

Physics & Simulation

@

e^+e^- Colliders

Shao-Feng Ge

(gesf02@gmail.com)

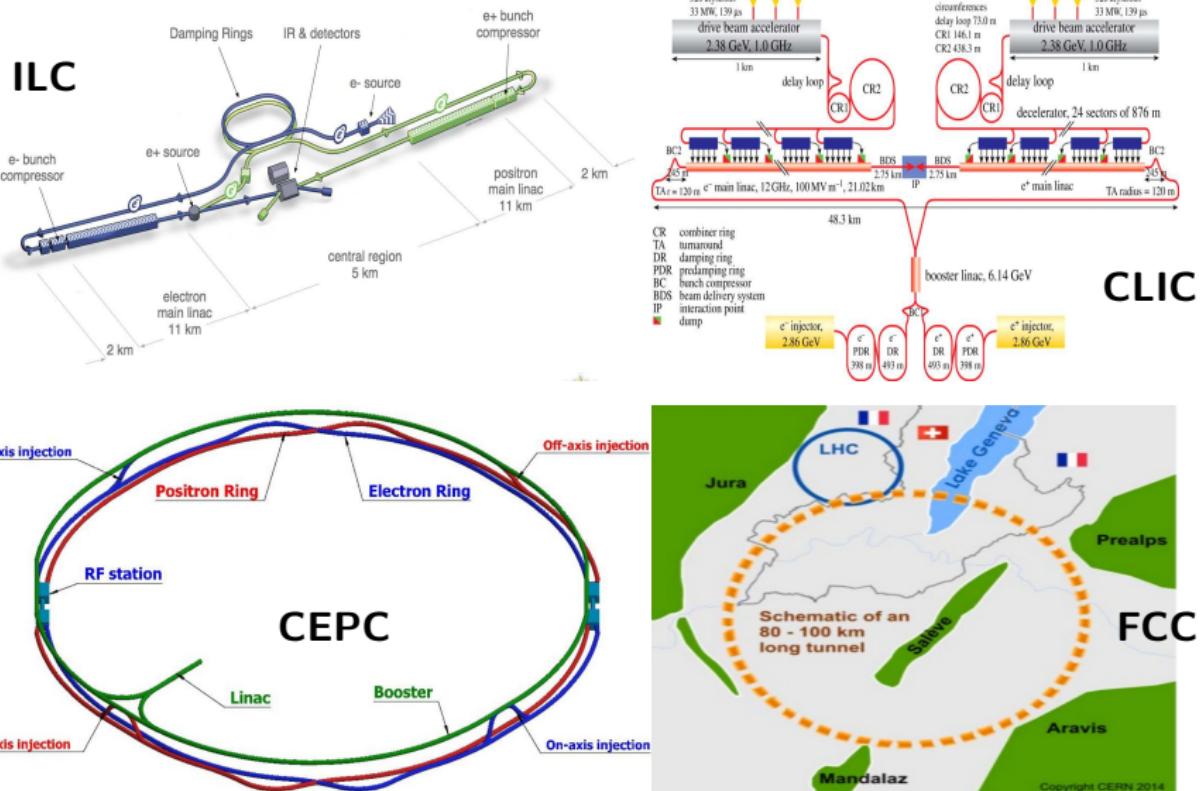
Kavli IPMU (WPI), UTIAS, The University of Tokyo, Japan
Department of Physics, University of California, Berkeley, USA

2018-11-22

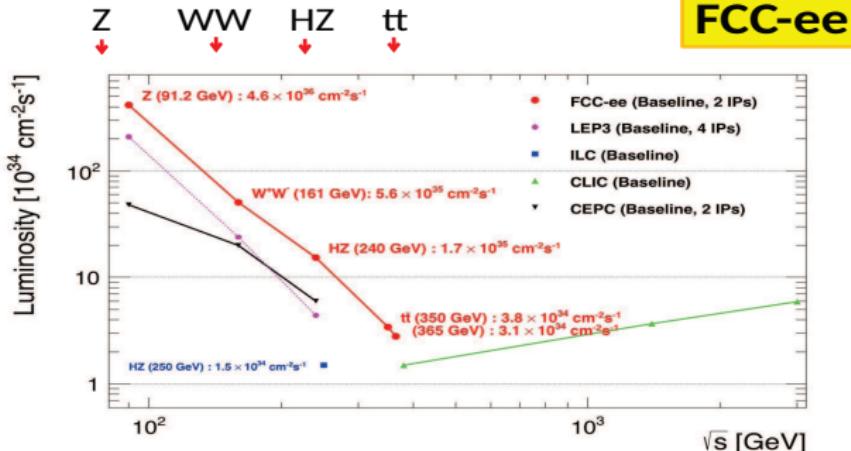
Outline

- **Basic Physics @ e^+e^- Colliders (CEPC, FCC-ee, ILC)**
 - Higgs Factory @ 240~250GeV
 - WW threshold scan $\sim 161\text{GeV}$
 - Z-Pole run @ 91GeV
 - ...
- **Parton Shower**
 - A black-box to even frequent users
 - Is it possible to make it transparent?
 - Analogy with particle decay
 - QED Parton Shower
- **Next Lecture**
 - More Features of Lepton Colliders
 - Precision measurement
 - Polarization
 - **Polarized Parton Shower: Mad-ee**
 - Spin
 - Azimuthal angle

Linear & Circular Lepton Colliders



Comparison of Different Lepton Colliders



Event statistics :

Z peak	$E_{cm} : 91 \text{ GeV}$	$5 \cdot 10^{12} e^+e^- \rightarrow Z$	LEP $\times 10^5$
WW threshold	$E_{cm} : 161 \text{ GeV}$	$10^8 e^+e^- \rightarrow WW$	LEP $\times 2 \cdot 10^3$
ZH threshold	$E_{cm} : 240 \text{ GeV}$	$10^6 e^+e^- \rightarrow ZH$	Never done
-tt threshold	$E_{cm} : 350 \text{ GeV}$	$10^6 e^+e^- \rightarrow tt$	Never done

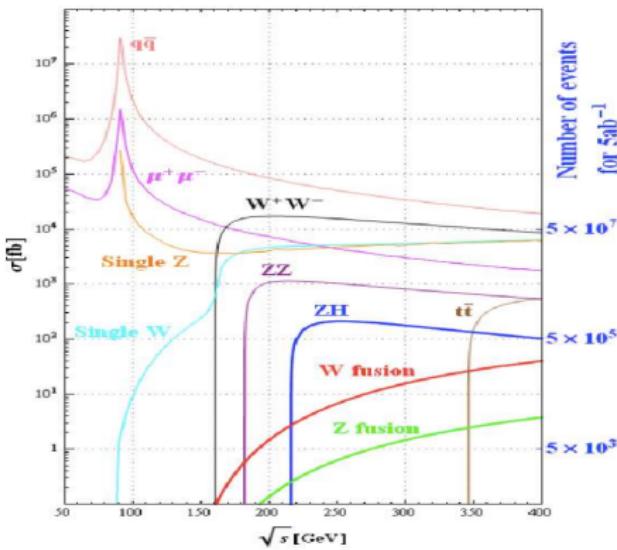
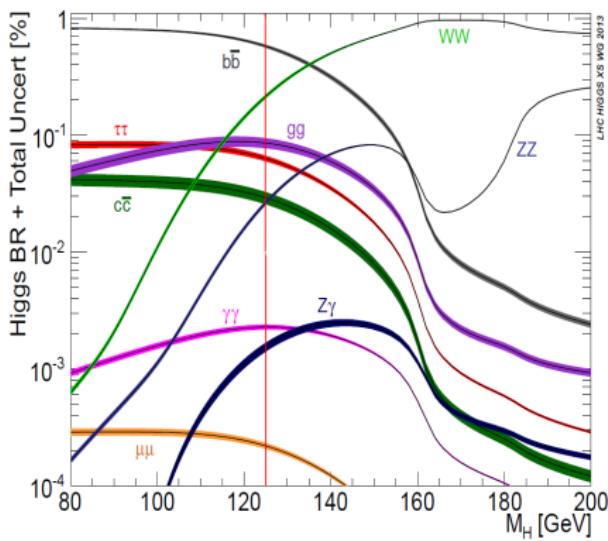
E_{CM} errors:

100 keV
300 keV
1 MeV
2 MeV

Great energy range for the heavy particles of the Standard Model.

Higgs Factory @ 250 GeV

- LHC tells us: $h(125)$ is **SM-like** → Dream Case for Experiments!
- ILC250 & CEPC produces $h(125)$ via $e^+e^- \rightarrow Zh, \nu\bar{\nu}h, e^+e^-h$
- Indirect Probe to New Physics. 5/ ab with 2 detectors in 10y → 10^6 Higgs → Relative Error $\sim 10^{-3}$.



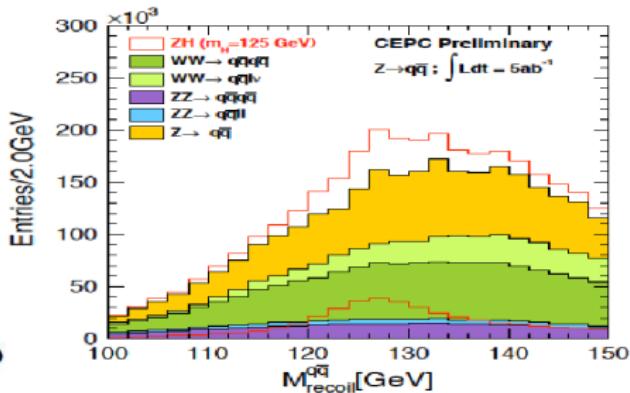
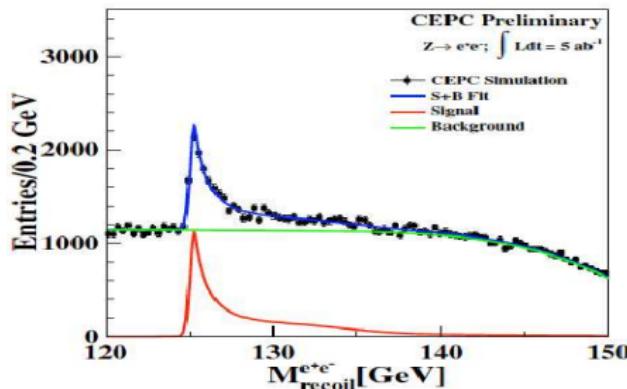
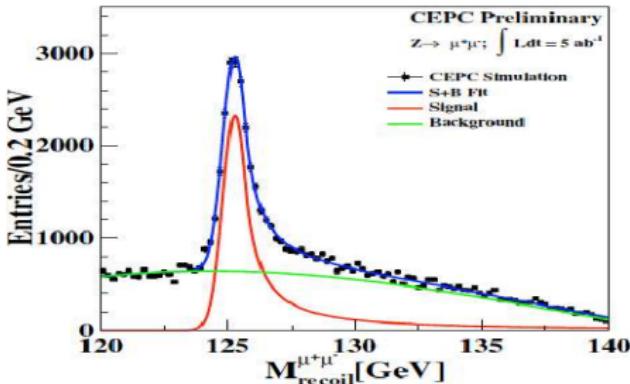
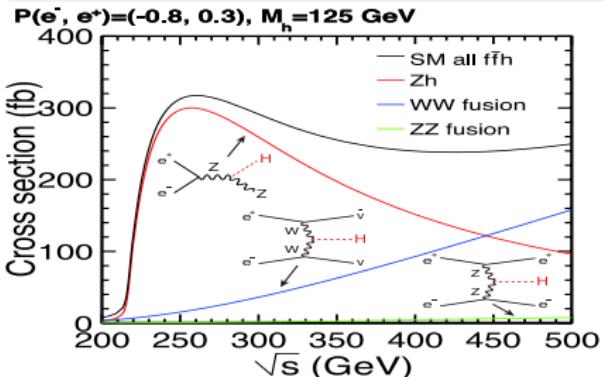
Mo, Li, Ruan & Lou, Chin.Phys.C 2015

Higgs discovery is not just about H particle

- Force Mediators
 - Gauge Forces – Spin-**1** Gauge Bosons
 - Gravity – Spin-**2** Graviton (Planck Scale?)
 - New Force – Spin-**0** Higgs Boson
- Deep understanding of Mass Generation
 - Yukawa Forces – Hierarchy & Mixing (Flavor Symmetries?)
 - Discrete v.s. Continuous
 - Full v.s. Residual [[1001.0940](#), [1104.0602](#), [1108.0964](#), [1308.6522](#)]
 - hWW , hZZ , $h\gamma\gamma$ & $hZ\gamma$
 - Higgs Self-Interaction Forces – h^3 & h^4 (concerns spontaneous EWSB and providing masses to all particles).
True Self-Interactions – Exactly the Same Quantum # (Spin & Charge)
- These new forces associated with spin-0 Higgs were **Never Seen Before**. Needs to test directly.
- Even within SM, we are strongly motivated to quantitatively test Higgs Couplings!

Recoil Mass

$$M_{\text{recoil}}^2 = (\sqrt{s} - E_{\text{ff}})^2 - p_{\text{ff}}^2$$



Precision Measurement of Higgs Decay

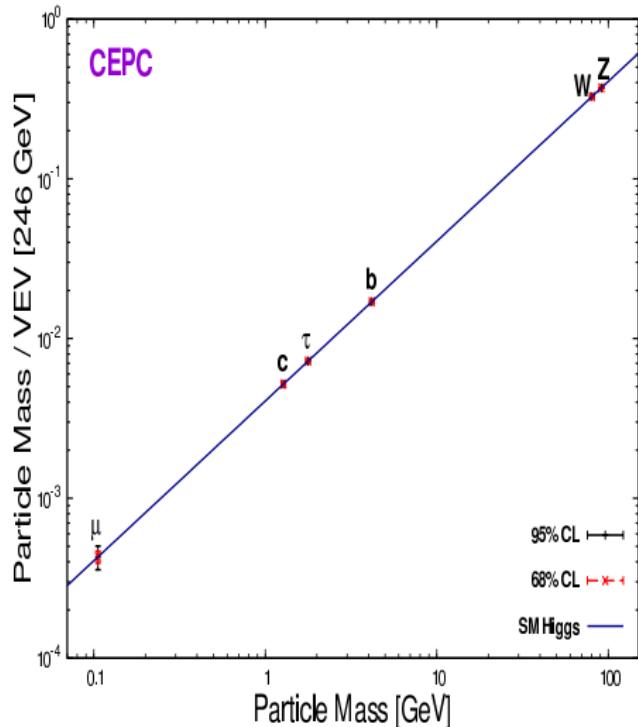
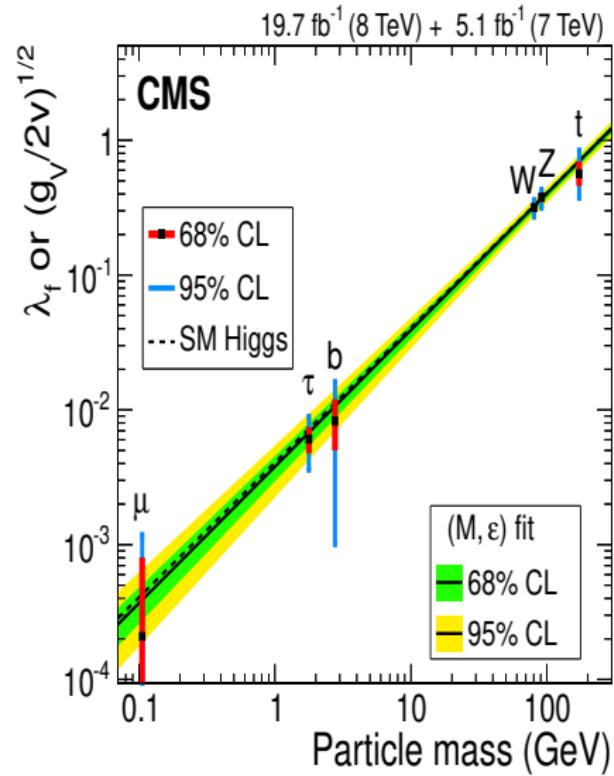
ΔM_h	Γ_h	$\sigma(Zh)$	$\sigma(\nu\bar{\nu}h) \times \text{Br}(h \rightarrow bb)$
5.0 MeV	2.6%	0.5%	2.8%
Decay Mode		$\sigma(Zh) \times \text{Br}$	Br
$h \rightarrow bb$		0.21%	0.54%
$h \rightarrow cc$		2.5%	2.5%
$h \rightarrow gg$		1.3%	1.4%
$h \rightarrow \tau\tau$		1.0%	1.1%
$h \rightarrow WW$		1.1%	1.2%
$h \rightarrow ZZ$		4.3%	4.3%
$h \rightarrow \gamma\gamma$		9.0%	9.0%
$h \rightarrow \mu\mu$		17%	17%
$h \rightarrow \text{invisible}$		–	0.14%

latest 1σ uncertainty
KITPC WS, July 28, 2016

SM Predictions

$\text{Br}(b\bar{b})$	$\text{Br}(c\bar{c})$	$\text{Br}(gg)$	$\text{Br}(\tau\bar{\tau})$	$\text{Br}(WW)$	$\text{Br}(ZZ)$	$\text{Br}(\gamma\gamma)$	$\text{Br}(\mu\bar{\mu})$	$\text{Br}(\text{inv})$
58.1%	2.10%	7.40%	6.64%	22.5%	2.77%	0.243%	0.023%	0

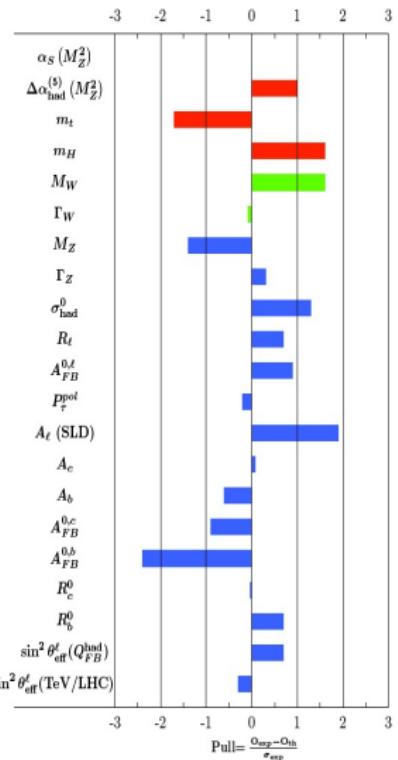
Precision on Higgs Couplings



Current Electroweak Precision Observables

input parameters

	Measurement	Posterior	Prediction	Pull
$\alpha_s(M_Z)$	0.1180 ± 0.0010	0.11800 ± 0.00094	0.1180 ± 0.0029	0.00
$\Delta\alpha_{\text{had}}^{(5)}(M_Z)$	0.027611 ± 0.000111	0.027576 ± 0.000106	0.02720 ± 0.00038	1.0
$M_Z [\text{GeV}]$	91.1875 ± 0.0021	91.1882 ± 0.0020	91.2005 ± 0.0091	-1.4
$m_t [\text{GeV}]$	172.8 ± 0.7	173.18 ± 0.66	176.27 ± 1.97	-1.7
$m_H [\text{GeV}]$	125.13 ± 0.17	125.13 ± 0.17	96.78 ± 18.23	1.6
$M_W [\text{GeV}]$	80.379 ± 0.012	80.3621 ± 0.0057	80.3570 ± 0.0065	1.6
$\Gamma_W [\text{GeV}]$	2.085 ± 0.042	2.08854 ± 0.00059	2.08855 ± 0.00059	-0.08
$\Gamma_Z [\text{GeV}]$	2.4952 ± 0.0023	2.49458 ± 0.00065	2.49446 ± 0.00069	0.3
$\sigma_h^0 [\text{nb}]$	41.540 ± 0.037	41.4924 ± 0.0077	41.4915 ± 0.0080	1.3
R_b^0	20.767 ± 0.025	20.7495 ± 0.0081	20.7482 ± 0.0086	0.7
$A_{FB}^{b,\ell}$	0.0171 ± 0.0010	0.01623 ± 0.00010	0.01622 ± 0.00010	0.9
$P_\tau^{\text{pol}} = \mathcal{A}_\ell$	0.1465 ± 0.0033	0.14710 ± 0.00046	0.14712 ± 0.00047	-0.2
$\mathcal{A}_\ell (\text{SLD})$	0.1513 ± 0.0021	0.14710 ± 0.00046	0.14714 ± 0.00049	1.9
\mathcal{A}_c	0.670 ± 0.027	0.66793 ± 0.00023	0.66793 ± 0.00023	0.08
\mathcal{A}_b	0.923 ± 0.020	0.934753 ± 0.000041	0.934754 ± 0.000041	-0.6
$A_{FB}^{0,c}$	0.0707 ± 0.0035	0.07369 ± 0.00024	0.07371 ± 0.00026	-0.9
$A_{FB}^{0,b}$	0.0992 ± 0.0016	0.10313 ± 0.00032	0.10315 ± 0.00034	-2.4
R_c^0	0.1721 ± 0.0030	0.172210 ± 0.000054	0.172211 ± 0.000054	-0.04
R_b^0	0.21629 ± 0.00066	0.21586 ± 0.00010	0.21585 ± 0.00010	0.7
$\sin^2 \theta_{\text{eff}}^{\text{lept}}(Q_{FB}^{\text{had}})$	0.2324 ± 0.0012	0.231512 ± 0.000059	0.231509 ± 0.000059	0.7
$\sin^2 \theta_{\text{eff}}^{\text{lept}}(\text{Tev/LHC})$	0.23143 ± 0.00027	0.231512 ± 0.000059	0.231516 ± 0.000060	-0.3



Satoshi Mishima @ CEPC workshop 2018

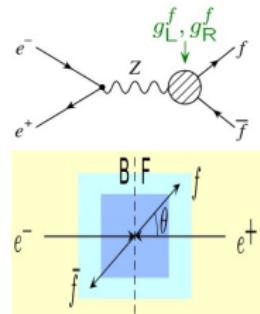
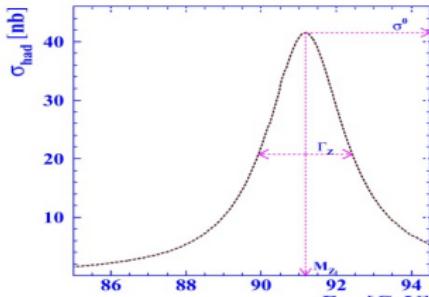
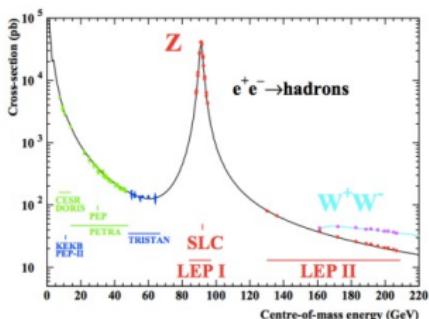
Improving Electroweak Precision Observables

	Current exp.	ILC/GigaZ	CEPC	FCC-ee
M_W [MeV]	15	3–4	1	1
M_Z [MeV]	2.3	–	0.5	0.1
Γ_Z [MeV]	2.3	0.8	0.5	0.1
$R_\ell = \Gamma_Z^{\text{had}} / \Gamma_Z^\ell [10^{-3}]$	25	10	2	1
$R_b = \Gamma_Z^b / \Gamma_Z^{\text{had}} [10^{-5}]$	66	14	4.3	6
$\sin^2 \theta_{\text{eff}}^\ell [10^{-5}]$	16	1.3	<1	0.5

→ Existing theoretical calculations adequate for LEP/SLC/LHC,
but not ILC/CEPC/FCC-ee!

A. Freitas @ CEPC workshop 2018

Z-Pole Measurements



$e^+e^- \rightarrow f\bar{f}$ for $E_{\text{CM}} \sim M_Z$:

- Mass M_Z
- Width $\Gamma_Z = \sum_f \Gamma_{ff}$
- Branching ratio $R_f = \Gamma_{ff}/\Gamma_Z$
- $\sigma^0 \approx \frac{12\pi \Gamma_{ee} \Gamma_{ff}}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} = \frac{12\pi}{M_Z^2} R_e R_f$

$$\Gamma_{ff} = C[(g_L^f)^2 + (g_R^f)^2]$$

of ν 's

$$A_{\text{FB}} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

$$\mathcal{A}_f = \frac{2(1 - 4\sin^2 \theta_{\text{eff}}^f)}{1 + (1 - 4\sin^2 \theta_{\text{eff}}^f)^2}$$

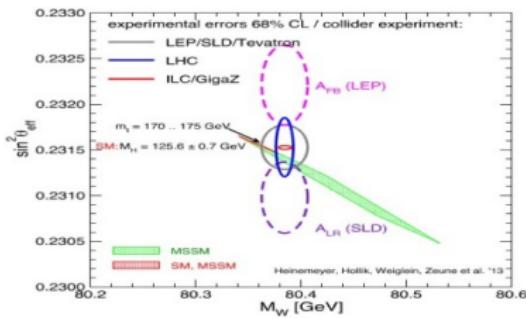
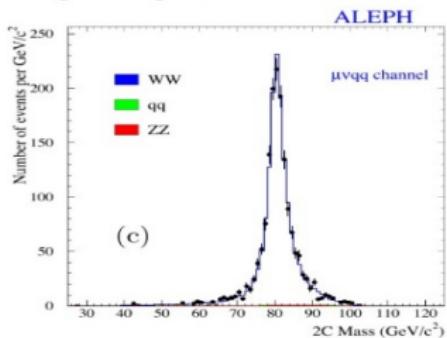
$$\sin^2 \theta_{\text{eff}}^f = \frac{g_R^f}{2|Q_f|(g_R^f - g_L^f)}$$

Freitas & Zhijun Liang @ CEPC workshop 2018

WW Threshold Scan

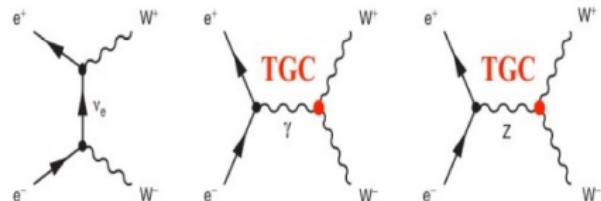
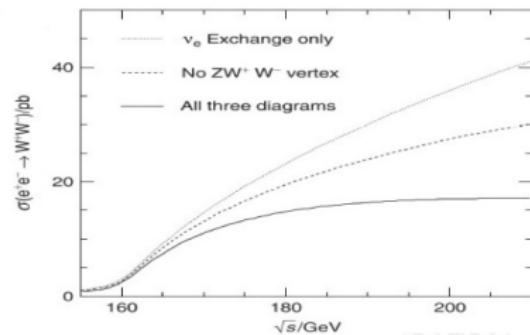
Direct Measurement

- Performed in ZH runs (240 GeV)
- Expected precision 2~3 MeV at CEPC



WW Threshold Scan

- WW threshold runs (157~172 GeV)
- Expected precision 1 MeV at CEPC



Pei-Zhu Lai @ CEPC workshop 2018

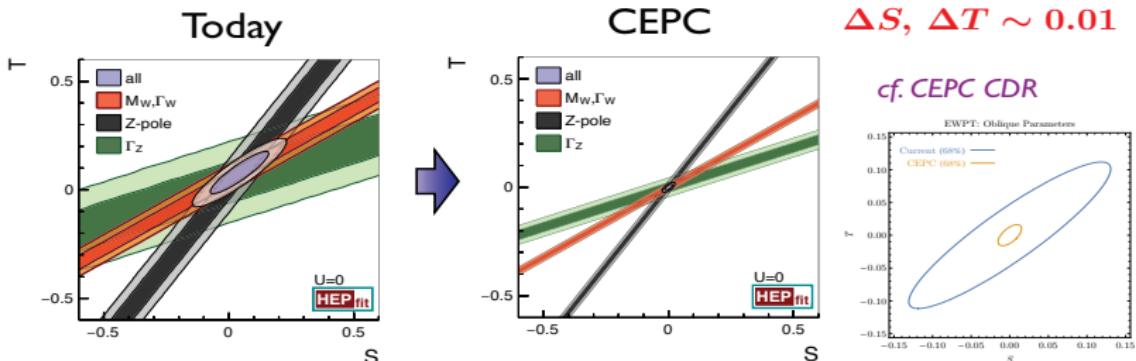
Oblique Parameters @ CEPC

Oblique parameters

preliminary

- EWPO depend on three combinations:

$$\begin{aligned}\delta M_W, \delta \Gamma_W &\propto -\mathbf{S} + 2c_W^2 \mathbf{T} + \frac{(c_W^2 - s_W^2) \mathbf{U}}{2s_W^2} \\ \delta \Gamma_Z &\propto -10(3 - 8s_W^2) \mathbf{S} + (63 - 126s_W^2 - 40s_W^4) \mathbf{T} \\ \text{others} &\propto \mathbf{S} - 4c_W^2 s_W^2 \mathbf{T}\end{aligned}$$

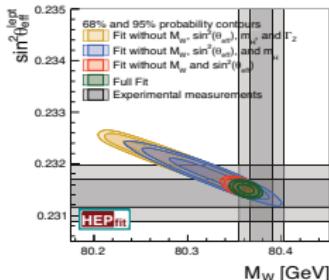


Z-Pole Precision Measurement @ CEPC

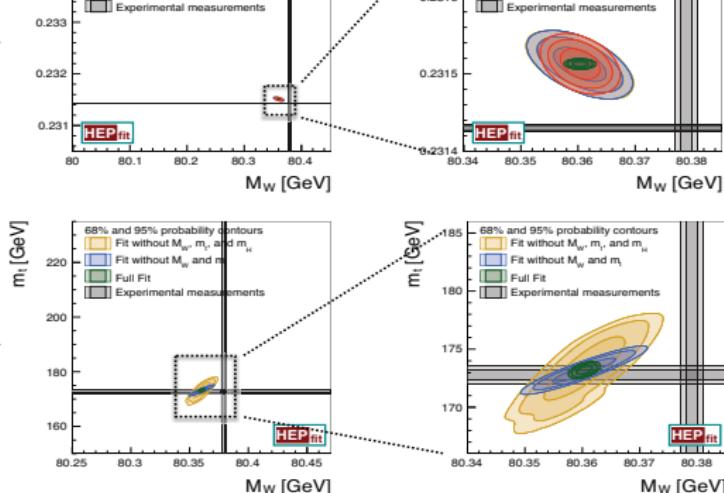
CEPC sensitivity

preliminary

Today



CEPC



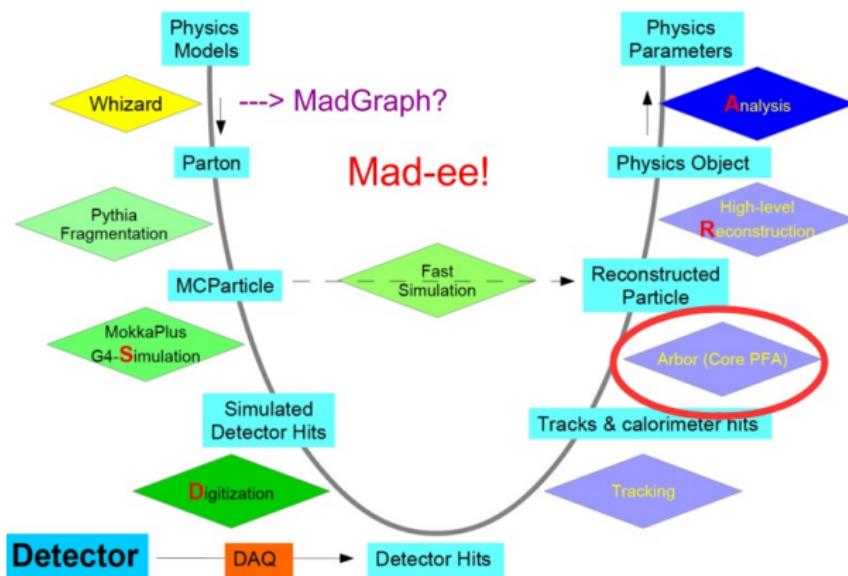
Rich Physics @ Lepton Colliders

- CEPC, FCC-ee, ILC
 - Z-Pole
 - WW Threshold scan
 - Higgsstrahlung
 - QCD & Flavor physics
- FCC-ee (ILC also?)
 - $t\bar{t}$ threshold scan
- CLIC (ILC also?)
 - Higgs self-coupling
 - New particle production
- 30TeV Lepton Collider

Indirect Probe of New Physics / Interpretation of Precision Data

See the EFT talk by Gauthier Durieux

The Full Simulation Flow



CEPC-SIMU-2017-001,
CEPC-SIMU-2017-002,
(DocDB id-167, 168, 173)

11/11/18

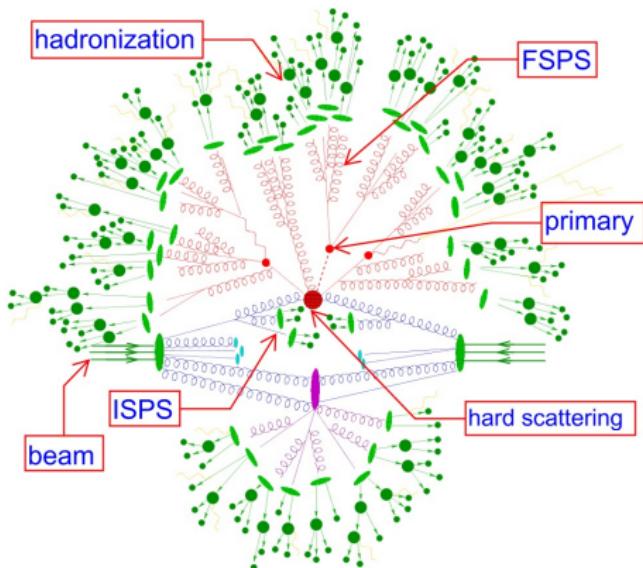
Generators (Whizard & Pythia)
Data format & management (LCIO & Marlin)
Simulation (MokkaC)
Digitizations
Tracking
PFA (Arbor)
Single Particle Physics Objects Finder (LICH)
Composed object finder (Coral)
Tau finder
Jet Clustering (FastJet)
Jet Flavor Tagging (LCFIPPlus)
Event Display (Druid)
General Analysis Framework (FSClasser)
Fast Simulation (Delphes + FSClasser)



4

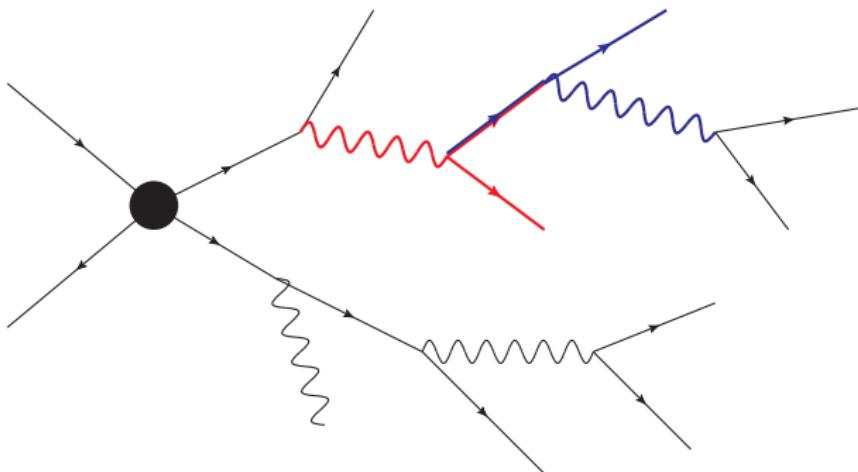
Manqi Ruan @ CEPC workshop 2018

Why we need Parton Shower?



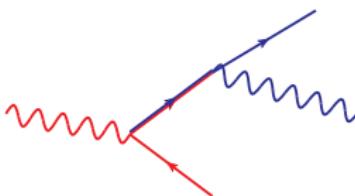
- Too complicated to calculate/simulate.
- Divide the simulation into several steps.
- The parton shower part has a large number of vertices.
- We need a model for parton shower!

The Basic Ingredients of Parton Shower



- **Parton shower = a sequence of $1 \rightarrow 2$ splittings**
- **Basic Ingredients**
 - $1 \rightarrow 2$ splitting (analogy with particles)
 - Grouping of $1 \rightarrow 2$ splitting (analogy with interactions)

Factorization: Cutting the Fermion Line

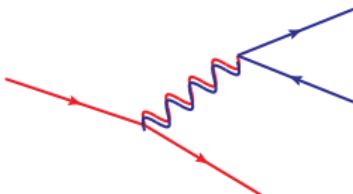


$$\mathcal{M}_{\gamma \rightarrow f\bar{f}\gamma} = \bar{u} \not{v} \frac{1}{q^2} \not{v} = \bar{u} \not{v} \frac{q^2}{q^2} \not{v} = \bar{u} \not{v} u \frac{1}{q^2} \bar{u} \not{v}, \quad d\Phi_3 = \not{d\Phi}_2' \frac{dq^2}{2\pi} \not{d\Phi}_2$$

$$|\mathcal{M}_{\gamma \rightarrow f\bar{f}\gamma}|^2 d\Phi_3 = |\mathcal{M}_{\gamma \rightarrow f'\bar{f}}|^2 \not{d\Phi}_2' \frac{dq^2}{(q^2)^2} |\mathcal{M}_{f' \rightarrow f\gamma}|^2 \not{d\Phi}_2$$

- The chain factorizes if the intermediate fermion is on-shell!
- For **on-shell fermion**, $\not{u}\bar{u} = q + m$ ($m \ll q$)
- Good assumption due to **pole structure** in the fermion propagator.

Factorization: Cutting the Photon Line



$$\mathcal{M}_{f \rightarrow f\bar{f}} = \bar{\mathbf{u}} \gamma^\mu \mathbf{u} \frac{g_{\mu\nu}}{q^2} \bar{\mathbf{u}} \gamma^\nu \mathbf{v} = \bar{\mathbf{u}} \gamma^\mu \mathbf{u} \frac{\epsilon_\mu \epsilon_\nu^*}{q^2} \bar{\mathbf{u}} \gamma^\nu \mathbf{v} = \bar{\mathbf{u}} \not{\epsilon} \mathbf{u} \frac{1}{q^2} \bar{\mathbf{u}} \not{\epsilon} \mathbf{v}$$

$$|\mathcal{M}_{f \rightarrow f\bar{f}}|^2 d\Phi_3 = |\mathcal{M}_{f \rightarrow f\gamma'}|^2 d\Phi'_2 \frac{dq^2}{2\pi(q^2)^2} |\mathcal{M}_{\gamma' \rightarrow f\bar{f}}|^2 d\Phi_2$$

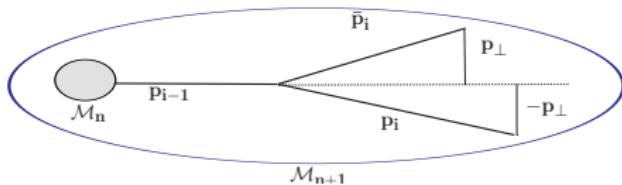
Home Work 1

- The chain factorizes if the intermediate photon is on-shell!
- For on-shell photon, $\epsilon_\mu \epsilon_\nu^* = g_{\mu\nu}$
- Good assumption due to pole structure in the photon propagator.

Phase Space & Variables

- Sudakov Basis: $p_i \equiv \alpha_i \hat{n}_+ + \beta_i \hat{n}_- + p_{\perp i}$ with $n_\perp = (1, 0, 0, \pm 1)$

$$\frac{d^4 p_i}{(2\pi)^4} (2\pi) \delta(p_i^2) = \frac{d\alpha_i d^2 \mathbf{p}_{\perp i}}{2\alpha_i (2\pi)^3}, \quad d\Phi_2^{(i)} = \frac{d\alpha_i}{4\pi\alpha_{i-1}} \frac{d\phi_i}{2\pi} = \frac{dz_i}{4\pi} \frac{d\phi_i}{2\pi}$$



$$d\sigma_{n+1} = \textcolor{red}{d\sigma_n} \frac{dq^2}{2\pi(q^2)^2} |\mathcal{M}_{1 \rightarrow 2}| \textcolor{blue}{d\Phi_2} = \textcolor{red}{d\sigma_n} \frac{dt}{t} \textcolor{teal}{P_{ij}}(z, \phi) \textcolor{teal}{dz} \frac{\textcolor{blue}{d\phi}}{2\pi}$$

where $z_i \equiv \frac{\alpha_i}{\alpha_{i-1}}$, $\textcolor{violet}{t} \propto \mathbf{q}^2 \propto \mathbf{p}_{\perp i}^2 / z(1-z)$ & $\textcolor{teal}{P_{ij}} \equiv \frac{|\mathcal{M}_{i \rightarrow j}|^2}{8\pi^2(\mathbf{q}^2)}$

- The chance for one more splitting $d\sigma_{n+1}$ is proportional to the chance $\textcolor{red}{d\sigma_n}$ times the **splitting kernel**.

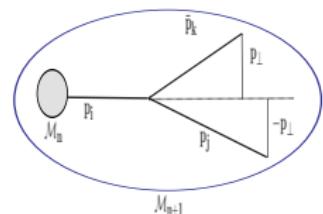
Home Work 2

Parton Shower vs Particle Decay

- For $i \rightarrow jk$ splitting

$$\frac{dN_i}{N_i} = -\frac{d\sigma_{n+1}}{d\sigma_n} = -\frac{dt}{t} P_{ij}(z, \phi) dz \frac{d\phi}{2\pi}$$

is the **branching probability**.



- Particle decay

$$\frac{dN}{N} = -\Gamma dt, \quad \frac{dN}{N} = -dt \frac{d^2\Gamma}{dEd\phi} dE \frac{d\phi}{2\pi}$$

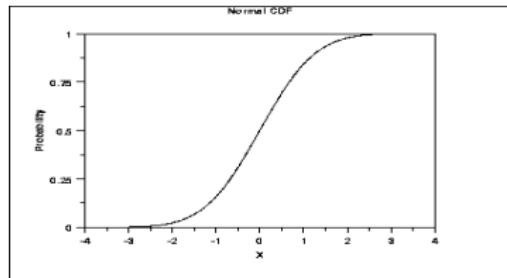
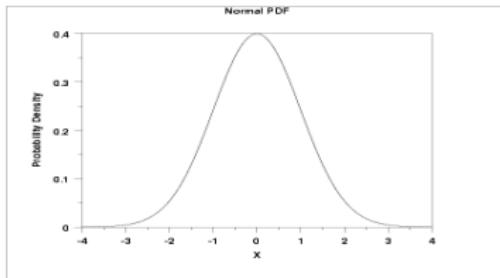
- Parton Shower vs Particle Decay

$$\frac{dt}{t} \leftrightarrow dt$$

$$P_{ij}(z, \phi) \leftrightarrow \frac{d^2\Gamma}{dEd\phi}$$
$$dz \leftrightarrow dE$$

Sampling Probability Distribution Function (PDF)

- 1D PDF $f(x)$



$$\text{Hit-and-loss } \frac{f(x)}{f_{\max}} \in [0, 1]$$

$$CDF(x) = \int_{x_{\min}}^x f(x') dx' \in [0, 1]$$

- 2D PDF \rightarrow 1D PDF

$$f(x, y) \rightarrow f(x) = \int f(x, y) dy \quad \text{and} \quad f(x_i, y)$$

- Multi-Dimensional PDF

Home Work 3

Markov Chain MC of Decays

- Multiple decays + Each decay has **multiple variables** (t_i, E_i, ϕ_i);
- First decide at what time t_i the next splitting happens

$$\frac{dN}{N} = -\Gamma dt \quad \Rightarrow \quad \text{PDF}(\mathbf{t}) \equiv \frac{N(t)}{N_0} = e^{-\Gamma t} \in (0, 1]$$

- Once t_i determined, sampling E according to 1D PDF

$$\text{PDF}(\mathbf{E})|_{t=t_i} \equiv \frac{d\Gamma(t_i)}{dE} = \int \frac{d^2\Gamma}{dEd\phi} \frac{d\phi}{2\pi}.$$

- Once (t_i, E_i) determined, sampling ϕ according to 1D PDF

$$\text{PDF}(\phi) \equiv \left. \frac{d^2\Gamma}{dEd\phi} \right|_{t=t_i, E=E_i}$$

Markov Chain MC of Parton Shower

- Multiple decays + Each decay has multiple variables ($\mathbf{t}_i, \mathbf{z}_i, \phi_i$);
- First decide at what time t_i the next splitting happens

$$\frac{dN_i}{N_i} = -\frac{dt}{t} \int \mathbf{P}_{ij}(\mathbf{z}, \phi) d\mathbf{z} \quad \Rightarrow \quad \Delta_i(\mathbf{t}) = e^{-\int \frac{dt}{t} P_{ij}(z) dz} \in (0, 1]$$

- Once t_i determined, sampling z according to 1D PDF

$$\mathbf{PDF}(\mathbf{z})|_{t=t_i} \equiv P_{ij}(z)$$

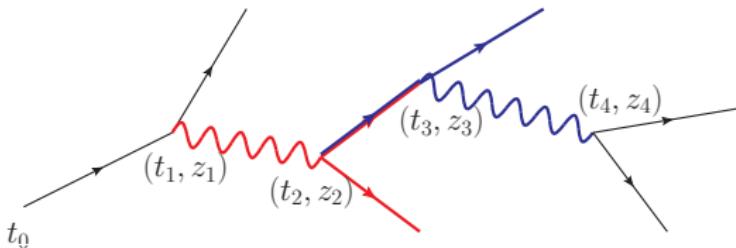
- If there are several species

$$\Delta_i(\mathbf{t}) = e^{-\int \frac{dt}{t} \sum_j P_{ij}(z) dz}$$

$$\Gamma_{ij} = \int P_{ij}(z) dz$$

$$\mathbf{PDF}(\mathbf{z})|_{t=t_i} \equiv P_{ij}(z)$$

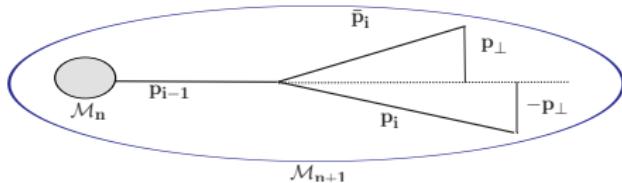
Energy Momentum Reconstruction



- Starting from \mathbf{t}_0 , we have sampled a series of $(\mathbf{t}_i, \mathbf{z}_i)$.
- Already know $\alpha_i = z_i \alpha_{i-1} = \prod_{j=0}^i z_j \alpha_0$
- Unknowns: Virtuality (p_i^2) & Transverse Momentum $\mathbf{p}_{\perp i}^2$
- Momentum not fully known!

$$p_i = \alpha_i n_+ + \beta_i n_- + p_{\perp i} \quad \text{with} \quad \beta_i = \frac{p_i^2 + \mathbf{p}_{\perp i}^2}{4\alpha_i}$$

Virtuality Reconstruction & Evolution Scale



- **Momentums**

$$p_i = \alpha_i n_+ + \beta_i n_- + p_{\perp i} \quad \text{with} \quad \beta_i = \frac{\mathbf{p}_i^2 + \mathbf{p}_{\perp i}^2}{4\alpha_i}$$

- **Momentum conservation** $\beta_{i-1} = \beta_i + \bar{\beta}_i$

$$\mathbf{p}_{i-1}^2 = \frac{\mathbf{p}_i^2}{z_i} + \frac{\bar{\mathbf{p}}_i^2}{1-z_i} + \frac{\mathbf{p}_{\perp i}^2}{z_i(1-z_i)}$$

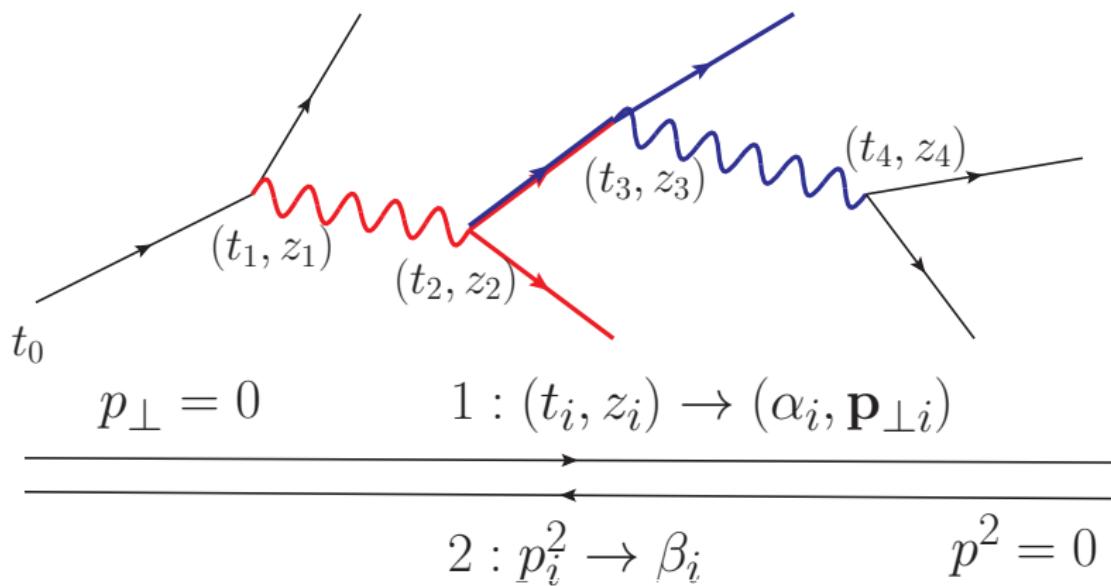
- **Evolution Variable** ($\mathbf{p}_{\perp i}$ is relative transverse momentum)

$$t_i \propto p_{i-1}^2 \propto \frac{\mathbf{p}_{\perp i}^2}{z_i(1-z_i)} \quad \Rightarrow \quad \mathbf{p}_{\perp i}^2 = t_i z_i \bar{z}_i$$

with $\mathbf{p}_i^2 = \bar{\mathbf{p}}_i^2 = 0$.

Home Work 4

Flow Chart



Multiple Partons following PDF $f_i(t, x)$

Splitting Kernels

- The sampling is determined by the Sudakov Factor

$$\Delta_i(t) = e^{-\int \frac{dt}{t} \sum_j P_{ij}(z) dz}$$

- Splitting Kernels $P_{ij} \equiv \frac{|\mathcal{M}_{i \rightarrow jk}|^2}{8\pi^2 q^2} = \frac{\alpha}{2\pi} \frac{1}{2q^2} \sum_{h_i, h_j, h_k} |\mathcal{M}_{i \rightarrow jk}|^2$
- QED has 3 kernels

$$P_{ee}(z) = \frac{\alpha}{2\pi} \frac{1+z^2}{1-z},$$

$$P_{e\gamma}(z) = \frac{\alpha}{2\pi} \frac{1+(1-z)^2}{z},$$

$$P_{\gamma e}(z) = \frac{\alpha}{2\pi} [z^2 + (1-z)^2].$$

- Divergences in P_{ee} & $P_{e\gamma}$.

Home Work 5

Phase Space Boundaries to remove Divergences

- Evolution Scale

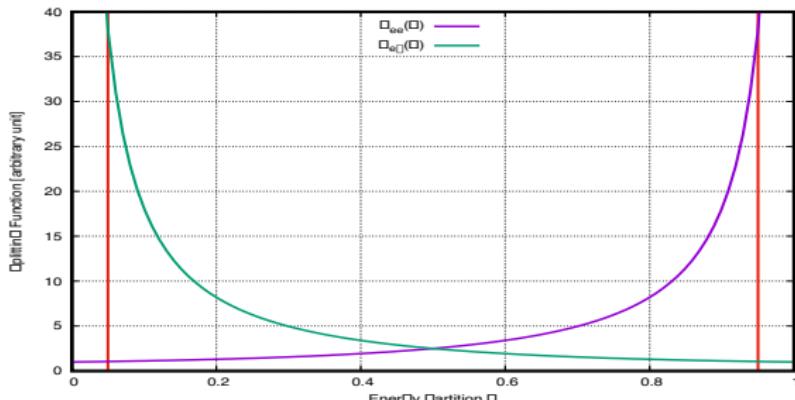
$$\textcolor{red}{t_0} > t_i > \textcolor{red}{t}_{\min}$$

- p_\perp Cut

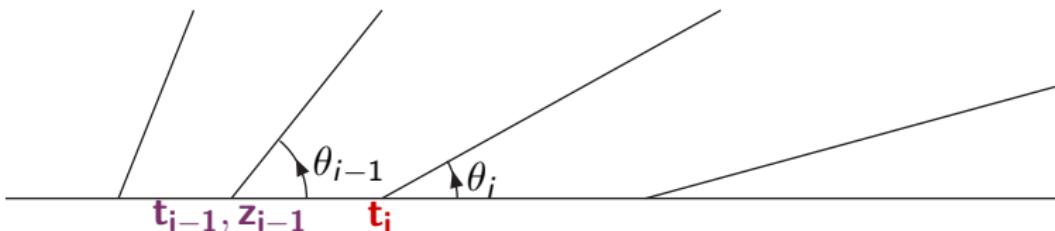
$$p_\perp^2 = z(1-z)t > \Lambda^2 \quad \Rightarrow \quad \textcolor{red}{z_-} \leq z \leq \textcolor{red}{z_+}$$

with

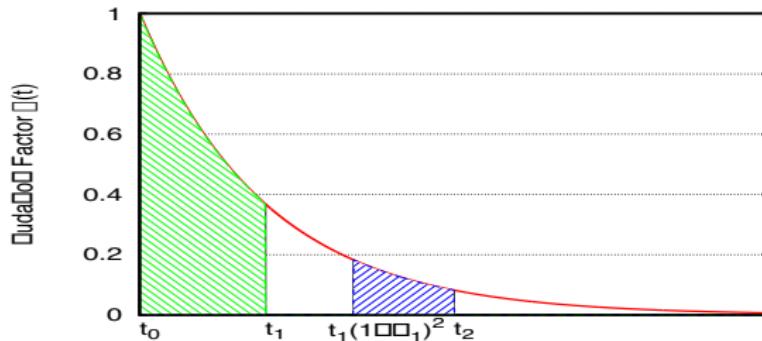
$$z_\pm = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{\Lambda^2}{t}}$$



Coherence & Angular Ordering



$$\theta_i \approx \frac{t_i}{2\alpha_i^2} = \frac{t_i}{2z_i^2\alpha_{i-1}^2}, \quad \theta_i < \theta_{i-1} \quad \Rightarrow \quad t_i < (1 - z_{i-1})^2 t_{i-1}.$$



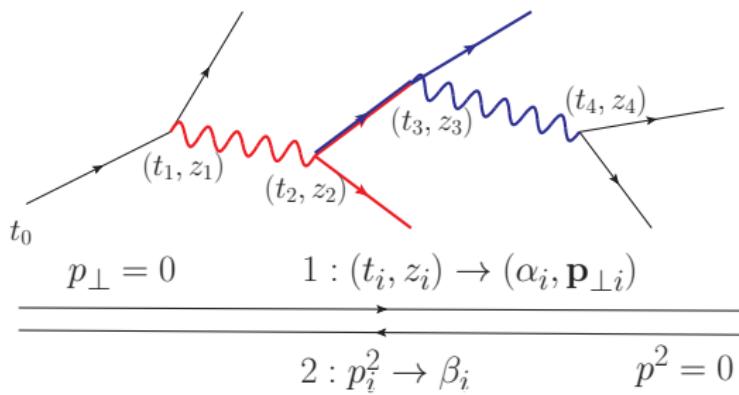
see also Peter Skands' talk

Final-State Parton Shower (FSPS)

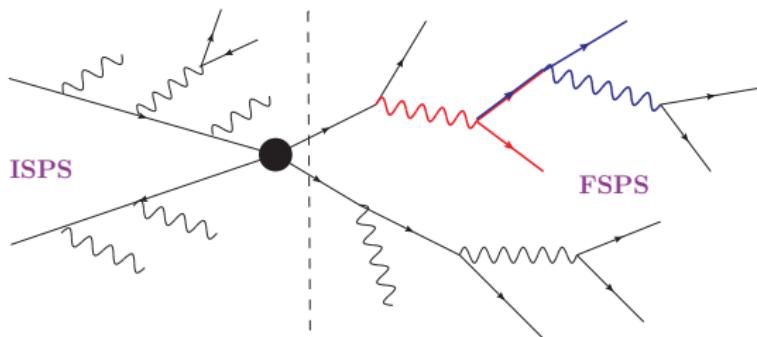
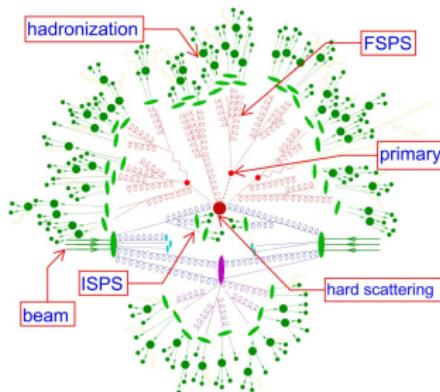
- Sudakov Factor

$$\Delta_i(t) = \exp \left[- \int_{t_0}^t \frac{dt'}{t'} \int_{z_{min}(t')}^{z_{max}(t')} dz \sum_j \mathbf{P}_{ij}(t', z) \right].$$

- Energy Momentum Construction



How about Initial-State Parton Shower?

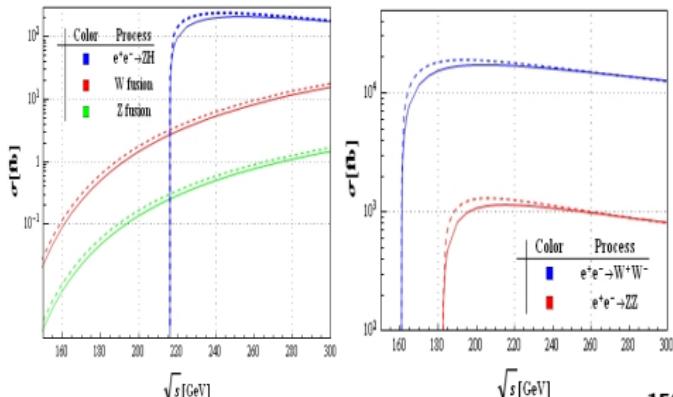


Importance of ISPS

1702.04827, Potter

\sqrt{s} [GeV]	Pol.	Process	σ [pb]	CEPC	σ [pb]	MG5
250	none	$e^+e^- \rightarrow \mu^+\mu^-, \tau^+\tau^-$	4.40	3.50		
250	none	$e^+e^- \rightarrow q\bar{q}$	50.2	11.3		
250	none	$e^+e^- \rightarrow ZZ$	1.03	1.10		
250	none	$e^+e^- \rightarrow WW$	15.4	16.5		
250	none	$e^+e^- \rightarrow Zh$	0.212	0.240		

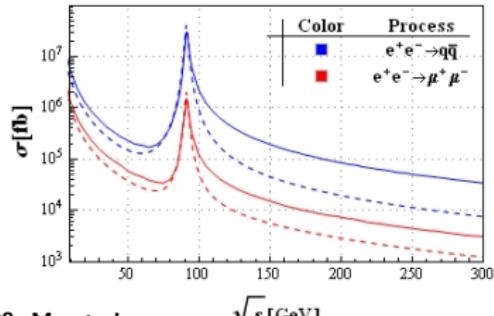
\sqrt{s} [GeV]	Pol.	Process	σ [pb]	ILC	σ [pb]	MG5
250	(+, -)	$e^+e^- \rightarrow Zh$	0.319	0.356		
250	(-, +)	$e^+e^- \rightarrow Zh$	0.206	0.240		
350	(+, -)	$e^+e^- \rightarrow t\bar{t}$	0.286	0.378		
350	(-, +)	$e^+e^- \rightarrow t\bar{t}$	0.137	0.166		
500	(+, -)	$e^+e^- \rightarrow t\bar{t}$	1.08	0.921		
500	(-, +)	$e^+e^- \rightarrow t\bar{t}$	0.470	0.436		



6 Conclusion

We have described the production of fast simulation background samples for new physics studies at a future e^+e^- collider like the ILC or CEPC. Events are generated for a variety of run scenarios with approximately five times the integrated luminosity envisaged by the most optimistic run scenario for each \sqrt{s} . The events are generated with MG5_aMC@NLO with detector simulation performed by Delphes using the DSid detector card. Finally, the samples are compared to the ILC background samples made for the DBD study and CEPC background samples.

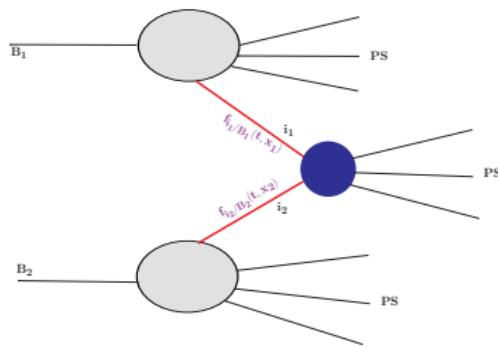
Systematic uncertainties associated with the MG5_aMC@NLO samples have been estimated. These samples lack a detailed simulation of initial state radiation and beamstrahlung. The $2f$ background from radiative return events is absent, and both pileup from bunch-bunch interactions and a re-



1505.01008, Mo et al

Parton Probability Distribution

- The result of parton shower is PDF $f_{i/B}(t, x)$

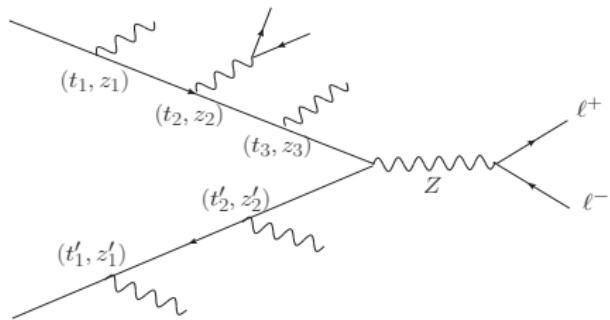


- This PDF is exactly the same as the one attached to beams!

$$\sigma_{B_1 B_2 \rightarrow X}(s) = \sum_{i_1 i_2} \int f_{i_1/B_1}(Q^2, x_1) f_{i_2/B_2}(Q^2, x_2) \sigma_{i_1 i_2 \rightarrow X} dx_1 dx_2$$

Difficulty in ISPS

$$\sigma_{B_1 B_2 \rightarrow X}(s) = \sum_{i_1 i_2} \int f_{i_1/B_1}(Q^2, x_1) f_{i_2/B_2}(Q^2, x_2) \sigma_{i_1 i_2 \rightarrow X} dx_1 dx_2$$



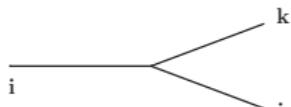
- Matching the center-of-mass energy $\hat{s} = x_1 x_2 s$.

$$x_1 = \prod_i z_i, \quad x_2 = \prod_i z'_i.$$

Very **inefficient** to shoot two random numbers from 2D plane to a single line $x_1 x_2 = \hat{s}/s$.

Backward Evolution for ISPS

- We start from the hard process with x_1 & x_2
- Cross section ratio for $i \rightarrow j$ evolution



$$\frac{dN_i}{N_i} = -\frac{d\sigma_{n+1}}{d\sigma_n} = -\frac{dt}{t} P_{ij}(z, \phi) dz \frac{d\phi}{2\pi}$$

- Event ($N_i = f_i \sigma_i$) ratio for backward $j \rightarrow i$ evolution

$$\frac{dN_j}{N_j} = -\frac{d\sigma_{n+1}}{d\sigma_n} \frac{f_i}{f_j} = -\frac{dt}{t} \frac{f_i(t, x/z)}{f_j(t, x)} P_{ij}(z, \phi) dz \frac{d\phi}{2\pi}$$

- Modified Sudakov Factor

$$\Pi_j(t, x) = \exp \left[- \int_{t_0}^t \frac{dt'}{t'} \int_{z_{min}(t')}^{z_{max}(t')} dz \sum_i \frac{f_i(t', x/z)}{f_j(t', x)} P_{ij}(t', z) \right].$$

Very intuitive picture

Formal Derivation (1)

- Both **flow-in** & **flow-out**

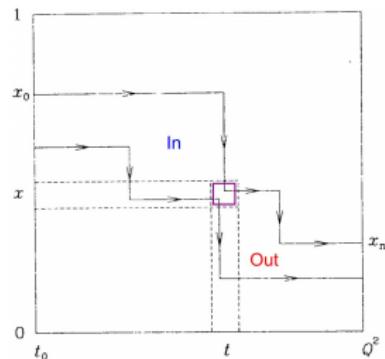
$$\delta f_i^{\text{in}}(t, x) = \frac{\delta t}{t} \int dx' \int_0^1 dz \sum_j P_{ij}(z) f_j(t, x') \delta(x' z - x)$$

$$\delta f_i^{\text{out}}(t, x) = \frac{\delta t}{t} \int dx' \int_0^1 dz \sum_j P_{ji}(z) f_i(t, x) \delta(x' - x z)$$

- Energy Fraction Constraint

$$\delta f_i^{\text{in}}(t, x) = \frac{\delta t}{t} \int_x^1 \frac{dz}{z} \sum_j P_{ij}(z) f_j\left(t, \frac{x}{z}\right),$$

$$\delta f_i^{\text{out}}(t, x) = \frac{\delta t}{t} f(t, x) \int_0^1 dz P(z).$$



- Combined

$$\delta f(t, x) = \delta f^{\text{in}}(t, x) - \delta f^{\text{out}}(t, x) = \frac{\delta t}{t} \int P(z) \left[\frac{1}{z} f\left(t, \frac{x}{z}\right) - f(t, x) \right] dz$$

Formal Derivation (2)

- The flow-out part as Sudakov Factor

$$t \frac{\partial f(t, x)}{\partial t} = \int_0^1 \frac{dz}{z} P(z) f\left(t, \frac{x}{z}\right) + f(t, x) \frac{t}{\Delta(t)} \frac{\partial \Delta(t)}{\partial t}$$

where $\Delta(t) \equiv e^{- \int \frac{dt}{t} P(z) dz}$ is the solution of $\frac{\partial \Delta}{\partial t} = -\frac{f}{t} \int P(z) dz$

- Then $\frac{f}{\Delta}$ follows similar equation

$$t \frac{\partial}{\partial t} \left(\frac{f}{\Delta} \right) = \frac{1}{\Delta} \int \frac{dz}{z} P(z) f\left(t, \frac{x}{z}\right).$$

- To be exactly the same, $\frac{f}{\Delta} \rightarrow \Pi(t, t'; x) \equiv \frac{f(t, x)\Delta(t')}{f(t', x)\Delta(t)}$

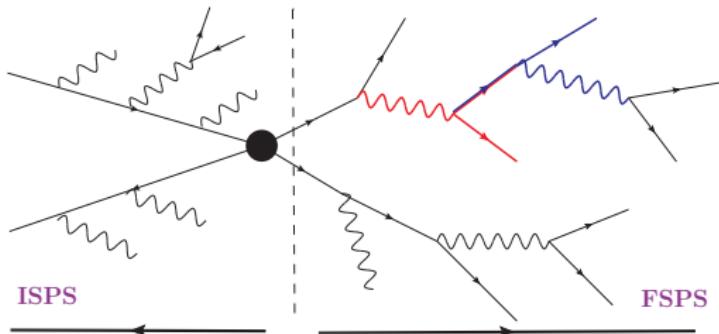
$$\frac{\partial \Pi(t, t'; x)}{\partial t} = -\frac{\Pi(t, t'; x)}{t} \int \frac{dz}{z} P(z) \frac{f(t, \frac{x}{z})}{f(t', x)}$$

where $\Pi(t, t_0; x) = e^{- \int_{t_0}^t P(z) \frac{f(t', x/z)}{f(t', x)} \frac{dz}{z} \frac{dt'}{t'}}$

Home Work 6

FSPS vs ISPS

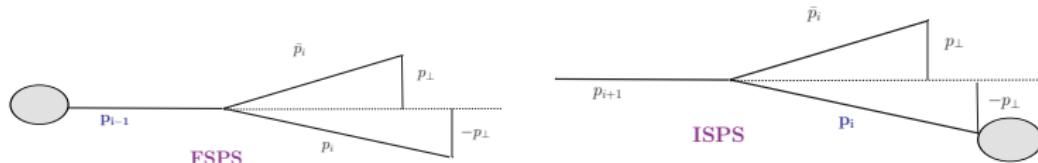
- Forward vs Backward



- $\Delta(t) \equiv e^{-\int \frac{dt}{t} P(z) dz}$ vs $\Pi(t, t_0; x) = e^{-\int_{t_0}^t P(z) \frac{f(t', x/z)}{f(t', x)} \frac{dz}{z} \frac{dt'}{t'}}$
 - **Weighted Splitting Kernels:** $P(z) \rightarrow P(z) \frac{f(t', x/z)}{f(t', x)}$
 - **Precalculated PDF as Input:** $f(t', x)$ & $f(t', x/z)$
 - **Extra Parameter:** x in $f(t', x)$ & $f(t', x/z)$
 - **Extra Pole Structure:** $dz \rightarrow \frac{dz}{z}$
- **Markov Chain Monte Carlo is the same**

Different Evolution Variables

- Same kinematics



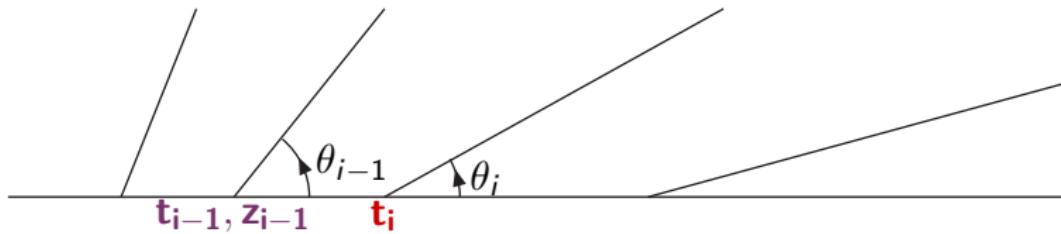
$$q_{i\mp 1}^2 = \frac{q_i^2}{z_i} + \frac{k_i^2}{1-z_i} + \frac{\mathbf{p}_{\perp i}^2}{z_i(1-z_i)}, \quad \mathbf{p}_{\perp i} = \mathbf{q}_{\perp i} - z_i \mathbf{q}_{\perp i\mp 1}$$

- Evolution Variable t vs Off-shellness

$$\text{FSPS} : t_i \equiv \frac{q_{i-1}^2}{z_i(1-z_i)} = \frac{\mathbf{p}_{\perp i}^2}{z_i^2(1-z_i)^2}$$

$$\text{ISPS} : t_i \equiv \frac{-q_i^2}{z_i(1-z_i)} = \frac{\mathbf{p}_{\perp i}^2}{z_i(1-z_i)^2}$$

Angular Ordering



- FSFS:

$$1 - \cos \theta_i = \frac{q_i \cdot k_i}{E_i E'_i} = \frac{1}{2} \frac{t_i}{E_{i-1}^2} \quad \Rightarrow \quad t_i < t_{i-1} z_{i-1}^2$$

- ISPS:

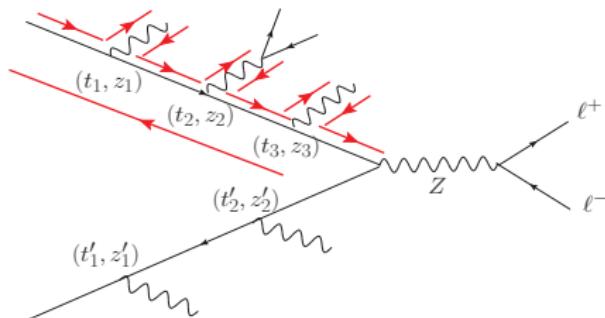
$$1 - \cos \theta_i \equiv \frac{q_i \cdot k_i}{E_i E'_i} = \frac{1}{2} \frac{t_i}{E_{i+1}^2} \quad \Rightarrow \quad t_i < \frac{t_{i-1}}{z_i^2}$$

Angular ordering is automatically implemented in ISPS.

Home Work 7

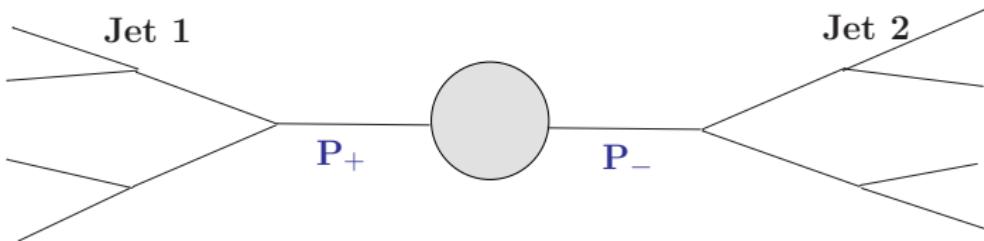
FSPS in ISPS

- The partner of ISPS can also experience further splitting



- After ISPS, each partner has a characteristic scale t_i
- Starting from t_i , we check whether the ISPS partners can experience FSPS splittings.
- The whole PS stops when no FSPS splitting is allowed.

Global Energy-Momentum Conservation (EMC)



- After PS, the primary parton develops into jets;
- Each jet has nonzero invariant mass \Leftrightarrow Primary parton becomes off-shell;
- The jet momentum \mathbf{P}_\pm is no longer matched with hard scattering;
- Take $e^+e^- \rightarrow \ell^+\ell^-$ for illustration

- Off-shell

$$P_\pm = \alpha_\pm^{(0)} \hat{n}_\pm \quad \rightarrow \quad P_\pm = \alpha_\pm \hat{n}_\pm + \frac{P_\pm^2}{4\alpha_\pm} \hat{n}_\mp$$

- Energy-Momentum Conservation

Home Work 8

$$P_+ + P_- = \frac{1}{2}\sqrt{s}(\hat{n}_+ + \hat{n}_-) \quad \Rightarrow \quad \alpha_\pm$$

Thank You!

Home Works

- ① Split Amplitudes & Phase Space
- ② Sudakov Basis & Variables
- ③ Sampling multi-dimension PDF
- ④ Kinematics of $1 \rightarrow 2$ splitting
- ⑤ Splitting kernels
- ⑥ Sudakov factors [FSPS & ISPS]
- ⑦ Angular ordering
- ⑧ Global energy-momentum conservation (EMC)