

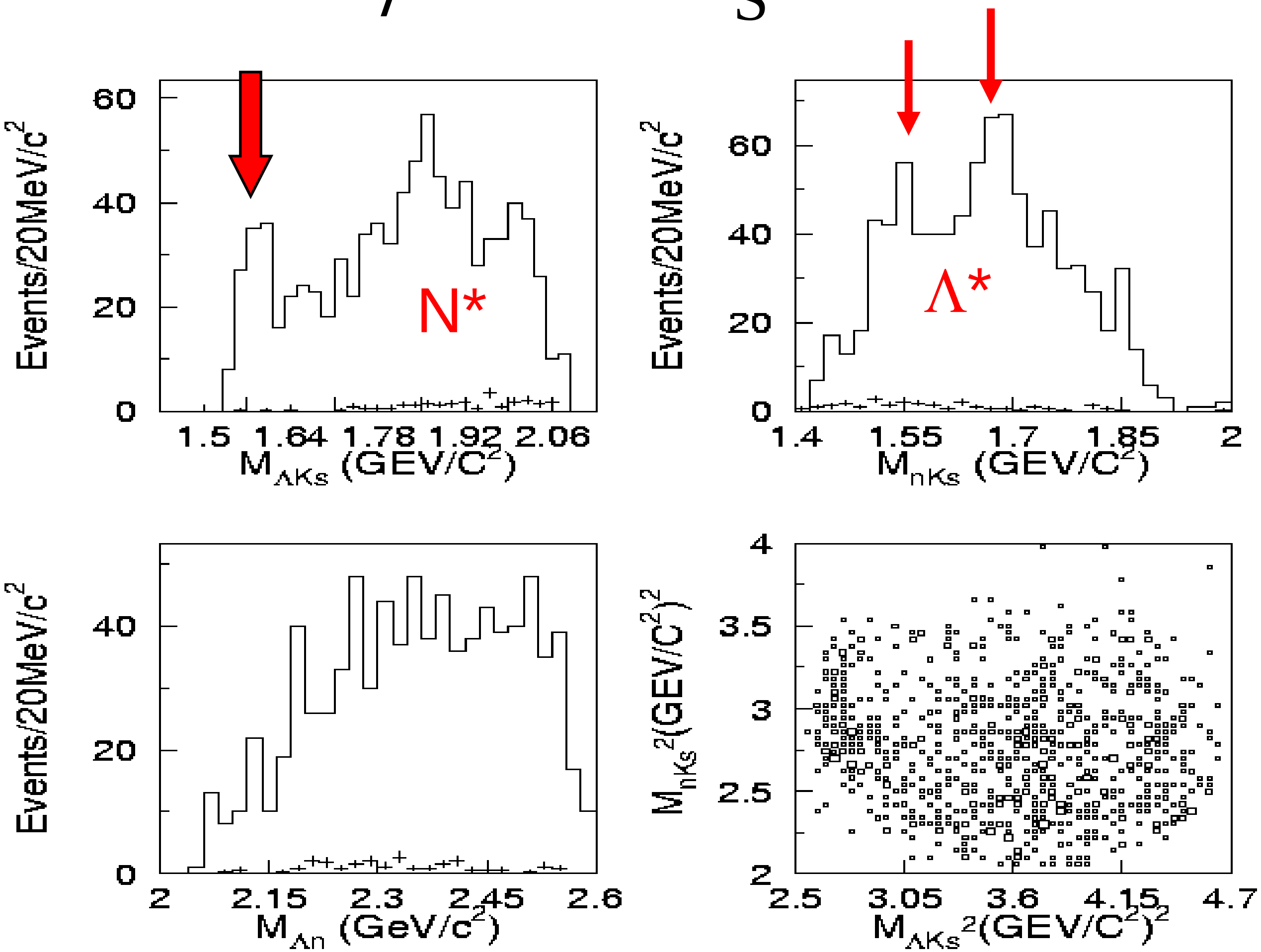
$K^- \bar{\Lambda}$ & $K^+ \Lambda$ — **the same interaction**

$P K^-$ for Λ^* — **$P K^+$ for pentaquarks**

Near-threshold enhancement in $M_{K\Lambda}$

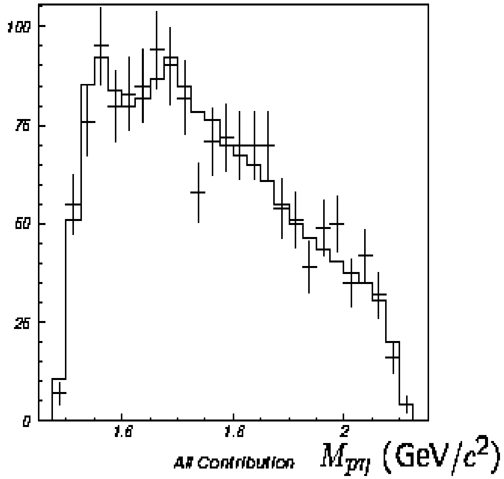


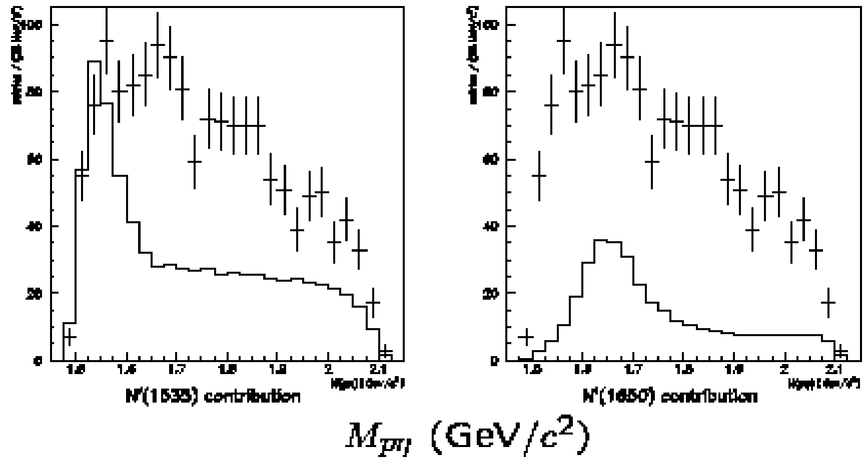
$J/\psi \to nK_S^0\overline{\Lambda}$



- **An enhancement near ΛK_S threshold is evident**
- **N* and Λ* found in the ΛK_S and nK_S spectrum**

PWA Results from $J/\psi \to p\bar{p}\eta$ (BESI 7.8M)





$N^*(1535)$ parameters	BES	PDG2000
Mass (MeV) Γ (MeV)	1530 ± 10 95 ± 25	$1520 - 1555$ $100 - 250$
$N^*(1650)$ parameters	BES	PDG2000
Mass (MeV) Γ (MeV)	1647 ± 20 145^{+80}_{-45}	$1640 - 1680$ $145 - 190$

BES Collaboration, Phys. Lett. B510 (2001) 75

B.C.Liu and B.S.Zou, Phys. Rev. Lett. 96 (2006) 042002

From relative branching ratios of

$J/\psi \rightarrow p\ \bar{N}^* \rightarrow p\ (K^-\ \bar{\Lambda})\ /\ p\ (\bar{p}\eta)$

↓

$g_{N^*K\Lambda} / g_{N^*p\eta} / g_{N^*p\pi} \sim 1.3 : 1 : 0.6$

↓

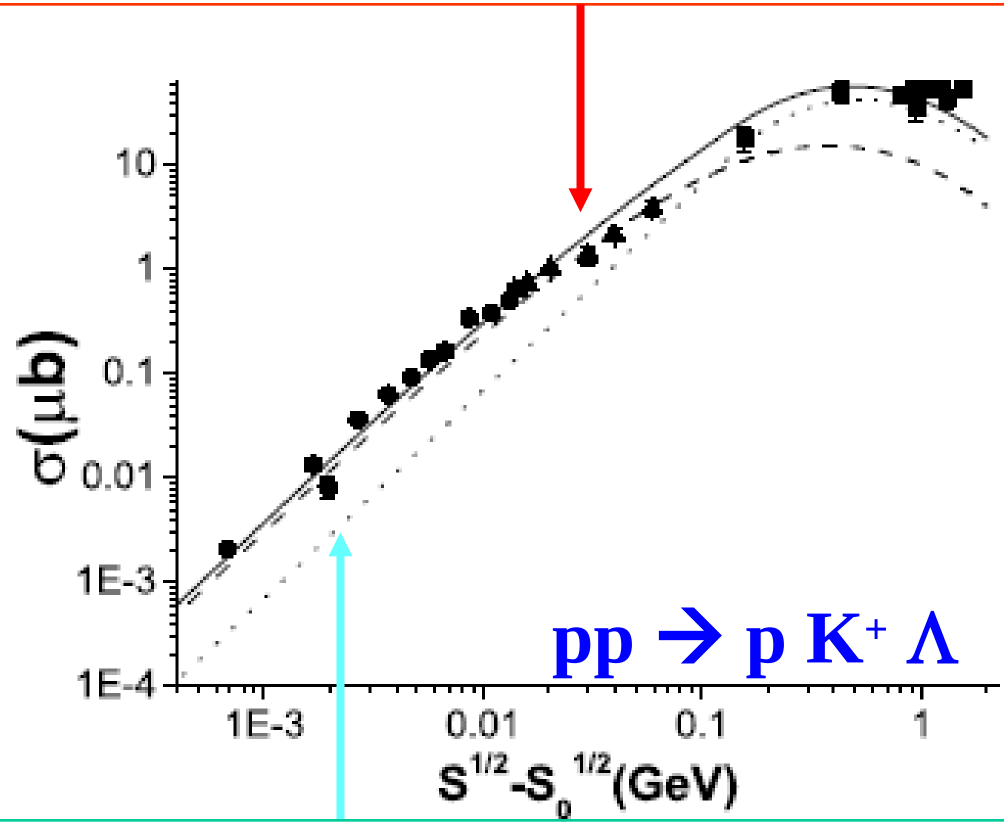
Smaller N*(1535) BW mass

previous results 0 ~ 2.6 from πN and γN data

Evidence for large $g_{N^*K\Lambda}$ from $pp \rightarrow p K^+ \Lambda$

Total cross section and theoretical results with N*(1535), N*(1650), N*(1710), N*(1720)

B.C.Liu, B.S.Zou, Phys. Rev. Lett. 96 (2006) 042002

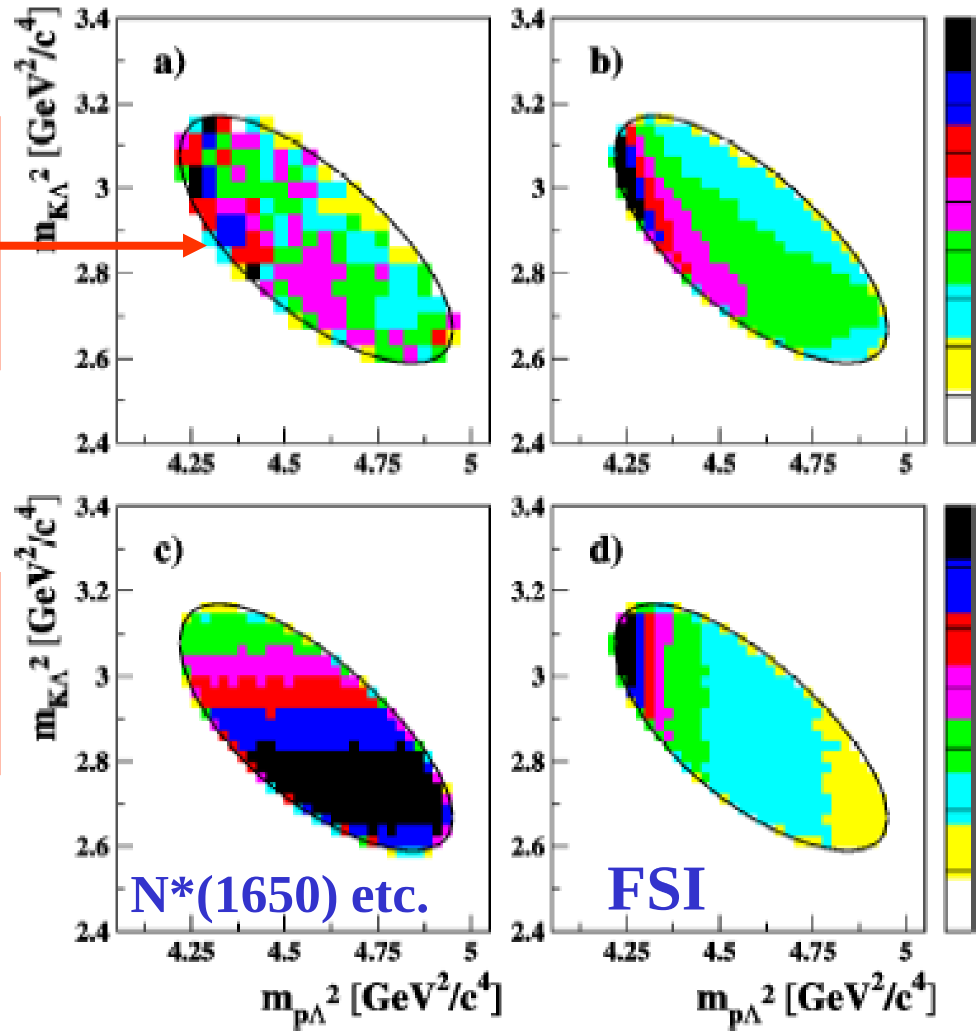


Tsushima,Sibirtsev,Thomas, PRC59 (1999) 369, without including N*(1535)

FSI vs N*(1535) contribution in pp → p K^+ Λ

B.C.Liu & B.S.Zou, Phys. Rev. Lett. 98 (2007) 039102 (reply)
A.Sibirtsev et al., Phys. Rev. Lett. 98 (2007) 039101 (comment)

COSY-TOF data
S. Abdel-Samad *et al.*,
Phys.Lett.B632:27(2006)



Both FSI & N*(1535) are needed !

Interference between N*(1535) and non-resonant FSI

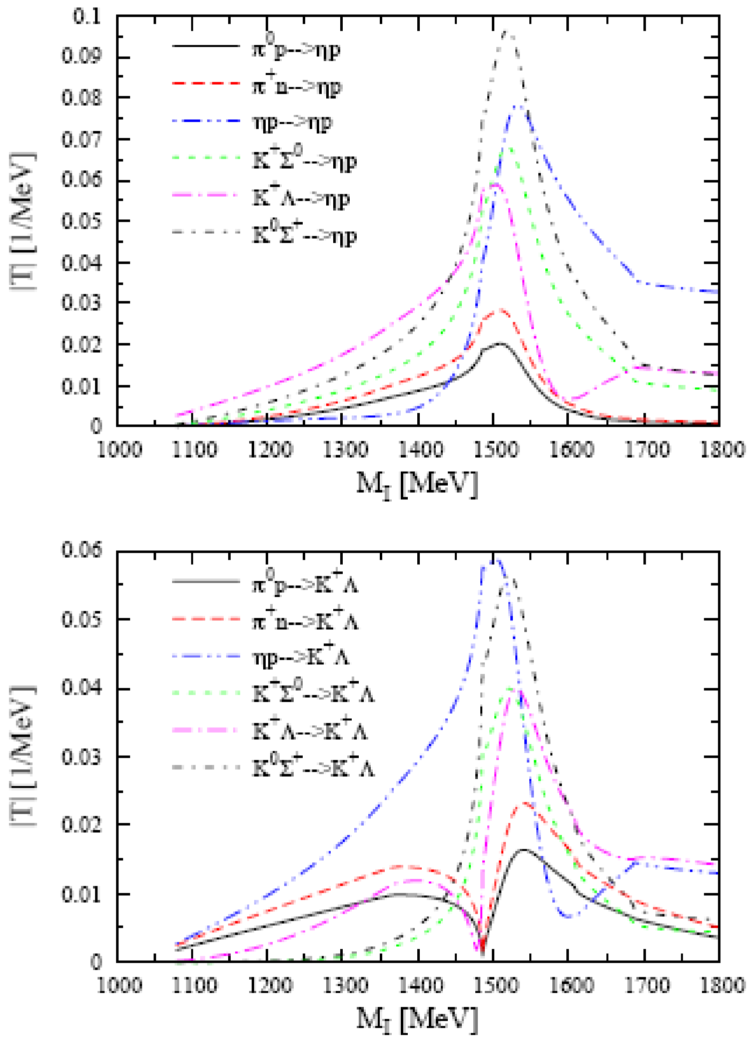


FIG. 1: The moduli of the transition amplitudes in different channels leading to the ηp and $K^+\Lambda$ final states.

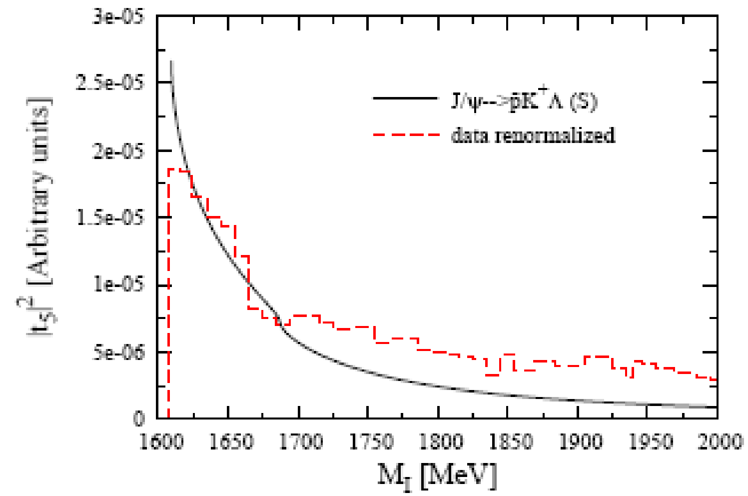


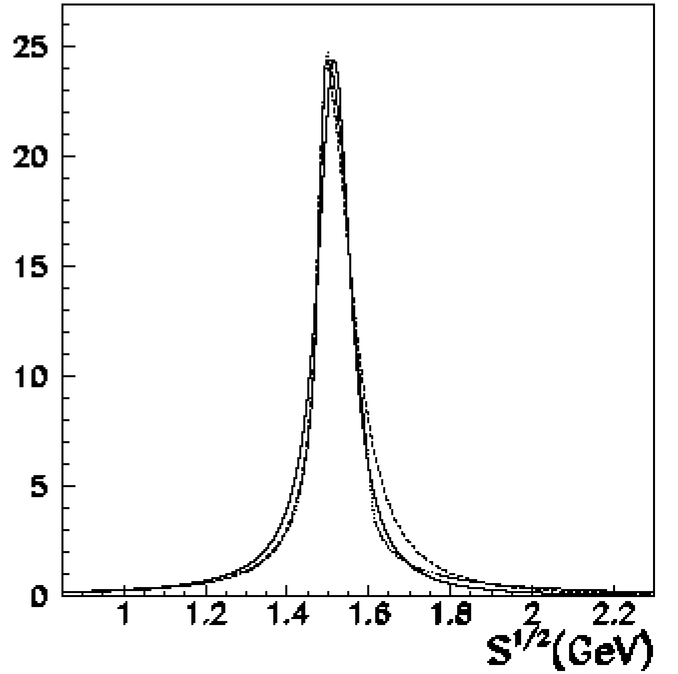
FIG. 4: Modulus squared of the amplitude of $J/\psi \to \bar{p}K^+\Lambda$ in comparison with the equivalent quantity obtained experimentally (integrated cross section weighted by phase space) [17].

$$R = \frac{|g_{N^*(1535)K\Lambda}|}{|g_{N^*(1535)\eta N}|} = 0.5 \sim 0.7.$$

L.S.Geng, E.Oset, B.S.Zou, M.Döring, arXiv:0807.2913v1 [hep-ph]

Mass of N*(1535)

$$BW(p_{N^*}) = \frac{1}{M_{N^*}^2 - s - iM_{N^*}\Gamma_{N^*}(s)}$$



(1)

$$\Gamma_{N^*}(s) = 98MeV$$

$$M_{N^*} = 1515MeV$$

(2) $$\Gamma_{N^*}(s) = \Gamma_{N^*}^0 \left(0.5\frac{\rho_{\pi N}(s)}{\rho_{\pi N}(M_{N^*}^2)} + 0.5\frac{\rho_{\eta N}(s)}{\rho_{\eta N}(M_{N^*}^2)}\right) = \Gamma_{N^*}^0\,[0.8\rho_{\pi N}(s) + 2.1\rho_{\eta N}(s)]$$

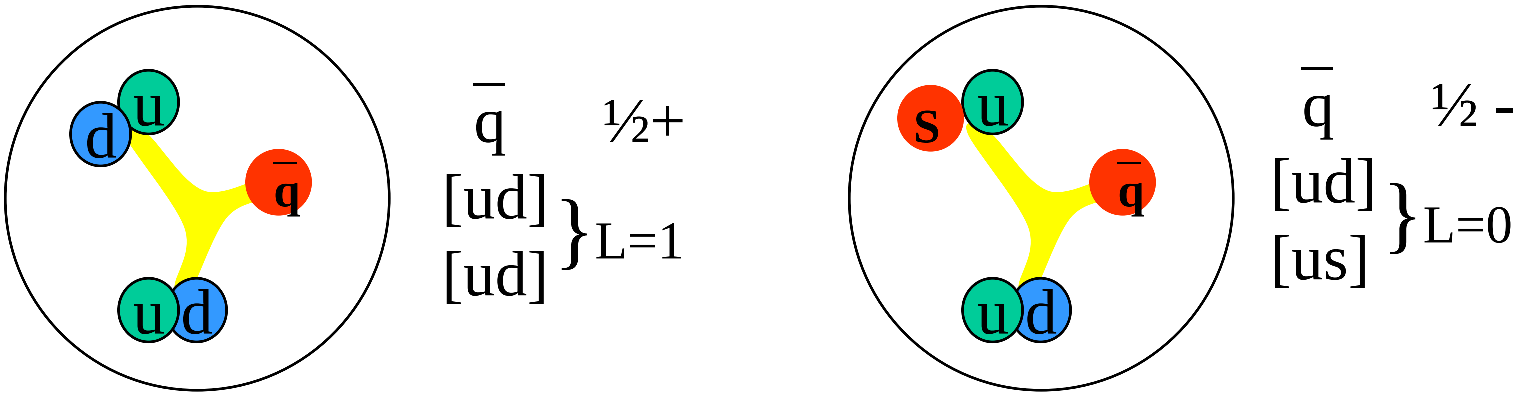
$$M_{N^*} = 1535MeV \text{ and } \Gamma_{N^*}^0 = 150MeV$$

(3) $$\Gamma_{N^*}(s) = \Gamma_{N^*}^0\,[0.8\rho_{\pi N}(s) + 2.1\rho_{\eta N}(s) + 3.5\rho_{\Lambda K}(s)]$$

$$M_{N^*} \approx 1400MeV$$

$$\Gamma_{N^*}^0 = 270MeV$$

Nature of N*(1535) and its 1/2⁻ octet partner



Zhang et al, hep-ph/0403210

N*(1535) ~ uud (L=1) + ε [ud][us] $\bar{s}$ + …

N*(1440) ~ uud (n=1) + ξ [ud][ud] $\bar{d}$ + …

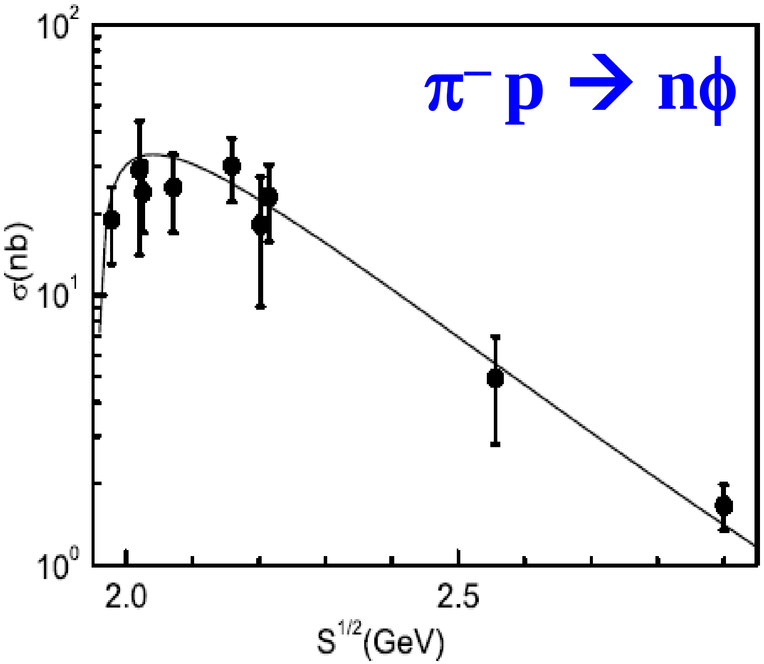
Λ*(1405) ~ uds (L=1) + ε [ud][su] $\bar{u}$ + …

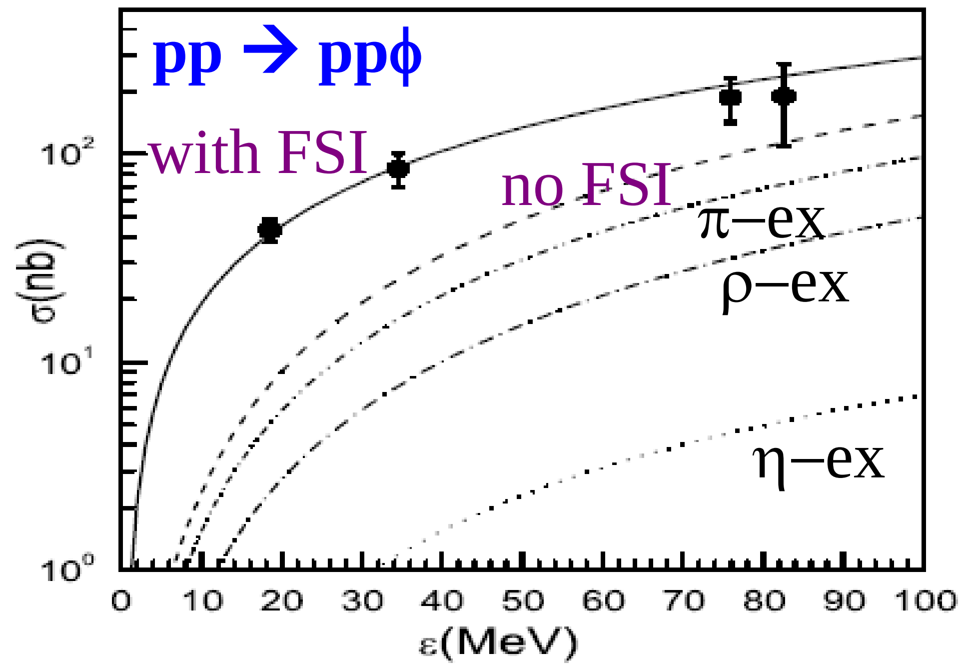
N*(1535): [ud][us] $\bar{s}$ → larger coupling to Nη, Nη', Nϕ & KΛ, weaker to Nπ & KΣ, and heavier !

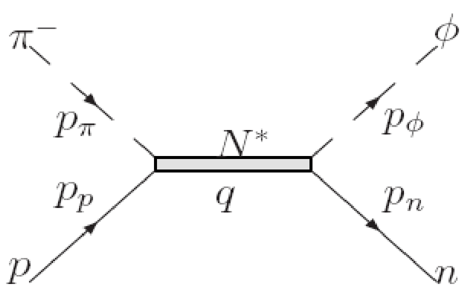
B.C.Liu, B.S.Zou, PRL 96(2006)042002

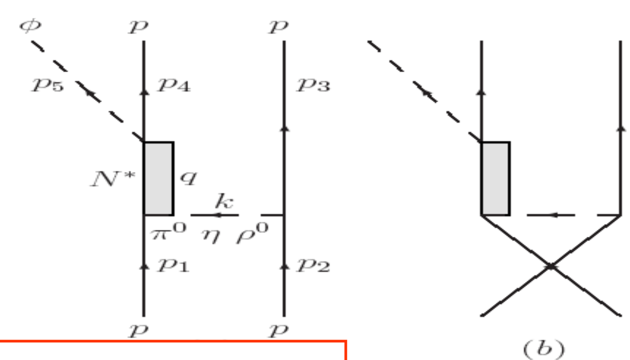
Evidence for large $g_{N^*N\phi}$ from $\pi^- p \to n\phi$ & $pp \to pp\phi$

Xie, Zou & Chiang, PRC77(2008)015206









Evasion of OZI rule by N*(1535) !

Sub-threshold $\Delta^{++*}(1620)$ in $pp \rightarrow nK^+\Sigma^+$

J.J.Xie, B.S.Zou, PLB649 (2007) 405

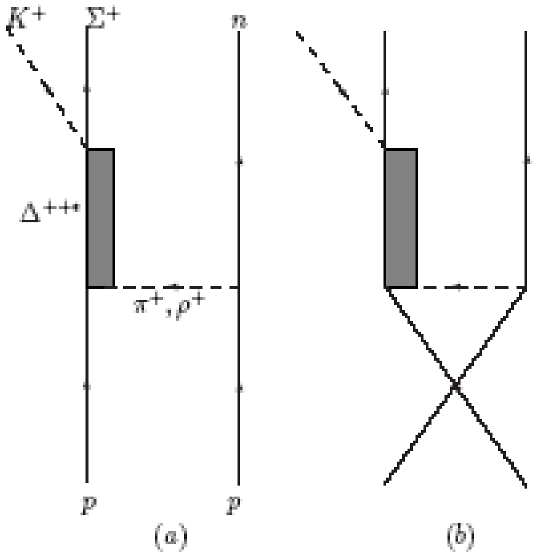
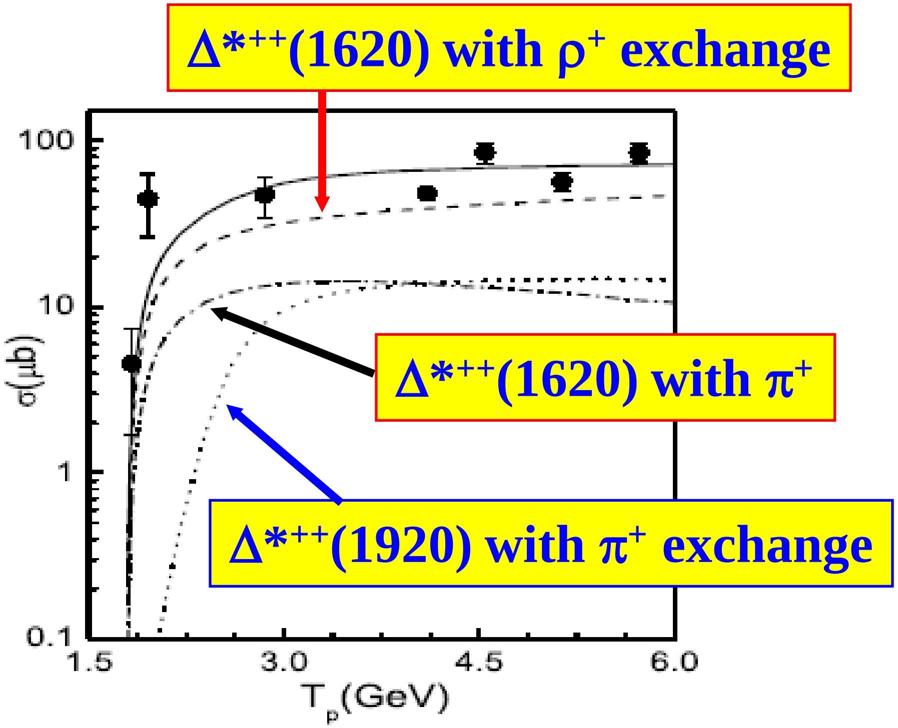


Figure 1: Feynman diagrams for $pp \rightarrow nK^+\Sigma^+$ reaction.



t-channel ρ-exchange plays important role !

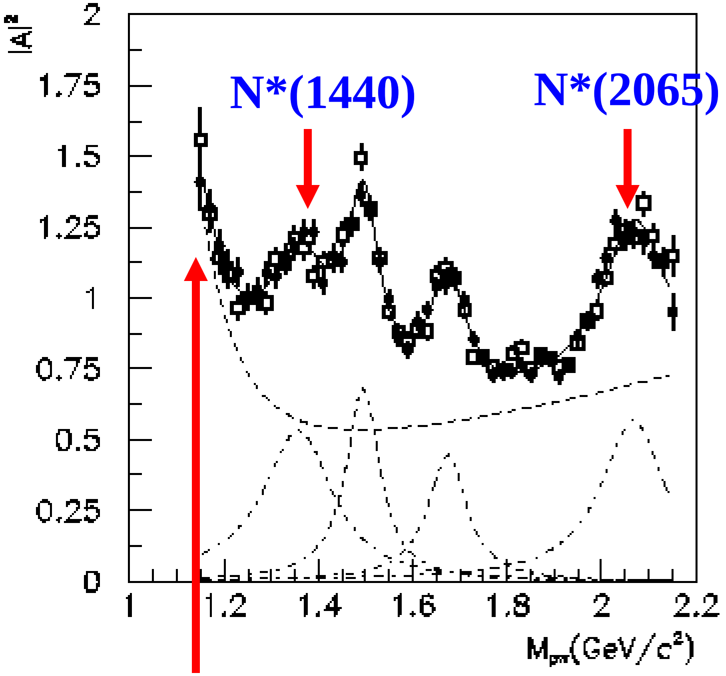
Summary for our study on $J/\psi \rightarrow P K^- \bar{\Lambda}$ and $P P \rightarrow P K^+ \Lambda$

1) **$K\Lambda$ near-threshold enhancement due to sub-threshold N*(1535) with large $g_{N^*K\Lambda}$**

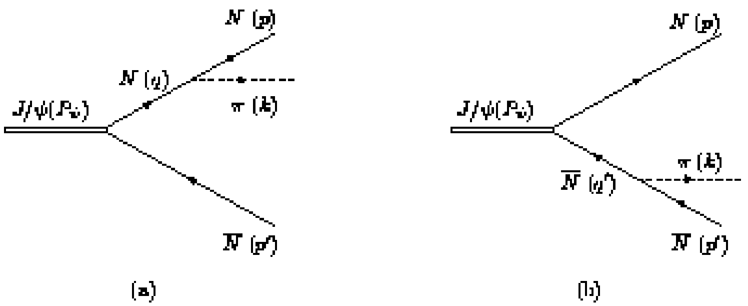
2) **Larger [ud][us] $\bar{s}$ component in N*(1535) makes it coupling stronger to strangeness and heavier !**

Observation of Two New N^* Peaks in $J/\psi \to p\pi^-\bar{n}$ and $\bar{p}\pi^+n$ Decays

BES Collaboration



The first experiment "see" N*(1440) and a "missing" N*(2065) peak



Nucleon-pole diagrams for $J/\psi \to \pi N\bar{N}$ decay.

Off-shell nucleon contribution

If fitting it with a simple BW formula, its mass and width are not compatible with any PDG known particle; and it has an "un-usual large BR" to πN !

But it is NOT a new resonance !

Comment on BR of sub-threshold resonances

$a_0(980)$ has large BR to $\bar{K}K$

X(1859) has large BR to $\bar{p}p$

Nucleon has large BR to πN

?

One should either change "large" to "zero"
or change "BR" to "coupling"