



# Overview of **MadGraph5\_aMC@NLO**

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CP3/CISM



# Plan

- Details of the computation
  - Evaluation of matrix-element
  - Phase-Space integration
- What is MG5\_aMC

# What are my goals

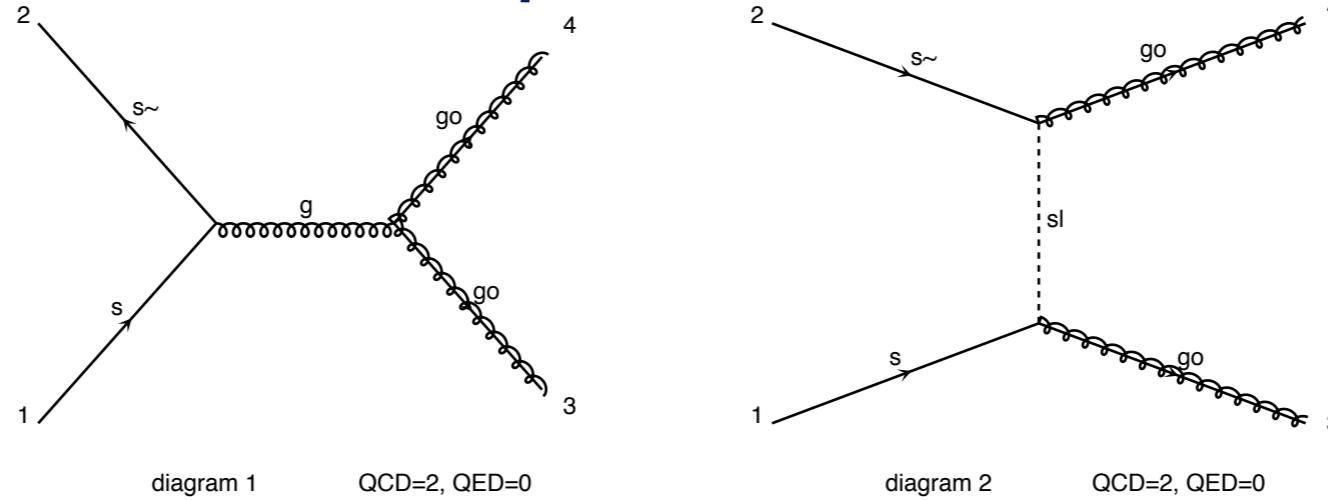
## Title

- Justify why **analytic** computation are **SLOWER** than **numerical** computation
- Justify why **adding cuts** to the code are **POSSIBLE** but can lead to **PROBLEM**
- Overview of MG5\_aMC
  - What does each package
- Details on small width handling

# Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$$|\mathcal{M}|^2$$

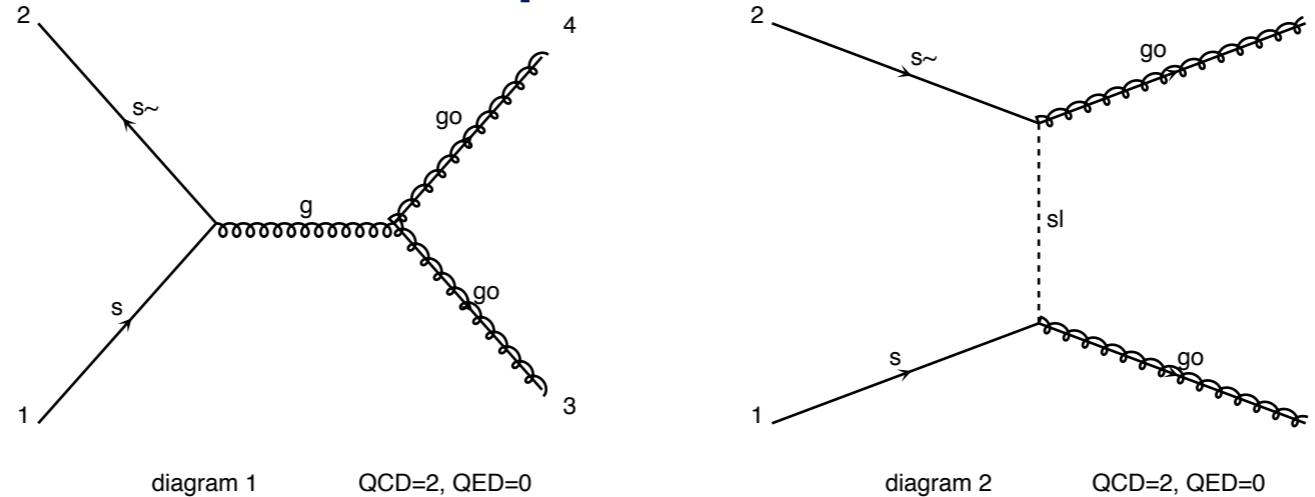
- Phase-Space Integration

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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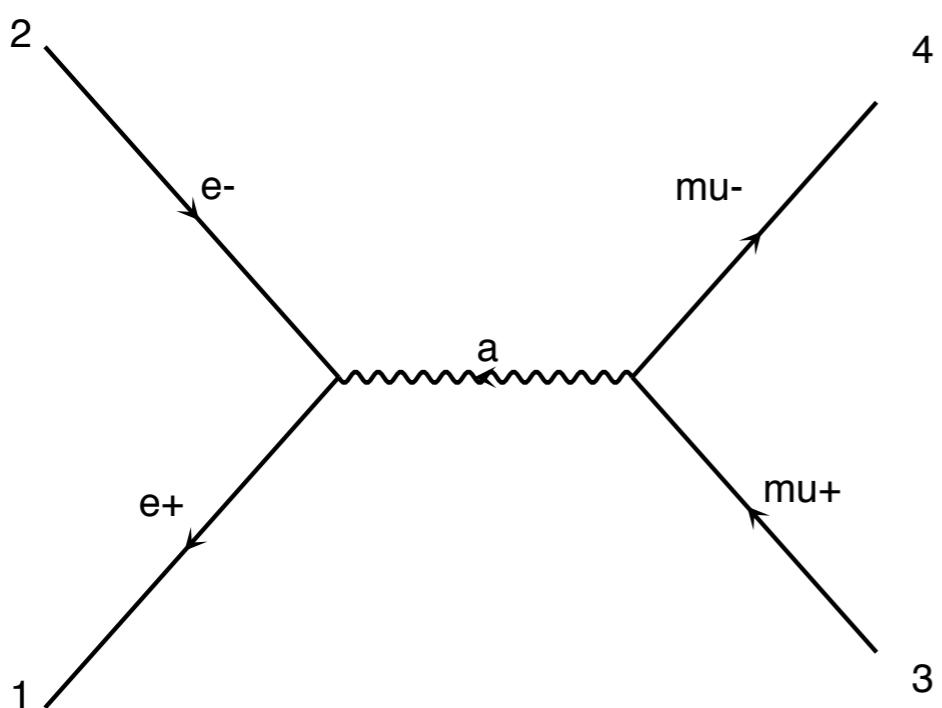
$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

Easy

Hard

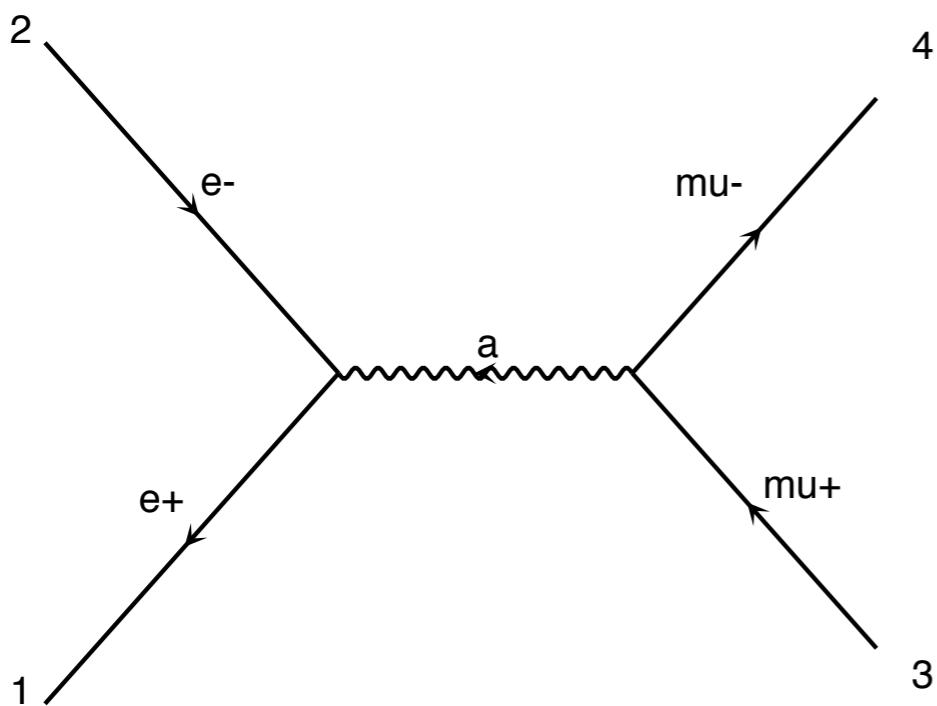
Very Hard  
(in general)

# Matrix Element



$$\mathcal{M} = e^2 (\bar{u} \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v} \gamma^\nu u)$$

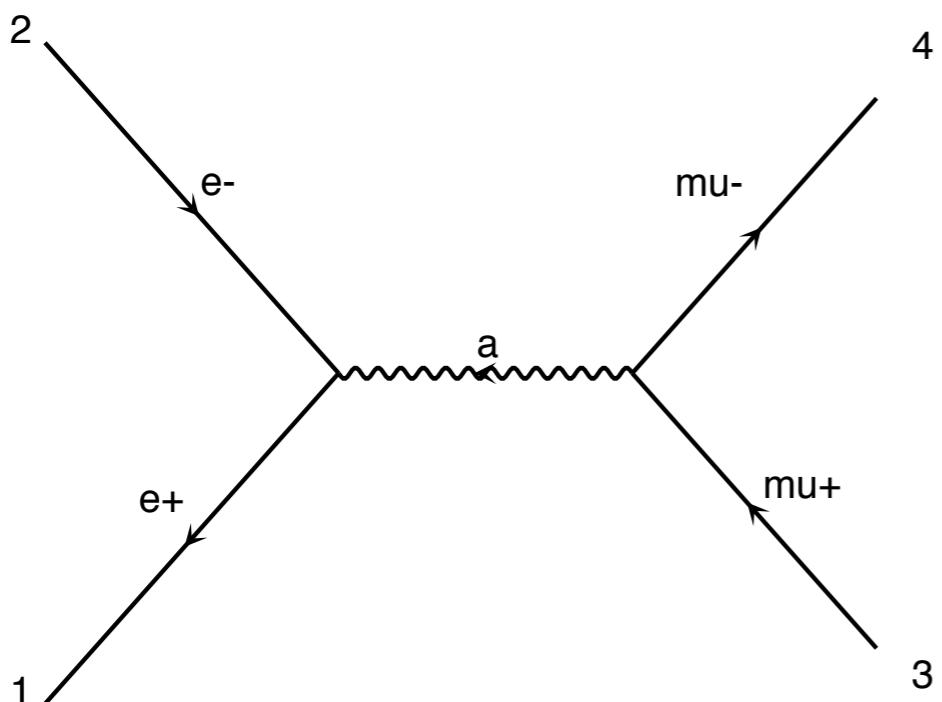
# Matrix Element



$$\mathcal{M} = e^2 (\bar{u} \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v} \gamma^\nu u)$$

$$\frac{1}{4} \sum_{pol} |\mathcal{M}|^2 = \frac{1}{4} \sum_{pol} \mathcal{M}^* \mathcal{M}$$

# Matrix Element

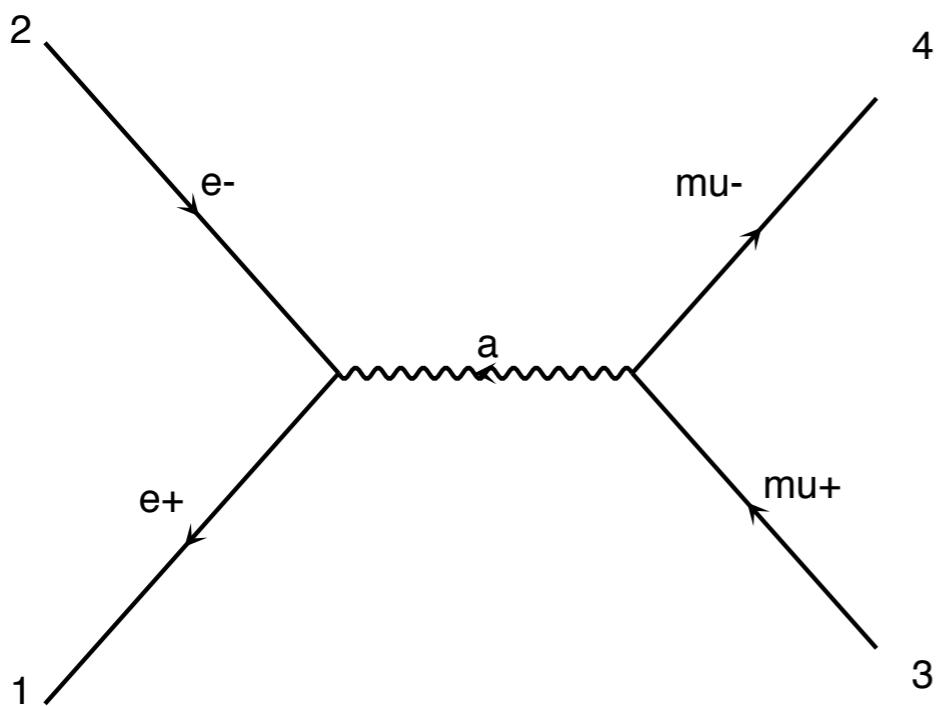


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# Matrix Element



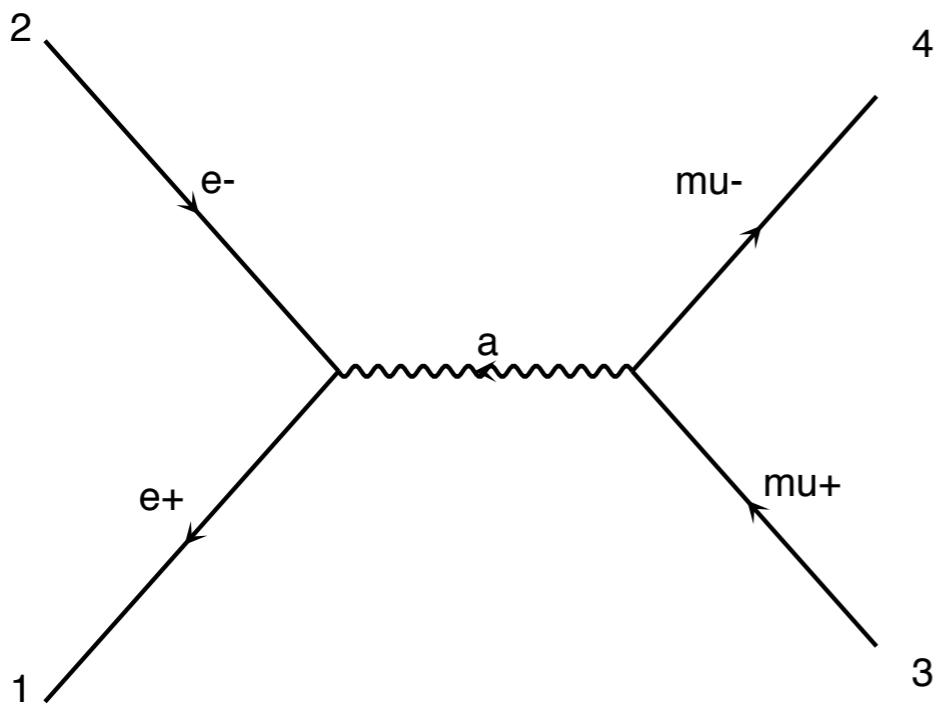
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→ 
$$\frac{e^4}{4q^4} Tr[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu] Tr[\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu]$$

# Matrix Element



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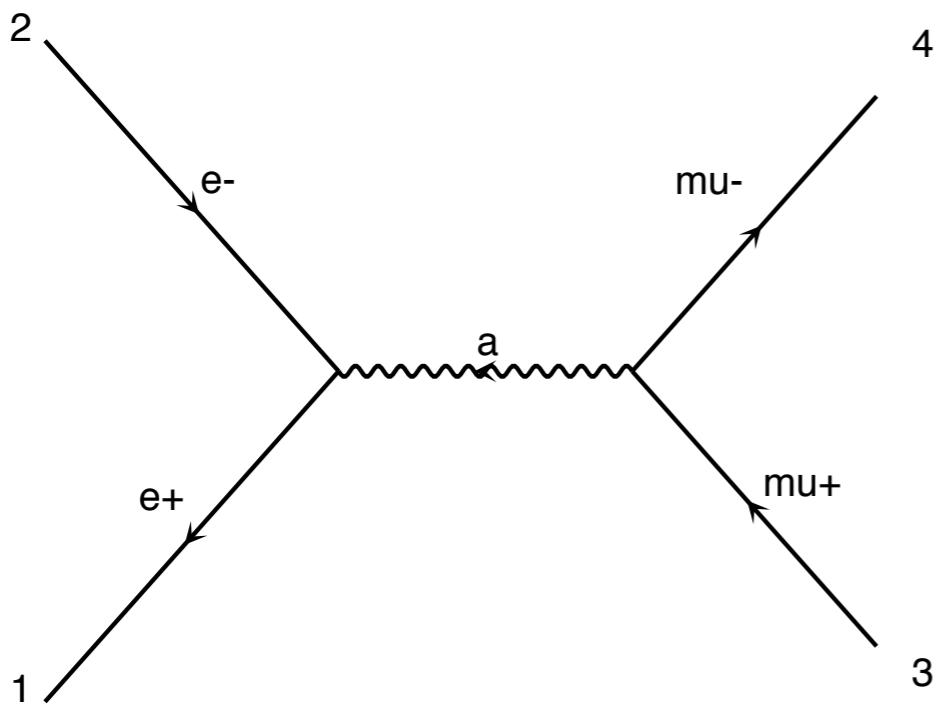
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$$\sum_{pol} \bar{u} u = p + m$$

$$\rightarrow \frac{e^4}{4q^4} Tr[\not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu] Tr[\not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu]$$

$$\rightarrow \frac{8e^4}{q^4} [(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)]$$

# Matrix Element



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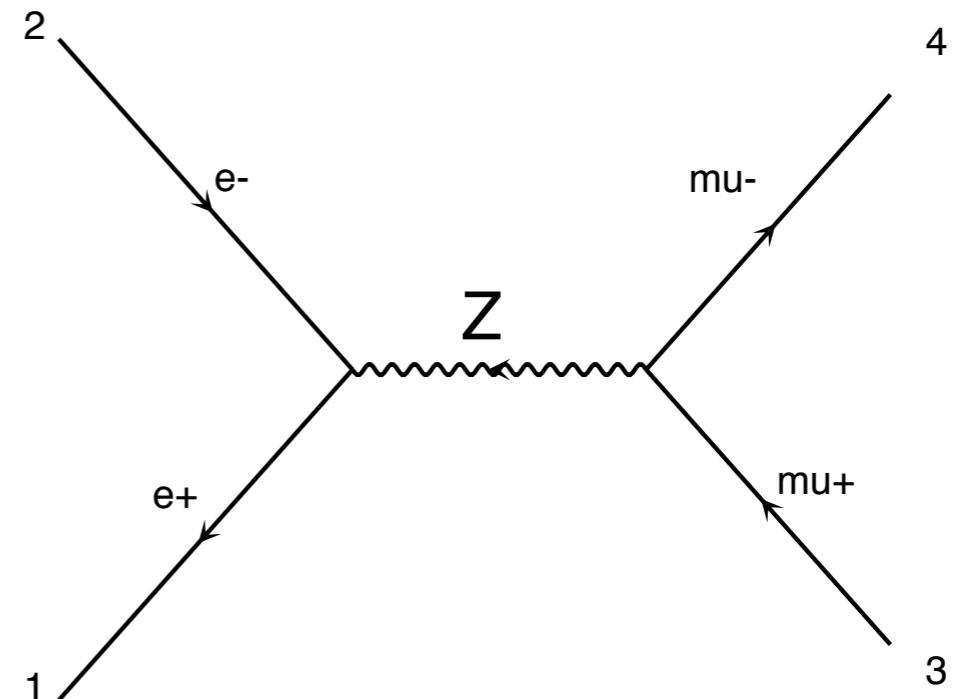
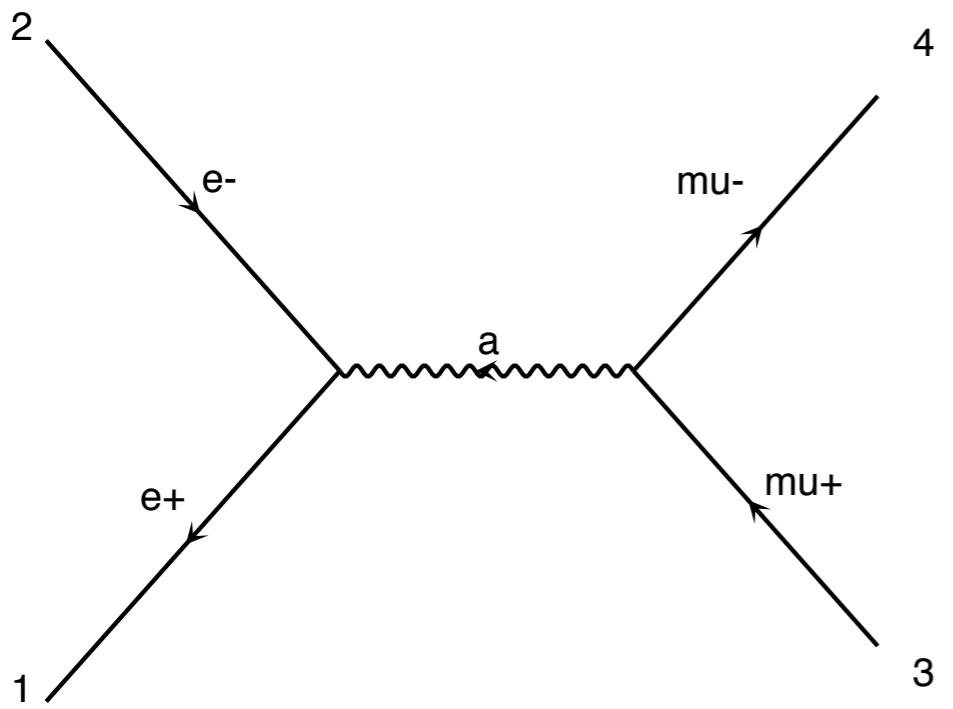
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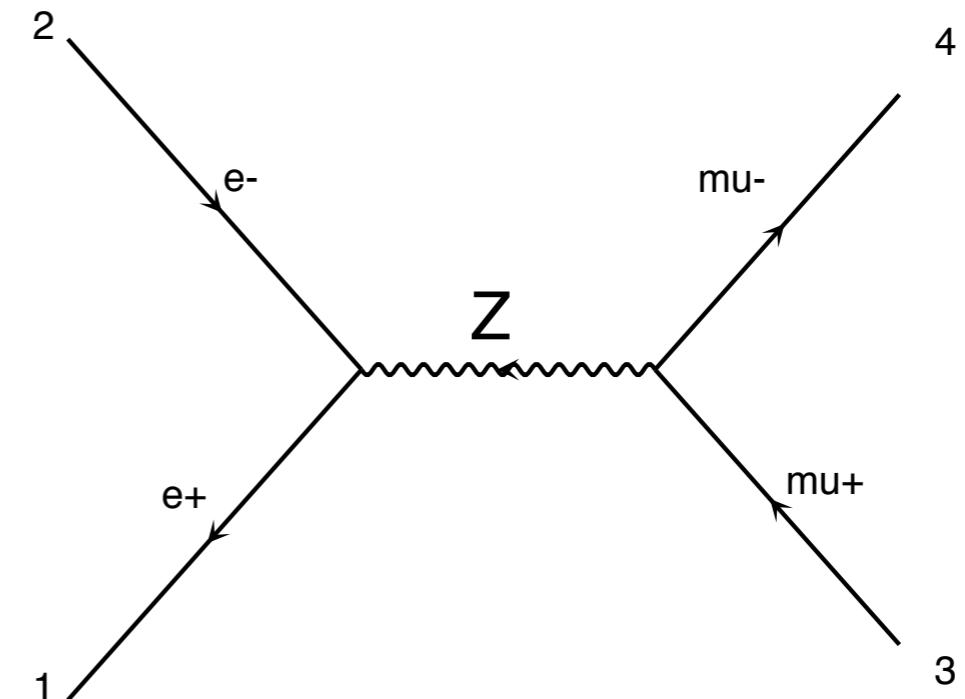
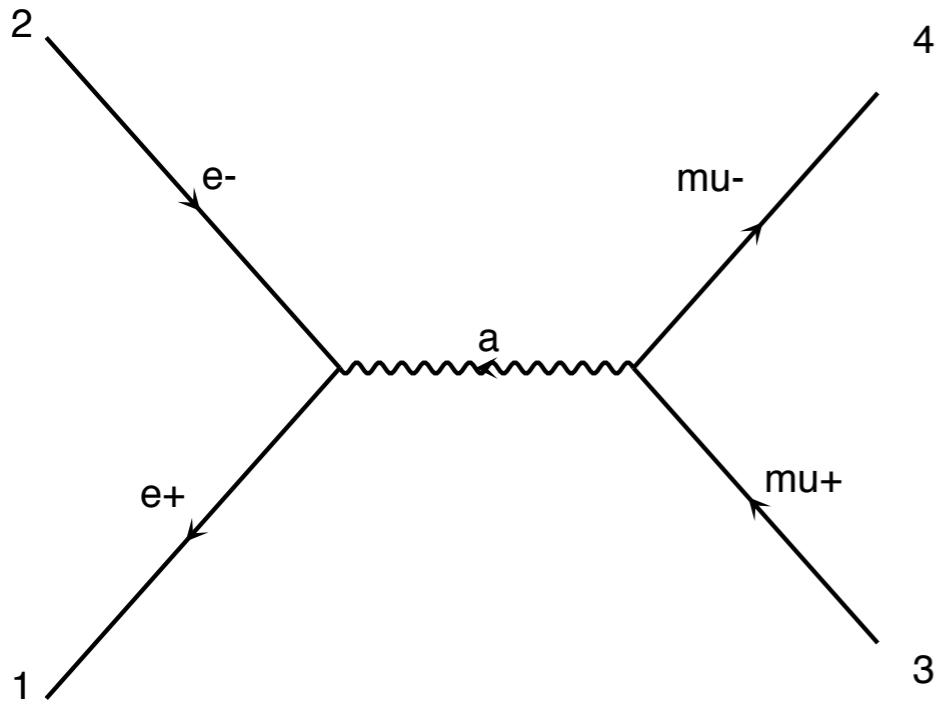
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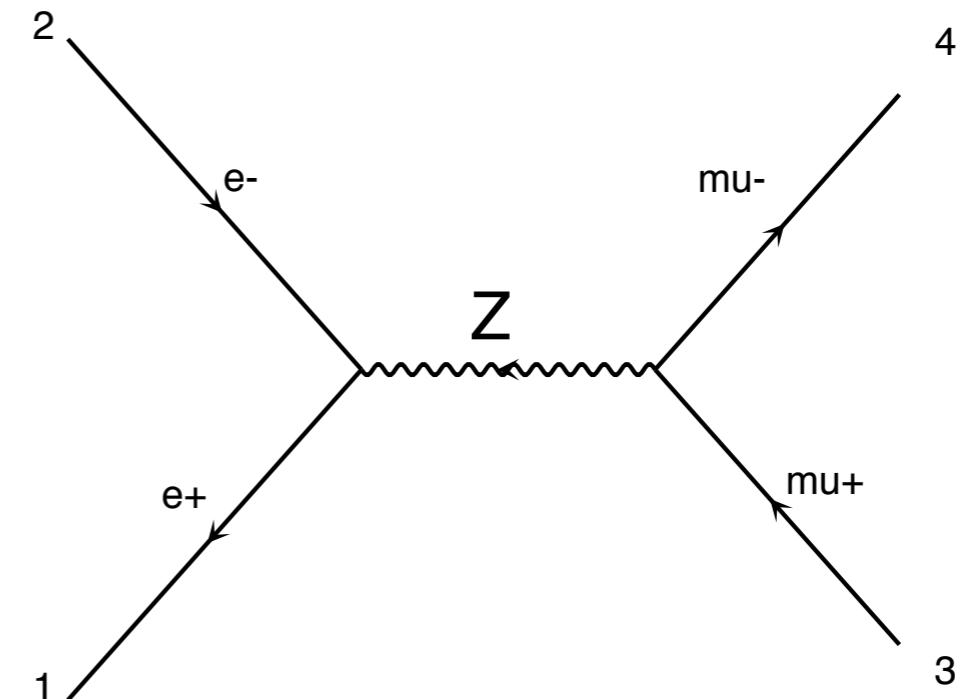
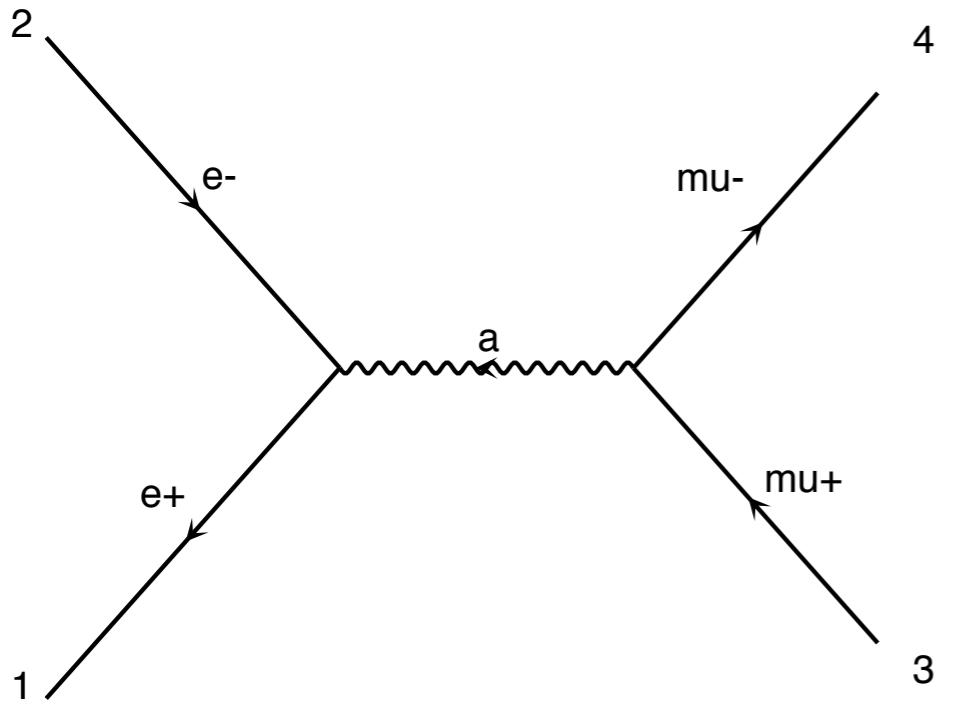
$$\rightarrow \frac{8e^4}{q^4} [(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)]$$

**Very Efficient**  
**(few computation to perform to get that number)**



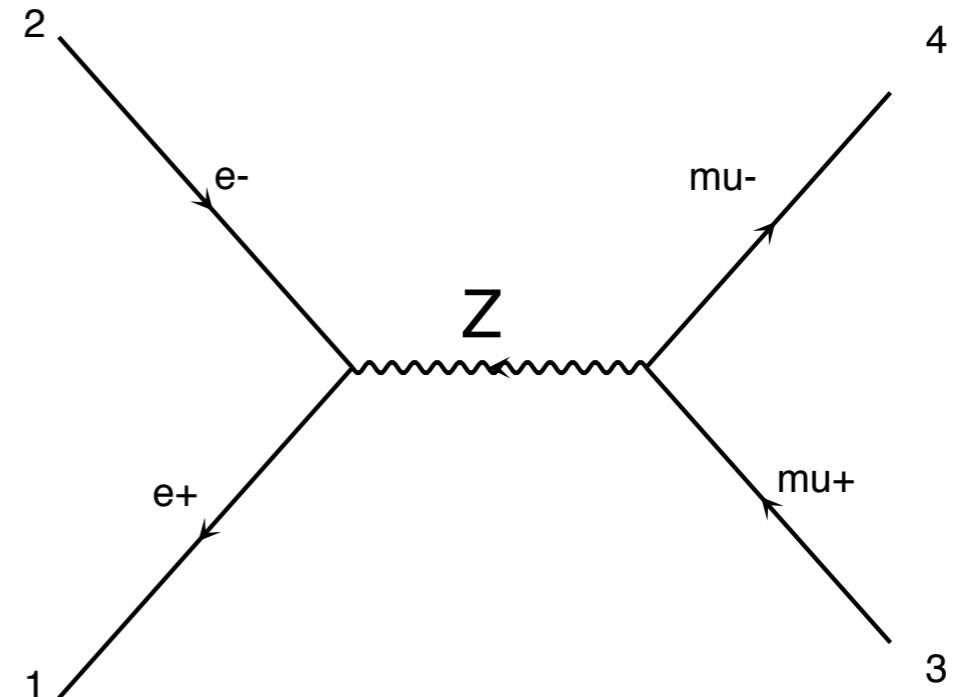
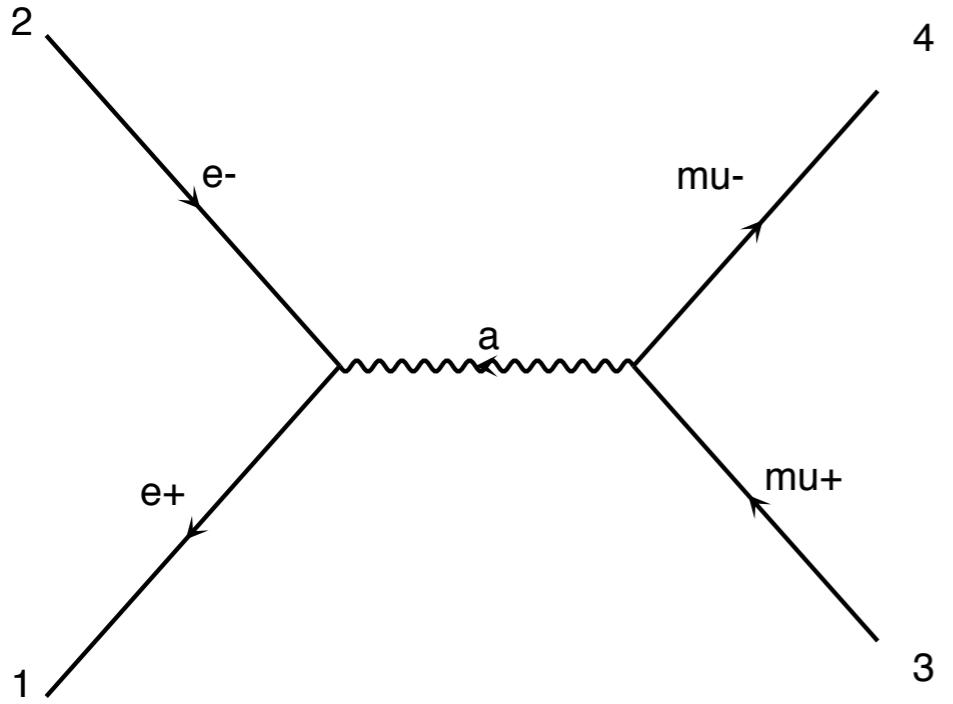


**Need to compute**  $|M_a|^2$   $|M_z|^2$   $2Re(M_a^* M_z)$



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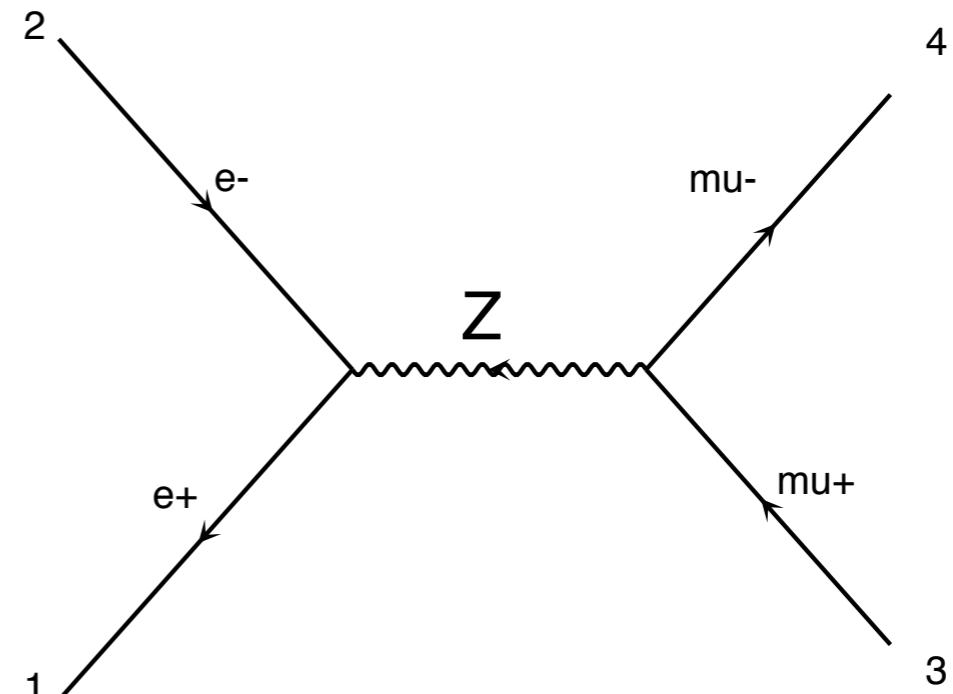
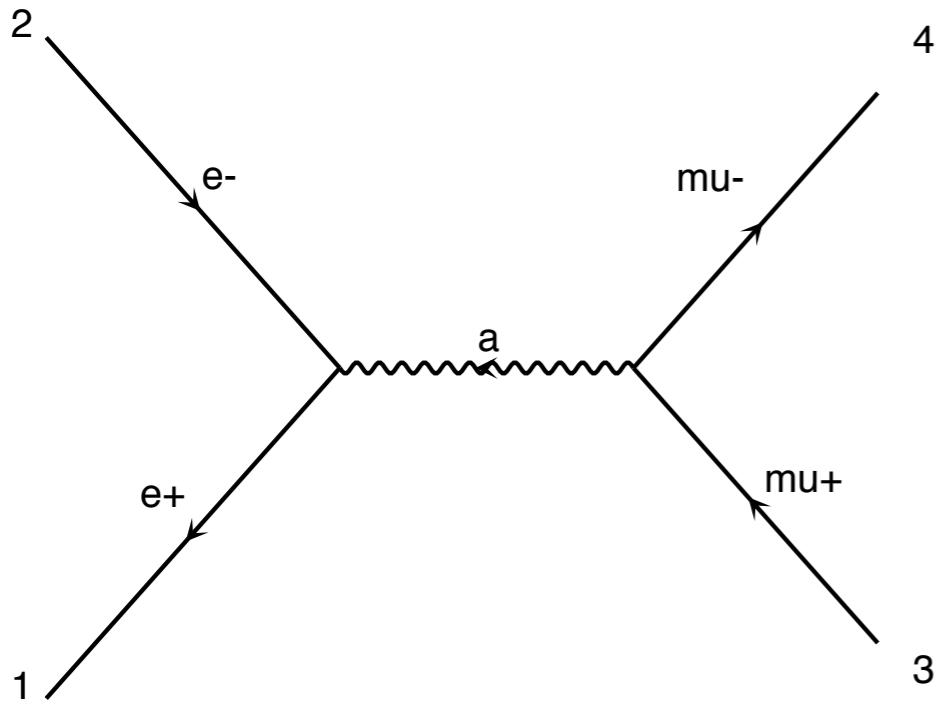
So for M Feynman diagram we need to compute  $M^2$   
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The number of diagram scales factorially with the number of particle



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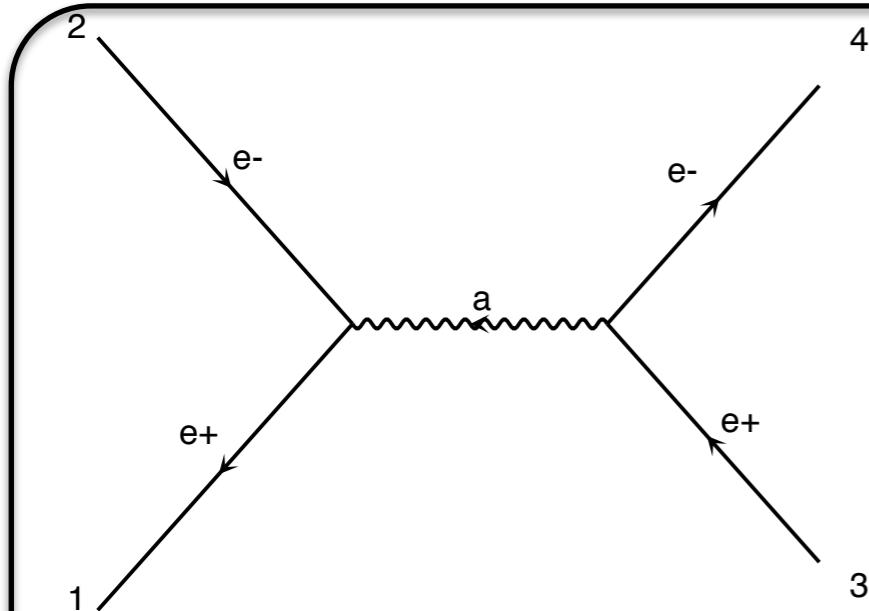
The number of diagram scales factorially with the number of particle

In practise possible up to  $2>4$

# Helicity

## Idea

- Evaluate  $\mathcal{M}$  for fixed helicity of external particles
  - Multiply  $\mathcal{M}$  with  $\mathcal{M}^*$  ->  $|\mathcal{M}|^2$
  - Loop on Helicity and average the results

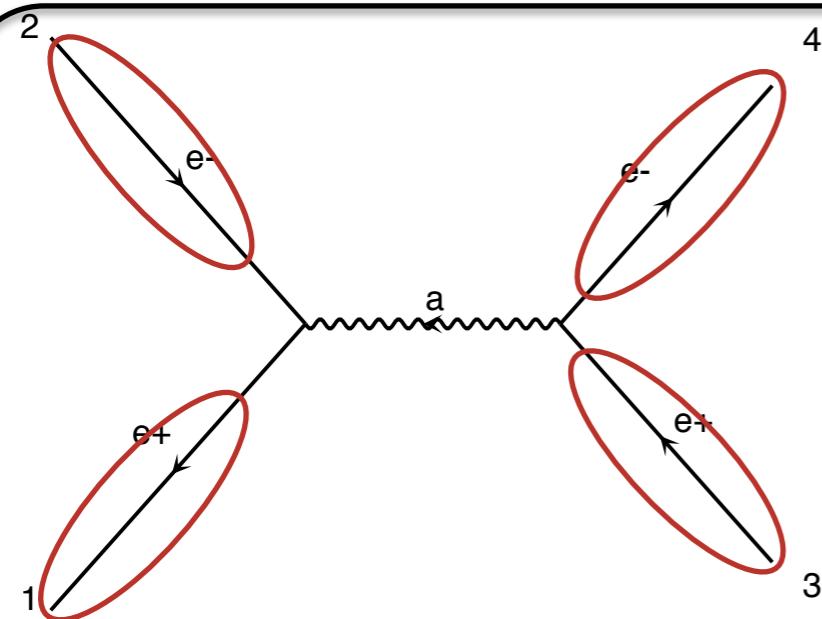


$$\mathcal{M} = (\bar{u} e \gamma^\mu v) \frac{g_{\mu\nu}}{q^2} (\bar{v} e \gamma^\nu u)$$

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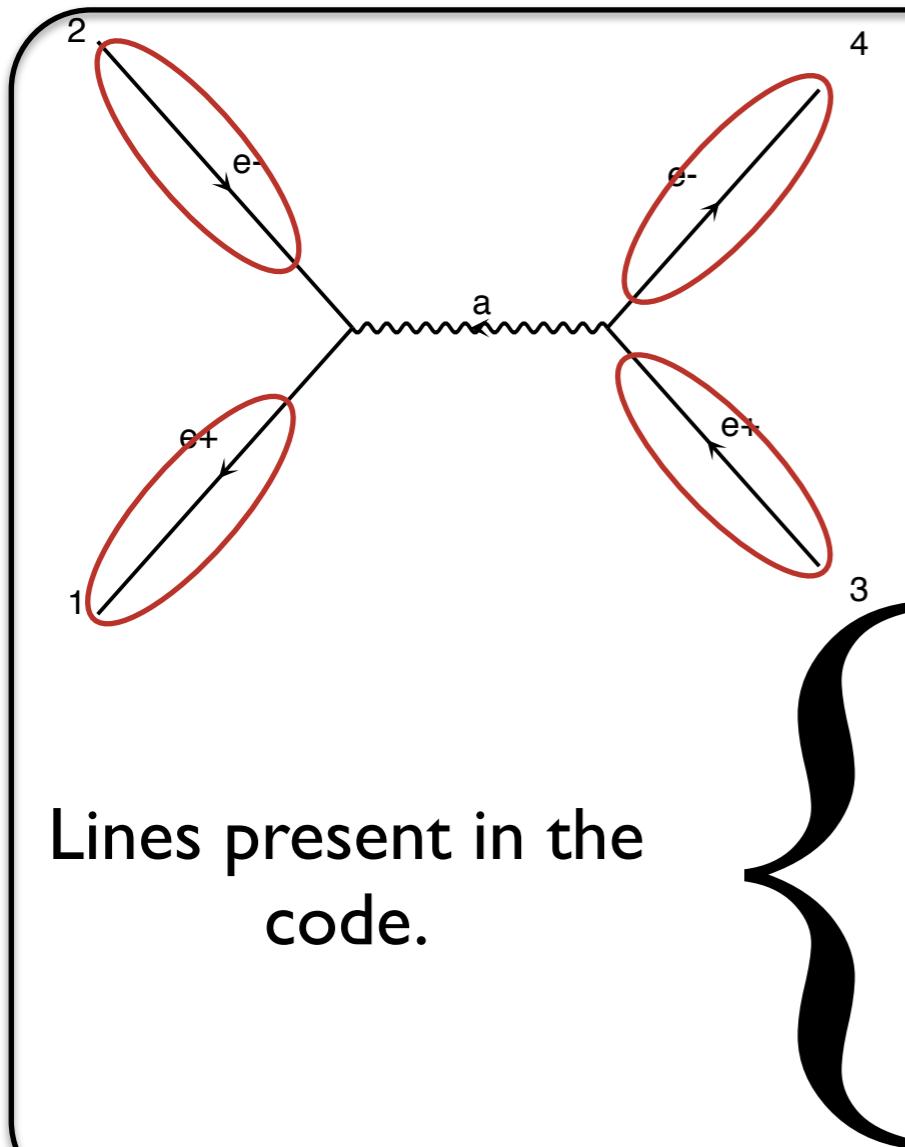
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*Numbers for given helicity and momenta*

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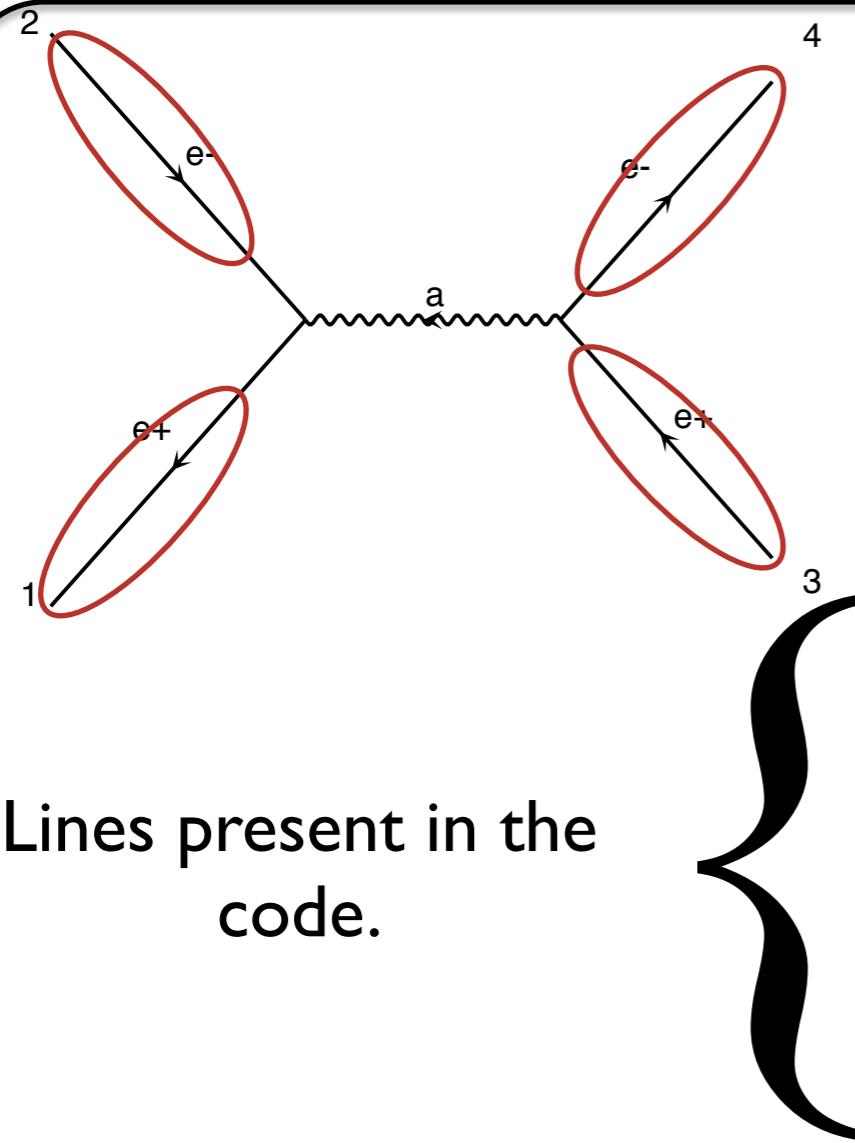
Numbers for given helicity and momenta

$$\begin{aligned}\bar{v}_1 &= fct(\vec{p}_1, m_1) \\ u_2 &= fct(\vec{p}_2, m_2) \\ v_3 &= fct(\vec{p}_3, m_3) \\ \bar{u}_4 &= fct(\vec{p}_4, m_4)\end{aligned}$$

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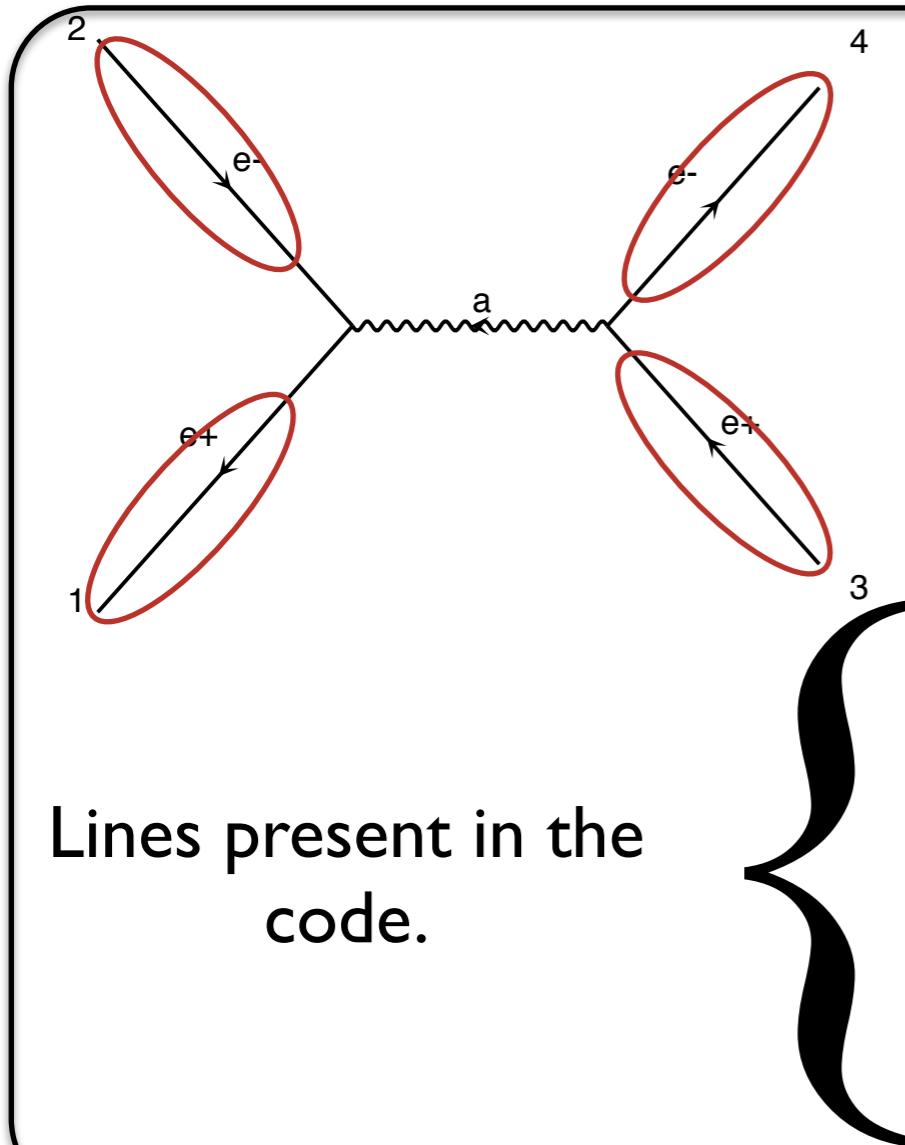
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$$\begin{aligned}u(p) &= \begin{pmatrix} \omega_{-\lambda}(p) \chi_\lambda(\vec{p}) \\ \omega_\lambda(p) \chi_\lambda(\vec{p}) \end{pmatrix} \\ \omega_\pm(p) &\equiv \sqrt{E \pm |\vec{p}|}. \\ \chi_+(\vec{p}) &= \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_z)}} \begin{pmatrix} |\vec{p}| + p_z \\ p_x + ip_y \end{pmatrix}, \\ \chi_-(\vec{p}) &= \frac{1}{\sqrt{2|\vec{p}|(|\vec{p}| + p_z)}} \begin{pmatrix} -p_x + ip_y \\ |\vec{p}| + p_z \end{pmatrix}.\end{aligned}$$

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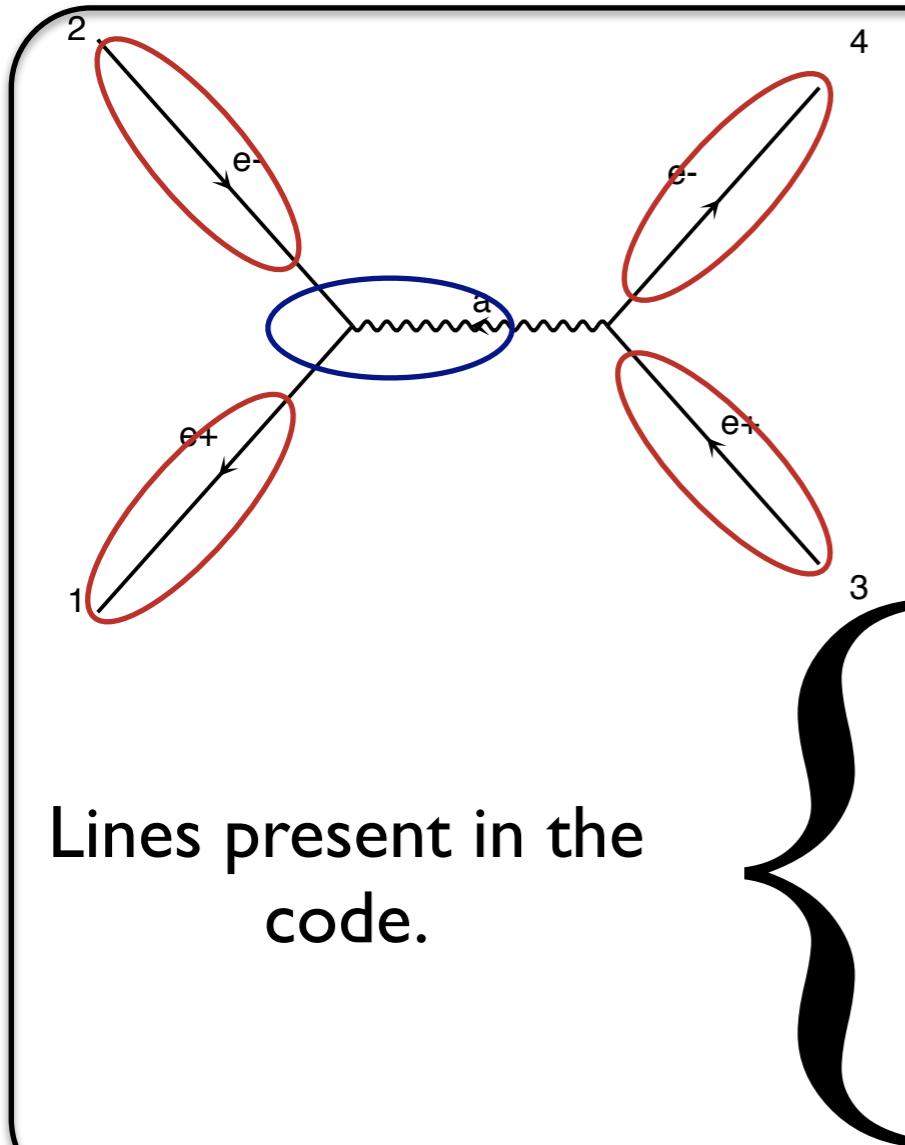
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$$\mathcal{M} = \bar{u} e \gamma^\mu v \frac{g_{\mu\nu}}{q^2} \bar{v} e \gamma^\nu u$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

$$\bar{v}_1 = fct(\vec{p}_1, m_1)$$

$$u_2 = fct(\vec{p}_2, m_2)$$

$$v_3 = fct(\vec{p}_3, m_3)$$

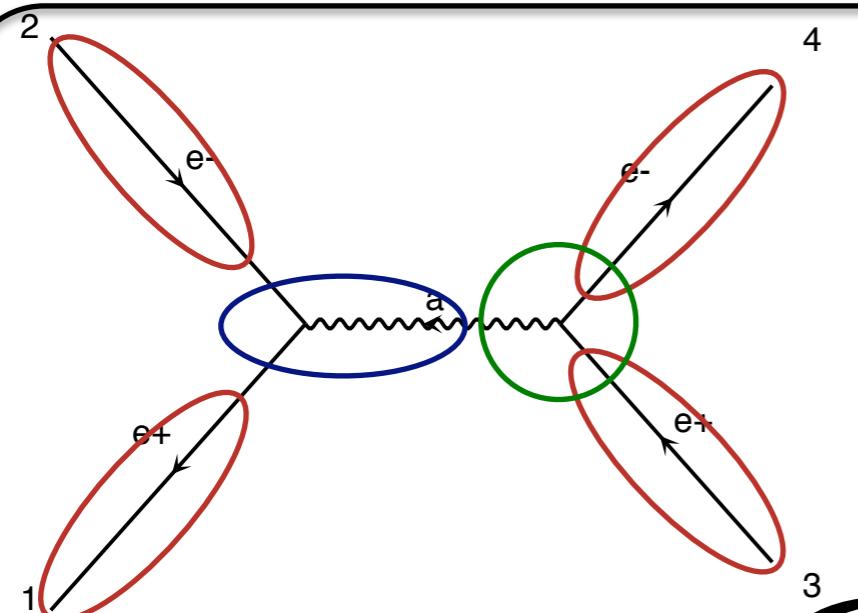
$$\bar{u}_4 = fct(\vec{p}_4, m_4)$$

$$W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a)$$

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Lines present in the code.

$$\mathcal{M} = \bar{u} e \gamma^\mu v \frac{g_{\mu\nu}}{q^2} (\bar{v} e \gamma^\nu u)$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

$$\bar{v}_1 = fct(\vec{p}_1, m_1)$$

$$u_2 = fct(\vec{p}_2, m_2)$$

$$v_3 = fct(\vec{p}_3, m_3)$$

$$\bar{u}_4 = fct(\vec{p}_4, m_4)$$

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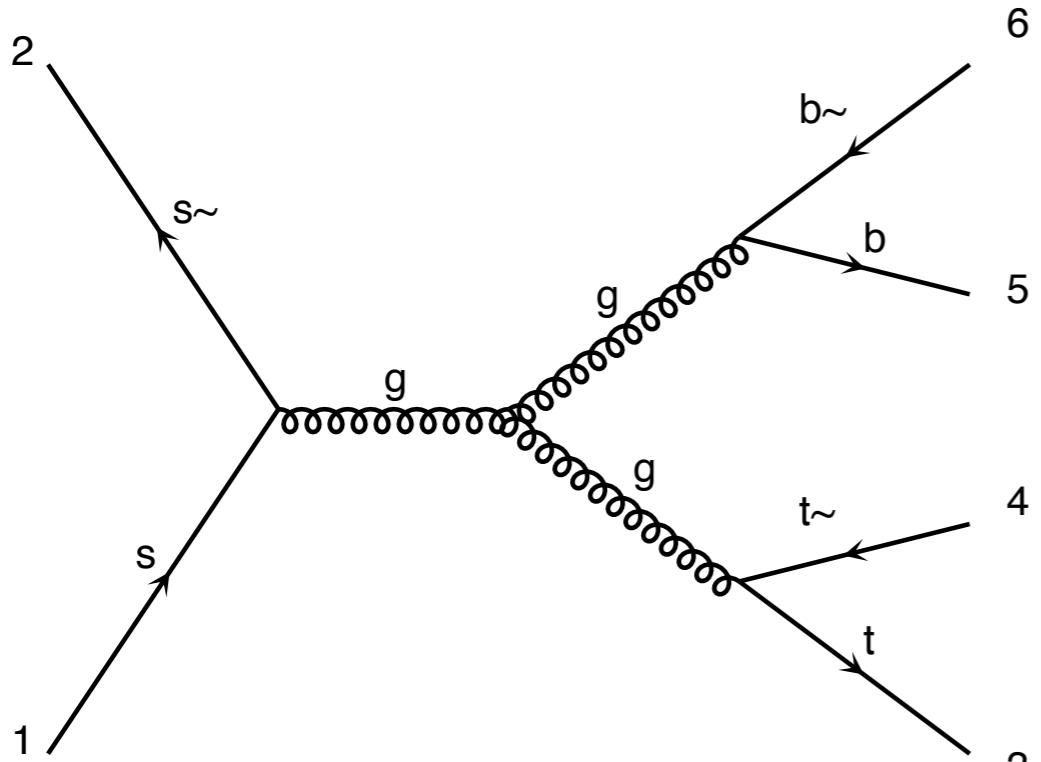
$$\mathcal{M} = fct(v_3, \bar{u}_4, W_a)$$

# Comparison

	M diag	N particle
Analytical	$M^2$	$(N!)^2$
Helicity	$M$	$(N!) 2^N$

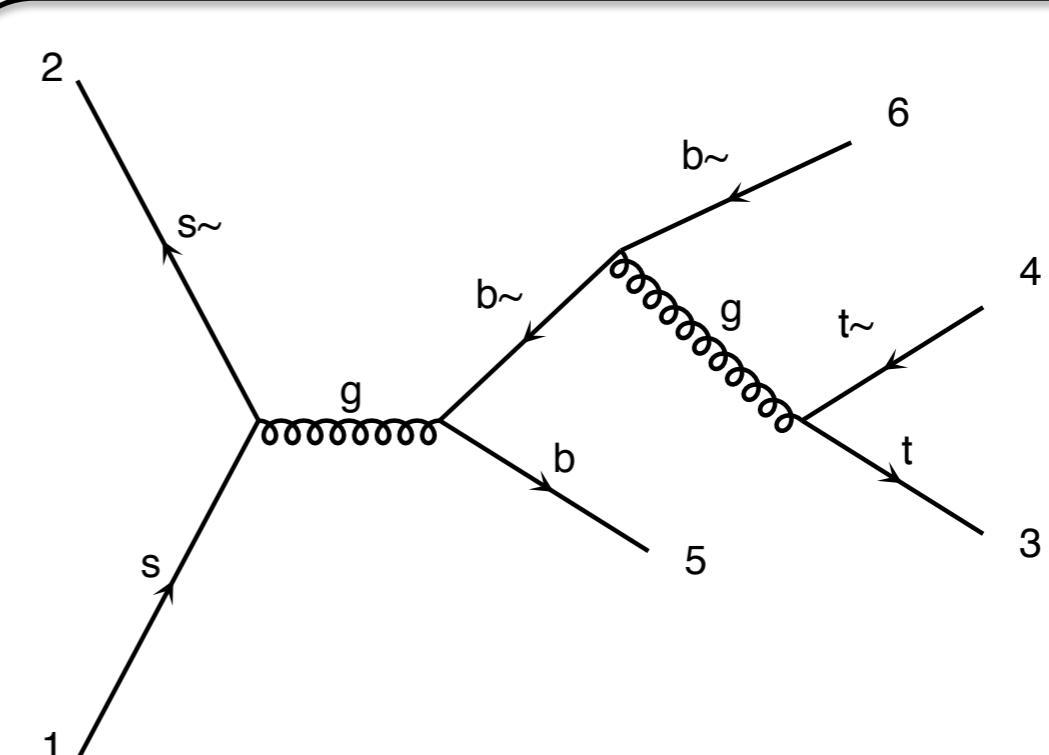
# Real case

Known



M1

Number of routines: 0



M2

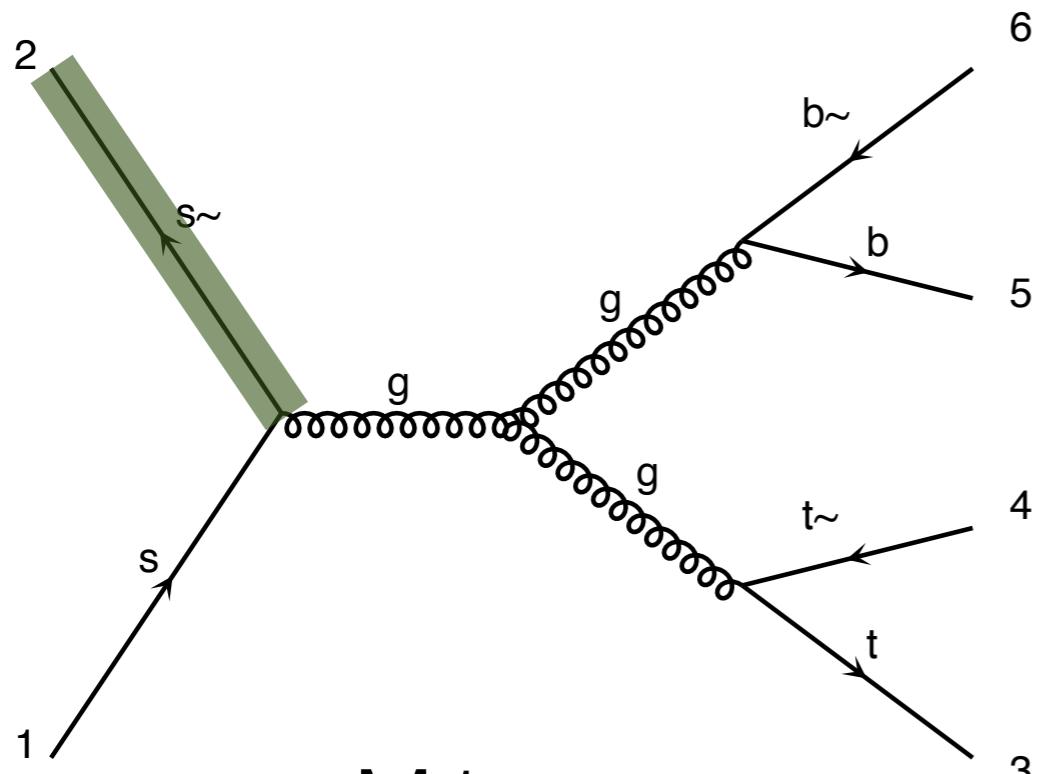
Number of routines: 0

Number of routines for both: 0

$$|M|^2 = |M_1 + M_2|^2$$

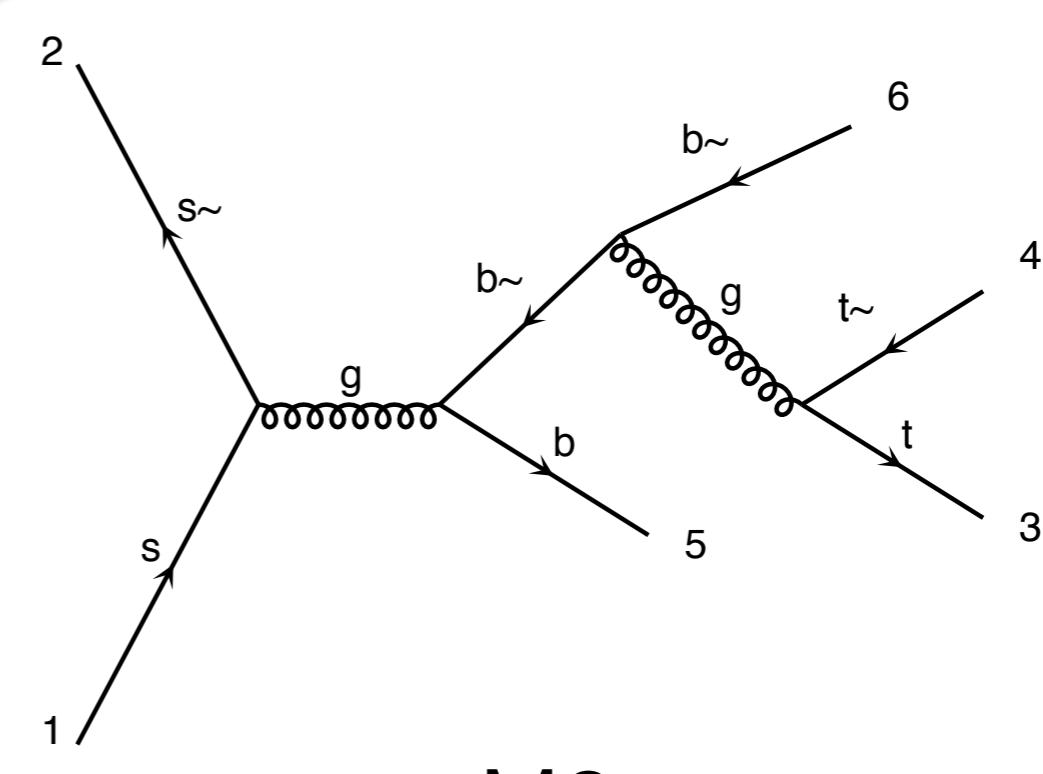
# Real case

Known



M1

Number of routines: 1



M2

Number of routines: 0

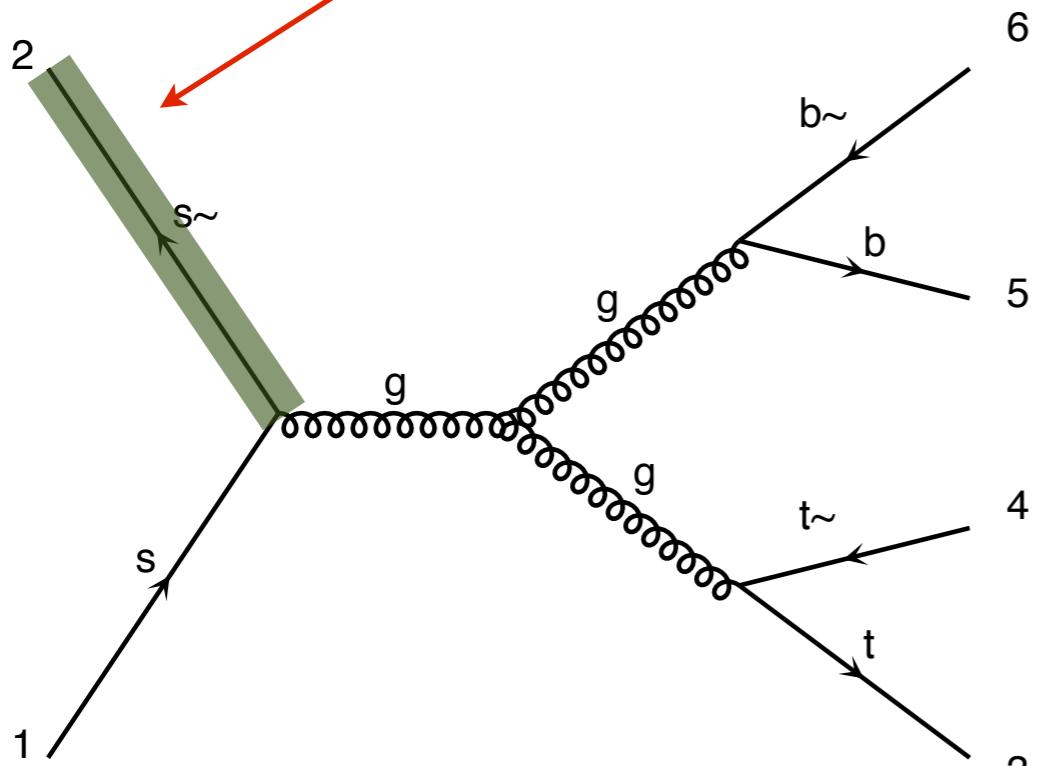
Number of routines for both: 1

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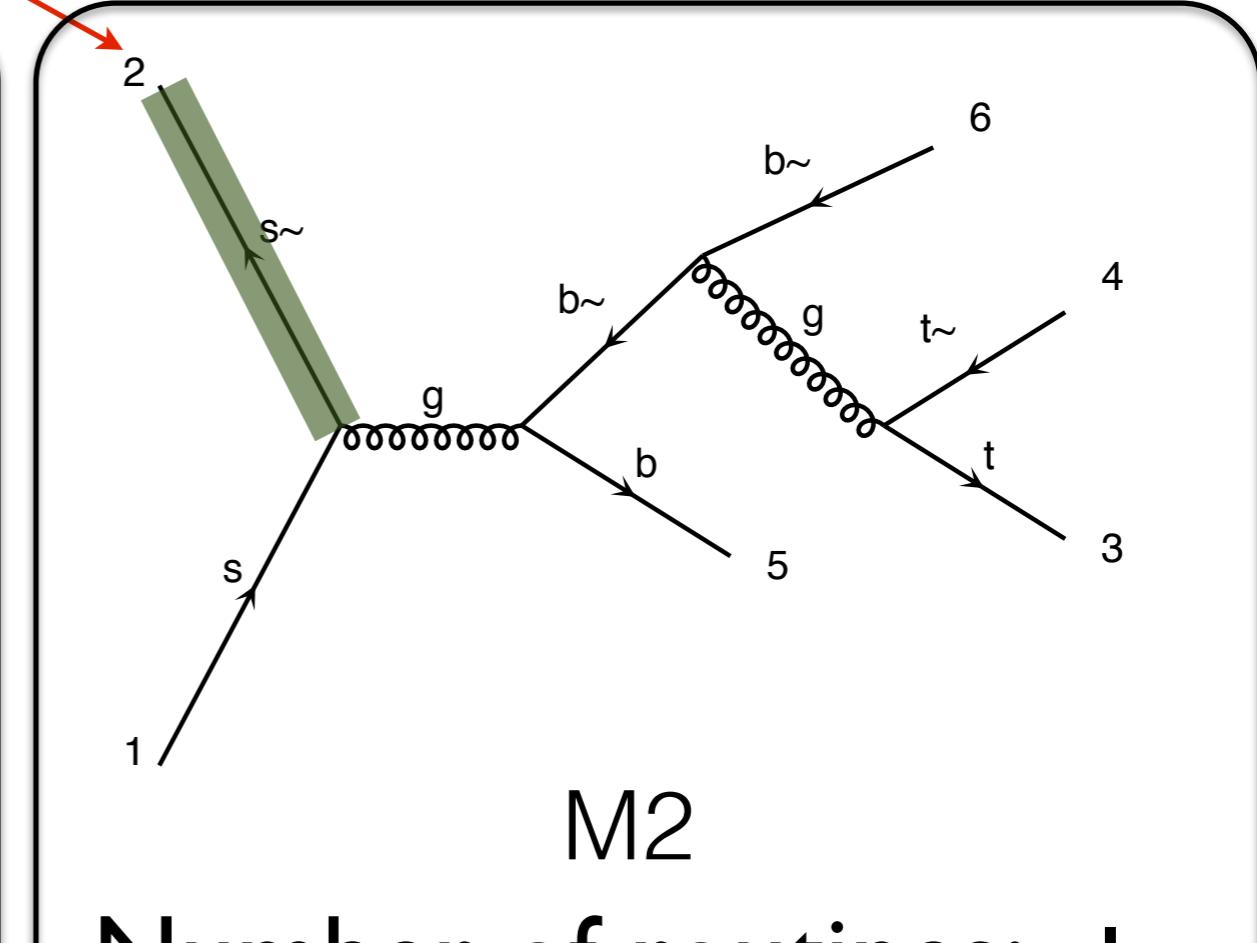
Identical

Known



M1

Number of routines: I



M2

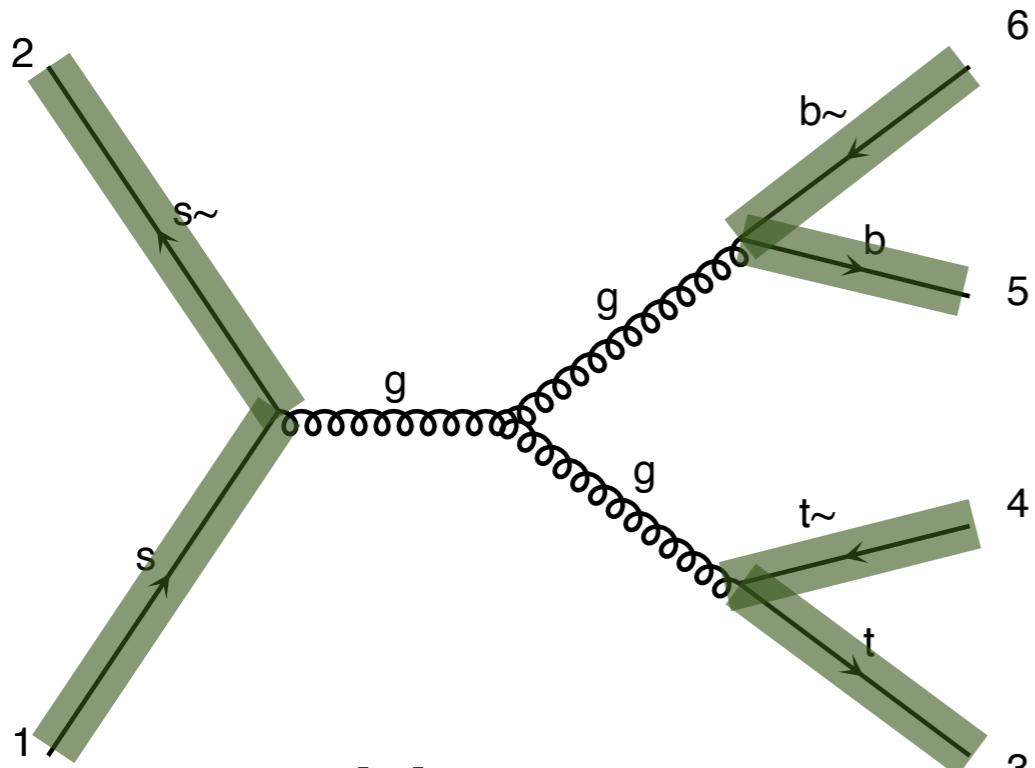
Number of routines: I

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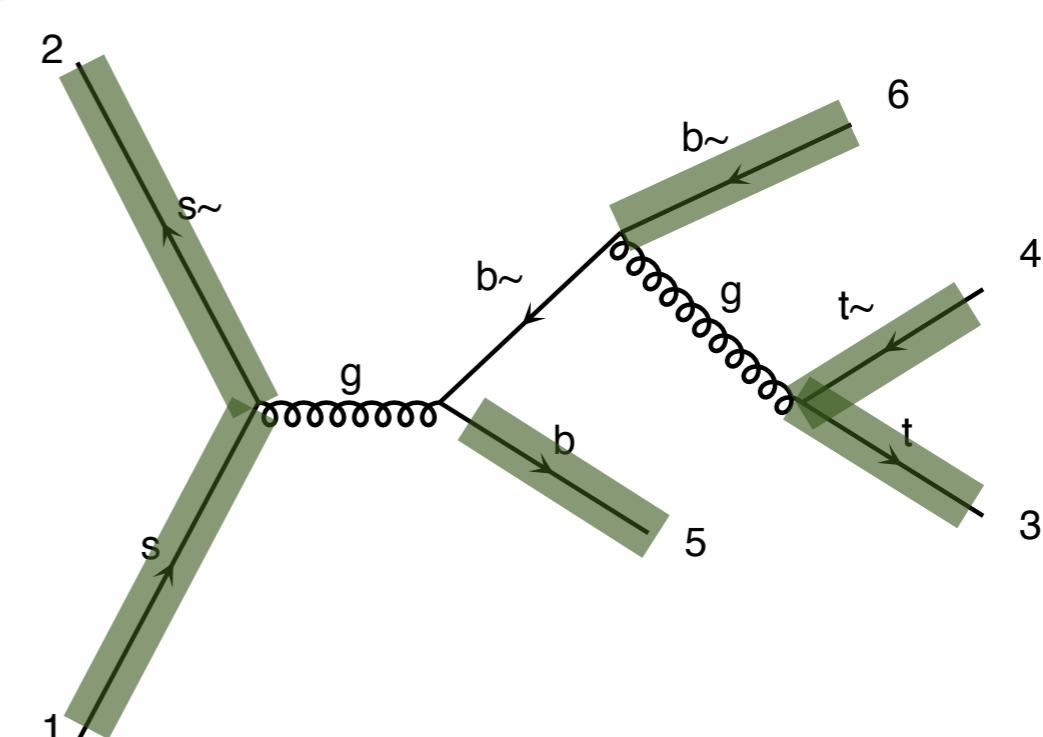
# Real case

Known



M1

Number of routines: 6



M2

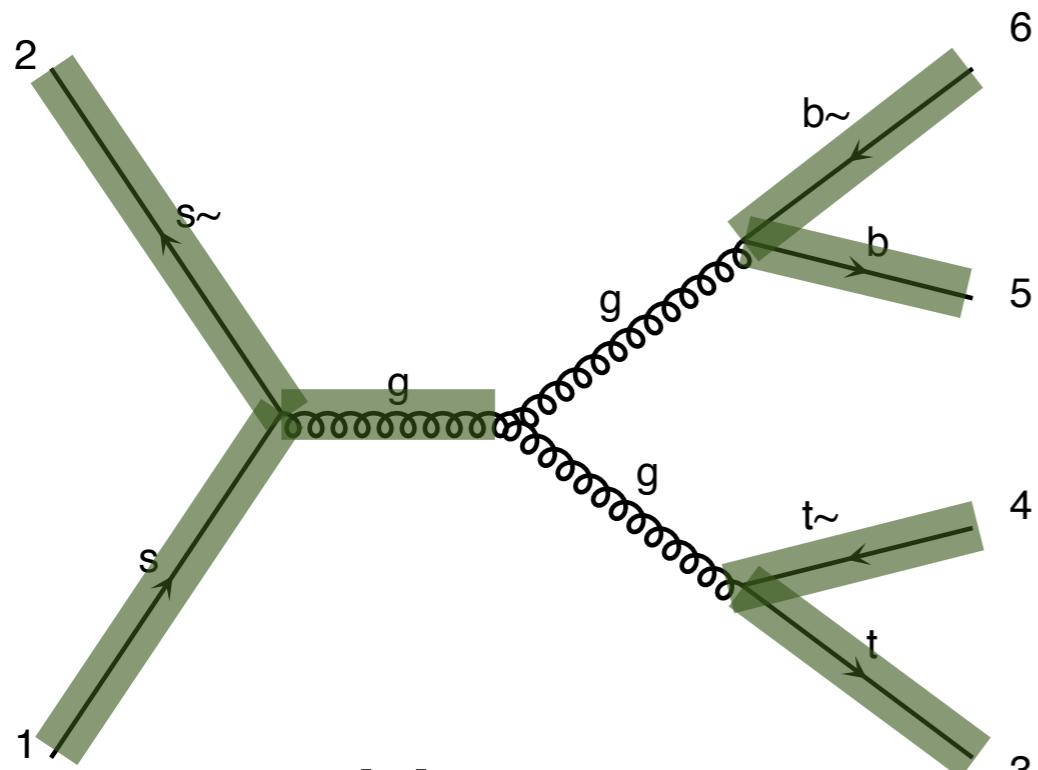
Number of routines: 6

Number of routines for both: 6

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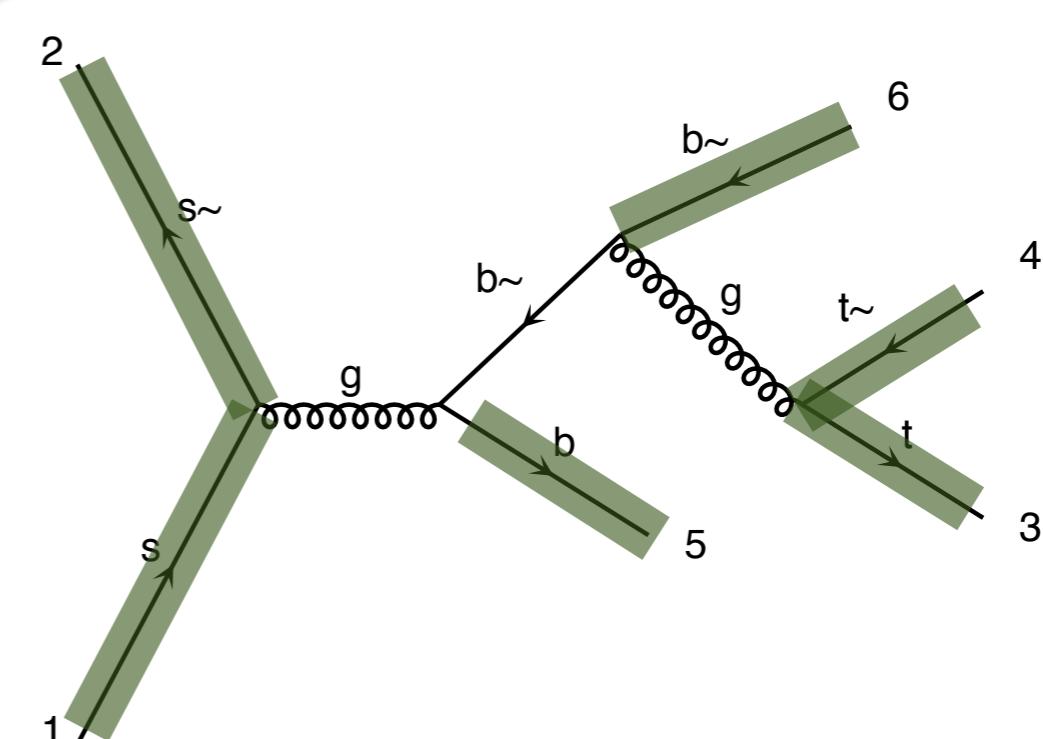
# Real case

Known



M1

Number of routines: 7



M2

Number of routines: 6

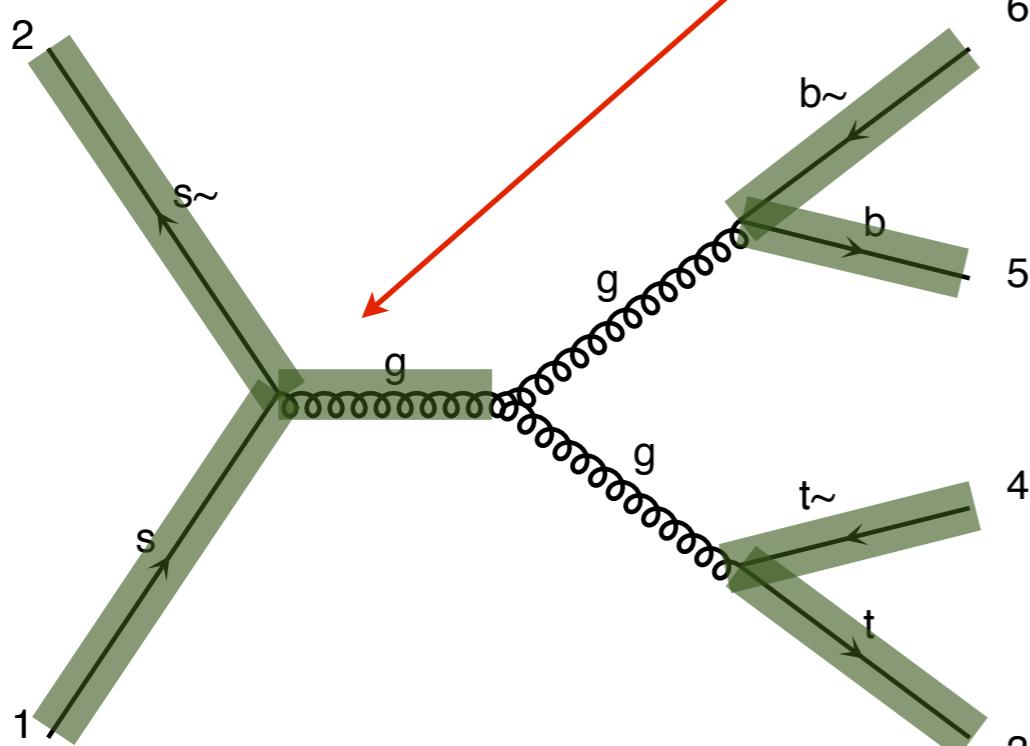
Number of routines for both: 7

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# Real case

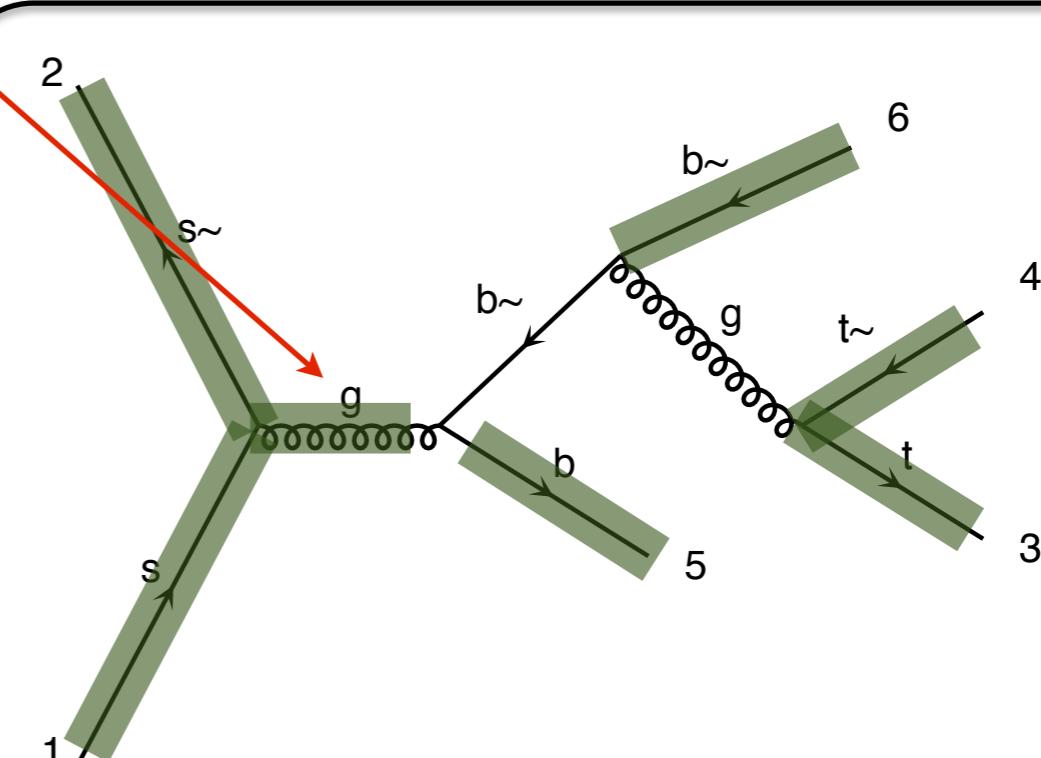
Known

Identical



M1

Number of routines: 7



M2

Number of routines: 7

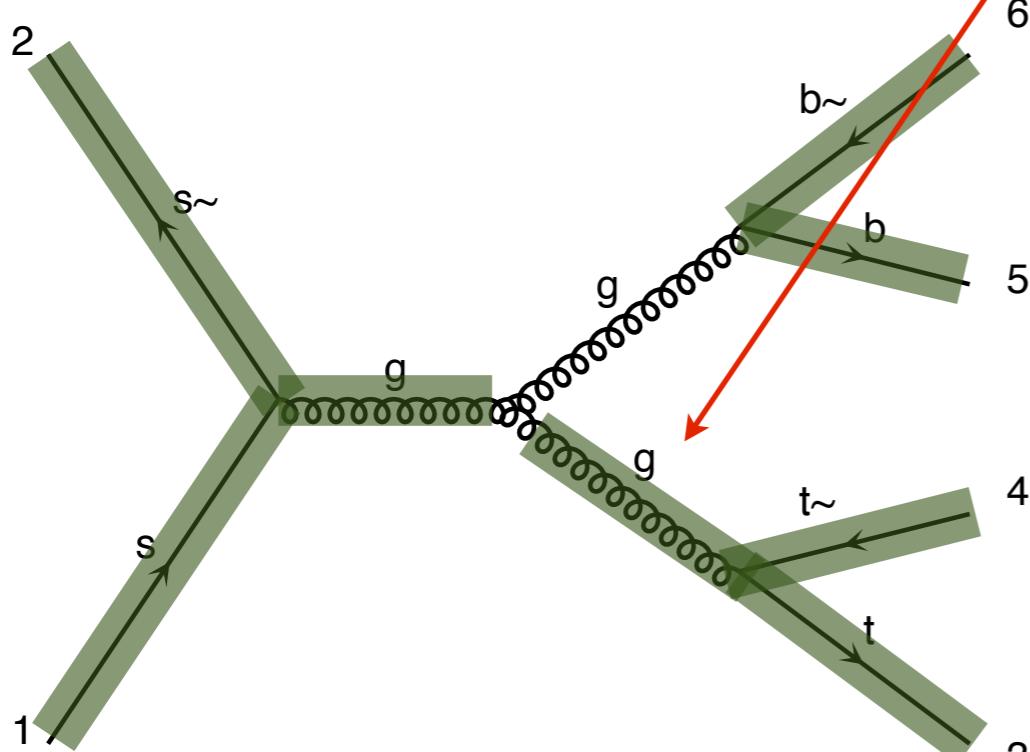
Number of routines for both: 7

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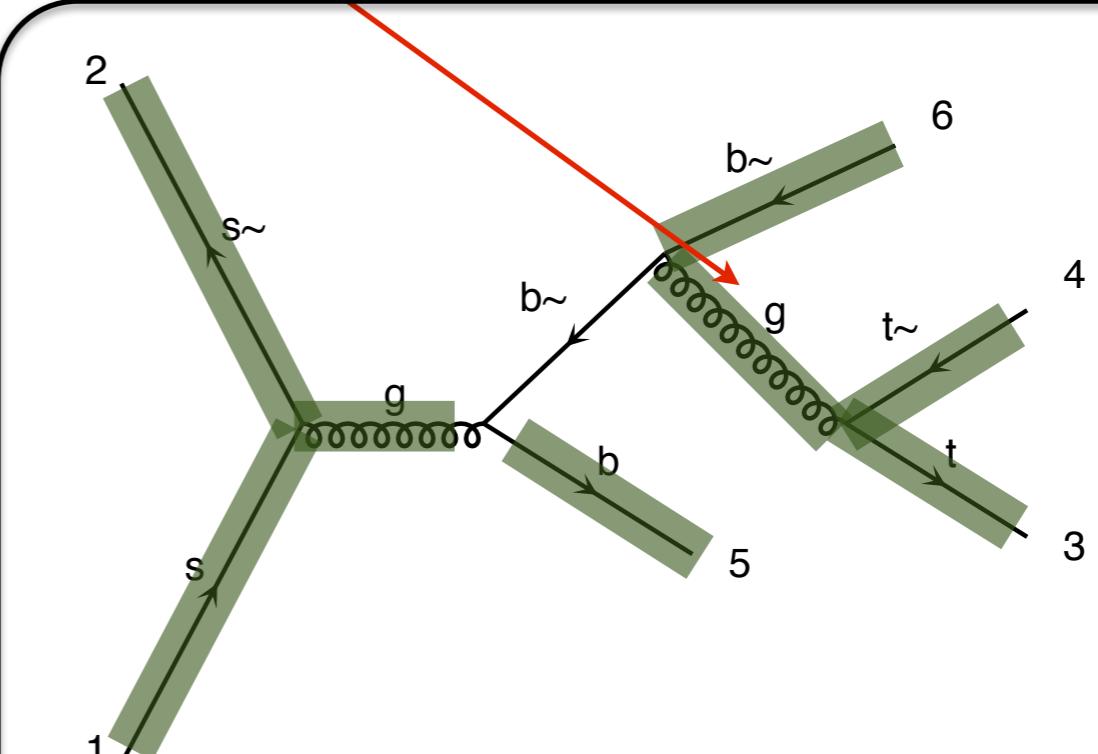
# Real case

Identical

Known



Number of routines: 8



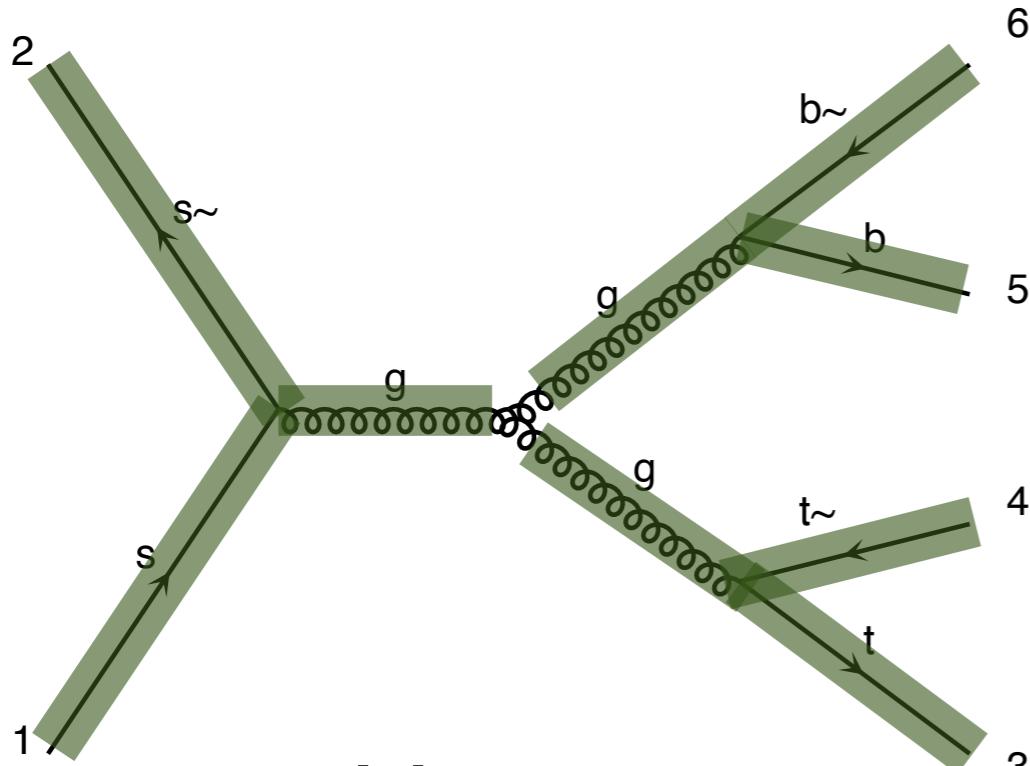
Number of routines: 8

Number of routines for both: 8

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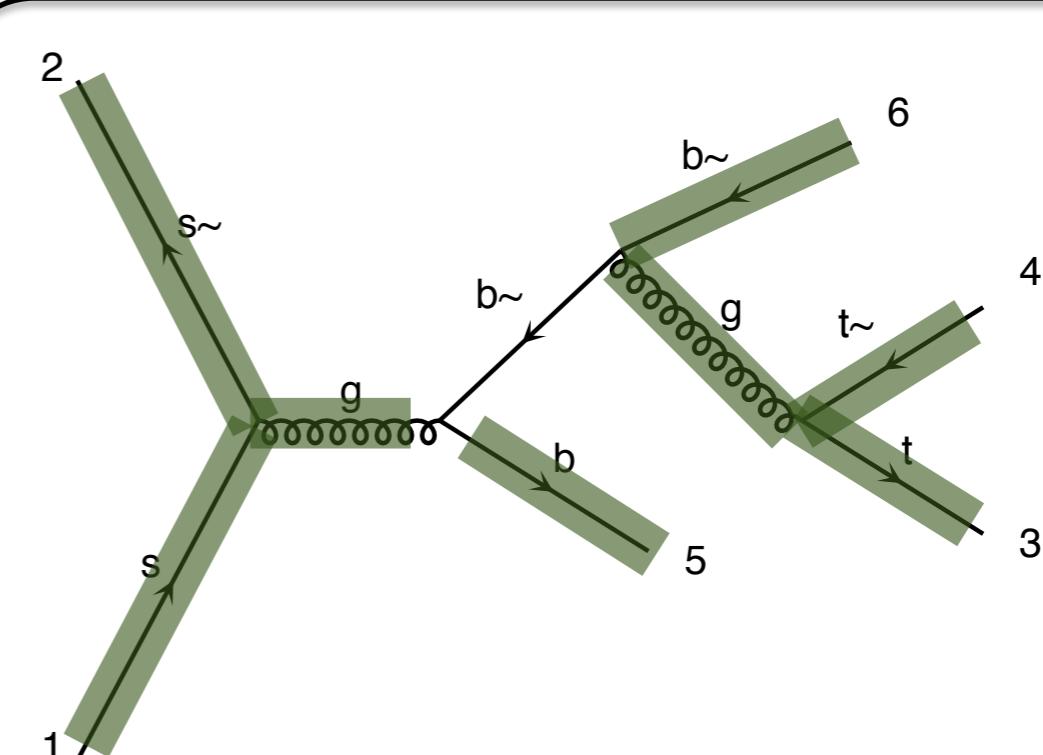
# Real case

Known



M1

Number of routines: 9



M2

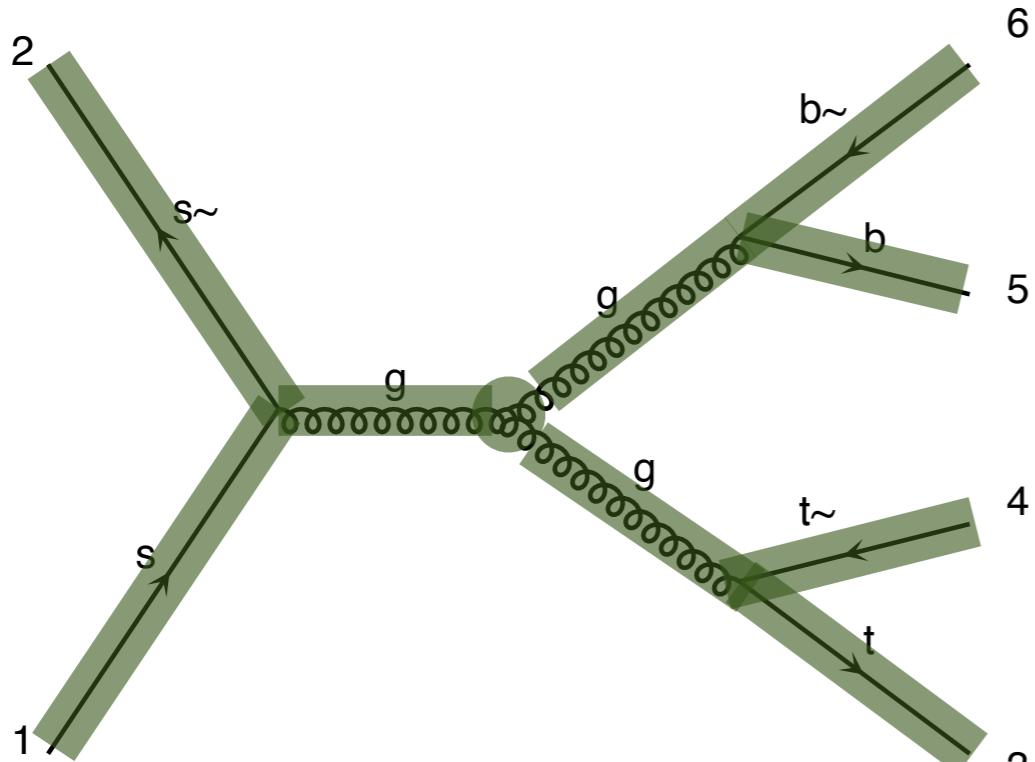
Number of routines: 8

Number of routines for both: 9

$$|M|^2 = |M_1 + M_2|^2$$

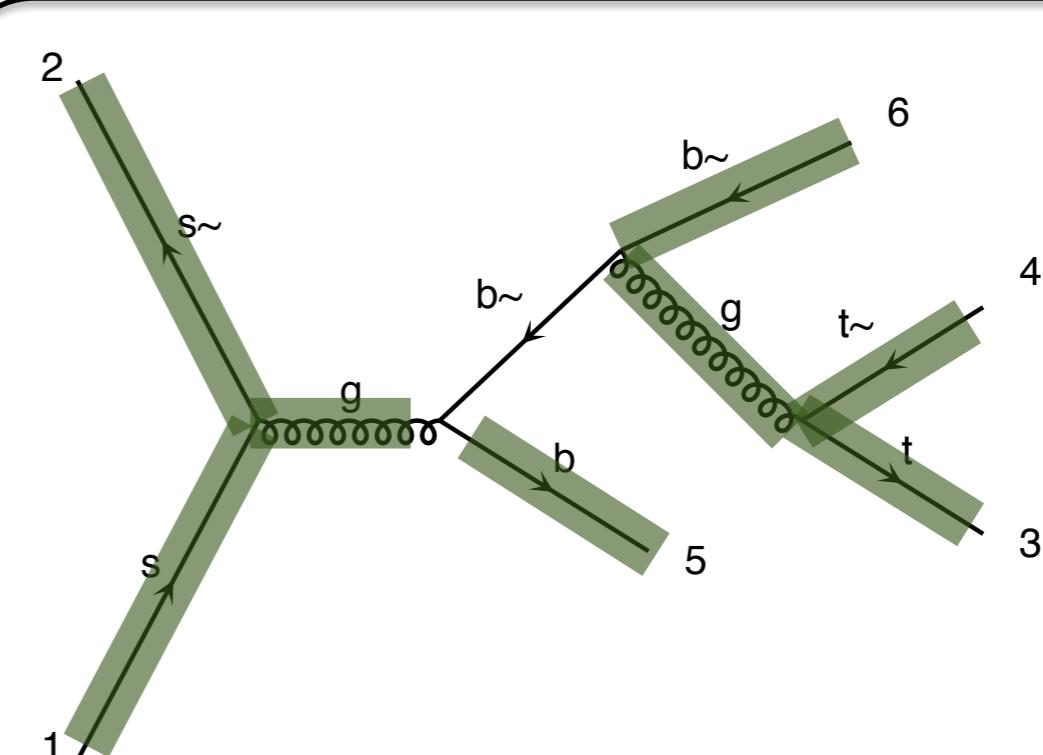
# Real case

Known



M1

Number of routines: 10



M2

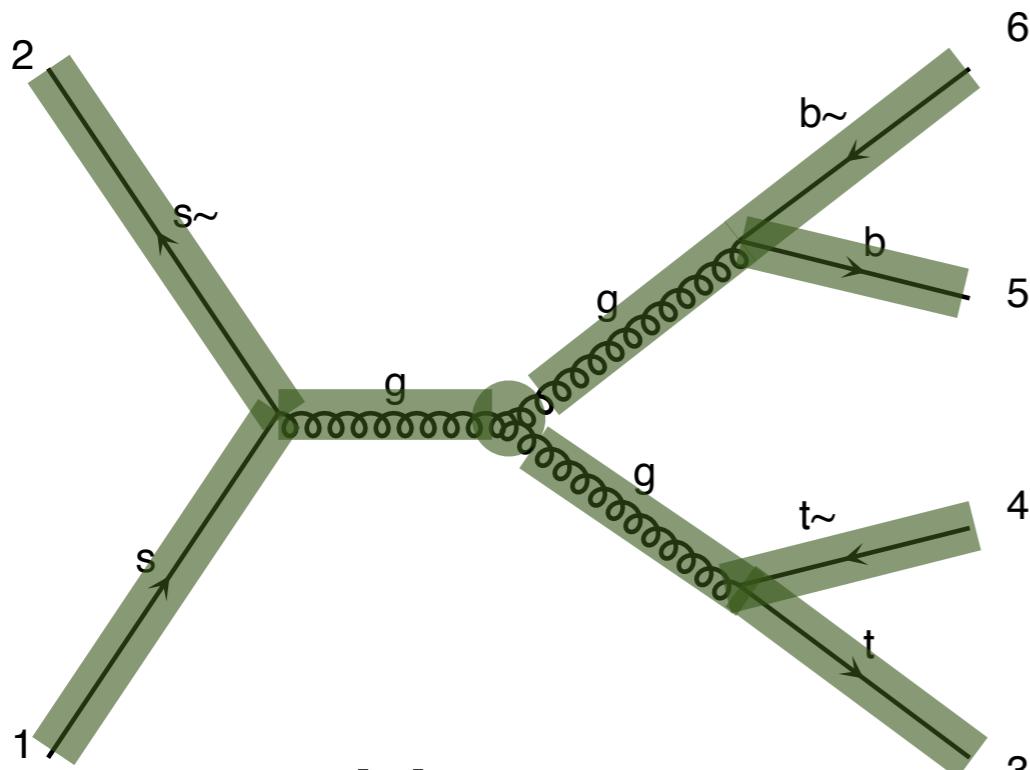
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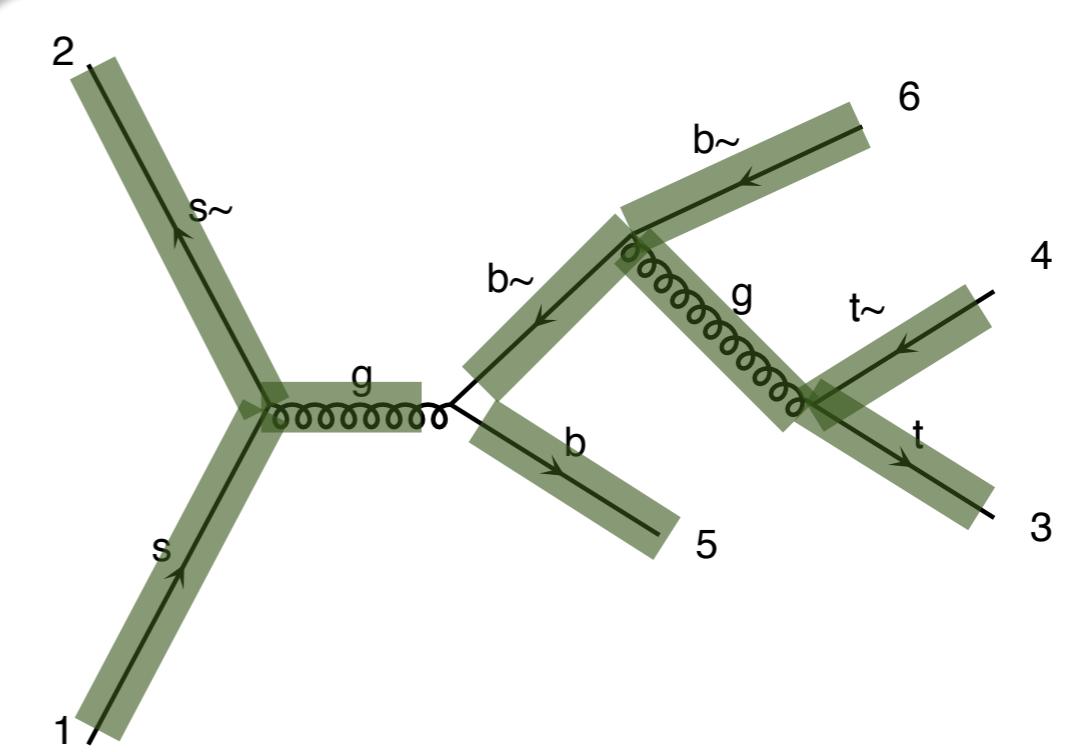
# Real case

Known



M1

Number of routines: 10



M2

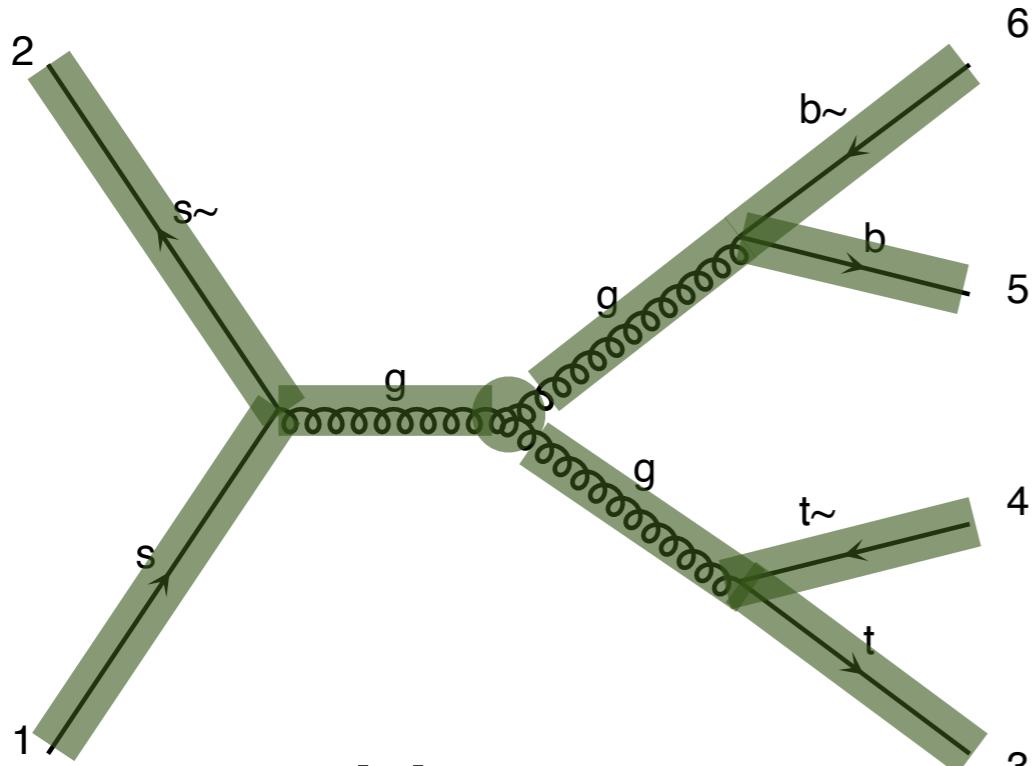
Number of routines: 9

Number of routines for both: 11

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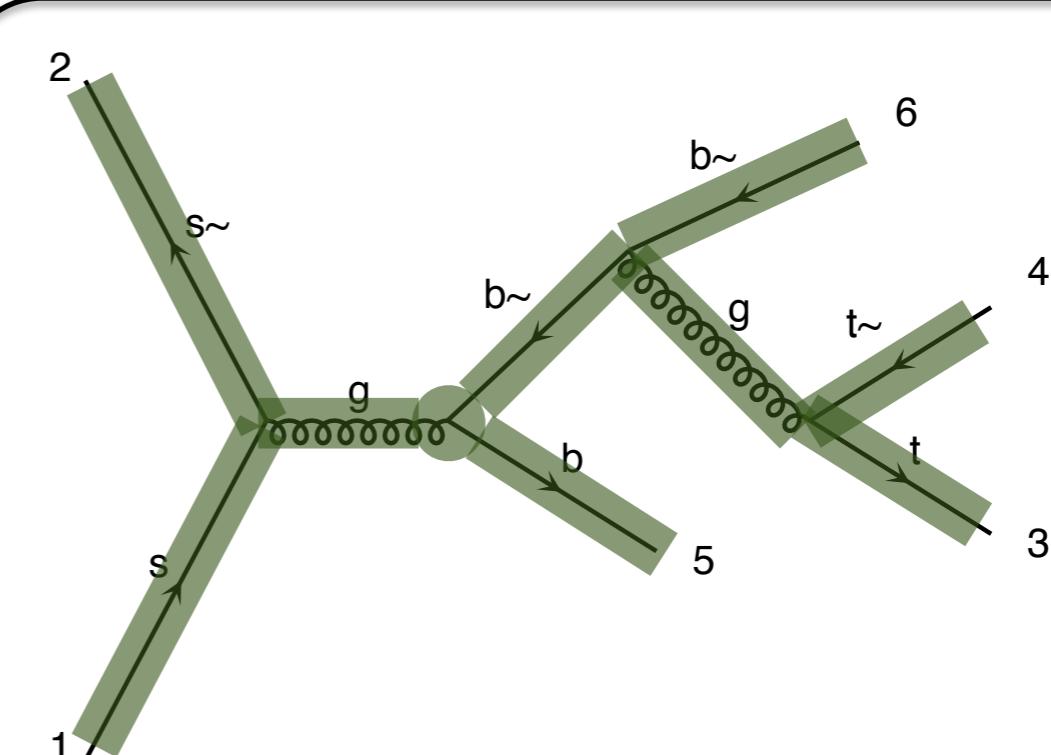
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Known



M1

Number of routines: 10



M2

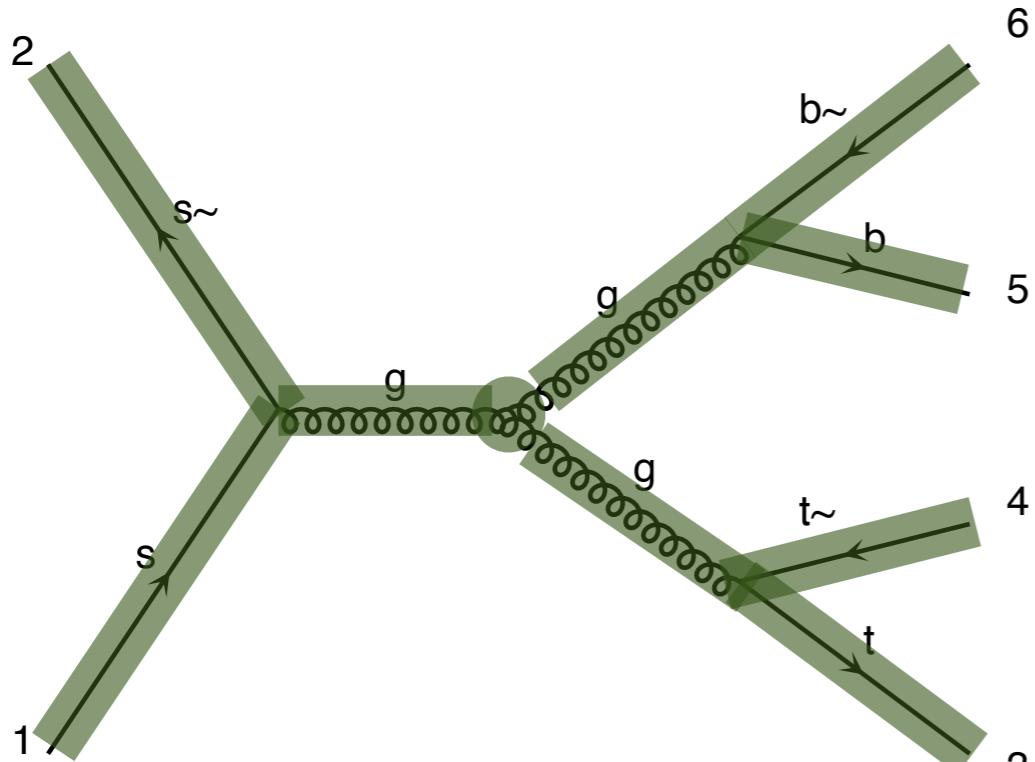
Number of routines: 10

Number of routines for both: 12

$$|M|^2 = |M_1 + M_2|^2$$

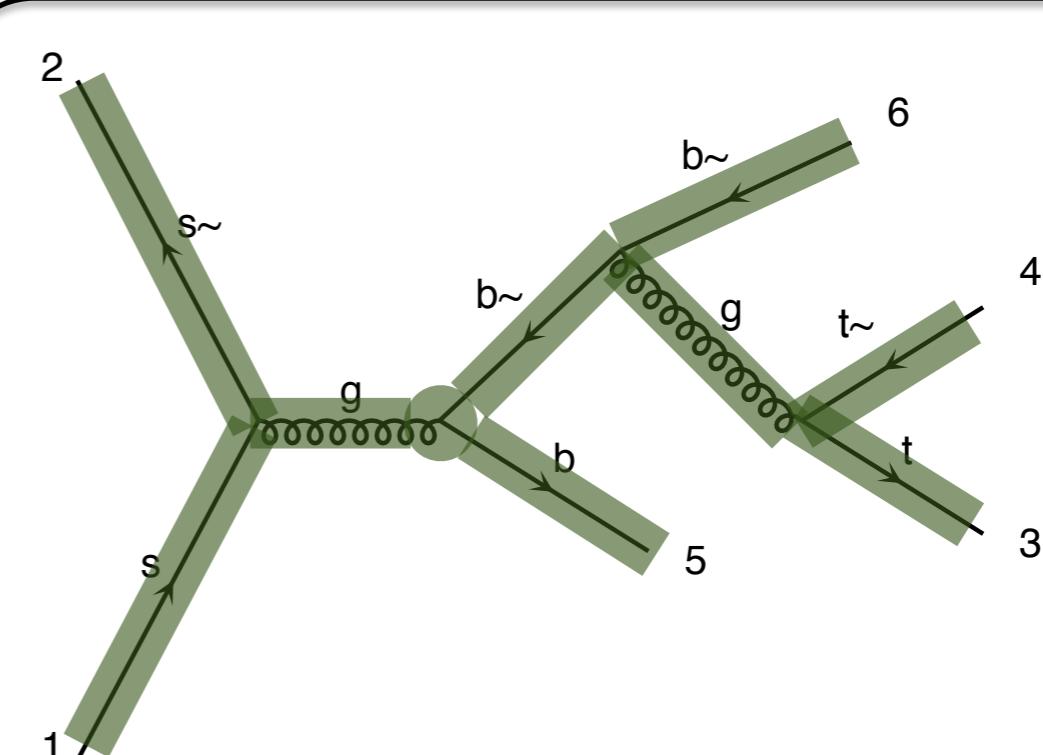
# Real case

Known



M1

Number of routines: 10  
 $2(N+1)$



M2

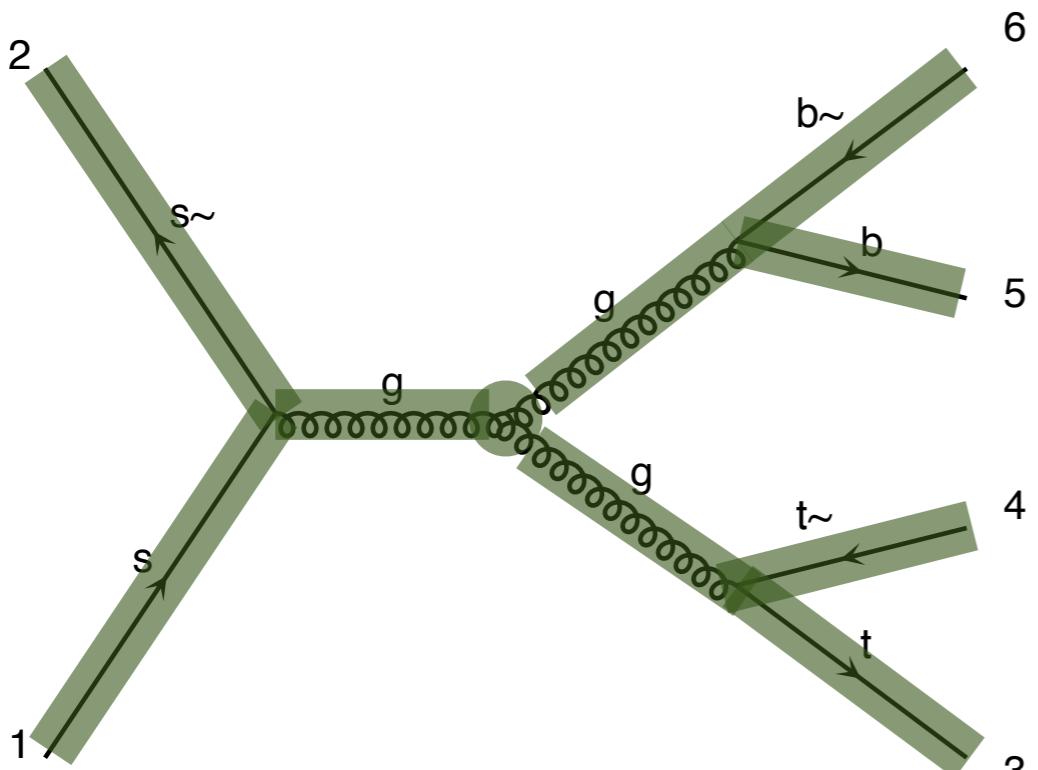
Number of routines: 10  
 $2(N+1)$

Number of routines for both: 12

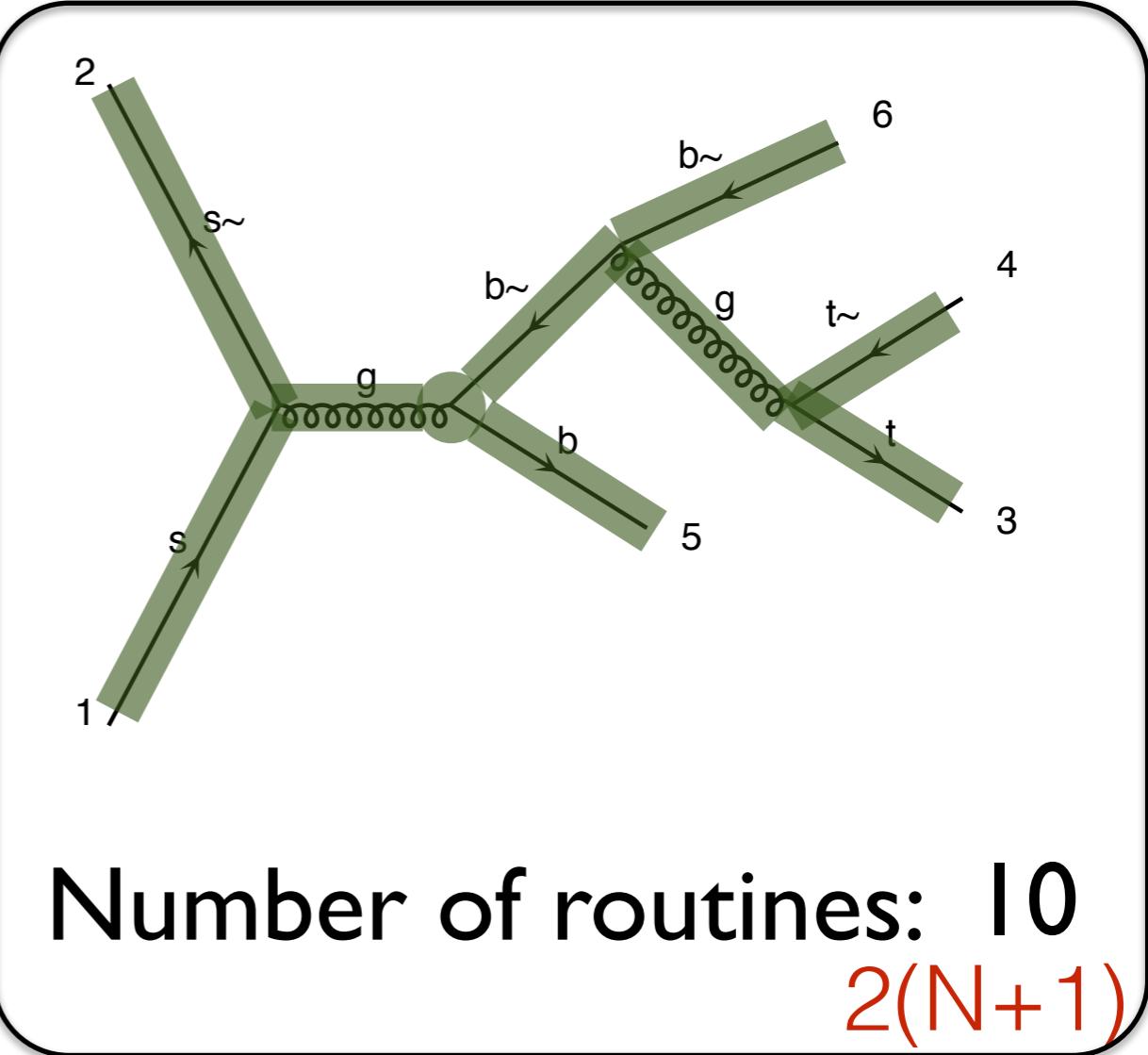
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# Real case

Known



Number of routines: 10  
 $2(N+1)$



Number of routines: 10  
 $2(N+1)$

Number of routines for both: 12

$$N! * 2(N+1) \longrightarrow N!$$

# Comparison

	M diag	N particle
Analytical	$M^2$	$(N!)^2$
Helicity	$M$	$(N!) 2^N$
Recycling	$M$	$(N - 1)! 2^{(N-1)}$

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Recursion Relation	$\log(M)$	$2^N 2^{(N-1)}$
		Not used in Madgraph

# Comparison

	M diag	N particle	2 > 6
Analytical	$M^2$	$(N!)^2$	1.6e9
Helicity	$M$	$(N!) 2^N$	1.0e7
Recycling	$M$	$(N - 1)! 2^{(N-1)}$	6.5e5
Recursion Relation	$\log(M)$	$2^N 2^{(N-1)}$	3.2e4

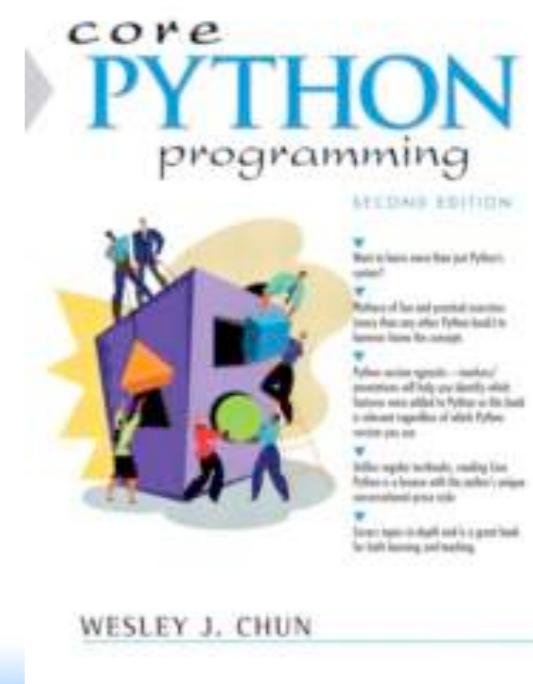
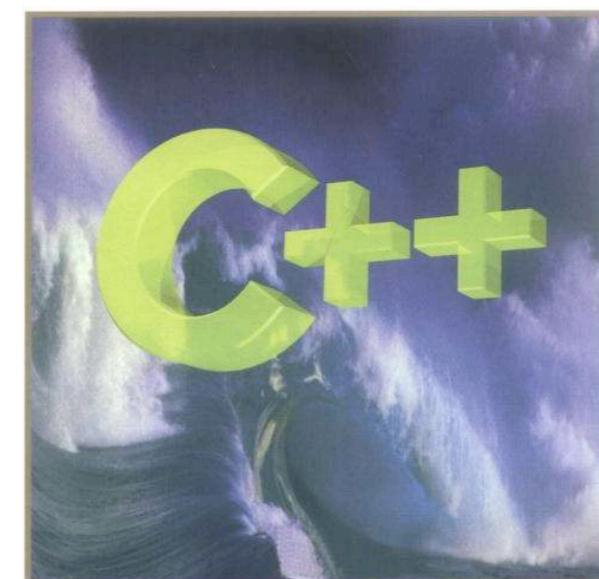
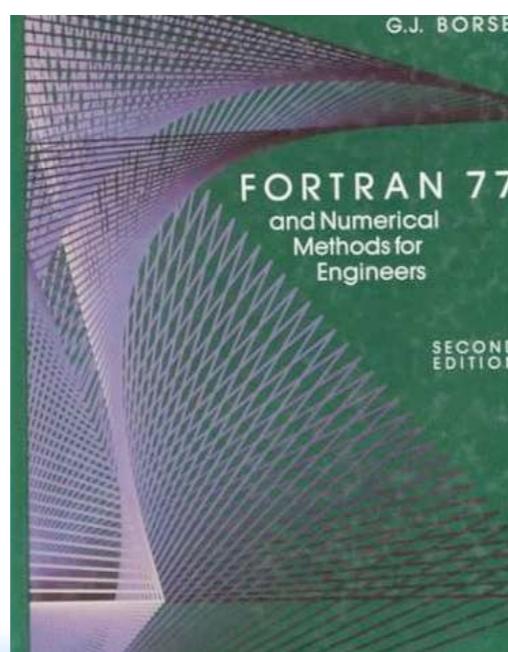


# ALOHA

~~ALOHA  
Google translate~~

From: [ UFO ]  To: Helicity

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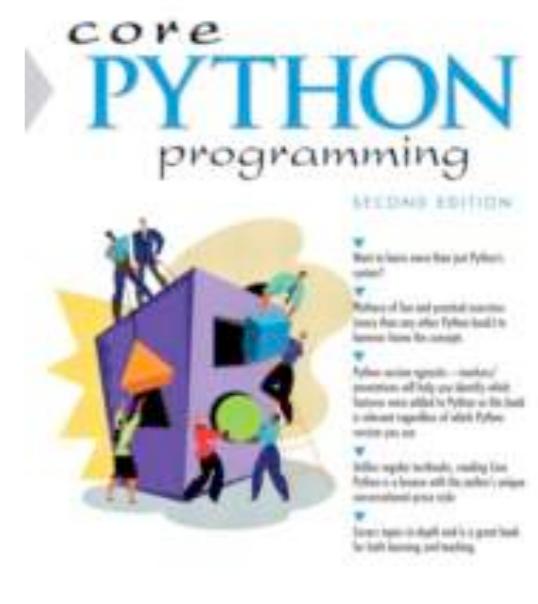
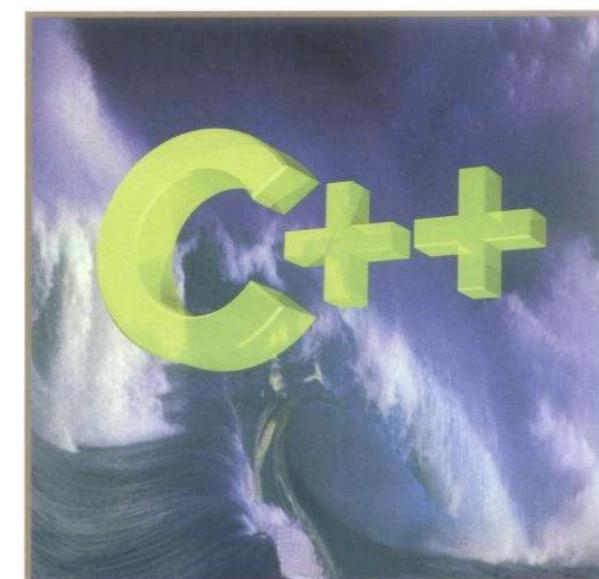
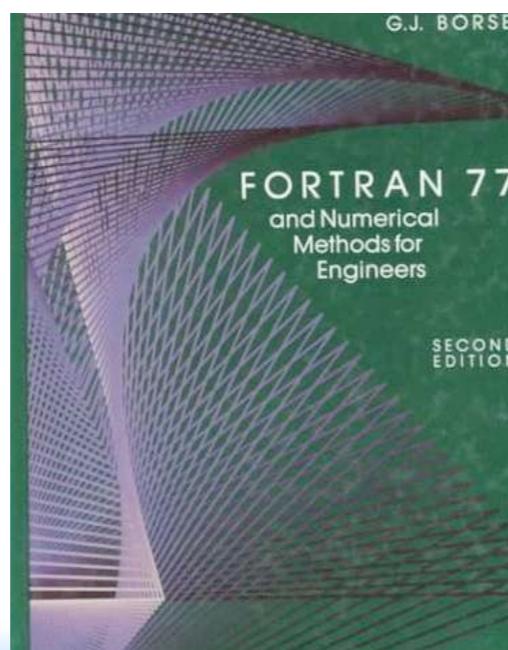
# ALOHA

~~ALOHA  
Google translate~~

From: [ UFO ]  To: Helicity

Basically, any new operator can be handle by MG5/Pythia8 out of the box!

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# To Remember

- Numerical computation faster than analytical computation
- We are able to compute matrix-element
  - for large number of final state
  - for any BSM theory

# Plan

- Details of the computation
  - Evaluation of matrix-element
  - Phase-Space integration
- What is MG5\_aMC?

# Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

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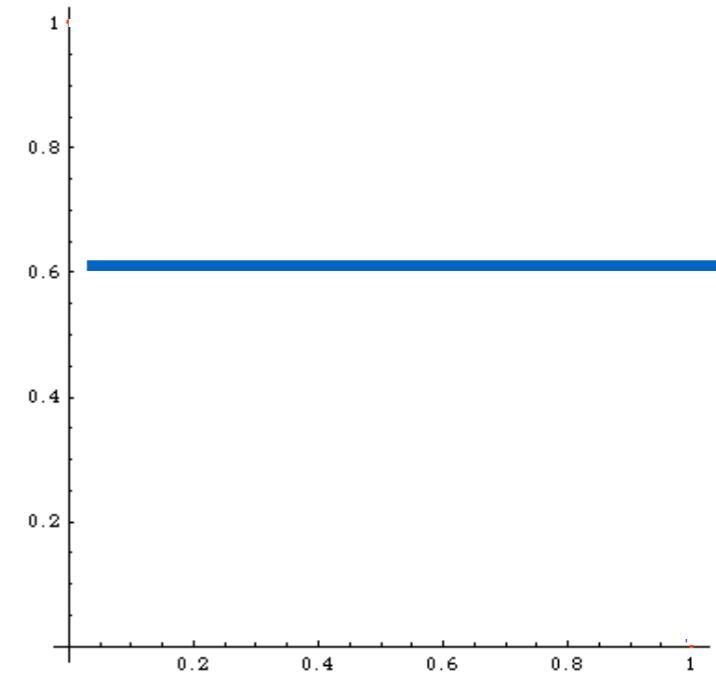
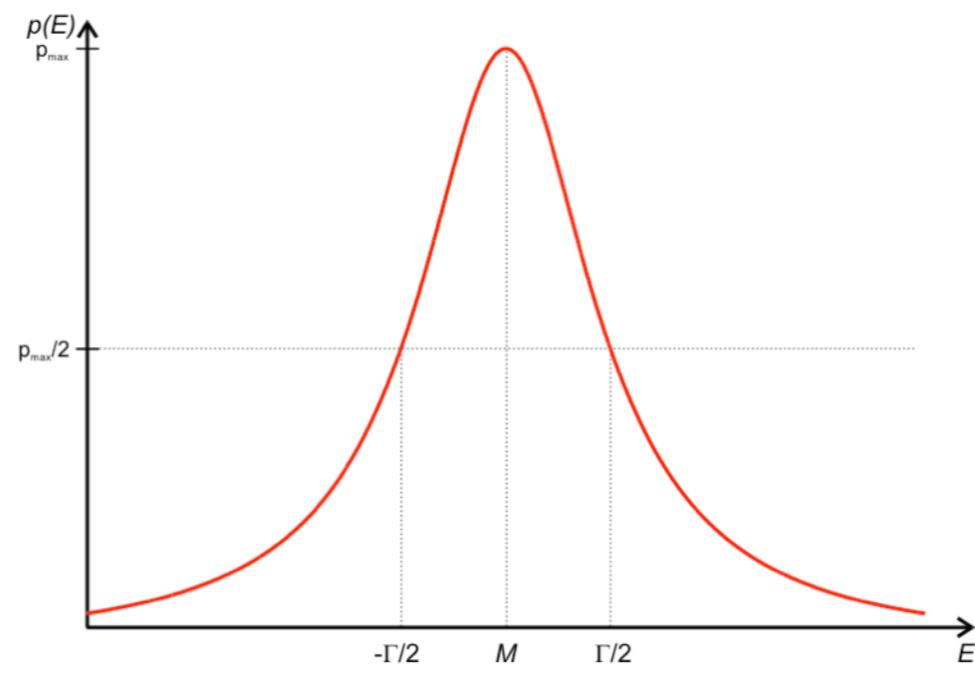
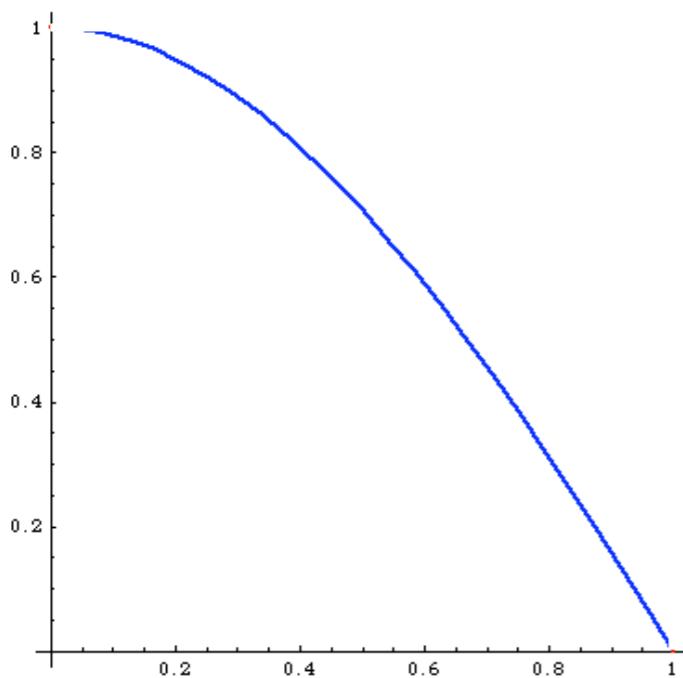
$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

$\dim[\Phi(n)] \sim 3n$

General and flexible method is needed

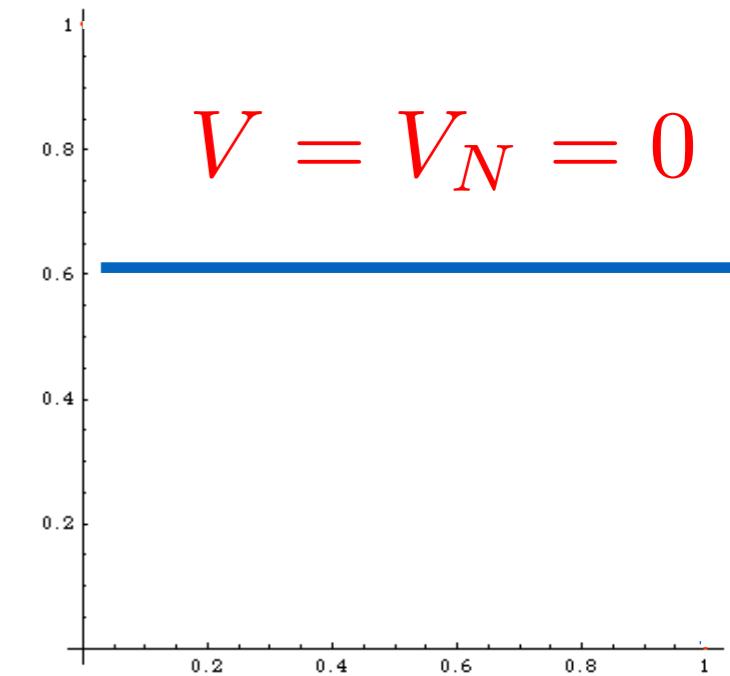
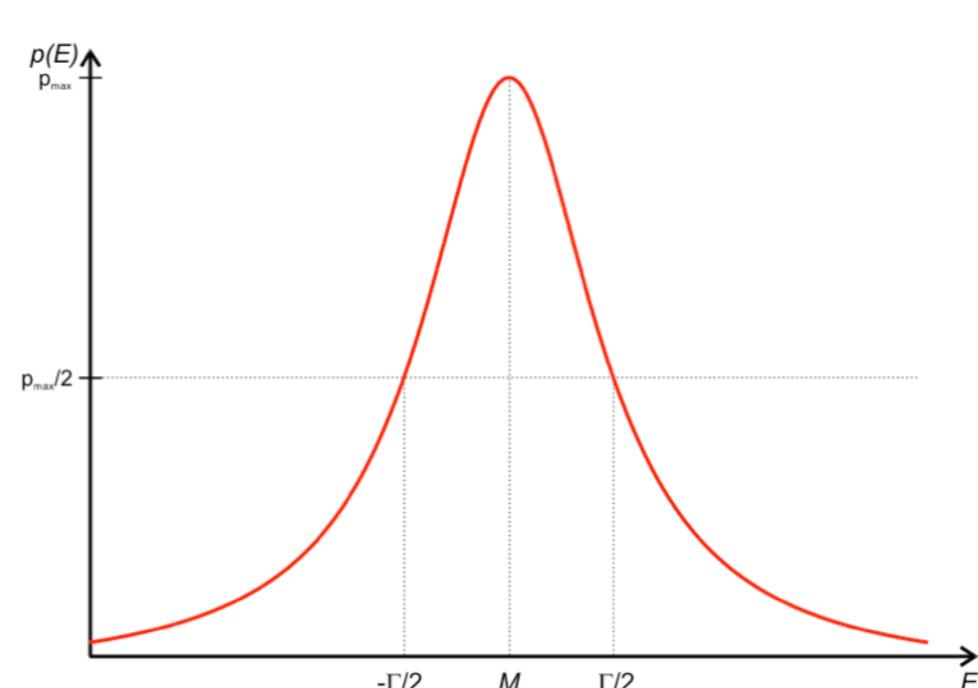
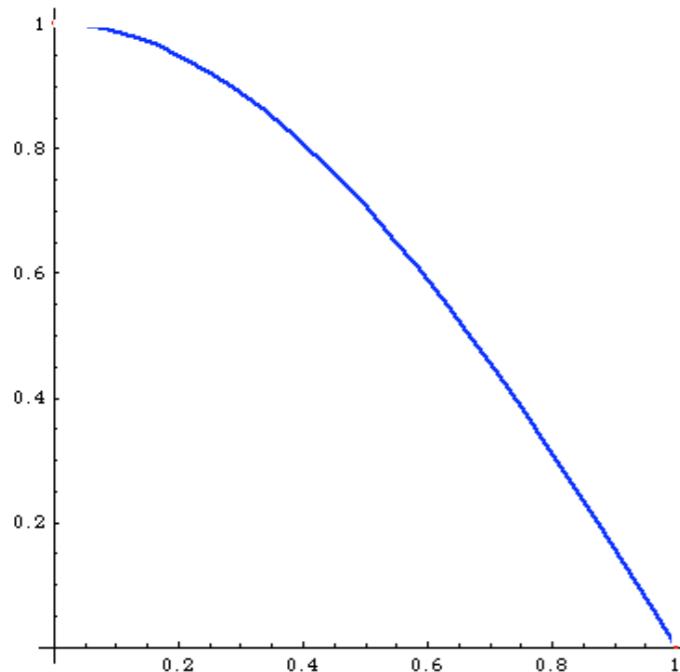
# Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x \quad \int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2} \quad \int dx C$$



# Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x \quad \int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2} \quad \int dx C$$

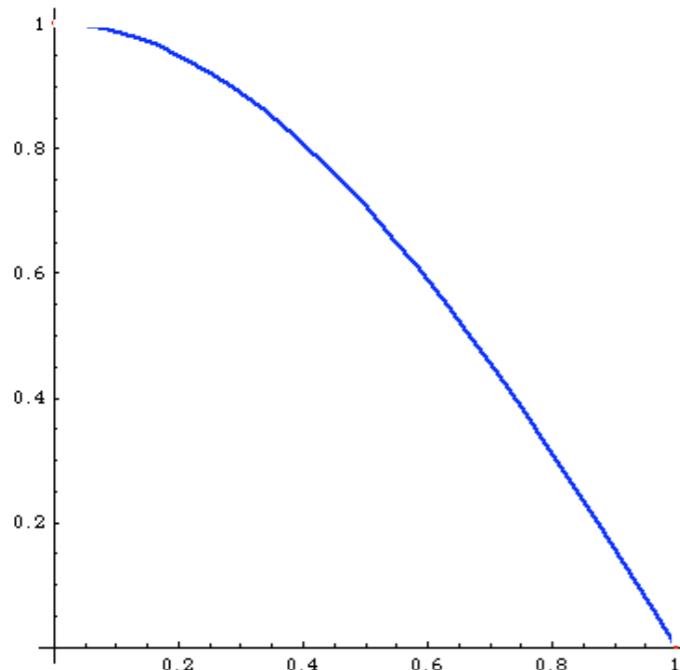


## Method of evaluation

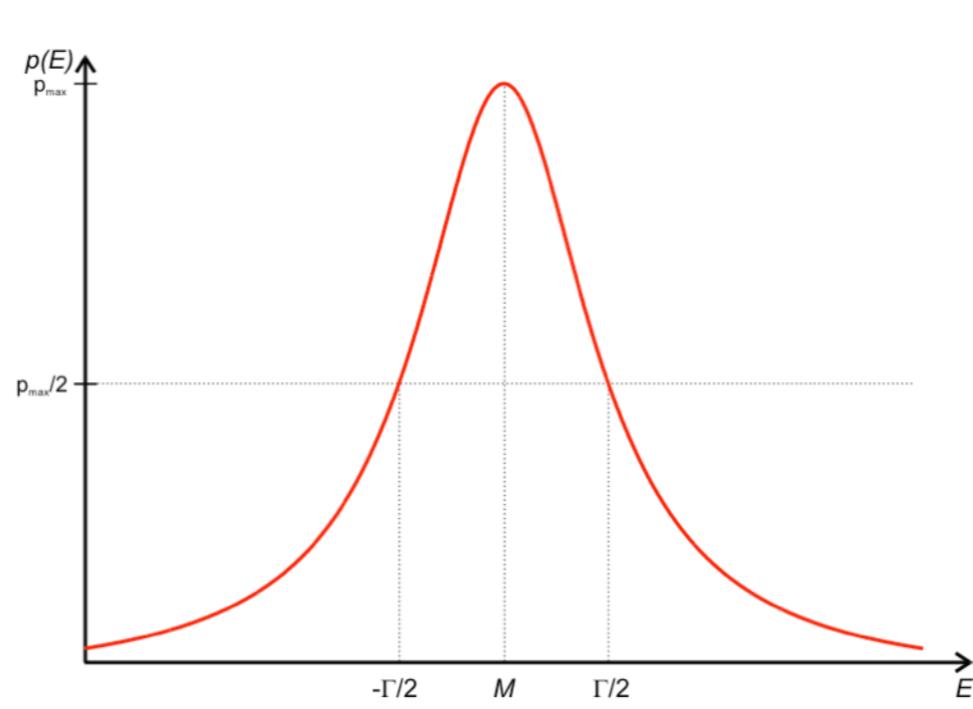
- MonteCarlo  $1/\sqrt{N}$
- Trapezium  $1/N^2$
- Simpson  $1/N^4$

# Integration

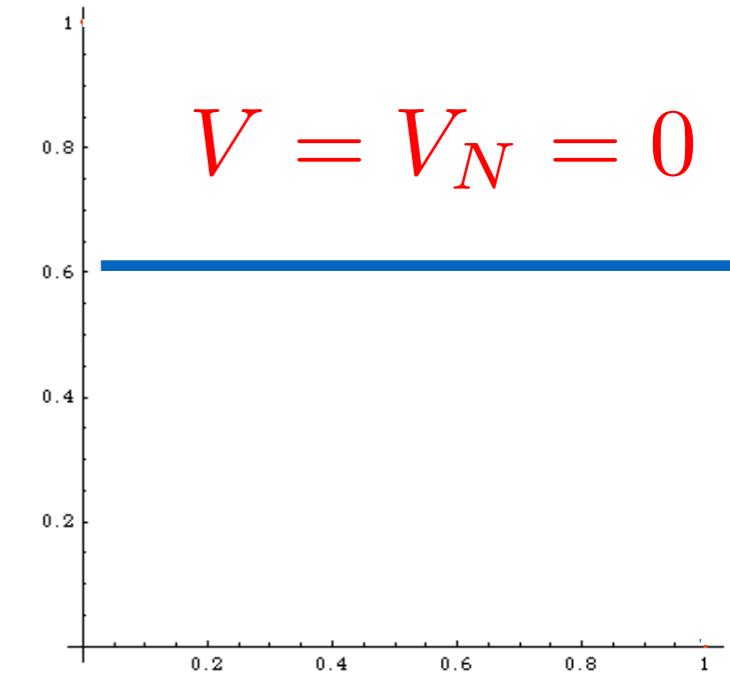
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$



$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



$$\int dx C$$



	<b>simpson</b>	<b>MC</b>
<b>3</b>	0,638	0,3
<b>5</b>	0,6367	0,8
<b>20</b>	0,63662	0,6
<b>100</b>	0,636619	0,65
<b>1000</b>	0,636619	0,636

## Method of evaluation

- MonteCarlo
- Trapezium
- Simpson

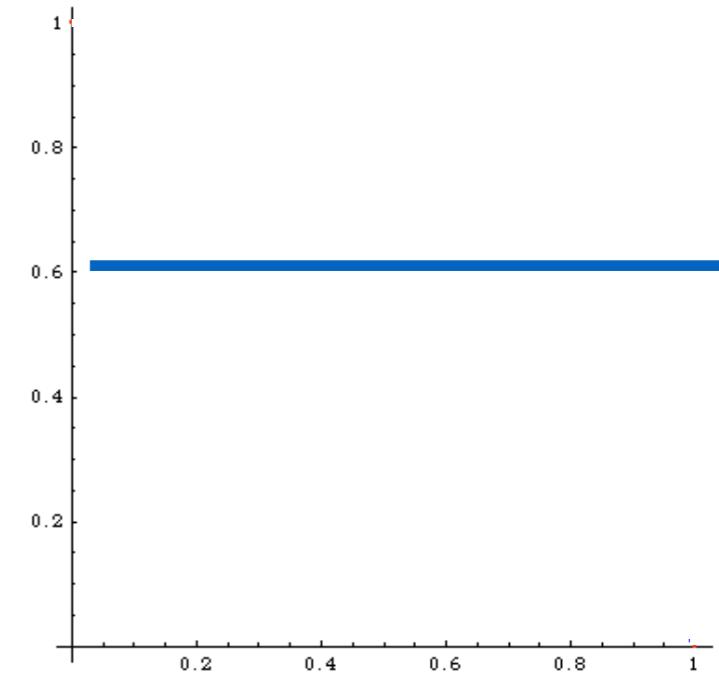
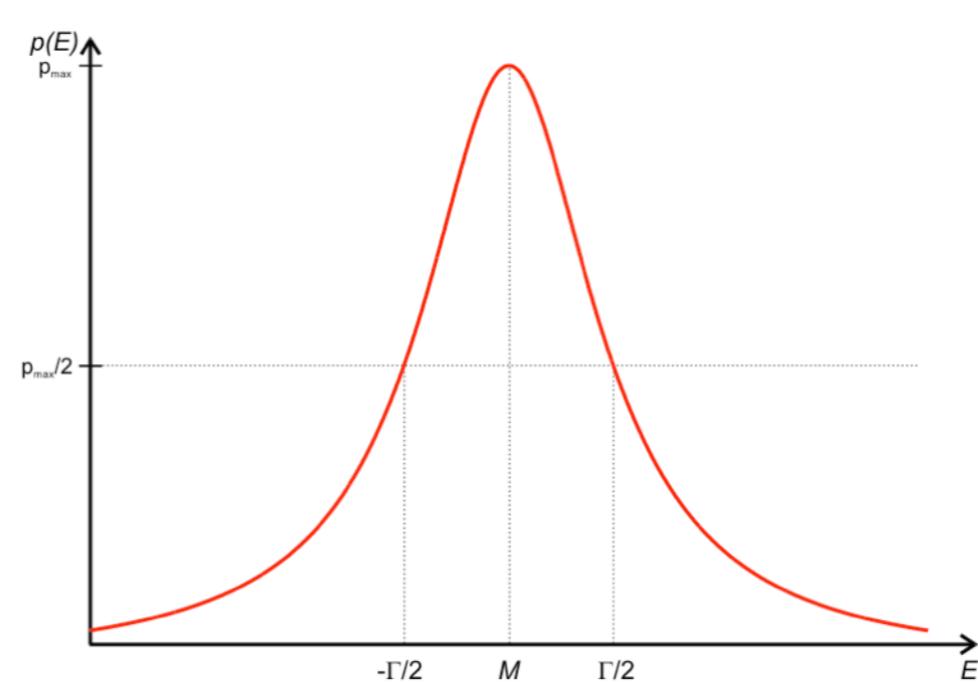
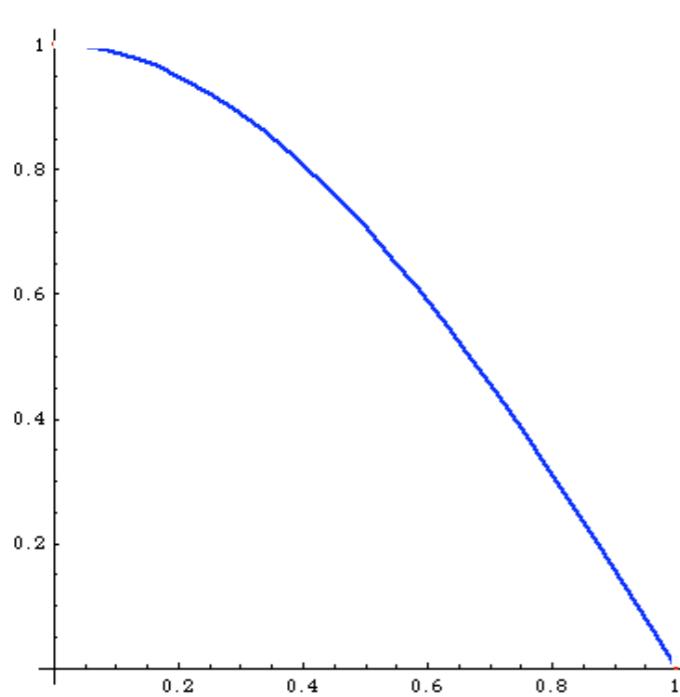
$1/\sqrt{N}$

$1/N^2$

$1/N^4$

# Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x \quad \int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2} \quad \int dx C$$



## Method of evaluation

- MonteCarlo
- Trapezium
- Simpson

$$1/\sqrt{N}$$

$$1/N^2$$

$$1/N^4$$

More Dimension



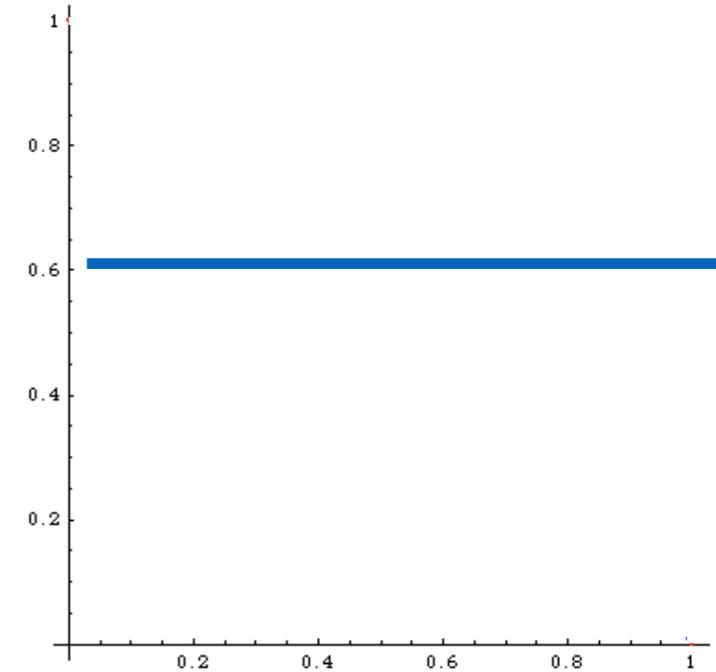
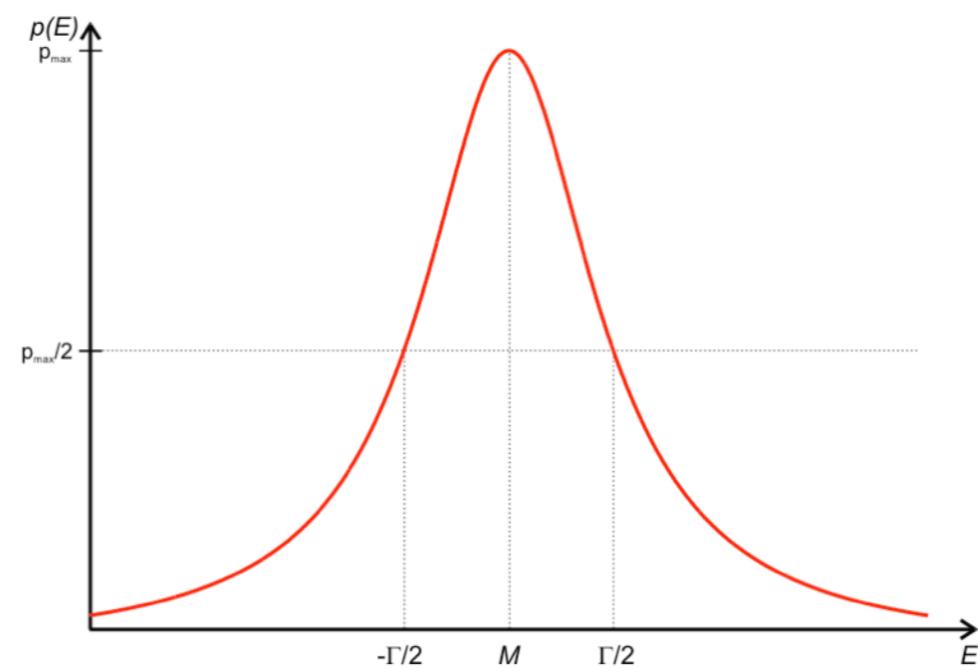
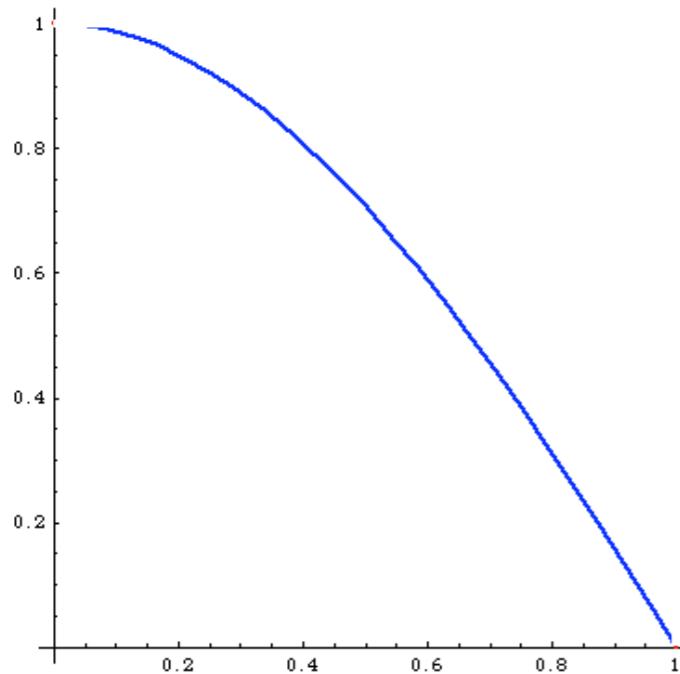
$$1/\sqrt{N}$$

$$1/N^{2/d}$$

$$1/N^{4/d}$$

# Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x \quad \int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2} \quad \int dx C$$

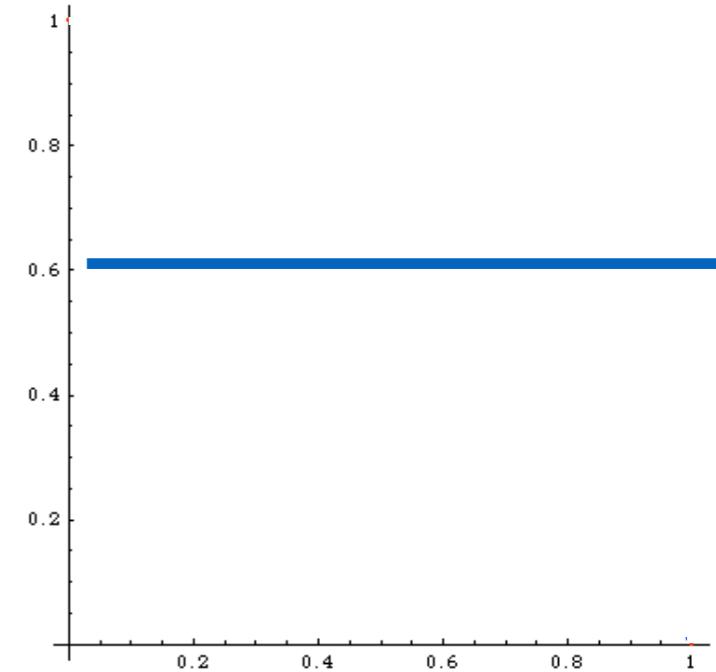
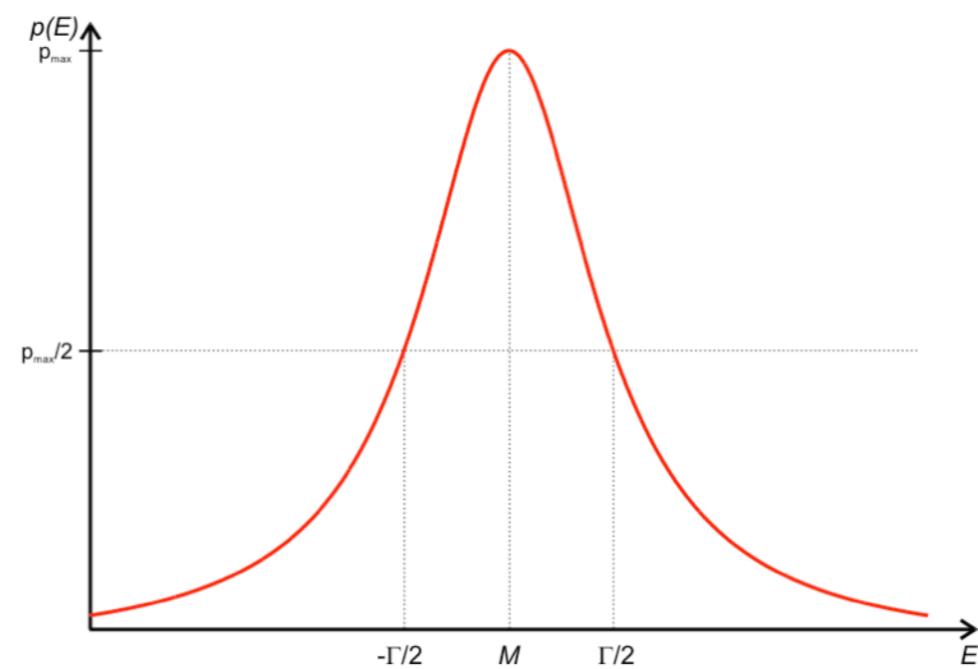
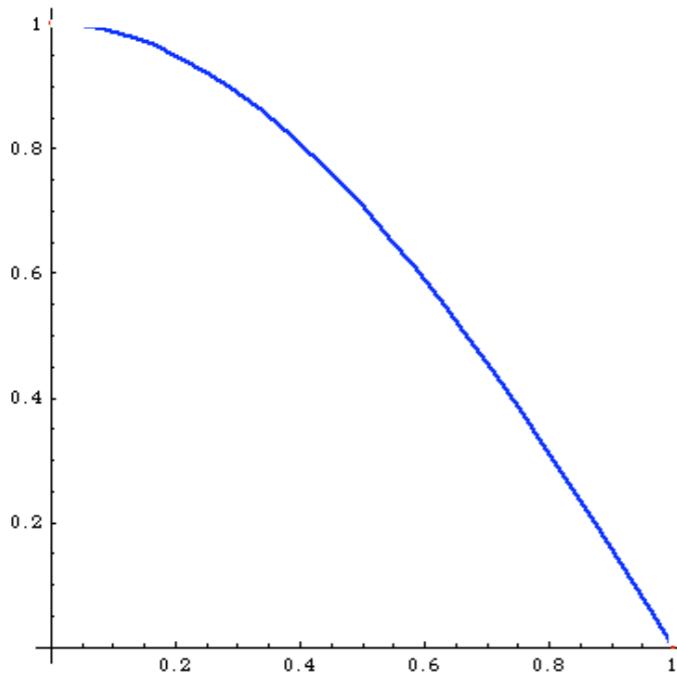


$$I = \int_{x_1}^{x_2} f(x) dx \quad \rightarrow \quad I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x)$$

$$V = (x_2 - x_1) \int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N} \sum_{i=1}^N [f(x)]^2 - I_N^2$$

# Integration

$$I = \int_0^1 dx \cos \frac{\pi}{2} x \quad \int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2} \quad \int dx C$$



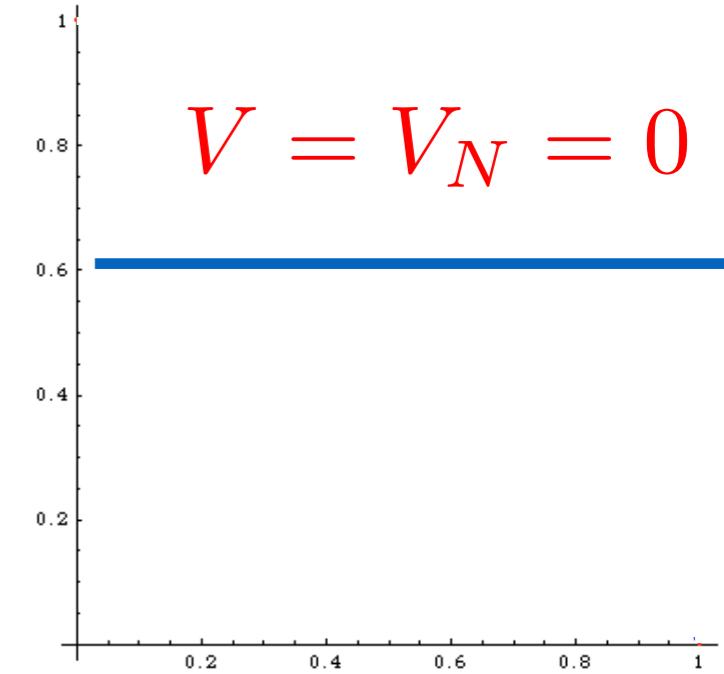
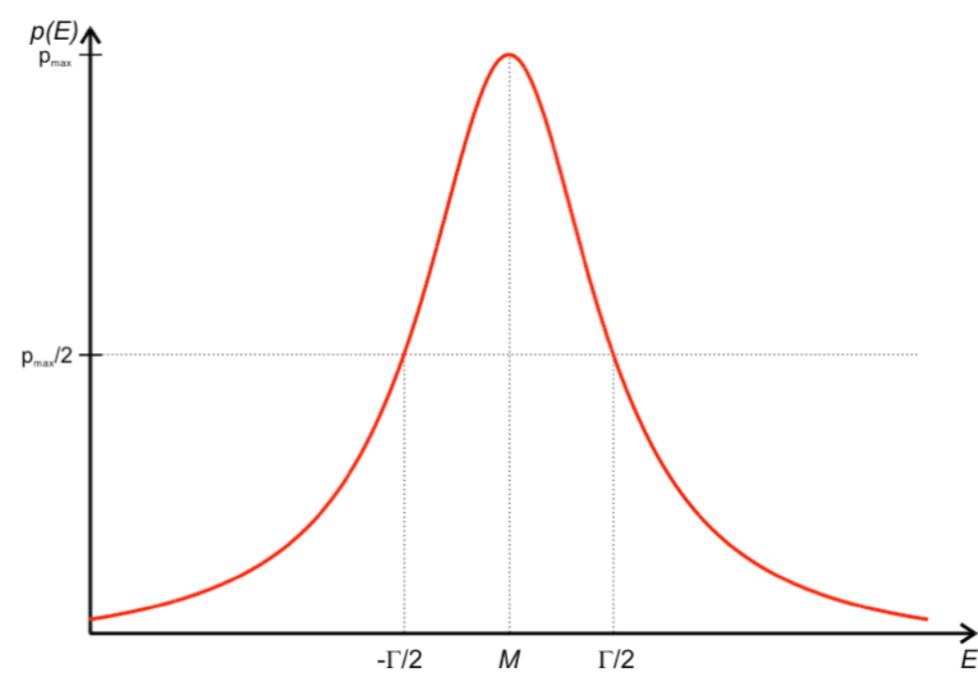
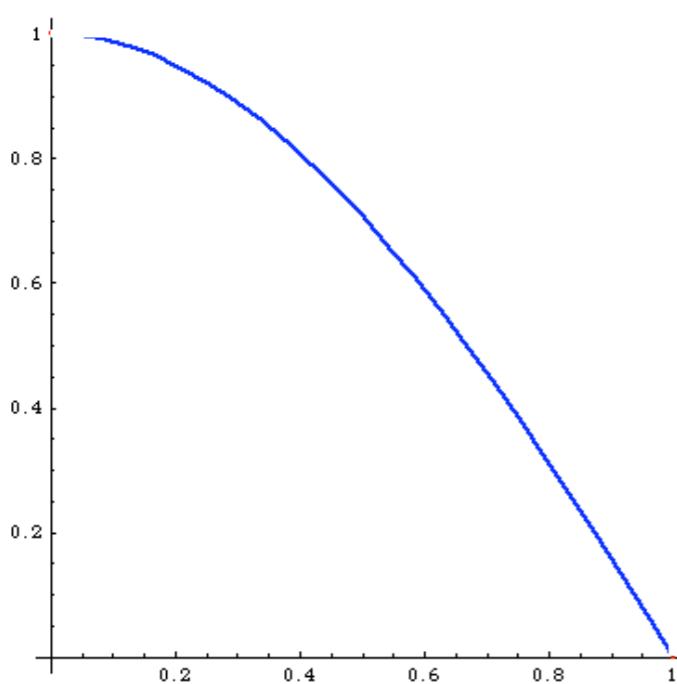
$$I = \int_{x_1}^{x_2} f(x) dx \quad \rightarrow \quad I_N = (x_2 - x_1) \frac{1}{N} \sum_{i=1}^N f(x)$$

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$$I = I_N \pm \sqrt{V_N/N}$$

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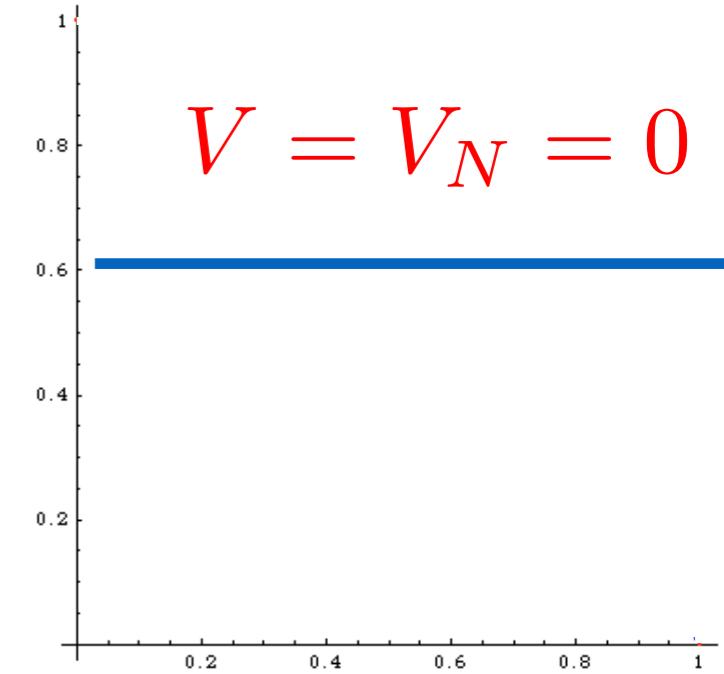
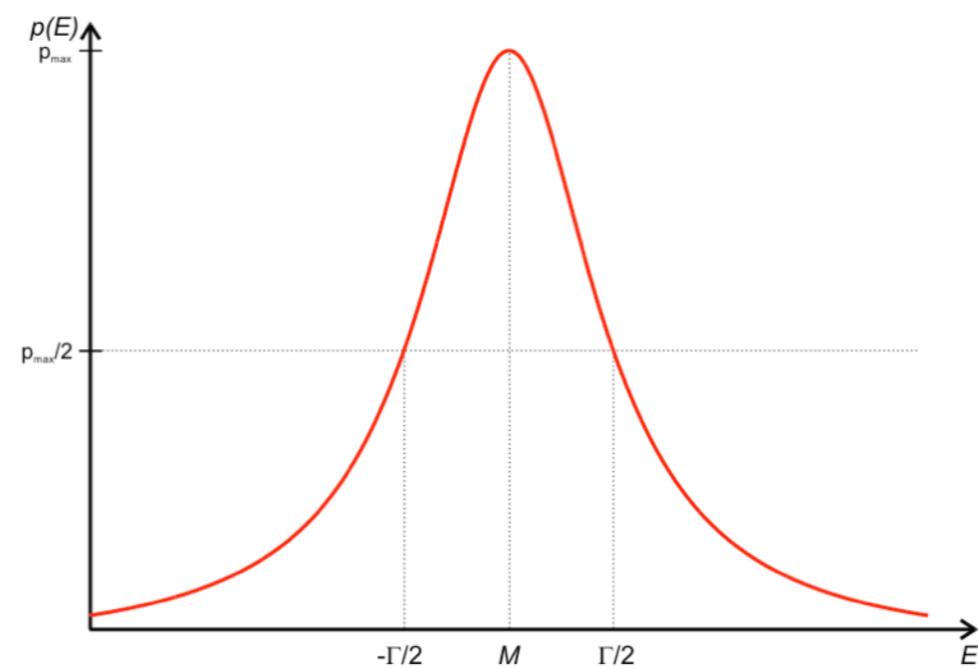
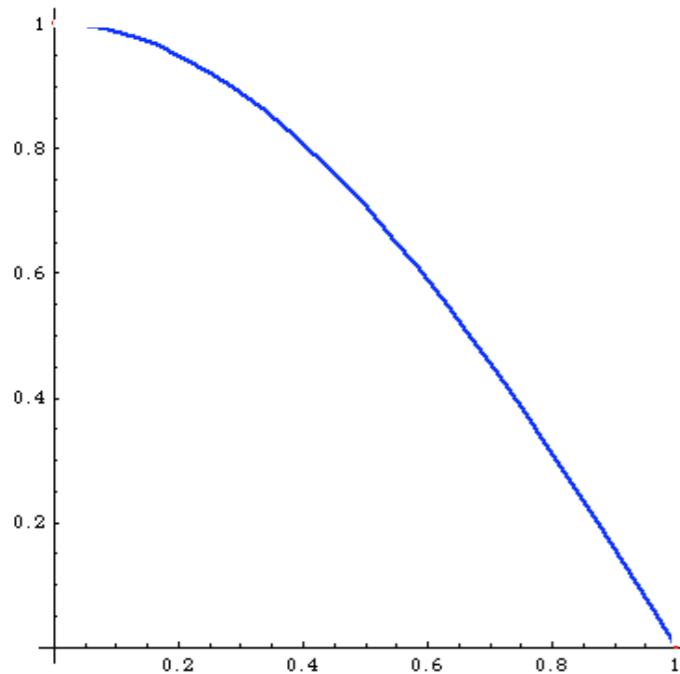
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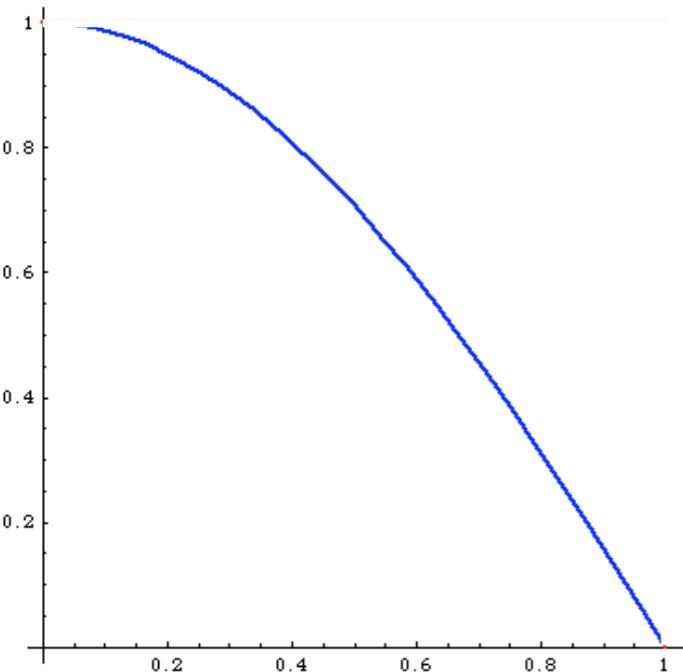


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$$I = I_N \pm \sqrt{V_N/N} \quad \text{Can be minimized!}$$

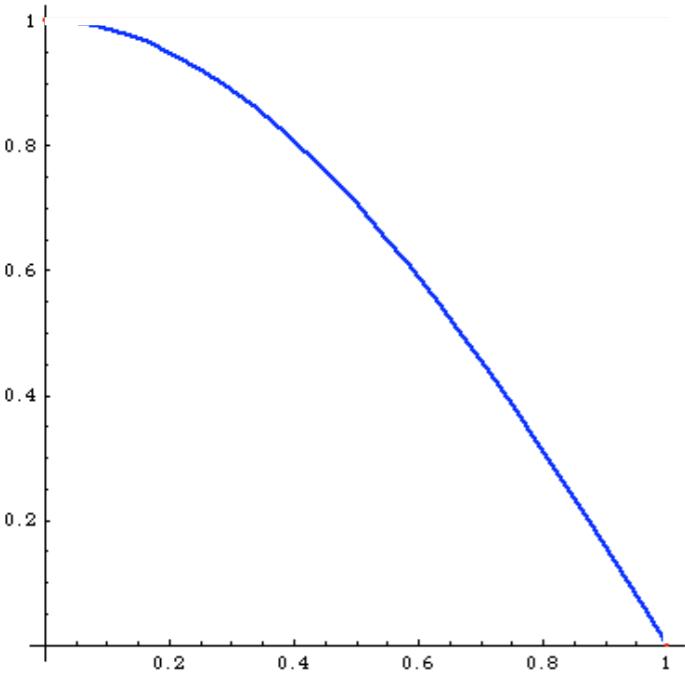
# Importance Sampling



$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

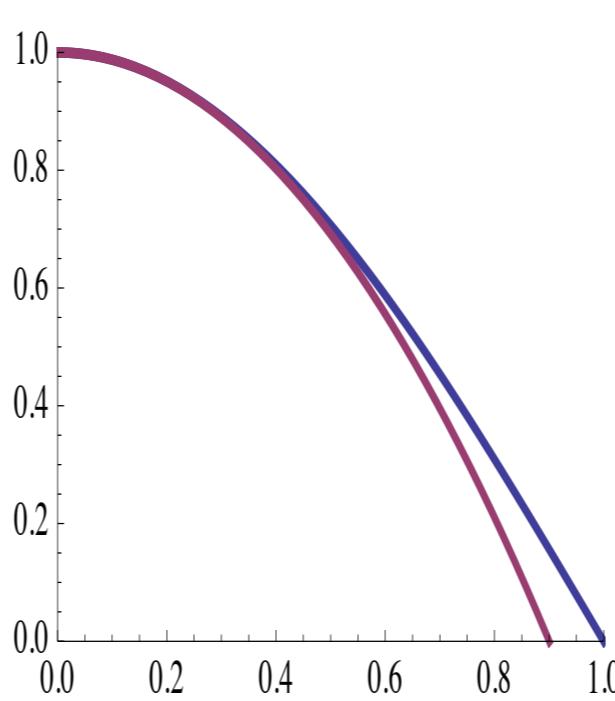
$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

# Importance Sampling



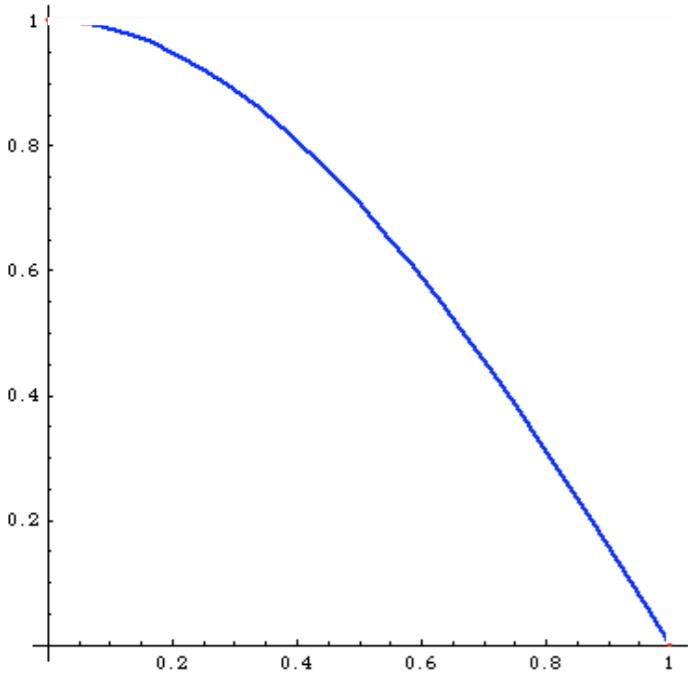
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

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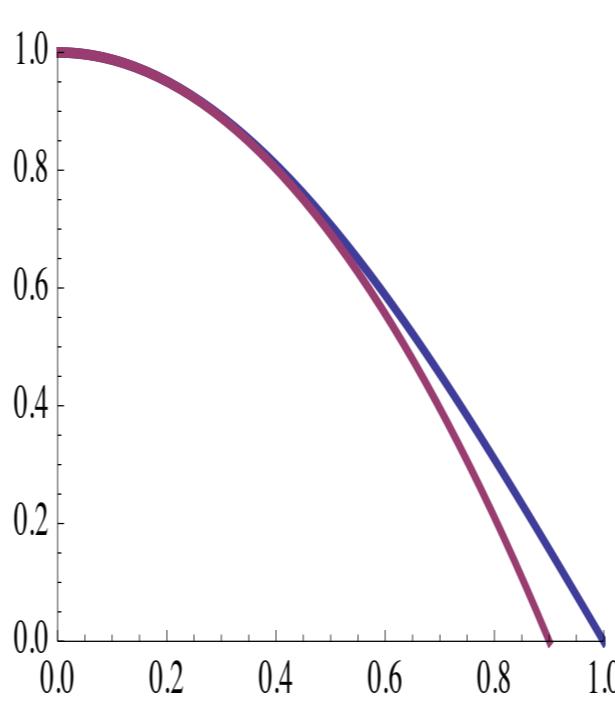
$$I = \int_0^1 dx (1 - cx^2) \frac{\cos\left(\frac{\pi}{2}x\right)}{(1 - cx^2)}$$

# Importance Sampling



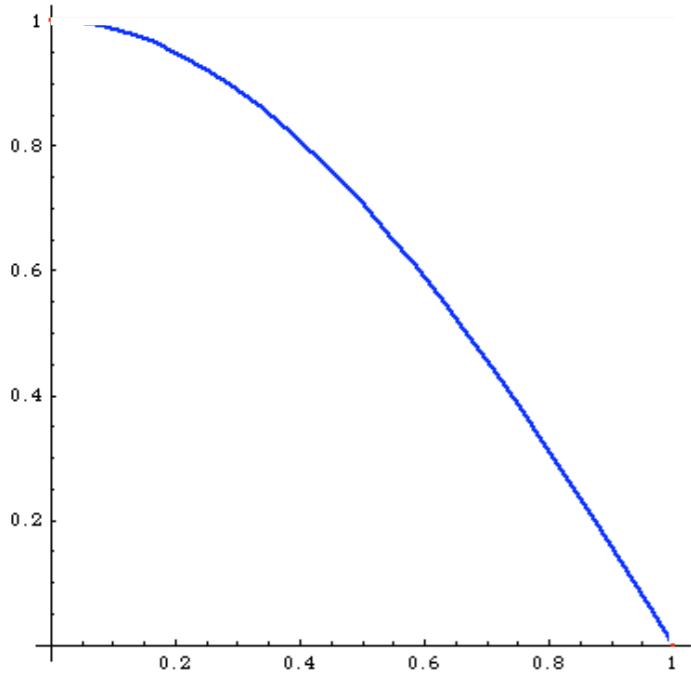
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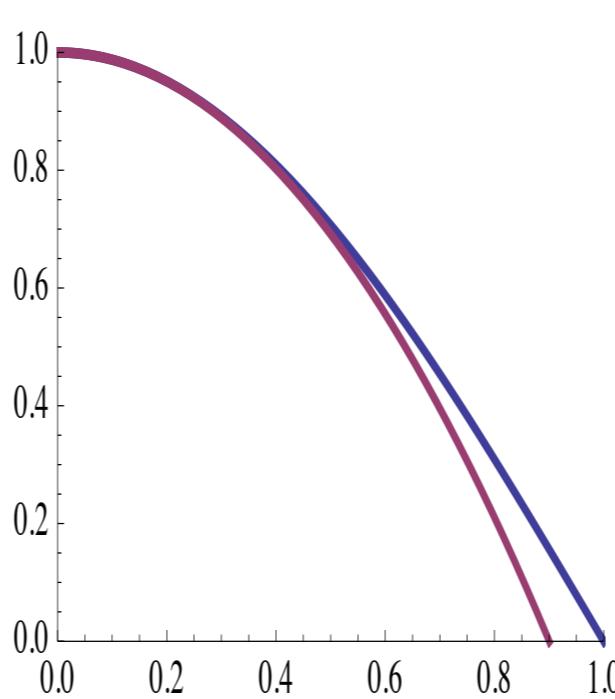
$$I = \int_0^1 dx (1 - cx^2) \frac{\cos\left(\frac{\pi}{2}x\right)}{(1 - cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2 c}$$

# Importance Sampling



$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

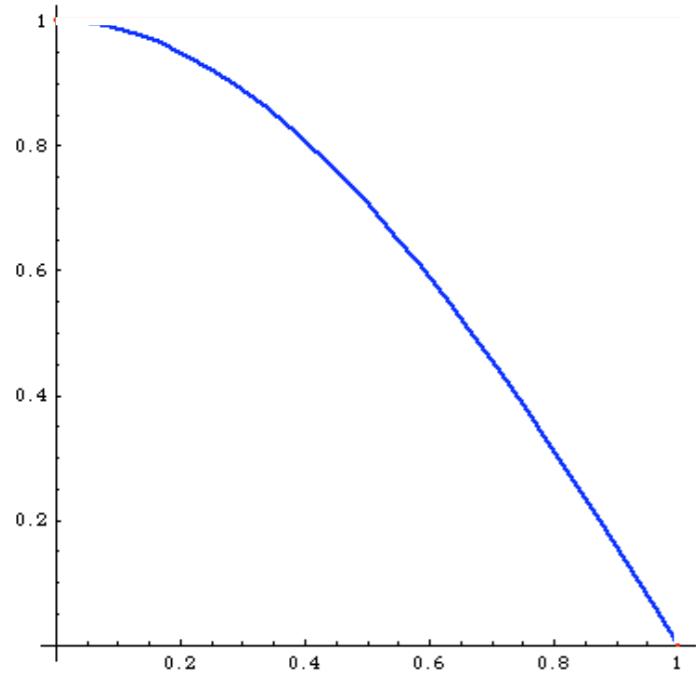
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$$I = \int_0^1 dx (1 - cx^2) \frac{\cos\left(\frac{\pi}{2}x\right)}{(1 - cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2 c}$$

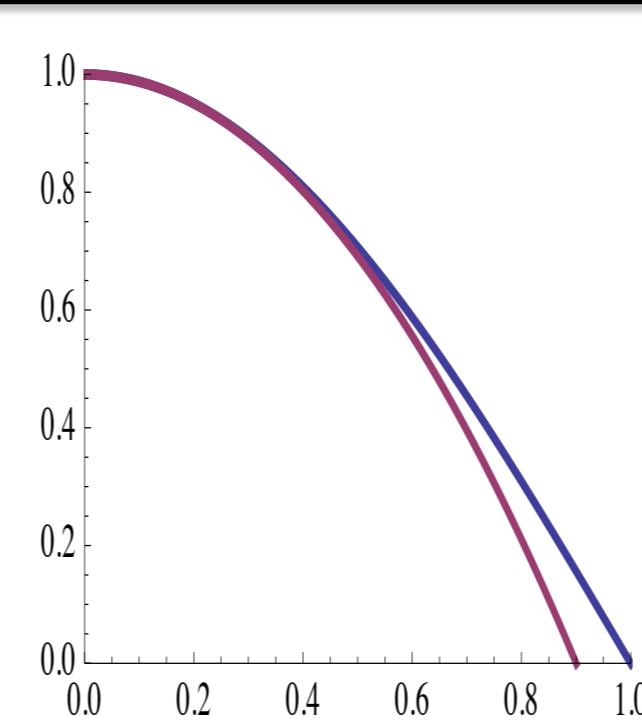
↗  $\simeq 1$

# Importance Sampling

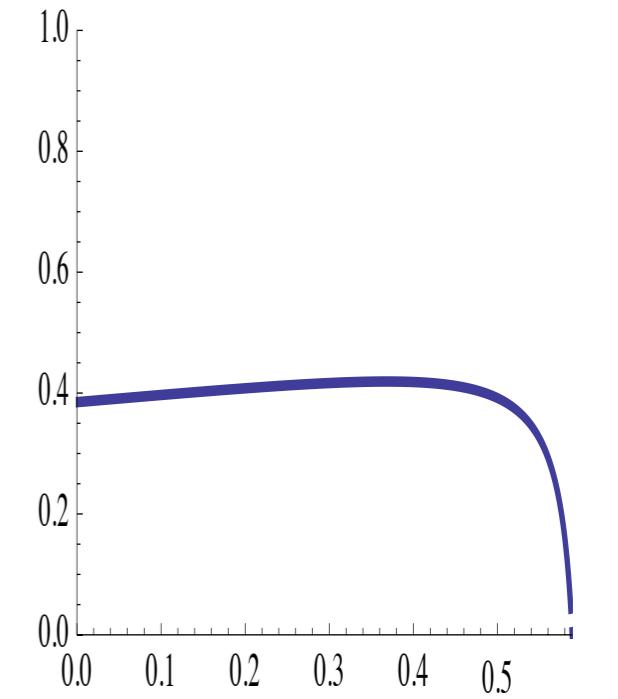


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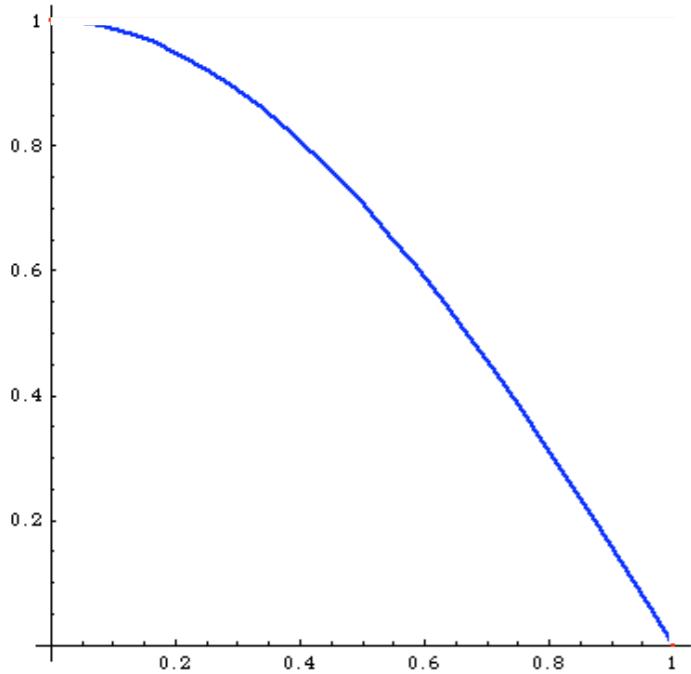


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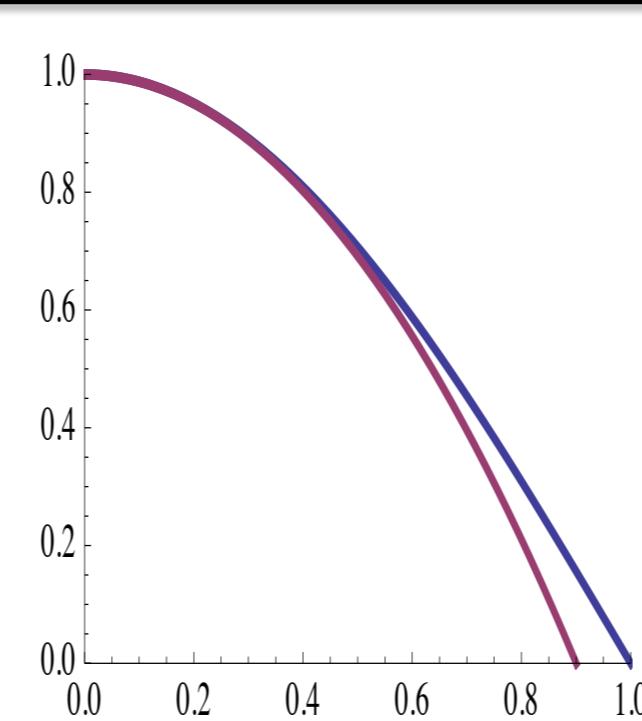
$\simeq 1$

# Importance Sampling



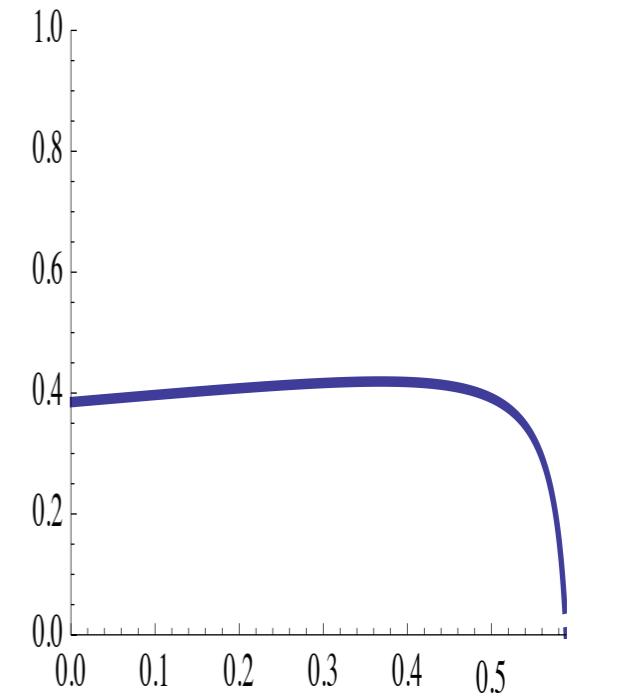
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$



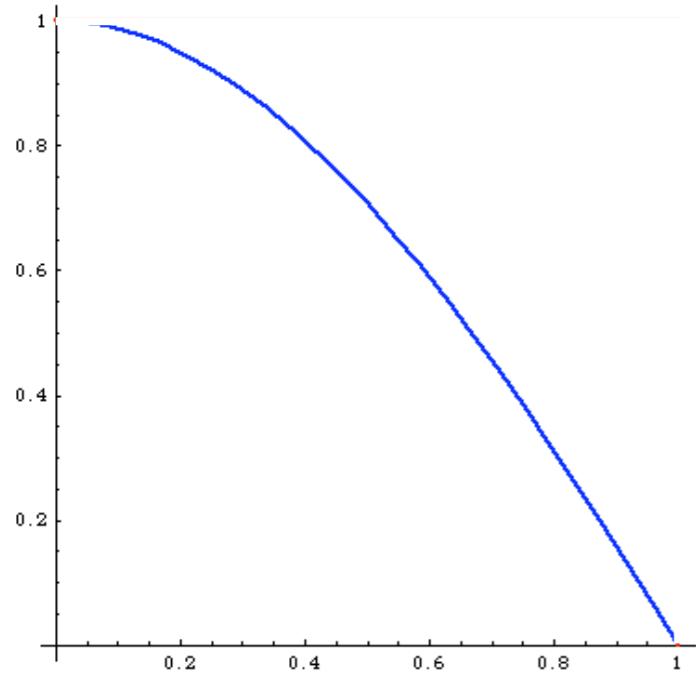
$$I = \int_0^1 dx (1 - cx^2) \frac{\cos\left(\frac{\pi}{2}x\right)}{(1 - cx^2)}$$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$



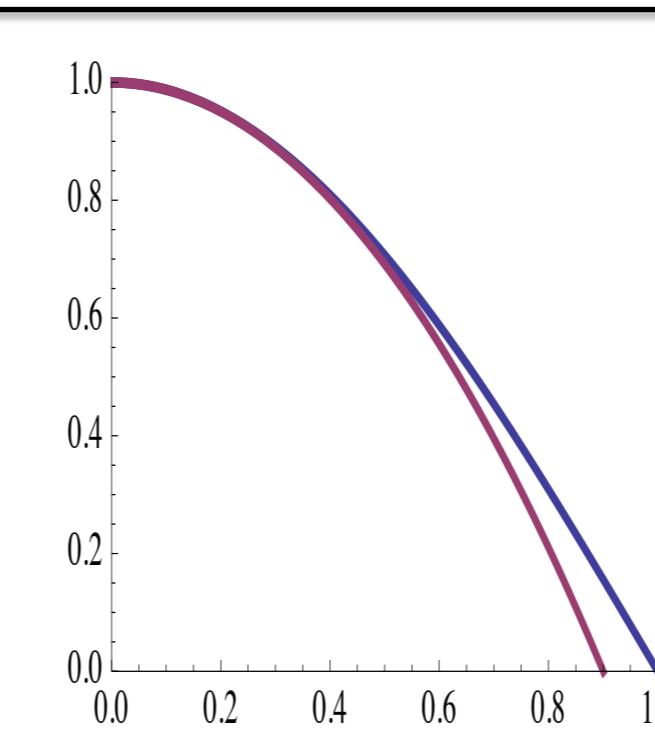
$$\int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2 c} \simeq 1$$

# Importance Sampling



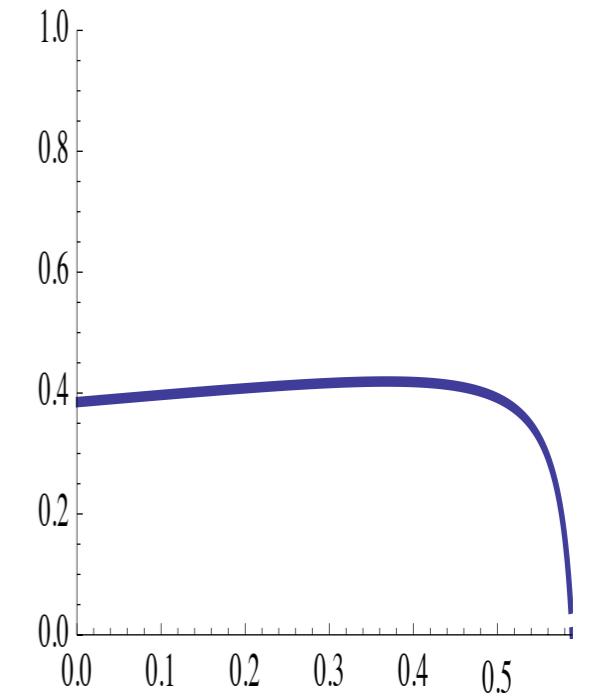
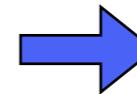
$$I = \int_0^1 dx \cos \frac{\pi}{2} x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$

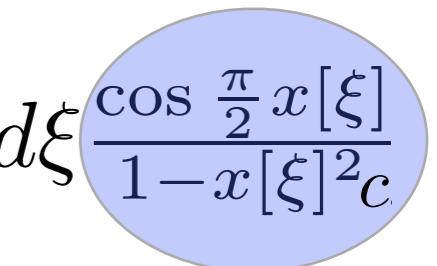


$$I = \int_0^1 dx (1 - cx^2) \frac{\cos(\frac{\pi}{2}x)}{(1 - cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos \frac{\pi}{2} x[\xi]}{1 - x[\xi]^2 c}$$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$



$\rightarrow \simeq 1$



The Phase-Space parametrization is important to have an efficient computation!

# Importance Sampling

## Key Point

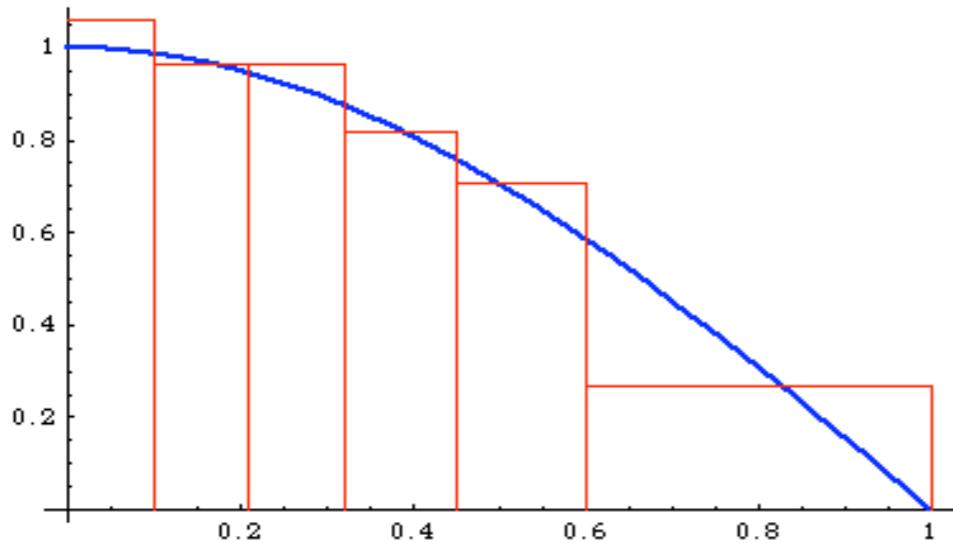
- Generate the **random point** in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is **flatter** in this new variable.
- Needs to know an **approximate function**.

## Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

## Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

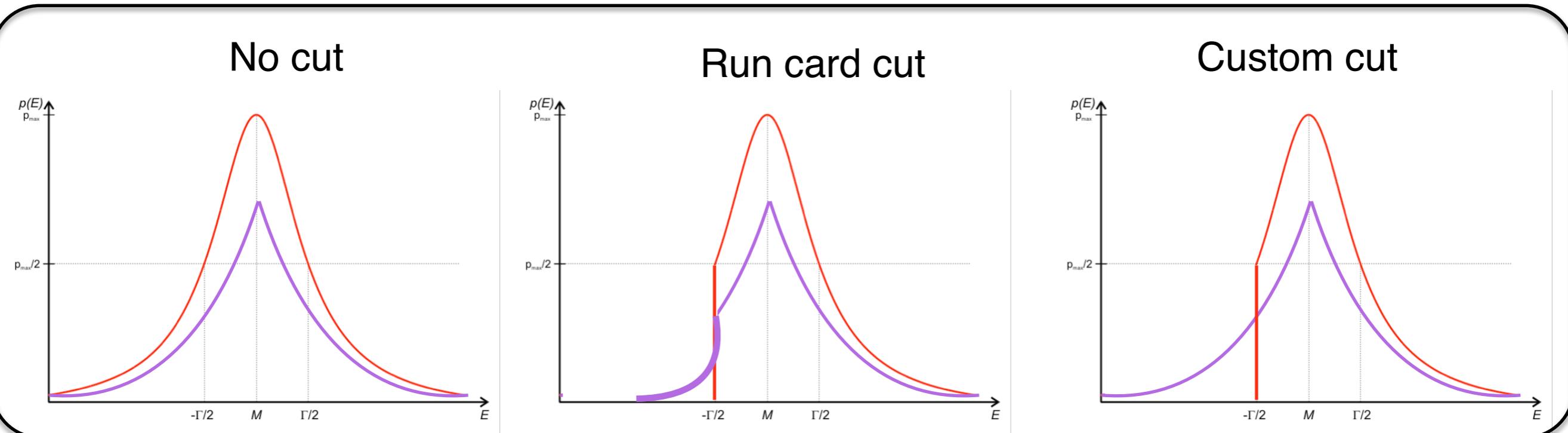


### Algorithm

1. Creates bin such that each of them have the same contribution.
  - Many bins where the function is large
2. Use the approximate for the importance sampling method.

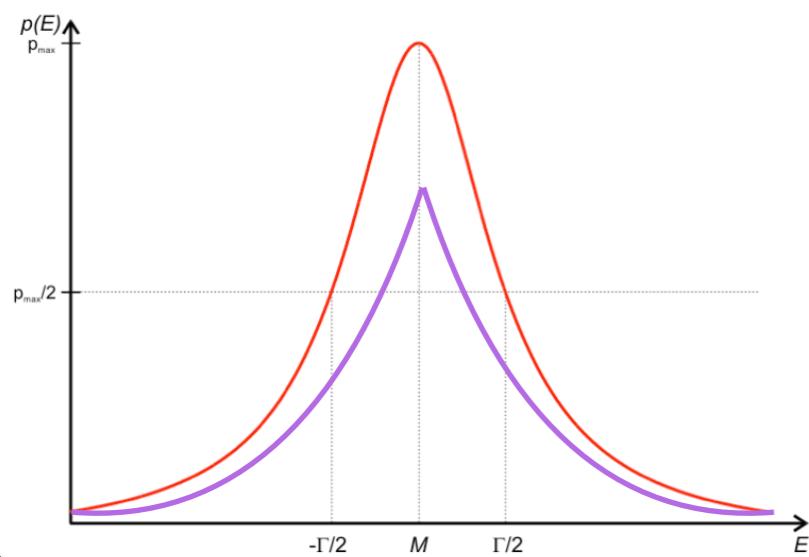
# Cut Impact

- Events are generated according to our best knowledge of the function
  - Basic cut include in this “best knowledge”
  - Custom cut are ignored

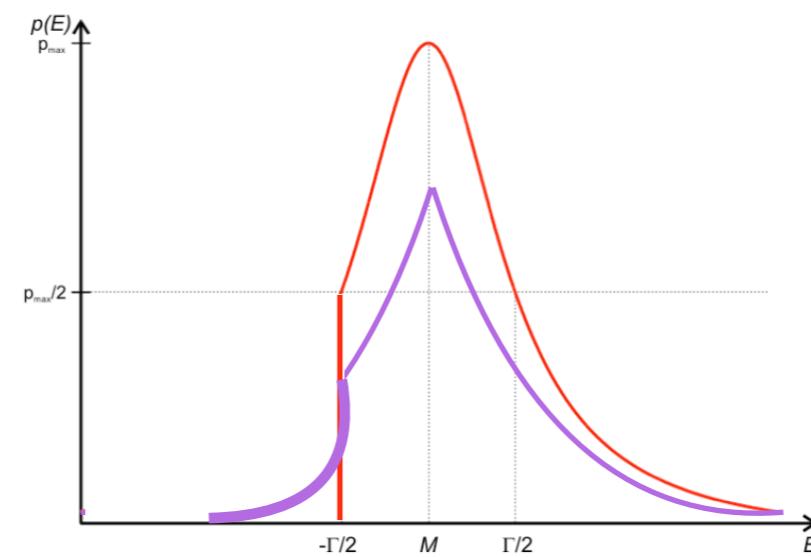


# Cut Impact

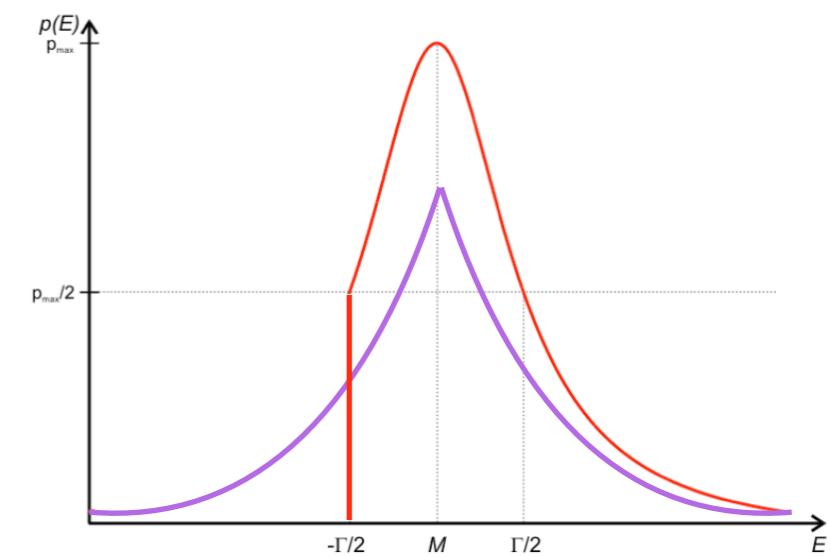
No cut



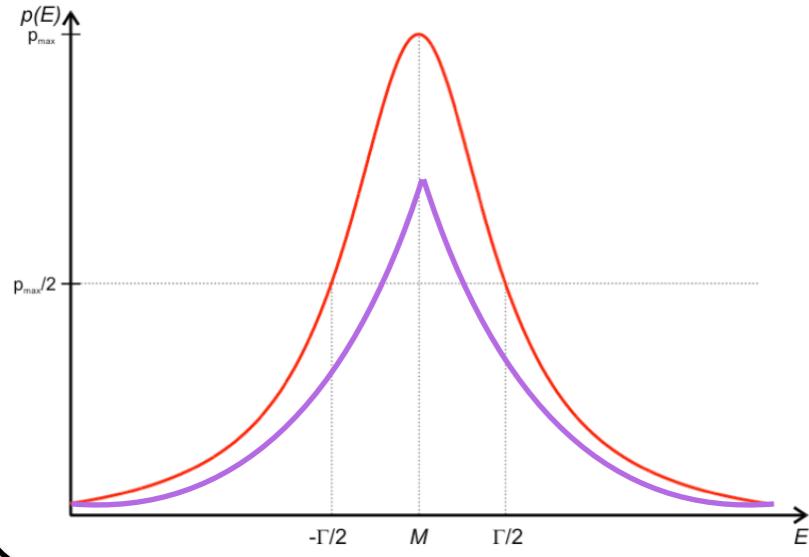
Run card cut



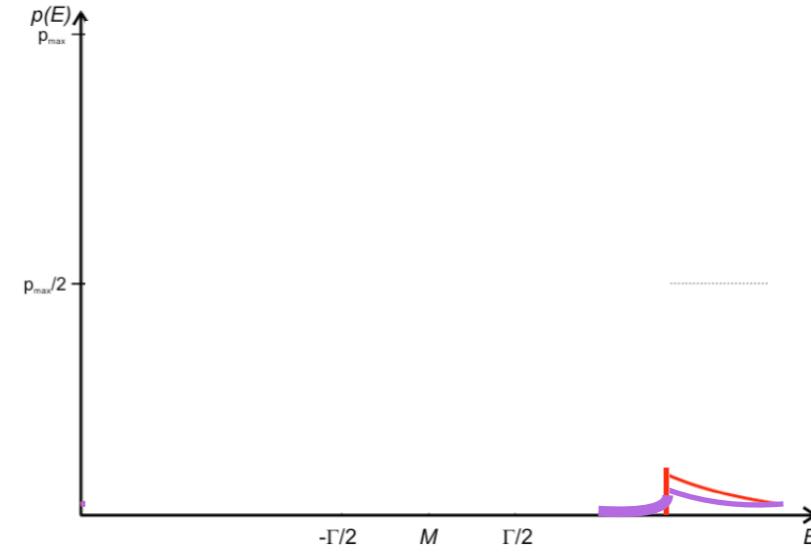
Custom cut



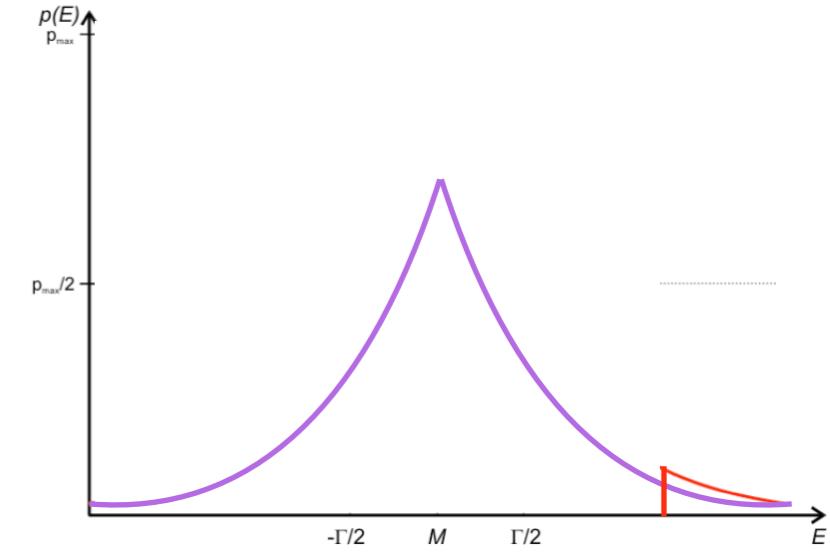
No cut



Run card cut

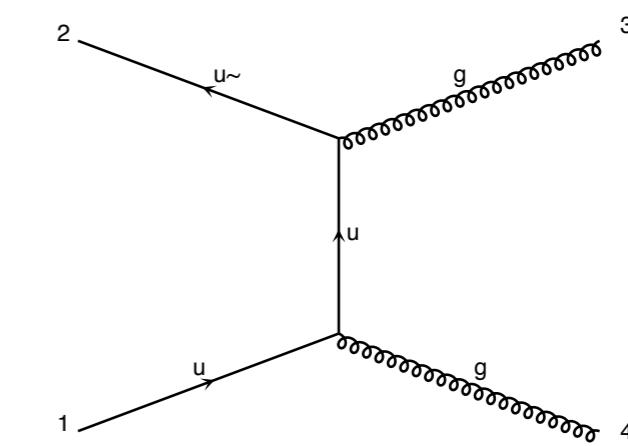
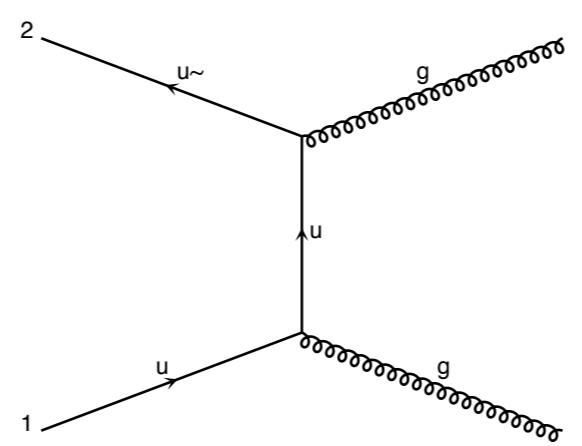
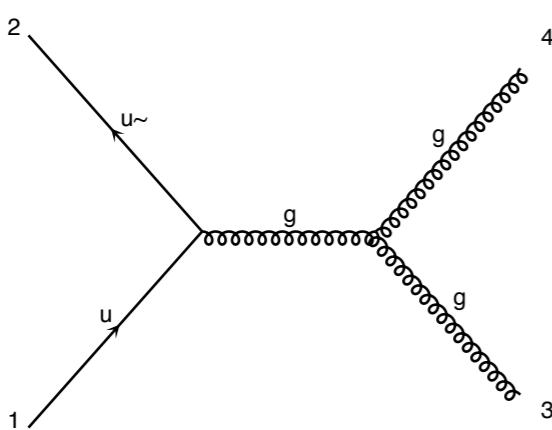


Custom cut



Might miss the contribution and think it is just zero.

# Example: QCD $2 \rightarrow 2$



$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2}$$

$$\propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2}$$

$$\propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

# Let's cut the problem in piece

$$|M_T|^2 = \frac{|M_1|^2 + |M_2|^2}{|M_1|^2 + |M_2|^2} |M_T|^2$$

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

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## Key Idea

- One diagram is manageable
- All other peaks taken care of by denominator sum

# Let's cut the problem in piece

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## Key Idea

- One diagram is manageable
- All other peaks taken care of by denominator sum

## N Integral

- Errors add in quadrature so no extra cost
- Parallel in nature

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

### P1\_qq\_wpwm

**s= 725.73 ± 2.07 (pb)**

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G2.2	<a href="#">377.6</a>	1.67	142.285	7941.0	21
G3	<a href="#">239</a>	1.16	220.04	10856.0	45.5
G1	<a href="#">109.1</a>	0.378	70.88	3793.0	34.8

term of the above sum.

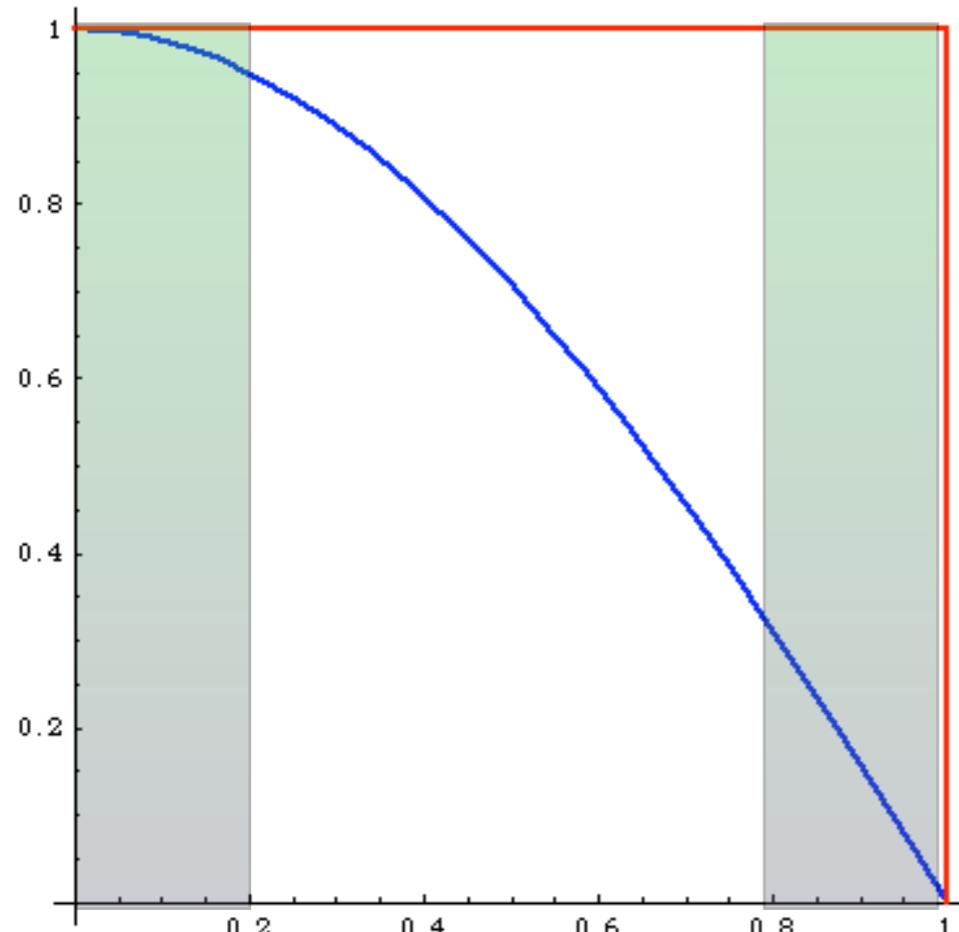
each term might not be gauge invariant

### P1\_gg\_wpwm

**s= 20.714 ± 0.332 (pb)**

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.2	<a href="#">20.71</a>	0.332	7.01	373.0	18

# Event generation

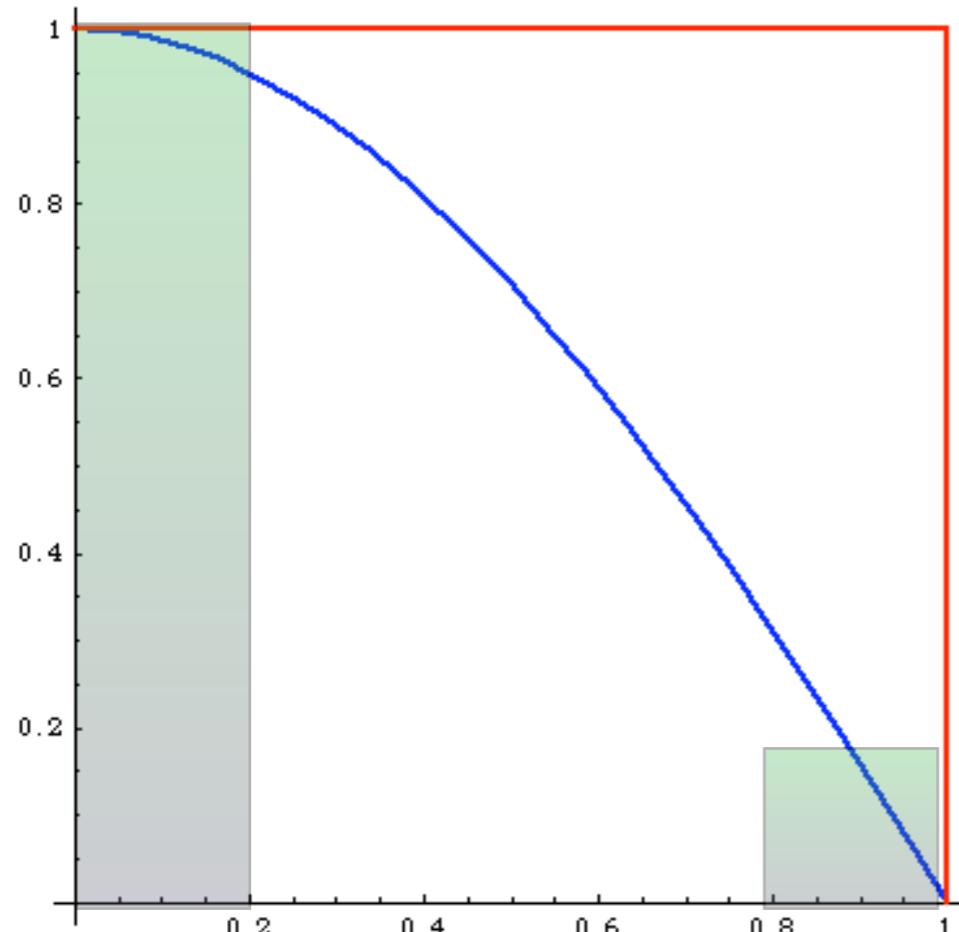


What's the difference between weighted and unweighted?

Weighted:

Same # of events in areas of phase space with very different probabilities:  
events must have different weights

# Event generation



What's the difference between weighted and unweighted?

Unweighted:

# events is proportional to the probability of areas of phase space:  
events have all the same weight ("unweighted")

Events distributed as in nature

# Event generation

$$\int f(x)dx = \frac{1}{N} \sum_{i=1}^N f(x_i)$$

# Event generation

$$\int f(x)dx = \frac{1}{N} \sum_{i=1}^N f(x_i) = \frac{1}{N} \sum_{i=1}^N \frac{f(x_i)}{\max(f)} \max(f)$$

# Event generation

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Number between 0 and 1 (assuming positive function)  
-> re-interpret as the probability to keep the events

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Let's reduce the sample size by playing the lottery.  
For each events throw the dice and see if we keep or reject the events

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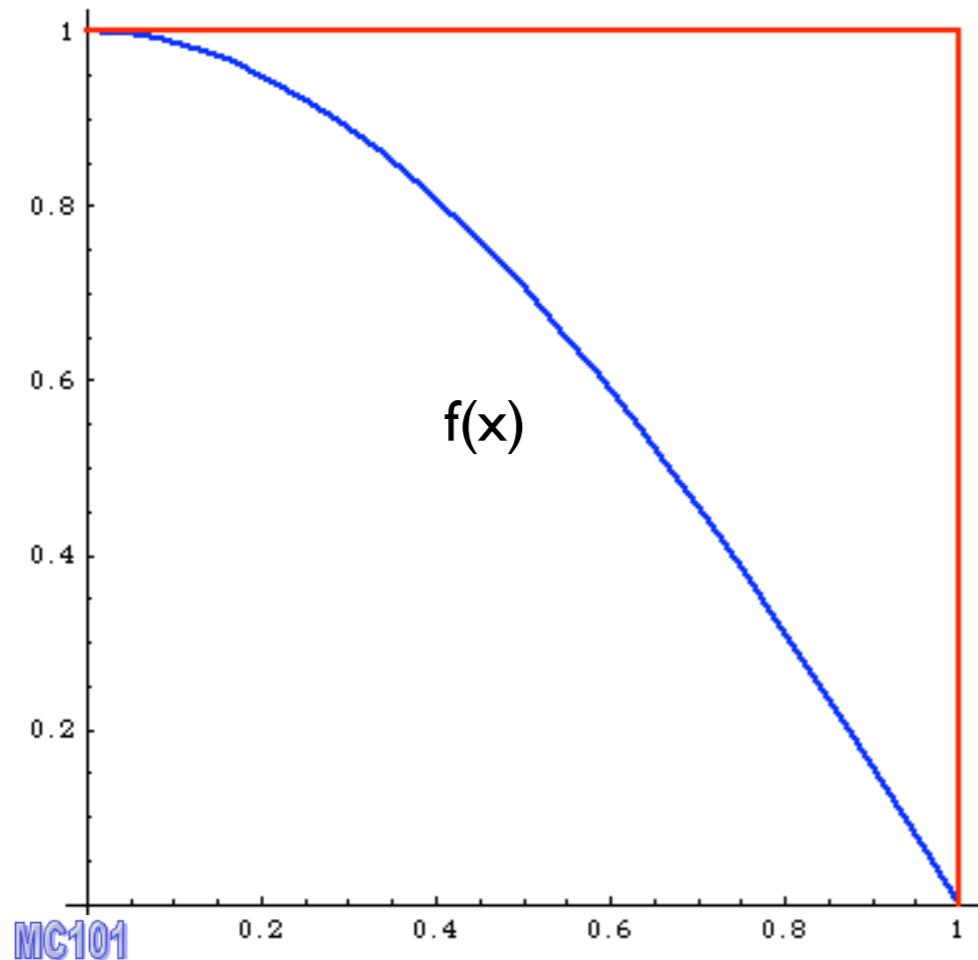
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$$\int f(x)dx = \frac{1}{N} \sum_{i=1}^N f(x_i) = \frac{1}{N} \sum_{i=1}^N P(x_i) \max(f) \simeq \frac{\max(f)}{N} \sum_{i=1}^n 1$$

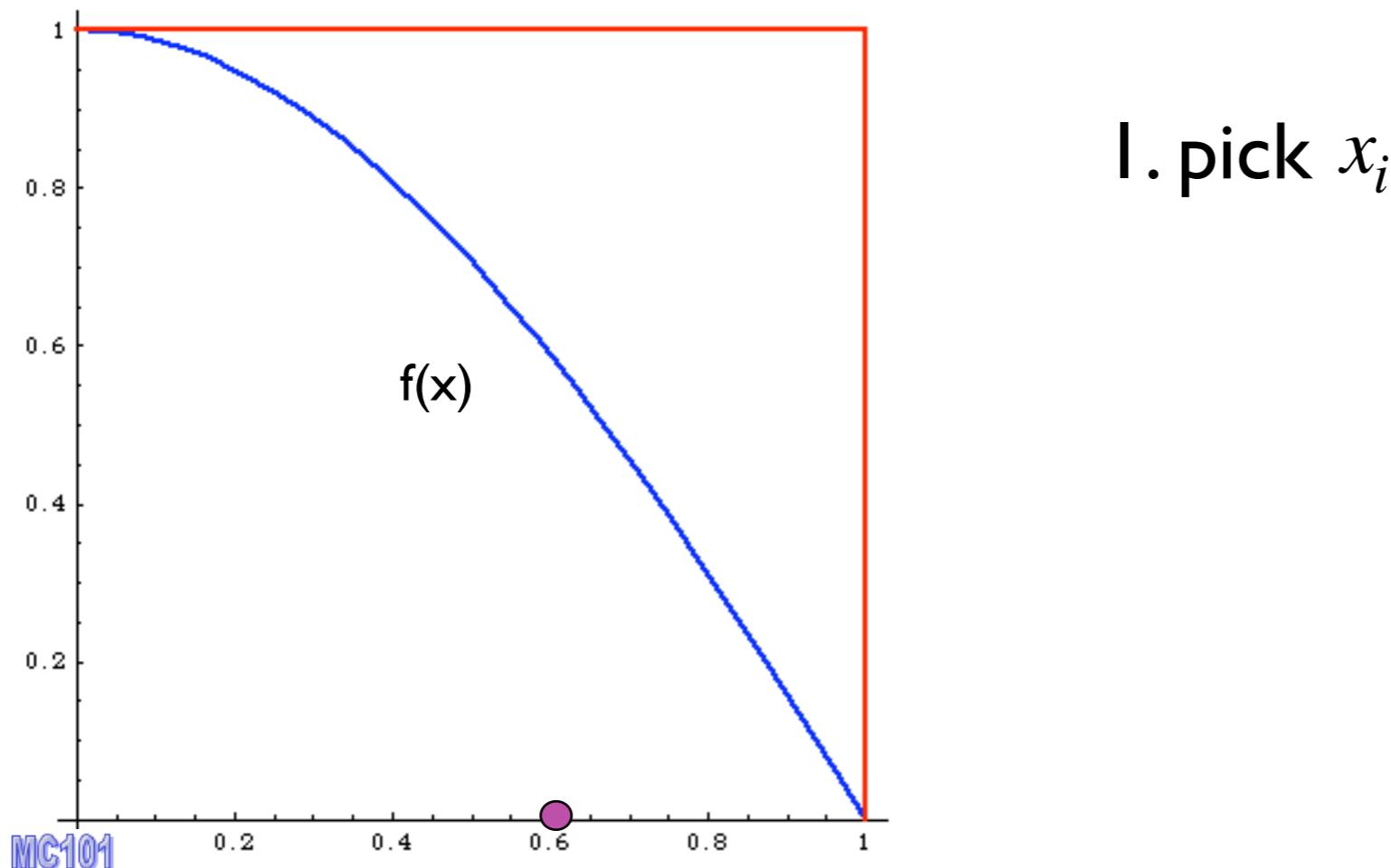
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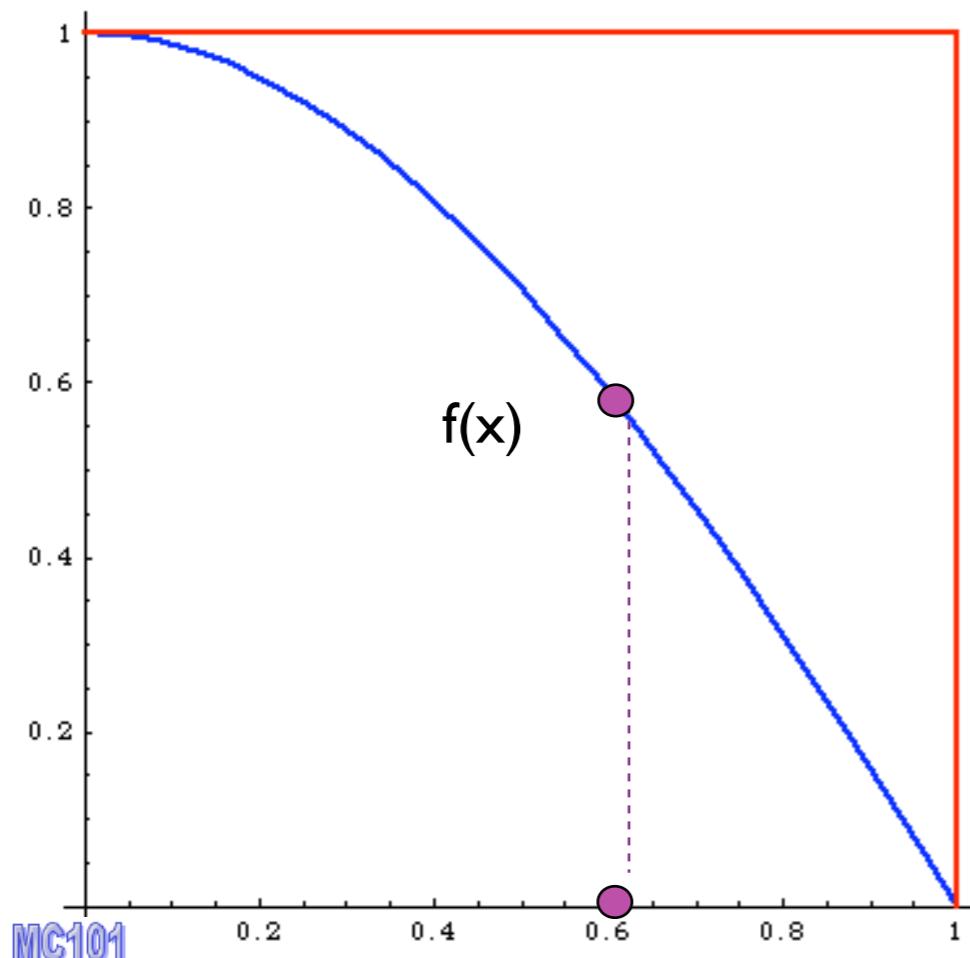
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# Event generation

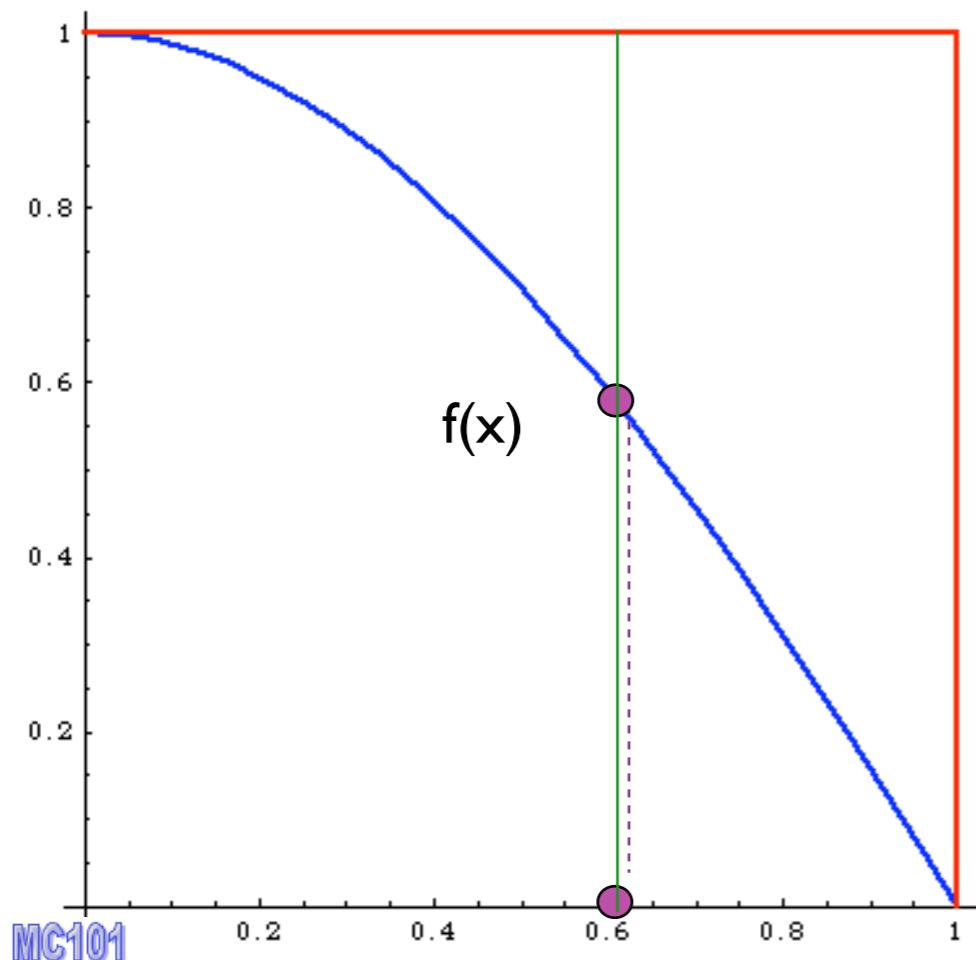
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1. pick  $x_i$
2. calculate  $f(x_i)$

# Event generation

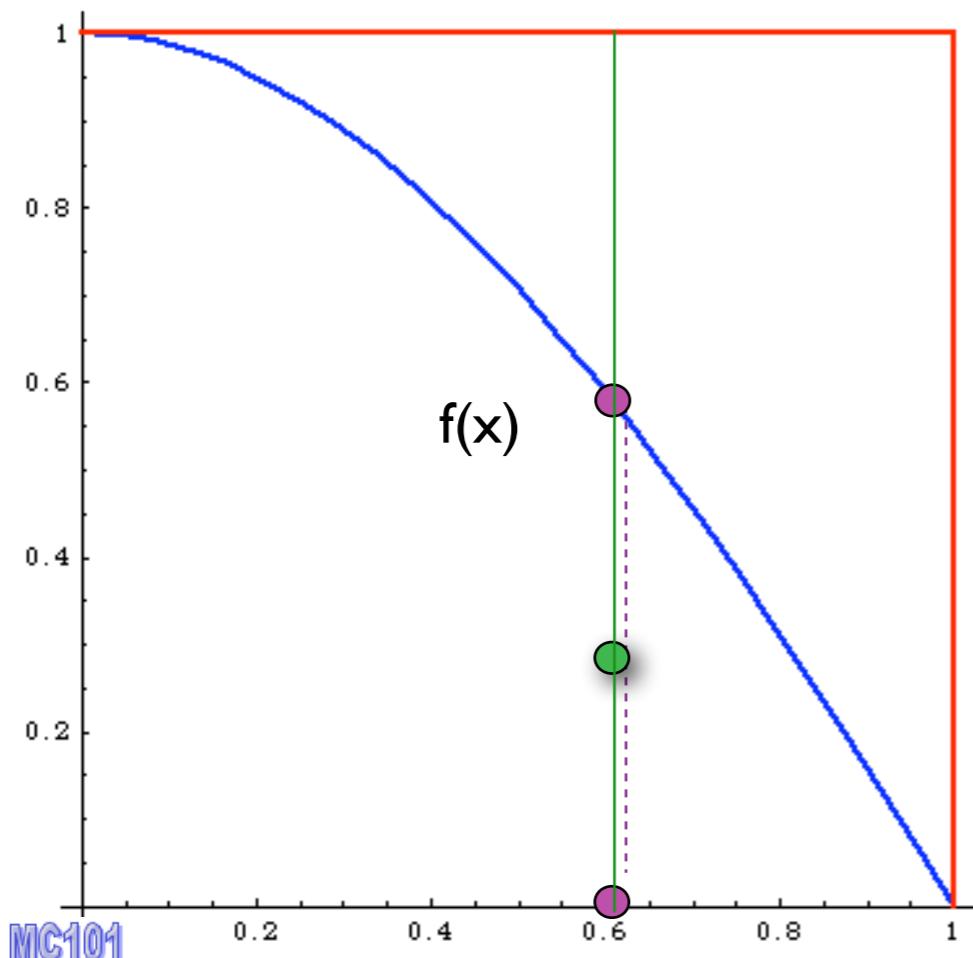
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2. calculate  $f(x_i)$
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# Event generation

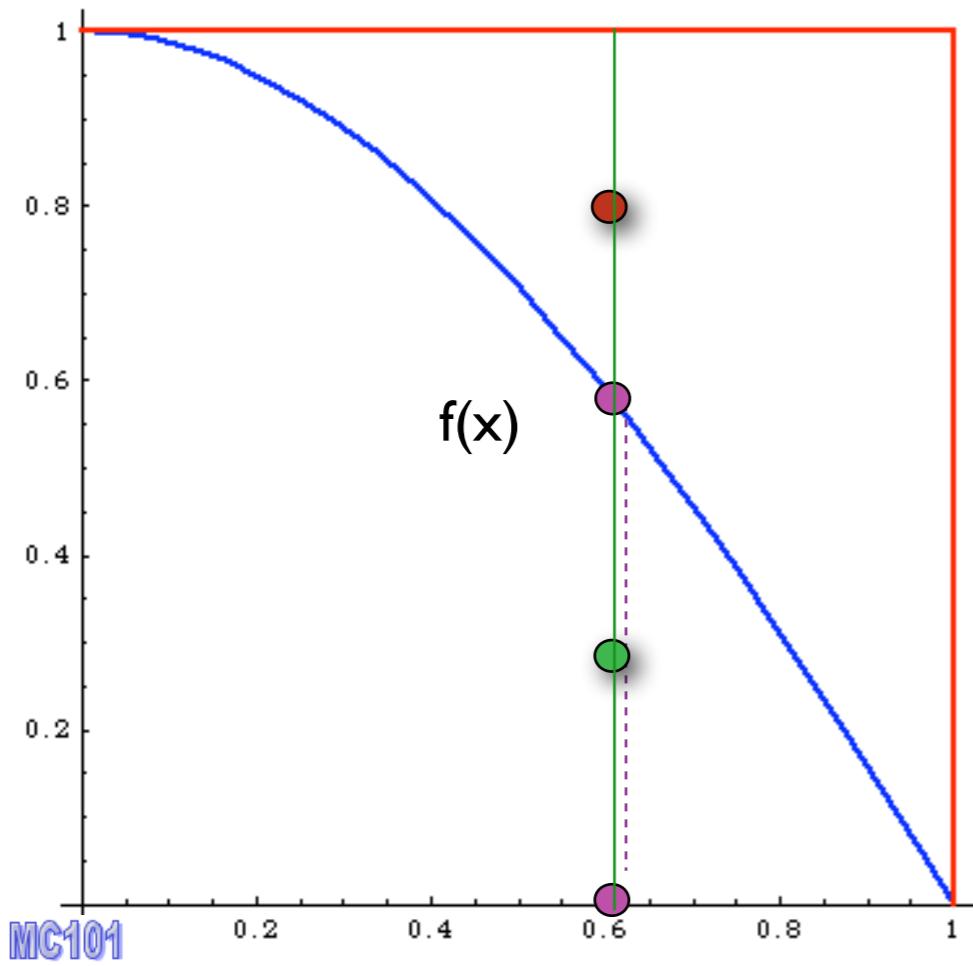
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if  $y < f(x_i)$  accept event,

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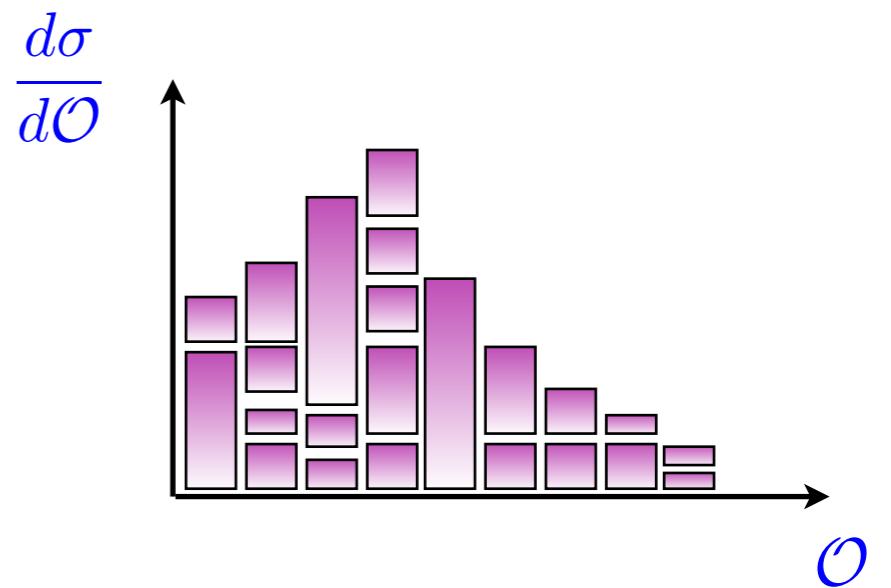
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# Event generation

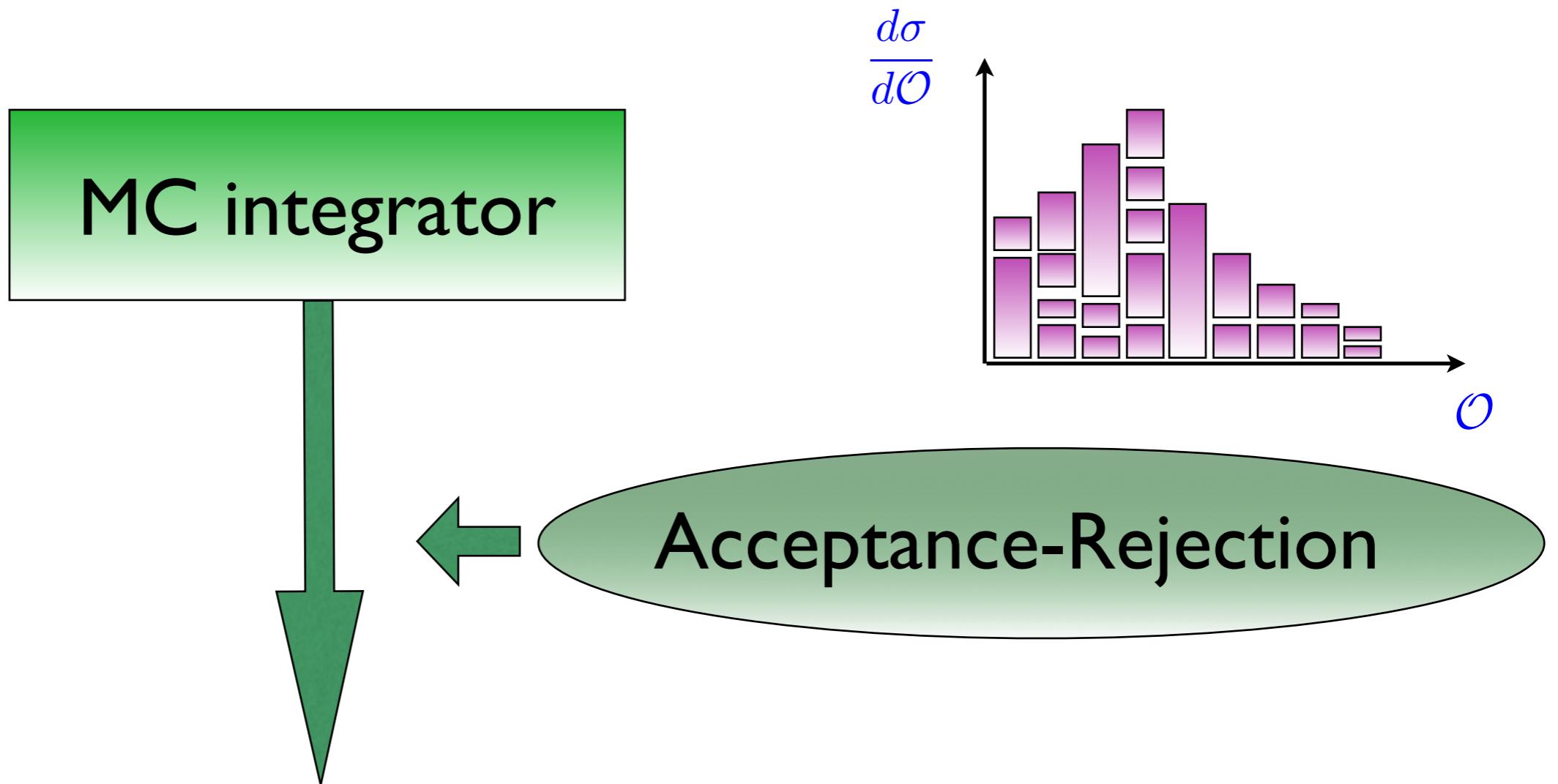
MC integrator

# Event generation

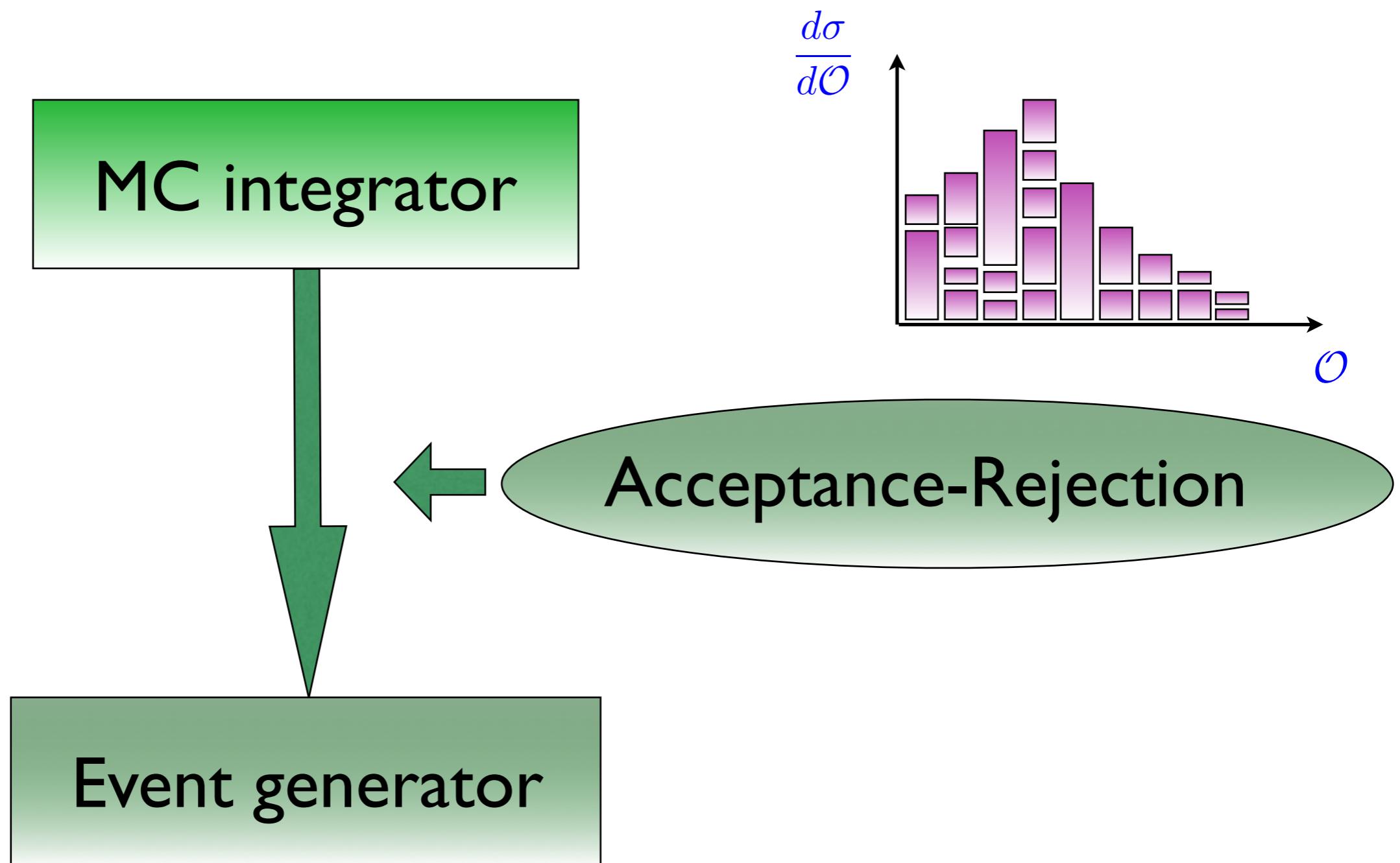
MC integrator



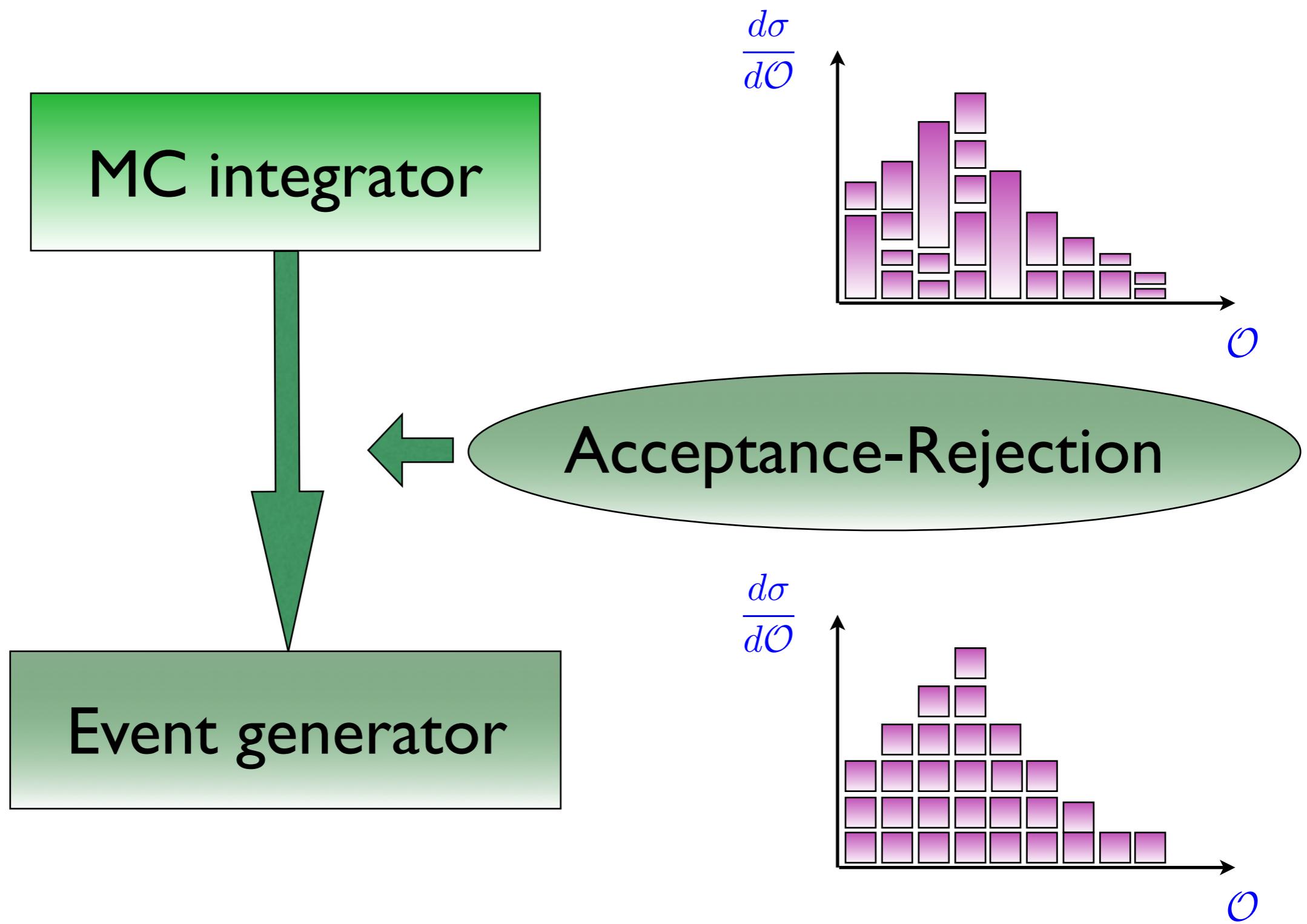
# Event generation



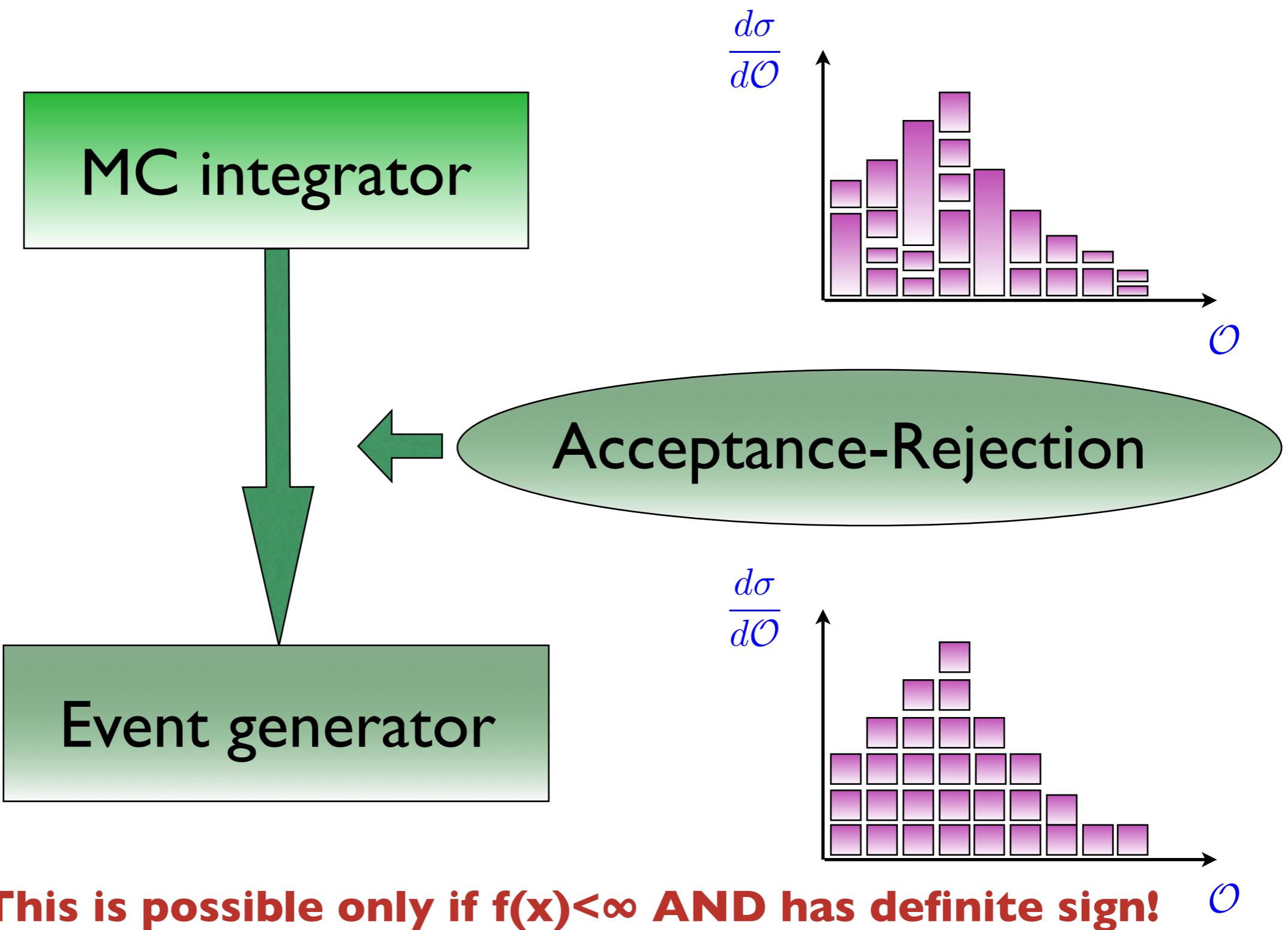
# Event generation



# Event generation



# Event generation



**This is possible only if  $f(x) < \infty$  AND has definite sign!**

# Monte-Carlo Summary

## Bad Point

- Slow Convergence (especially in low number of Dimension)
- Need to know the function
  - Impact on cut

# Monte-Carlo Summary

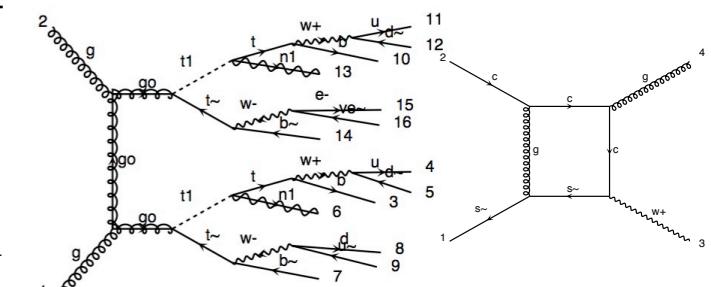
## Bad Point

- Slow Convergence (especially in low number of Dimension)
- Need to know the function
  - Impact on cut

## Good Point

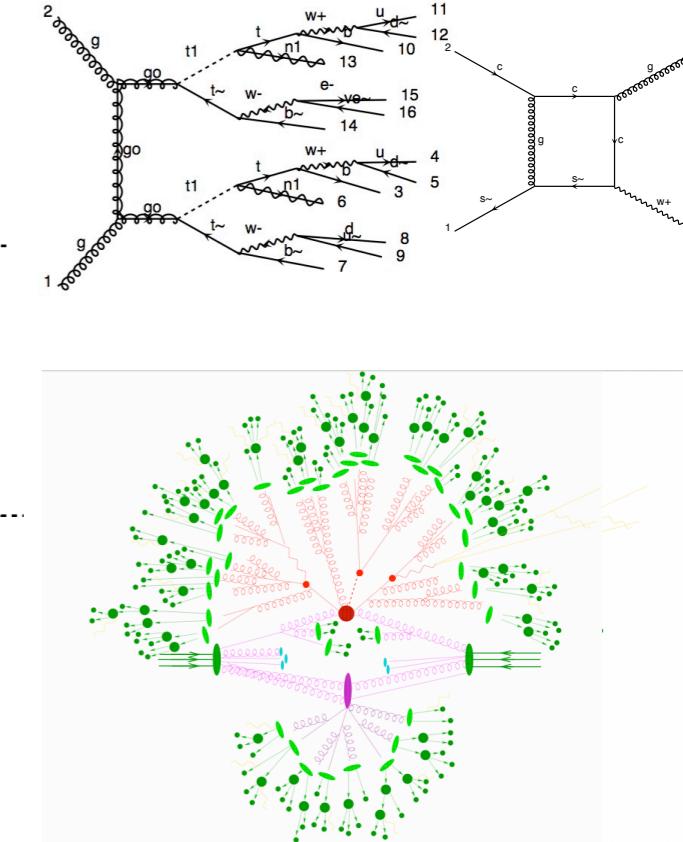
- Complex area of Integration
- Easy Error estimate
- quick estimation of the integral
- Possibility to have unweighted events

# Type of generation

	Tree (B)SM	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	Loop Induced (B)SM
Fix Order	✓	✓	✓	✓	✓
$\hat{\sigma} = \sigma^{\text{Born}} \left( 1 + \frac{\alpha_s}{2\pi} \sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma^{(3)} + \dots \right)$	LO predictions	NLO corrections	NNLO corrections	N3LO or NNNLO corrections	

# Type of generation

	Tree (B)SM	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	Loop Induced (B)SM
Fix Order	✓	✓	✓	✓	✓
+Parton Shower	✓	✓	✓	✗	✓



The figure shows two parts illustrating particle generation. The top part is a Feynman diagram for a loop-induced process, featuring gluons (g), top quarks (t), W bosons (W+/-), and other fermions like electrons (e-). Lines are numbered 1 through 16. The bottom part is a 3D visualization of a particle shower, showing a central red point from which many colored lines (green, blue, pink) radiate outwards, representing the distribution of generated particles.

# Type of generation

	Tree (B)SM	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	Loop Induced (B)SM
Fix Order	✓	✓	✓	✓	✓
+Parton Shower	✓	✓	✓	✗	✓
Merged Sample	✓	✓	?	✗	✓

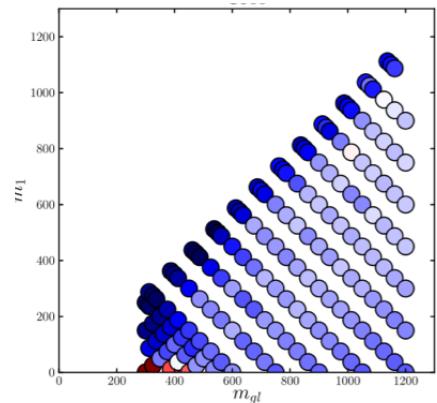
# LO Feature

# LO Feature

Auto-Width

$$\Gamma = ?$$

Parameter scan

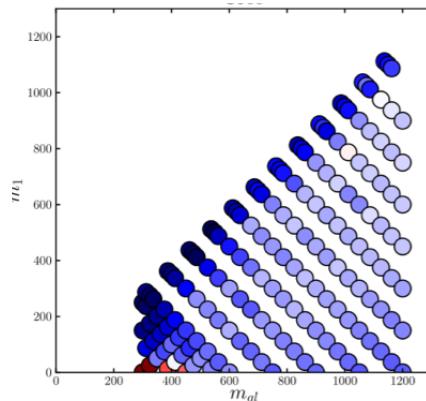


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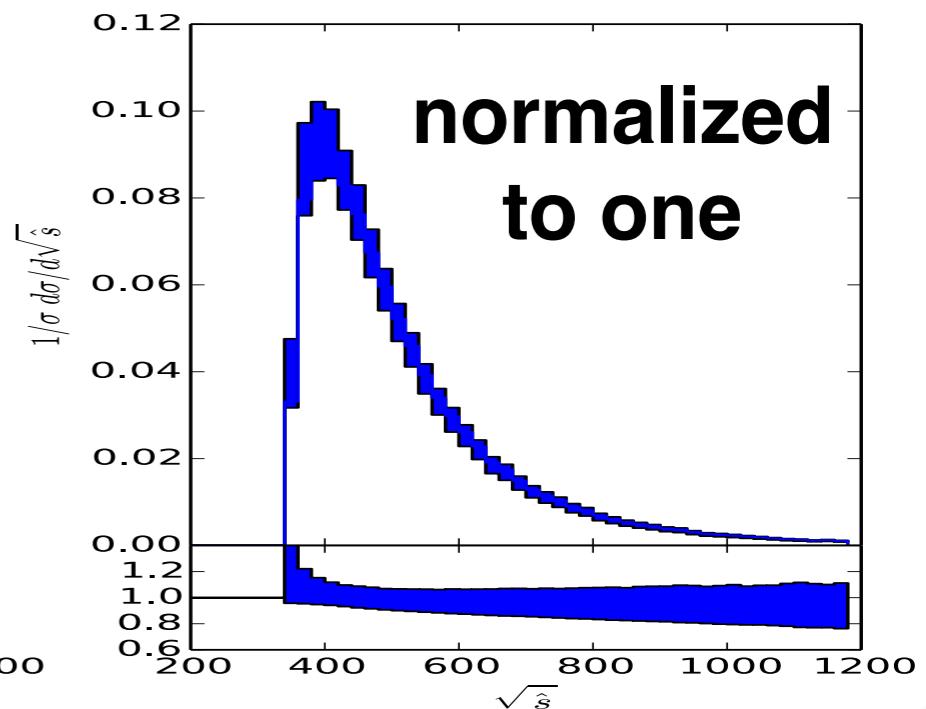
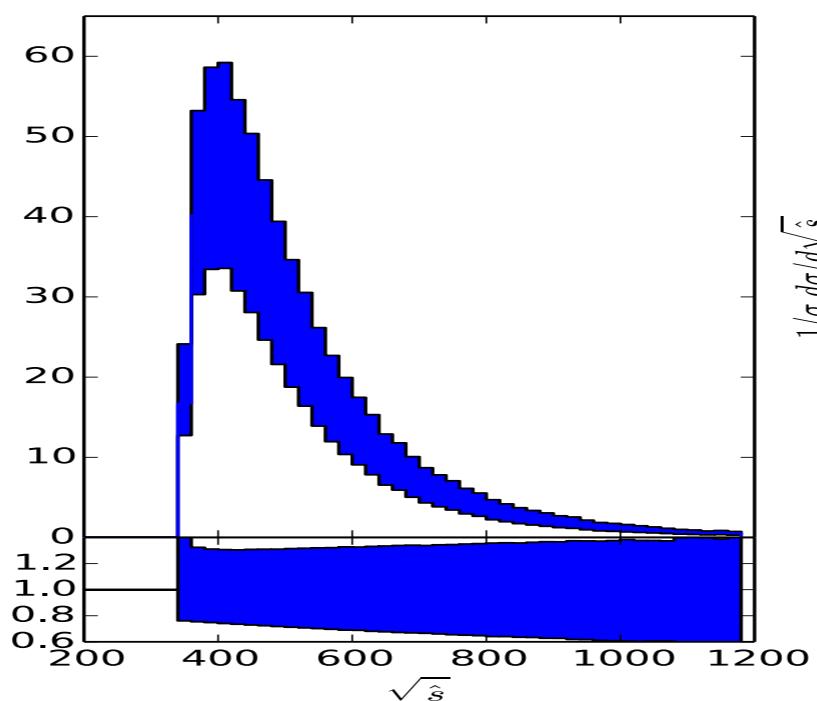
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Systematics



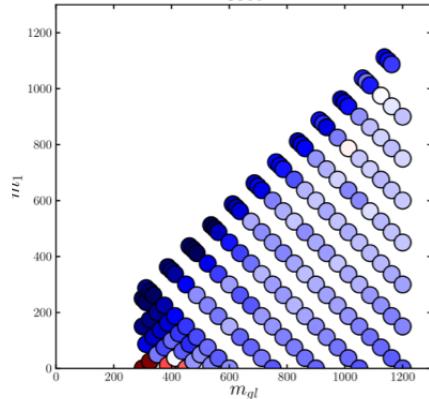
BSM re-weighting  
 $|M_{new}|^2 / |M_{old}|^2$

# LO Feature

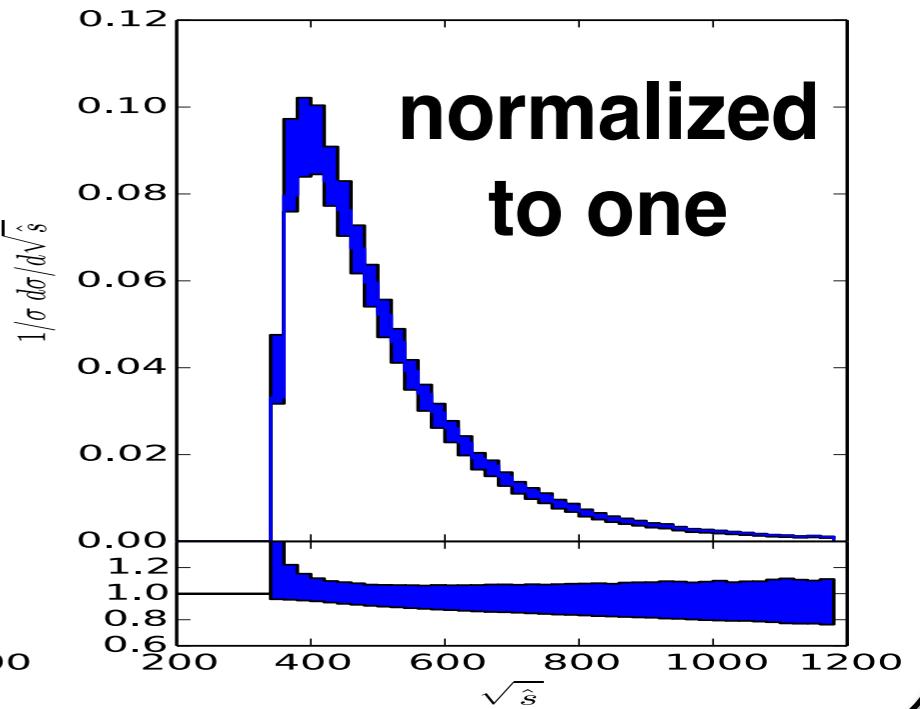
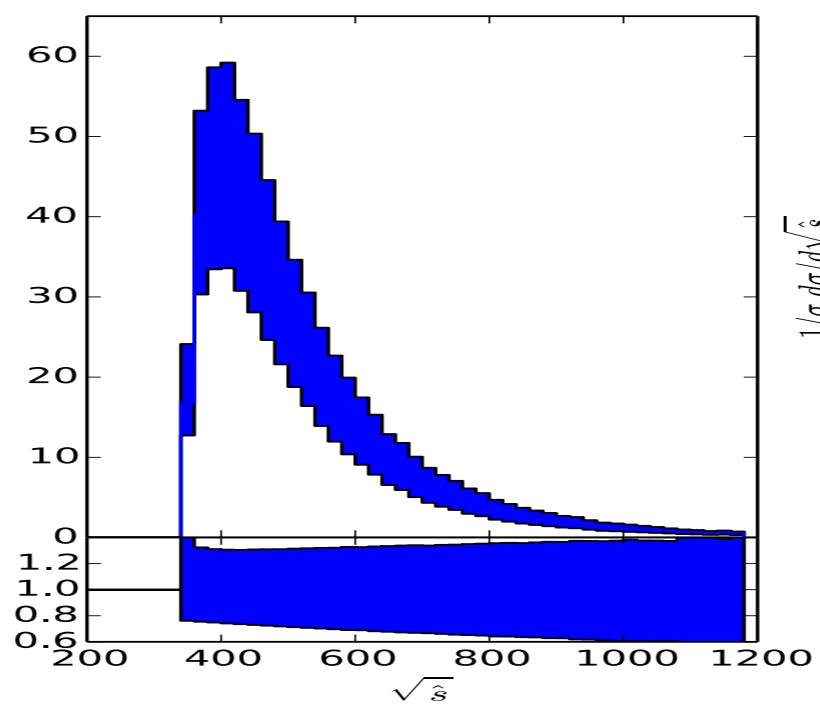
Auto-Width

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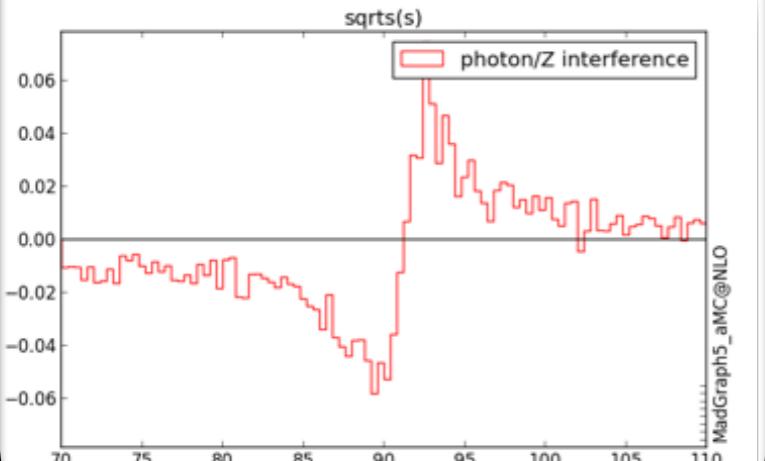
Parameter scan



Systematics



Interference



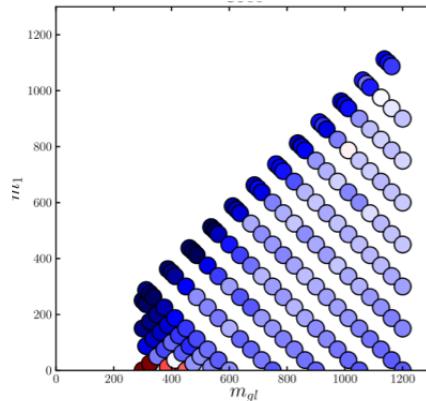
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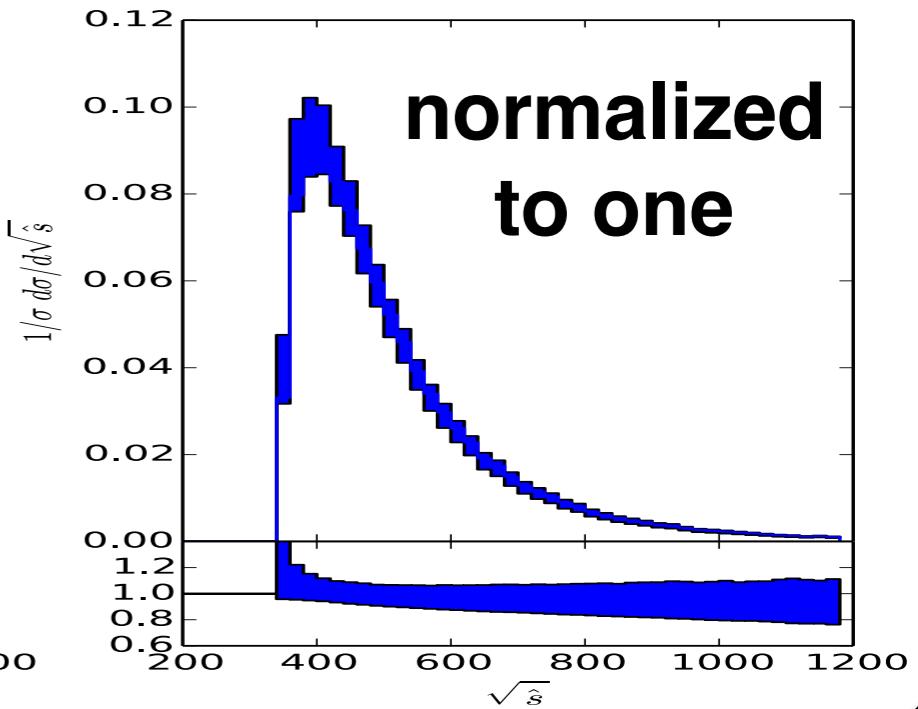
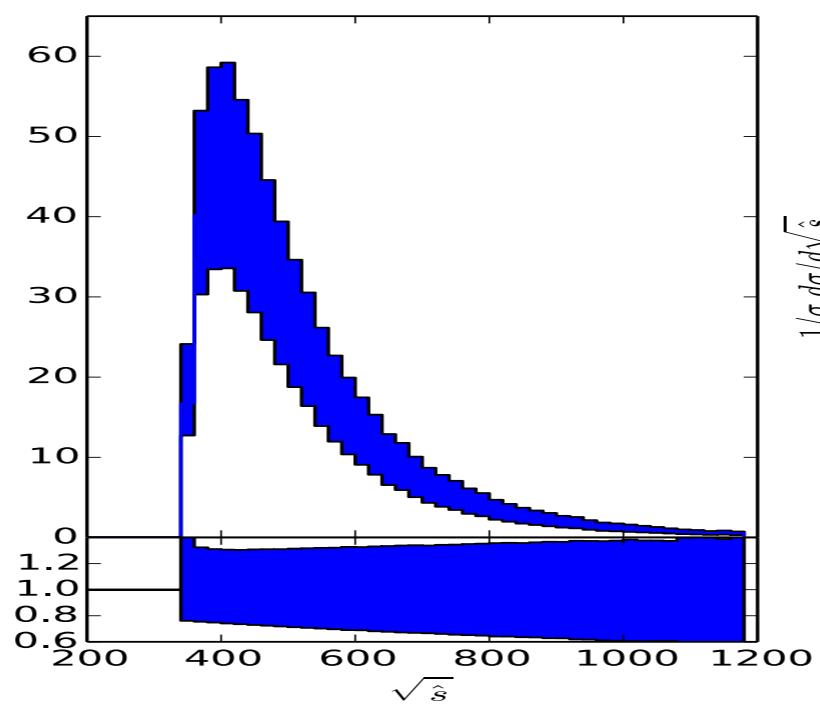
Auto-Width

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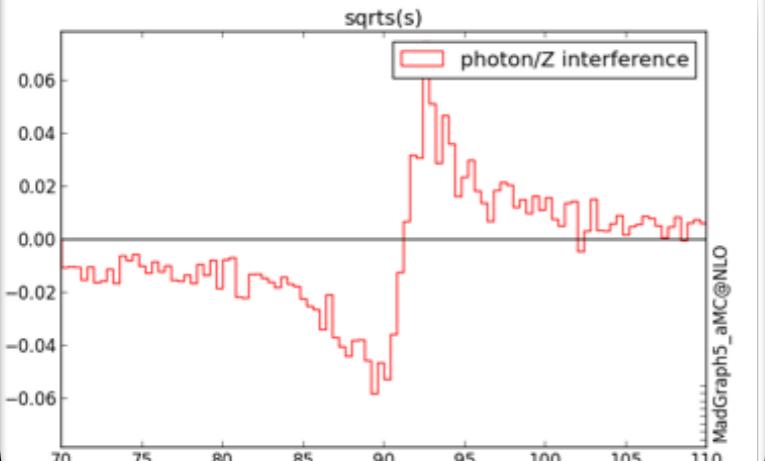
Parameter scan



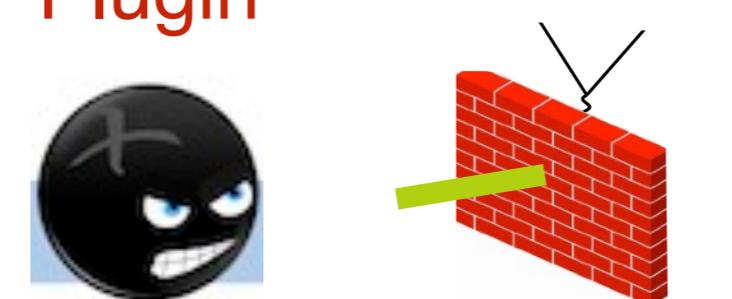
Systematics



Interference



Plugin



Interface

**MAD**  
Analysis 5



BSM re-weighting

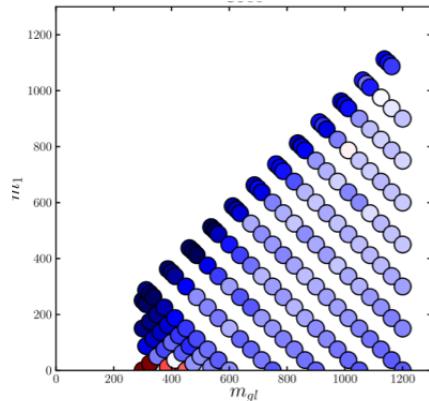
$$|M_{new}|^2 / |M_{old}|^2$$

# LO Feature

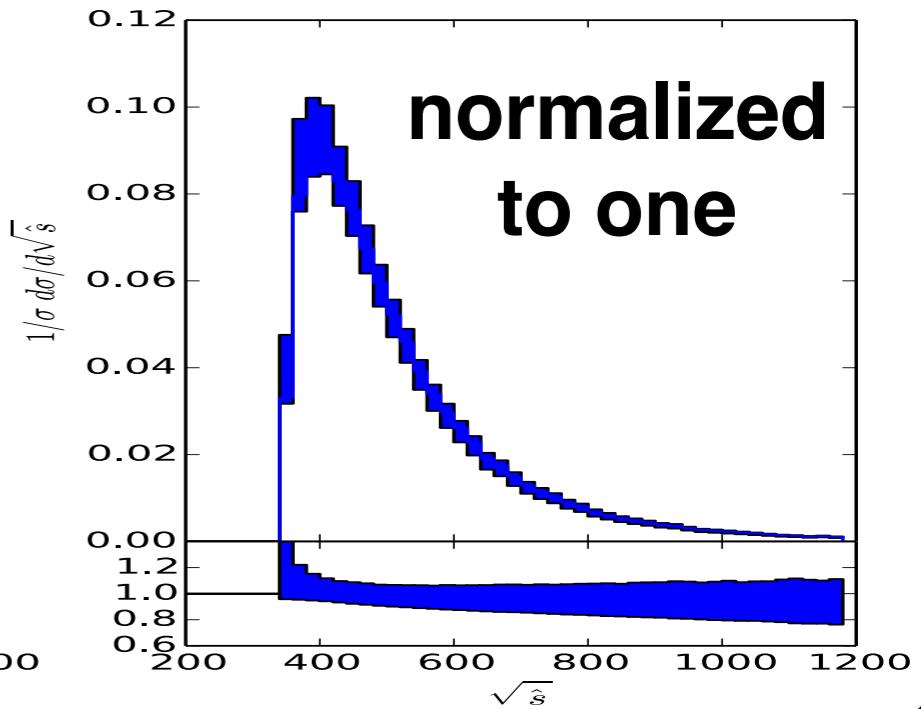
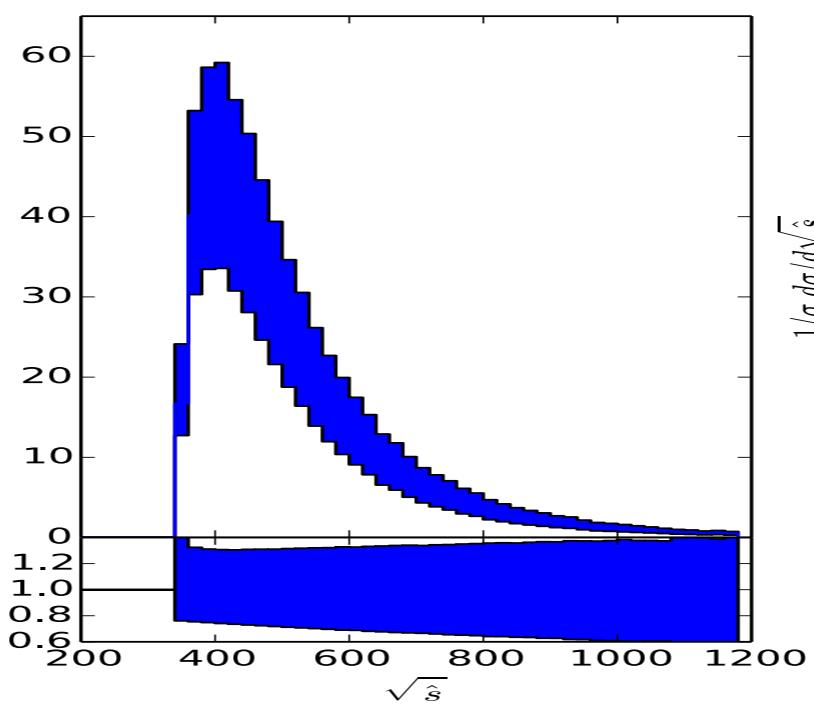
Auto-Width

$$\Gamma = ?$$

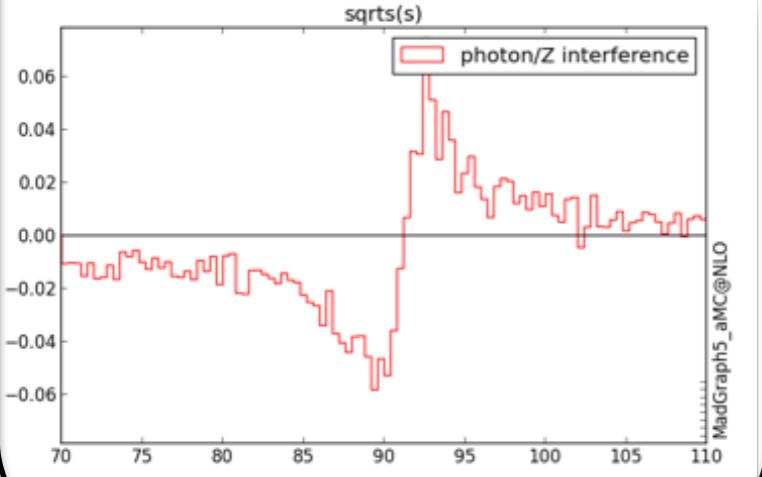
Parameter scan



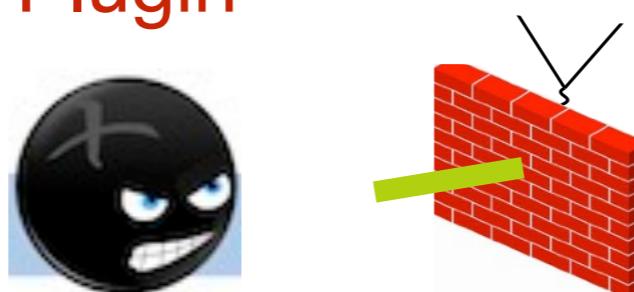
Systematics



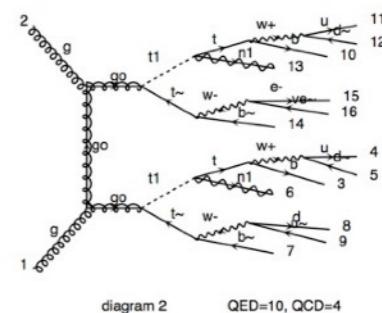
Interference



Plugin



Narrow-width



Interface

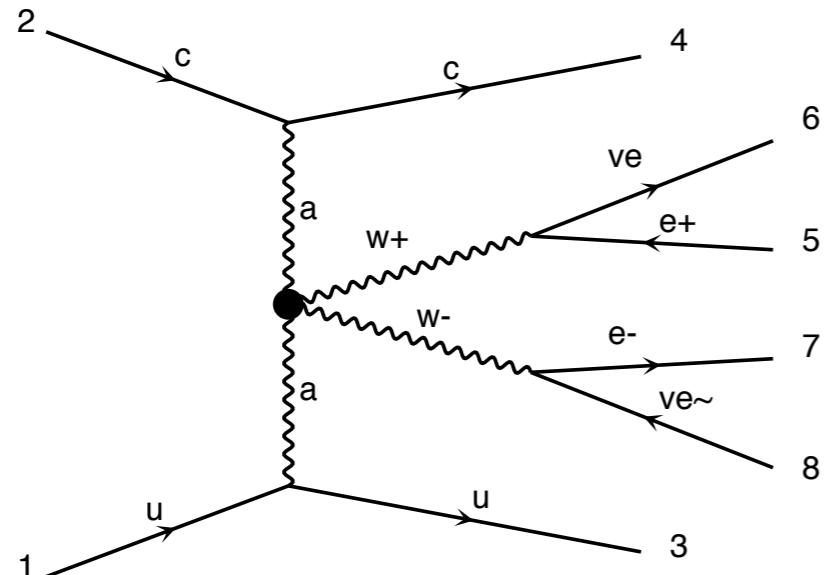


BSM re-weighting

$$|M_{new}|^2 / |M_{old}|^2$$

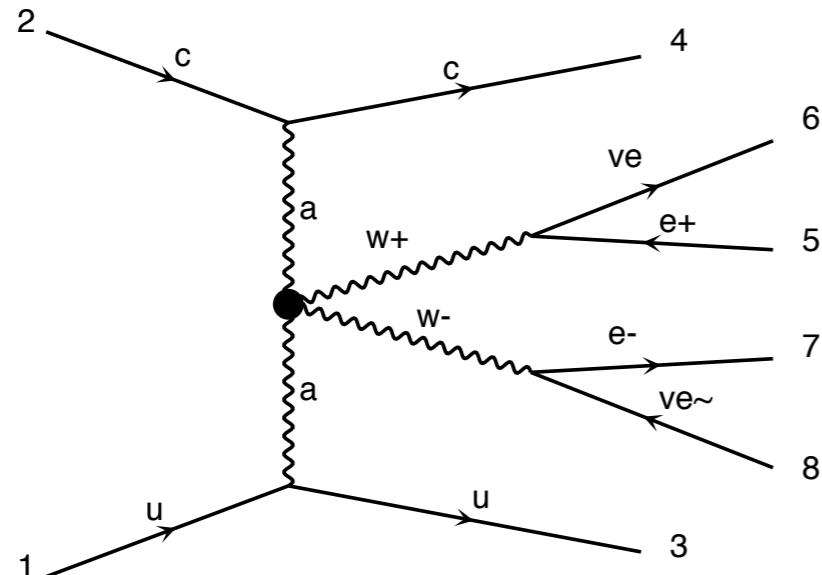
# Decay

## Resonant Diagram

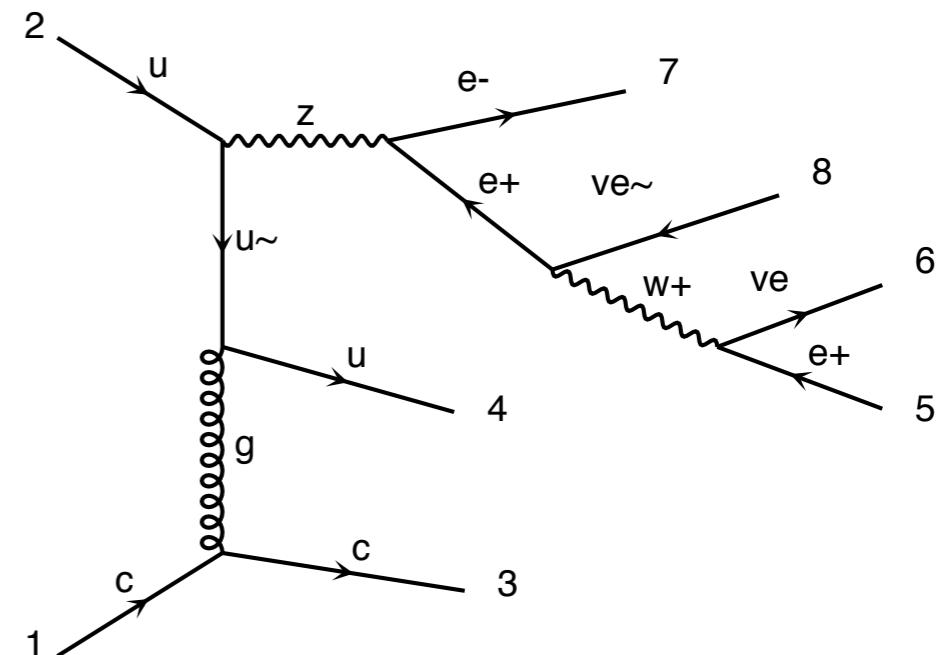


# Decay

## Resonant Diagram



## Non Resonant Diagram

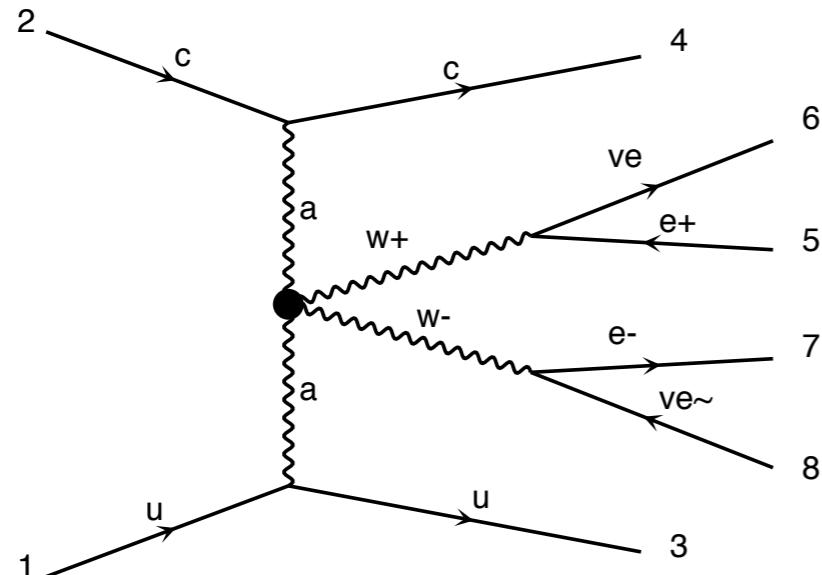


## Problem

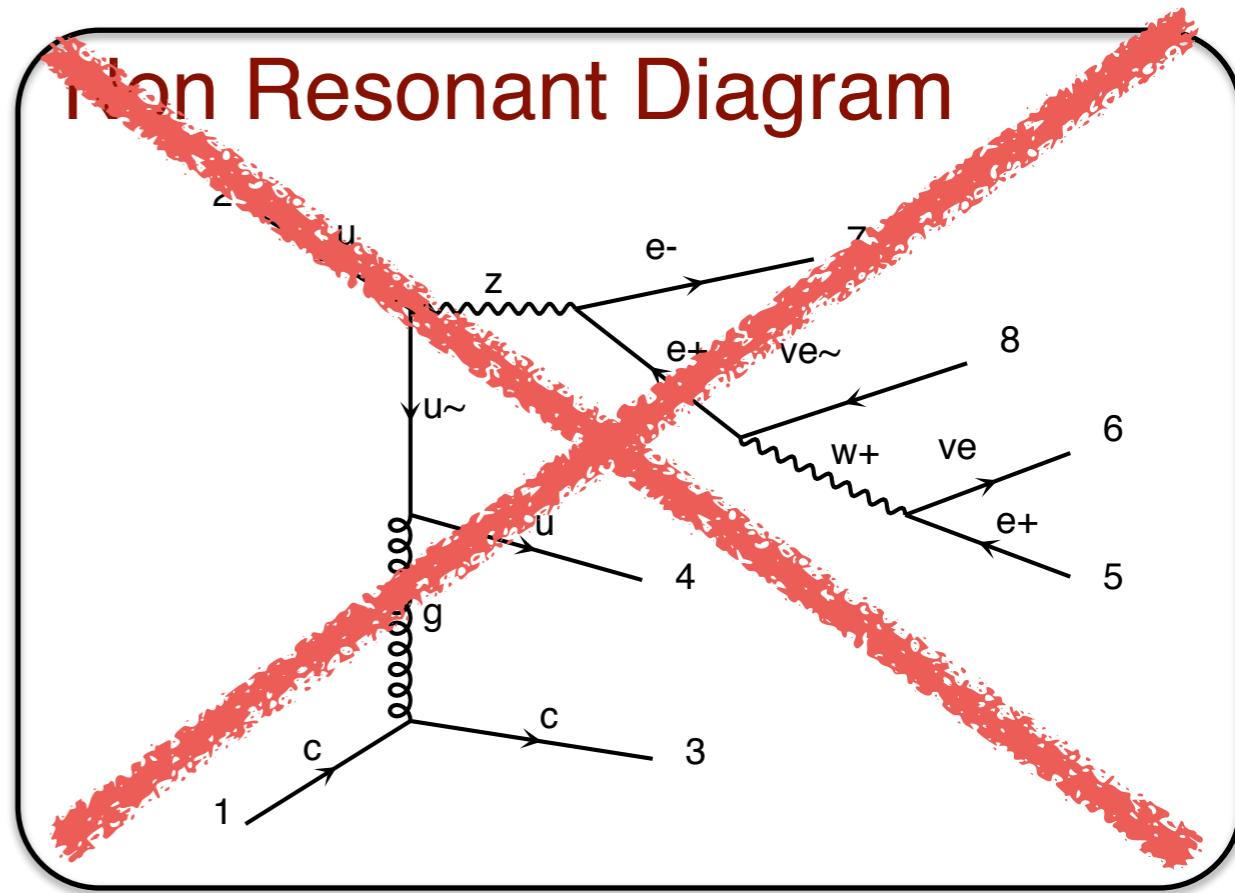
- Process complicated to have the full process  
→ Including off-shell contribution

# Decay

## Resonant Diagram



## Non Resonant Diagram



## Problem

- Process complicated to have the full process
  - Including off-shell contribution

## Solution

- Only keep on-shell contribution

# Narrow-Width Approx.

## Theory

$$\int dq^2 \left| \frac{1}{q^2 - M^2 - iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma} \delta(q^2 - M^2)$$

$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}\left(\frac{\Gamma}{M}\right))$$

## Comment

# Narrow-Width Approx.

## Theory

$$\int dq^2 \left| \frac{1}{q^2 - M^2 - iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma} \delta(q^2 - M^2)$$

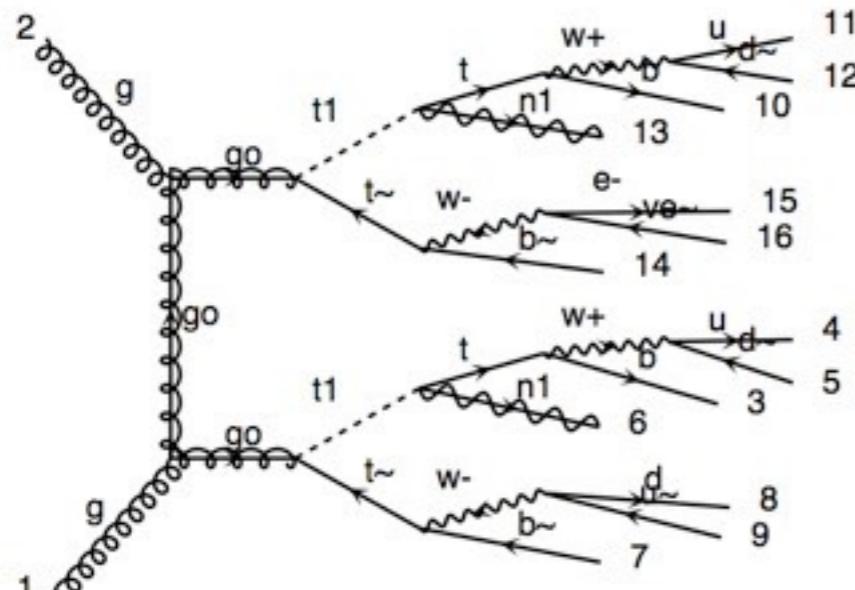
$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}\left(\frac{\Gamma}{M}\right))$$

## Comment

- This is an **Approximation!**
- This force the particle to be on-shell!
  - Recover by re-introducing the Breit-wigner up-to a cut-off

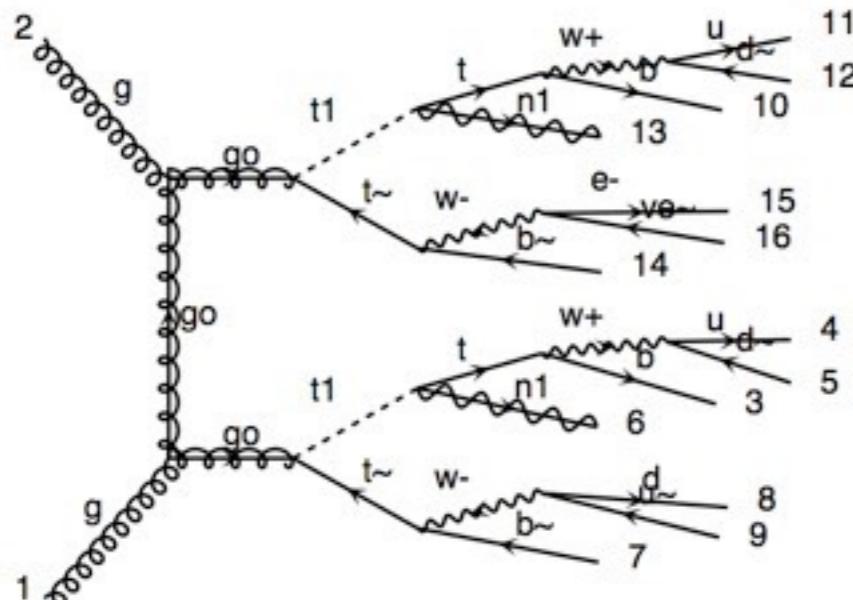
# Decay chain

- $P P \rightarrow t \bar{t} \sim w^+, (t \rightarrow w^+ b, w^+ \rightarrow l^+ \nu_l), \backslash$   
 $(\bar{t} \rightarrow w^- b \bar{\sim}, w^- \rightarrow j j), \backslash$   
 $w^+ \rightarrow l^+ \nu_l$



# Decay chain

- $p\ p \rightarrow t\ t^{\sim} w^+, (t \rightarrow w^+ b, w^+ \rightarrow l^+ \nu_l), \backslash$   
 $(t^{\sim} \rightarrow w^- b^{\sim}, w^- \rightarrow j\ j), \backslash$   
 $w^+ \rightarrow l^+ \nu_l$

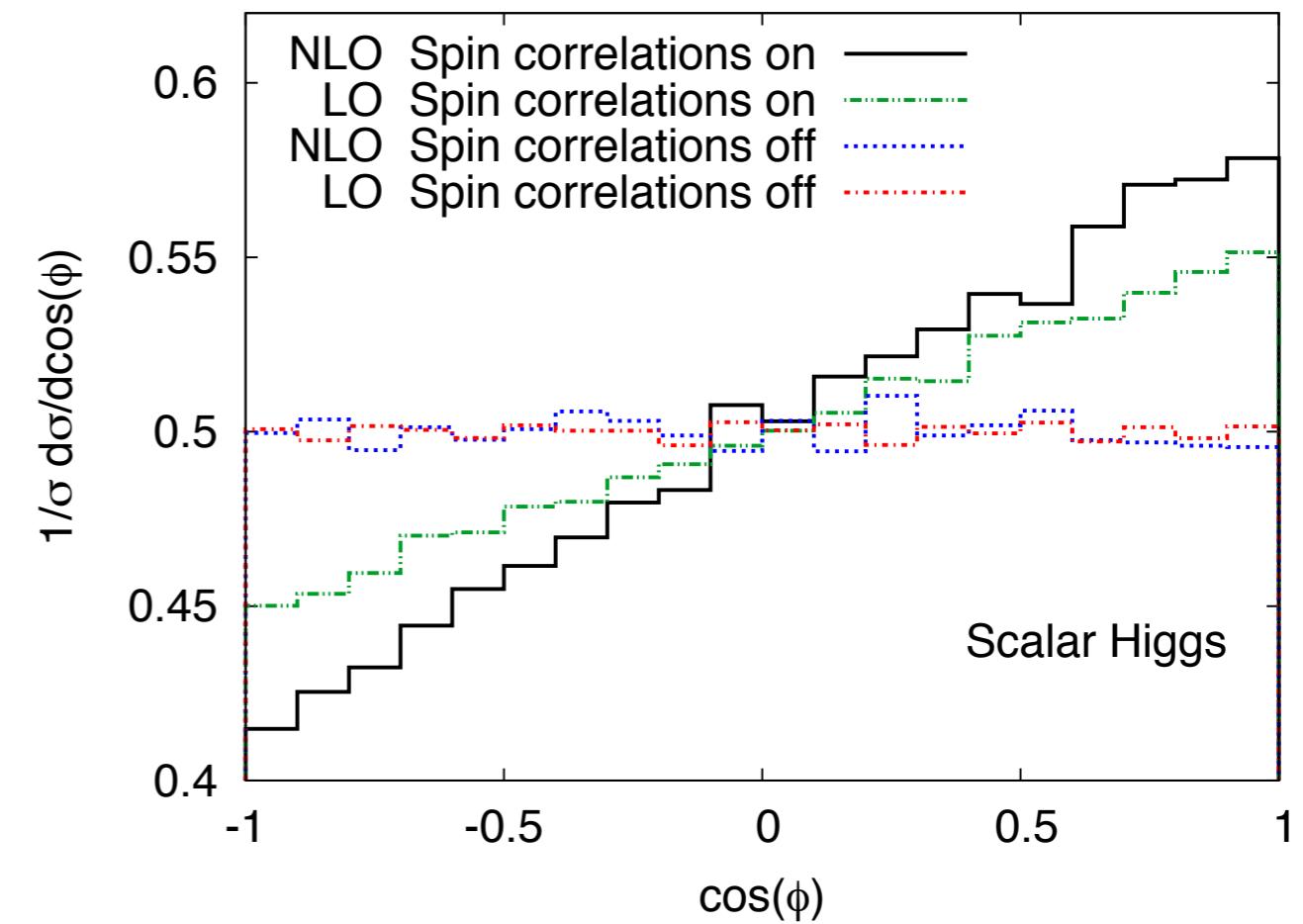
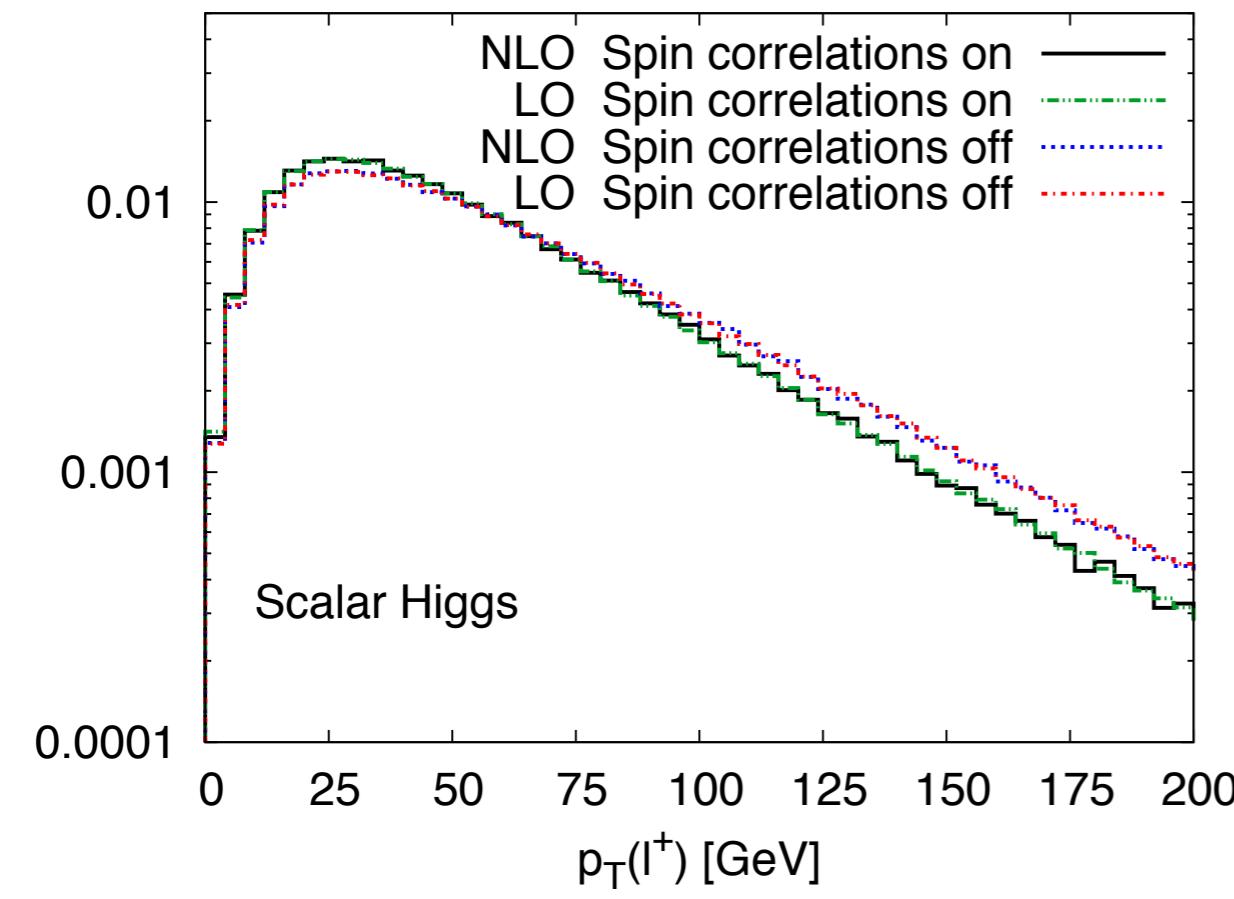
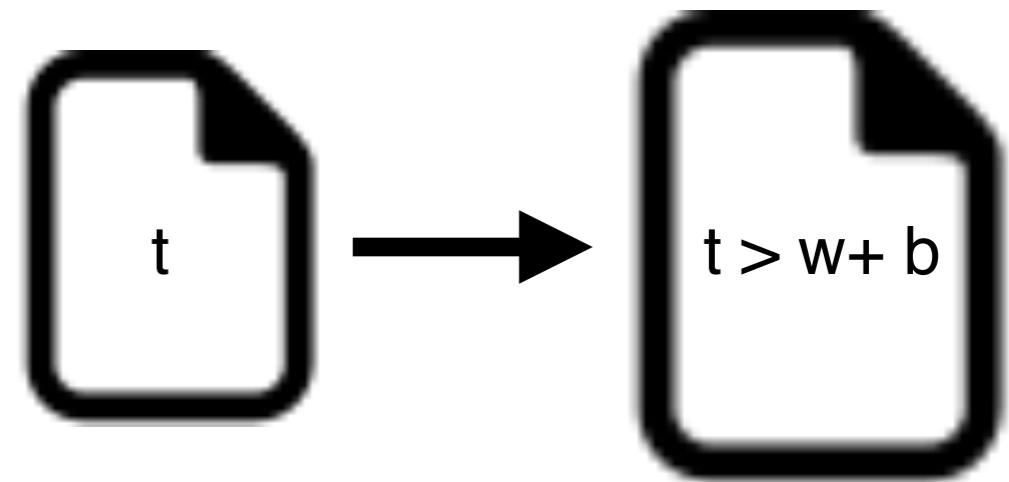


very long  
decay chains possible to simulate  
directly in MadGraph!

- Full spin-correlation
- Off-shell effects (up to cut-off)
- NWA not used for the cross-section

# MadSpin

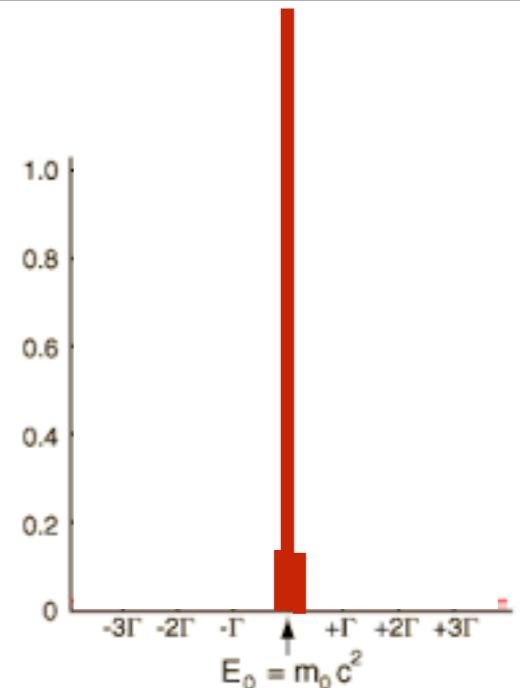
Decay as post-processing  
Independently of event generation  
But same accuracy (spin-correlation)  
Use NWA for cross-section



# Very small width

$$\Gamma < 10^{-8}M$$

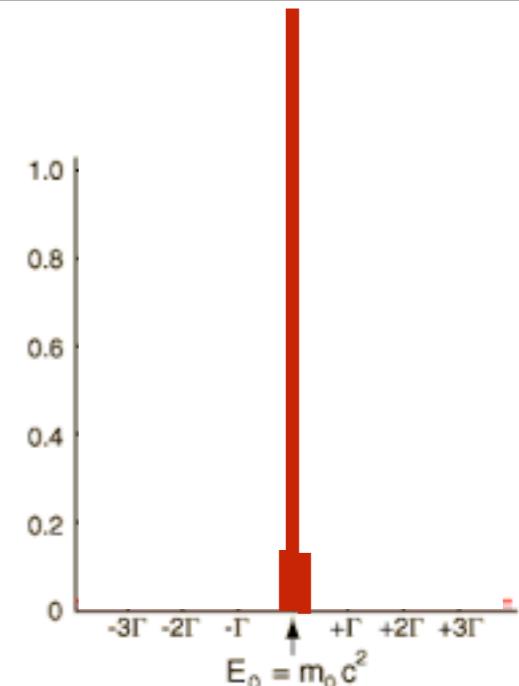
- Slows down the code
- Can lead to numerical instability



# Very small width

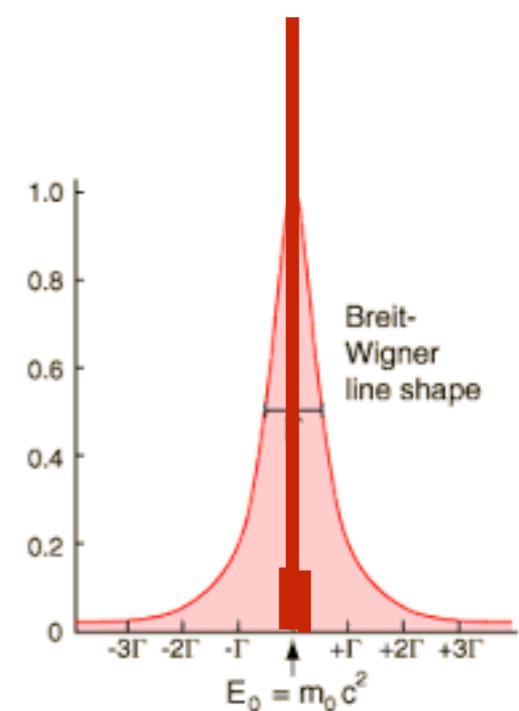
$$\Gamma < 10^{-8}M$$

- Slows down the code
- Can lead to numerical instability



## Solution

- Use a Fake-Width for the evaluation of the matrix-element
- Correct cross-section according to NWA formula  $\frac{\Gamma_{fake}}{\Gamma_{true}}$



# What to remember



- Analytical computation can be slower than numerical method
- Any BSM model are supported (at LO)
- Phase Space integration are slow
  - need knowledge of the function
  - cuts can be problematic
- Event generation are from free.
- All this are automated in MG5\_aMC@NLO
- Important to know the physical hypothesis

# Plan

- What is MG5\_aMC?
- Details of the computation
  - Evaluation of matrix-element
  - Phase-Space integration
- Tools/functionality of MG5\_aMC

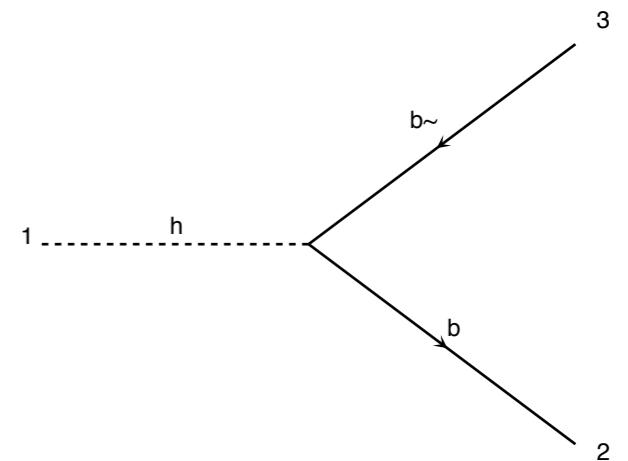
# List of package

- SysCalc (computation of systematics)
- MadWidth (computation of width in NWA)
- MadSpin (decay with full spin-correlation)
- Re-Weighting (change of the weight of an event)
- Shower / Detector Interface
- MadWeight (Matrix-Element Method)
- Interference
- MadAnalysis5
- Tau Decay
- MadDM
- GPU

# 2-body decay

2 body decay

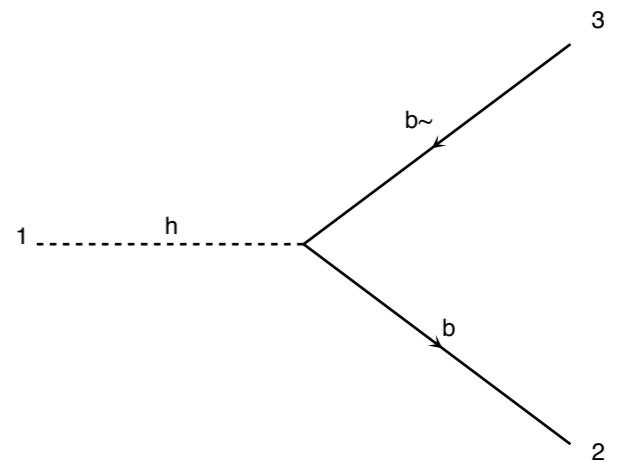
$$\Gamma = \frac{1}{2MS} \int d\Phi_2 |\mathcal{M}|^2$$



# 2-body decay

## 2 body decay

$$\Gamma = \frac{1}{2MS} \int d\Phi_2 |\mathcal{M}|^2$$



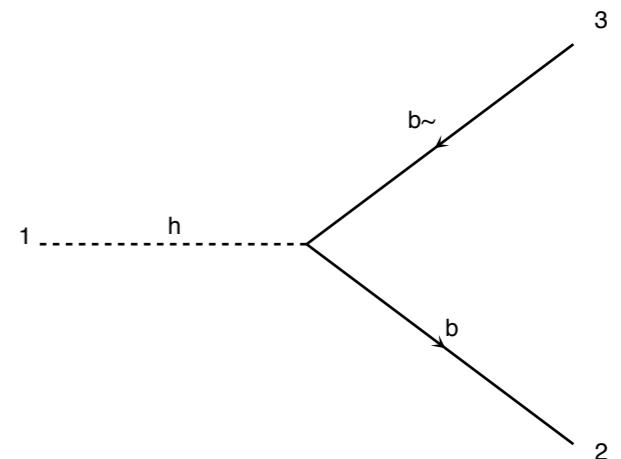
- By Lorentz Invariance the matrix element is constant over the phase-space.

$$\Gamma = \frac{\sqrt{\lambda(M^2, m_1^2, m_2^2)} |\mathcal{M}|^2}{16\pi S M^3}$$
$$\lambda(M^2, m_1^2, m_2^2) = (M^2 - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2$$

# 2-body decay

## 2 body decay

$$\Gamma = \frac{1}{2MS} \int d\Phi_2 |\mathcal{M}|^2$$



- By Lorentz Invariance the matrix element is constant over the phase-space.

$$\Gamma = \frac{\sqrt{\lambda(M^2, m_1^2, m_2^2)} |\mathcal{M}|^2}{16\pi S M^3}$$
$$\lambda(M^2, m_1^2, m_2^2) = (M^2 - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2$$

- Calculable analytically by FeynRules

# MadWidth

hep-ph/1402.1178

2-body

- Use FeynRules formula (instantaneous)

# MadWidth

hep-ph/1402.1178

## 2-body

- Use FeynRules formula (instantaneous)

## Fast-Estimation of 3-body

- Only use 2-body decay and PS factor

# MadWidth

hep-ph/1402.1178

## 2-body

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Relevant?

# MadWidth

hep-ph/1402.1178

## 2-body

- Use FeynRules formula (instantaneous)

## Fast-Estimation of 3-body

- Only use 2-body decay and PS factor

Relevant?

No

DONE

# MadWidth

hep-ph/1402.1178

## 2-body

- Use FeynRules formula (instantaneous)

## Fast-Estimation of 3-body

- Only use 2-body decay and PS factor

Relevant?

Maybe

No

## Channel Generation

- Remove Sequence of 2-body/radiation diagram

DONE

# MadWidth

hep-ph/1402.1178

## 2-body

- Use FeynRules formula (instantaneous)

## Fast-Estimation of 3-body

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Relevant?

Maybe

No

## Channel Generation

- Remove Sequence of 2-body/radiation diagram

DONE

## Estimation of 3-body

- Based on the diagram. Approx. PS/Matrix-Element

Relevant?

# MadWidth

hep-ph/1402.1178

## 2-body

- Use FeynRules formula (instantaneous)

## Fast-Estimation of 3-body

- Only use 2-body decay and PS factor

Relevant?

Maybe

No

## Channel Generation

- Remove Sequence of 2-body/radiation diagram

DONE

## Estimation of 3-body

- Based on the diagram. Approx. PS/Matrix-Element

Relevant?

No

# MadWidth

hep-ph/1402.1178

## 2-body

- Use FeynRules formula (instantaneous)

## Fast-Estimation of 3-body

- Only use 2-body decay and PS factor

Relevant?

Maybe

No

## Channel Generation

- Remove Sequence of 2-body/radiation diagram

DONE

## Estimation of 3-body

- Based on the diagram. Approx. PS/Matrix-Element

Relevant?

Yes?

## Numerical Integration

# MadWidth

hep-ph/1402.1178

## 2-body

- Use FeynRules formula (instantaneous)

## Fast-Estimation of 4 body

- Only use 2-body decay and PS factor

Relevant?

Maybe

No

## Channel Generation

- Remove Sequence of 2-body/radiation diagram

DONE

## Estimation of 4 body

- Based on the diagram. Approx. PS/Matrix-Element

Relevant?

Yes?

## Numerical Integration

# MadWidth

## Limitation

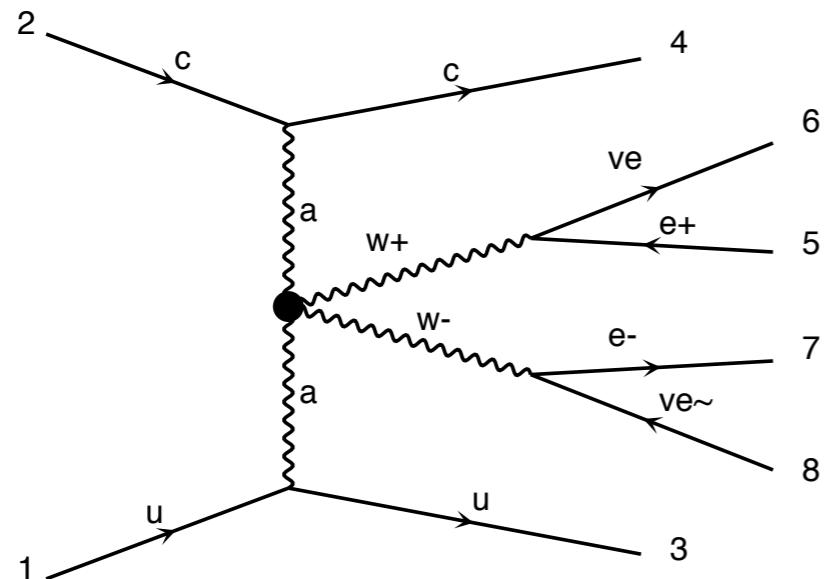
- Only LO
- No Loop Induced Decay
- Valid in Narrow-width approximation
- No hadronization effect

## Lesson

- Be aware of code limitation
- Quite often those are not checked.

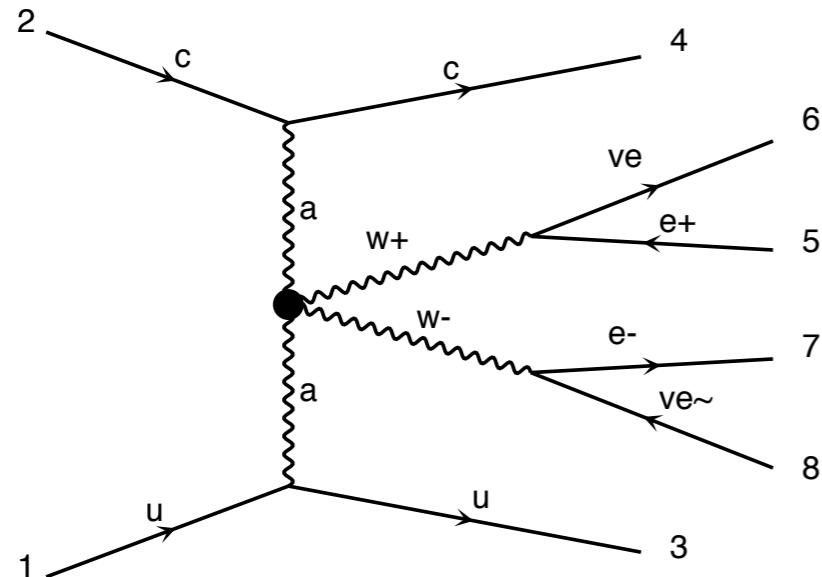
# Decay

## Resonant Diagram

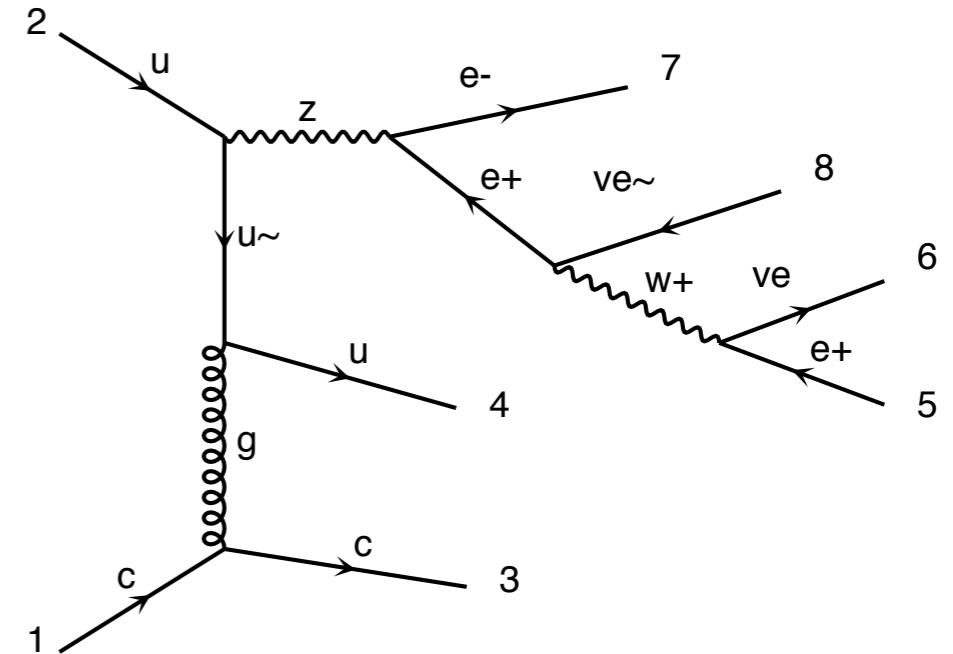


# Decay

## Resonant Diagram



## Non Resonant Diagram

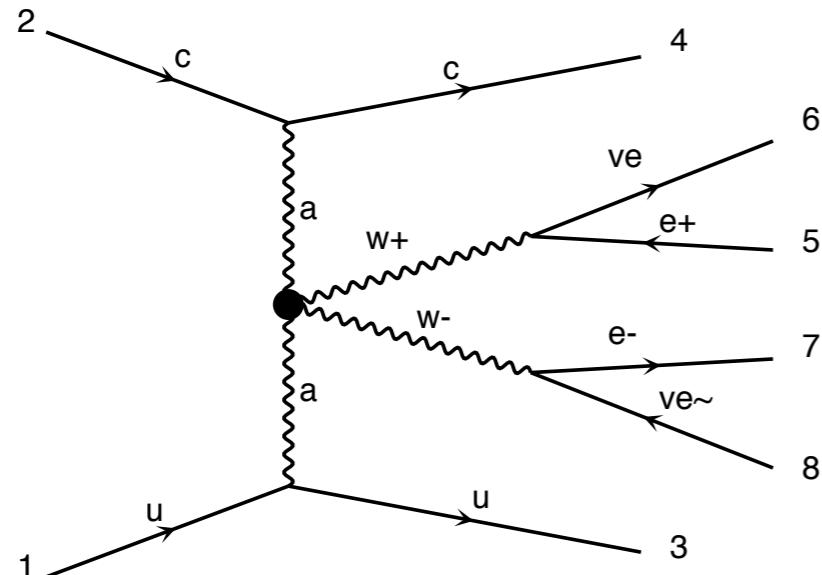


## Problem

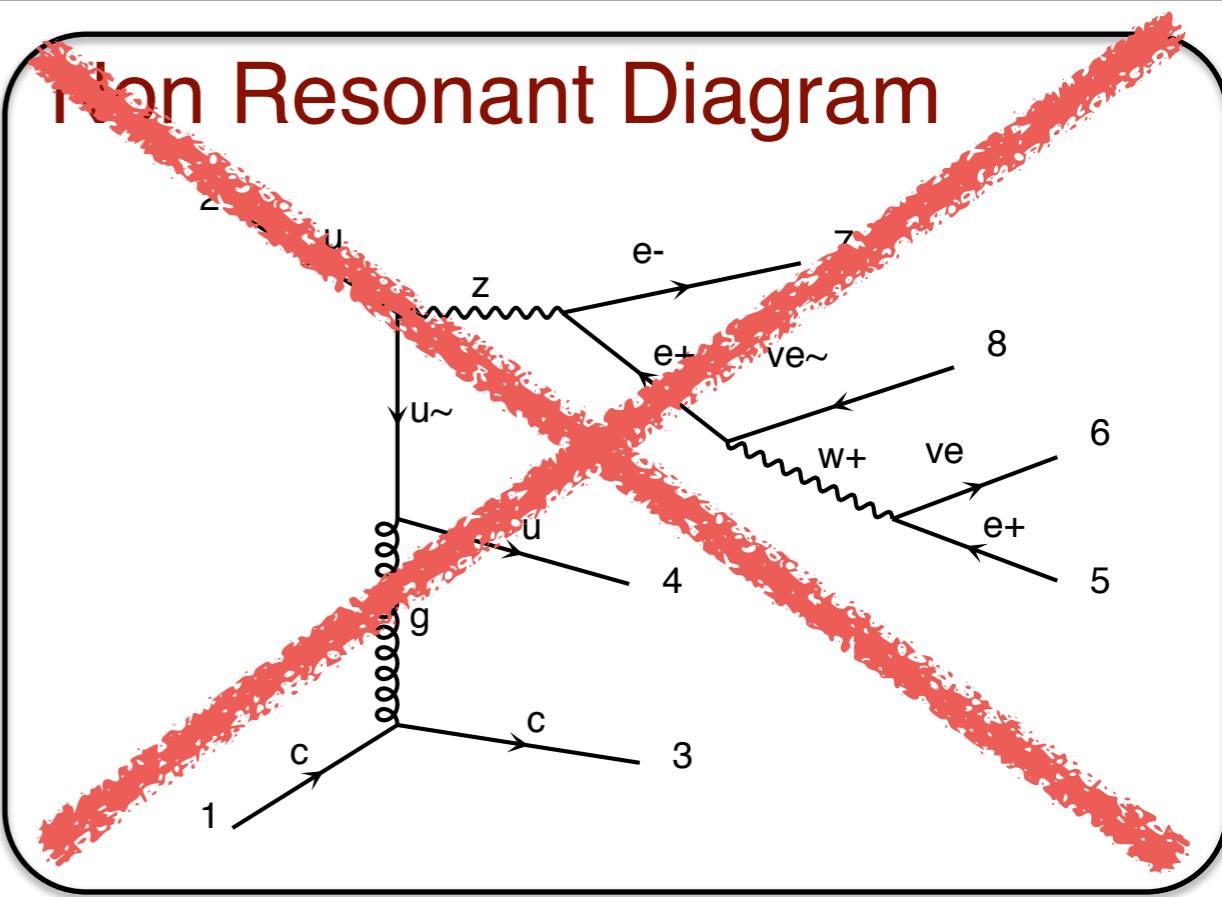
- Process too complicated to compute all the Feynman Diagram

# Decay

## Resonant Diagram



## Non Resonant Diagram



## Problem

- Process too complicated to compute all the Feynman Diagram

## Solution

- Only keep on-shell contribution

# Narrow-Width Approx.

## Theory

$$\int dq^2 \left| \frac{1}{q^2 - M^2 - iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma} \delta(q^2 - M^2)$$

$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}\left(\frac{\Gamma}{M}\right))$$

## Comment

# Narrow-Width Approx.

## Theory

$$\int dq^2 \left| \frac{1}{q^2 - M^2 - iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma} \delta(q^2 - M^2)$$

$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}\left(\frac{\Gamma}{M}\right))$$

## Comment

- This is an **Approximation!**
- This force the particle to be on-shell!
  - Recover by re-introducing the Breit-wigner up-to a cut-off

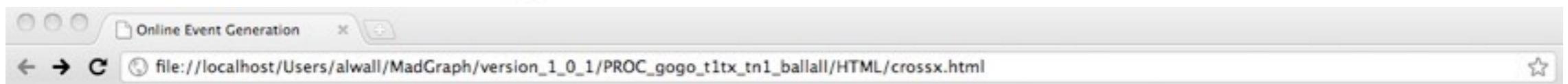
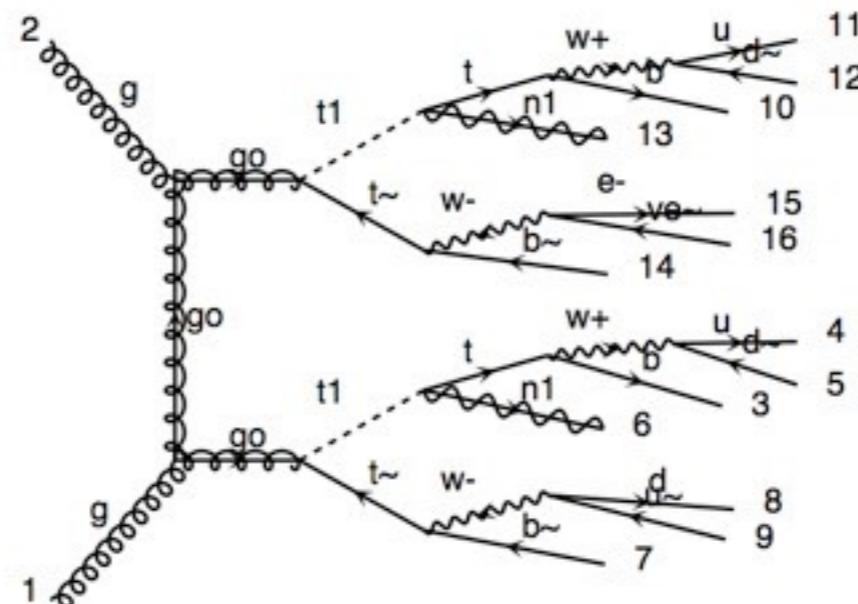
# Decay chains

- $p p \rightarrow t \bar{t} \sim w^+, (t \rightarrow w^+ b, w^+ \rightarrow l^+ \nu l), \backslash$   
 $(\bar{t} \rightarrow w^- b \bar{\sim}, w^- \rightarrow j j), \backslash$   
 $w^+ \rightarrow l^+ \nu l$
- Separately generate core process and each decay
  - Decays generated with the decaying particle as resulting wavefunction
- Iteratively combine decays and core processes
- Difficulty: Multiple diagrams in decays

# Decay chains

- Decay chains retain **full matrix element** for the diagrams compatible with the decay
  - **Full spin correlations** (within and between decays)
  - **Full width effects**
- However, no interference with non-resonant diagrams
  - Description only valid close to pole mass
  - Cutoff at  $|m \pm n\Gamma|$  where n is set in run\_card.

# Decay chains



**Results for  $g g \rightarrow go go , (go \rightarrow t1 t\sim , t\sim \rightarrow b\sim all\ all / h+ , (t1 \rightarrow t n1 , t \rightarrow b\ all\ all / h+))$  in the mssm**

## Available Results

Links	Events	Tag	Run	Collider	Cross section (pb)	Events
<a href="#">results banner</a>	Parton-level <a href="#">LHE</a>	fermi	<a href="#">test</a>	p p 7000 x 7000 GeV	.33857E-03	10000

[Main Page](#)

Thanks to developments in MadEvent, also (very) long decay chains possible to simulate directly in MadGraph!

# MadSpin

[Artoisenet, OM et al. 1212.3460]

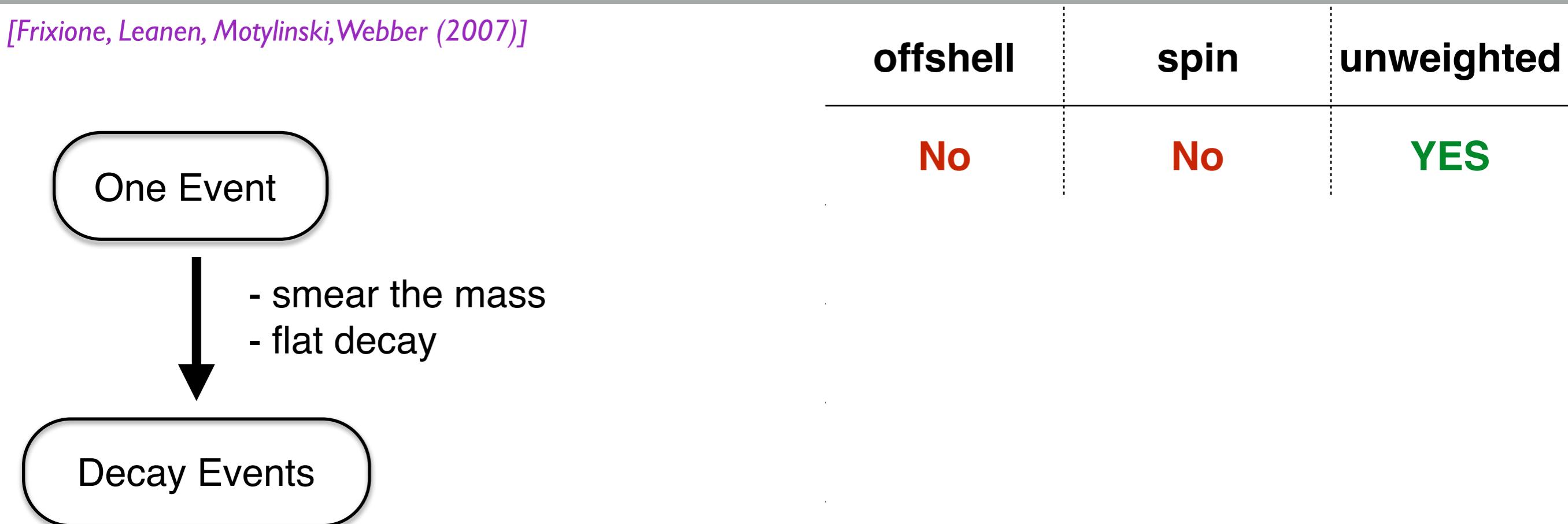
[Frixione, Leanen, Motylinski, Webber (2007)]

	offshell	spin	unweighted
One Event	No	No	YES

# MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]



# MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]

One Event

- smear the mass
- flat decay

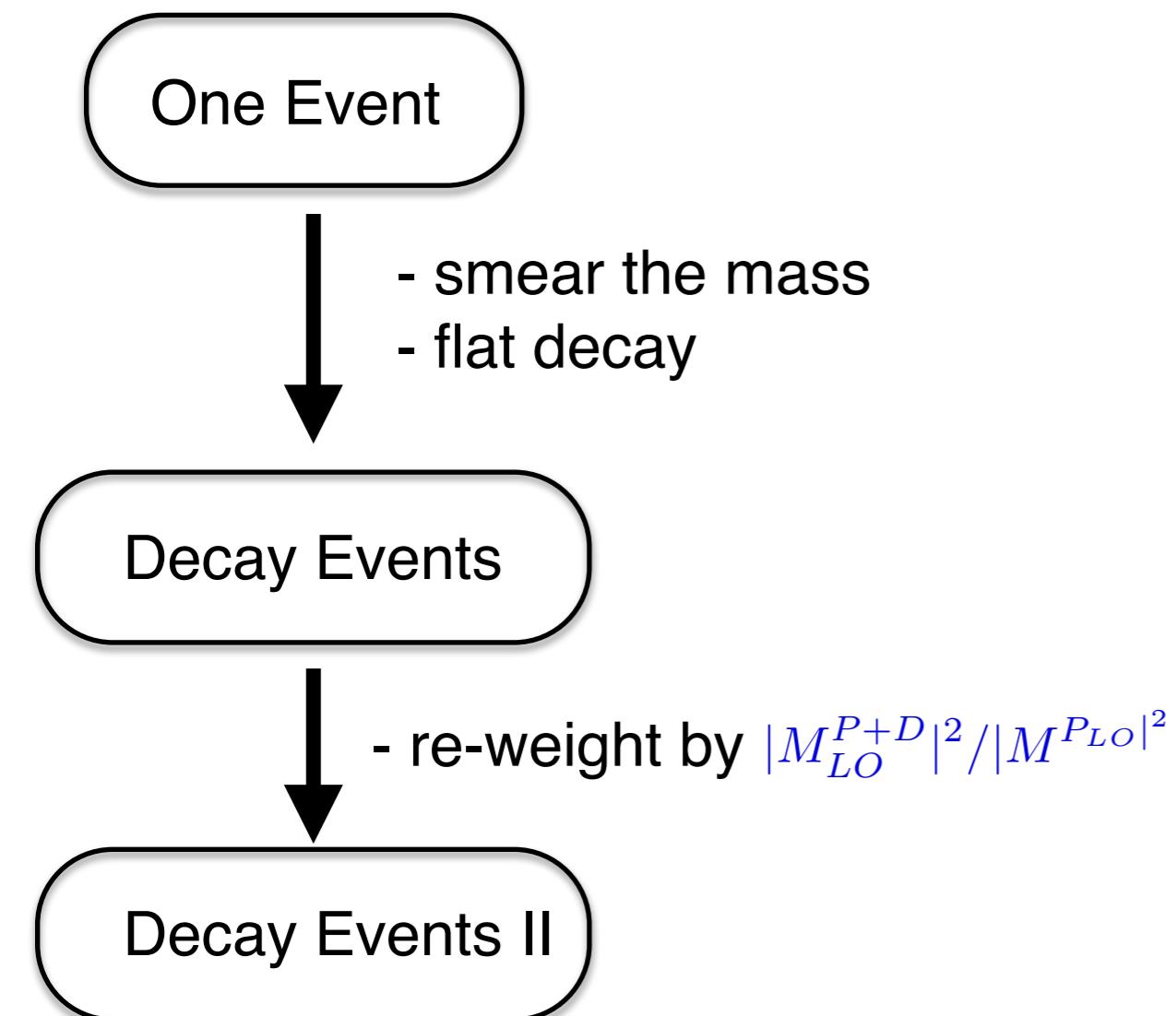
Decay Events

	offshell	spin	unweighted
offshell	No	No	YES
spin	YES	No	No

# MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]

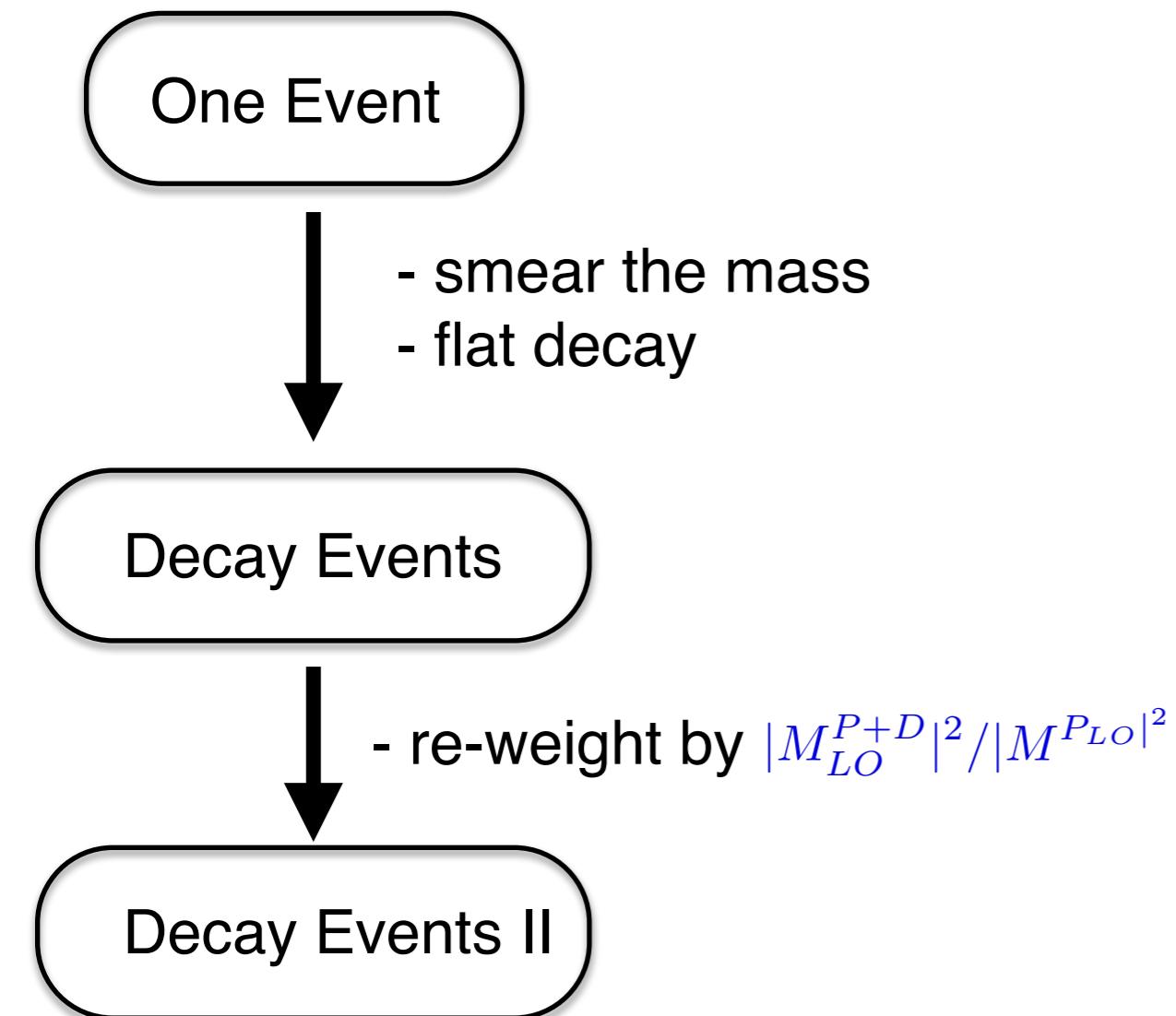


	offshell	spin	unweighted
offshell	No	No	YES
spin	YES	No	No

# MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]

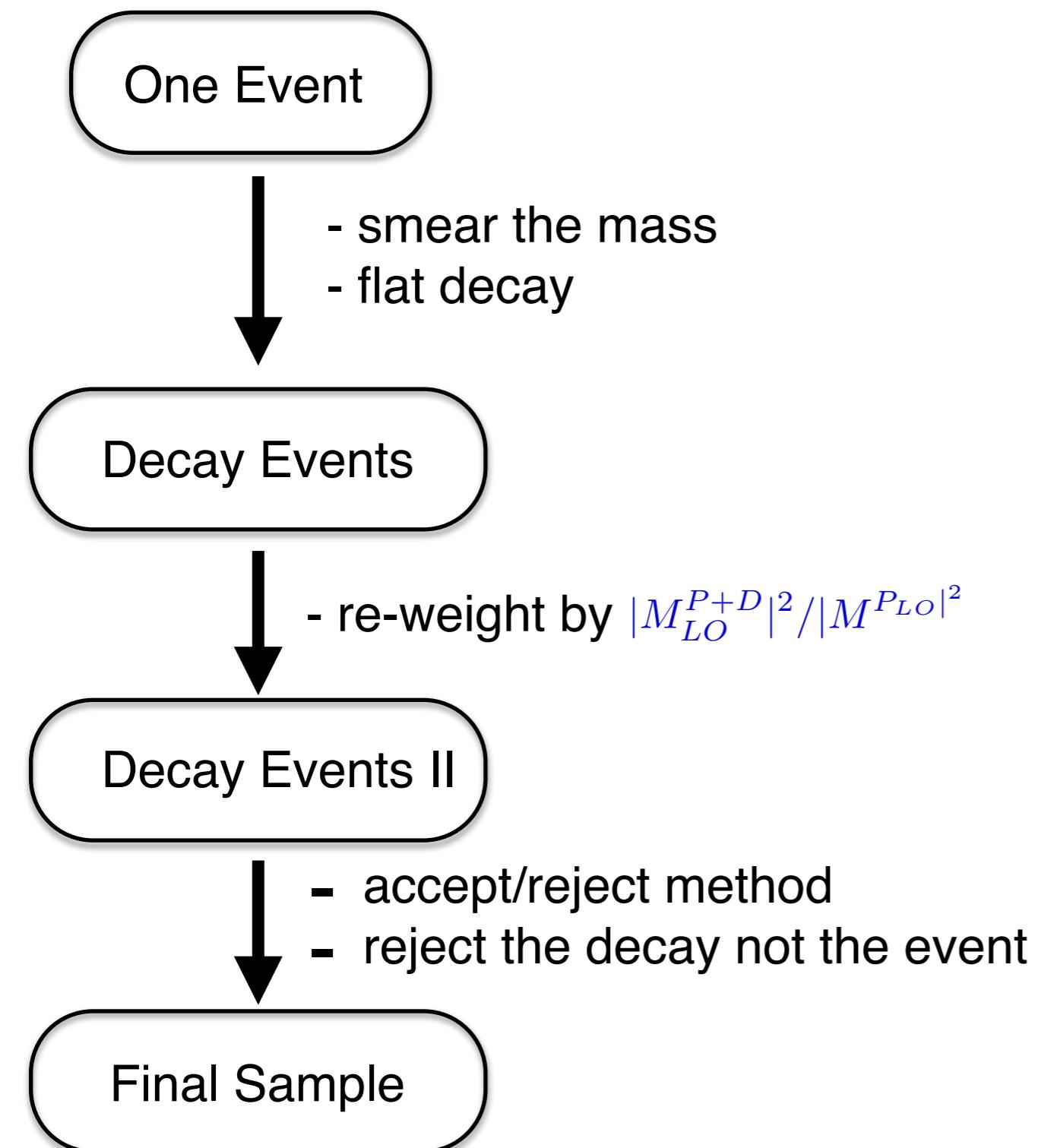


	offshell	spin	unweighted
offshell	No	No	YES
spin	YES	No	No
unweighted	YES	YES	No

# MadSpin

[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]

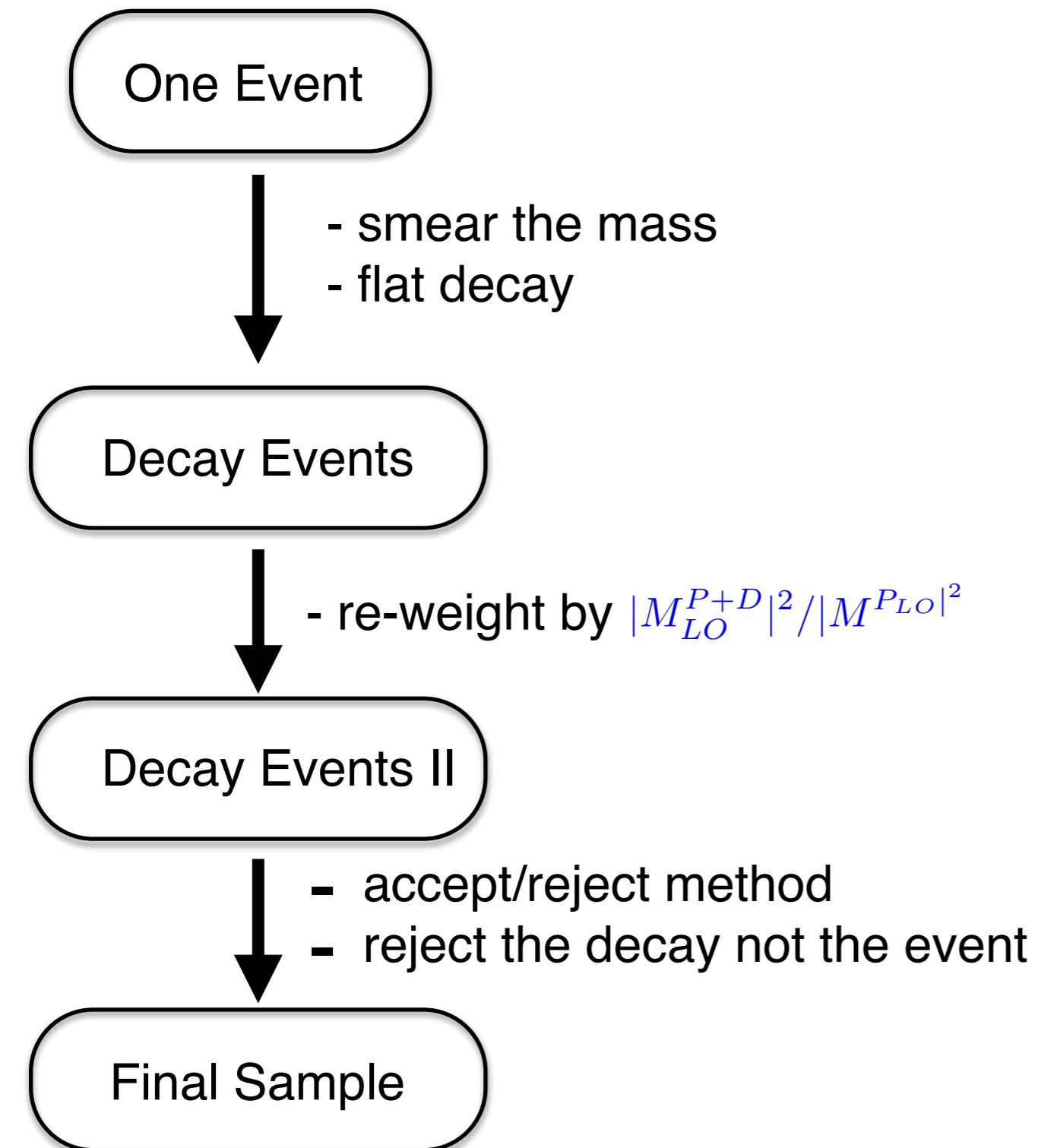


	offshell	spin	unweighted
offshell	No	No	YES
spin	YES	No	No
unweighted	YES	YES	No

# MadSpin

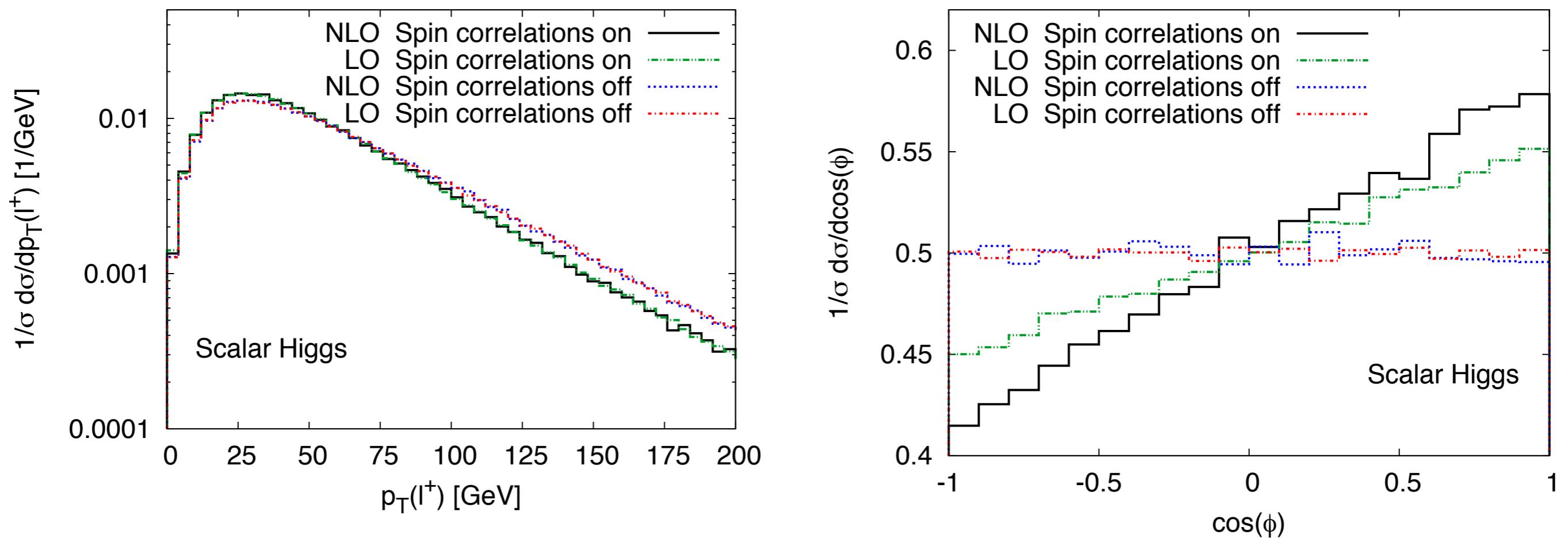
[Artoisenet, OM et al. 1212.3460]

[Frixione, Leanen, Motylinski, Webber (2007)]



	offshell	spin	unweighted
No	No	YES	
YES	No	No	
YES	YES	YES	No
YES	YES	YES	YES

# TTH Example



# **Tutorial**

**Olivier Mattelaer  
IPPP/Durham**

# Tutorial map

## Learning MG5

- follow the built-in tutorial
- cards meaning
- meaning of QCD/QED
- details of syntax (\$/)
- script
- width computation
- decay chain

## BSM CASE

- check the model
- width computation
- signal generation
  - decay chain
- merging sample generation
- background/NLO generation

# Learning MG5\_aMC

# Where to find help?

- Ask me
- Use the command “help” / “help XXX”
  - “help” tell you the next command that you need to do.
- Launchpad:
  - <https://answers.launchpad.net/madgraph5>
  - FAQ: <https://answers.launchpad.net/madgraph5/+faqs>

# What are those cards?

---

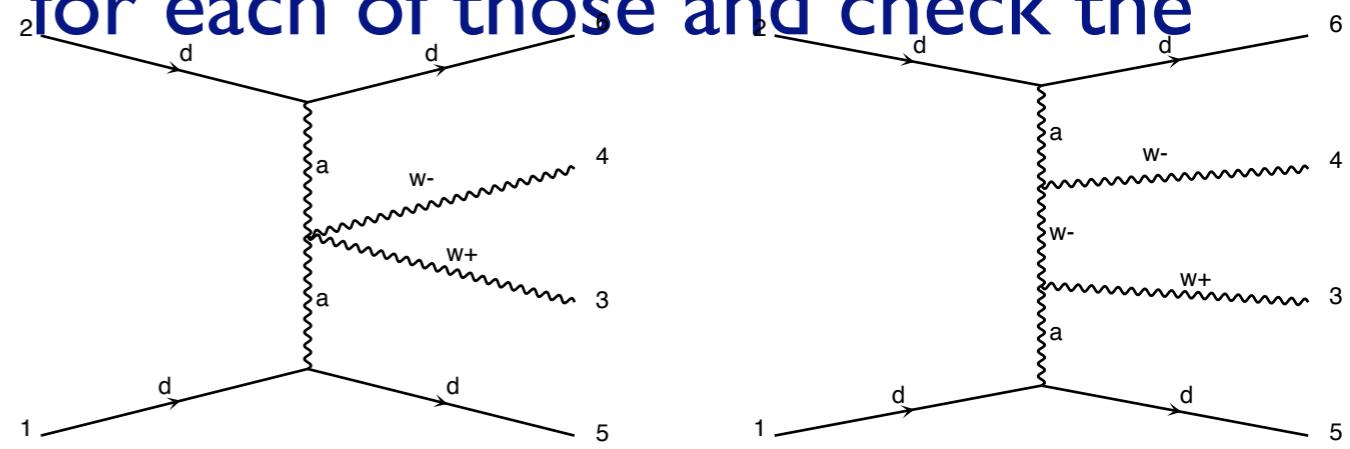
- Read the Cards and identify what they do
  - `param_card`: model parameters
  - `run_card`: beam/run parameters and cuts
    - <https://answers.launchpad.net/madgraph5/+faq/2014>

# Exercise II: Cards Meaning

- How do you change
  - top mass
  - top width
  - W mass
  - beam energy
  - pt cut on the lepton

# Exercise II : Syntax

- What's the meaning of the order →  $\text{P P} > \text{t t} \sim \text{QCD=0}$   
→  $\text{QED/QCD}$   
→  $\text{P P} > \text{t t} \sim \text{QED}<=2$
- What's the difference between →  $\text{P P} > \text{t t} \sim \text{QCD}^{\wedge}2==2$ 
  - $\text{P P} > \text{t t} \sim$
  - $\text{P P} > \text{t t} \sim \text{QED}=2$
  - $\text{P P} > \text{t t} \sim \text{QED}=0$
- Compute the cross-section for each of those and check the diagram
- Generate VBF process



# Exercise III: Syntax

- Generate the cross-section and the distribution (invariant mass) for
  - $P P > e^+ e^-$
  - $P P > z, z > e^+ e^-$
  - $P P > e^+ e^- \$ z$
  - **Hint:** To plot automatically distributions:  
 $P P > e^+ e^- / z$   
mg5> install MadAnalysis
- Use the invariant mass distribution to determine the

# Exercise IV: Automation/Width

- Compute the cross-section for the top pair production for 3 different mass points.
  - Do NOT use the interactive interface
    - hint: you can edit the param\_card/run\_card via the “set” command [After the launch]
    - hint: All command [including answer to question] can be put in a file. (run ./bin/mg5 PATH\_TO\_FILE)

## Examples

File:

```
import model EWDim6
generate p p > z z
output TUTO_DIM6
launch
  set nevents 5000
  set MZ 100
```

How to Run: ./bin/mg5\_amc PATH

# Exercise V: Decay Chain

- Generate  $p\ p \rightarrow t\ t^{\sim} h$ , fully decayed (fully leptonic decay for the top)
  - Using the decay-chain formalism
  - Using MadSpin
- Compare cross-section
  - which one is the correct one?
  - Why are they different?
- Compare the shape.

# BSM Tutorial

# Exercise I: Check the model validity

- Check the model validity:

- check  $p \cdot p > uv \cdot uv^\sim$
- check  $p \cdot p > ev \cdot ev^\sim$
- check  $p \cdot p > t \cdot t^\sim \cdot p_1 \cdot p_2$

- This checks

- gauge invariance
- lorentz invariance
- that various way to compute the matrix element provides the same answer

# Exercise II: Width computation

- Check with MG the width computed with FR:
  - generate uv > all all; output; launch
  - generate ev > all all; output; launch
  - generate p1 > all all; output; launch
  - generate p2 > all all; output; launch
- Check with MadWidth
  - compute\_widths uv ev p1 p2
  - (or Auto in the param\_card)

FR Number
0.0706 GeV
0.00497 GeV
0 GeV
0.0224 GeV

- $M_{uv} = 400 \text{ GeV}$      $M_{ev} = 50 \text{ GeV}$      $\lambda = 0.1$
- $m_1 = 1 \text{ GeV}$      $m_2 = 100 \text{ GeV}$      $m_{l2} = 0.5 \text{ GeV}$

## Exercise III:

- Compute cross-section and distribution
  - uv pair production with decay in top and  $\Phi_1/\Phi_2$  (semi leptonic decay for the top)
- Hint: The width of the new physics particles has to be set correctly in the param\_card.
  - You can either use “Auto” arXiv:1402.1178
  - or use the value computed in exercise 1
- Hint: For sub-decay, you have to put parenthesis:
  - example:  
 $p\ p > t\ t \sim w^+, (t > w^+ b, w^+ > e^+ \nu e), (t \sim > b \sim w^-, w^- > j\ j), w^+ > l^+ \nu l$



- Use MadSpin! arXiv:1212.3460
  - Use Narrow Width Approximation to factorize production and decay
- instead of
  - $p\ p \rightarrow t\ t^{\sim} w^+$ , ( $t \rightarrow w^+ b$ ,  $w^+ \rightarrow e^+ \nu e$ ), ( $t^{\sim} \rightarrow b^{\sim} w^-$ ,  $w^- \rightarrow j\ j$ ),  
 $w^+ \rightarrow l^+ \nu l$
- Do
  - $p\ p \rightarrow t\ t^{\sim} w^+$
- At the question:

The following switches determine which programs are run:

1 Run the pythia shower/hadronization:	pythia=OFF
2 Run PGS as detector simulator:	pgs=OFF
3 Run Delphes as detector simulator:	delphes=NOT INSTA
4 Decay particles with the MadSpin module:	madspin=OFF
5 Add weight to events based on coupling parameters:	reweight=OFF

Either type the switch number (1 to 5) to change its default setting,  
or set any switch explicitly (e.g. type 'madspin=ON' at the prompt)  
Type '0', 'auto', 'done' or just press enter when you are done.  
[0, 1, 2, 4, 5, auto, done, pythia=ON, pythia=OFF, ... ] [60s to answer]
- At the next question edit the madspin\_card and define the decay

# Exercise IV: generate multiple multiplicity sample for pythia8

- We will do MLM matching
  - in the run\_card.dat ickkw=1
  - the matching scale (Qcut) will be define in pythia
    - in madgraph we use xqcut which should be smaller than Qcut (but at least 10-20 GeV)

# Exercise V: Have Fun

---

- Simulate Background
- Go to NLO (ask me the model)
- ...

# Solution Learning MG5\_aMC

# Exercise II: Cards Meaning

- How do you change
  - top mass
  - top width
  - W mass
  - beam energy
  - pt cut on the lepton



Param\_card

Run\_card

## ● top mass

```
#####
## INFORMATION FOR MASS
#####
Block mass
 5 1.73000e+02 # MT
 15 1.73000e+02 # MT
 23 9.118800e+01 # MZ
 25 1.200000e+02 # MH
## Dependent parameters, given by model restrictions.
## Those values should be edited following the
## analytical expression. MG5 ignores those values
## but they are important for interfacing the output of MG5
## to external program such as Pythia.
 1 0.000000 # d : 0.0
 2 0.000000 # u : 0.0
 3 0.000000 # s : 0.0
 4 0.000000 # c : 0.0
 11 0.000000 # e- : 0.0
 12 0.000000 # ve : 0.0
 13 0.000000 # mu- : 0.0
 14 0.000000 # vm : 0.0
 16 0.000000 # vt : 0.0
 21 0.000000 # g : 0.0
 22 0.000000 # a : 0.0
 24 80.419002 # w+ : cmath.sqrt(MZ__exp__2/2. + cmath.sqrt(MZ__exp__4/4. - (aEW*cmath.pi*MZ__exp__2)/(Gf*sqrt__2)))
```

```

#####
## INFORMATION FOR MASS
#####
Block mass
 5 4.70000e+00 # MB
 6 1.73000e+02 # MT
15 1.77700e+00 # MTA
23 9.11800e+01 # MZ
25 1.20000e+02 # MH
## Dependent parameters, given by model restrictions.
## Those values should be edited following the
## analytical expression. MG5 ignores those values
## but they are important for interfacing the output of MG5
## to external program such as Pythia.
 1 0.00000 # d : 0.0
 2 0.00000 # u : 0.0
 3 0.00000 # s : 0.0
 4 0.00000 # c : 0.0
11 0.00000 # e- : 0.0
12 0.00000 # ve : 0.0
13 0.00000 # mu- : 0.0
14 0.00000 # vm : 0.0
16 0.00000 # vt : 0.0
21 0.00000 # g : 0.0
22 0.00000 #
24 80.419002 # w+ : cmath.sqrt(MZ__exp__2/2. + cmath.sqrt(MZ__exp__4/4. - (aEW*cmath.pi*MZ__exp__2)/(Gf*sqrt__2)))

```

W Mass is an internal parameter!

**MG5 didn't use this value!**

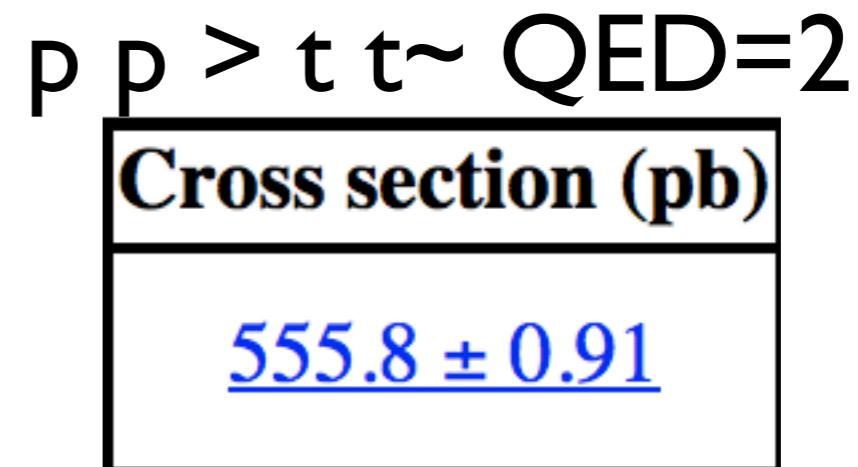
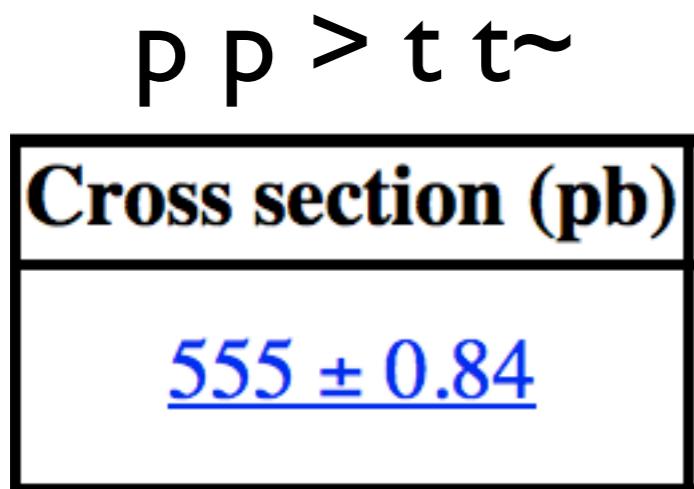
So you need to change MZ or Gf or alpha\_EW

# Exercise III: Syntax

- What's the meaning of the order QED/QCD
- What's the difference between
  - $p\ p > t\ t\sim$
  - $p\ p > t\ t\sim \text{QED}=2$
  - $p\ p > t\ t\sim \text{QED}=0$
  - $p\ p > t\ t\sim \text{QCD}^{\wedge 2}==2$

# Solution I : Syntax

- What's the meaning of the order QED/QCD
  - By default MG5 takes the lowest order in QED!
  - $p\ p > t\ t\sim \Rightarrow p\ p > t\ t\sim \text{QED}=0$
  - $p\ p > t\ t\sim \text{QED}=2$ 
    - additional diagrams (photon/z exchange)



No significant QED contribution

- $\text{QED} \leq 2$  is the SAME as  $\text{QED}=2$ 
  - quite often source of confusion since most of the people use the = syntax
- $\text{QCD}^2=2$ 
  - returns the interference between the QCD and the QED diagram

Cross section (pb)
<u>5.455e-17 ± 4.7e-19 ± systematics</u>

# Solution | Syntax

- generate  $p p \rightarrow w^+ w^- jj$ 
  - 76 processes

- generate  $p p \rightarrow w^+ w^- jj$  QED = 4
  - 1432 diagrams
  - None of them are VBF
  - 76 processes
  - 5332 diagrams
  - VBF present! + those not VBF

- generate  $p p \rightarrow w^+ w^- jj$  QCD = 2
  - 76 processes
  - 5332 diagrams

- generate  $p p \rightarrow w^+ w^- jj$  QED = 2
  - 76 processes
  - 1432 diagrams
  - None of them are VBF

- generate  $p p \rightarrow w^+ w^- jj$  QCD = 0
  - 60 processes
  - 3900 diagrams
  - VBF present!

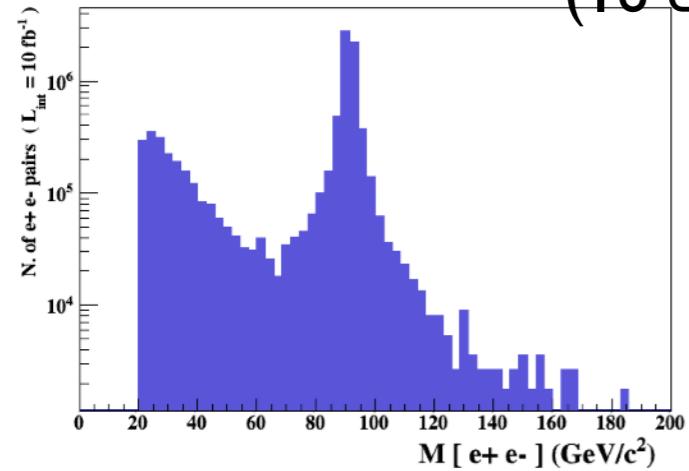
- generate  $p p \rightarrow w^+ w^- jj$  QCD = 4
  - 76 processes
  - 5332 diagrams

# Exercise IV: Syntax

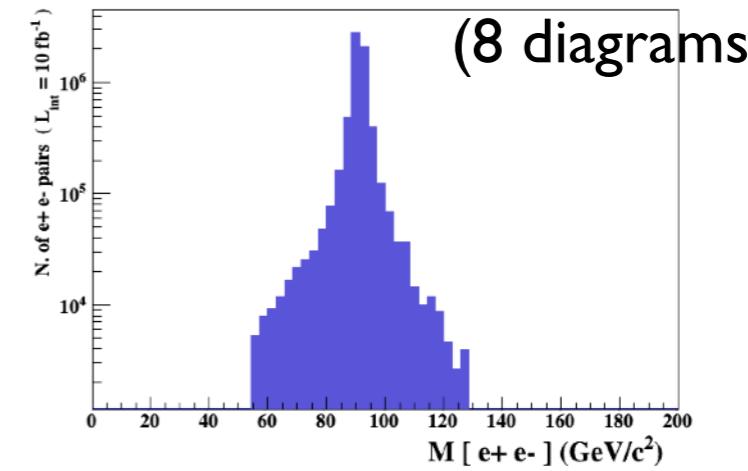
- Generate the cross-section and the distribution (invariant mass) for
  - $p p \rightarrow e^+ e^-$
  - $p p \rightarrow z, z \rightarrow e^+ e^-$
  - $p p \rightarrow e^+ e^- \$ z$
  - $p p \rightarrow e^+ e^- / z$

**Hint :**To have automatic distributions:  
mg5> install MadAnalysis

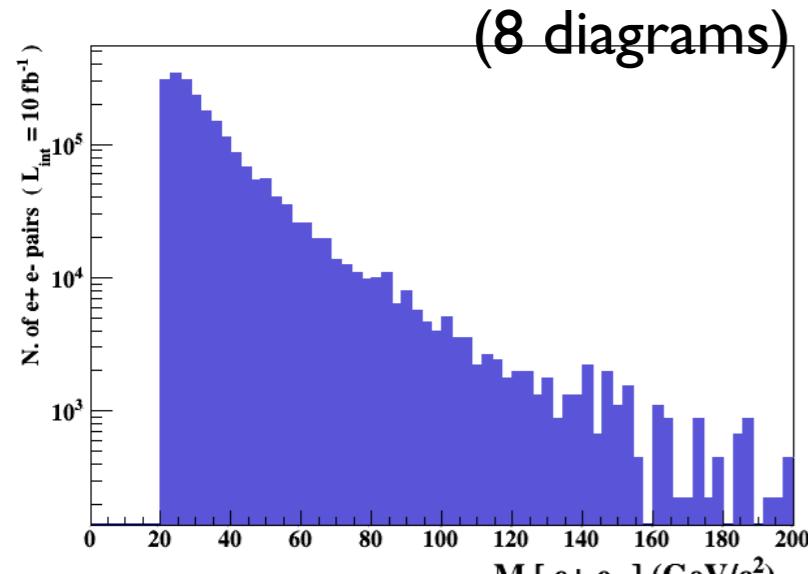
$\text{P P} > \text{e+ e-}$   
(16 diagrams)



$\text{P P} > z, z > \text{e+ e-}$   
(8 diagrams)

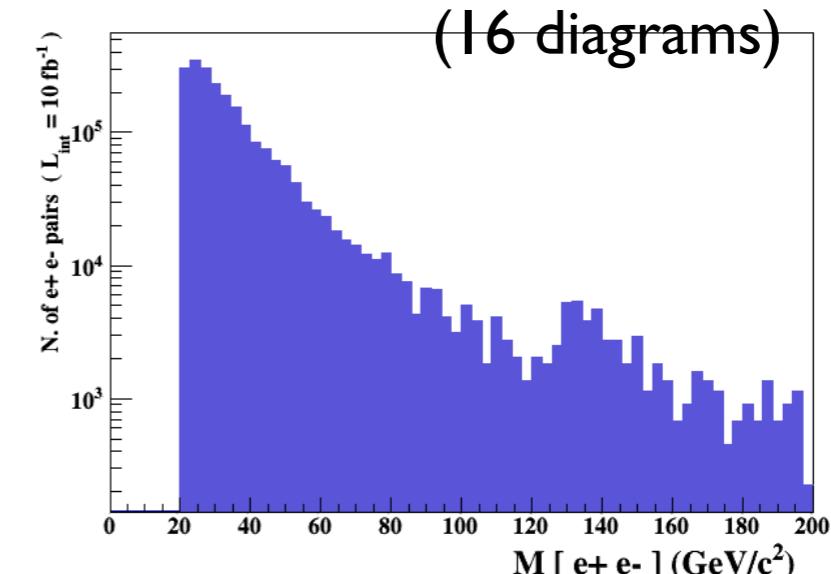


$\text{P P} > \text{e+ e- } /z$



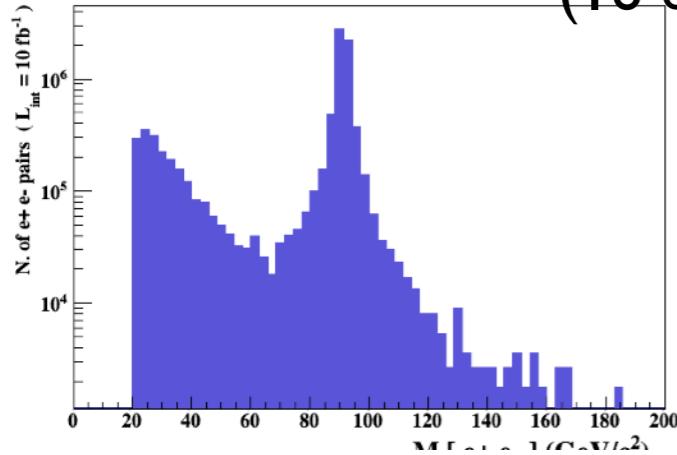
No Z

$\text{P P} > \text{e+ e- } \$ z$

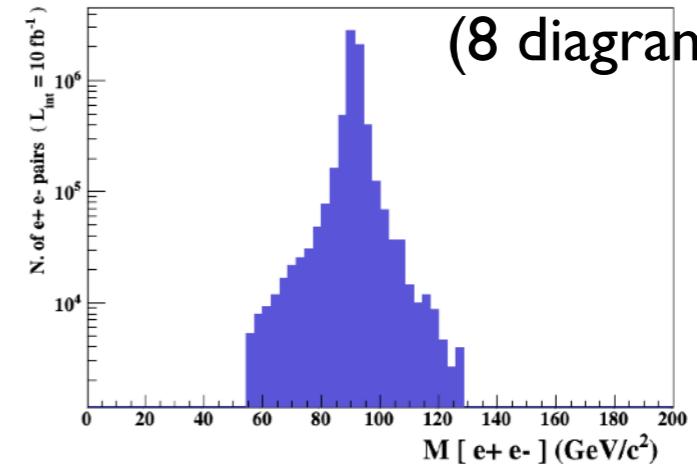


Z- onshell veto

$p\ p > e^+ e^-$   
(16 diagrams)



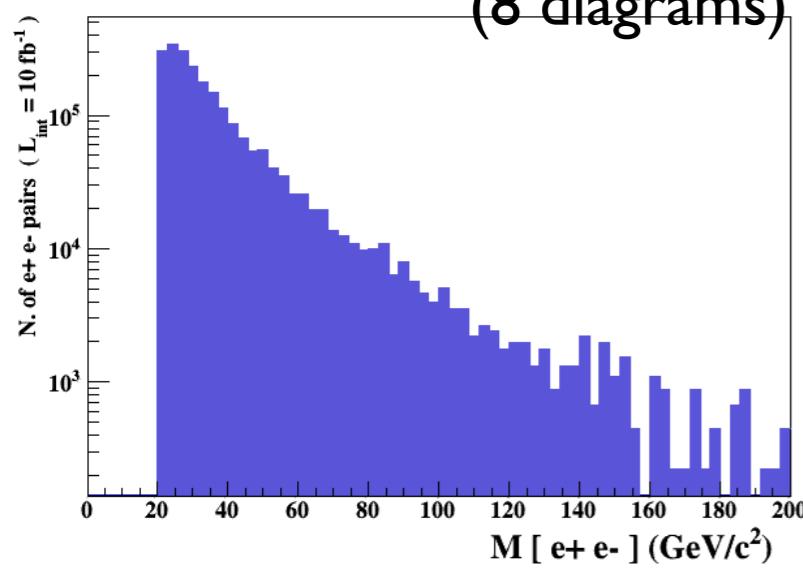
$p\ p > z, z > e^+ e^-$   
(8 diagrams)



Correct Distribution

$p\ p > e^+ e^- / z$

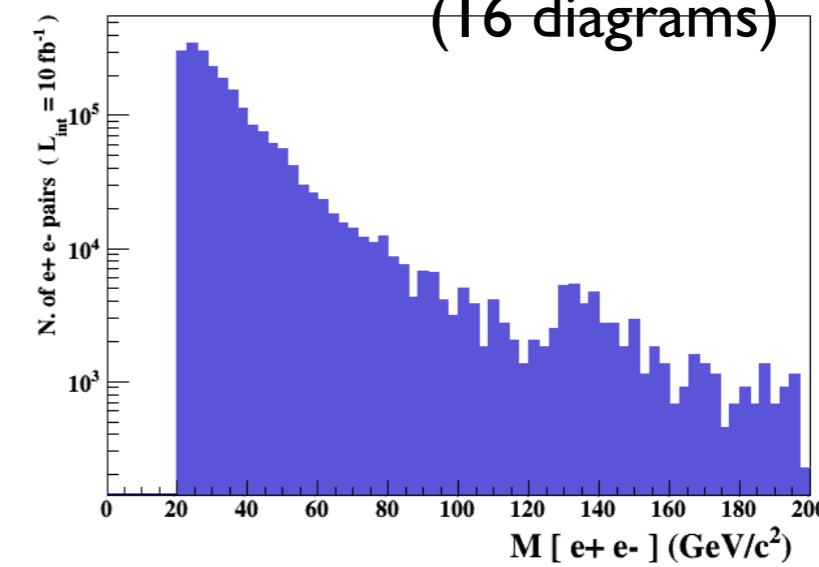
(8 diagrams)



No Z

$p\ p > e^+ e^- \$ z$

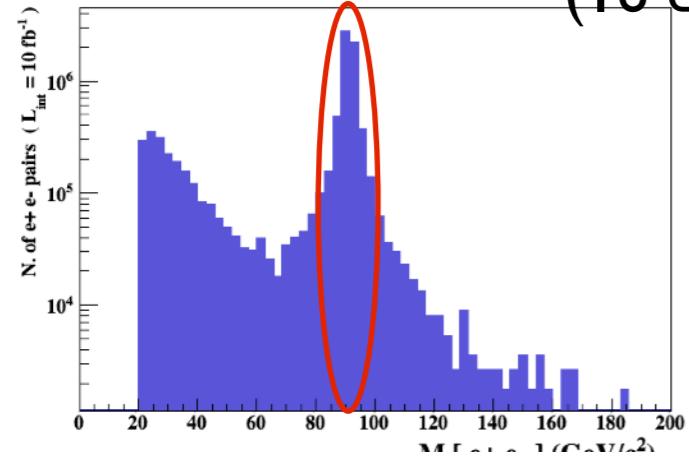
(16 diagrams)



Z- onshell veto

$p p > e^+ e^-$

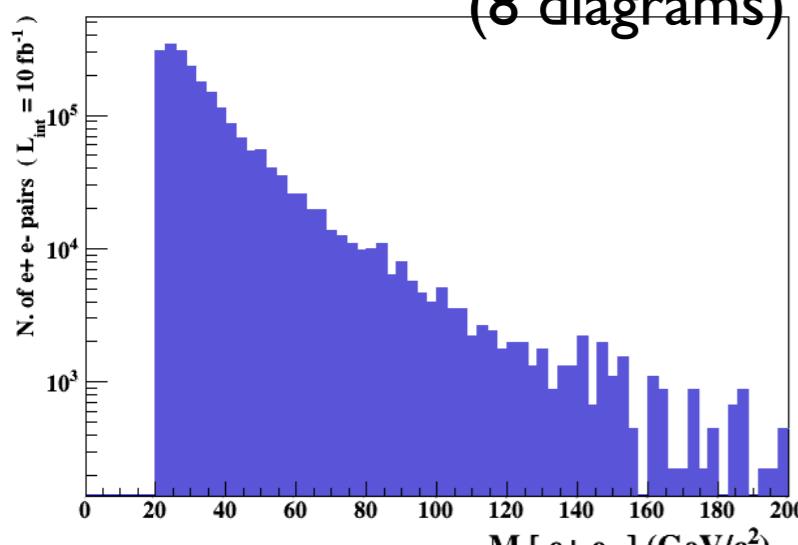
(16 diagrams)



Correct Distribution

$p p > e^+ e^- / z$

(8 diagrams)

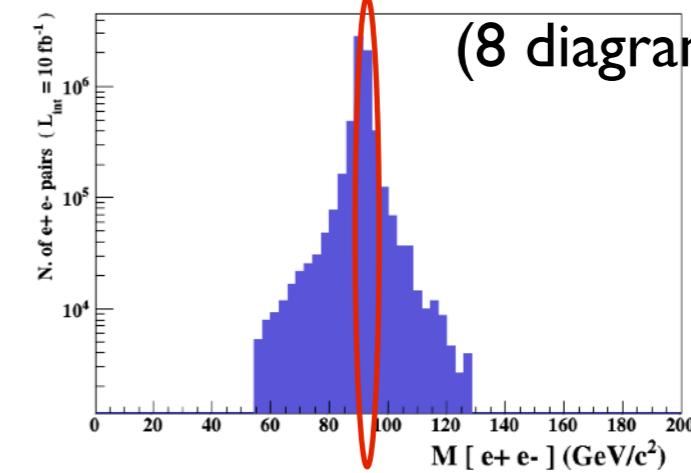


No Z

Z Peak

$p p > z, z > e^+ e^-$

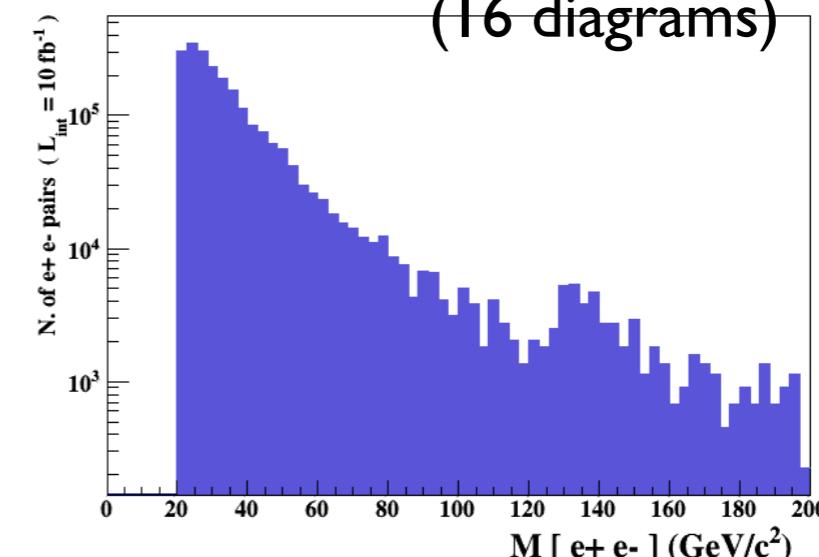
(8 diagrams)



$p p > e^+ e^- \$ z$

NO Z Peak

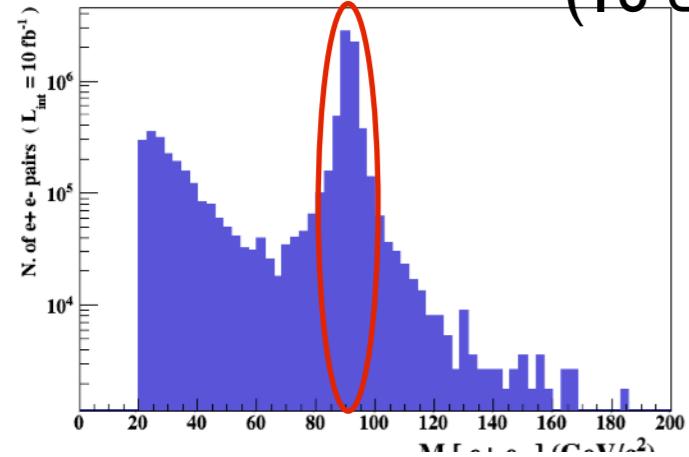
(16 diagrams)



Z- onshell veto

$p p > e^+ e^-$

(16 diagrams)

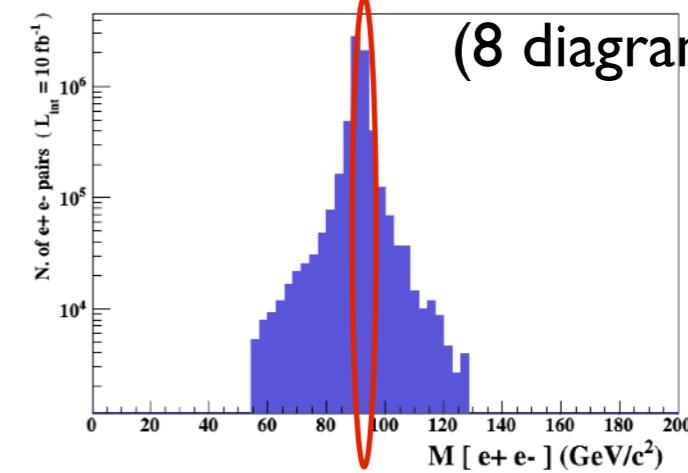


Correct Distribution

Z Peak

$p p > z, z > e^+ e^-$

(8 diagrams)

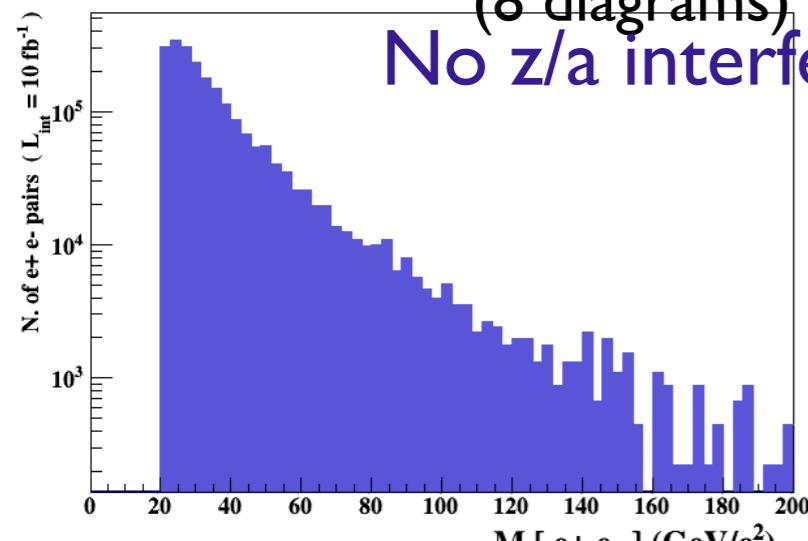


$p p > e^+ e^- / z$

NO Z Peak

(8 diagrams)

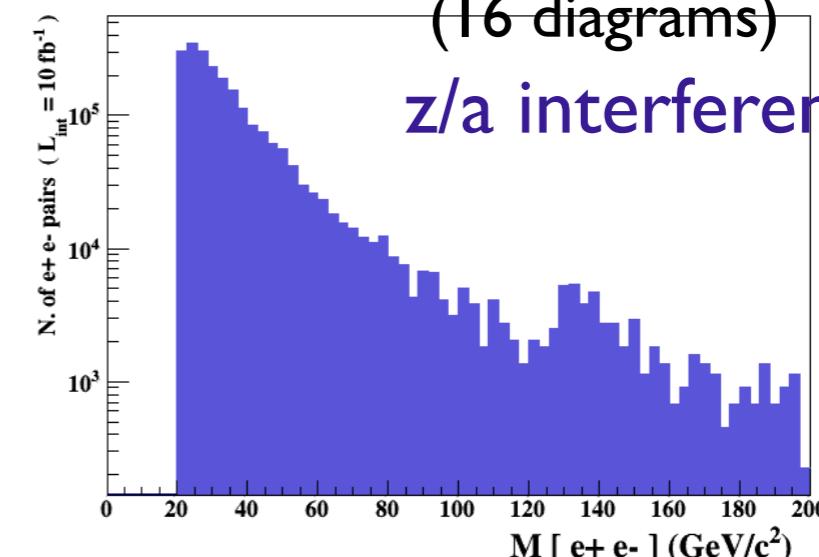
No z/a interference



No Z

$p p > e^+ e^- \$ z$

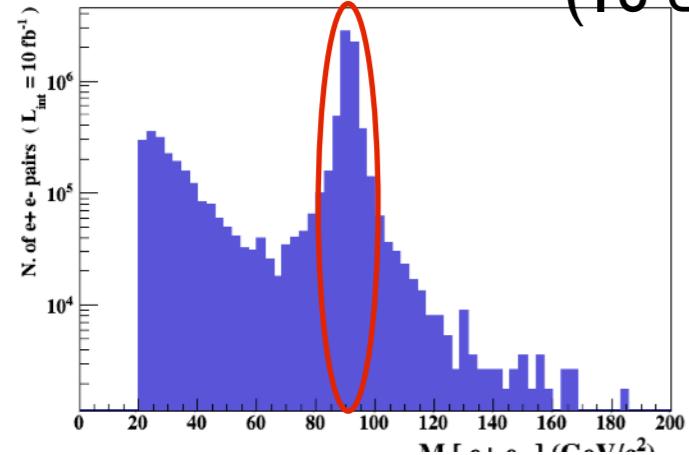
(16 diagrams)  
z/a interference



Z- onshell veto

$p p > e^+ e^-$

(16 diagrams)

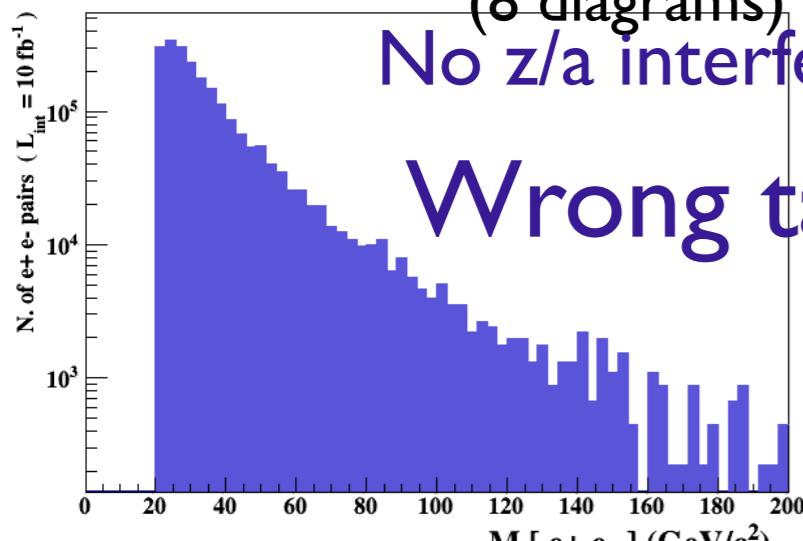


Correct Distribution

$p p > e^+ e^- / z$

(8 diagrams)  
No z/a interference

Wrong tail

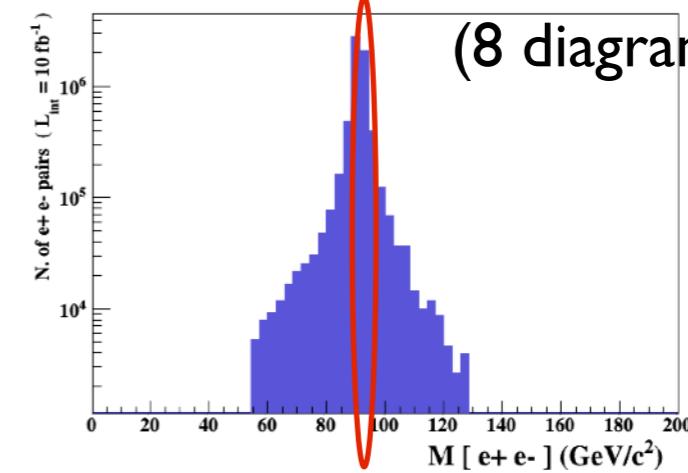


No Z

Z Peak

$p p > z, z > e^+ e^-$

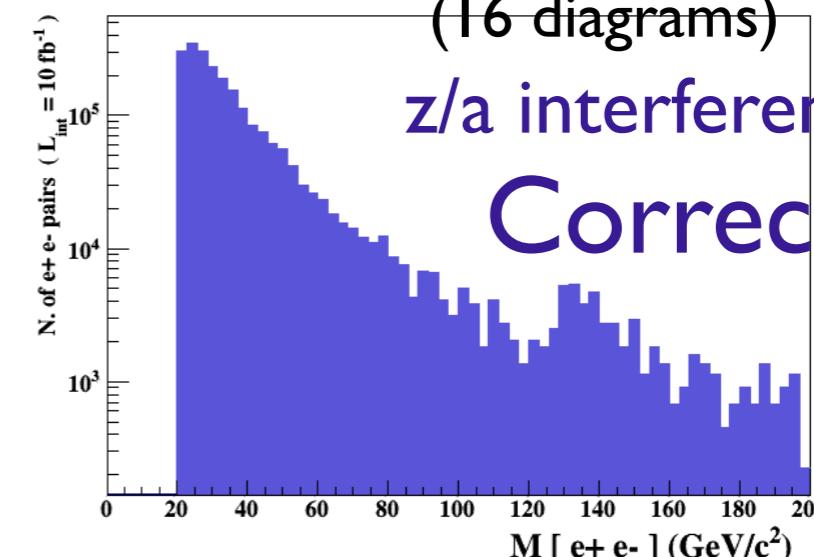
(8 diagrams)



$p p > e^+ e^- \$ z$

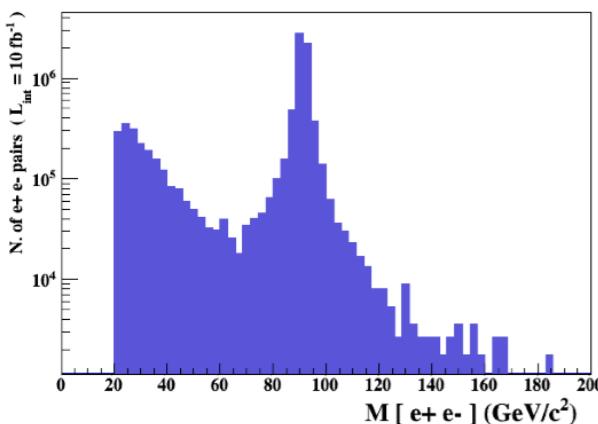
(16 diagrams)  
z/a interference

Correct tail



Z- onshell veto

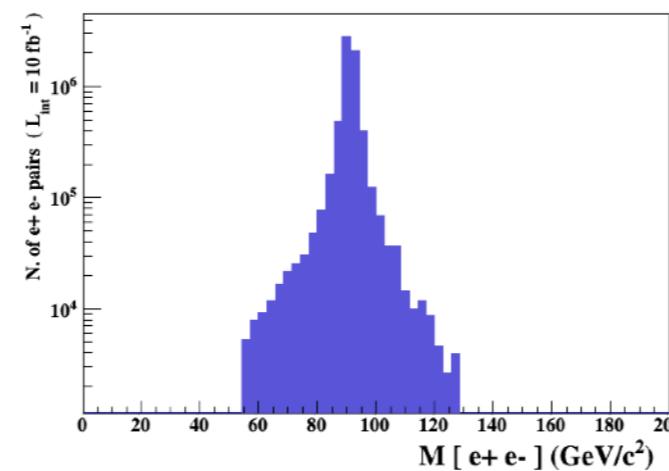
$P\bar{P} > e^+ e^-$



ical dis-

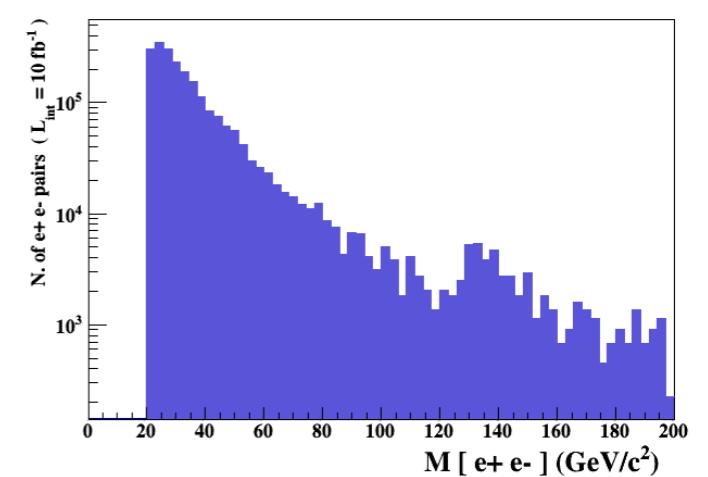
(16 diagrams) two other one.

$P\bar{P} > Z, Z > e^+ e^-$



(8 diagrams)

$P\bar{P} > e^+ e^- \$ z$



(16 diagrams)

- The “\$” forbids the  $Z$  to be onshell but the photon invariant mass can be at  $|M - M| < BW_{cut} * 1$
- The “/” is to be avoided if possible since this leads to violation of gauge invariance.

# WARNING

---

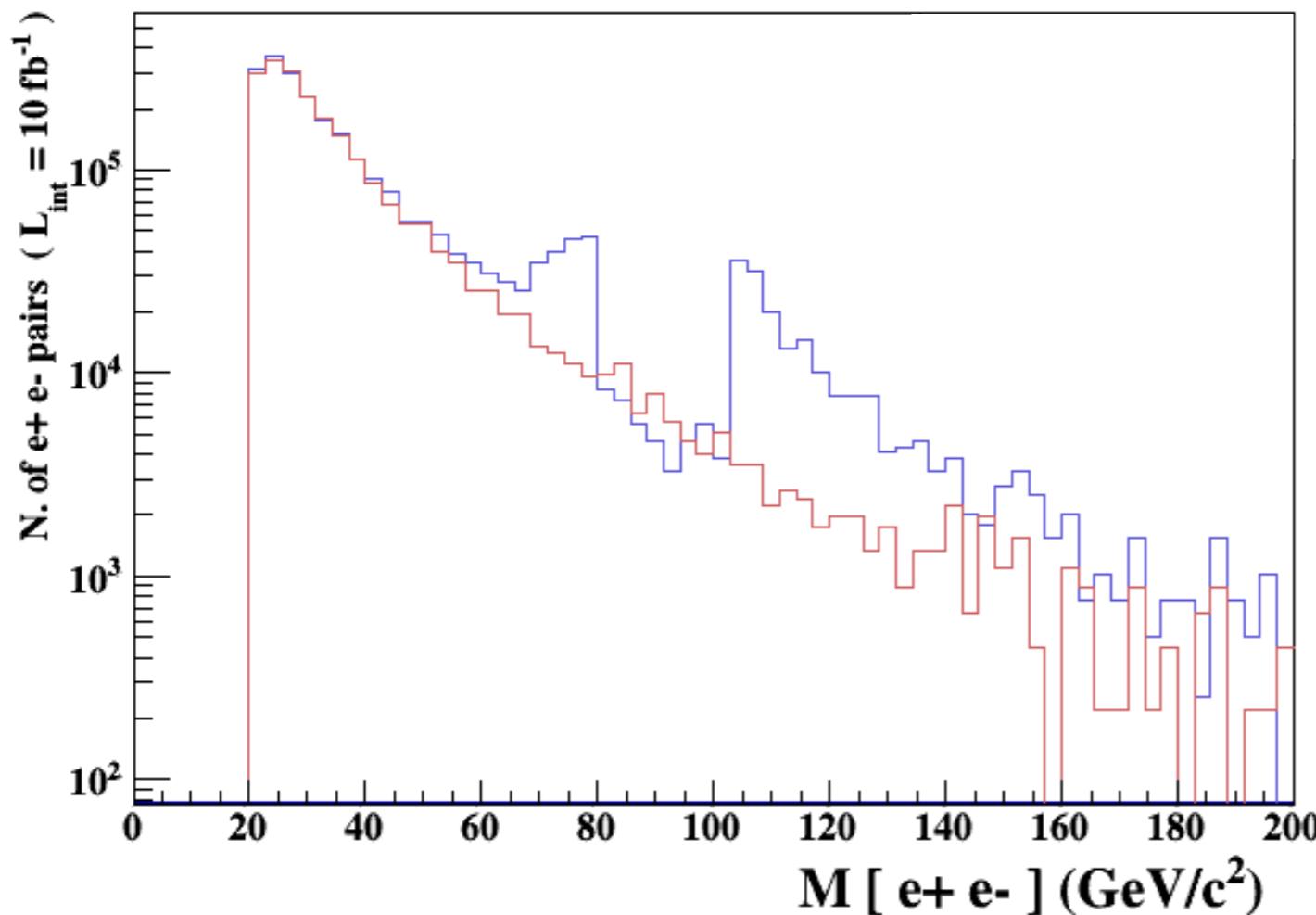
- NEXT SLIDE is generated with `bw_cut =5`
- This is **TOO SMALL** to have a physical meaning (15 the default value used in previous plot is better)
- This was done to illustrate more in detail how the “\$” syntax works.

See previous slide warning

$p p > e^+ e^- / Z$

(red curve)

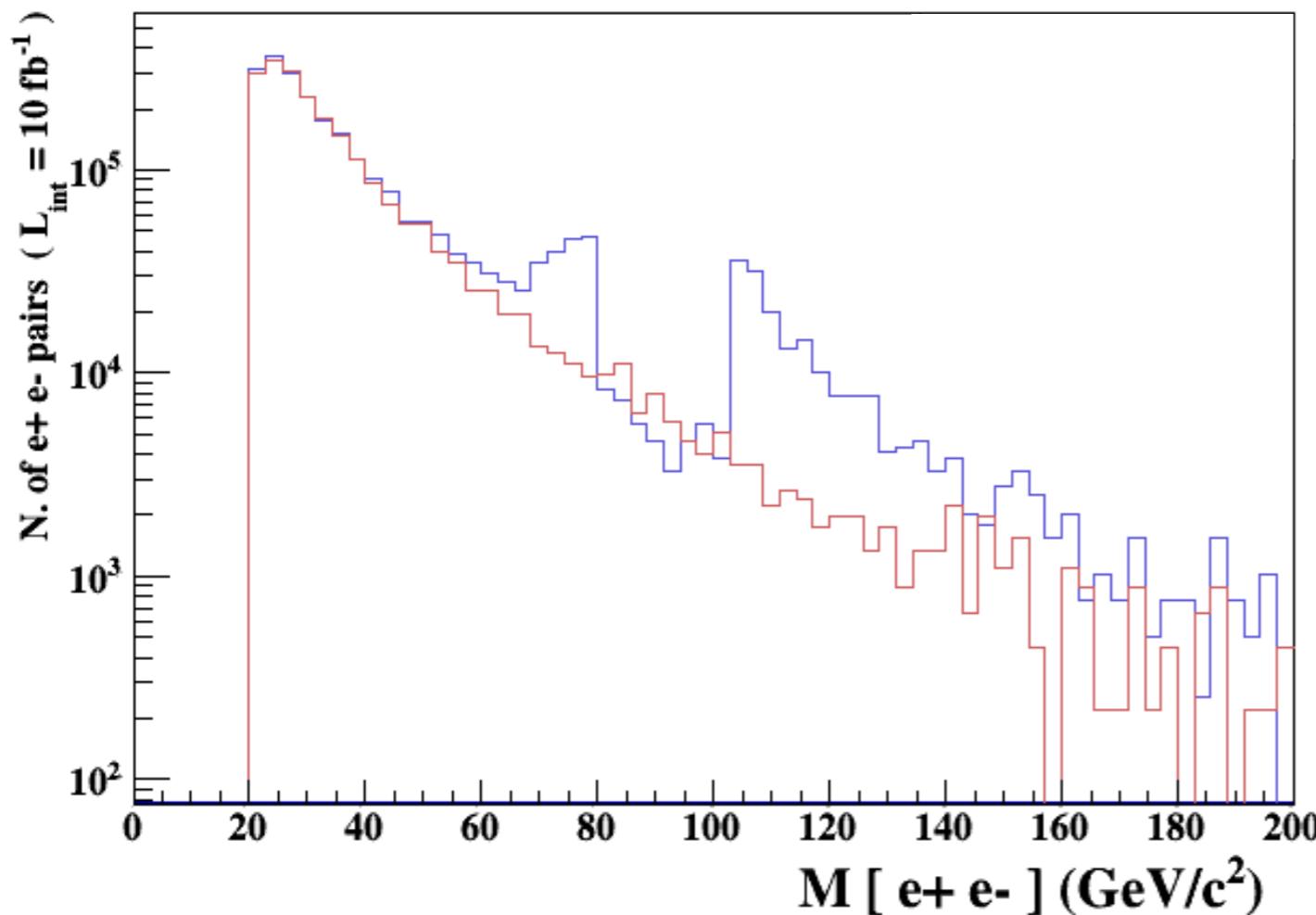
(blue curve)



See previous slide warning

$p p > e+ e- / Z$   
(red curve)

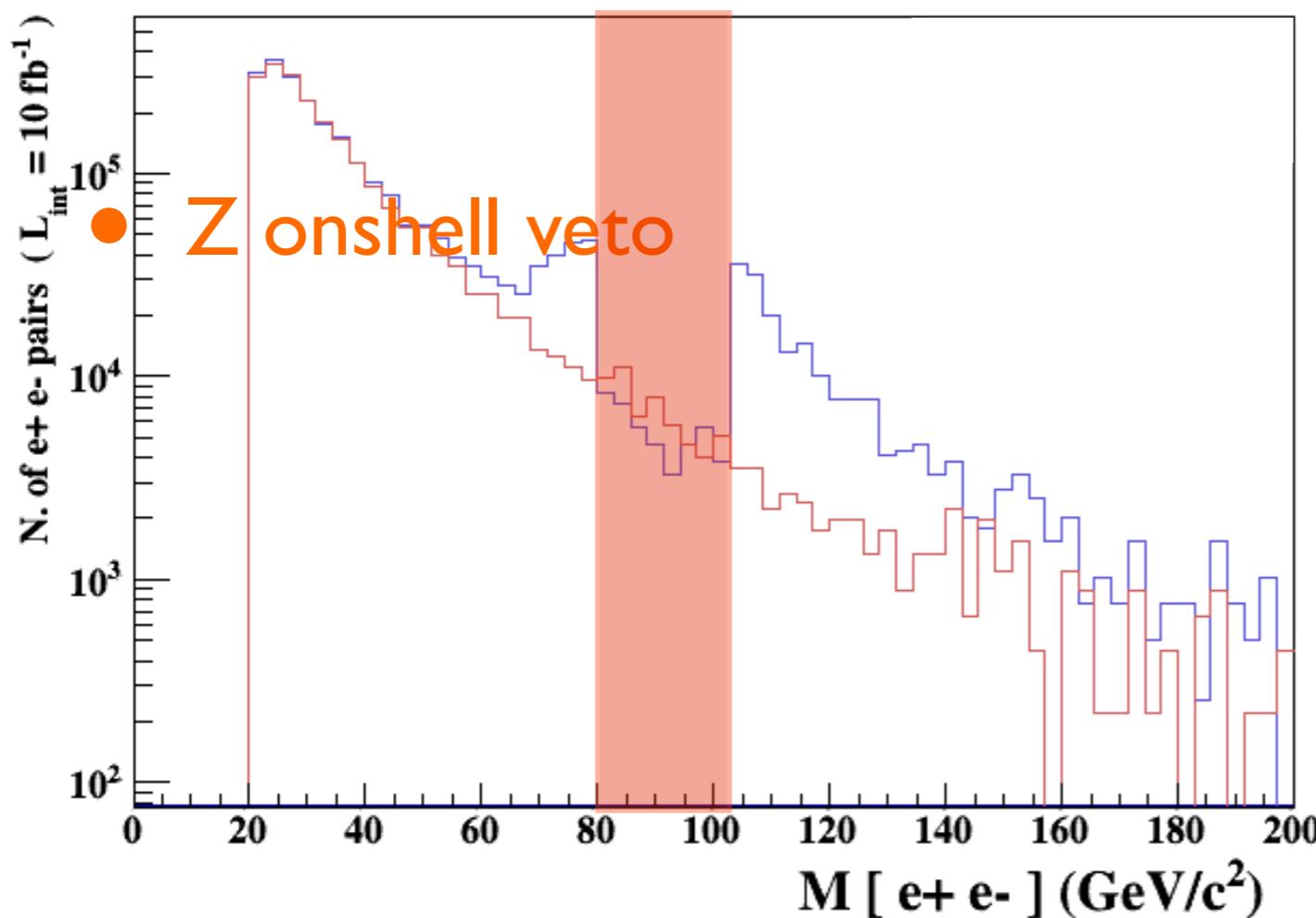
adding  $p p > e+ e- + Z$   
(blue curve)



See previous slide warning

$p p > e+ e- / Z$   
(red curve)

adding  $p p > e+ e- + Z$   
(blue curve)

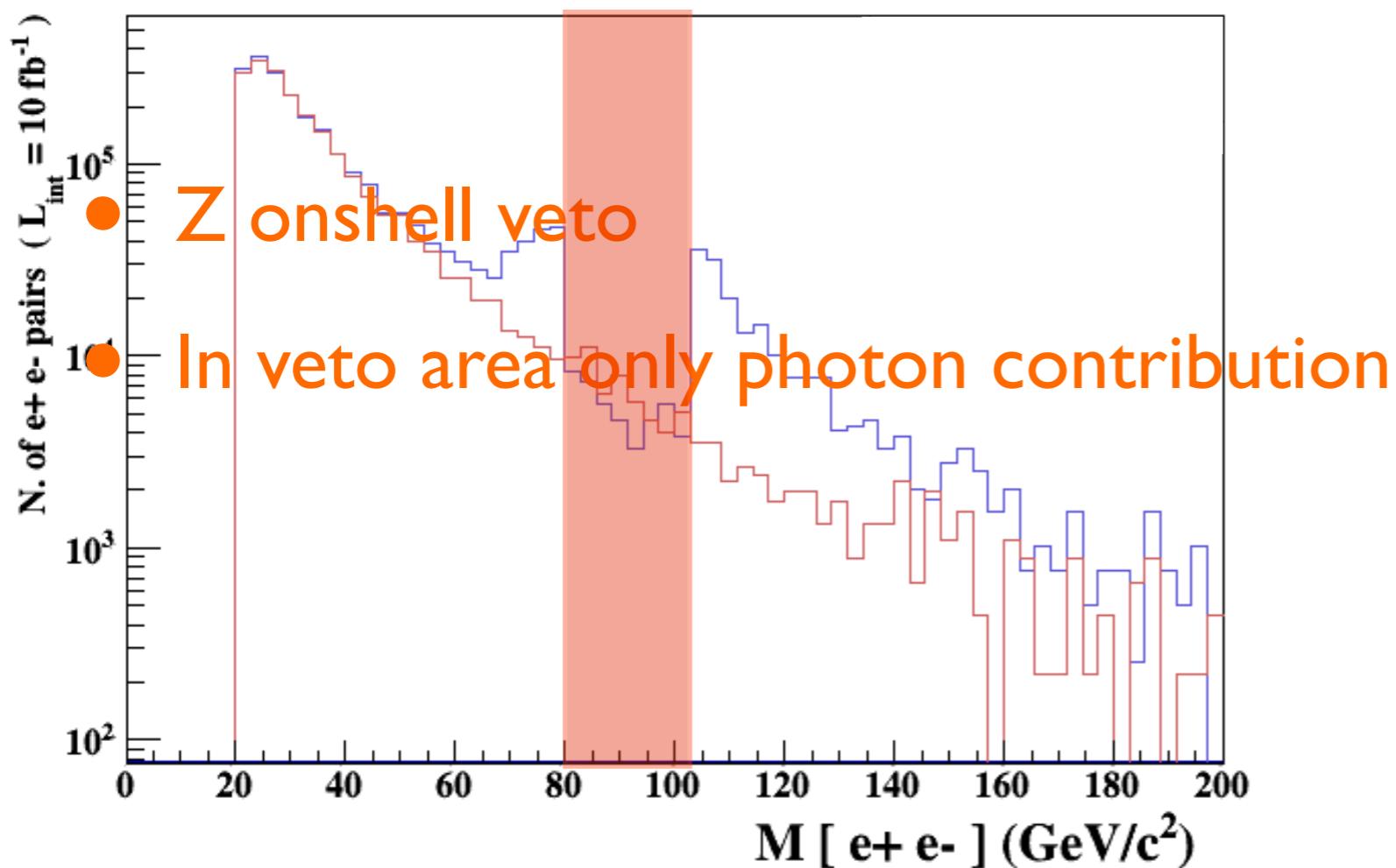


5 times width area

See previous slide warning

$p p > e+ e- / Z$   
(red curve)

adding  $p p > e+ e- + Z$   
(blue curve)

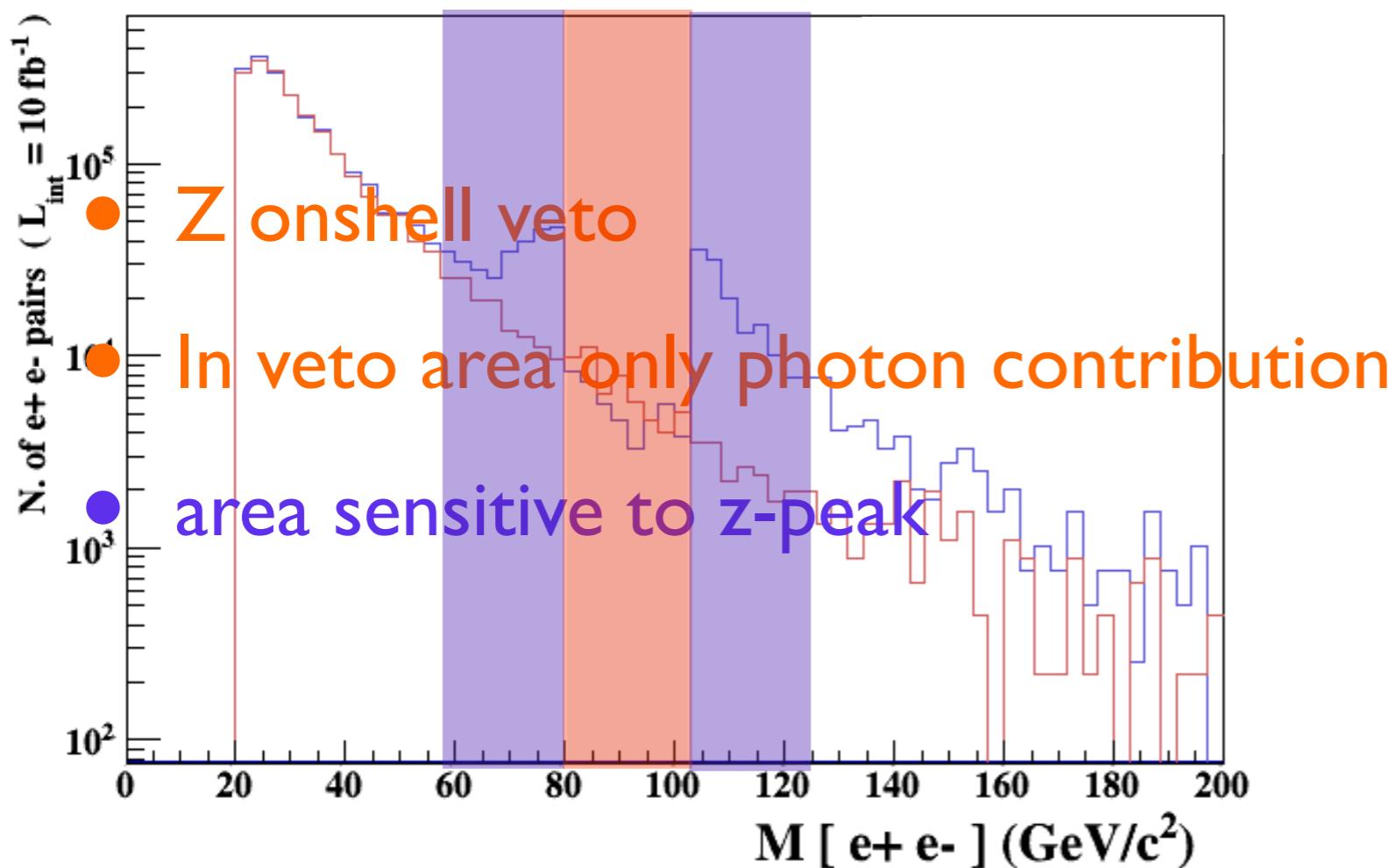


5 times width area

See previous slide warning

$p p > e+ e- / Z$   
(red curve)

adding  $p p > e+ e- + Z$   
(blue curve)



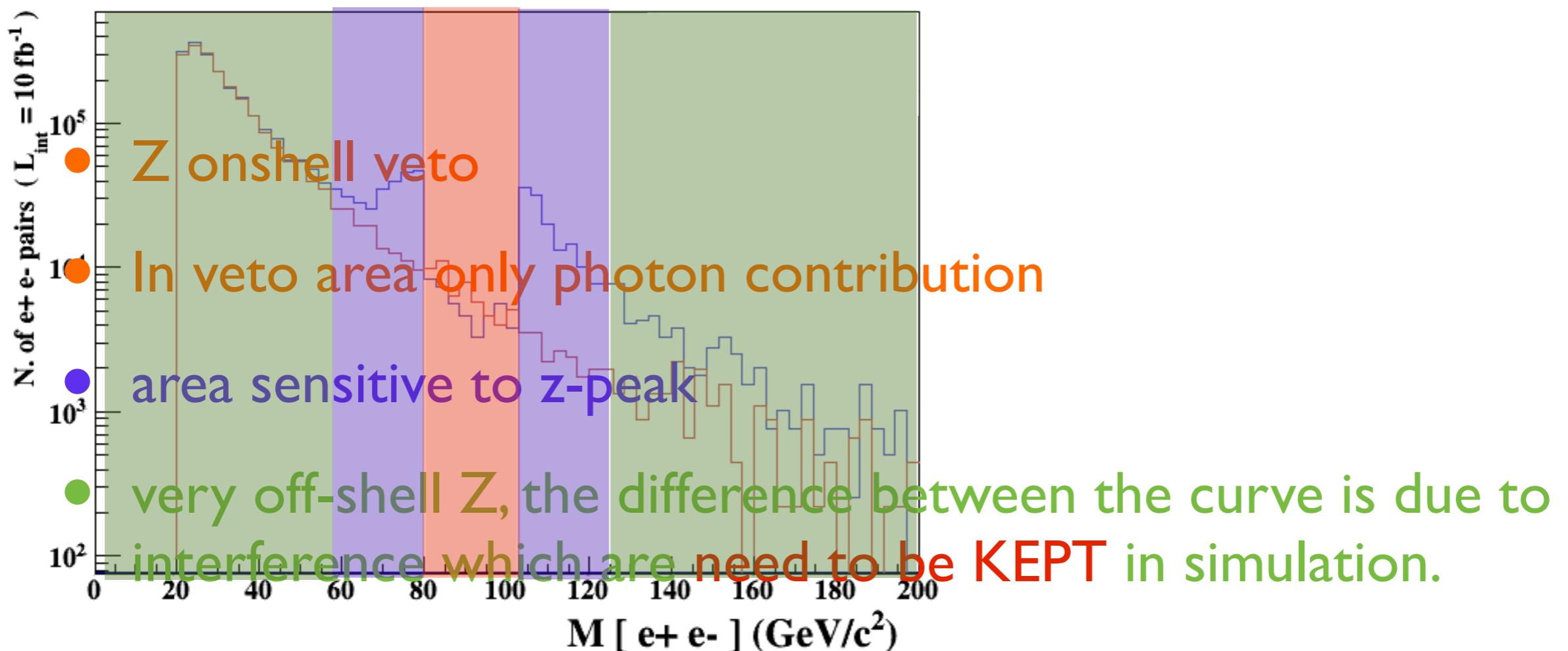
5 times width area

15 times width area

# See previous slide warning

$p p > e+ e- / Z$   
(red curve)

adding  $p p > e+ e- + Z$   
(blue curve)



5 times width area

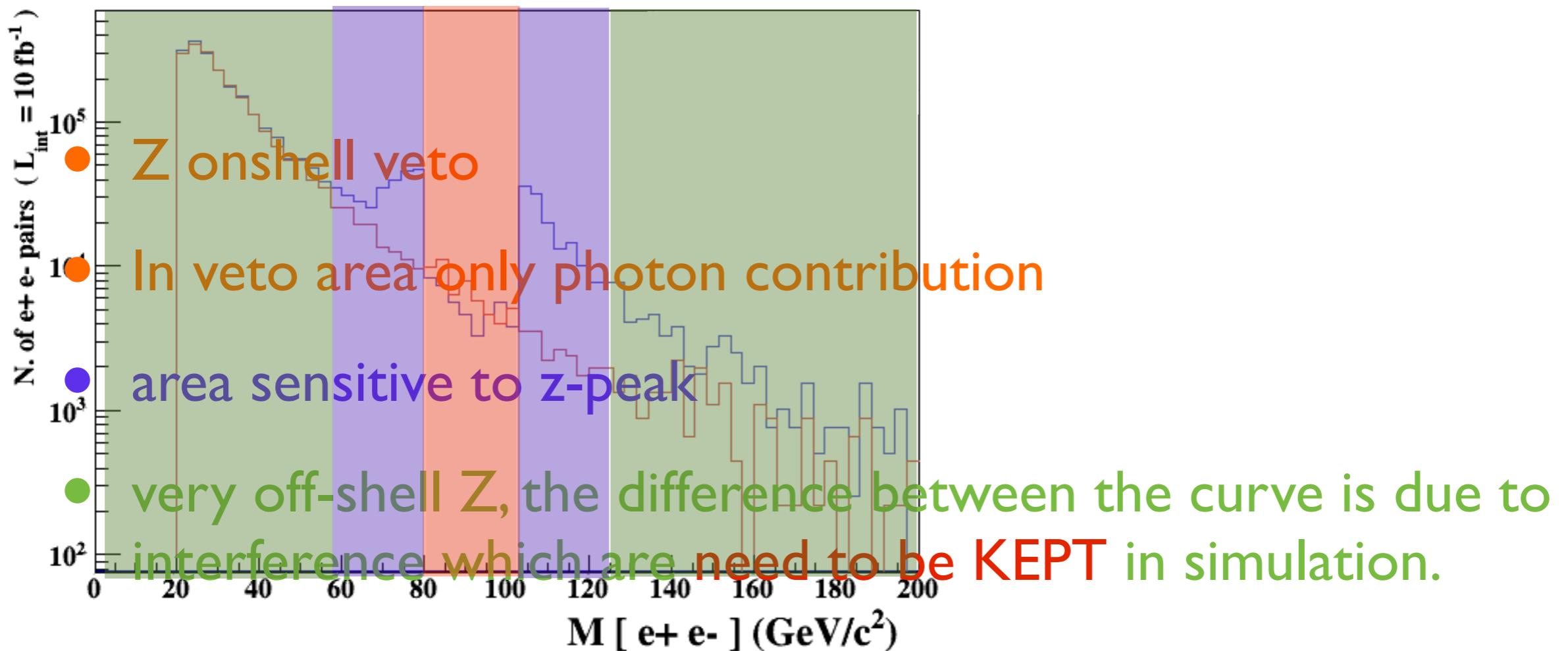
15 times width area

>15 times width area

See previous slide warning

$p p > e+ e- / Z$   
(red curve)

adding  $p p > e+ e- \$ Z$   
(blue curve)



5 times width area

15 times width area

>15 times width area

The “\$” can be used to split the sample in BG/SG area

- Syntax Like
  - $p\ p > z > e^+ e^-$  (ask one S-channel  $z$ )
  - $p\ p > e^+ e^- / z$  (forbids any  $z$ )
  - $p\ p > e^+ e^- \$\$ z$  (forbids any  $z$  in s-channel)
- ARE NOT GAUGE INVARIANT !
- forgets diagram interference.
- can provides un-physical distributions.

- Syntax Like
  - $p\ p > z > e^+ e^-$  (ask one S-channel z)
  - $p\ p > e^+ e^- / z$  (forbids any z)
  - $p\ p > e^+ e^- \$\$ z$  (forbids any z in s-channel)

- ARE NOT GAUGE INVARIANT !
- forgets diagram interference.

Avoid Those as much as possible!  
Those can provides un-physical distributions.

- Syntax Like
  - $p\ p > z > e^+ e^-$  (ask one S-channel  $z$ )
  - $p\ p > e^+ e^- / z$  (forbids any  $z$ )
  - $p\ p > e^+ e^- \$\$ z$  (forbids any  $z$  in s-channel)

- ARE NOT GAUGE INVARIANT !
- forgets diagram interference.

Avoid Those as much as possible!  
can provides un-physical distributions.

check physical meaning and gauge/Lorentz invariance if you do.

- Syntax like
  - $p\ p > z, z > e^+ e^-$  (on-shell z decaying)
  - $p\ p > e^+ e^- \$ z$  (forbids s-channel z to be on-shell)
- Are linked to cut  $|M^* - M| < BW_{cut} * \Gamma$
- Are more safer to use
- Prefer those syntax to the previous slides one

# Exercise V: Automation

---

- Look at the cross-section for the previous process for 3 different mass points.
  - **hint:** you can edit the param\_card/run\_card via the “set” command [After the launch]
  - **hint:** All command [including answer to question] can be put in a file.

# Exercise V: Automation

```
import model sm
generate p p > t t~
output
launch
set mt 160
set wt Auto
done
launch
set mt 165
set wt Auto
launch
set mt 170
set wt Auto
launch
set mt 175
set wt Auto
launch
set mt 180
set wt Auto
launch
set mt 185
set wt Auto
```

- Run it by:
  - `./bin/mg5 PATH`
  - (smarter than `./bin/mg5 < PATH`)
  - If an answer to a question is not present: Default is taken automatically

# Exercise VI: Decay

## MadSpin

- MadSpin Card  
`generate p p > t t~ h`

→ `decay t > w+ b, w+ > e+ ve`  
→ `decay t~ > w- b~, w- > e- ve~`

2m18.214s  
0.004707

## MadGraph

- `generate p p > t t~ h, (t > w+ b, w+ > e+ ve), (t~ > w- b~, w- > e- ve~), h > b b~`

9m30.806s  
0.003014

Different here because of cut (not cut should be applied since 2.3.0)