Outline

- Introduction to perturbative QCD:
  asymptotic freedom, IR-safety, factorization, S/W jets
- Jet physics
  algorithms, jet functions, factorization
- Jet substructure: hadrons inside a jet, jet mass, grooming
- Jets in HI collisions

Bibliography

more references later

pQCD & factorization

- G. Sterman TASI '95 hep-ph/9606312
- Dave Soper hep-ph/9702203
- Collins, Soper, Sterman hep-ph/0409313
- CTEQ cteq.org
- Richard Field: Applications of perturbative QCD
- Vogelsang / Stratmann

SCET

- C. Bauer / I. Stewart lecture notes & videos
- Becher, Broggio, Ferroglia: arXiv: 1410.1892

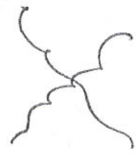
Jets & substructure

- Salam: PhD level lectures
- Larkoski, Moult, Nachman: arXiv 1709.04464

The QCD Lagrangian

$$\mathcal{L} = \bar\psi(i\partial^\mu \gamma_\mu - m)\psi - g_s\bar\psi\gamma^\mu T_a \psi A_\mu^a - \frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$$

- SU(3) non-abelian gauge group
  $\psi(x) \to e^{i\alpha_a(x) T_a}\psi(x)$, T_a : generators : $8 = N_c^2-1$ 3x3 matrices ; $[T_a, T_b] = i f_{abc} T_c$
- like QED $g_s T_a$  $g f_{abc}$  
- experimentally 10 % 3-jet events   LEP

→ later quark/gluon discrimination

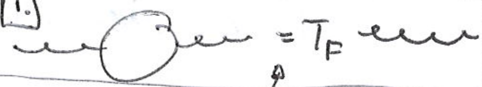
Color algebra ... $C_F, C_A, T_F N_f$:

$i,j = 1, N_c (=3)$
$a = 1, 8 (= N_c^2-1)$

$i \longrightarrow j = \delta_{ij}$  $= T^a_{ij}$

a ~~~~ b $= \delta_{ab}$  $= i f^{abc}$

e.g.  $= \mathrm{tr}[T^a] = 0$

$\boxed{1.}$  $= T_F$ ~~~~

$(\& N_f \ldots)$ $\mathrm{tr}[T^a T^b] = T_F\,\delta^{ab}$

$[T^a, T^b] = i f^{abc} T^c$

$(T^a)^\dagger = T^a$, $\mathrm{tr}[T^a] =$

hermitian & traceless

normalization $\mathrm{tr}[T^a T^b] = T_F\,\delta^{ab}$

with $T_F = 1/2$

i) show $i f^{abc} = 2\,\mathrm{tr}([T^a, T^b] T^c)$

$$[T^a, T^b] = i f^{abc} T^c \quad | \cdot T^d$$

$$[T^a, T^b] T^d = i f^{abc} T^c T^d \quad | \,\mathrm{tr}$$

$$\mathrm{tr}([T^a, T^b] T^d) = i f^{abc} \underbrace{\mathrm{tr}[T^c T^d]}_{= \delta^{cd}/2}$$

$$\Rightarrow 2\,\mathrm{tr}([T^a, T^b] T^d) = i f^{abd}$$ ✓ ⇒ f anti-sym. in all indices

ii) Fierz / completeness relation $\quad T^a_{il}\, T^a_{jk} = \frac{1}{2}\delta_{ik}\delta_{jl} - \frac{1}{2N}\delta_{il}\delta_{jk}$

use T^a, $\mathbb{1}$ as basis for hermitian $N\times N$ matrices:

Ansatz: $A_{ij} = c_0\,\delta_{ij} + \sum_{a=1}^{N^2-1} c_a T^a_{ij}$ ⊛ $\Rightarrow$ determine coefficients

⊛ $\cdot\,\delta_{ij} \Rightarrow \quad A_{ij}\delta_{ij} = A_{ii} = c_0\delta_{ii} + \sum_a c_a \underbrace{T^a_{ii}}_{=0} = c_0 N$

⊛ $\cdot\, T^b_{ji} \Rightarrow \quad A_{ij}T^b_{ji} = 0 + \sum_a c_a \underbrace{T^a_{ij}T^b_{ji}}_{=\mathrm{tr}[T^aT^b]=\delta^{ab}/2} = c_b/2 \quad \Rightarrow c_a = A_{ij}T^a_{ji}$

$\Rightarrow$ back to ⊛: $\quad A_{ij} = \frac{A_{ll}}{N}\delta_{ij} + 2A_{lm}T^b_{ml}T^b_{ij}$ ← [implicit sum over b]

now factor out A_{lm}: $\quad A_{lm}\left(\delta_{li}\delta_{jm} - \frac{1}{N}\delta_{ij}\delta_{lm} - 2T^b_{ml}T^b_{ij}\right) = 0$

↑ arbitr. $\Rightarrow (\ldots) = 0$ ✓

12.  $= C_F$ ——— $\qquad T^a_{ik}T^a_{kj} = C_F\delta_{ij}$ $\qquad$ | & is Casimir op. [...]

use Fierz: $T^a_{ik}T^a_{kj} = \frac{1}{2}\delta_{ij}\delta_{kk} - \frac{1}{2N}\delta_{ik}\delta_{kj} = \frac{1}{2}\delta_{ij}N - \frac{1}{2N}\delta_{ij}$

$= \delta_{ij}\left(\frac{N}{2} - \frac{1}{2N}\right) = \delta_{ij}\,\frac{N^2-1}{2N} \equiv \delta_{ij}C_F$ ✓

13.  $= C_A$ 〰〰 $\qquad f^{acd}f^{bcd} = C_A\delta^{ab}$

(sketch): $\quad = -4\,\mathrm{tr}(T^a[T^c,T^d])\,\mathrm{tr}(T^b[T^c,T^d])$

$= -8\left(\mathrm{tr}(T^aT^cT^d)\cdot\mathrm{tr}(T^bT^cT^d) - \mathrm{tr}(T^aT^cT^d)\,\mathrm{tr}(T^bT^dT^c)\right)$

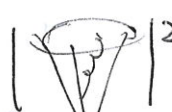
use Fierz to compute product of traces e.g. $(T^a_{ij}T^c_{jl}T^d_{li})(T^b_{\alpha\beta}T^c_{\beta\gamma}T^d_{\gamma\alpha})$

$= T^a_{ij}T^b_{\alpha\beta}\underbrace{T^c_{jl}T^c_{\beta\gamma}}_{\text{Fierz}}\underbrace{T^d_{li}T^d_{\gamma\alpha}}$

$= \ldots = -\frac{1}{2N}\mathrm{tr}(T^aT^b) = \frac{-1}{4N}\delta^{ab}$

find $\Rightarrow f^{abc}f^{bcd} = -8\left(-\frac{1}{4N}\delta^{ab} - \frac{1}{8}\left(N - \cancel{\frac{2}{N}}\right)\delta^{ab}\right) = N\delta^{ab} \equiv C_A\delta^{ab}$ ✓

later e[...]



→ know th[...] there is a[...] color factor[...] C_F relativ[...] to LO 

[...]