Qualitative Arguments

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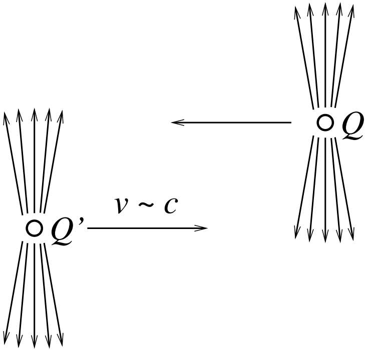
How to think about the initial state factorization – QED analogy



- Weizsäcker-Williams field – Highly contracted in the z direction
- Coulomb potential in the rest frame of the charge

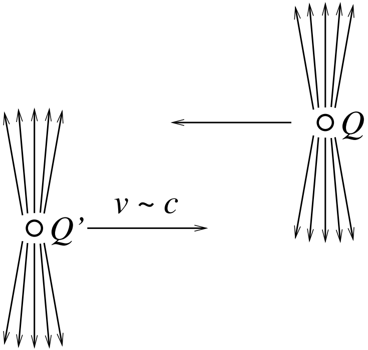
$$A^0 = Q/|\mathbf{r}|$$

- In the moving frame

$$A^\mu(x') = \Lambda^\mu_\nu A^\nu(x(x'))$$

- The coordinate in the moving frame $x' = (t, x, y, z)$. This corresponds to the rest frame position $x = (\gamma(t - zv), x, y, \gamma(z - tv))$.

How to think about the initial state factorization – QED analogy



- Weizsäcker-Williams field – Highly contracted in the z direction
- Coulomb potential in the rest frame of the charge

$$A^0 = Q/|\mathbf{r}|$$

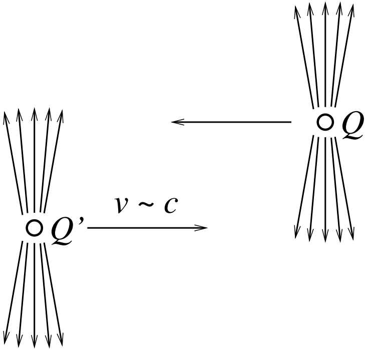
- In the moving frame

$$A^\mu = \frac{Q(\gamma, 0, 0, \gamma v)}{\sqrt{(z - vt)^2\gamma^2 + \mathbf{x}_\perp^2}}$$

- Pure gauge in the $v \to 1$ limit

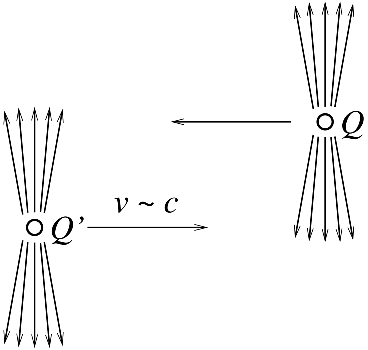
$$A^\mu \approx \frac{Q(1, 0, 0, 1)}{|z - vt|} = Q\partial_\mu \ln|z - vt|$$

How to think about the initial state factorization – QED analogy



- Weizsäcker-Williams field – Highly contracted in the z direction
  $F^{\mu\nu} \approx 0$ unless $z \approx vt$
- In the rest frame: Coulomb field is made up of space-like virtual photons $q^\mu q_\mu = -\mathbf{q}^2$ with $q_0 = 0$.
- In the Lab frame:
  $q'^\mu = (q^z \sinh\eta, \mathbf{q}_\perp, q^z \cosh\eta)$
- For large η,
  $|\Delta E| = |q^- - |\mathbf{q}|| \sim e^{-\eta}\mathbf{q}^2/q_z$
  $\Longrightarrow \Delta t \sim 1/|\Delta E| \sim e^\eta q_z/\mathbf{q}^2 \Longrightarrow$ virtual photons look almost like real photons.

How to think about the initial state factorization – QED analogy



- Weizsäcker-Williams field – Highly contracted in the z direction

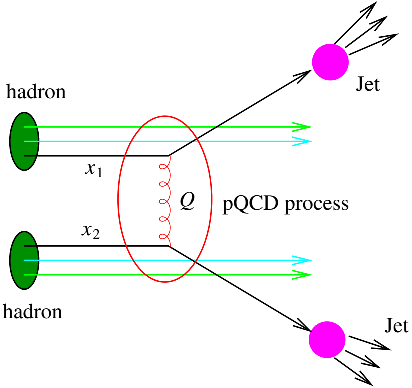
  $F^{\mu\nu} \approx 0$ unless $z \approx vt$

- To a first approximation, the approaching particles *do not* know about each other until they are on top of each other.

- Initial photon momentum distribution *factorizes*: $F(x_1, x_2) = f(x_1)f(x_2)$ but this is not exact.

- In QCD, color neutrality of hadrons help.

Factorization Theorem



Hadron-Hadron Jet production scheme:

$$\sigma = \int_{abcd} f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \sigma_{ab\to cd} D_{C/c}(z_C, Q)$$

Factorization Theorem

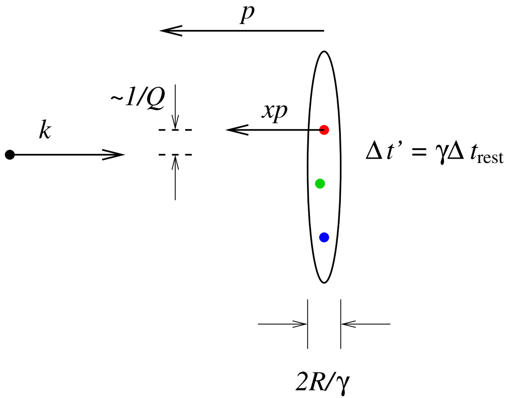
- How realistic pQCD calculations are done

$$\sigma_{hh'\to C+X} = \int_{abcd} dx_1 dx_2 f_{a/h}(x_1, Q_f) f_{b/h'}(x_2, Q_f) \sigma_{ab\to cd}(Q_R) D_{C/c}(z_C, Q_f')$$

- $f_{a/h}(x_1, Q_f)$: Parton distribution function. Probability to have a parton type a with the momentum fraction x_1 in a hadron h. Depends on the factorization scale Q_f.
- $\sigma_{ab\to cd}(Q_R)$: Parton-parton scattering cross-section.
- $D_{C/c}(z_C, Q_f')$: Fragmentation function. Probability to create a hadron type C our of parton type c carrying the momentum fraction z_c.

Rough Understanding of the Factorization Theorem

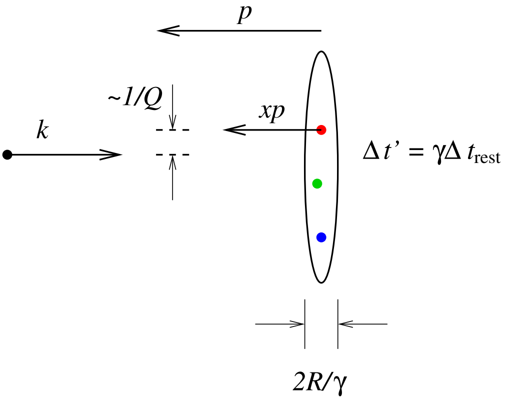
Example: Electron-Hadron collision



1. No interaction before the collision
2. Interaction vis exchaning a virtual photon with the virtuality Q
3. Hard scattering:
   Probability to find a *single* quark within $d \sim 1/Q$ with a momentum fraction x

Rough Understanding of the Factorization Theorem

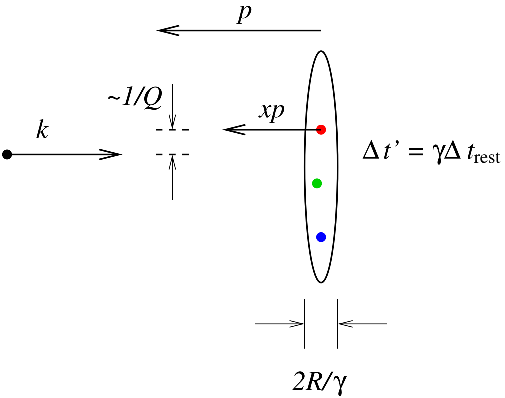
Example: Electron-Hadron collision



1. Probability to find two *quantum mechanically interfereing* quarks within d:
   $P(r < d) \sim \dfrac{d^2}{R^2} \sim \dfrac{1}{(QR)^2}$
   Small if $Q \gg 1/R$
   $\Longrightarrow$ Probability picture for single particle collisions valid $\Longrightarrow$ PDF

Rough Understanding of the Factorization Theorem

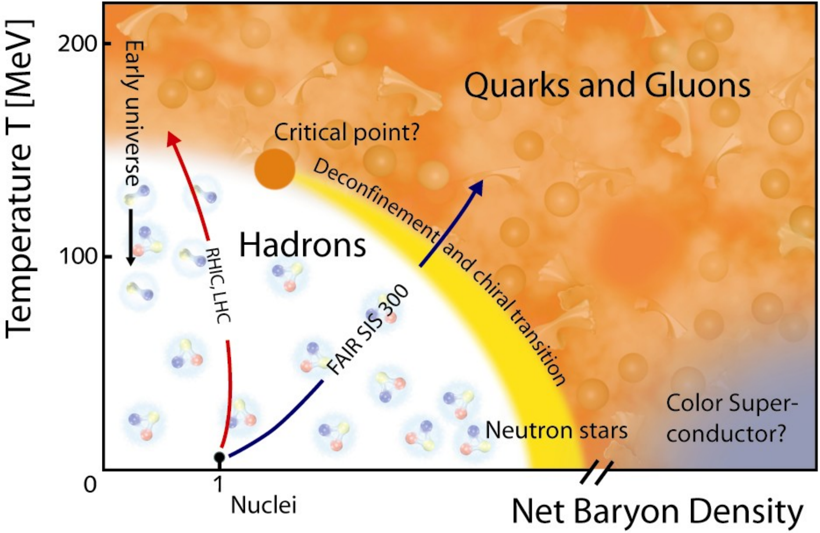
Example: Electron-Hadron collision



1. Scattering time scale: $\tau_{\text{crossing}} = R/\gamma$
2. Hadronization time scale: $\gamma/\Lambda_{\text{QCD}} \gg R/\gamma \implies$ No interference $\implies$ Proability ficture valid $\implies$ FF

QGP Properties

QCD Phase Diagram



Picture credit: GSI (www.gsi.de)

Nobelprize.org

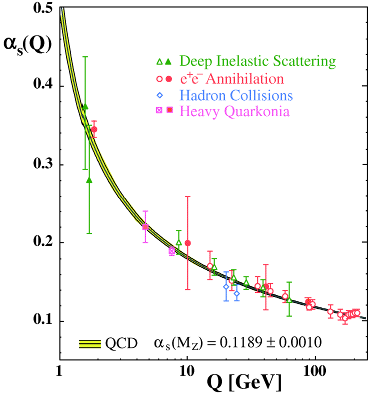
The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"

David J. Gross	H. David Politzer	Frank Wilczek
1/3 of the prize	1/3 of the prize	1/3 of the prize
USA	USA	USA
University of California, Kavli Institute for Theoretical Physics Santa Barbara, CA, USA	California Institute of Technology (Caltech) Pasadena, CA, USA	Massachusetts Institute of Technology (MIT) Cambridge, MA, USA
b. 1941	b. 1949	b. 1951

Titles, data and places given above refer to the time of the award.
Photos: Copyright © The Nobel Foundation

QCD is asymptotically free.



Bethke, hep-ex/0606035

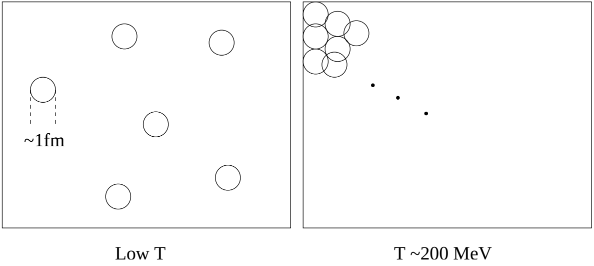
At high T

- Running coupling

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda_{\rm QCD}^2)}$$

  - When $Q \sim \Lambda_{\rm QCD} \sim 200\,\mathrm{MeV}$, the above expression blows up: Not physical. Indicates breakdown of perturbation theory. Hadrons.
  - Perturbative QCD is a theory of quarks and gluons *not* hadrons.
  - At high T, $Q \sim T$.
  - Possible phase transition around $T \sim \Lambda_{\rm QCD}$?
  - If $Q \sim T \to \infty$, $\alpha_s \to 0$: Weakly coupled
  - At $Q \sim$ few GeV, $\alpha_s \sim 0.2 - 0.4$

Another estimate of $T_{\text{transition}}$



- Density: Consider a pion gas.

$$n = 3 \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{E_p/T} - 1} = 0.37\, T^3$$

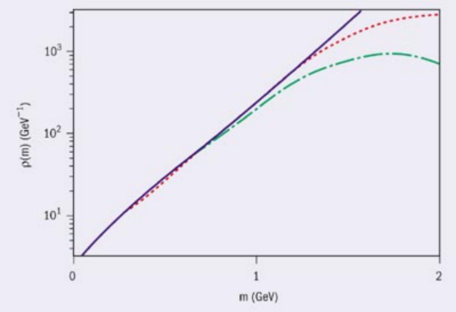
As T becomes larger, more and more pair creation results.

- Inter particle distance:

$$l_{\text{inter}} = n^{1/3} = 1.4/T$$

At $T = 200\,\text{MeV}$, $l_{\text{inter}} \approx 1.4\,\text{fm} \approx d_\pi$

Hadronic density of states $\rho(m) \sim e^{m/T_H}$:



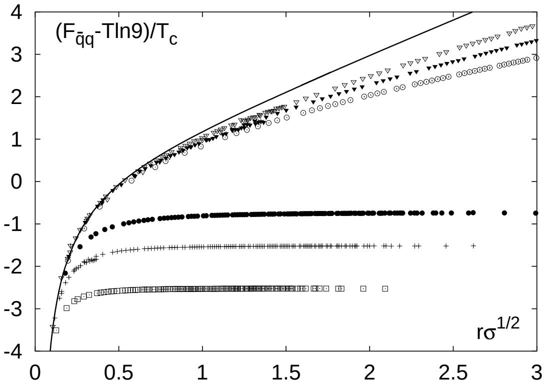
The smoothed mass spectrum of hadronic states as a function of mass. Experimental data: long-dashed green line with the 1411 states known in 1967; short-dashed red line with the 4627 states of 1996. The solid blue line represents the exponential fit yielding T_H=158 MeV. *CERN Courier, Sept, 2003*

- $\sum_m \int_p \rho(m) e^{-E_p/T}$: Not well defined when $T > T_H$ for hadronic matter.
- Phase transition around T_H: Hagedorn temperature $\approx$ 160 MeV

Story so far

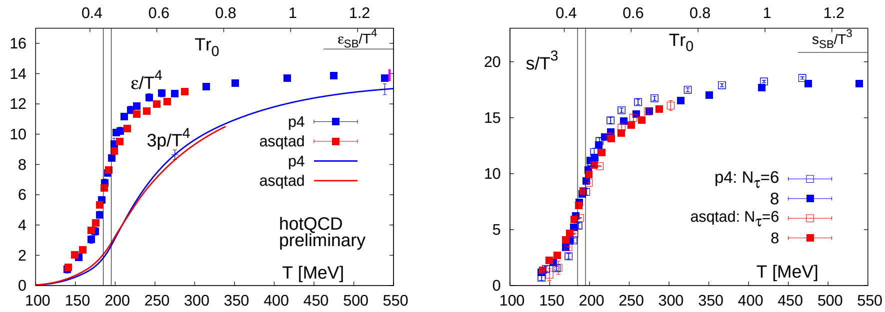
- Perturbative calculation possible much above $Q = \Lambda_{\rm QCD}$
- $Q \sim T$ at high T
- If T is much above the binding energy of hadrons
  $\Longrightarrow$ Deconfinement
- At high enough T, the system is a plasma of weakly interacting quarks and gluons
- All the above arguments are plausible but not a proof

Lattice QCD Evidence



- F. Karsch, hep-lat/0403016. The color averaged heavy quark free energy at temperatures $T/T_c = 0.9,\ 0.94,\ 0.98,\ 1.05,\ 1.2,\ 1.5$ (from top to bottom) obtained in quenched QCD.

Lattice QCD Evidence of QGP



- From HotQCD Collaboration (C. DeTar, arXiv:0811.2429)
- “Cross-over” between 185 - 195 MeV

Expected properties

- High number density

$$
\begin{aligned}
n &\approx (24+16)\int \frac{d^3p}{(2\pi)^3}\, e^{-p/T} \approx 4\,T^3 \\
&= 4\left(\frac{T}{200\,\mathrm{MeV}}\right)^3 \mathrm{fm}^{-3}
\end{aligned}
$$

- High energy density

$$
\begin{aligned}
\varepsilon &\approx (24+16)\int \frac{d^3p}{(2\pi)^3}\, p\, e^{-p/T} \approx 12\,T^4 \\
&= 2.4\left(\frac{T}{200\,\mathrm{MeV}}\right)^4 \mathrm{GeV}/\mathrm{fm}^3
\end{aligned}
$$

Simple Estimates

With $\hbar = c = 1$

- 1 mole of hydrogen atom: 6.02×10^{23} atoms $= 1$ g (Avogadro's number)
- 1 hydrogen atom $m_p \approx (1/6) \times 10^{-23}\,\mathrm{g} = (1/6) \times 10^{-26}\,\mathrm{kg}$
- $m_p = 940\,\mathrm{MeV} \approx 1\,\mathrm{GeV}$
- $E = mc^2$: $1\,\mathrm{GeV} \approx (1/6) \times 10^{-26}\,\mathrm{kg}$

$$
\begin{aligned}
2.4\,\mathrm{GeV/fm^3} &= 0.4 \times 10^{-26}\,\mathrm{kg}/(10^{-13}\,\mathrm{cm})^3 \\
&= 0.4 \times 10^{-26+39}\,\mathrm{kg/cm^3} \\
&= 4 \times 10^{12}\,\mathrm{kg/cm^3}
\end{aligned}
$$

- Typical human: $\sim 100\,\mathrm{kg}$

$$
2.4\,\mathrm{GeV/fm^3} \sim 4 \times 10^{10}\,\mathrm{human/cm^3}
$$

Simple Estimates

With $\hbar = c = 1$

- Another way of looking at the energy density

$$2.4\,\mathrm{GeV}/\mathrm{fm}^3 = 4 \times 10^{12}\,\mathrm{kg}/\mathrm{cm}^3$$

- Restoring $c = 3 \times 10^8$ m/s,

$$2.4\,\mathrm{GeV}/\mathrm{fm}^3 = 4 \times 10^{12} \times (9 \times 10^{16})\,\mathrm{J/cm}^3 = 3.6 \times 10^{29}\,\mathrm{J/cm}^3$$

- World energy consumption (2008):

$$144\,\mathrm{pWh} = 144 \times 10^{15} \times 3.6 \times 10^3\,\mathrm{J} = 5.2 \times 10^{20}\,\mathrm{J}$$

- A cubic centimeter of QGP can power the world for about 70 million years.

Simple Estimates

With $\hbar = c = 1$

- Pressure $P \approx \epsilon/3$

$$P = 0.8\,\mathrm{GeV}/\mathrm{fm}^3 \approx 1.3 \times 10^{12}\,\mathrm{kg}/\mathrm{cm}^3 = 1.3 \times 10^{18}\,\mathrm{kg/m}^3$$

- SI Unit for pressure: $\mathrm{Pa} = \mathrm{N/m}^2 = \mathrm{kg/m/s}^2$
- Restoring $c = 3 \times 10^8\,\mathrm{m/s}$,

$$P \approx 1.3 \times 10^{18} \times (9 \times 10^{16})\,\mathrm{kg}/\mathrm{m/s}^2 \approx 10^{35}\,\mathrm{Pa} \approx 10^{30}\,\mathrm{atm}$$

How do you achieve high temperature?

- Temperature = energy (1 eV $\approx 12,000$K)
- More usefully, the energy density:

$$\varepsilon = g \int \frac{d^3p}{(2\pi)^3} E_p \, e^{-E_p/T} \approx \frac{3g}{\pi^2} T^4$$

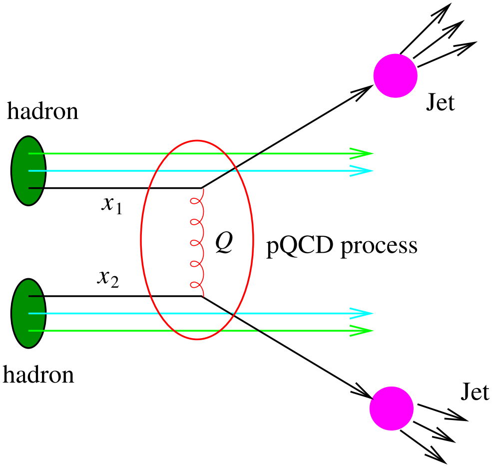
- To get high temperature: Get high energy density $\Longrightarrow$ Cram maximum possible energy into the smallest possible volume while randomizing the momenta $\Longrightarrow$ Relativistic heavy ion collisions.
- What to expect: $dN/d\eta$ and $dE/d\eta$ grow something like $(\ln s)^n$ with $n \sim 1$ $\Longrightarrow$ T should behave something like $(\ln s)^n$ with $n \sim 1$

Observable Consequence

- High temperature $\Longrightarrow$ Thermal photons
- High density $\Longrightarrow$ *Jet quenching*
- High pressure $\Longrightarrow$ Hydrodynamic flow
  - The size of the eliptic flow depends on the shear viscosity η.
  - If weakly coupled, $\eta/s \gg 1$: $\approx$ Ideal gas
  - If stronlgy coupled, $\eta/s \ll 1$: $\approx$ Perfect (Ideal) fluid.
- Neutrality $\Longrightarrow$ Tight unlike-sign correlation
- Critical point $\Longrightarrow$ Large momentum fluctuations

Jet Quenching
– Schematic Ideas

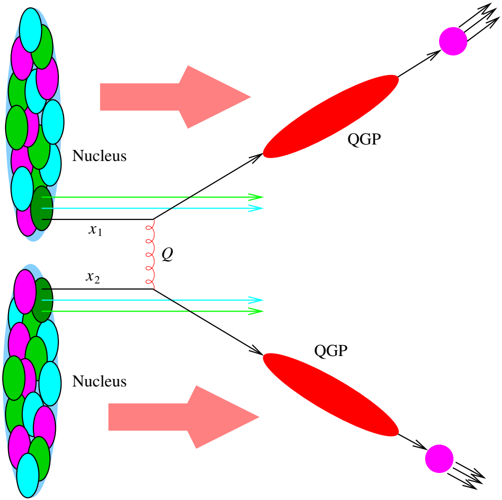
Hadronic Jet production



Hadron-Hadron Jet production scheme:

$$\frac{d\sigma}{dt} = \int_{abcd} f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab\to cd}}{dt} D(z_c, Q)$$

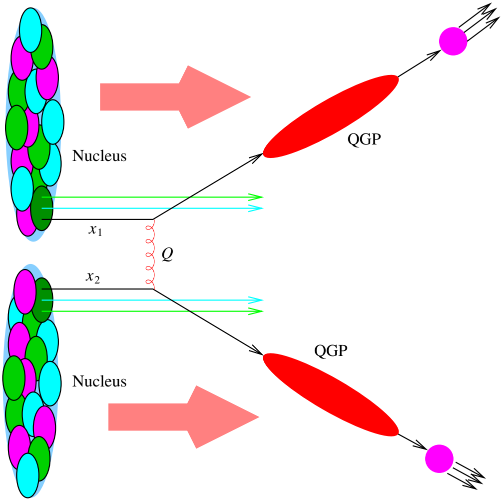
Heavy Ion Collisions



What we want to study:

- How does QGP modify jet property?

Heavy Ion Collisions



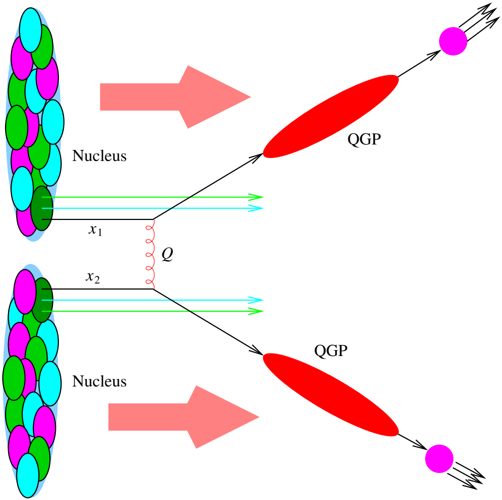
What we want to study:

- How does QGP modify jet property?

Complications:
How well do we know the *initial condition?*

- Nuclear initial condition?
- What happens to a jet between the production and the formation of (hydrodynamic) QGP?

Heavy Ion Collisions



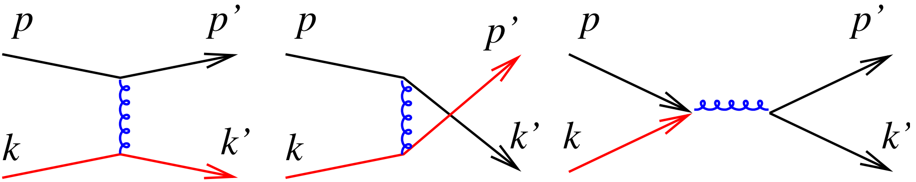
Schematically,

$$\frac{d\sigma_{AB}}{dt} = \int_{\text{geometry}} \int_{abcd} \times f_{a/A}(x_a, Q_f) f_{b/B}(x_b, Q_f) \times \frac{d\sigma_{ab\to cd}}{dt} \times \mathcal{P}(x_c \to x'_c | T, u^\mu) \times D(z'_c, Q)$$

$\mathcal{P}(x_c \to x'_c | T, u^\mu)$: Medium modification of high energy parton property $\Longrightarrow$ Jet quenching

Elastic Energy Loss

Mandelstam variables

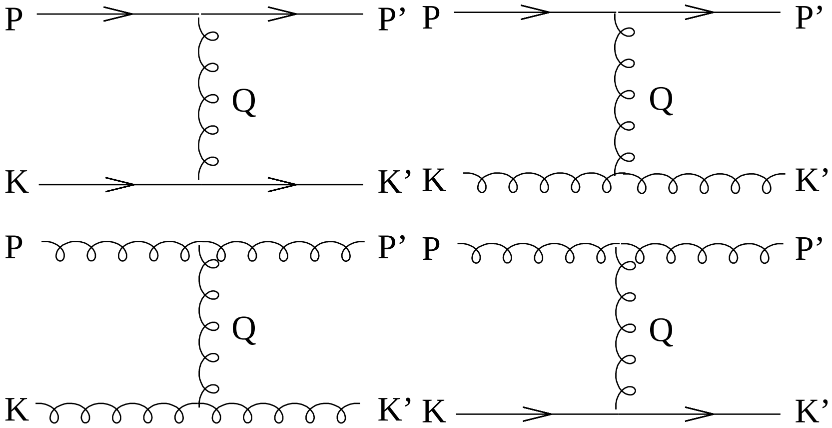


t –channel $\quad\quad$ u–channel $\quad\quad$ s –channel

$\hat{t} = (p-p')^2 \quad\quad \hat{u} = (p-k')^2 \quad\quad \hat{s} = (p+k)^2$

Elastic scattering rate

Coulombic t-channel dominates



Must be cut-off at a medium soft scale $|t_{\rm min}| \sim m_D^2$

Physical origin of Debye mass m_D

- E & M
- Potential in a thermal system

$$\nabla^2\Phi(\mathbf{r}) = -\rho_0(\mathbf{r}) - \delta\rho(\mathbf{r})$$

- Medium composed of many charged particles

$$\delta\rho(\mathbf{r}) = qn_+(\mathbf{r}) - qn_-(\mathbf{r})$$

- Boltzmann Density:

$$\begin{aligned}
n_\pm(\mathbf{r}) &= \int \frac{d^3k}{(2\pi)^3}\, e^{-E/T} \\
&= \int \frac{d^3k}{(2\pi)^3}\, e^{-\sqrt{k^2+m^2}}\, e^{\mp q\Phi(\mathbf{r})/T} \\
&= n_0(T) e^{\mp q\Phi(\mathbf{r})/T} \\
&\approx n_0(T)(1 \mp q\Phi(\mathbf{r})/T)
\end{aligned}$$

Physical origin of Debye mass

- E & M
- Boltzmann Density:

$$n_{\pm}(\mathbf{r}) \approx n_0(T)(1 \mp q\Phi(\mathbf{r})/T)$$

- Linearized equation for the potential:

$$\nabla^2\Phi - m_D^2\Phi \approx -\rho_0(\mathbf{r})$$

where

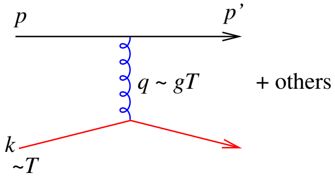
$$m_D^2 = 2q^2(n_0(T)/T) \sim \alpha T^2$$

- For a static point particle source,

$$\Phi = \alpha\frac{e^{-m_D r}}{r}$$

Range: $\sim 1/m_D$

Rough Idea - Elastic energy loss(Following Bjorken)



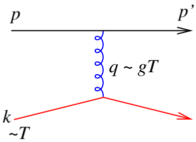
- What we want to calculate: $\dfrac{dE}{dt}$
- Mean free time

$$\frac{1}{\tau_{\text{mft}}} = n_{\text{scatt}}\,\sigma_{\text{el}}\,v_{\text{rel}}$$

- Energy loss rate

$$\frac{dE}{dt} \approx \frac{\Delta E}{\tau_{\text{mft}}}$$

where ΔE is the average energy loss per collision.



+ others

- Kinetic theory def: (Recall $n(x) = \int (d^3k/(2\pi)^3)\, f(x,k)$)

$$\frac{1}{\tau_{\rm mft}} = \langle n_{\rm scatt}\, v_{\rm rel}\, \sigma_{\rm el}\rangle = \int \frac{d^3k}{(2\pi)^3} f_{\rm scatt}(x,k) v_{\rm rel}(p,k) \sigma_{\rm el}(p,k)$$

- Mean free time in the massless limit

$$\frac{1}{\tau_{\rm mft}} \equiv \int \frac{d^3k}{(2\pi)^3} f_{\rm scatt}(k)\,(1-\cos\theta_{pk}) \int d\hat{t}\, \frac{d\sigma_{\rm el}}{d\hat{t}}$$

- Energy loss per unit time in the massless limit

$$\frac{dE}{dt} = \langle \frac{\Delta E}{\tau_{\rm mft}} \rangle = \int \frac{d^3k}{(2\pi)^3} f_{\rm scatt}(k)\,(1-\cos\theta_{pk}) \int d\hat{t}\, \frac{d\sigma_{\rm el}}{d\hat{t}} (E_p - E_{p'})$$

Estimation of ℓ_{mfp}

- Elastic cross-section (Coulombic) $\dfrac{d\sigma}{d\hat{t}} \approx C_R \dfrac{2\pi\alpha_s^2}{\hat{t}^2}$
- With thermal $f_{\mathrm{scatt}}(x, k)$, this yields

$$\frac{1}{\tau_{\mathrm{mft}}} \approx \int \frac{d^3k}{(2\pi)^3} f_{\mathrm{scatt}}(x, k)(1 - \cos\theta_{pk}) \int d\hat{t}\, C_R \frac{2\pi\alpha_s^2}{\hat{t}^2} \sim \alpha_s T$$

with the IR cut off given by the Debye mass scale $m_D^2 \sim \alpha_s T^2$

Rough Idea - Elastic energy loss(Following Bjorken)

- Elastic cross-section (Coulombic) $\dfrac{d\sigma_{\rm el}}{d\hat{t}} \approx C_R \dfrac{2\pi\alpha_s^2}{\hat{t}^2}$
- When $|\mathbf{p}| \gg |\mathbf{k}|$, one can approximate $(1 - \cos\theta_{pk})\Delta E \approx -\hat{t}/2k$
- With thermal $f_{\rm scatt}(x,k)$, this yields

$$\left(\frac{dE}{dt}\right)_{\rm coll} \sim \int \frac{d^3k}{(2\pi)^3} f_{\rm scatt}(x,k)/k \int d\hat{t}\,\frac{\alpha_S^2}{\hat{t}} \sim \alpha_S^2 T^2 \ln(ET/m_D^2)$$

More precisely,

$$\frac{dE}{dt} = \frac{1}{2E_p}\int_{k,k',p'} \delta^4(p+k-p'-k')\,(E_p - E_{p'})|M|^2 f(E_k)[1 \pm f(E_{k'})]$$
$$= C_r\pi\,\alpha_s^2 T^2 \left[\ln(ET/m_g^2) + D_r\right]$$

where C_r and D_r are channel dependent $O(1)$ constants.

For instance, see Qin et al. Phys. Rev. Lett. 100, 072301 (2008)

Before I begin...

McGill is in Montréal, Québec, Canada

Montreal
QC
Canada

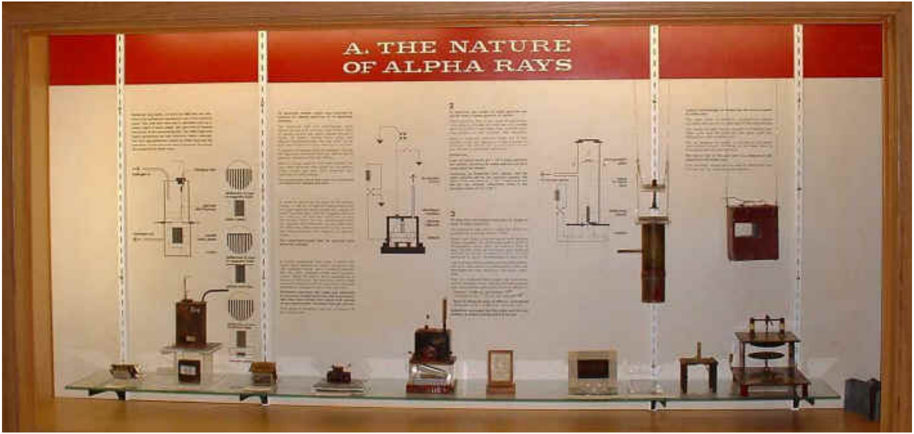
McGill is in Montréal, Québec, Canada

McGill is in Montréal, Québec, Canada

McGill is in Montréal, Québec, Canada

Mr. McGill going home after a hard day's work.

McGill is in Montréal, Québec, Canada



Rutherford carried out his Nobel (1908) winning work at McGill (1898-1907).
His *original* equipments on display

McGill Team

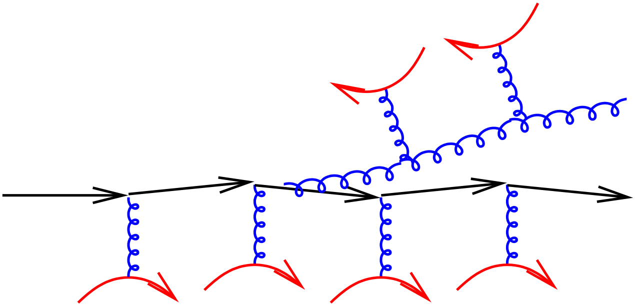
- Charles Gale
- Sangyong Jeon
- *Li yan*
- *Alina Czajka*
- *Dani Pablos*
- *Shuzhe Shi*
  (Joining in September)

- Chanwook Park
- Mayank Singh
- Scott McDonald
- Siggi Hauksson
- Igor Kozlov
- Rouzbeh Modarresi-Yazdi
- Jessica Churchil
- Matt Heffernan
- Melissa Mendes
  (Joining in September)

Alumni: G .Qin, A .Majumder, B. Schenke, G. Denicol, M. Luzum, C. Shen, G. Vujanovic, J.F. Paquet, S. Ryu, ...

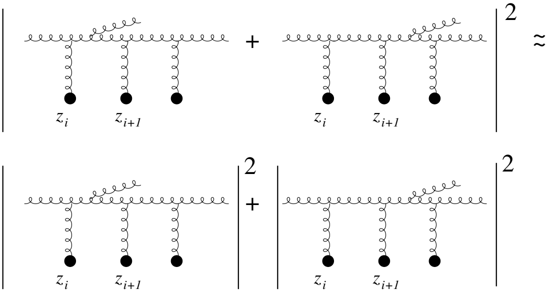
Radiational Energy Loss

Process to study



- Radiative (Inelastic) energy loss via collinear gluon emission

Incoherent emission



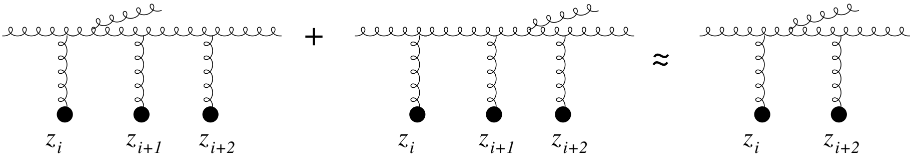
- $|\sum_n T_n|^2 \approx \sum |T_n|^2$
- Interference terms $T_n^* T_m$ with $n \neq m$ negligible. (Large phase change between scatterings)
- Single emission probability scales like the number of scatterers:

$$\mathcal{P}_{\mathrm{N_{sc}}} \approx N_{\mathrm{sc}} \mathcal{P}_1$$

- In a unit length, there are $N_{\mathrm{sc}} = \dfrac{1}{\ell_{\mathrm{mfp}}}$ number of scatterers.

Coherent emission

- If there is a destructive interference,



- Single emission probability scales like

$$\mathcal{P}_{N_{\rm sc}} \approx \frac{N_{\rm sc}}{N_{\rm coh}} \mathcal{P}_1$$

where $N_{\rm coh}$ is the number of scattering centers that destructively interfere. (Small phase change between scatterings)

- The medium's power to induce radiation is *reduced*.
$\Longrightarrow$ Landau-Pomeranchuck-Migdal (LPM) effect
- Define the coherence length

$$\ell_{\rm coh} = \ell_{\rm mfp} N_{\rm coh}$$

Effective Emission rate

- Incoherent Emission rate:

$$\frac{d\mathcal{P}}{dt} \approx \frac{C}{\ell_{\mathrm{mfp}}}\mathcal{P}_1$$

- Coherent Emission rate:

$$\frac{d\mathcal{P}}{dt} \approx \frac{C}{\ell_{\mathrm{coh}}}\mathcal{P}_1$$

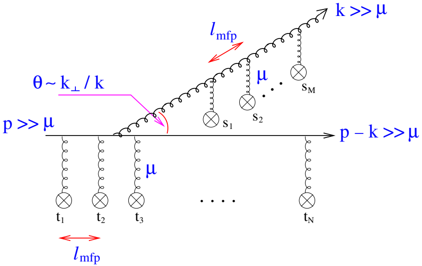
- $\mathcal{P}_1$: Bethe-Heitler (BH, Single emission off of one scatterer)

$$\mathcal{P}_1 \sim \left.\frac{dN_g}{d\omega}\right|_{BH} \approx \frac{\alpha_S N_c}{\pi\omega}$$

for small ω

Coherent scattering can be important

Following BDMPS



- What we need to calculate R_{AA}:

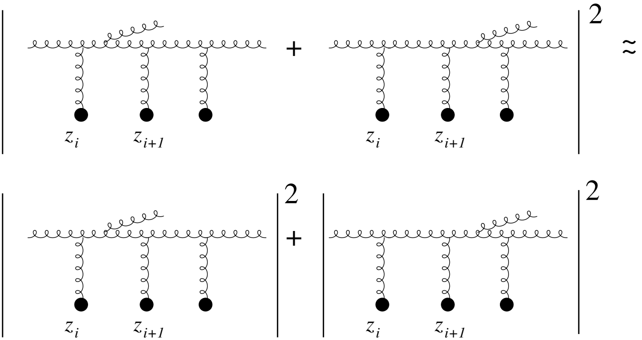
  Differential gluon radiation rate $\omega \dfrac{dN_g}{d\omega dt}$

  Medium dependence comes through the scattering time (length) scale

$$\omega \frac{dN_g}{d\omega dt} \approx \frac{\omega}{\ell_{sc}} \left. \frac{dN_g}{d\omega} \right|_{\mathrm{BH}}$$

Length Scales

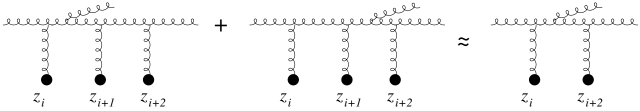
Following BDMPS



- If all scatterings are incoherent ($\ell_{\rm mfp} > \ell_{\rm coh}$),

$$\ell_{sc} = \ell_{\rm mfp} \approx \tau_{\rm mft}$$

Following BDMPS



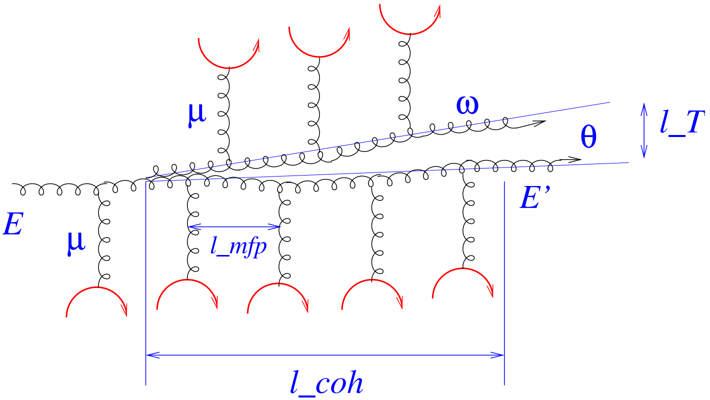
- If $\ell_{\rm coh} \ge \ell_{\rm mfp} \implies$ LPM effect:
  All scatterings within $\ell_{\rm coh}$ effectively count as a single scattering.
- $\ell_{sc} = \ell_{\rm coh}$

Estimation of $\ell_{\rm mfp}$

- Elastic cross-section (Coulombic) $\dfrac{d\sigma}{d\hat{t}} \approx C_R \dfrac{2\pi\alpha_s^2}{\hat{t}^2}$
- With thermal $f_{\rm scatt}(x,k)$, this yields

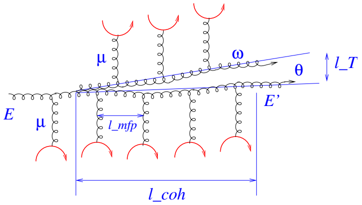
$$\frac{1}{\tau_{\rm mft}} \approx \int \frac{d^3k}{(2\pi)^3} f_{\rm scatt}(x,k)(1-\cos\theta_{pk}) \int d\hat{t}\, C_R \frac{2\pi\alpha_s^2}{\hat{t}^2} \sim \alpha_s T$$

Estimation of ℓ_{coh}



- E: Original parton energy
- ω: Energy of the radiated gluon
- μ: Typical transverse momentum transfer
- $E \gg \omega \gg \mu$

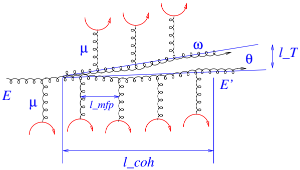
Estimation of ℓ_{coh}



- The radiated gluon random walks away from the original parton. Original parton's trajectory is less affected since $\omega \ll E$
- From the geometry $\theta \approx \dfrac{k_T^g}{\omega}$ and $\theta \approx \dfrac{\ell_T}{\ell_{\text{coh}}}$
- Separation condition: ℓ_T is longer than the transverse size of the radiated gluon: $\ell_T \approx 1/k_T^g$
- Putting together,

$$\ell_{\text{coh}} \approx \frac{\omega}{(k_T^g)^2}$$

Estimation of $\ell_{\rm coh}$



- We have: $\ell_{\rm coh} \approx \dfrac{\omega}{(k_T^g)^2}$
- After suffering $N_{\rm coh}$ collisions (random walk),

$$\langle (k_T^g)^2 \rangle = N_{\rm coh}\mu^2 = \frac{\ell_{\rm coh}}{\ell_{\rm mfp}}\mu^2 = \ell_{\rm coh}\left(\frac{\mu^2}{\ell_{\rm mfp}}\right) = \ell_{\rm coh}\,\hat{q}$$

- $\hat{q}$: Transport coefficient. Momentum transfer squared per elastic collision $\Longrightarrow$ QGP property
- $\ell_{\rm coh} \approx \dfrac{\omega}{(k_T^g)^2}$ becomes, with $\hat{q} = \mu^2/\ell_{\rm mfp}$ and $E_{\rm LPM} = \mu^2 \ell_{\rm mfp}$,

$$\ell_{\rm coh} \approx \ell_{\rm mfp}\sqrt{\frac{\omega}{E_{\rm LPM}}} = \sqrt{\frac{\omega}{\hat{q}}}$$

Length scales

Coherence length: $\dfrac{\ell_{\rm coh}}{\ell_{\rm mfp}} \approx \sqrt{\dfrac{\omega}{E_{\rm LPM}}}$

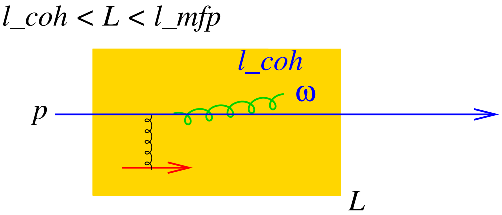
Key quantity: $E_{\rm LPM} = \mu^2 \ell_{\rm mfp} \sim T$ in pert. thermal QCD.

- L: The size of the medium
- $\mu^2 \sim m_D^2 \sim \alpha_s T^2$
- $\ell_{\rm mfp} \sim 1/(\alpha_s T)$: The mean free path for elastic collisions
- $\ell_{\rm coh} \sim \ell_{\rm mfp} \sqrt{\dfrac{\omega}{T}}$
- $\ell_{\rm coh} > \ell_{\rm mfp}$ when $\omega > T$
- $\ell_{\rm coh} > L$ when $\omega > E_L$ with $E_L = \alpha_s^2 T^3 L^2 = T(L/\ell_{\rm mfp})^2$

Thin Plasma Radiation Rate

Length scale hierarchies

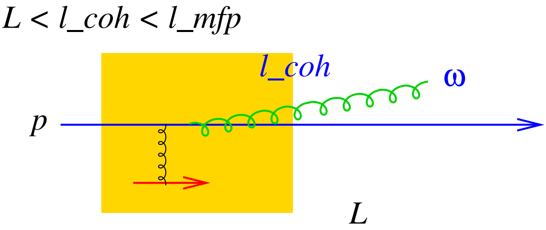
Thin plasma $L < \ell_{\rm mfp}$



- $\omega < T(L/\ell_{\rm mfp})^2 < T$
- Mostly 0 or 1 collision and radiation occurs *inside* the medium
  $\Longrightarrow$ The daughter can undergo another splitting

Length scale hierarchies

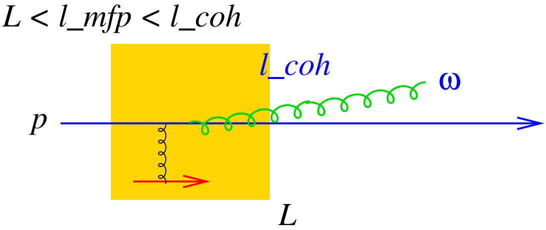
Thin plasma $L < \ell_{\rm mfp}$



- $T(L/\ell_{\rm mfp})^2 < \omega < T$
- Mostly 0 or 1 collision and a single radiation occurs *outside* the medium

Length scale hierarchies

Thin plasma $L < \ell_{\mathrm{mfp}}$



- $\omega > T$
- Mostly 0 or 1 collision and a single radiation occurs *outside* the medium

Thin plasma energy loss

Thin plasma $L < \ell_{\mathrm{mfp}}$

- Elastic energy loss possible:

$$\frac{dE}{dt} \sim \alpha_s^2 T^2 \ln(ET/m_D^2)$$

- Radiational energy loss possible
  - Opacity expansion works
  - Since 1 scattering should dominate, the basic process is BH
  - Recall that when deriving Fermi's Golden rule,

$$P_{\Delta E}(t) \propto \int d\Delta E\, \rho(E - \Delta E)|M|^2 \frac{1 - \cos(\Delta E t)}{\Delta E^2}$$

    where $\rho(E - \Delta E)$ is the phase space of the final state
  - Energy conserving δ-function appears only in the $t \to \infty$ limit
  - The $(1 - \cos(\Delta E t))/\Delta E^2$ part is the finite length (time) correction

Thin plasma energy loss

Thin plasma $L < \ell_{\mathrm{mfp}}$

- Factor out the $t \to \infty$ parts. That is, the BH σ_{BH}.
- Thin plasma radiation rate = BH * (Finite length (time) corr.)

$$\omega \frac{dN_g}{d\omega dt} \propto \frac{\alpha_s C_R}{\pi t} \int d\Delta E\, C(\Delta E, \omega, m_D) \frac{1 - \cos(\Delta E\, t)}{\Delta E^2}$$

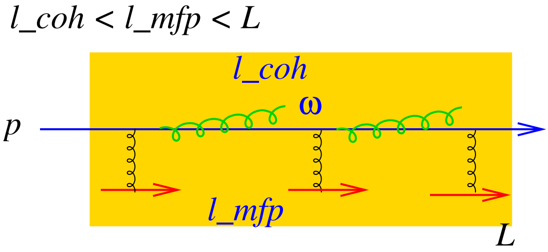
where $C(\Delta E, \omega, m_D)$ depends on the process

- This is basically the GLV (Gyulassy, Levai, Vitev) formalism with a suitable $C(q_\perp^2)$ related to $v(\mathbf{q}_\perp) = 1/(q_\perp^2 + m_D^2)$

Thick Plasma Radiation Rate

Length scale hierarchies

Thick plasma $\ell_{\rm mfp} < L$

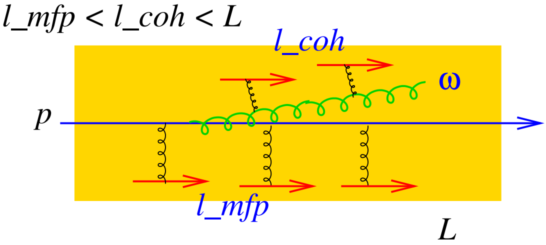


- $\omega < T < T(L/\ell_{\rm mfp})^2$
- Multiple independent collisions and radiations occur *inside* the medium
- Spectrum: Bethe-Heitler per $\ell_{\rm mfp}$

$$\omega \left. \frac{dN_g}{d\omega dt} \right|_{BH} \approx \frac{\alpha_s N_c}{\pi \ell_{\rm mfp}} = O(\alpha_s^2)$$

Length scale hierarchies

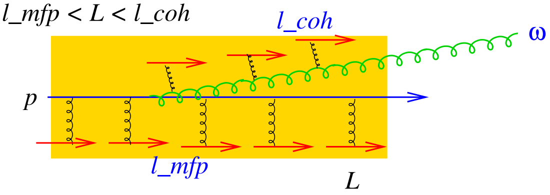
Thick plasma $\ell_{\mathrm{mfp}} < L$



- $T < \omega < T(L/\ell_{\mathrm{mfp}})^2$
- Multiple elastic collisions can occur and multiple radiations can occur, too. But coherence matters.
- The daughter can undergo another splitting
- AMY/BDMPS/HT regime
- Spectrum: BH * $(\ell_{\mathrm{mfp}}/\ell_{\mathrm{coh}})$: $\omega \left.\dfrac{dN_g}{d\omega dt}\right|_{BDMPS} \approx \dfrac{\alpha_s N_c}{\pi \ell_{\mathrm{mfp}}}\sqrt{\dfrac{T}{\omega}}$

Length scale hierarchies

Thick plasma $\ell_{\mathrm{mfp}} < L$



- $\omega > T(L/\ell_{\mathrm{mfp}})^2$
- Multiple elastic collisions can occur, but only one radiation *outside* the medium
- The energy at the splitting is *not* the energy you finally observe because of multiple $2 \to 2$ scatterings $\Longrightarrow$ Not a free propagation any more
- The radiation rate: $\omega \dfrac{dN_g}{d\omega dt} \sim \dfrac{\alpha_s N_c}{\pi L}$

What we learned so far (Thick Plasma)

Coherence length

$$\frac{\ell_{\text{coh}}}{\ell_{\text{mfp}}} \approx \sqrt{\frac{\omega}{E_{\text{LPM}}}}$$

Key quantity: $E_{\text{LPM}} = \mu^2 \ell_{\text{mfp}} \sim T$ in pert. thermal QCD.
Related: $\hat{q} = E_{\text{LPM}}/\ell_{\text{mfp}}{}^2 = \mu^2/\ell_{\text{mfp}}$ (average momentum transfer squared per elastic collision)

- $\ell_{\text{coh}} < \ell_{\text{mfp}}$
  Soft gluon emission, $\omega < \mu^2 \ell_{\text{mfp}} \sim T$
  $\Longrightarrow$ Coherence matters not. BH should suffice. No need to resum.

- $L > \ell_{\text{coh}} > \ell_{\text{mfp}}$ where L is the length of the medium
  Hard gluon emission, $E \gg \omega > \mu^2 \ell_{\text{mfp}} \sim T$,
  $\Longrightarrow$ Coherence matters. Resummation needed.

What we have learned so far (Thick Plasma)

Summary of E-loss rates per scattering

Elastic energy loss

$$\frac{dE}{dt} \sim \alpha_s^2 T^2 \ln(ET/m_D^2)$$

For the emission of *a single gluon,* while traversing a medium of length $L \gg \ell_{\rm mfp} \sim 1/(\alpha_s T)$ and temperature T

$$\omega \frac{dN}{d\omega dt} \approx \frac{\alpha_s}{\pi}\frac{N_c}{\ell_{\rm mfp}} = O(\alpha_s^2) \;\text{ for }\; 0 < \omega < T$$

$$\omega \frac{dN}{d\omega dt} \approx \frac{\alpha_s}{\pi}\frac{N_c}{\ell_{\rm mfp}}\sqrt{\frac{T}{\omega}} \;\text{ for }\; T < \omega < T(L/\ell_{\rm mfp})^2$$

$$\omega \frac{dN}{d\omega dt} \approx \frac{\alpha_s}{\pi}\frac{N_c}{L} \;\text{ for }\; T(L/\ell_{\rm mfp})^2 < \omega < E \;\text{ if }\; T(L/\ell_{\rm mfp})^2 < E$$

$$\frac{dN}{d\omega dt} \approx \frac{\alpha_s}{\pi|\omega|}\frac{N_c}{\ell_{\rm mfp}} e^{-|\omega|/T} \;\text{ for }\; \omega < 0 \;\; \text{Absorption}$$

Rough Idea – Multiple Emissions (Poisson ansatz)

- The radiation rate $\dfrac{dN(E,\omega)}{d\omega dt}$: Original parton energy E, energy of the emitted gluon ω
- The rate equation (for a single process)

$$\frac{dN(E)}{dt} = \int d\omega \, \frac{dN(E+\omega,\omega)}{d\omega dt} N(E+\omega) - \int d\omega \, \frac{dN(E,\omega)}{d\omega dt} N(E)$$

[Gain] [Loss]

- *If* the rate $dN(E,\omega)/d\omega dt$ is independent of E (the large E limit), the solution is

$$N(E,t) = \int d\epsilon \, D(\epsilon,t) \, N_0(E+\epsilon)$$

where

$$D(\epsilon,t) \;=\; e^{-\int d\omega \int_0^t dt' \frac{dN}{d\omega dt'}} \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int d\omega_i \int_0^t dt' \frac{dN}{d\omega_i dt'} \right] \delta\left(\epsilon - \sum_{i=1}^{n} \omega_i\right)$$

Rough Idea – Multiple Emissions (Poisson ansatz)

- Poisson Ansatz: Even when $dN/d\omega dt$ does depend on the parton energy E, use

$$N(E,t) = \int d\epsilon\, D(\epsilon,t)\, N_0(E+\epsilon)$$

with

$$D(\epsilon,t) = e^{-\int d\omega \int_0^t dt' \frac{dN(E,\omega)}{d\omega dt'}} \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \int d\omega_i \int_0^t dt' \frac{dN(E,\omega)}{d\omega_i dt'} \right] \times \delta\left(\epsilon - \sum_{i=1}^{n} \omega_i\right)$$

Understanding the behavior of R_{AA}

Rough Idea - The behavior of R_{AA}

- R_{AA}: Interplay between $\frac{dN}{d\omega dt}$ and the stiff $\frac{dN}{dp_T}$ for high p_T particles
- High p_T spectrum with $p_T = E$:

$$N_0(E) = \frac{dN}{dp_T} \sim \frac{1}{E^n}$$

  with $n \sim 10$

- At the parton level

$$R_{AA} \sim \frac{N(E)}{N_0(E)} = \int d\epsilon\, D(\epsilon, t)\, \frac{N_0(E+\epsilon)}{N_0(E)}$$
$$= \int d\epsilon\, D(\epsilon, t)\, (1+\epsilon/E)^{-n}$$

Rough Idea - The behavior of R_{AA}

- R_{AA}: Interplay between $\dfrac{dN}{d\omega dt}$ and the stiff $\dfrac{dN}{dp_T}$ for high p_T particles
- More generally, let

$$\frac{N_0(E+\epsilon)}{N_0(E)} = 1 - \nu(\epsilon/E)$$

- In the limit $\epsilon \ll E$, one can show

$$\begin{aligned} R_{AA}(E) &= \frac{N(E)}{N_0(E)} \approx \exp\left(-\int_{-\infty}^{\infty} d\omega \int_0^L dt\, \frac{dN_{\text{inel+el}}}{d\omega dt} (1 - (1 - \nu'(0)\omega/E)) \right) \\ &\approx \exp\left(- \frac{\nu'(0)\Delta E(L)}{E} \right) \end{aligned}$$

  where L is the length of the medium.
- $\nu'(0)$: Stiffness of the spectrum. The stiffer the spectrum, the more sensitive R_{AA} to $\Delta E/E$
- $\omega < 0$: Absorption

Rough Idea - The behavior of R_{AA}

For $N_0(E) \approx 1/E^n$, we have

$$\ln R_{AA}(E) \approx -\frac{n\Delta E(L)}{E}$$

where L is the length of the medium.

For the elastic energy loss,

$$\frac{dE}{dt} = C_r \pi\, \alpha_s^2 T^2 \left[\ln(ET/m_g^2) + D_r\right]$$

For the radiation rate, use simple estimates

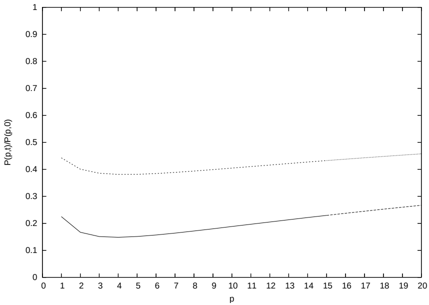
$$\omega \frac{dN}{d\omega dt} \approx \frac{\alpha_s}{\pi} \frac{N_c}{\ell_{\rm mfp}} \quad \text{for} \quad 0 < \omega < \ell_{\rm mfp}\mu^2$$

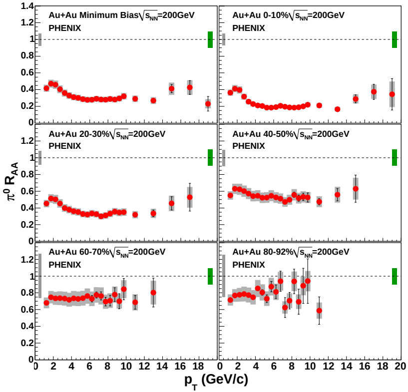
$$\omega \frac{dN}{d\omega dt} \approx \frac{\alpha_s}{\pi} N_c \sqrt{\frac{\mu^2}{\ell_{\rm mfp}\omega}} \quad \text{for} \quad \ell_{\rm mfp}\mu^2 < \omega < \ell_{\rm mfp}\mu^2 (L/\ell_{\rm mfp})^2$$

$$\omega \frac{dN}{d\omega dt} \approx \frac{\alpha_s}{\pi} \frac{N_c}{L} \quad \text{for} \quad \ell_{\rm mfp}\mu^2 (L/\ell_{\rm mfp})^2 < \omega < E$$

$$\omega \frac{dN}{d\omega dt} \approx -\frac{\alpha_s}{\pi} \frac{N_c}{\ell_{\rm mfp}} e^{-|\omega|/T} \quad \text{for} \quad \omega < 0$$

Rough Idea - The behavior of R_{AA}

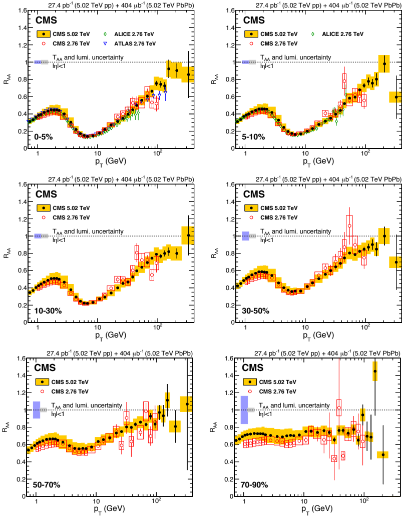




- Upper line: Without elastic
- Lower line: With elastic
- Flat R is produced in both cases up to $O(10\ T)$.
- R just not that sensitive to p in the RHIC-relevant range.

Rough Idea - The behavior of R_{AA}

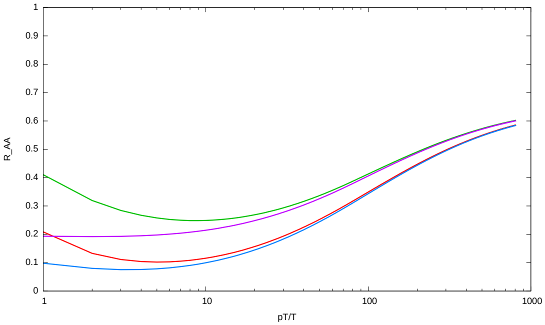
CMS:



arXiv:1611.01664

No longer flat. Slow rise for $p_T \gtrsim 10\,\text{GeV}$.
Can we understand these features?

Rough Idea - The behavior of R_{AA}



- Red: Elastic on, thermal absorption on
- Blue: Elastic on, thermal absorption off
- Green: Elastic off, thermal absorption on
- Magenta: Elastic off, thermal absorption off
- Dip, rise, levelling-off roughly reproduced
- *No dip if thermal absorption is turned off*

Flat then slow rise

We have $\ln R_{AA}(E) \approx -\dfrac{n\Delta E(L)}{E}$

- If $E < E_{\rm LPM} = \mu^2 \ell_{\rm mfp} \sim T$,

$$\ln R_{AA} \approx -L \int_0^E d\omega \, \frac{dN}{d\omega dt} \left(\frac{n\omega}{E}\right) \approx \frac{nL}{E} \int_0^E d\omega\omega \left(\frac{\alpha_S}{\pi\omega} \frac{N_c}{\ell_{\rm mfp}}\right) \sim \text{Const.}$$

  Flat R_{AA}

- If $T < E < T(L/\ell_{\rm mfp})^2$,

$$\begin{aligned}
\ln R_{AA} &\approx -\frac{nL}{E} \int_0^{E_{\rm LPM}} d\omega\omega \left(\frac{\alpha_S}{\pi\omega} \frac{N_c}{\ell_{\rm mfp}}\right) \\
&\quad -\frac{nL}{E} \int_{E_{\rm LPM}}^{E} d\omega\omega \left(\frac{\alpha_S}{\pi\omega} N_c \sqrt{\frac{\mu^2}{\ell_{\rm mfp}\omega}}\right) \\
&\approx -\frac{n\alpha_S N_c}{\pi} \frac{L}{\ell_{\rm mfp}} \left(2\sqrt{\frac{T}{E}}\right)
\end{aligned}$$

  with $E_{\rm LPM} \sim T$. Slowly rising R_{AA}

Plateau at high p_T

- If $\ell_{\rm coh} > L$, effectively only a single scattering happens. $\Longrightarrow$ Goes back to BH

If $E > T(L/\ell_{\rm mfp})^2 = E_L$,

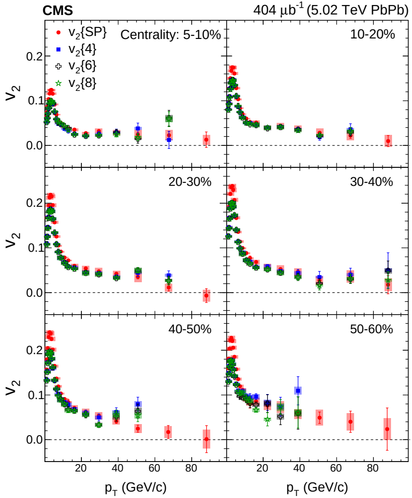
$$
\begin{aligned}
\ln R_{AA} \approx & -\frac{nL}{E}\int_0^{E_{\rm LPM}} d\omega\omega \left(\frac{\alpha_S}{\pi\omega}\frac{N_c}{\ell_{\rm mfp}}\right) \\
& -\frac{nL}{E}\int_{E_{\rm LPM}}^{E_L} d\omega\omega \left(\frac{\alpha_S}{\pi\omega}\frac{N_c}{\ell_{\rm mfp}}\sqrt{\frac{E_{\rm LPM}}{\omega}}\right) \\
& -\frac{nL}{E}\int_{E_L}^{E} d\omega\omega \left(\frac{\alpha_S}{\pi\omega}\frac{N_c}{L}\right) \\
\approx & -n\frac{\alpha_S N_c}{\pi}\left(1+\frac{E_L}{E}\right)
\end{aligned}
$$

This is *approximately constant* for large E.

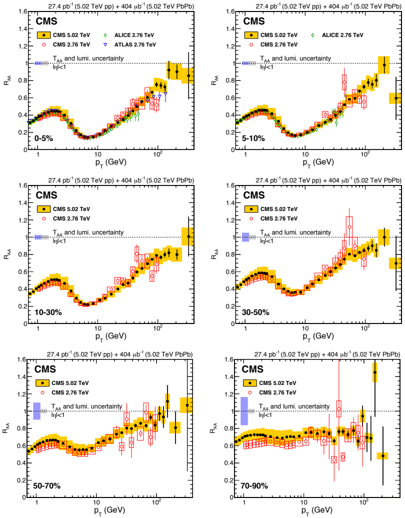
What is R_{AA} telling us?

- Dip-rise-flat feature qualitatively understandable
- Opaque medium
- Density of the medium
- Dip in R_{AA}: Could be an indirect indication of the initial temperature.
- Plateau at high p_T: Could be an indication that $\ell_{\rm coh} > L$ is reached.

Understanding high p_T part of v_2

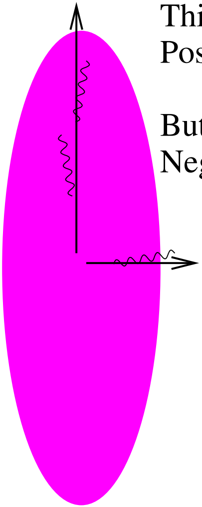


[CMS: arXiv:1702.00630]



[CMS: arXiv:1611.01664]

- v_2 and R_{AA}: Is there a relationship?



This jet loses more energy:
Positive v2

But it radiates more photons:
Negative photon v2

Understanding high p_T part of v_2

- Start with an isotropic distribution of high energy particles
- After going through the almond:

$$\begin{aligned} p_x &= E - \Delta E_x \\ p_y &= E - \Delta E_y \end{aligned}$$

That is,

$$p_x^2 \approx E^2 - 2\Delta E_x E$$

- Elliptic flow definition:

$$\begin{aligned} v_2 &= \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \\ &\sim \frac{2\Delta E_y E - 2\Delta E_x E}{2E^2} \\ &= \left(\frac{\Delta E_y - \Delta E_x}{E} \right) \end{aligned}$$

Approx. relationship between R_{AA} and v_2 at high p_T

- BDMPS: If $dN/p_T dp_T \approx 1/p_T^n$, $\ln R_{AA} \approx -n\dfrac{\Delta E}{E}$
- If $E < E_{\rm LPM} = \mu^2 \ell_{\rm mfp}$, $\ln R_{AA} \approx -\dfrac{n}{E}\dfrac{\alpha_S N_c}{\pi}\dfrac{L}{\ell_{\rm mfp}}$

$$v_2 \sim \left(\frac{\Delta E_y - \Delta E_x}{E}\right) \propto (L_y - L_x)$$

Flat v_2

- If $E_{\rm LPM} < E < E_L = L^2\mu^2/\ell_{\rm mfp}$, $\ln R_{AA} \sim -\dfrac{n\alpha_S N_c}{\pi}\dfrac{L}{\ell_{\rm mfp}}\left(2\sqrt{\dfrac{E_{\rm LPM}}{E}}\right)$

$$v_2 \sim \left(\frac{\Delta E_y - \Delta E_x}{E}\right) \propto (L_y - L_x)\sqrt{\frac{\hat{q}}{E}}$$

Slowly falling v_2

What I have been trying to do

Hopefully,

- Make you excited about studyig heavy ion physics
- Make you see the *big picture*
- Make you *think about* physics in general
- Make you *think about* jet physics in particular
- Convince you that *understanding* physics is a lot of fun and can lead to new insights and discoveries

Before I end ...

Success in your Physics career

Disclaimer: These are my own thoughts. Everyone is different. Take these with a grain of salt.

- Passion for Physics!
- Communication skill – Improve your English
  - Writing skill – Writing guide books are actually helpful
    A good one: *BUGS in Writing: A Guide to Debugging Your Prose*, by Lyn Dupre
  - Presentation skill – Have a look at R. Geroch's *"Suggestions for Giving Talks"*, arXiv:gr-qc/9703019v1. Scripts help including possible Q & A.
  - Debate skill – Practice thinking in English. Don't beat around the bushes. Get to the point.
  - Social communication skill – Read novels (paperbacks are better), watch sitcoms, know the culture, slang, ...

Writing/Presentation skill

Approach it as if you're writing a story

Story

- Introduction – Make the reader interested in the rest of the story
- Expanding the story – Main characters, main events, conflicts, puzzles, plot twists, ...
- Resolution – Story escalates to the ultimate resolution by a big battle, saved by the heroes/heroines.
- Ending – Tie up loose ends. Make the reader want to read the sequel.

Article/Talk

- Introduction – Make the reader interested in the rest of the paper/talk
- Expanding the point – Main physics points, main data, conflicts, puzzles, plot twists, ...
- Resolution – What big physics the new data/theory illuminates/resolves. Saved by the heroes/heroines.
- Conclusion – Tie up loose ends. Make the reader want to read the sequel.