Precise measurement of m_W using threshold scan method

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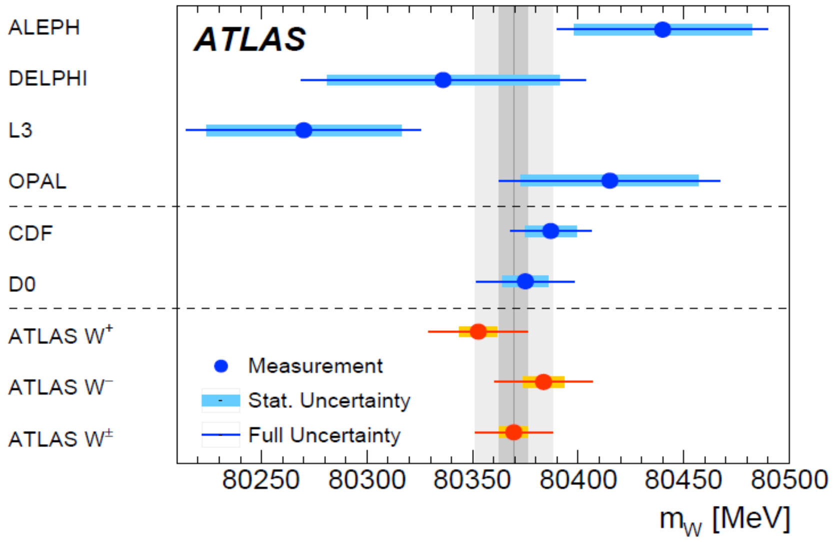
Outline

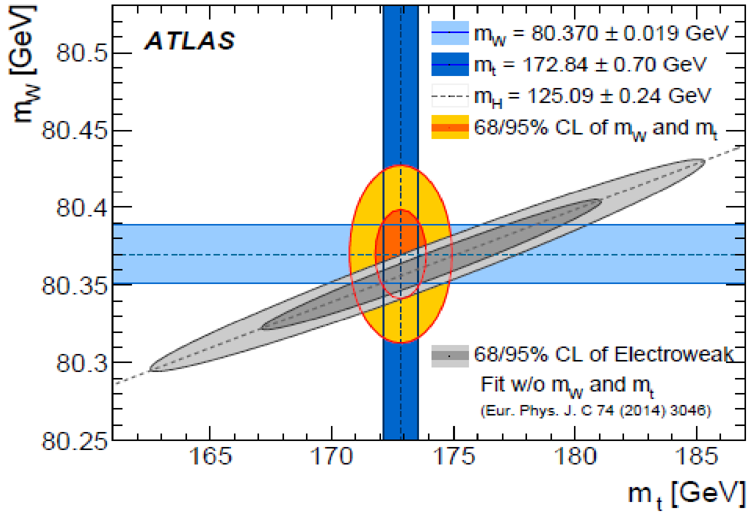
- Motivation
- Methodology
- Theoretical tool
- Statistical and systematic uncertainties
- Data taking schemes
- Summary

Motivation

https://arxiv.org/abs/1701.07240

- The m_W plays a central role in precision EW measurements and in constraint on the SM model through global fit.
- The direct measurement suffers the large systematic uncertainty, such as radiative correction, EW corrections, modeling of hadronization.
- For the threshold scan method, the precision is limited by the statistics of data and the accelerator performance (this work).





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Methodology

- Why?

$$\sigma_{WW}(m_W, \Gamma_W, \sqrt{s}) = \frac{N_{obs}}{L\epsilon P} \qquad (P = \frac{N_{WW}}{N_{WW}+N_{bkg}})$$

so m_W (Γ_W) can be obtained by fitting the N_{obs}, with the theoretical formula σ_{WW}

- How?

$\Delta m_W, \Delta\Gamma_W$						
N_{obs}	L	ϵ	P	E	E_{BS}	……

In general, these uncertainties are dependent on $\sqrt{s}$, so it is a optimization problem when considering the data taking.

- If …, then?

With the configurations of $L, \Delta L, \Delta E$ …, we can obtain: m_W~?

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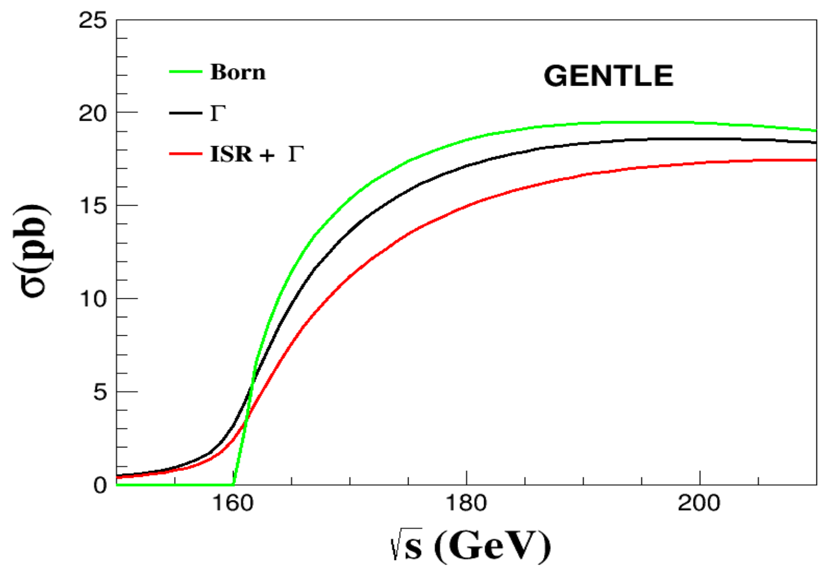
Theoretical Tool

- The σ_{WW} is a function of $\sqrt{s}$, m_W and Γ_W, which is calculated with the GENTLE package in this work

	CC11	ISR	Coulumb	EW	QCD
Gentle	√	√	√	√	√

- The ISR correction is also calculated by convoluting the Born cross sections with QED structure funtion, with the radiator up to NL $O(\alpha^2)$ and $O(\beta^3)$



1. On the QED radiator at order α^3
2. Higher Order Radiative Correction

Statistical and systematic uncertainties

Statistical uncertainty

- $\Delta\sigma_{WW} = \sigma_{WW} \times \frac{\Delta N_{WW}}{N_{WW}} = \sigma_{WW} \times \frac{\sqrt{N_{WW}+N_{bkg}}}{N_{WW}}$

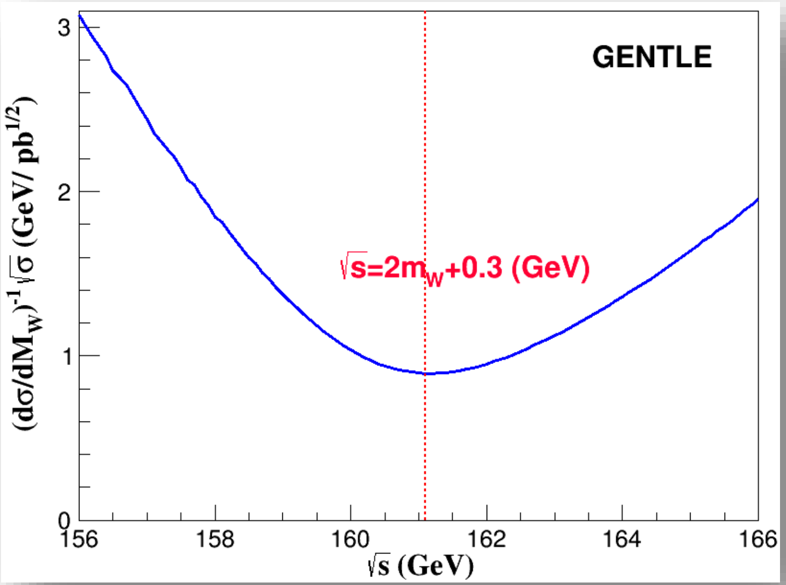
$$= \sqrt{\frac{\sigma_{WW}}{L\epsilon P}} \qquad (P = \frac{N_{WW}}{N_{WW}+N_{bkg}})$$

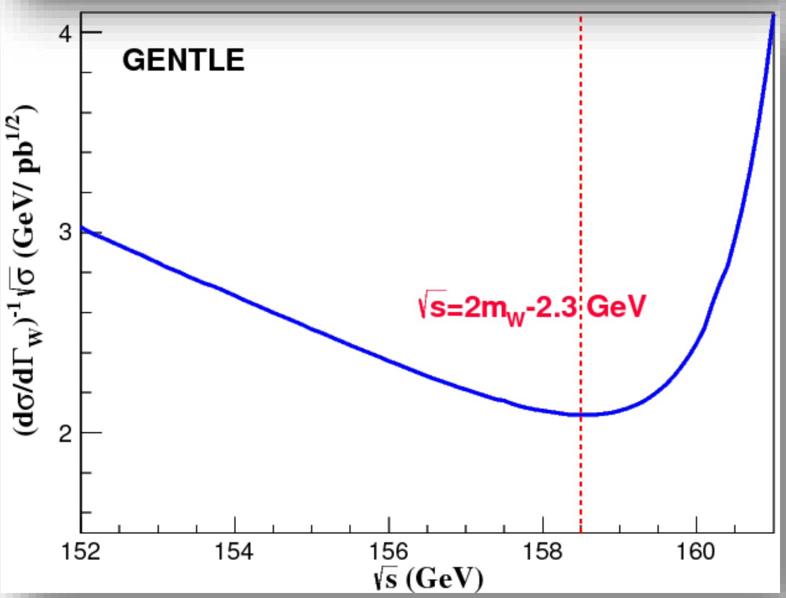
- $\Delta m_W = \left(\frac{\partial\sigma_{WW}}{\partial m_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial\sigma_{WW}}{\partial m_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$

- $\Delta\Gamma_W = \left(\frac{\partial\sigma_{WW}}{\partial\Gamma_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial\sigma_{WW}}{\partial m_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$

With L=3.2ab^{-1}, ϵ=0.8, P=0.9:

Δm_W=0.6 MeV, $\Delta\Gamma_W$=1.4 MeV (individually)





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Statistical uncertainty

- When there are more than one data point, we can measure both m_W and Γ_W.
- With the chisquare defined as:

$$\chi^2 = \sum_i \frac{(N_{\text{fit}^i} - N^i_{\text{obs}})^2}{N^i_{\text{obs}}} = \frac{(\mathcal{L}\epsilon P)^i(\sigma^i_{\text{fit}} - \sigma^i_{\text{obs}})^2}{\sigma^i_{\text{obs}}}$$

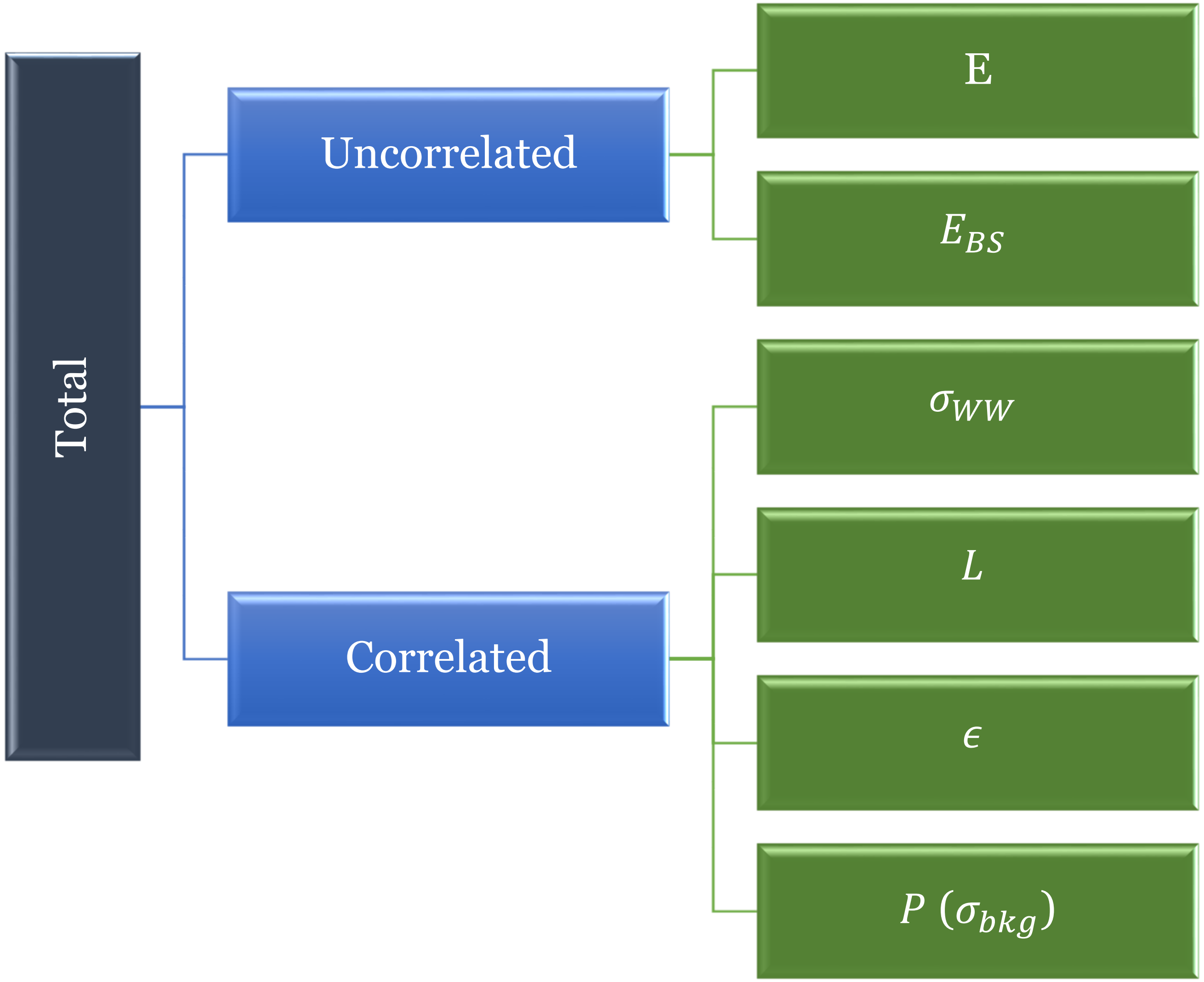
the error matrix is in the form:

$$V = \frac{1}{2} \times \begin{pmatrix} \frac{\partial^2 \chi 2}{\partial m_W^2} & \frac{\partial^2 \chi 2}{\partial m_W \partial \Gamma_W} \\ \frac{\partial^2 \chi 2}{\partial m_W \partial \Gamma_W} & \frac{\partial^2 \chi 2}{\partial m_{\Gamma_W}^2} \end{pmatrix}^{-1} = \sum_i \begin{pmatrix} \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\text{obs}}}(\frac{\partial \sigma}{\partial m_W})^2 & \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\text{obs}}} \frac{\partial \sigma}{\partial m_W} \frac{\partial \sigma}{\partial \Gamma_W} \\ \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\text{obs}}} \frac{\partial \sigma}{\partial m_W} \frac{\partial \sigma}{\partial \Gamma_W} & \frac{(\mathcal{L}\epsilon P)^i}{\sigma^i_{\text{obs}}}(\frac{\partial \sigma}{\partial m_W})^2 \end{pmatrix}^{-1}$$

- When the number of fit parameter reduce to 1:

$$\Delta m_W = \left(\frac{\partial \sigma_{WW}}{\partial m_W}\right)^{-1} \times \Delta\sigma_{WW} = \left(\frac{\partial \sigma_{WW}}{\partial m_W}\right)^{-1} \times \sqrt{\frac{\sigma_{WW}}{L\epsilon P}}$$

Systematic uncertainty



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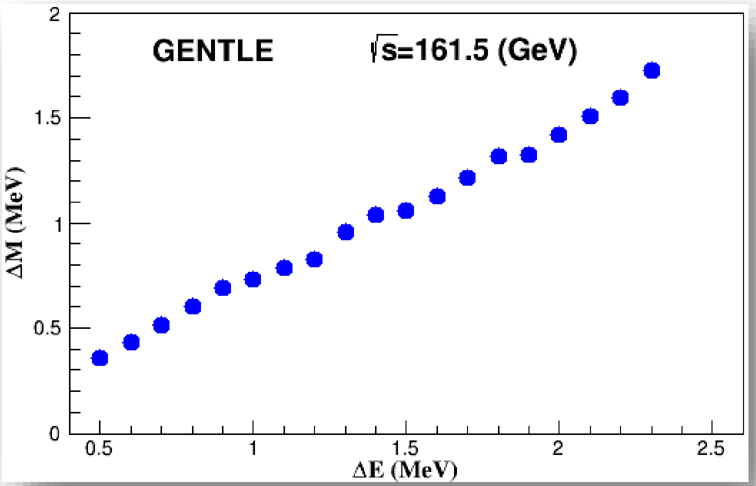
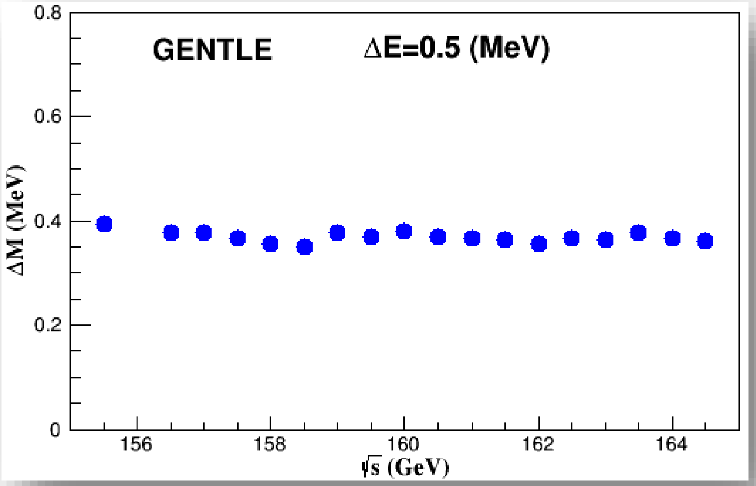
Beam energy uncertainty ΔE

- With ΔE, the total energy becomes:

$$E = G\left(E_p, \Delta E\right) + G(E_m, \Delta E)$$

- E is used in the data simulation, and $E_0 = E_p + E_m$ is for the fit formula.
- The Δm_W will be large when ΔE increase, and almost independent with $\sqrt{s}$.





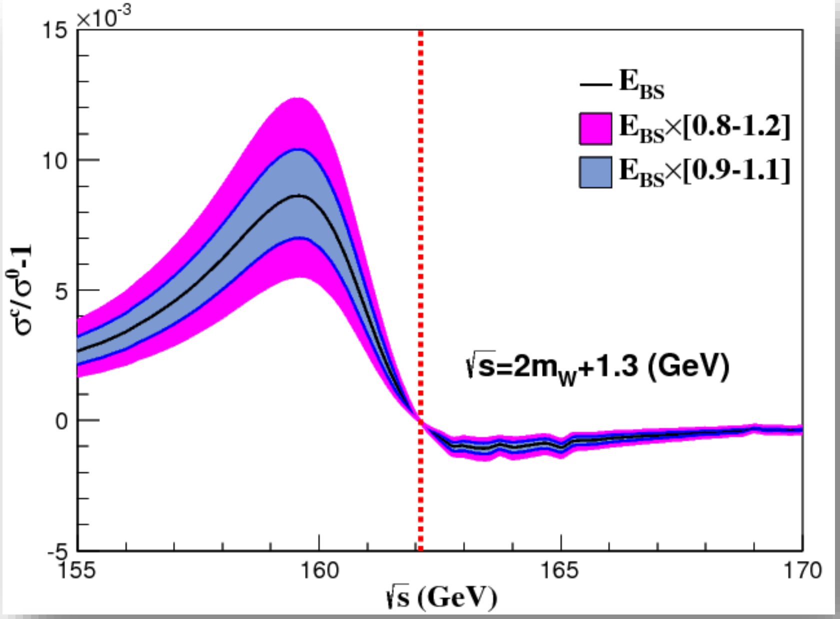
Beam energy spread uncertainty ΔE_{BS}

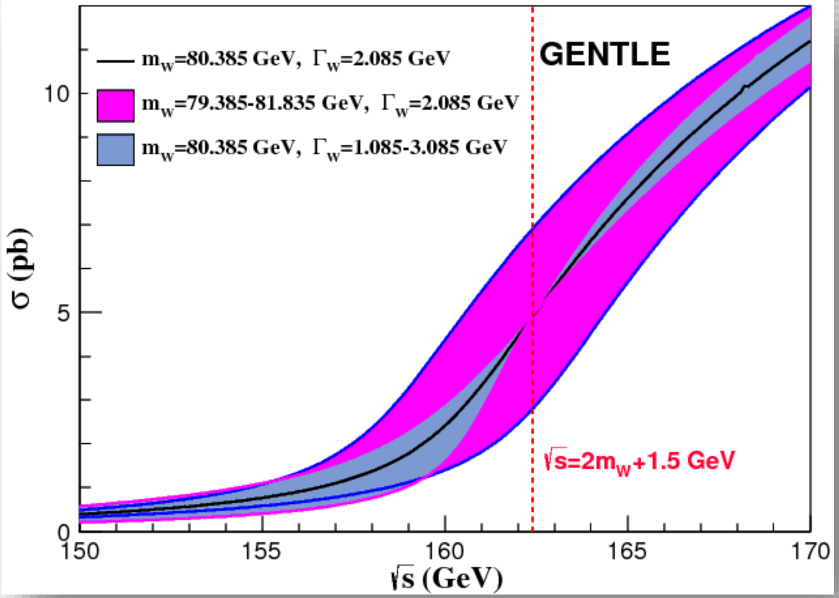
- With E_{BS}, the σ_{WW} becomes:

$$\sigma_{WW}(E) = \int_0^{\infty} \sigma_{WW}(E') \times G(E,E')dE'$$

$$\approx \int_{E-6\sqrt{2}\Delta E_{BS}}^{E+6\sqrt{2}\Delta E_{BS}} \sigma(E') \times \frac{1}{\sqrt{2\pi}\sqrt{2}E_{BS}} e^{\frac{-(E-E')^2}{2(\sqrt{2}E_{BS})^2}} dE'$$

- $E_{BS} + \Delta E_{BS}$ is used in the simulation, and E_{BS} is for the fit formula.
- The m_W insensitive to ΔE_{BS} when taking data around 162.1 GeV





Correlated sys. uncertainty

- The correlated sys. uncertainty includes: $\Delta L,\ \Delta\sigma_{WW},\ \Delta\epsilon,\ \Delta P\ldots$
- Since $N_{obs} = L \cdot \sigma \cdot \frac{\epsilon}{P}$, these uncertainties affect m_W and Γ_W in same way.
- We take L as an example, and use the total correlated sys. uncertainty in data taking optimization:

$$\sigma^{sys}(corr) = \sqrt{\Delta L^2 + \Delta\sigma_{WW}^2 + \Delta\epsilon^2 + \Delta P^2}$$

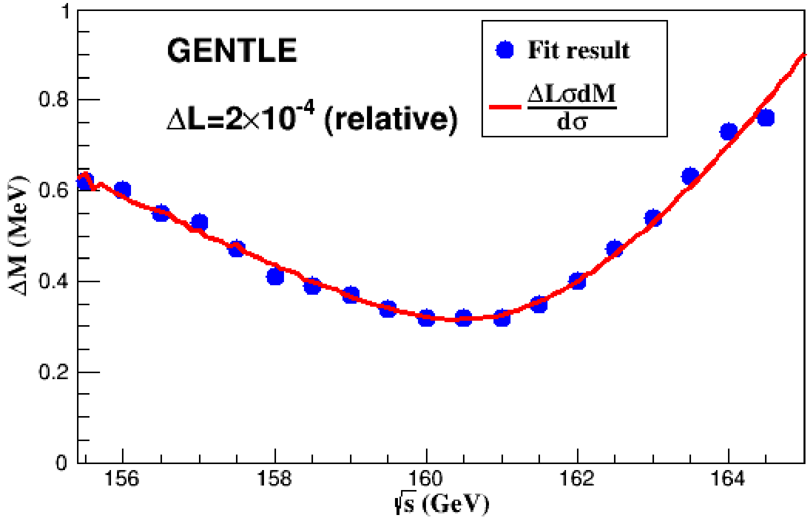
Correlated sys. uncertainty ΔL (1)

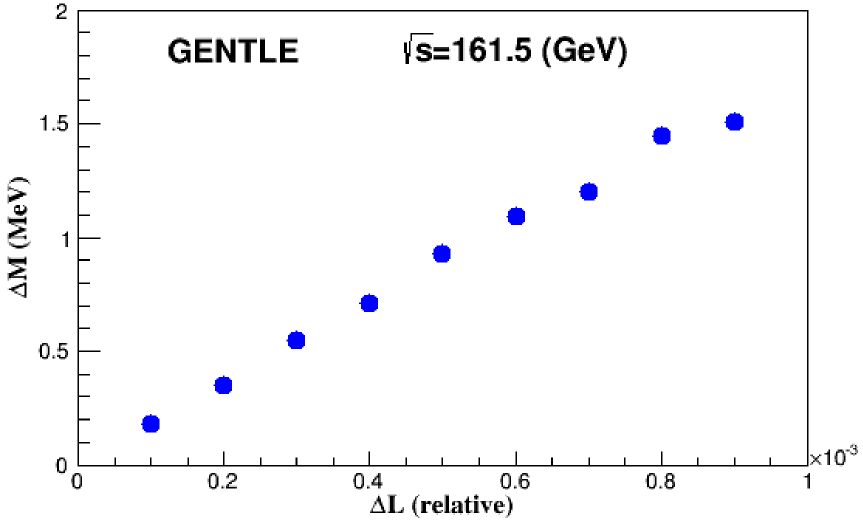
- With ΔL (relative), the L becomes:

$$L = G(L^0, \Delta L \cdot L^0)$$

  L is used for simulation, and L^0 is for fit

$$\Delta m_W = \frac{\partial m_W}{\partial \sigma_{WW}} \sigma \Delta L$$





- The Δm_W almost increases linearly along with ΔL

Correlated sys. uncertainty ΔL (2)

- If there is more than 1 data taking point, the correlated sys. uncertainty can be constructed into the χ^2:

$$\chi^2 = \sum_{i}^{n} \frac{(y_i - h \cdot x_i)^2}{\delta_i^2} + \frac{(h-1)^2}{\delta_c^2}$$

y_i, x_i are the true and fit results, h is a free parameter, δ_i and δ_c are the independent and correlated uncertainties.

- There will be no bias in the fit result with this method, and the $\Delta m_W(\Delta L)$ will be reduced.

Data taking scheme

Data taking scheme

One point

- Smallest Δm_W, $\Delta\Gamma_W$ (stat.)
- Large sys. Uncertainties
- Only for m_W or Γ_W, without correlation

Two points

- Measure m_W and Γ_W simultanously
- Without the correlation

Three points

- Measure m_W and Γ_W simultaneously, with the correlation
- Maybe increase the Δm_W, $\Delta\Gamma_W$ (stat.)

With $L = 3.2\ ab^{-1}, \epsilon P = 0.72$

Taking data at one point (just for m_W)

There are two special energy points :

- The one which most statistical sensitivity to m_W:

  Δm_W(stat.) ~0.59 MeV at E=161.2 GeV

  (with $\Delta\Gamma_W$ and ΔE_{BS} effect)

- The one Δm_W(stat)~0.68 MeV at $E \approx 162.5$ GeV

  (with small $\Delta\Gamma_W, \Delta E_{BS}$ effects)

With ΔL ($\Delta\sigma_{WW}, \Delta\epsilon, \Delta P$)<$10^{-4}$, σ^{sys}(corr)<2×10^{-4}
ΔE=0.5MeV, ΔE_{BS}=10^{-2}, $\Delta\Gamma_W$=42MeV)

$\sqrt{s}$(GeV)	161.2	162.5
σ^{sys}(corr)	0.35	0.44
ΔE	0.36	0.37
ΔE_{BS}	0.12	-
$\Delta\Gamma_W$	8	-
Stat.	0.59	0.68
Δm_W(MeV)	8	0.9

Taking data at two energy points

- To measure Δm_W and $\Delta\Gamma_W$, we scan the energies and the luminosity fraction of the two data points:

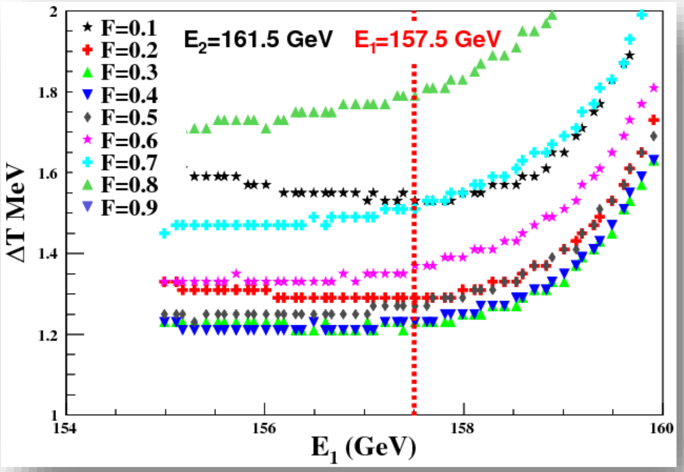
  1. $E_1, E_2 \in [155,\ 165]$ GeV, $\Delta E = 0.1$ GeV

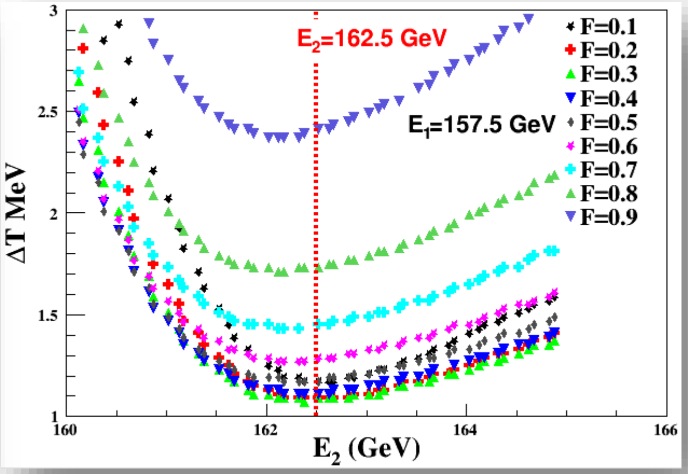
  2. $F \equiv \left(\frac{L_1}{L_2}\right) \in (0, 1)$, $\Delta F = 0.05$

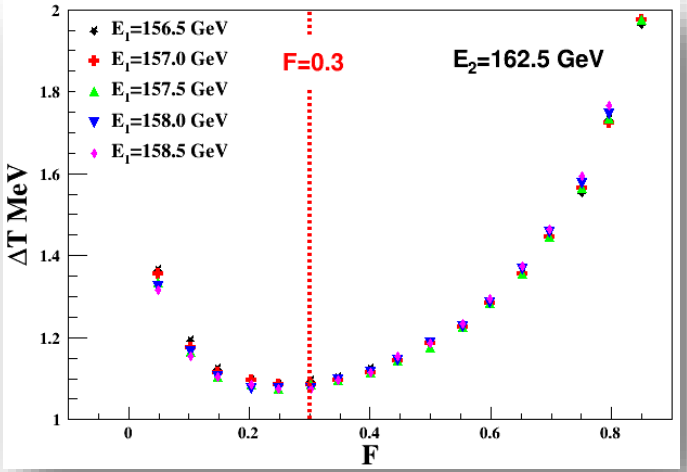
- Then we define the object function: $T = \mathrm{m_W} + 0.1\Gamma_W$ to optimize the scan parameters (assume m_W is more important than Γ_W).

Taking data at two energy points

- The 3D scan is performed, we just use 2D plots to illustrate the optimization results;
- When draw the ΔT change with one parameter, another is fixed with scanning of the third one;
- E_1=157.5 GeV, E_2=162.5 GeV (around $\frac{\partial \sigma_{WW}}{\partial \Gamma_W}=0$, $\frac{\partial \sigma_{WW}}{\partial E_{BS}}=0$) and F=0.3 are taken as the result.







$\Delta L\ (\Delta\sigma_{WW}, \Delta\epsilon, \Delta P)<10^{-4}$
$\sigma^{sys}(\text{corr})<2\times10^{-4}$
$E_{BS}=1.6\times10^{-3}$
$\Delta E=0.5\text{MeV}$
$\Delta\Gamma_W=42\text{MeV}$
$\Delta E_{BS}=0.01$

(MeV)	σ^{sys}(corr)	ΔE	ΔE_{BS}	Stat.	Total
Δm_W	0.48	0.38	-	0.81	1.02
$\Delta\Gamma_W$	0.22	0.54	0.88	1.06	2.9

Taking data at three energy points

- Fit parameters: m_W, Γ_W, h (associated with σ_{sys}^{corr})
- Scan parameters: $\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \mathrm{F}_1, \mathrm{F}_2$ ($\mathrm{F}_1 = \frac{\mathrm{L}_1}{L_2 + L_3}$, $F_2 = \frac{L_2}{L_3}$)
- Scan procedure:

  A. $E_1, E_2, E_3 \in (154, 165)$GeV, $F_1, F_2 \in (0,1)$, $\Delta E_i = 1$, $\Delta F_i = 0.1$ (σ_{stat})

  B. $E_1 \in (154, 160), E_2, E_3 \in (160, 164), F_1 \in (0,0.5), F_2 \in (0, 1), \Delta F_2 = 0.2$ (add σ_{sys}^{corr})

  C. Obtain the $\Delta \mathrm{m_W}, \Delta\Gamma_W$ with optimization result from step B ($\sigma_{stat} + \sigma_{sys}^{corr} + \Delta E + \Delta E_{BS}$)

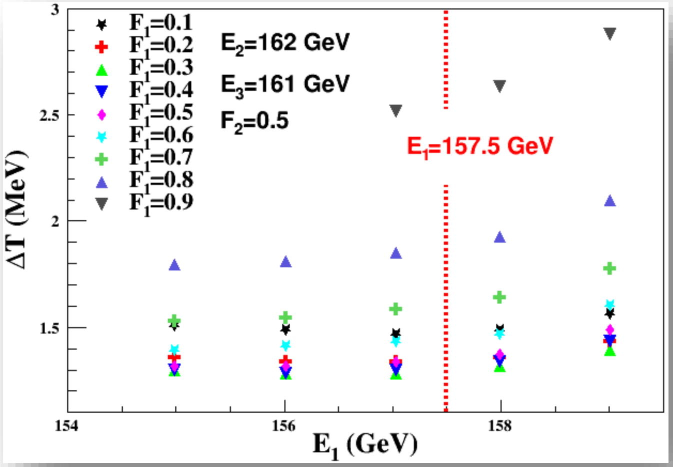
Taking data at three energy points

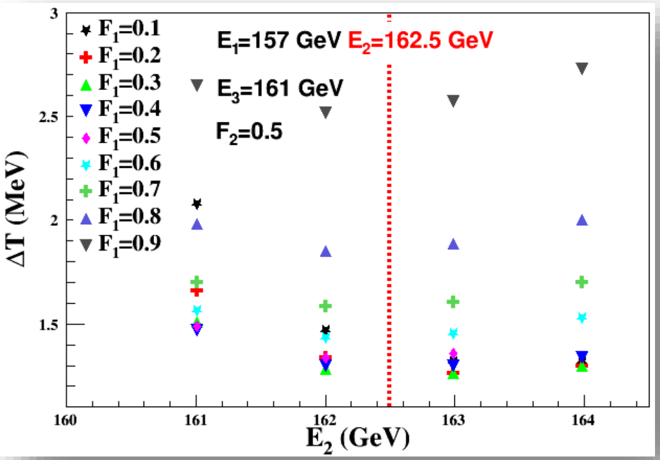
The optimized results:

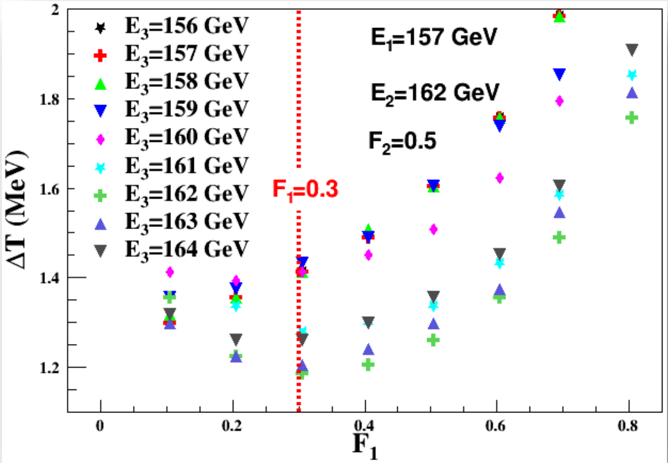
E_1	157.5 GeV
E_2	162.5 GeV
F_1	0.3
E_3	161.5 GeV
F_2	0.9

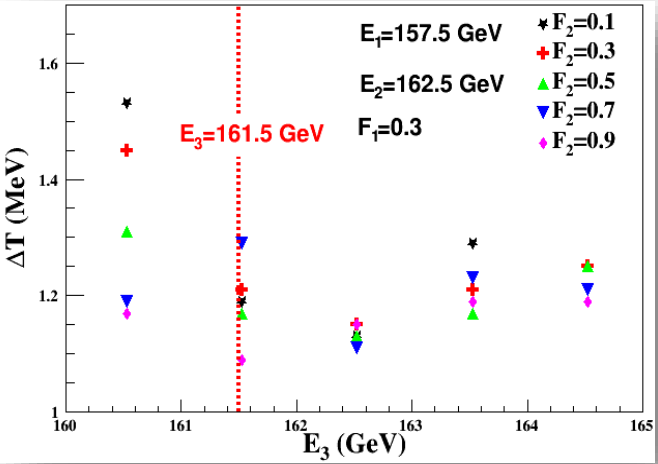
$\Delta m_W \sim 1$ MeV
$\Delta\Gamma_W \sim 2.8$ MeV

$\Delta L\ (\Delta\sigma_{WW}, \Delta\epsilon, \Delta P) < 10^{-4}$
$\sigma^{sys}(\text{corr}) < 2\times10^{-4}$
$E_{BS} = 1.6\times10^{-3}$
$\Delta E = 0.5$MeV
$\Delta\Gamma_W = 42$MeV
$\Delta E_{BS} = 0.01$









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Summary

- The precise measurement of m_W (Γ_W) is studied (threshold scan method)

- Different data taking schemes are investigated, based on the stat. and sys. uncertainties analysis.

- With the configurations :

$L_{tot} = 3.2\ ab^{-1}, \epsilon P = 0.72\ , \sigma_{sys}^{corr} = 2 \times 10^{-4}$

ΔE=0.5 MeV, E_{BS}=1.6$\times 10^{-3}$, ΔE_{BS}=0.01

Data points	Δm_W (MeV)	$\Delta\Gamma_W$ (MeV)
1	0.9	-
2	1.0	2.9
3	1.0	2.8

Thank you !

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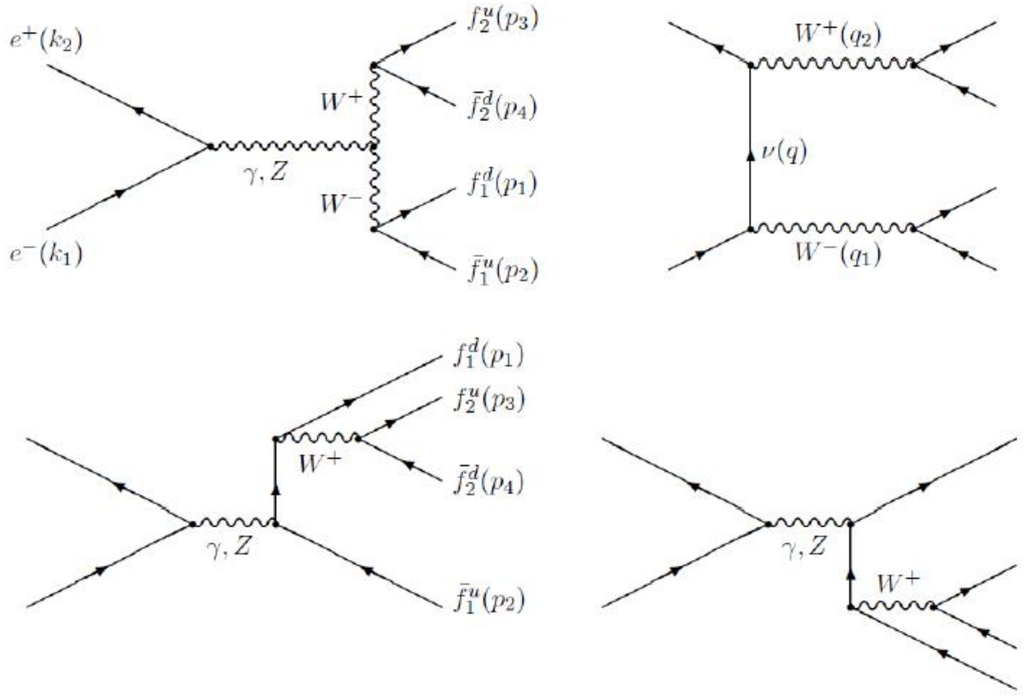
Backup

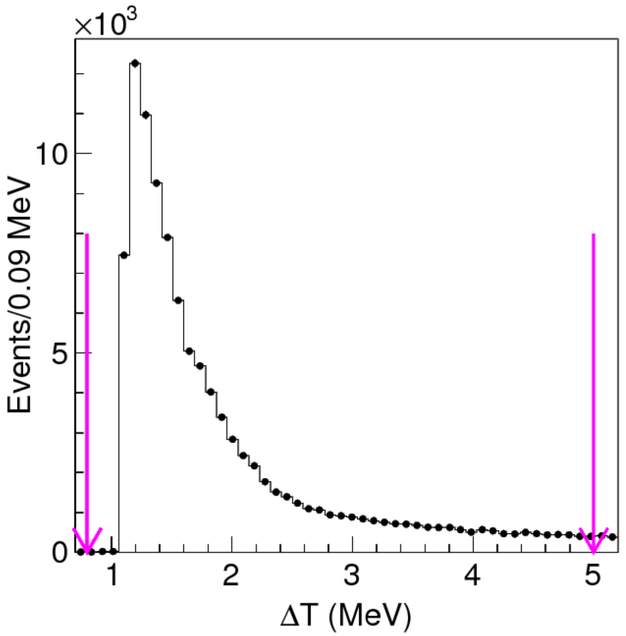
Theoretical Tool

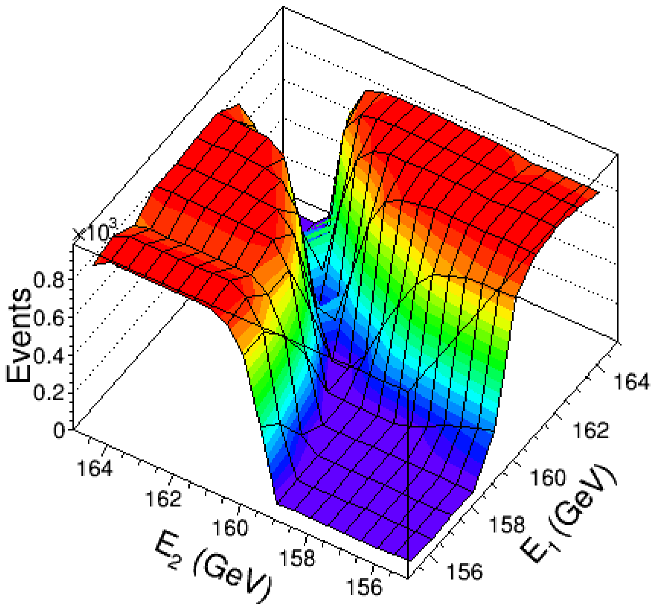
- Process: CC11, the minimal gauge-invariant subset of Feyman diagrams
- QED corrections: ISR, FSR, Coulomb, EM interaction of W pair ….
- EW correction: effective scale of the W pair production and decay process
- QCD correction



Optimizing results for two data points

E_1, E_2





The z axis is the accumulation of the fit results

$\Delta T \in (0.8, 3)$MeV is required in further study

The normal distribution of $E_1 : E_2$ is break, and divide into two parts.

E_1<160 GeV, E_2>160 GeV is used

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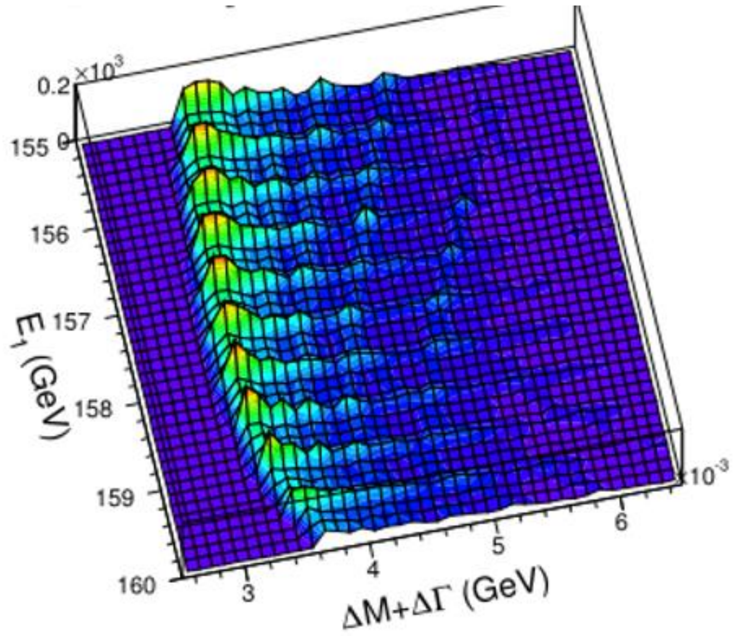
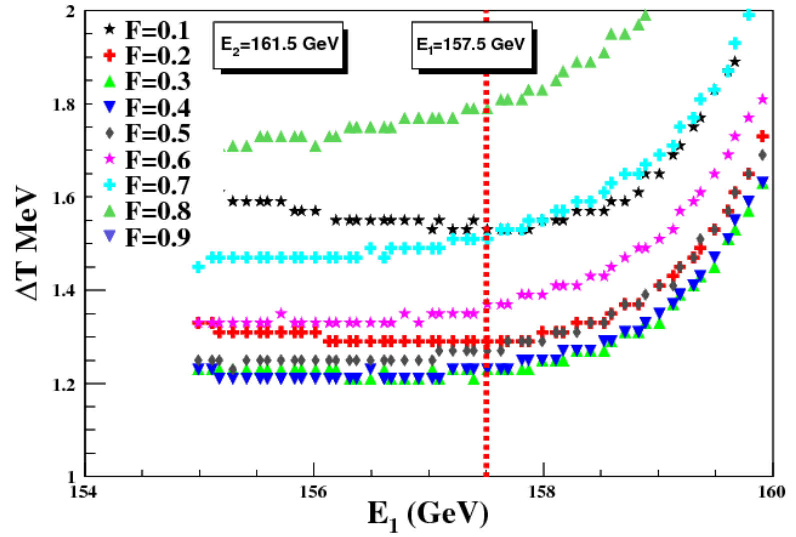
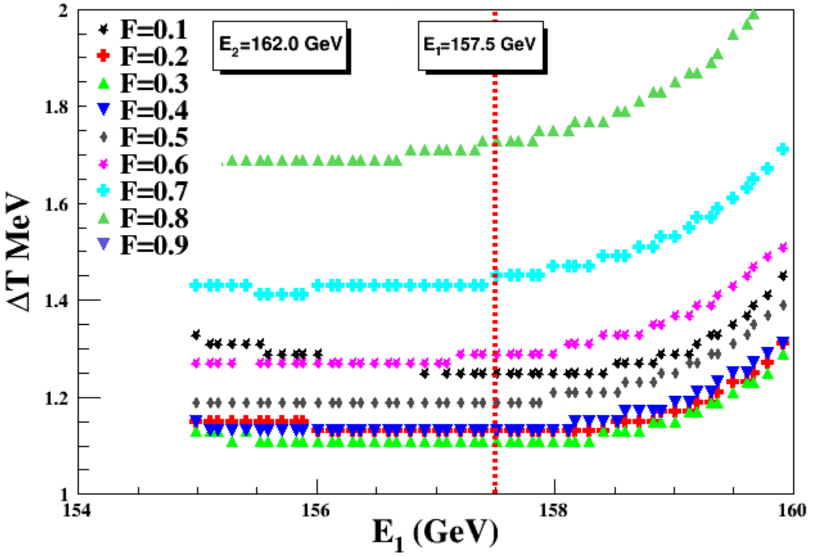
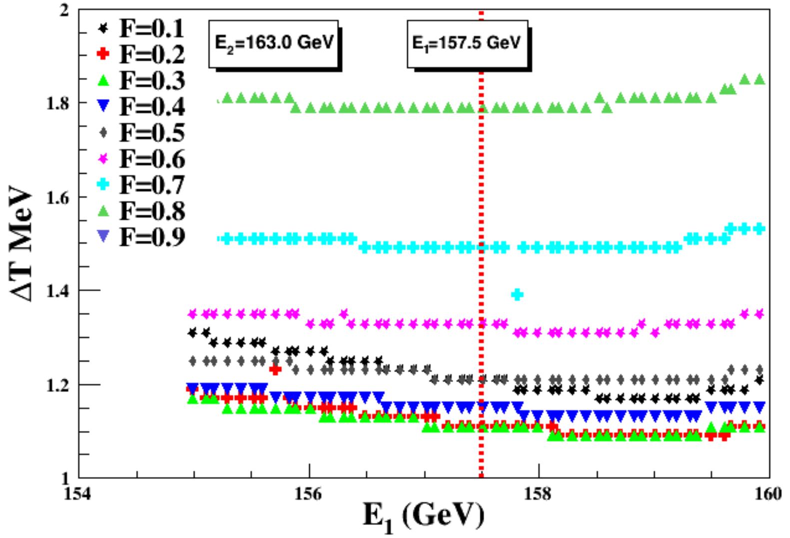
E_1









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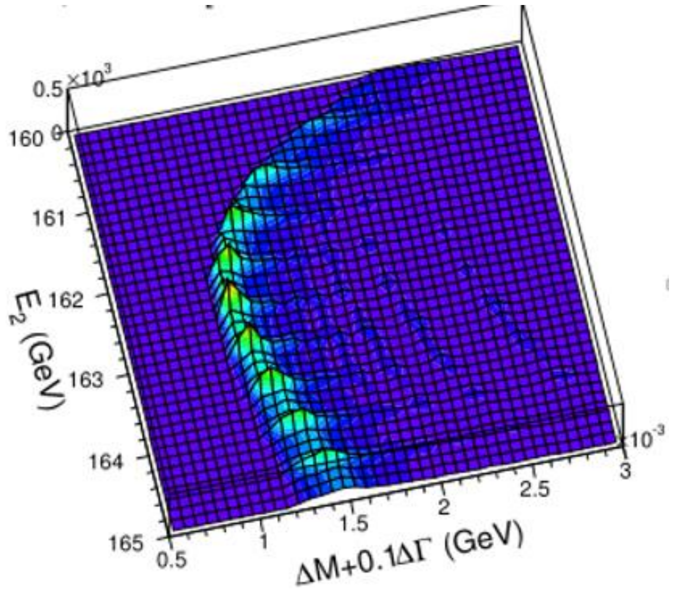
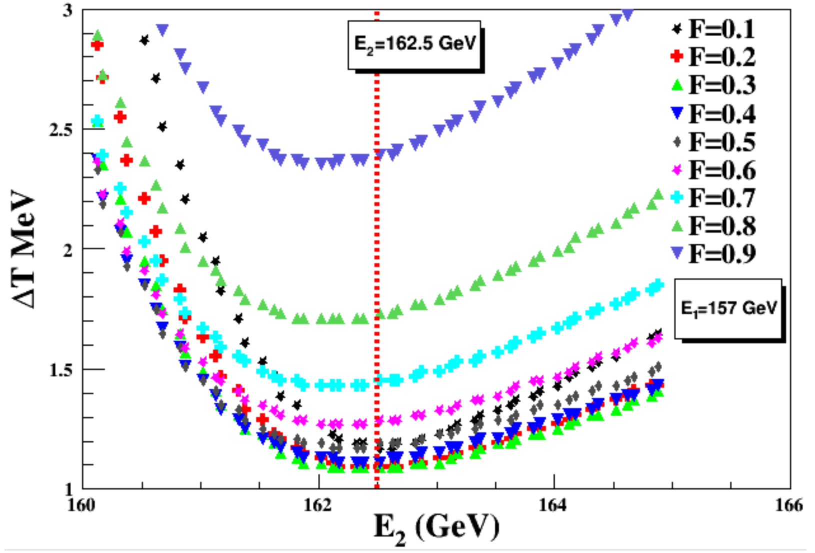
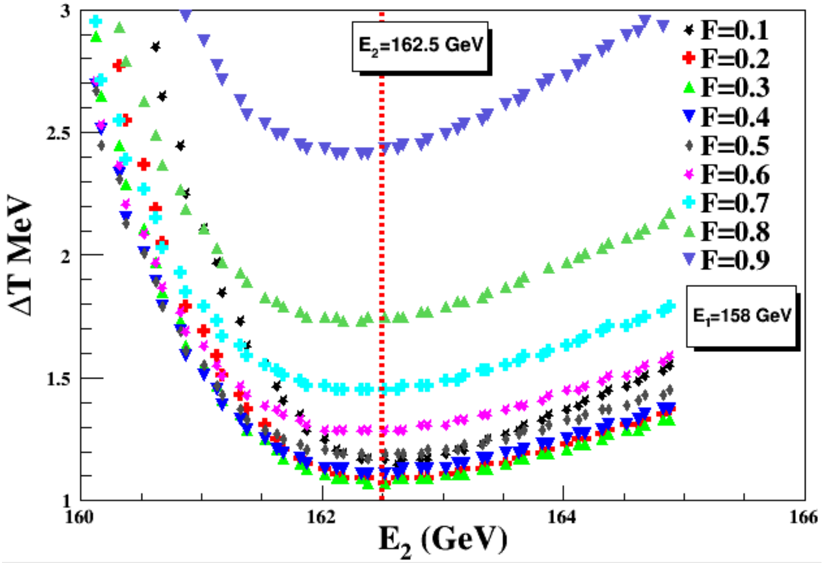
E_2



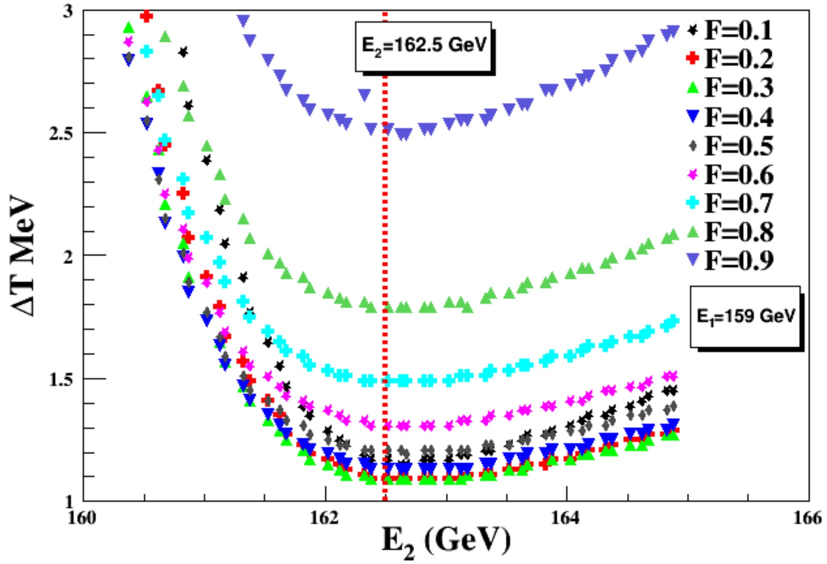
0.5 ×10³
E₂ (GeV)
ΔM+0.1ΔΓ (GeV)



ΔT MeV
E₂ (GeV)
E₂=162.5 GeV
E₁=157 GeV
F=0.1
F=0.2
F=0.3
F=0.4
F=0.5
F=0.6
F=0.7
F=0.8
F=0.9

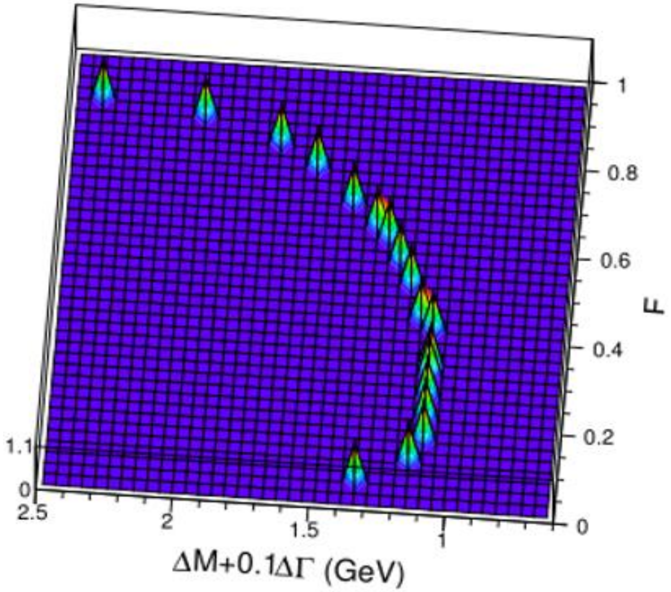


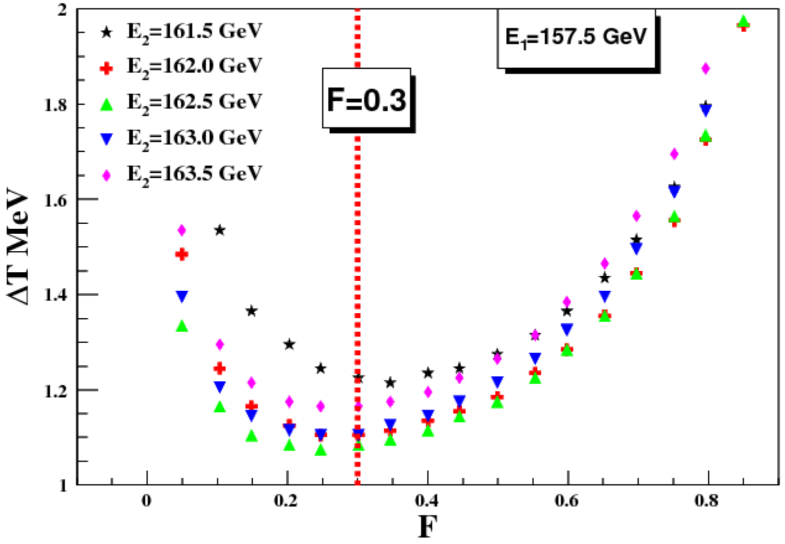
ΔT MeV
E₂ (GeV)
E₂=162.5 GeV
E₁=158 GeV
F=0.1
F=0.2
F=0.3
F=0.4
F=0.5
F=0.6
F=0.7
F=0.8
F=0.9

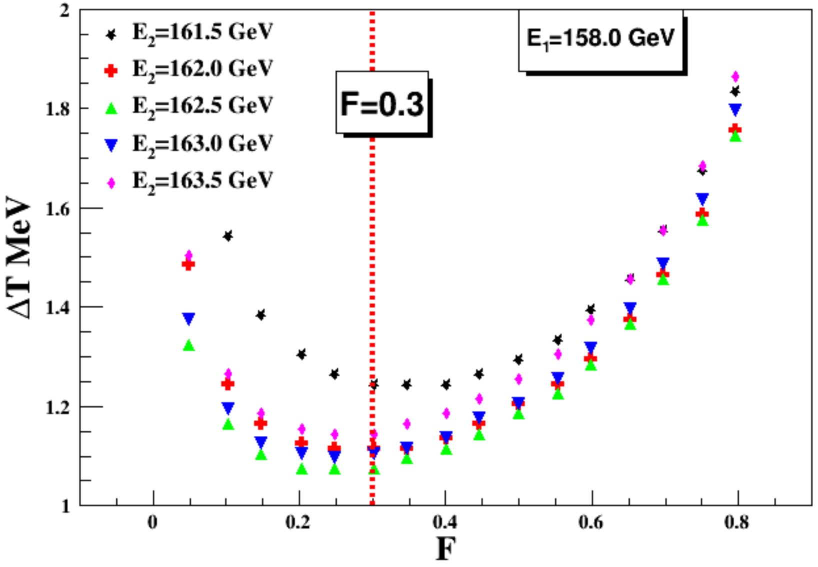


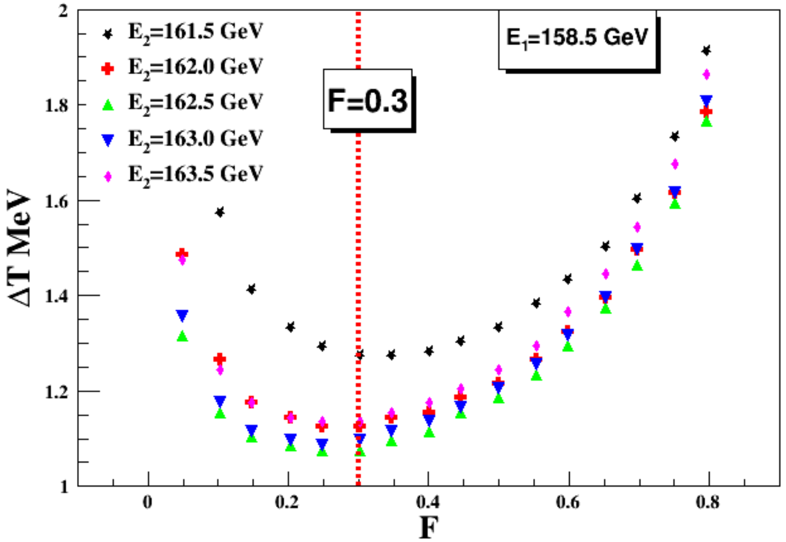
ΔT MeV
E₂ (GeV)
E₂=162.5 GeV
E₁=159 GeV
F=0.1
F=0.2
F=0.3
F=0.4
F=0.5
F=0.6
F=0.7
F=0.8
F=0.9

F









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With : E_1=157.5 GeV, E_2=162.5 GeV, σ^{sys}(corr.) = 2×10^{-4}(relative), ΔE_{BS}=1.6×10^{-3}(relative), ΔE=0.5 MeV

F	Δm_W (MeV)						$\Delta\Gamma_W$ (MeV)					
	Stat.	Sys.				Total	Stat.	Sys.				Total
		σ(corr.)	ΔE	ΔE_{BS}	σ_{tot}^{sys}			σ(corr.)	ΔE	ΔE_{BS}	σ_{tot}^{sys}	
0.1	0.71	0.47	0.35	–	0.92	0.92	4.6	0.31	0.52	0.43	0.74	4.7
0.15	0.73	0.47	0.37	–	0.94	0.94	3.7	0.28	0.52	0.55	0.8	3.8
0.2	0.76	0.45	0.37	–	0.96	0.96	3.3	0.26	0.52	0.60	0.84	3.4
0.25	0.78	0.46	0.37	–	0.98	0.98	3.0	0.23	0.51	0.76	0.94	3.1
0.3	0.81	0.48	0.38	–	1.02	1.02	2.7	0.22	0.54	0.88	1.06	2.9

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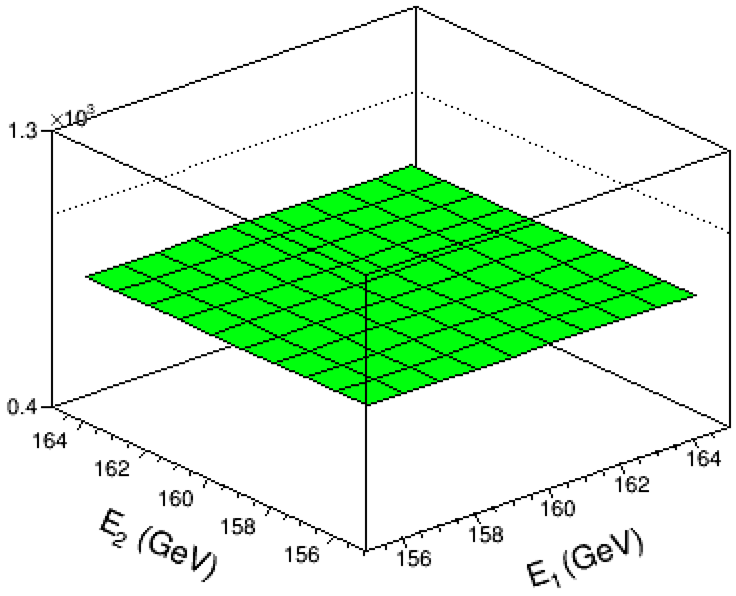
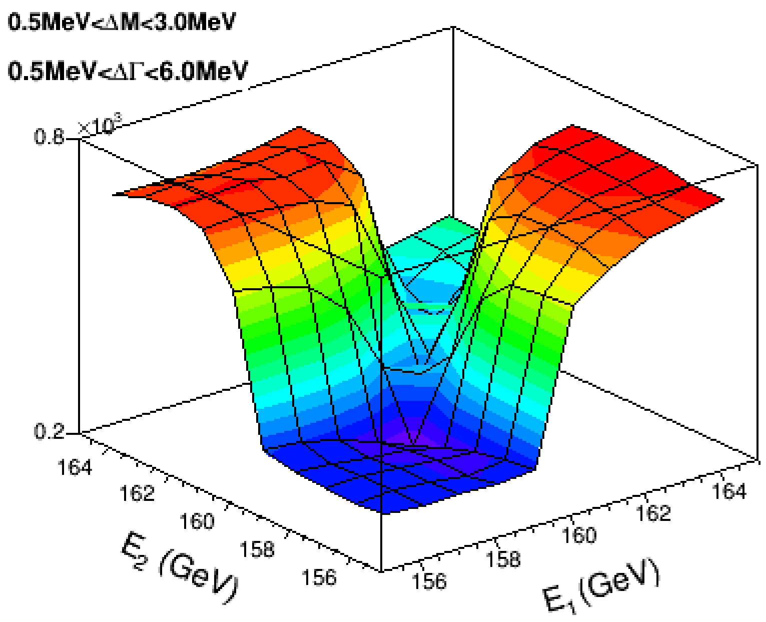
Optimizing results for three data points

Step A: E_1, E_2





The z axis is the acumulation of the fit result. The edge of the distributions will affect the optimization results.

E_1<160, E_2>160 GeV is used in further optimization

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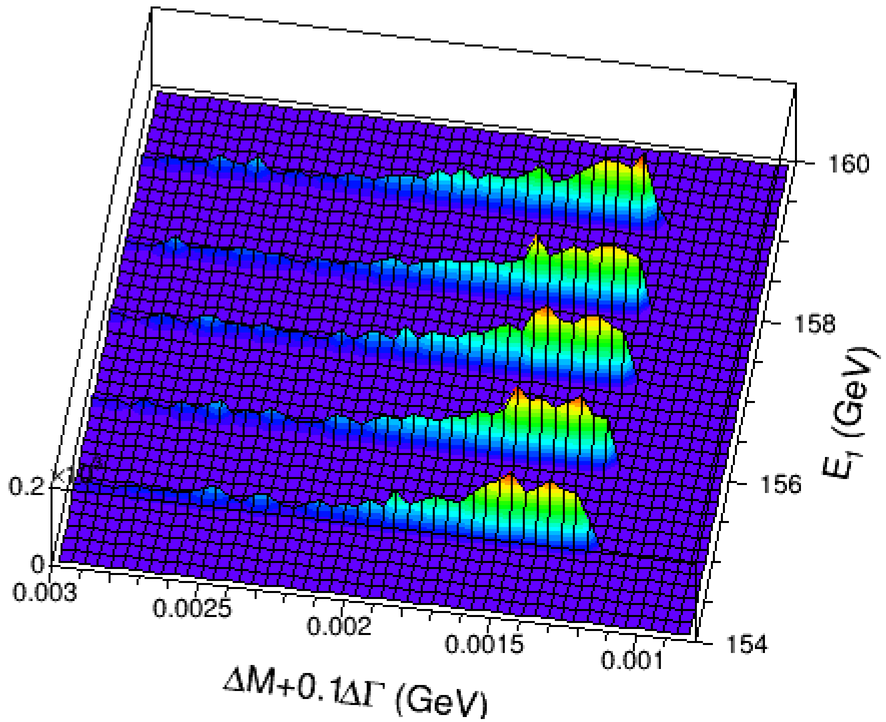
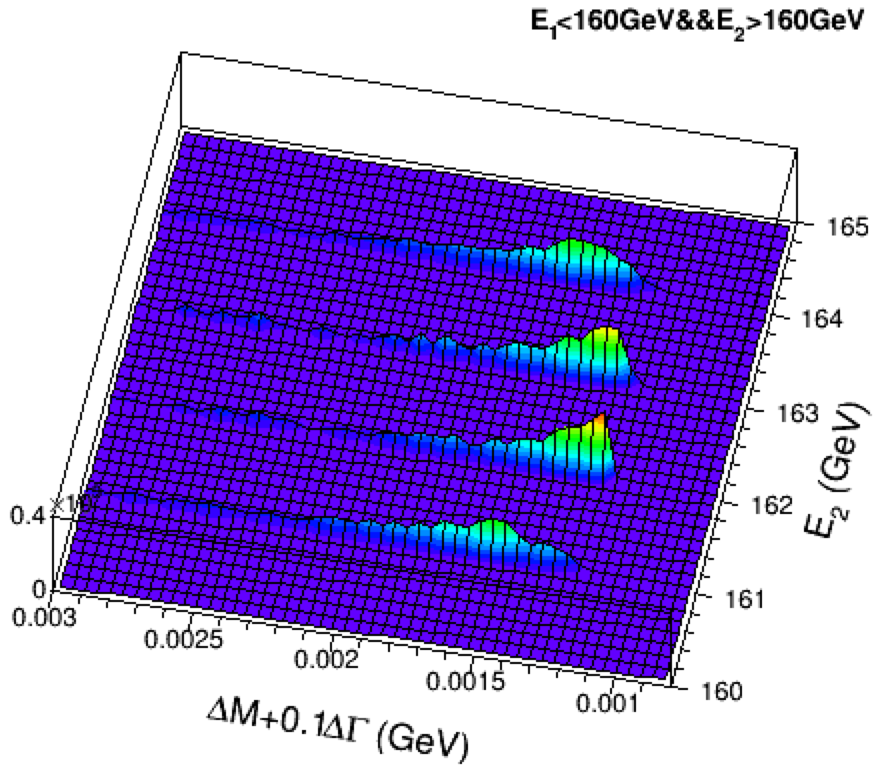
Step A: E_1, E_2

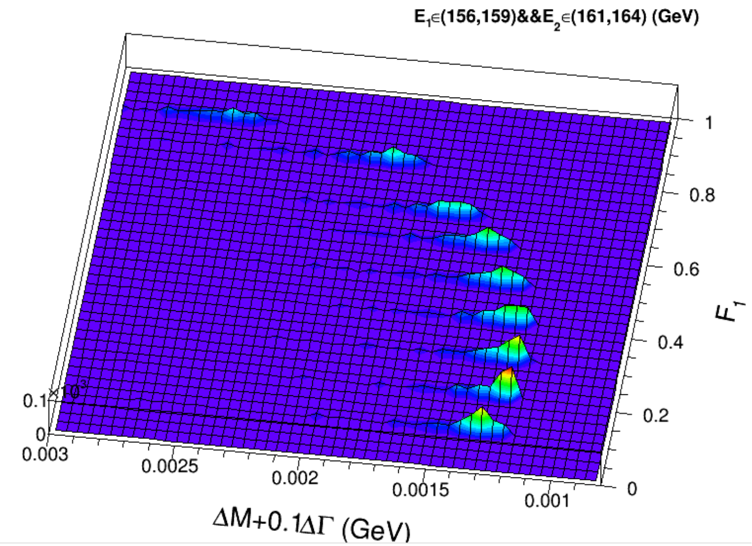




The optimal regions of E_1, E_2 are similar as two data points:

E_1~(157,158) GeV, E_2~(162, 163)GeV

Step A: F_1



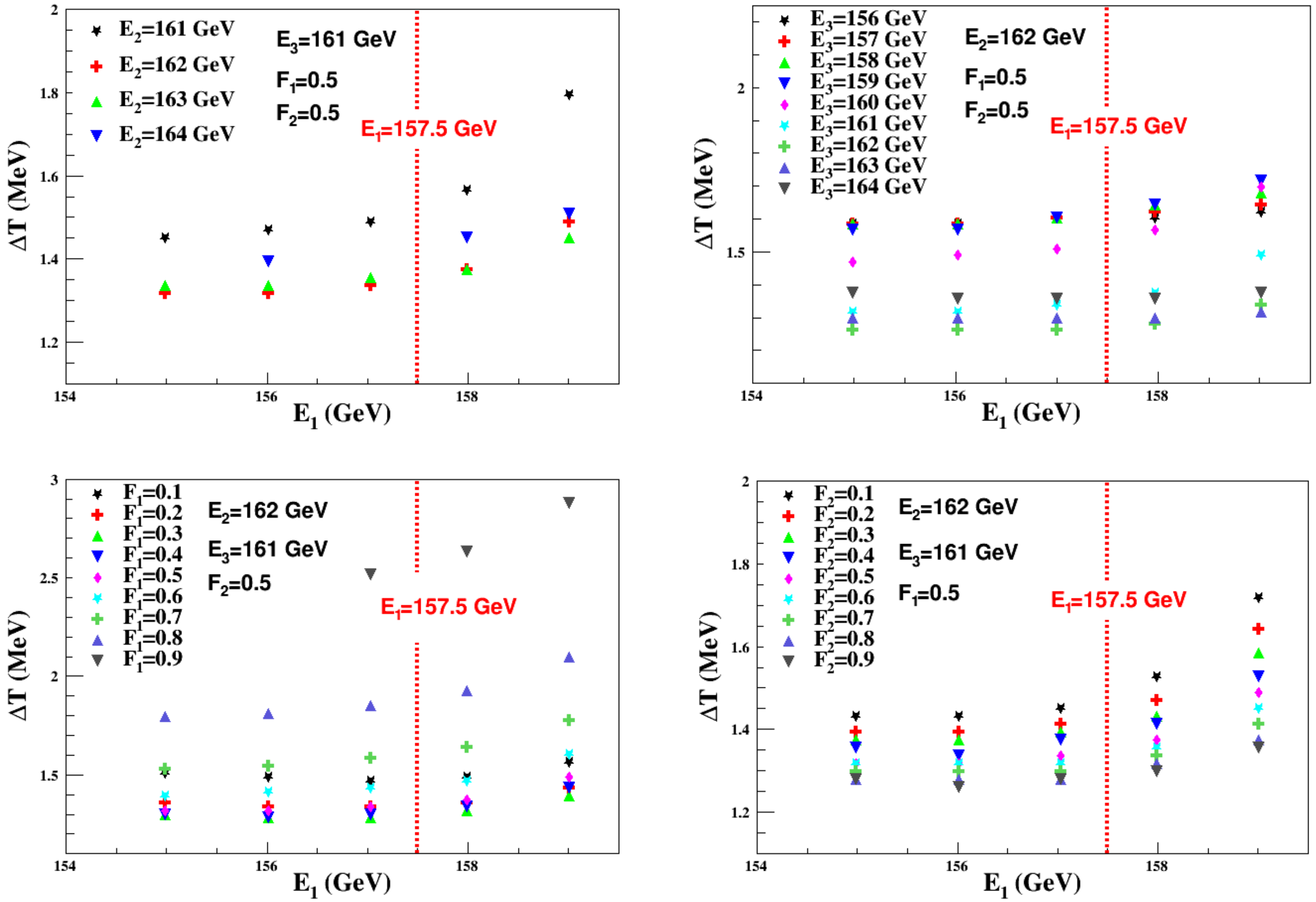
The optimal region of F_1 is similar as two data points: $F_1 \sim 0.3$

Optimization of E_1

- Default values:
  E_2=162 GeV
  E_3=161 GeV
  $F_1 = F_2$= 0.5

- We change one variable with fixing other three, and get the ΔT along E_1 distributions.

- E_1=157.5 GeV is taken as the optimized result.

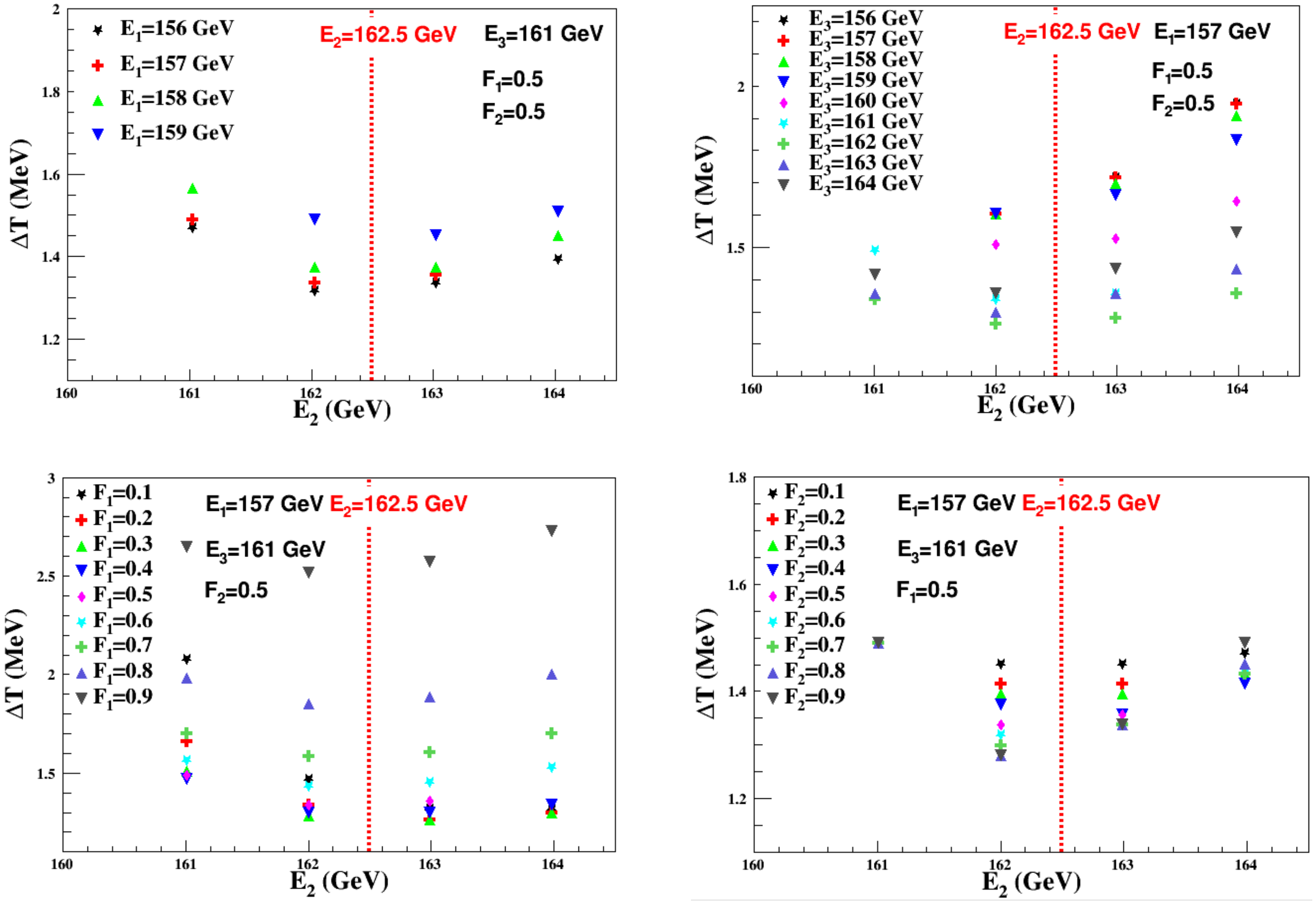


Optimization of E_2

- Default values:
  E_1=157 GeV
  E_3=161 GeV
  $F_1 = F_2$= 0.5

- We change one variable with fixing other three, and get the ΔT along E_2 distributions.

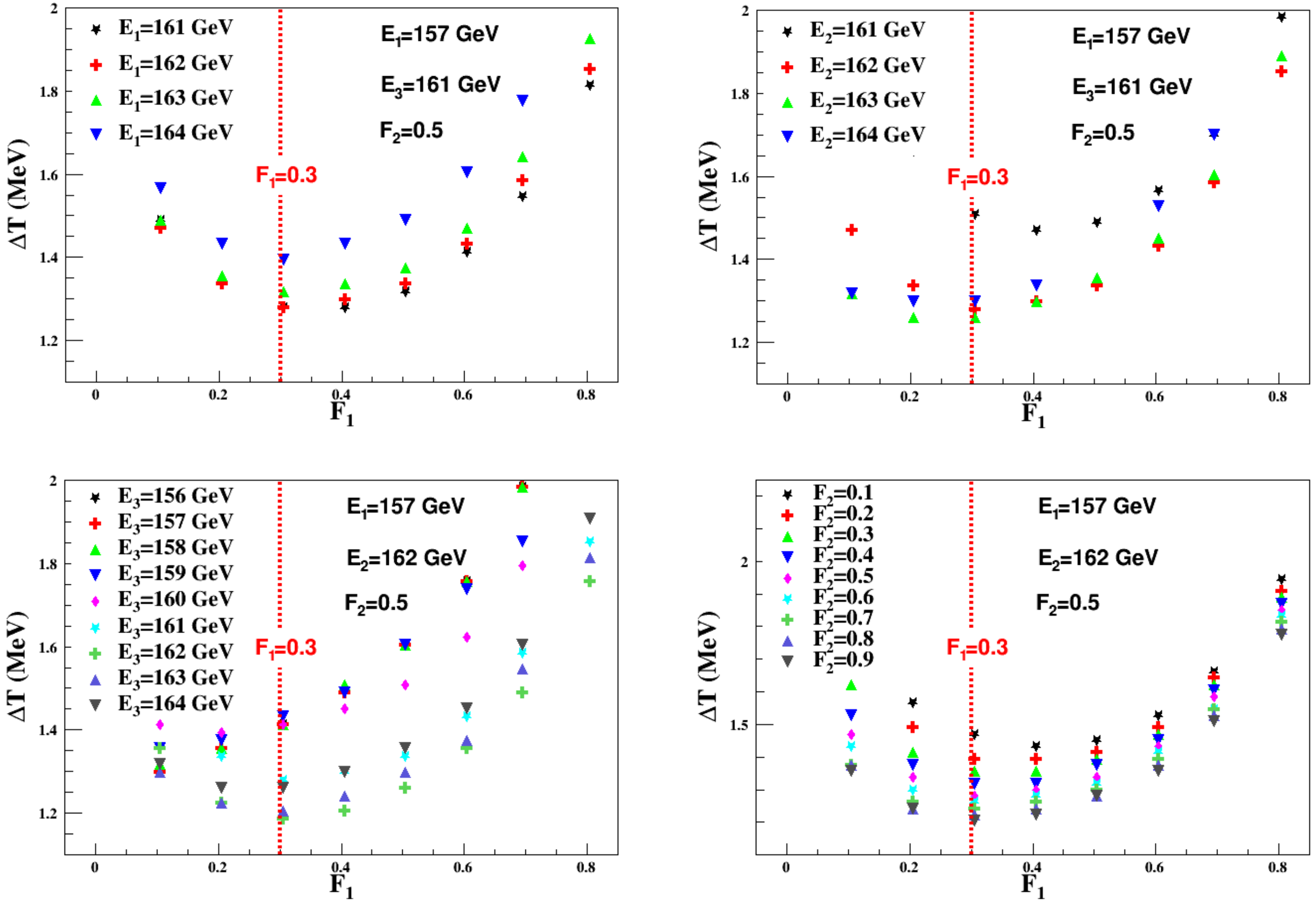
- E_2=162.5 GeV is taken as the optimized result.



Optimization of F_1

- Default values:
  E_1=157 GeV
  E_2=162 GeV
  E_3=161 GeV
  F_2= 0.5
- We change one variable with fixing other three, and get the ΔT along E_2 distributions.

- F_1=0.3 is taken as the optimized result.



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Step B

- Use the rough results from step A, the requirements below are used:

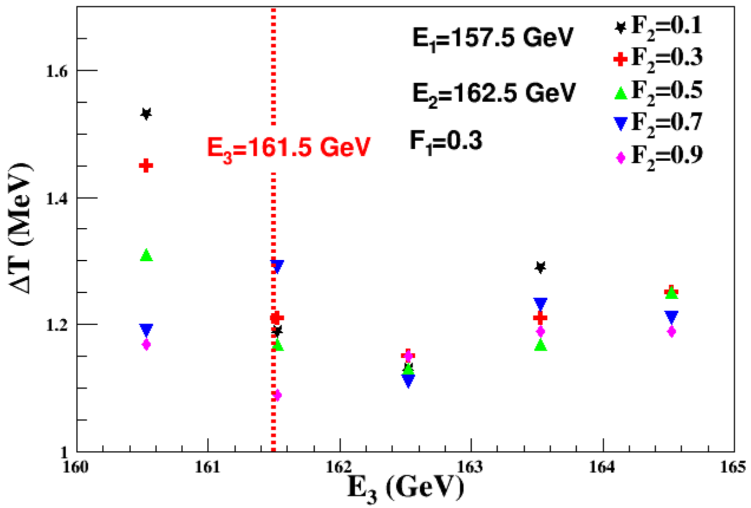
$$E_1 \in (155,160)$$
$$E_2 \in (160, 164)$$
$$E_3 \in (160, 164)$$
$$F_1 = 0.3, F_2 \in (0, 1)$$

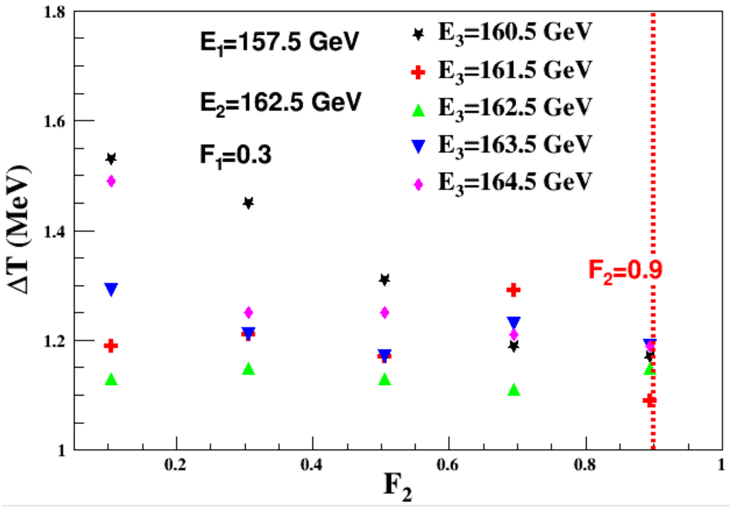
  the σ_{sys}^{corr} is considered in the fit.

- For each specific scan, 200 samplings are used, $\sigma_{WW} \sim G(\sigma_{WW}^0, \sigma_{sys}^{corr})$

- So we can get the results by fitting the distributions of m_W, Γ_W of the specific scan results.

Optimization of E_3 and F_2





E_3=161.5 GeV and F_2=0.9 are taken as the optimized results

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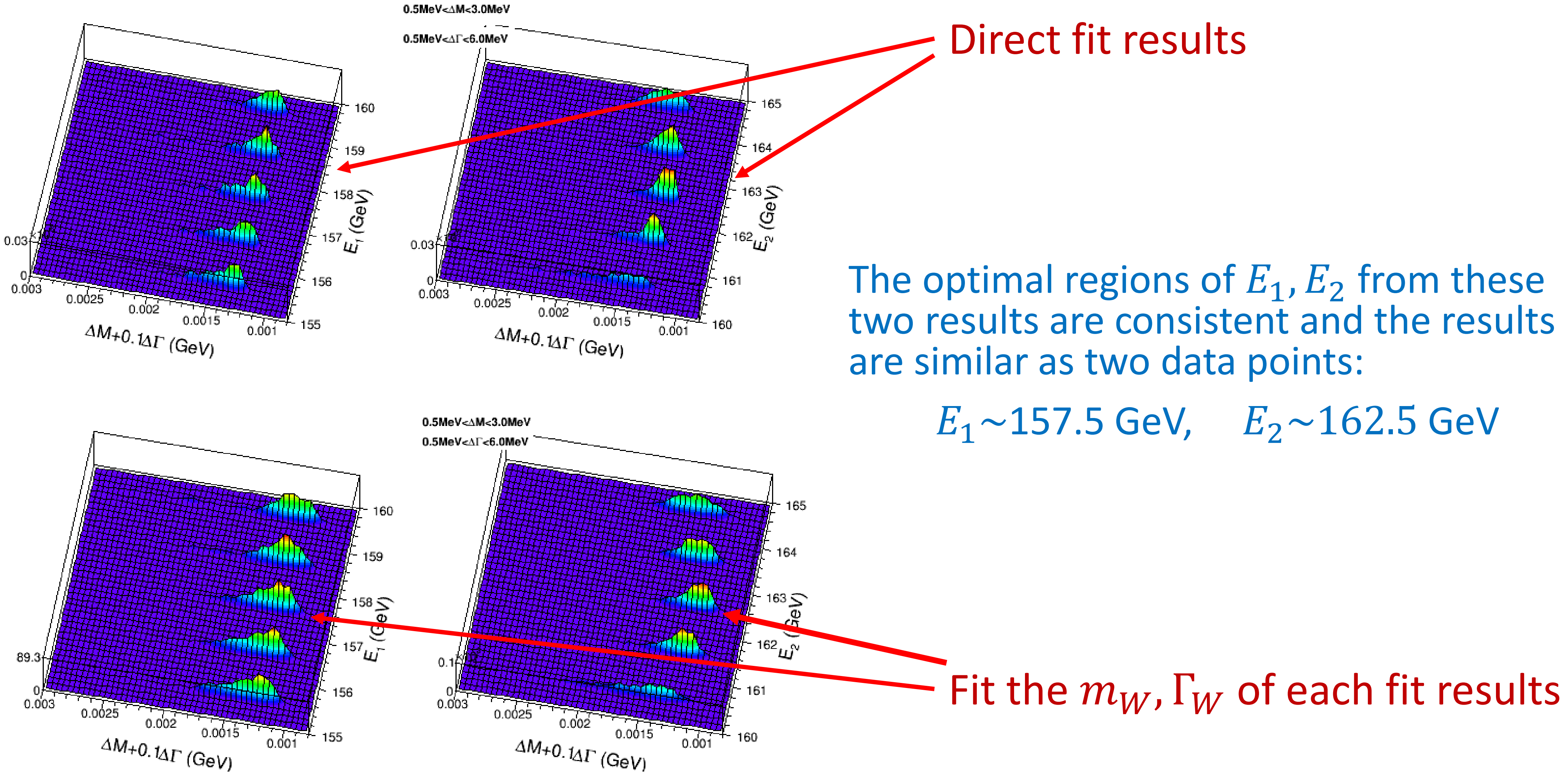
Step B: E_1, E_2

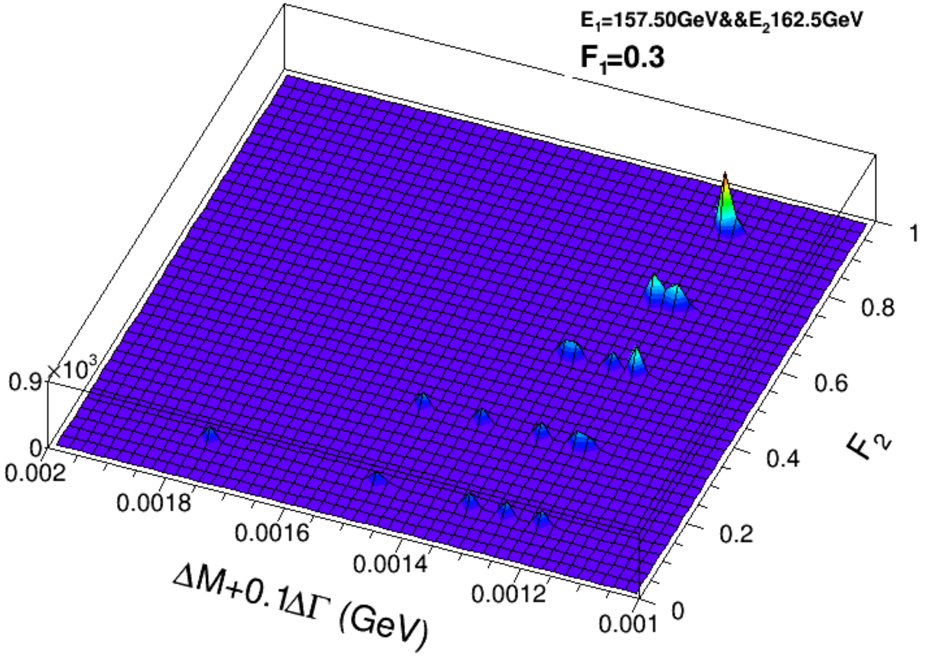


The optimal regions of E_1, E_2 from these two results are consistent and the results are similar as two data points:

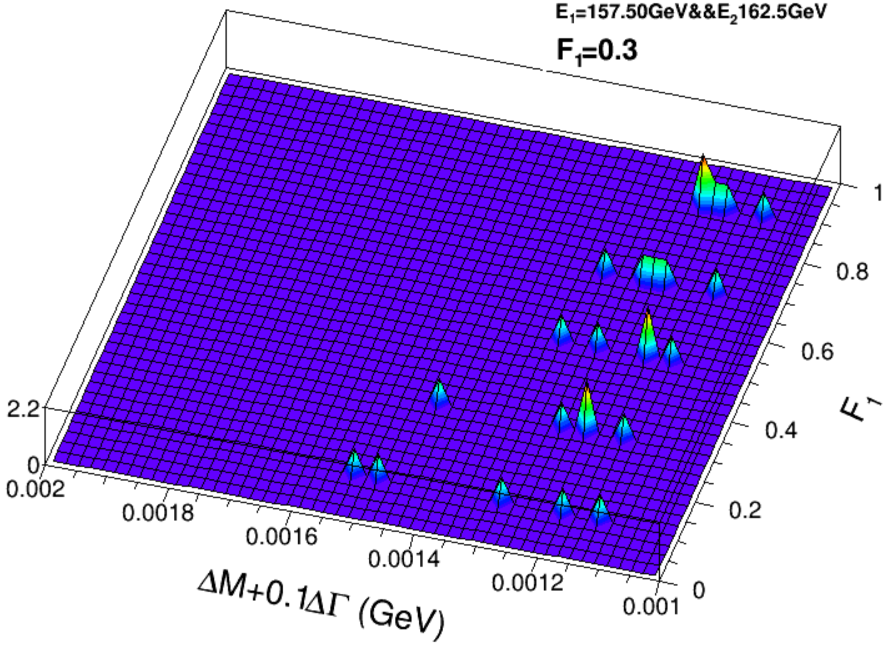
$E_1 \sim 157.5$ GeV, $E_2 \sim 162.5$ GeV

Step B: F_2

Direct fit results



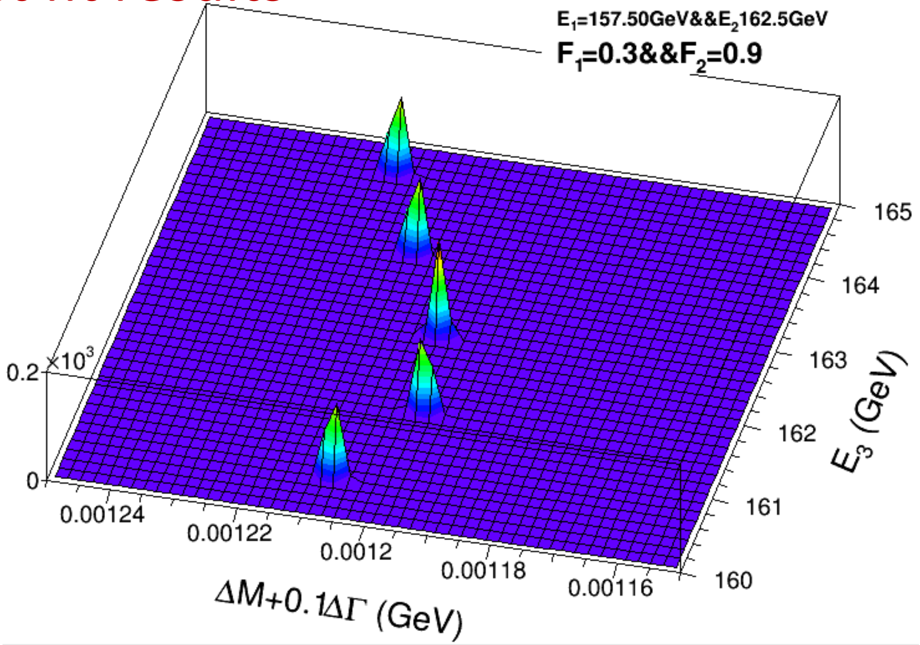
Fit the m_W, Γ_W of each fit results



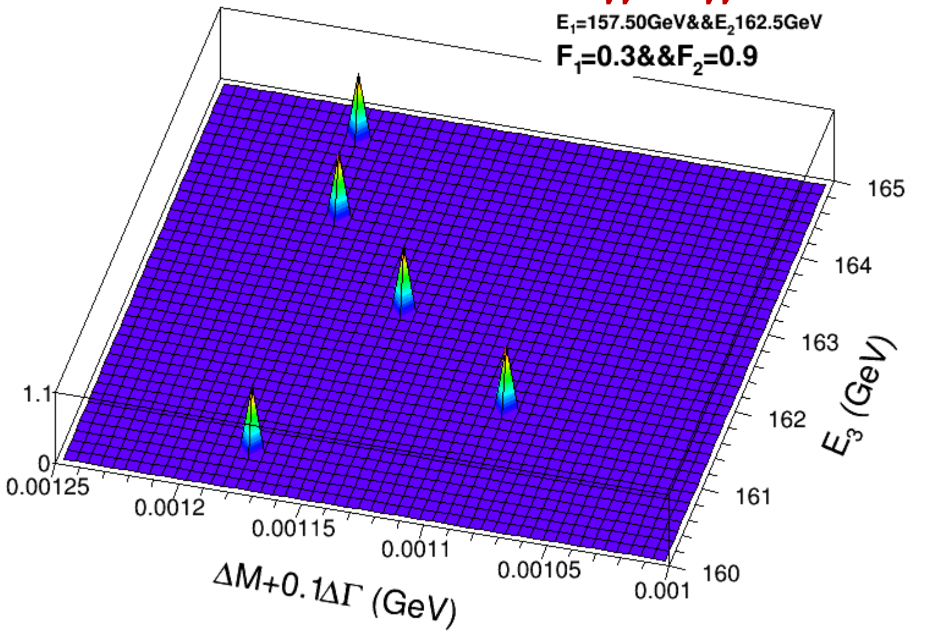
The $F_2 = 0.9$ is used in further study

Step B: E_3

Direct fit results



Fit the m_W, Γ_W of each fit results



The minimal result favors $E_3 \sim 161.5$ GeV

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Step C

- Use the rough results from step B, the configurations below are used:

  $E_1 = 157.5$, $E_2 = 162.5$, $E_3 = 161.5$, $F_1 = 0.3$, $F_2 = 0.9$

  $\sigma_{sys}^{corr} = 2 \times 10^{-4}$, $\Delta E = 0.5$ MeV, $\mathrm{E_{BS}} = 1.6 \times 10^{-3}$, $\Delta E_{BS} = 0.01$

- $\sigma_{WW} \sim G\left(\sigma_{WW}^0, \sigma_{sys}^{corr}\right)$, $E \sim G\left(E_p^0, \Delta E\right) + G\left(E_m^0, \Delta E\right)$, E_p^0 and E_m^0 are smeared with E_{BS},

$$\mathrm{E_{BS}} \sim G\left(E_{BS}^0, \Delta E_{BS}\right)$$

- By 500 samplings, we fit the distributions of m_W, Γ_w, and the corresponding uncertainties are : $\Delta m_W \sim 1$ MeV, $\Delta\Gamma_W \sim 2.8$ MeV

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