Basic BES PWA procedure (Part B)

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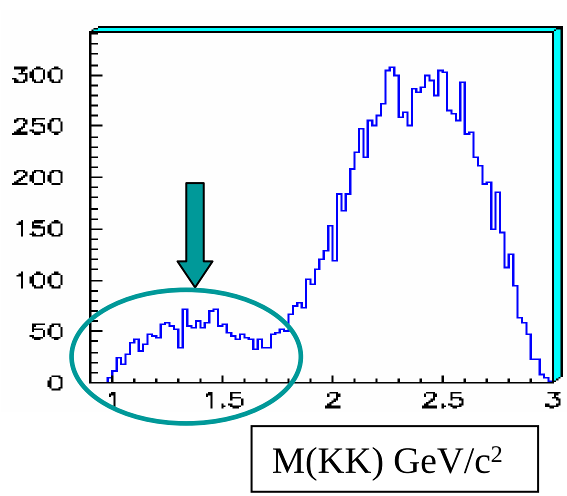
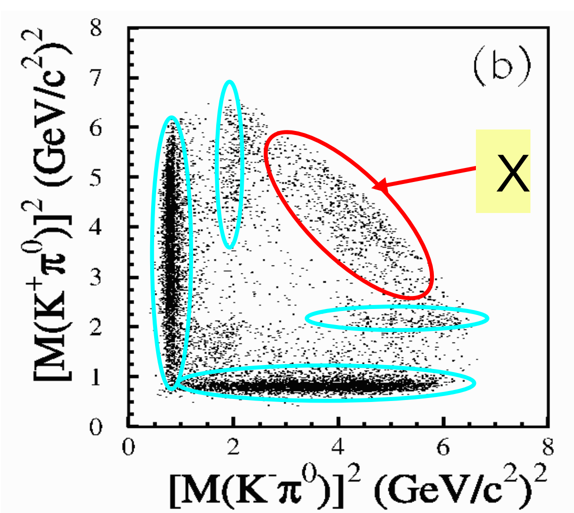
How to get the "best" solution

$J/\psi \to KK\pi^0$:

$$J/\psi \to X\pi^0, \quad X \to K^+K^-;$$
$$J/\psi \to \rho\pi^0, \quad \rho \to K^+K^-;$$
$$J/\psi \to K^{*\pm}K^{\mp}, \quad K^{*\pm} \to K^{\pm}\pi^0.$$
$$\rho: \ 1^-, 3^-, 5^- \quad \ldots$$
$$K^*: \ 1^-, 2^+, 3^- \quad \ldots$$





According to PDG, the possible resonance including:

K*(892), K*(1410), K*(1680)

K^*_2(1430), K^*_2(1980),

K^*_3(1780),

K^*_4(2045)

ρ(770), ρ(1450), ρ(1700), ρ(1900), ρ(2150)

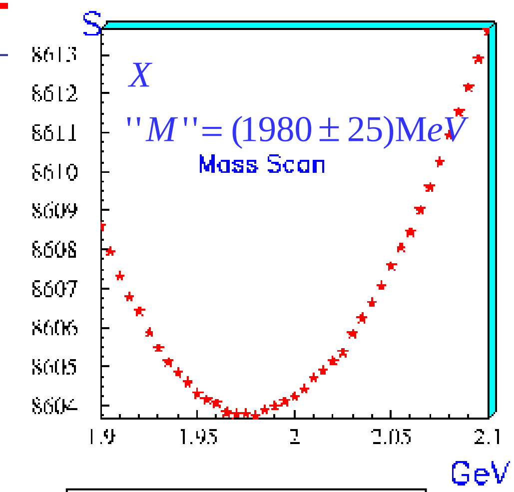
ρ_3(1690), ρ_3(1990), ρ_3(2250),

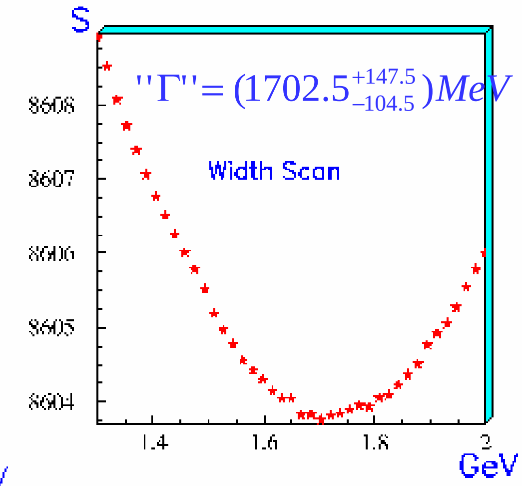
According to the statistical significance of each resonance, determine which one is needed and finally determine the "best" solution.

The results of the “best” solution

- **In this combination:**
  - **Including**
    - **K*(892),**
    - **K*(1410),**
    - **X,**
    - **ρ (1700)**
    - **phase space(p.s.)**

- **Scan the mass and width for the unknown resonance X.**





Pole position:

$$M-\frac{i}{2}\Gamma$$
$$=(1576^{+49}_{-55}(stat))-\frac{i}{2}(818^{+22}_{-23}(stat))MeV/c^2$$

Branching ratio:

$$BR(J/\psi\rightarrow X\pi^0).BR(X\rightarrow K^+K^-)$$
$$=(8.5\pm 0.6(stat))\times 10^{-4}$$

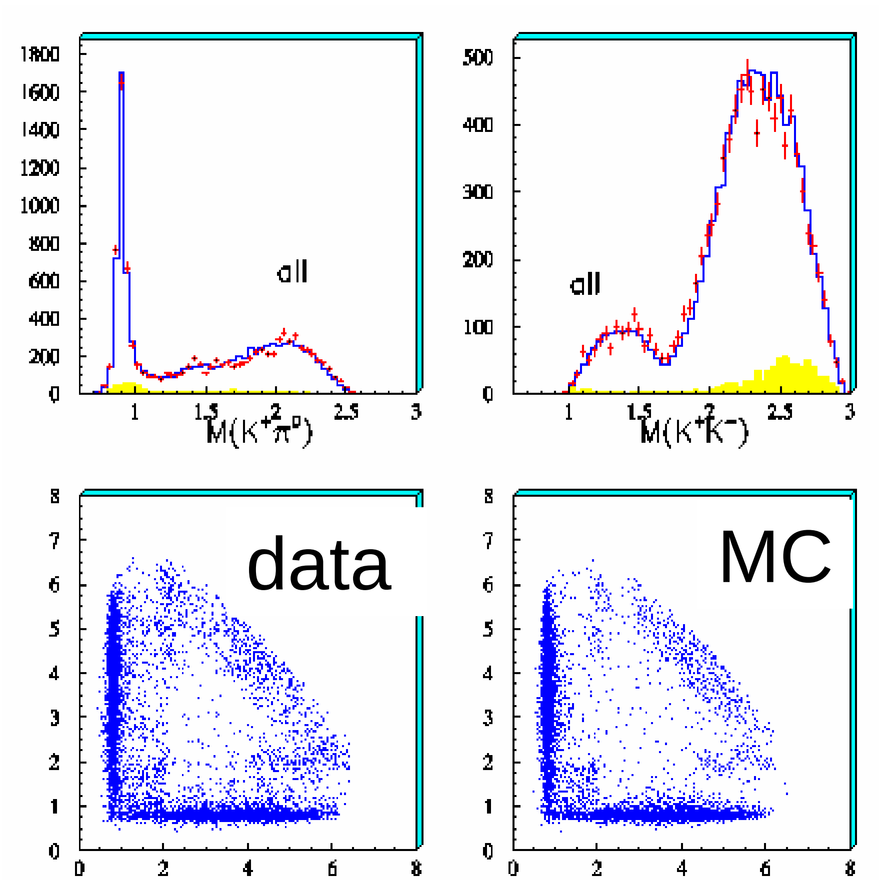
Spin-parity:

$$J^{pc}=1^{--}$$

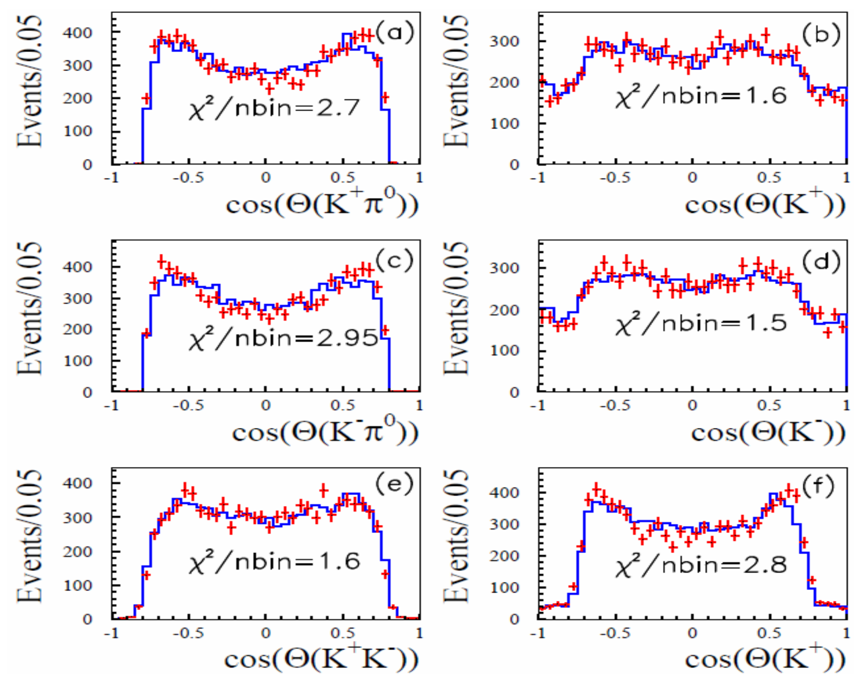
Projections of the fitting results

error bar: data(red)
histogram: projection of fitting results

no.	meson	percent	events
1	K1*(892)+	41.217	4068.07
2	K1*(892)-	41.185	4064.99
3	K1*(1410)+	6.361	627.83
4	K1*(1410)-	6.444	636.07
19	rh1(X)0	44.611	4403.07
20	rh1(1700)0	3.480	343.52
23	inter-bg20	17.226	1700.21



Mass spectrum and Dalitz plot



Angular distrubtions

Various checks about the result:

- This is the part which consume the most time of the whole analysis.
  - Statistical significance of each resonance in the "best" solution
  - Statistical significances of other resonances
  - Try to describe the broad structure by known resonances or their interferences
  - J^{pc} of X
- Above three items may prove the combination will be the best one.

Statistical significance check

resonance	ΔS	Δndf	significance
K*(892), K*(1410), X, ρ (1700) , p.s.			
Remove ρ (1700)	28	2	7.2 σ
Remove X	533	4	>>10 σ
Remove K*(892)	11438	2	>>10 σ
Remove K*(1410)	465	2	>>10 σ
Remove P.S.	254	2	>>10 σ

→ **All above resonances have significances lager than 5σ.**

Describe it by known resonances

- It is unlikely to be ρ(1450), because:
  - The parameters of the X are incompatible with ρ(1450).
  - ρ(1450) has very small fraction to KK. From PDG:

    $BR(\rho(1450) \rightarrow K^+K^-) < 1.6 \times 10^{-3}$ (95%C.L.)

- It cannot be fitted with the interference of ρ(770) , ρ(1900) and ρ(2150):
  - The log-likelihood value is worsen by 85.

→ The broad resonant structure, X, can not be described by any known resonances or their interferences.

Other resonances

For other resonances, such as $K^*_2(1430)$, $K^*(1680)$, $K^*(2075)$, ρ (770), ρ (1900) and ρ (2150), the statistical significances are less than 5σ,

➔These resonances are not included in the "best" solution, but their impacts will be considered into systematic uncertainty.

J^{pc} check

For parity conservation, the J^{pc} of X can only be 1^{--}, 3^{--}, 5^{--},…

If J^{pc} is changed from 1^{--} to 3^{--} , the log-likelihood is worsen by 325.

Even higher spin states are unlikely at such a low mass .

➔ J^{pc} (X) should be 1^{--}

Statistics uncertainties

Statistical error

- 当子样容量很大时，**u=-2lnλ=2(lnL$_m$(b+s)-L$_m$(b)** 渐进地服从χ^2**(r)** 分布。
- 利用似然比统计量计算信号统计显著性**S**的表达式:

$$\int_{-S}^{+S}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx = 1-P(u_{obs}) = \int_0^{u_{obs}}\chi^2(u;r)du$$

- 对于**r=1**的特殊情形，有

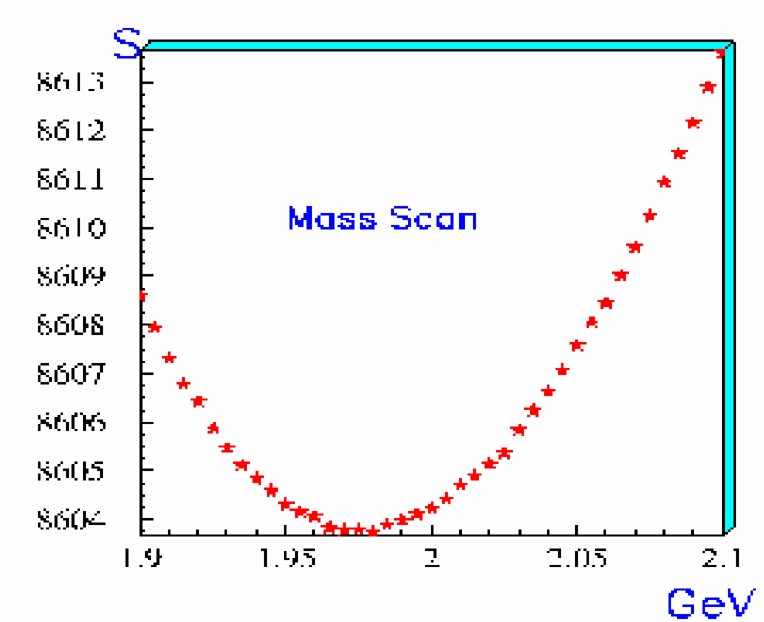
$$\int_{-S}^{+S}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx = \int_0^{u_{obs}}\chi^2(u;r)du = 2\int_0^{\sqrt{u_{obs}}}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx$$

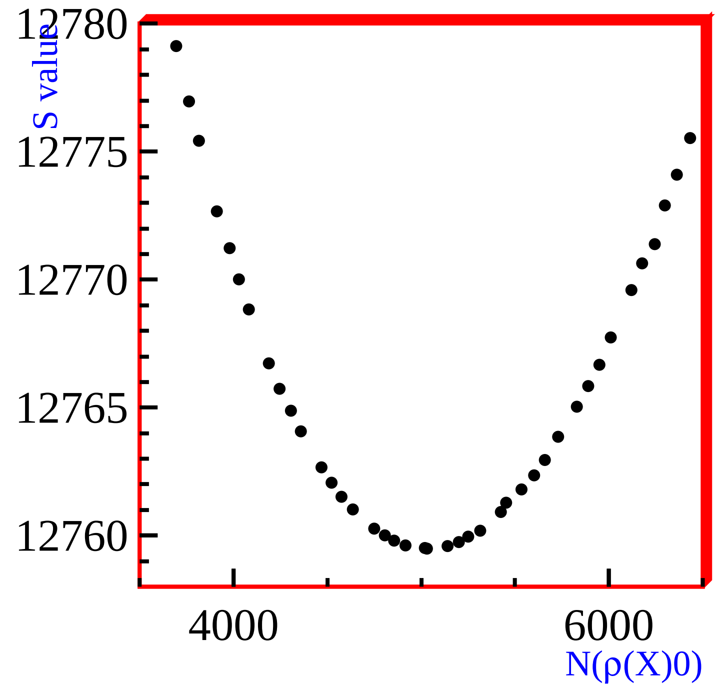
$$S = \sqrt{u_{obs}} = [2(lnL_m(s+b) - ln_m(b))]^{1/2}$$

- **If require S=1σ,**

then lnL$_m$(b+s)-L$_m$(b) =0.5.

Statistical error





Mass vs. S=-lnL

Δ S =0.5 → Δmass

From J/ψ→Ks K π,
The number of X resonance versus S=-lnL

Δ S =0.5 → ΔN →Δ BR

Systematic uncertainties

Systematic uncertainties

- Uncertainty from each resonance in the fit.
- Uncertainties from of other resonances.
- Uncertainty from background level
- The impact from different BW form of X
- The impact from MDC wire resolution simulation
- Uncertainty from X's mass and width

Systematic uncertainty from each resonance in the fit

resonance	Mass(MeV)(PDG)	Γ(MeV)(PDG)	Mass(MeV)(scanned)	ΓMeV) (scanned)
$K^\star(892)$	891.7±0.3	50.8±0.9	891.1±0.8	53.1±1.1
$K^\star(1410)$	1415±15	232±21	1438±6	158±5
$\rho(1700)$	1720±20	250±100	1705±9	239±17
X(fitted)	1576^{+49}_{-55}	818^{+22}_{-23}	1622±14	860±11

➔The differences caused by the parameters of $K^*(892)$, $K^*(1410)$ and ρ(1700) are included into systematic uncertainties.

Systematic uncertainty from inclusions of other resonances

Inclusions of other resonances will cause the dominant uncertainties of the parameters of X.

resonance	ΔS	$\Delta M(MeV)$(pole)	$\Delta\Gamma$(MeV)(pole)	$(\Delta B.R./B.R.)\%$
best solution				
add $K_2^*(1430)$	-0.1	-8.0	-17.6	-1.4
add $K^*(1680)$	-14.0	52.4	9.2	-2.1
add $K^*(2075)$	-1.6	-26.6	-52.2	0.9
add $\rho(770)$	-13.0	-58.0	-111.6	-29.9
add $\rho(1900)$	-0.1	-7.0	40.4	2.8
add $\rho(2150)$	-2.3	51.4	9.4	26.8
total		${}^{+73.4}_{-64.7}$	${}^{+42.5}_{-124.5}$	${}^{+27.0}_{-30.0}$

➔Their impacts are considered into the systematic uncertainties.

Systematic uncertainty from background level

bkg source	B.R.(%)	Δ Mass (MeV)	$\Delta\,\Gamma$ (MeV)
change $\rho\pi$ bkg by 10%	± 2.6	± 1.6	± 4.0
change π^0 sideband bkg by 50%	± 2.7	± 1.6	± 4.0
total affect	± 3.5	± 2.4	± 5.6

➔The influence from background uncertainty is small and will be taken into the systematic uncertainty.

Systematic uncertainty from different BW form

The BW form of X is changed by different forms :

$$\begin{cases} BW(R) = \frac{1}{s - M_R^2 + i\sqrt{s}\Gamma_R(s)} \\ \Gamma_R(s) = \Gamma_R(\frac{M_R^2}{s})(\frac{p(s)}{p(M_R^2)})^{2l+1} \end{cases} \longrightarrow BW(R) = \frac{1}{s - M_R^2 + iM_R\Gamma_R}$$

$$M - \frac{i}{2}\Gamma = (1584 \pm 39(stat)) - \frac{i}{2}(821 \pm 72(stat))MeV/c^2$$

$$BR(J/\psi \to X\pi^0).BR(X \to K^+K^-) = (8.3 \pm 0.6(stat)) \times 10^{-4}$$

➔**The difference caused by different BW form is small relatively and still will be considered into systematic uncertainty.**

Systematic uncertainty from MDC wire resolution simulation

There are two different methods to simulate MDC wire resolution at BESII. The difference of them will cover the uncertainty of tracking and kinematical fit.

If two different MC samples are used to calculate total cross section, the parameters of X will be changed.

➔The impact from MDC wire resolution simulation is considered into systematic uncertainty.

Systematic uncertainty from X's mass and width

The parameters of X given by the "best" solution:

$$M-\frac{i}{2}\Gamma=(1576^{+49}_{-55}(stat))-\frac{i}{2}(818^{+22}_{-23}(stat))MeV/c^2$$

$$BR(J/\psi\to X\pi^0).BR(X\to K^+K^-)=(8.5\pm0.6(stat))\times10^{-4}$$

➔The BR will be changed when the pole position of X is changed by 1σ, and the impact will be considered into BR's systematic uncertainty.

Systematic uncertainties

sources	Mass (MeV) (pole)	Width (MeV) (pole)	B.R.(%)
Parameters of each resonance in the fit	46.0	42.0	7.0
Inclusions of other resonances	+73.4 -64.7	+42.5 -124.5	+27.0 -30.0
Background level	+2.4	5.6	3.5
BW formulae	8.0	-5.0	2.1
MDC wire resolution simulation	-44.2	-20.2	10.1
X's M,Γ uncertainty			+6.1 -25.6
photon efficiency			4.0
particle identification			2.0
$N_{J/\psi}$			4.7
Total	+98 -91	+64 -133	+31 -42

Considering the systematic uncertainties, the parameters of X given by PWA are listed here:

$$M - \frac{i}{2}\Gamma$$
$$= (1576^{+49}_{-55}(stat)^{+98}_{-91}(syst)) - \frac{i}{2}(818^{+22}_{-23}(stat)^{+64}_{-133}(syst)) MeV/c^2$$

$$BR(J/\psi \to X\pi^0).BR(X \to K^+K^-)$$
$$= (8.5 \pm 0.6(stat)^{+2.7}_{-3.6}(syst)) \times 10^{-4}$$

$$J^{pc} = 1^{--}$$

Summary

- **In this report, basic BES PWA procedure has been introduced briefly, including**
  - **How to construct the likelihood function**
  - **How to deal with the background subtraction**
  - **How to get the statistical significance**
  - **How to get the “best” solution based on the PWA**
  - **Checks on the “best” solution**
  - **Statistical error of the result**
  - **Systematic error of the result**

- **Sincerely hope this report will be a little helpful for you to understand/perform the PWA procedure.**

Thank you!

About ρ(1700)

In this "best" solution, the significance of ρ(1700) is 7.2σ

Because data-MC inconsistencies are not included in the fit, the significance of the ρ(1700) may not be high enough to conclude that its inclusion in the fit is necessary. Therefore we list the results of the fit without ρ(1700).

$$M-\frac{i}{2}\Gamma = (1428^{+17}_{-18}(stat))-\frac{i}{2}(1073^{+29}_{-24}(stat))MeV/c^2$$

$$BR(J/\psi \to X\pi^0).BR(X \to K^+K^-)=(6.3\pm0.6(\mathrm{stat}))\times10^{-4}$$

Conclusion and discussion

- **A broad 1^{--} resonant structure around 1.5GeV/c^2 is observed in the K^+K^- mass spectrum in $J/\psi \rightarrow K^+K^-\pi^0$**

- **The parameters of the resonance given by PWA are not compatible with any known meson resonances.**

- **Broad width is expected for a multi-quark state!**

- **To understand the nature of the broad 1^{--} peak, it is important to search for a similar structure in $J/\psi \rightarrow K_sK\pi$ decay to determine its isospin.**