Measurements of Cross Sections for $e^+e^- \rightarrow \mu^+\mu^-$ at Energies from 3.8 to 4.6 GeV

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Outline

- Motivation
- Measurements of cross sections
  - Data & Monte Carlo Events
  - Measuring cross sections
- Analysis of cross sections
  - $Br(\psi_s(4040, 4160, 4416) \rightarrow \mu^+\mu^-)$
  - Phase angle of $\psi_s(4040, 4160, 4416) \rightarrow \mu^+\mu^-$
  - Evidence for structure $S(4230) \rightarrow \mu^+\mu^-$
- Summary

Motivation

Measurements of cross sections for $e^+e^- \to \mu^+\mu^-$ could give some important information about vector meson production and decays

- **Up to now, no branching fraction of heavy ψ_s[$\psi(4040)$, $\psi(4160)$ and $\psi(4415)$] electromagnetic di-muon decays is available in Review of Particle Physics or in Particle Physics BookLet 2016.**
- **Measure the branching fractions of the heavy ψ_s electromagnetic di-muon decays.**
- **Measure the phase angles of the heavy ψ_s relative to the continuum $e^+e^- \to \mu^+\mu^-$ process.**
- **Search for some new structure(s) in the decay of $S \to \mu^+\mu^-$.**
- **...**

These improve our knowledge for better understanding of the vector meson production in e^+e^- annihilation, as well as for better understanding of the vector meson decays

Data Samples and Software

- **Data**
  - Data taken at energies from 3.8~4.4 GeV in 2013
  - Data taken at energies from 3.85~4.6 GeV in 2013
  - Data taken at energies from 4.4~4.6 GeV in 2014
  - Data taken at 4.009 GeV (481.96 pb^{-1})

- **Software**
  - BOSS version of 6.6.4.p01
  - Monte Carlo events are generated with Babayaga
  - Other Monte Carlo simulated events are generated with the KKMC + BesEvtGen

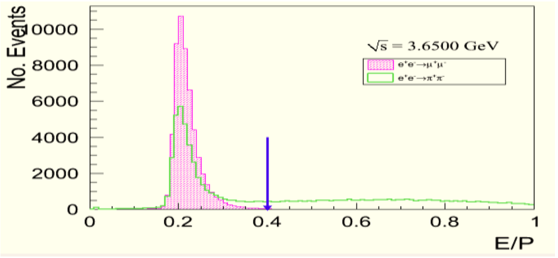
Event Selection

➢ Charged track

- $|\cos\theta| < 0.8$
- $|V_{xy}| < 1$ cm
- $|V_z| < 10$ cm
- $N_{Good} = 2, \qquad \sum_i Q = 0$

➢ Selection of $\mu^{+/-}$

- $|T_1 - T_2| < 2$ ns (T$_1$ and T$_2$ are the time from TOF)
- $(p_{\mu+} + p_{\mu-}) > 0.9E_{cm}$
- $0.05 < E_{EMC}/p < 0.40$



➢ 4C-Kinematic fit

- $\chi^2_{\ 4C\text{-}Fit} < 30$ (to reject KK and $p\bar{p}$ backgrounds)

No. of candidates for $e^+e^- \rightarrow \mu^+\mu^-$

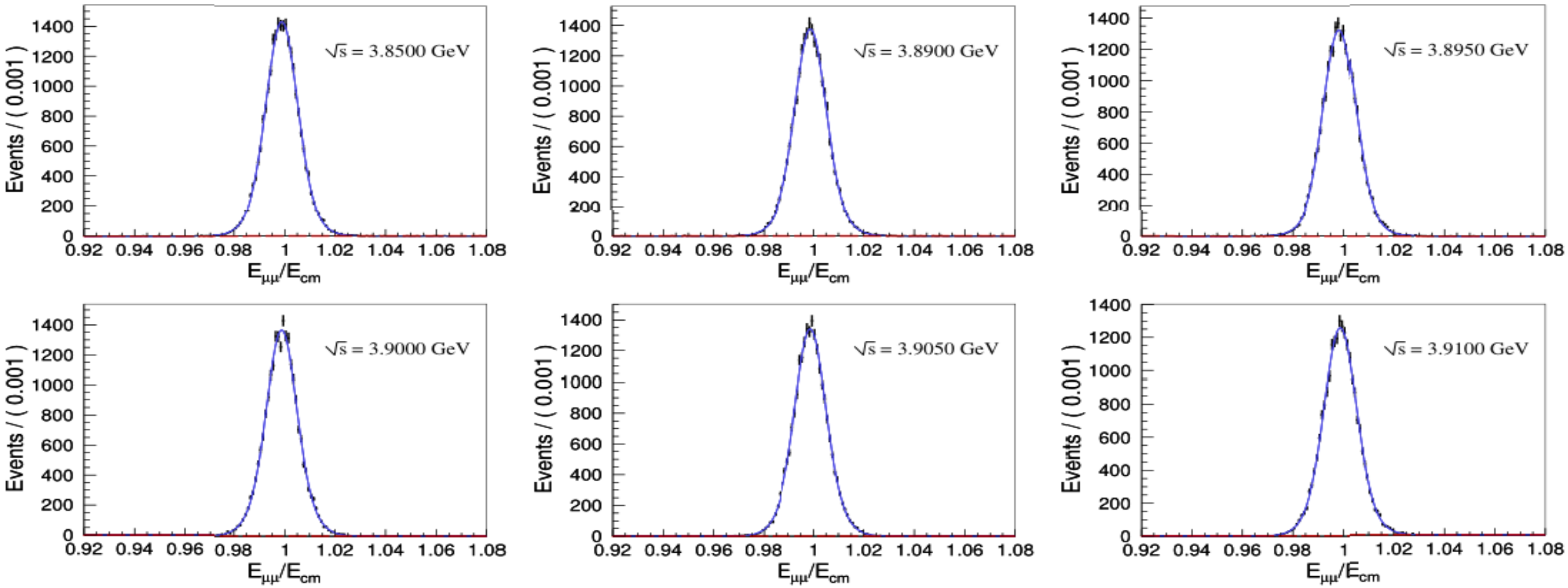
- To separate $e^+e^- \rightarrow \mu^+\mu^-$ from $e^+e^- \rightarrow K^+K^-$ and $e^+e^- \rightarrow p\bar{p}$, we examine the energy distributions of the accepted events satisfying the selection criteria. If a K or p is misidentified as a μ, the energy of the charged track would be lower than that expected. This allows us to separate μ/K/p.

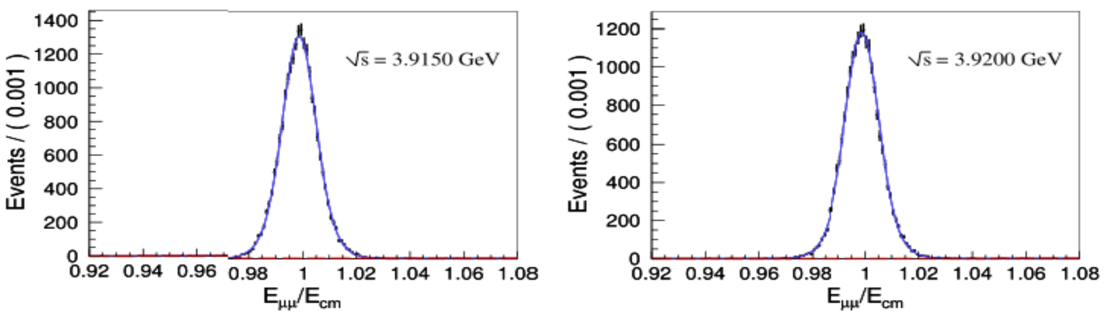
- We defined a quantity $E_{measured}/E_{cm}$ to examine the energy distributions of the selected events from data samples collected at different energies.

- The number of candidates for $e^+e^- \rightarrow \mu^+\mu^-$ is obtained by fitting the distributions of the normalized energies ($E_{measured}/E_{cm}$) of the final states satisfying the selection criteria.

Numbers of candidates for $e^+e^- \rightarrow \mu^+\mu^-$

- $E_{measured}/E_{cm}$





Signal shape:
Double Gaussian function
Background shape:
Polynomial

(More distributions of the ratio and fits are shown in the backup slides)

Background Estimate

➢ No. of background events

The events satisfying the selection criteria still contain some backgrounds. The number of these background events can be subtracted from the events with

$$N^b = \sum_i^N L \times \sigma^i \times \eta_{i \to \pi\pi(KK,\ ee\mu\mu)}$$

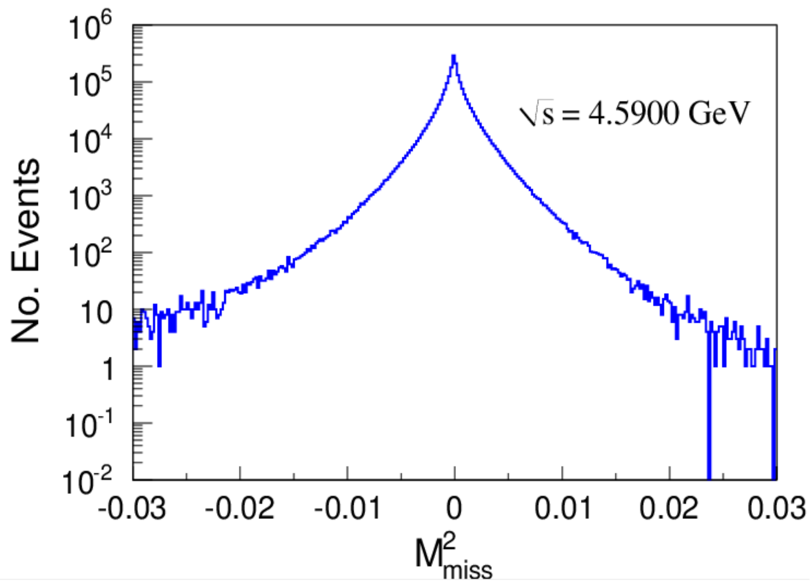
L : luminosity
N^b : the number of background
σ^i : cross section of i^{th} Bkg. source
η : mis-identification rate

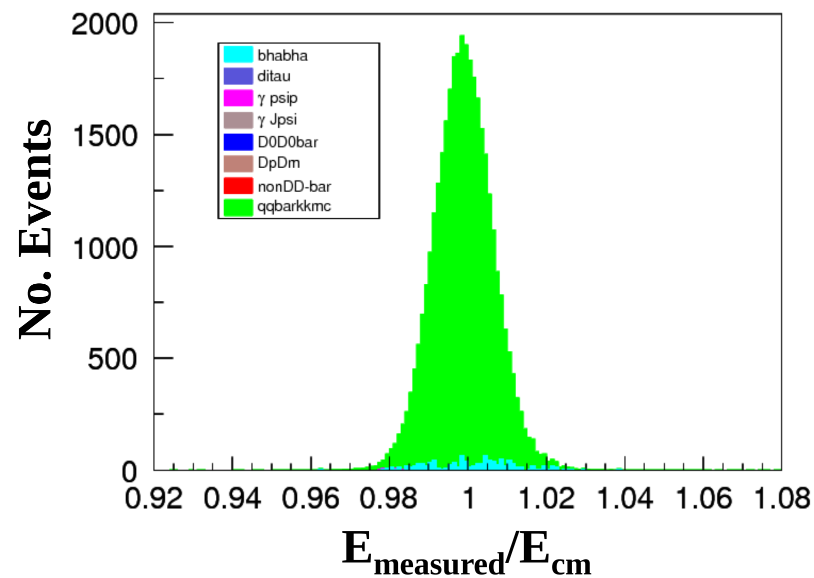
➢ Possible backgrounds

- ✓ $e^+e^- \to \mu^+\mu^-$ **is mis-identified from** $e^+e^- \to \pi^+\pi^-$**, KK,** $p\bar{p}$
- ✓ $e^+e^- \to \mu^+\mu^-$ **is mis-identified from** $e^+e^- \to \pi^+\pi^-\pi^0$
- ✓ $e^+e^- \to \mu^+\mu^-$ **is mis-identified from** $e^+e^- \to e^+e^- X$

Main Backgrounds

- **Possible backgrounds**
  - ✓ **The distribution of the $M^2_{missing}$ of the events satisfying the selection criteria shows that no the $e^+e^-\to\pi^+\pi^-\pi^0$ event is satisfied with the selection criteria**
  - ✓ **We study possible backgrounds with all kinds of Monte Carlo simulated event samples generated at 3.773 GeV and other E_{cm}**

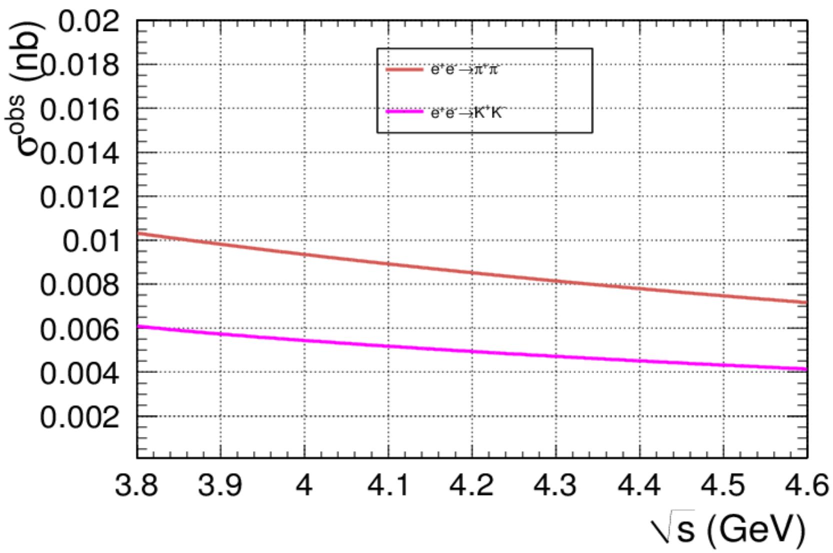




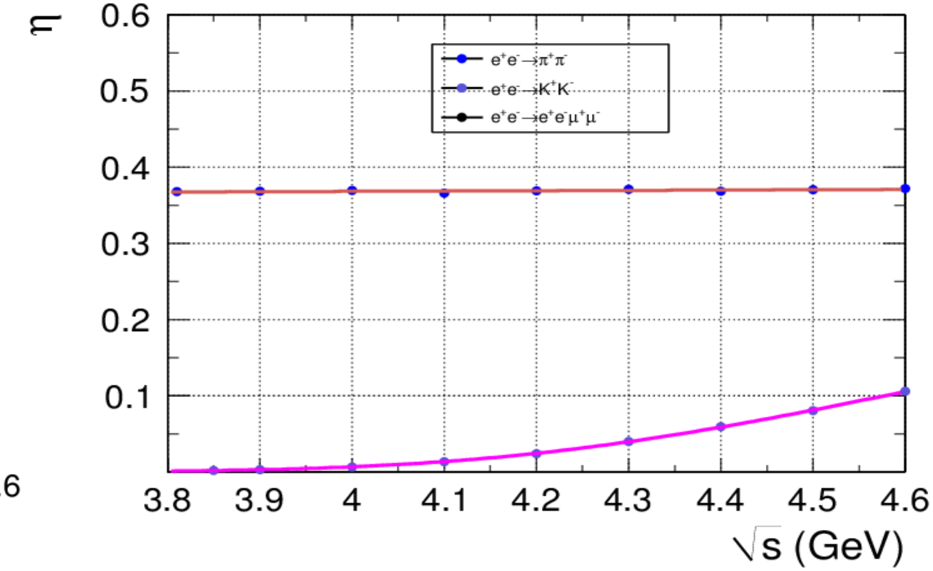
The Monte Carlo studies shows that the $e^+e^-\to\pi^+\pi^-$ is the mainly backgrounds from the continuum $e^+e^-\to q\bar{q}$ events

Number of Backgrounds

➢ **Cross sections for $e^+e^- \rightarrow \pi^+\pi^-$**



➢ **Misidentification rate**



Using this cross-section shape and η shape, we can estimate the number of background events at energies from 3.8 to 4.6 GeV

➢ **Number of the background events**

$$N^b = L \times \sigma^{e^+e^- \to \pi^+\pi^-} \times \eta$$

Observed Cross Sections

➢ **Cross Sections**

$$\sigma^{obs}_{e^+e^- \to \mu^+\mu^-} = \frac{N^{\mathrm{obs}} - N^{\mathrm{b}}}{L \times \varepsilon}$$

where N^{obs} is the number of candidates for $e^+e^- \rightarrow \mu^+\mu^-$ observed, N^b is the number of background events, L is the integrated luminosity, ε is the detection efficiency

✓ **Luminosity**

Measured with $e^+e^-(\gamma) \rightarrow e^+e^-$, or $e^+e^- \rightarrow \gamma\gamma$

✓ **Efficiency**

Determined with Babayaga, and/or with KKMC+BesEvtGen

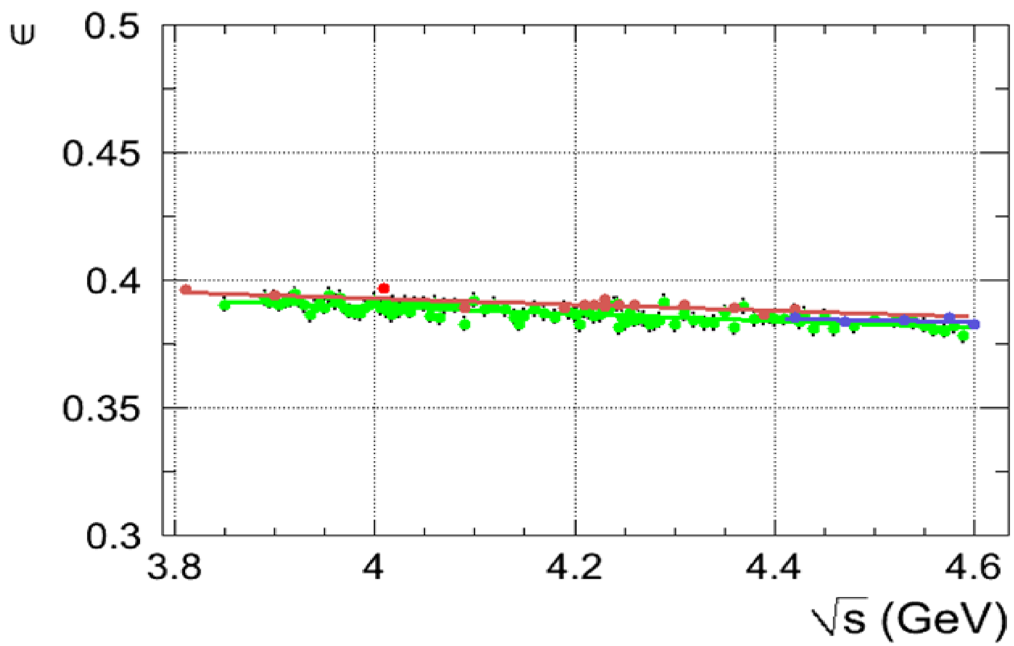
MC Events & Selection Efficiency

- **Monte Carlo simulated signal events**

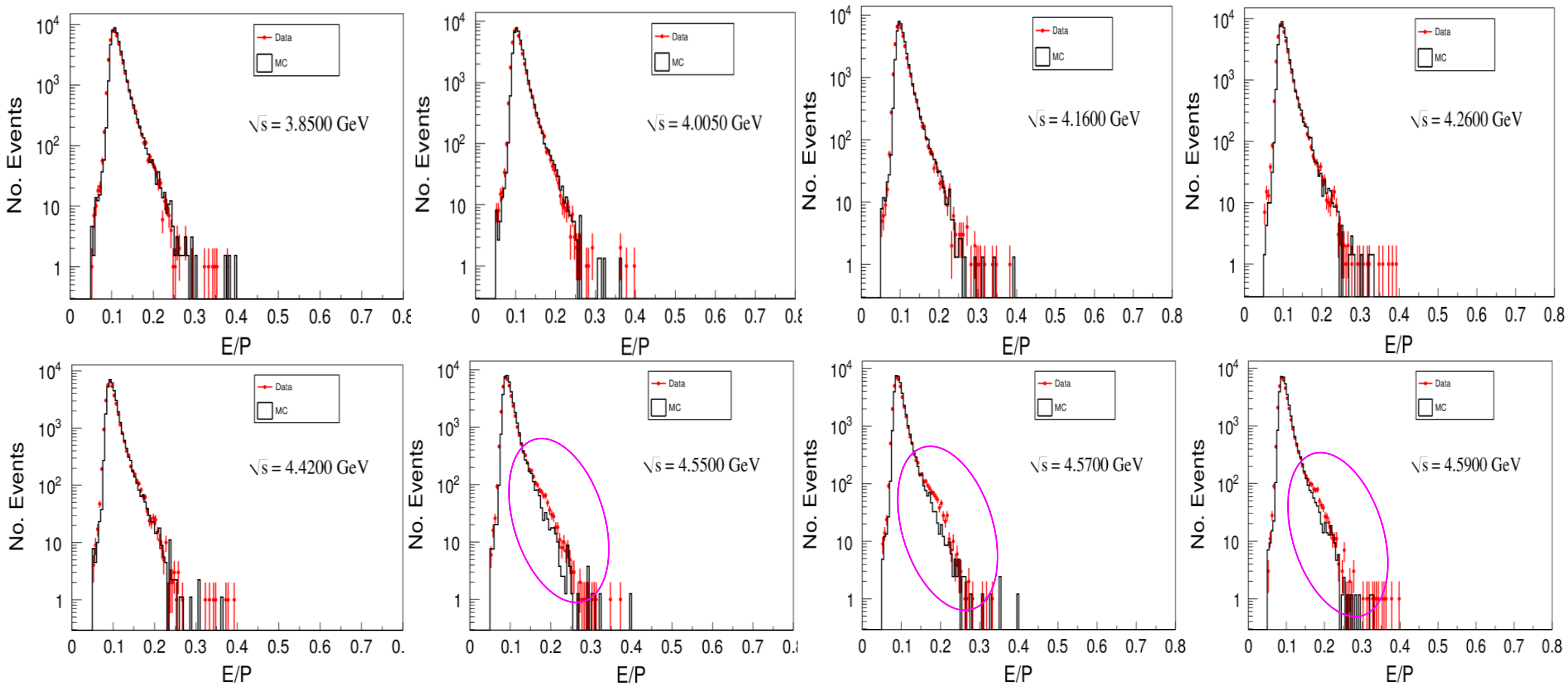
  At each energy point of the R scan data taken at energies from 3.85~4.6 GeV in 2013, we generated 40000 Monte Carlo signal events for $e^+e^- \rightarrow \mu^+\mu^-$; We generated 100000 Monte Carlo signal events at the energies from 3.8~4.6 GeV for the XYZ scan data taken in 2013 in 2014.

- **Selection efficiencies**

  Analyzing these Monte Carlo simulated signal events yields selection efficiencies ε at each of these energies.



Comparison of E_{EMC}/p between Data & MC



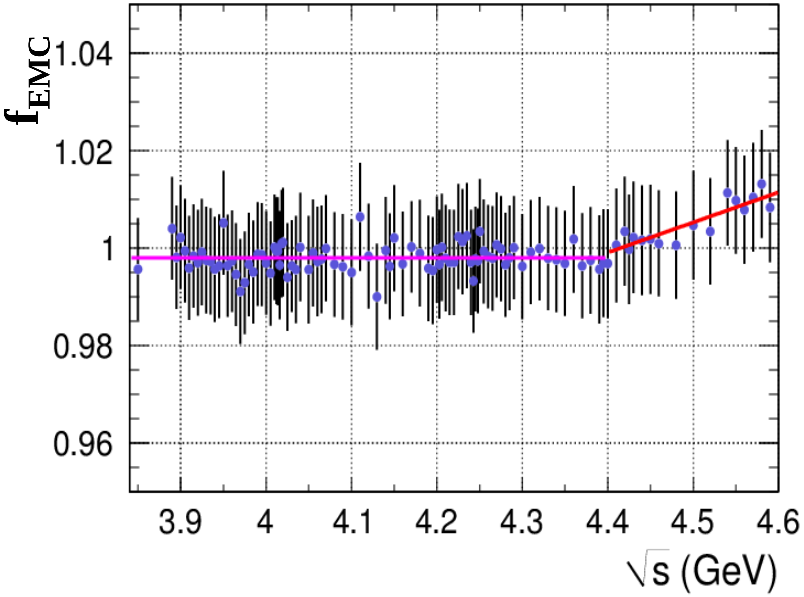
The ratio of E_{EMC}/p between the data and the Monte Carlo simulated events at energies above 4.45 GeV are difference. These should be corrected in determination of the cross sections

Correction Factor f_{EMC} to ε

- **Correction factor f_{EMC} to selection efficiency**

The difference in ratio of E_{EMC}/p between the data and the Monte Carlo simulated events varies with energy above 4.4 GeV, resulting under estimate of the selection efficiency.

By comparing the distributions of the E_{EMC}/p from both the data and the Monte Carlo simulated events, we obtain an energy dependent correction factor f_{EMC} to the selection efficiencies.

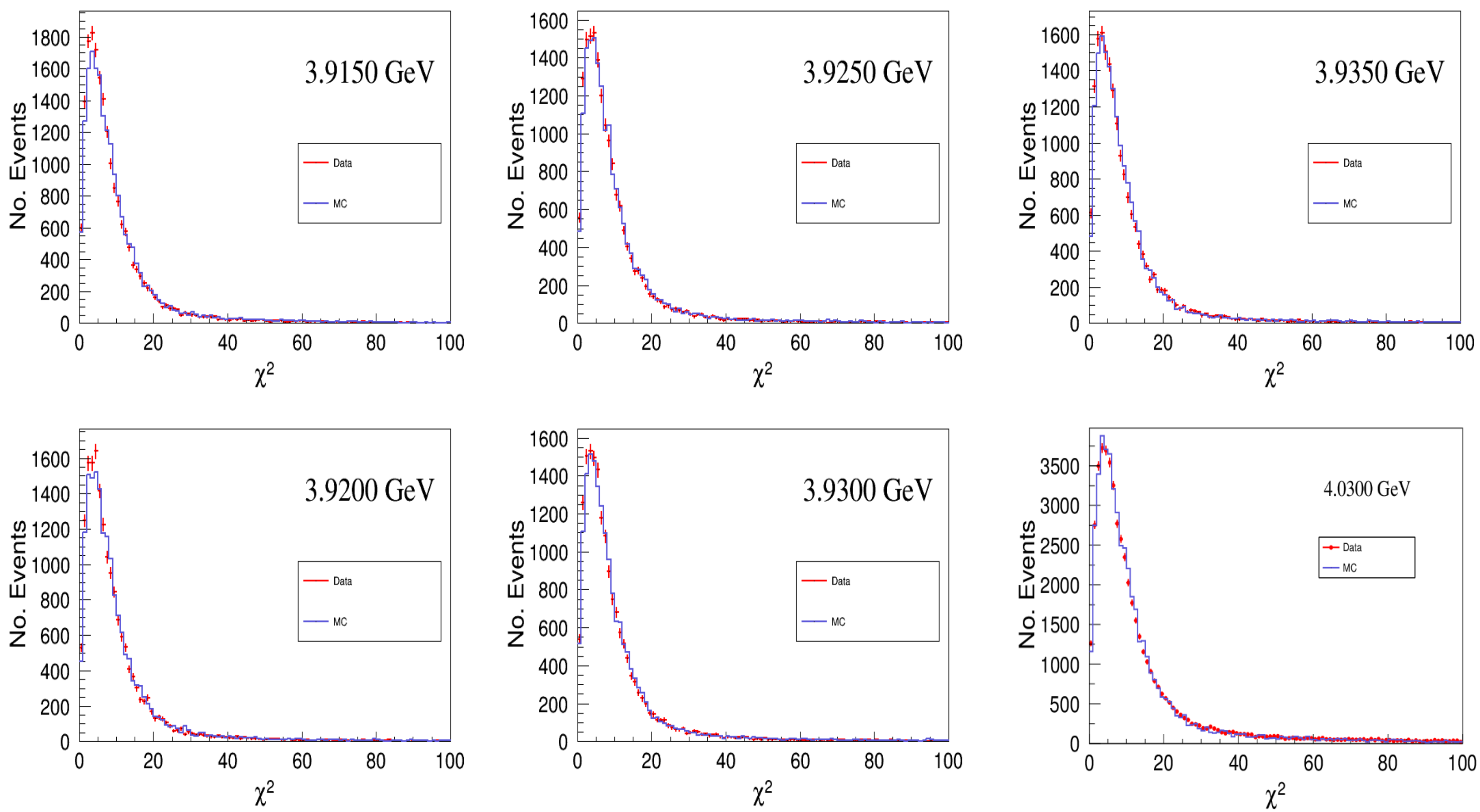


- **Observed cross section**

$$\sigma^{obs}_{e^+e^-\to\mu^+\mu^-} = \frac{N^{\mathrm{obs}} - N^{\mathrm{b}}}{L\times\varepsilon\times f_{\mathrm{EMC}}}$$

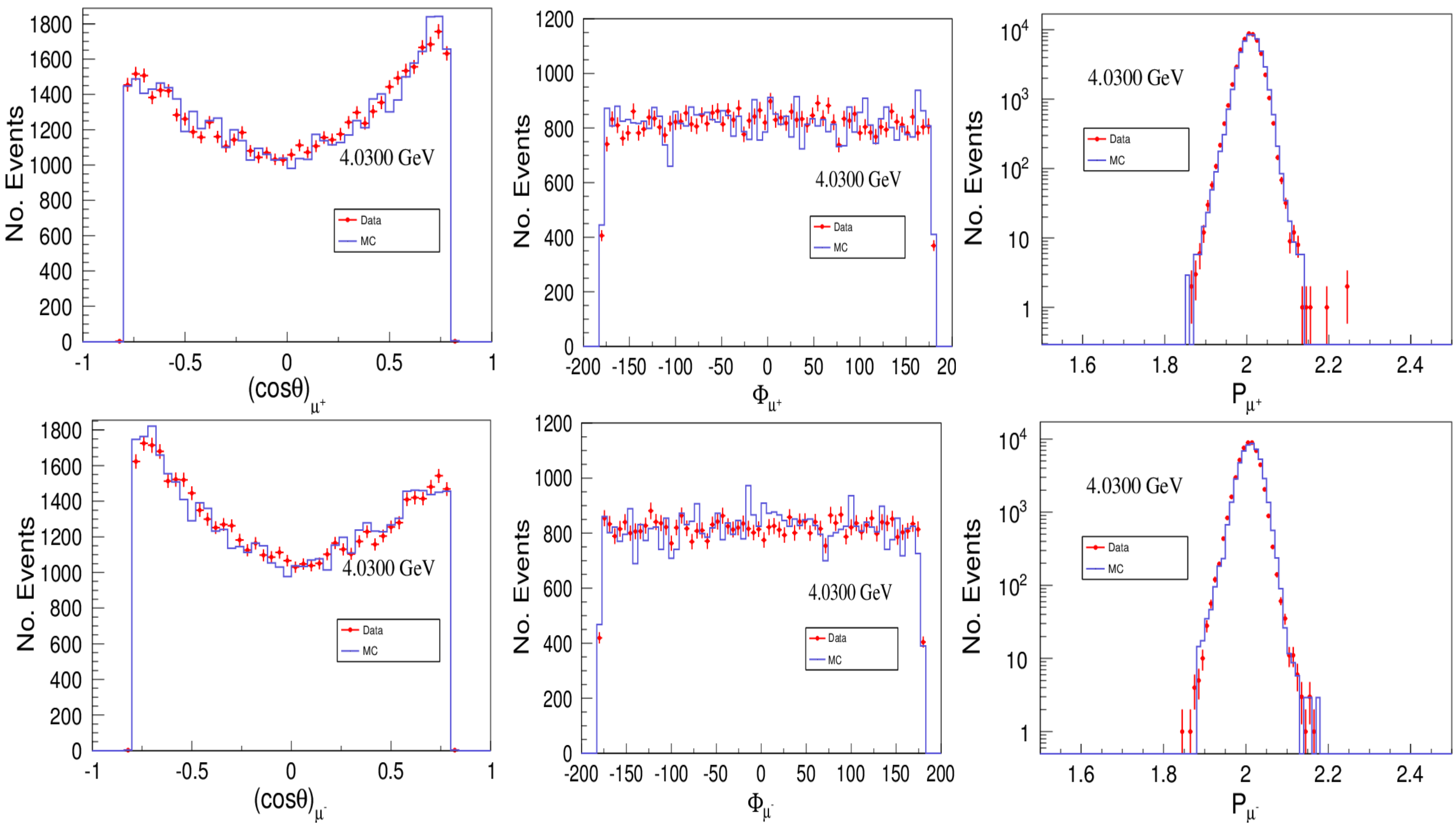
Compare Data and MC

- χ^2 distributions from both the data and the MC events at a few energies (as an example)



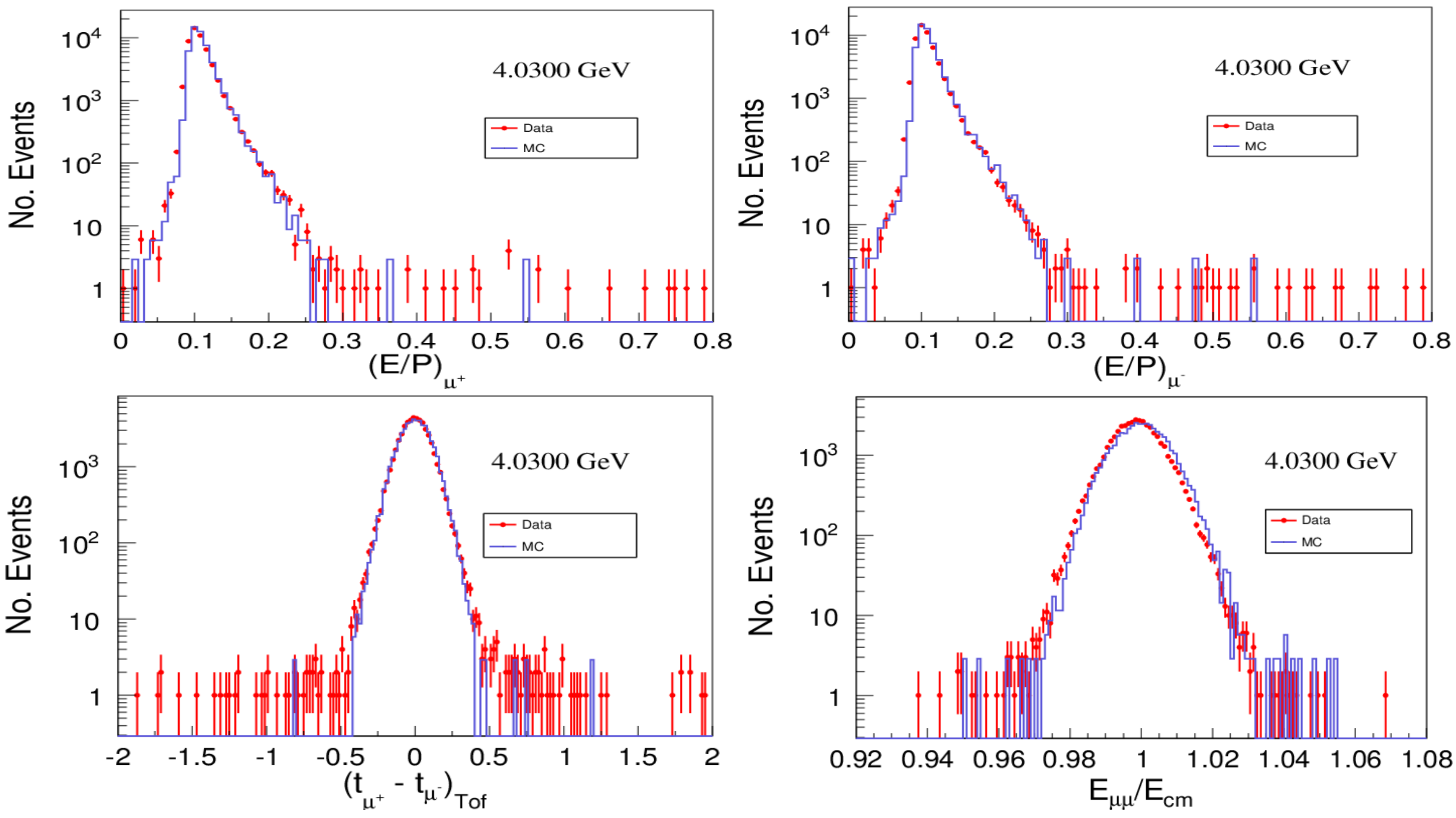
Comparison of Data and MC

- **Distributions of cosθ, ϕ and momentum of μ for both the data and the MC simulated events**



Comparison of Data and MC

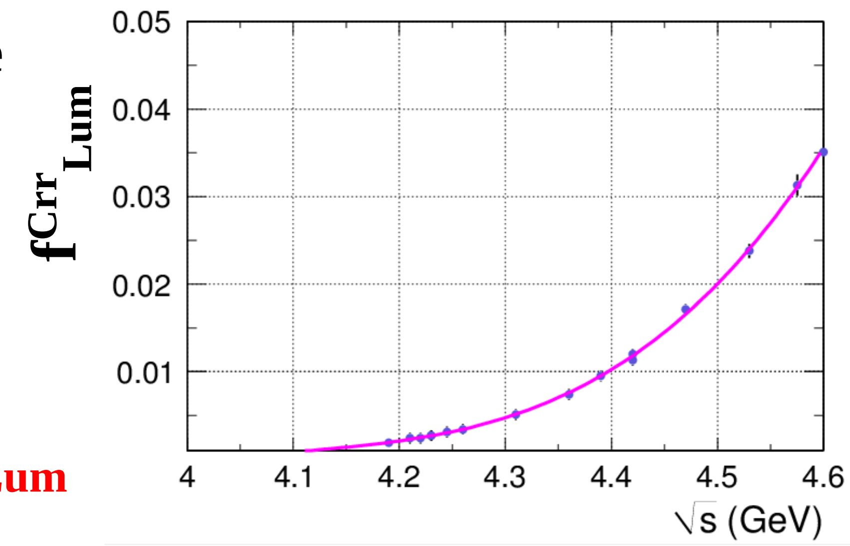
- Distributions of E_{EMC}/p, $T_{\mu+}$ -$T_{\mu-}$ and $E_{measured}/E_{cm}$ of μ^+ (μ^-) for both the data and the MC simulated events



Luminosities and Their Corrections

- ✓ **The published luminosities of the data samples are used in the analysis**
- ✓ **Yifan Yang, Guangyi Tang, and Changzheng Yuan found that the published luminosities of the data samples suffer from some problems due to missing readout some crystal in a reconstructed cluster. They gave a set of correction factors of the published luminosities. [see their talk given at the BES-III collaboration meeting in December 3, 2017]**
- ✓ **Using these correction factors, we obtained the 'right' luminosities of the data samples collected at energies above 4.1 GeV in 2013 and 2014， which are**

$$L = (1.0 + f^{Crr}_{Lum})\, L^{Published}_{Lum}$$



The Data Collected in 2013 and 2014

As an example, we list the published and corrected luminosities of the data collected at 10 energy points for XYZ scan

Ecm (GeV)	Lum (nb^{-1})	factor	Lum_Cor (nb^{-1})
3.8100	50540.00 ± 30.00	1.0000	50540.00 ± 30.00
4.0900	52630.00 ± 30.00	1.0008	52670.02 ± 30.02
4.2200	54130.00 ± 30.00	1.0025	54265.09 ± 30.07
4.2450	55590.00 ± 40.00	1.0031	55760.01 ± 40.12
4.3600	539840.00 ± 100.00	1.0076	543967.88 ± 100.76
4.4200	1028890.00 ± 130.00	1.0119	1041085.06 ± 131.54
4.4700	109940.00 ± 40.00	1.0166	111765.67 ± 40.66
4.5300	109980.00 ± 40.00	1.0241	112628.23 ± 40.96
4.5750	47670.00 ± 30.00	1.0311	49153.85 ± 30.93
4.6000	566930.00 ± 110.00	1.0356	587131.29 ± 113.92

$L^{Published}_{Lum}$ L

The Data Collected in 2013

As an example, we list the published and corrected luminosities of the data collected at 10 energy points for R scan

Ecm (GeV)	Lum (nb^{-1})	factor	Lum_Cor (nb^{-1})
3.8500	7967.00 $\pm$ 18.00	1.0000	7967.00 $\pm$ 18.00
3.9300	6735.00 $\pm$ 16.00	1.0000	6735.00 $\pm$ 16.00
3.9800	7851.00 $\pm$ 19.00	1.0000	7851.00 $\pm$ 19.00
4.0180	6968.00 $\pm$ 17.00	1.0000	6968.00 $\pm$ 17.00
4.0700	7271.00 $\pm$ 17.00	1.0000	7271.00 $\pm$ 17.00
4.1600	7954.00 $\pm$ 19.00	1.0015	7966.19 $\pm$ 19.03
4.2200	7935.00 $\pm$ 19.00	1.0025	7954.80 $\pm$ 19.05
4.3300	8657.00 $\pm$ 21.00	1.0061	8709.50 $\pm$ 21.13
4.4200	7519.00 $\pm$ 18.00	1.0119	7608.12 $\pm$ 18.21
4.5900	8162.00 $\pm$ 20.00	1.0338	8437.70 $\pm$ 20.68
	$L^{Published}_{Lum}$		L

The Data Collected in 2013, 2014

➤ **Observed cross sections at 10 energy points**

$$\sigma^{obs} = \frac{N^{obs} - N^{b}}{L \times \varepsilon \times f_{EMC}}$$

E_{cm} (GeV)	$L^{published}_{lum}$ (nb^{-1})	N^{obs}	N^{b}	N^{net}	ε	σ (nb)
3.8100	50540.00 ± 30.00	157055.3 ± 396.4	193.7 ± 0.1	156861.6 ± 396.4	0.3917	7.9234 ± 0.0206
4.0900	52670.02 ± 30.02	139732.4 ± 374.0	175.8 ± 0.1	139556.6 ± 374.0	0.3900	6.7936 ± 0.0186
4.2200	54265.09 ± 30.07	135992.3 ± 368.9	170.2 ± 0.1	135822.1 ± 368.9	0.3888	6.4373 ± 0.0178
4.2450	55760.01 ± 40.12	137613.9 ± 371.1	172.8 ± 0.1	137441.1 ± 371.1	0.3885	6.3442 ± 0.0177
4.3600	543967.88 ± 100.76	1263429.6 ± 1124.6	1593.8 ± 0.3	1261835.8 ± 1124.6	0.3872	5.9906 ± 0.0055
4.4200	1041085.06 ± 131.54	2325166.2 ± 1525.6	2932.3 ± 0.4	2322233.9 ± 1525.6	0.3835	5.8159 ± 0.0039
4.4700	111765.67 ± 40.66	245807.7 ± 498.9	307.5 ± 0.4	245500.2 ± 498.9	0.3855	5.6981 ± 0.0118
4.5300	112628.23 ± 40.96	242316.6 ± 492.6	300.7 ± 0.1	242015.9 ± 492.6	0.3869	5.5543 ± 0.0115
4.5750	49153.85 ± 30.93	104300.9 ± 323.1	128.2 ± 0.1	104172.7 ± 323.1	0.3879	5.4634 ± 0.0173
4.6000	587131.29 ± 113.92	1234901.2 ± 1111.8	1511.3 ± 0.1	1233389.9 ± 1111.8	0.3884	5.4079 ± 0.0050

The Data Collected in 2013

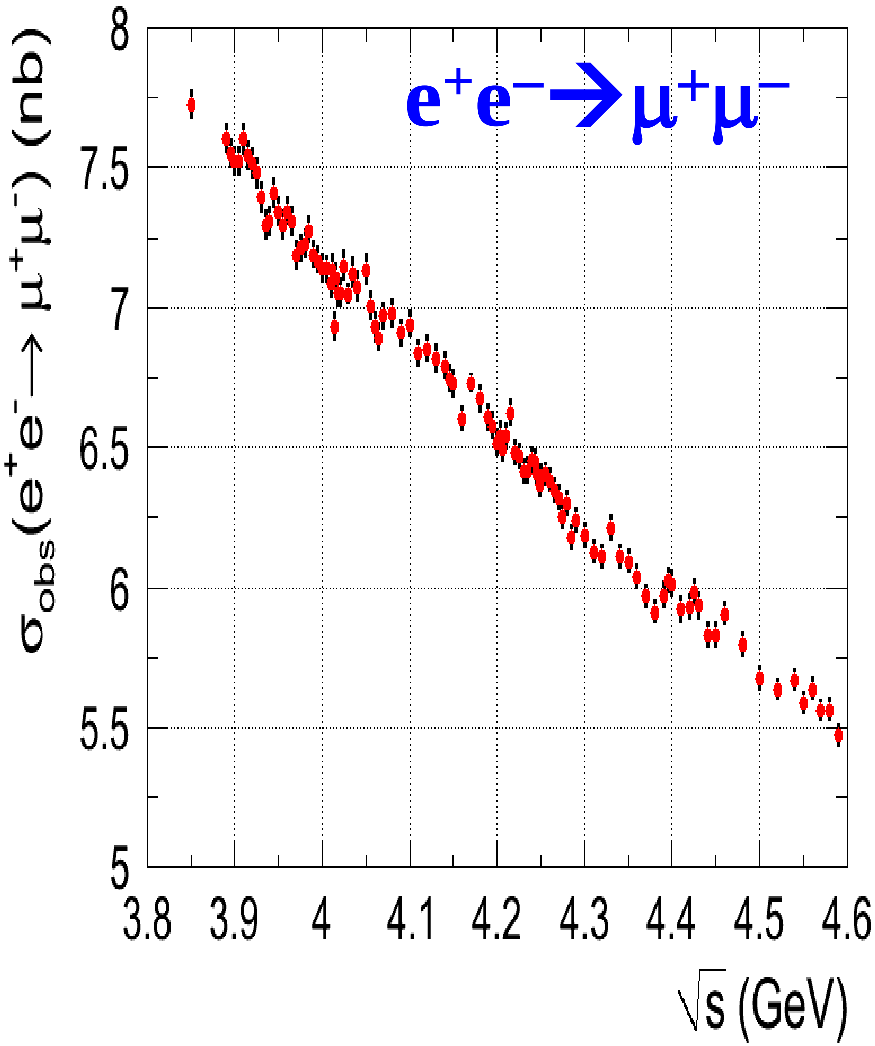
- **Observed cross sections at 10 energy points**

E_{cm} [GeV]	$L^{published}_{Lum}$ [nb^{-1}]	N^{obs}	N^{b}	N^{net}	ε	σ [nb]
3.8500	7967.00 $\pm$ 18.00	23954.3 $\pm$ 154.9	29.5 $\pm$ 0.1	23924.8 $\pm$ 154.9	0.3888	7.7233 $\pm$ 0.0530
3.9300	6735.00 $\pm$ 16.00	19358.1 $\pm$ 139.2	24.0 $\pm$ 0.1	19334.1 $\pm$ 139.2	0.3883	7.3926 $\pm$ 0.0560
3.9800	7851.00 $\pm$ 19.00	22035.1 $\pm$ 148.5	27.3 $\pm$ 0.1	22007.8 $\pm$ 148.5	0.3878	7.2280 $\pm$ 0.0518
4.0180	6968.00 $\pm$ 17.00	19055.8 $\pm$ 138.0	23.8 $\pm$ 0.1	19032.0 $\pm$ 138.0	0.3874	7.0500 $\pm$ 0.0539
4.0700	7271.00 $\pm$ 17.00	19630.0 $\pm$ 140.2	24.2 $\pm$ 0.1	19605.8 $\pm$ 140.2	0.3869	6.9689 $\pm$ 0.0524
4.1600	7966.19 $\pm$ 19.03	20321.4 $\pm$ 142.6	25.5 $\pm$ 0.1	20295.9 $\pm$ 142.6	0.3858	6.6034 $\pm$ 0.0490
4.2200	7954.80 $\pm$ 19.05	19883.3 $\pm$ 141.1	24.7 $\pm$ 0.1	19858.6 $\pm$ 141.1	0.3851	6.4821 $\pm$ 0.0486
4.3300	8709.50 $\pm$ 21.13	20786.1 $\pm$ 144.3	25.7 $\pm$ 0.1	20760.4 $\pm$ 144.3	0.3837	6.2118 $\pm$ 0.0457
4.4200	7608.12 $\pm$ 18.21	17320.5 $\pm$ 131.6	21.5 $\pm$ 0.1	17299.0 $\pm$ 131.6	0.3834	5.9311 $\pm$ 0.0473
4.5900	8437.70 $\pm$ 20.68	17797.6 $\pm$ 133.6	21.8 $\pm$ 0.1	17775.8 $\pm$ 133.6	0.3851	5.4711 $\pm$ 0.0433

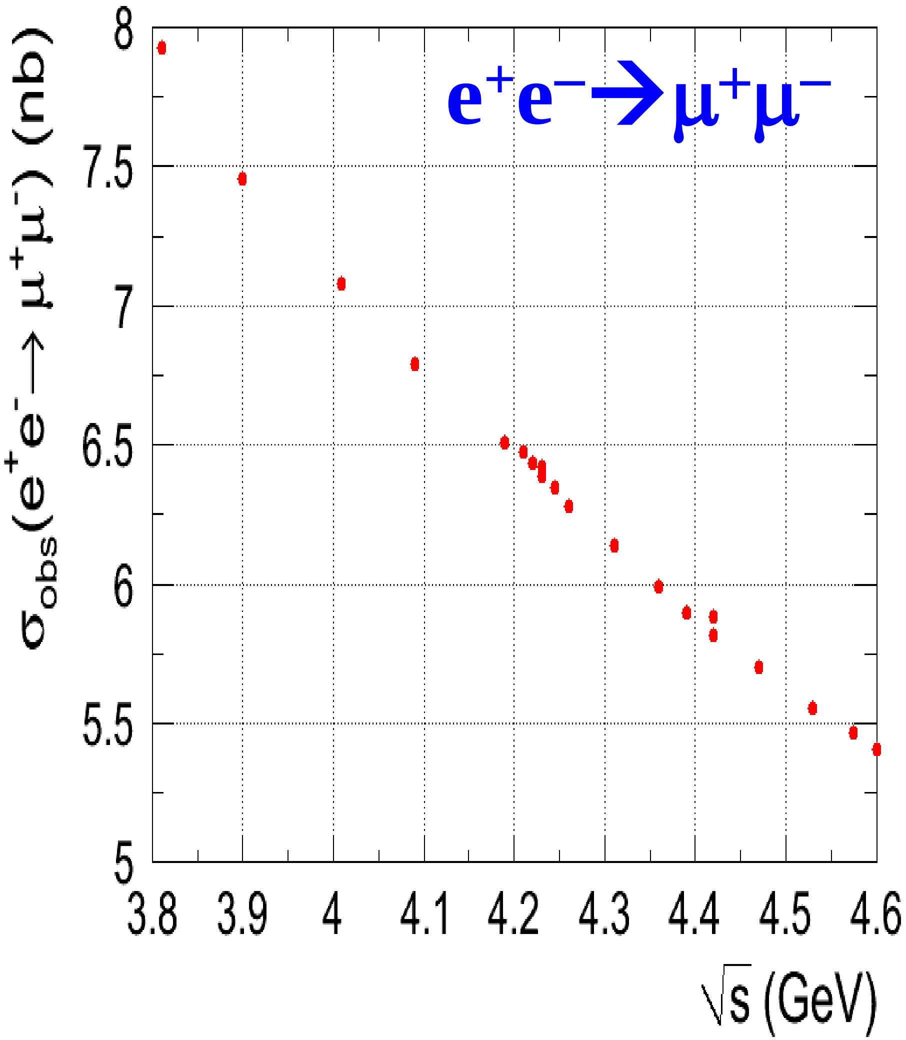
Similarly, we determine the observed cross sections at all other energies

Observed Cross Sections

- ➢ **R scan data**



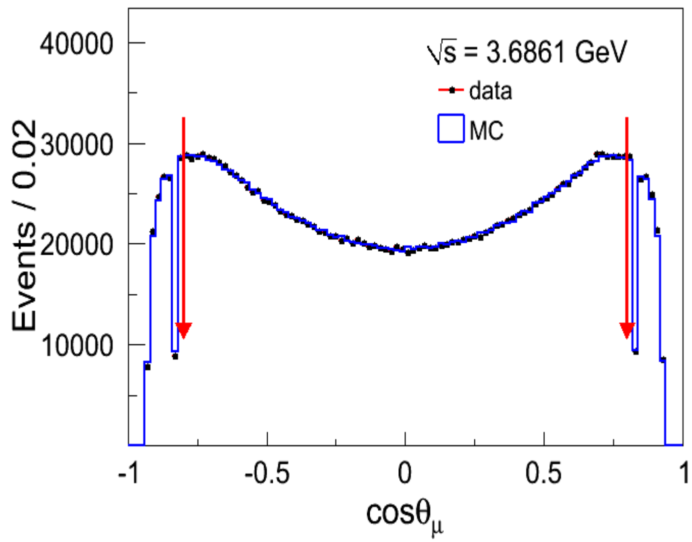
- ➢ **XYZ scan data**



Systematic Uncertainties

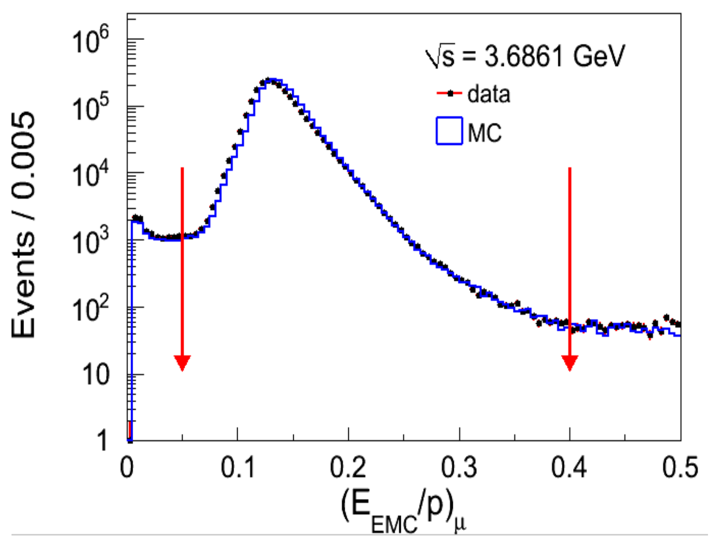
- ✓ **$|\cos\theta| < 0.80$ cut**

  We select $\psi(3686) \to J/\psi\pi^+\pi^-$ with $J/\psi \to \mu^+\mu^-$ samples, and generate Monte Carlo simulated events. By comparing the distribution of $|\cos\theta|$ between the data and the MC events, we find that the difference in $|\cos\theta|$ is $(-0.15 \pm 0.04)\%$, which is assigned as the systematic uncertainty



- ✓ **$E_{EMC}/p > 0.05$ and $E_{EMC}/p < 0.4$ cut**

  By comparing the distribution of E_{EMC}/p between the data and the MC events, we find that the difference in the E_{EMC}/p is $(-0.12 \pm 0.01)\%$, which is assigned as the systematic uncertainty



Systematic Uncertainties

✓ $|\Delta TOF| < 2.0$ ns cut

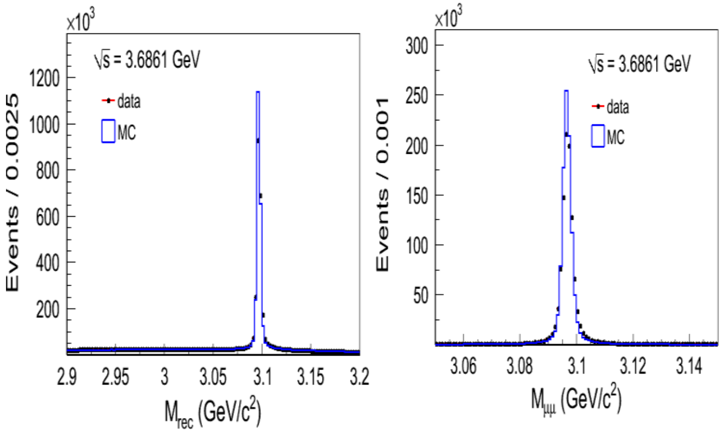
At present we measure the difference in the cross section at 4.230 GeV with this cut and without this cut. We take the difference as the estimated uncertainty due to this cut, which is -0.48%.

✓ $(P_{\mu+} + P_{\mu-}) > 0.9E_{cm}$ cut

We also measure the difference in the cross section at 4.230 GeV with this cut and without this cut, and take the difference as the estimated uncertainty due to this cut, which is -0.59%.

✓ 4-C kinematic fit

We select/generate $\psi(3686) \rightarrow J/\psi\pi^+\pi^-$ with $J/\psi \rightarrow \mu^+\mu^-$ samples, and compare the number of events satisfying K.F. requirement. The difference between the data and the MC is $(0.97 \pm 0.26)\%$, which is assigned as the systematic uncertainty due to the K.F.



Systematic Uncertainties

- ✓ **Fit to $E_{\mu\mu}/E_{cm}$**
  - **(1) Bin size : 0.001 GeV/bin → 0.0027 GeV/bin;**
  - **(2) Fit range : [0.92, 1.08] → [0.96, 1.04];**
  - **(3) Background PDF: 1-order polynomial → 0-order polynomial;**
  - **(4) Signal PDF : Double-Gaussian function → MC shape;**

	Bin size	Fit range	Background PDF	Signal PDF	Total
σ_{sys}	-0.59%	-0.65%	-0.59%	-0.63%	1.23%

- ✓ **Tracking efficiency**
  1.0% per track

(M.Ablikim et al.(BESIII Collaboration), Phys Rev.Lett.110,252001(2013))

- ✓ **Luminosity**
  1.0%

(Chin. Phys. C 39, 093001 (2015))
(M. Ablikim et al. (BESIII Collaboration), Chin. Phys. C 41, 063001 (2017))

- ✓ **Generator**
  1.0%

[https://www2.pv.infn.it/~hepcomplex/babayaga.html]

Systematic Uncertainties

Source	Systematic uncertainty (%)
$\lvert\cos\theta\rvert < 0.80$	0.15
$E_{EMC}/p > 0.05$ and $E_{EMC}/p < 0.4$	0.12
$\lvert\Delta Tof\rvert < 2.0$	0.48
$(P_{\mu+} + P_{\mu-}) > 0.9E_{cm}$	0.59
4-C kinematic fit	0.97
Fit to $E_{\mu\mu}/E_{cm}$	1.23
Tracking efficiency	2.0
Luminosity	1.0
Generator	1.0
Sum	3.01

The total systematic uncertainty in the measured cross section is 3.01%

Analysis of the Cross Sections

A χ^2 fit is performed to the observed cross sections

- **Objective χ^2 function**

$$\chi^2 = \sum_{i=1}^{n} \frac{(\sigma_i^{\text{obs}} - \sigma_i^{\text{expect}})^2}{\Delta^2_{\sigma_i^{obs}}}$$

Where σ_i^{obs} is the observed cross section, σ_i^{th} is the expected cross section, and Δ_i is the uncertainty of the σ_i^{obs} at ith energy point. The effects of correlation between point and point on the fit parameters are estimated by off-set method (see M. Botje, J. Phys. G 28 779(2002)]" for detail)

- **Dressed cross section**

$$\sigma^{\text{dress}} = \left| A_{con} + e^{i\varphi_1} A_1 + e^{i\varphi_2} A_2 + e^{i\varphi_3} A_3 + \ldots + e^{i\varphi_n} A_n \right|^2$$

$$A_{con} = \sqrt{\frac{f_{con}}{E^2_{cm}}} \qquad A_j = \frac{\sqrt{12\pi \Gamma_j^{ee} \Gamma_j^{uu}}}{E^2_{cm} - M_j^2 + i\Gamma_j^{tot} M_j}, \ (j = 1,2,\ldots n)$$

This definition is consistent with ϕ=0°, where ϕ is the angle between continuum e^+e^- → $\mu^+\mu^-$ and $\psi(3686)$ → $\mu^+\mu^-$

$$A_j = \frac{\sqrt{12\pi \Gamma_j^{ee} \Gamma_j^{\mu\mu}}}{M_j^2 - E^2_{cm} - i\Gamma_j^{tot} M_j}$$

(if using this form of the B-W amplitude, the phase angle would be shifted by 180°)

Analysis of the Cross Sections

- **Observed cross section**

$$\sigma^{\text{expect}}_{e^+e^-\to\mu^+\mu^-}(s)=\int_0^\infty ds' G(s,s')\int_0^{x_{\max}} dx \;\; F(x,s)\;\; \sigma^{\text{dress}}(s(1-x))$$

$$G(s,s')=\frac{1}{\sqrt{2\pi}\sigma_{E_{\text{BEPC-II}}}}\exp\left[-\frac{(\sqrt{s}-\sqrt{s'})^2}{2\sigma^2_{E_{\text{BEPC-II}}}}\right]$$

$$x=1-\frac{s'}{s}$$

Effective c.m. energy

Nominal c.m. energy

$$F(x,s)=\beta x^{\beta-1}\delta^{V+S}+\delta^{H}$$

Kuraev & Fadin

$$\beta=\frac{2\alpha}{\pi}\left(\ln\frac{s}{m_e^2}-1\right)$$

$$\delta^{V+S}=1+\frac{3}{4}\beta+\frac{\alpha}{\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)-\frac{\beta^2}{24}\left(\frac{1}{3}\ln\frac{s}{m_e^2}+2\pi^2-\frac{37}{4}\right)$$

$$\delta^{H}=\delta_1^{H}+\delta_2^{H} \qquad \delta_1^{H}=-\beta\left(1-\frac{x}{2}\right)$$

$$\delta_2^{H}=\frac{1}{8}\beta^2\left[4(2-x)\ln\frac{1}{x}-\frac{1+3(1-x)^2}{x}\ln(1-x)-6-x\right]$$

Measuring Br., Phase Angle and Searching for New Structure

- **Two hypotheses**

We fit these cross sections with two hypotheses

1. **Cnt + ψ(3686) +Rs(3770)+ψ(4040)+ψ(4160)+ψ(4415)**
2. **Cnt + ψ(3686) +Rs(3770)+ψ(4040)+ψ(4160)+ψ(4415)+S_1 +(S_2)**

S1 (S2) is any other structure(s) lying in the energy range from 3.8 to 4.6 GeV

In the fit, the branching fractions for ψ(3686) and Rs(3770) decays to $\mu^+\mu^-$ are fixed at values obtained by analyzing other data samples

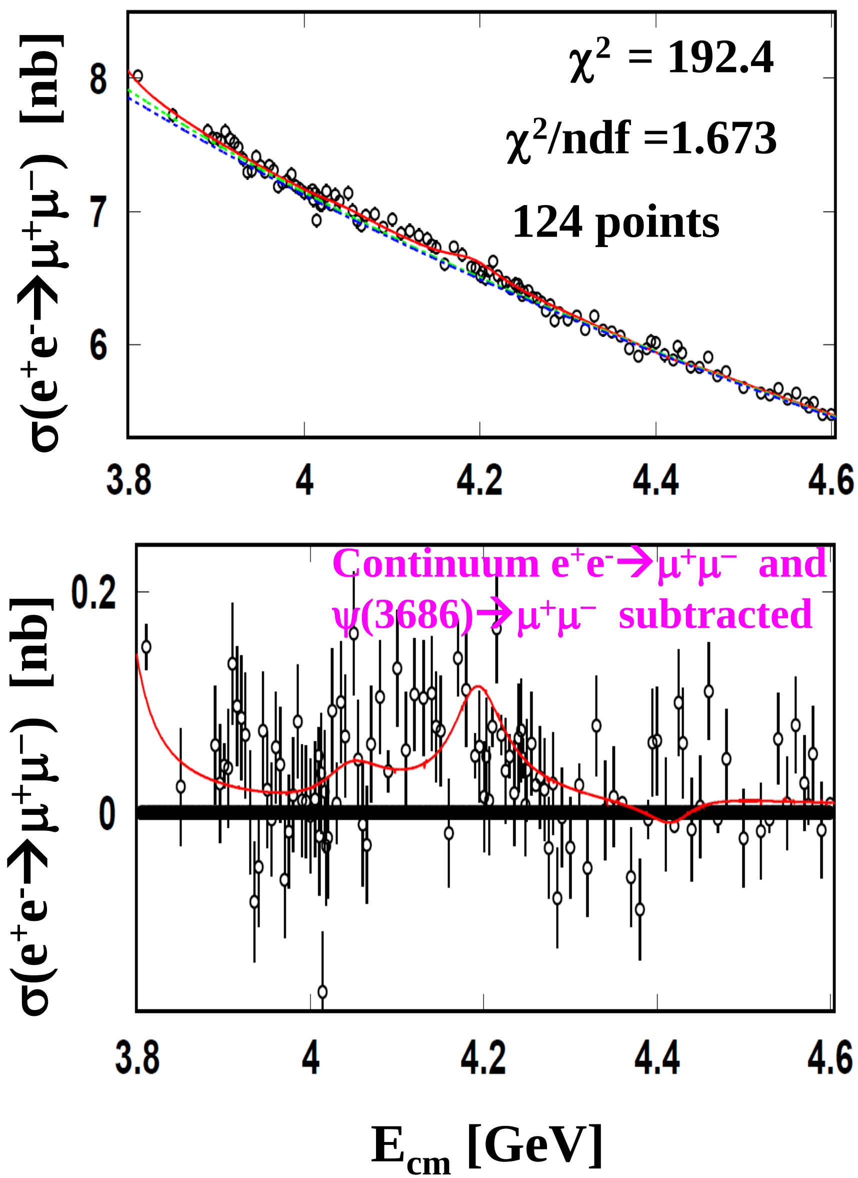
- **Find the better hypothesis**

By comparing the fit χ^2 obtained with these hypotheses, we find the better fit to these cross sections and search for new structure(s)

Fit to the Observed Cross Sections

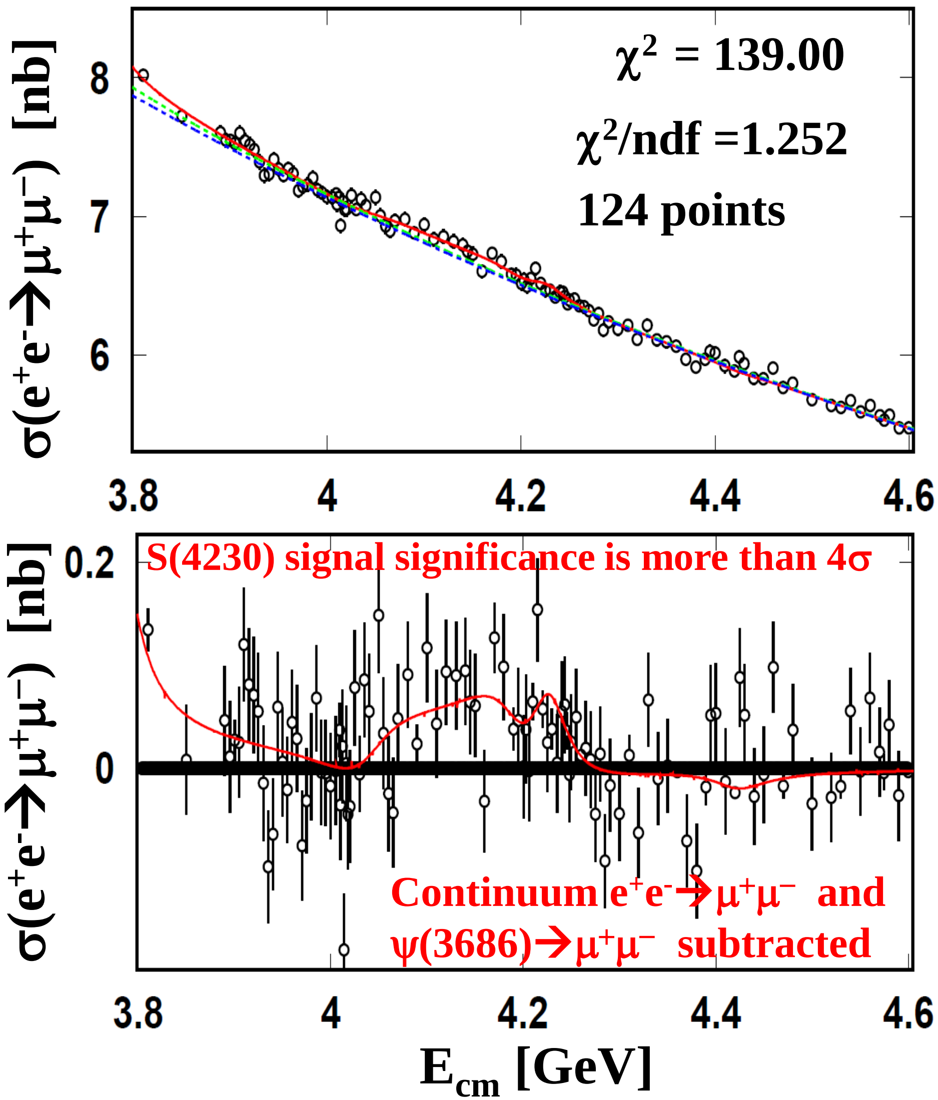
✓ **Cnt + ψ(3686)+Rs(3770) +ψ(4040)+ψ(4160)+ψ(4415)**

Parameters	Solution#1
$M_{\psi(4040)}$ [MeV]	4039.0 (fixed)
$\Gamma^{tot}_{\psi(4040)}$ [MeV]	80.0 (fixed)
$\Gamma^{ee}_{\psi(4040)}$ [keV]	0.86 (fixed)
$Br[\psi(4040)\to\mu^+\mu^-]$ [$\times 10^{-4}$]	$0.088\pm0.098\pm\Delta_{sys}$
$\Phi_{\psi(4040)}$	$(-229.1\pm25.2\pm\Delta_{sys})$ °
$M_{\psi(4160)}$ [MeV]	4419.0 (fixed)
$\Gamma^{tot}_{\psi(4160)}$ [MeV]	70 (fixed)
$\Gamma^{ee}_{\psi(4160)}$ [keV]	0.48 (fixed)
$Br[\psi(4160)\to\mu^+\mu^-]$ [$\times 10^{-4}$]	$1.479\pm0.032\pm\Delta sys$
$\Phi_{\psi(4160)}$	$(-270.6\pm7.4\pm\Delta_{sys})$ °
$M_{\psi(4415)}$ [MeV]	4421.0 (fixed)
$\Gamma^{tot}_{\psi(4415)}$ [MeV]	62.0 (fixed)
$\Gamma^{ee}_{\psi(4415)}$ [keV]	0.58 (fixed)
$Br[\psi(4415)\to\mu^+\mu^-]$ [$\times 10^{-4}$]	$0.080\pm0.042\pm\Delta_{sys}$
$\Phi_{\psi(4415)}$	$(-55.2\pm20.9\pm\Delta_{sys})$ °



Fit to the Observed Cross Sections

✓ Cnt + ψ(3686)+Rs(3770) +ψ(4040) +ψ(4160)+ψ(4415) + S_1



Parameters	Solution#1
$M_{\psi(4040)}$ [MeV]	4039.0 (fixed)
$\Gamma^{tot}_{\psi(4040)}$ [MeV]	80.0 (fixed)
$\Gamma^{ee}_{\psi(4040)}$ [keV]	0.86 (fixed)
Br[$\psi(4040) \to \mu^+\mu^-$] [$\times 10^{-4}$]	$0.167 \pm 0.128 \pm \Delta_{sys}$
$\Phi_{\psi(4040)}$	$(-28.1 \pm 25.7 \pm \Delta_{sys})$ °
$M_{\psi(4160)}$ [MeV]	4419.0 (fixed)
$\Gamma^{tot}_{\psi(4160)}$ [MeV]	70 (fixed)
$\Gamma^{ee}_{\psi(4160)}$ [keV]	0.48 (fixed)
Br[$\psi(4160) \to \mu^+\mu^-$] [$\times 10^{-4}$]	$0.882 \pm 0.953 \pm \Delta sys$
$\Phi_{\psi(4160)}$	$(-166.3 \pm 32.9 \pm \Delta_{sys})$ °
$M_{\psi(4415)}$ [MeV]	4421.0 (fixed)
$\Gamma^{tot}_{\psi(4415)}$ [MeV]	62.0 (fixed)
$\Gamma^{ee}_{\psi(4415)}$ [keV]	0.58 (fixed)
Br[$\psi(4415) \to \mu^+\mu^-$] [$\times 10^{-4}$]	$0.038 \pm 0.030 \pm \Delta_{sys}$
$\Phi_{\psi(4415)}$	$(-100.4 \pm 34.9 \pm \Delta_{sys})$ °
$M_{\psi(4230)}$ [MeV]	**$4227.7 \pm 9.1 \pm \Delta_{sys}$**
$\Gamma^{tot}_{\psi(4230)}$ [MeV]	**$42.3 \pm 15.2 \pm \Delta_{sys}$**
$\Gamma^{ee}_{\psi(4230)} \times B_{\psi(4230)}$ [eV]	**$0.052 \pm 0.038 \pm \Delta sys$**
$\Phi_{\psi(4230)}$	**$(-105.5 \pm 33.6 \pm \Delta_{sys})$ °**

Comparison with the PDG2016

Parameters	Solution#1	PDG2016
$M_{\psi(4040)}$ [MeV]	4039.0 (fixed)	4039 ± 1
$\Gamma^{tot}_{\psi(4040)}$ [MeV]	80.0 (fixed)	80 ± 10
$\Gamma^{ee}_{\psi(4040)}$ [keV]	0.86 (fixed)	0.86 ± 0.07
$Br[\psi(4040) \to \mu^+\mu^-]$ [$\times 10^{-4}$]	$0.167 \pm 0.128 \pm \Delta_{sys}$	0.107 ± 0.016 [for $\psi(4040) \to e^+e^-$]
$\Phi_{\psi(4040)}$	$(-28.1 \pm 25.7 \pm \Delta_{sys})$ °	N/A
$\Gamma^{ee}_{\psi(4160)} \times B_{\psi(4160)}$ [eV]		
$M_{\psi(4160)}$ [MeV]	4419.0 (fixed)	4191 ± 5
$\Gamma^{tot}_{\psi(4160)}$ [MeV]	70 (fixed)	70 ± 10
$\Gamma^{ee}_{\psi(4160)}$ [keV]	0.48 (fixed)	0.48 ± 0.22
$Br[\psi(4160) \to \mu^+\mu^-]$ [$\times 10^{-4}$]	$0.882 \pm 0.953 \pm \Delta sys$	0.069 ± 0.033 [for $\psi(4160) \to e^+e^-$]
$\Phi_{\psi(4160)}$	$(-166.3 \pm 32.9 \pm \Delta_{sys})$ °	N/A
$\Gamma^{ee}_{S1} \times B_{S1}$ [eV]		
$M_{\psi(4415)}$ [MeV]	4421.0 (fixed)	4421 ± 4
$\Gamma^{tot}_{\psi(4415)}$ [MeV]	62.0 (fixed)	62 ± 20
$\Gamma^{ee}_{\psi(4415)}$ [keV]	0.58 (fixed)	0.58 ± 0.07
$Br[\psi(4415) \to \mu^+\mu^-]$ [$\times 10^{-4}$]	$0.038 \pm 0.030 \pm \Delta_{sys}$	0.094 ± 0.032 [for $\psi(4415) \to e^+e^-$]
$\Phi_{\psi(4415)}$	$(-100.4 \pm 34.9 \pm \Delta_{sys})$ °	N/A

Summary

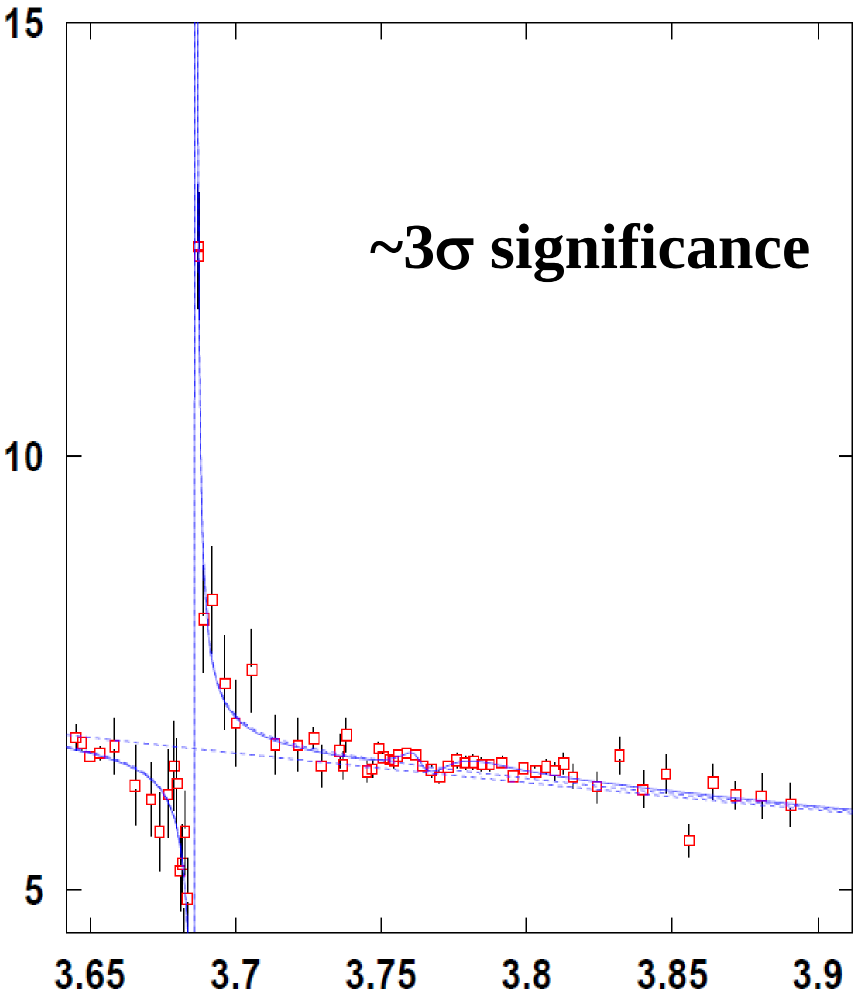
- **We measured the observed cross sections for $e^+e^- \to \mu^+\mu^-$ at energies from 3.8 to 4.6 GeV (with Yang's correction factors to the published luminosities of the data samples).**

- **We made the first measurements of the branching fractions of heavy ψ_s [$\psi(4040)$, $\psi(4160)$, and $\psi(4415)$] electromagnetic di-muon decays.**

- **We made the first measurements of the electromagnetic phase angles of the heavy ψ_s relative to the continuum $e^+e^- \to \mu^+\mu^-$.**

- **We may observe an evidence for $S(4230) \to \mu^+\mu^-$ with a signal significance of more than 4σ by analyzing these observed cross sections.**

Evidence for S(3760)→$\mu^+\mu^-$ (+ $\pi^+\pi^-$)



One of two solutions:

Br[$\psi(3686)\rightarrow\mu^+\mu^- + \pi^+\pi^-$]

$=(7.3\pm0.3\pm\Delta sys)\times10^{-3}$

$\phi=(7.8\pm3.7\pm\Delta_{sys})^\circ$

$M_{s1}=(3762.6\pm2.8\pm\Delta_{sys})$ MeV

$\Gamma_{s1}=(8.5\pm3.5\pm\Delta_{sys})$ MeV

$\Gamma^{ee}_{S1}\times B_{S1}=0.021\pm0.014$ eV

$\Phi_1=(148.5\pm42.9\pm\Delta_{sys})^\circ$

$M_{\psi(3770)}=3773.15$ MeV (fixed)

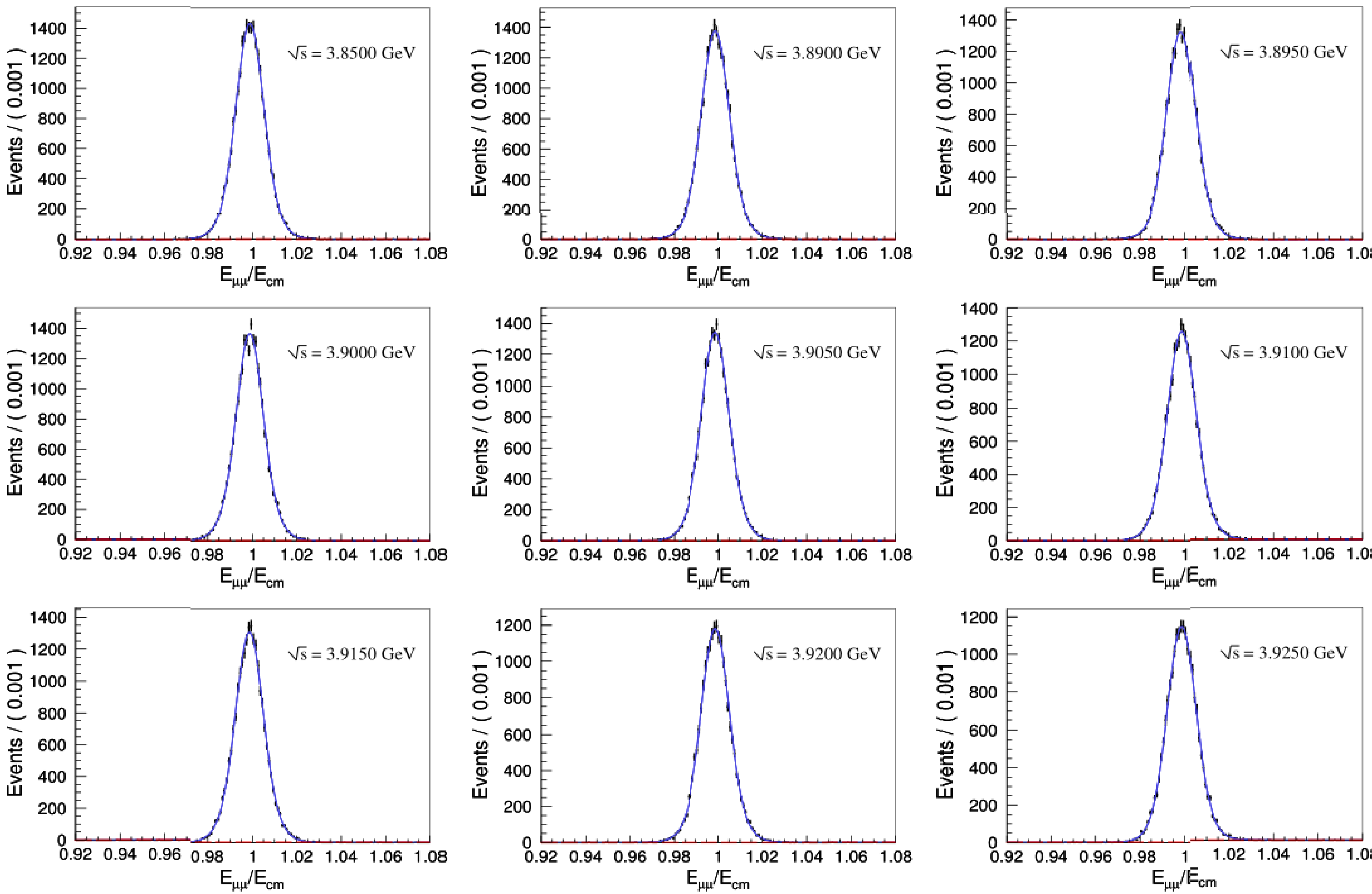
$\Gamma_{\psi(3770)}=27.3$ MeV (fixed)

$\Gamma^{ee}_{\psi(3770)}=265$ eV (fixed)

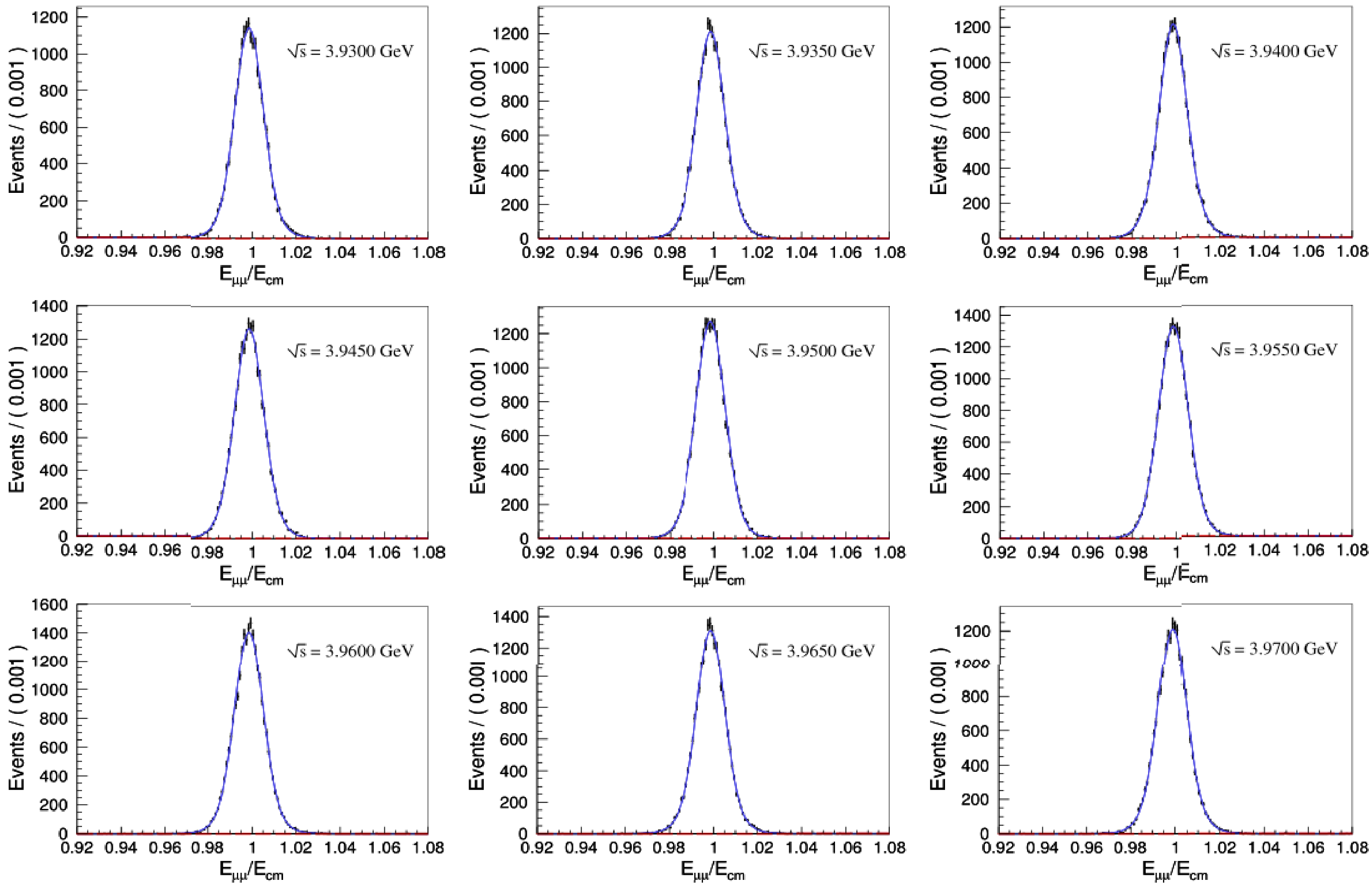
$B_{\psi(3770)}=(2.31\pm1.32\pm\Delta_{sys})\times10^{-4}$

$\Phi_2=(23.6\pm18.1\pm\Delta_{sys})^\circ$

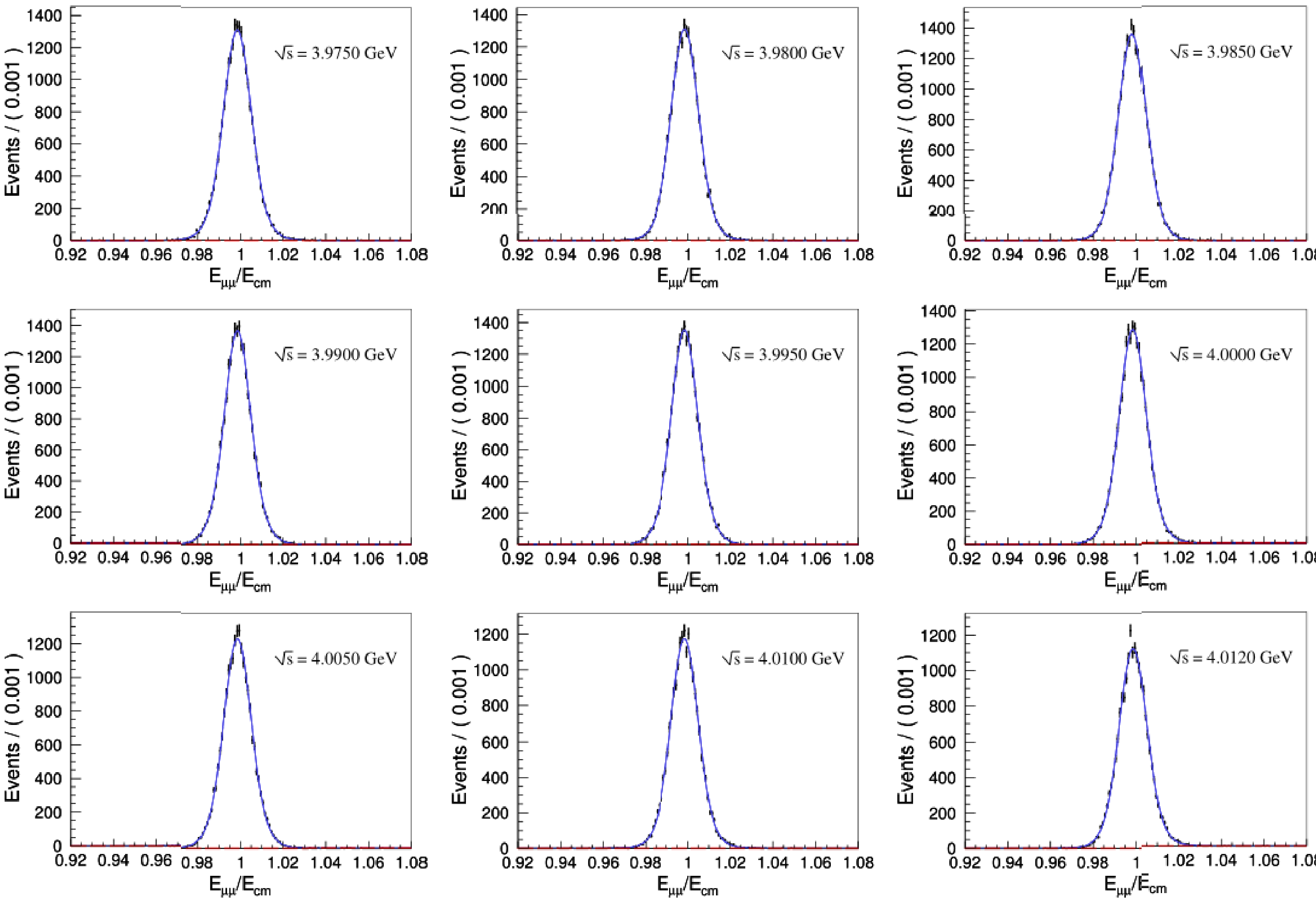
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



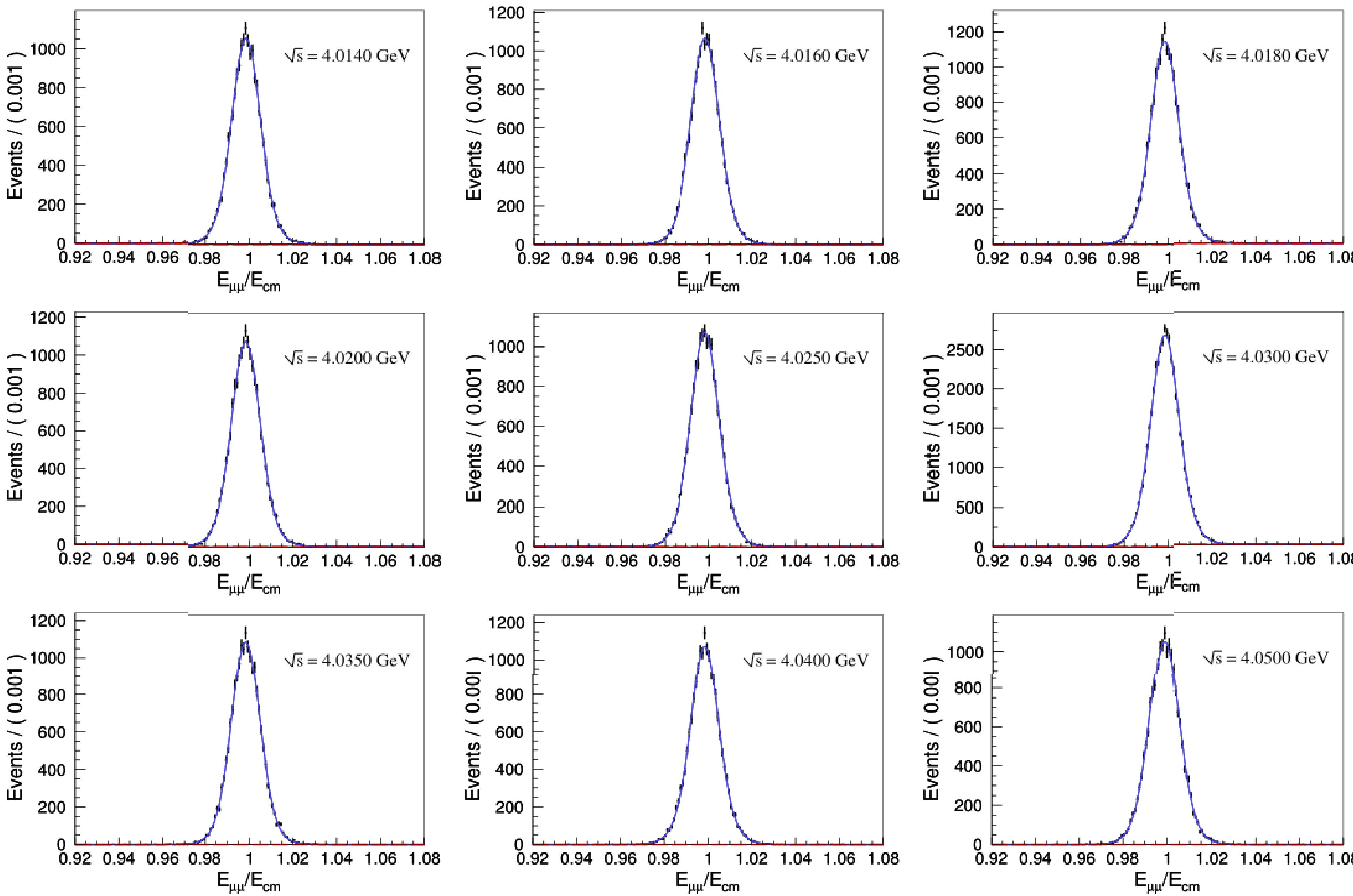
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



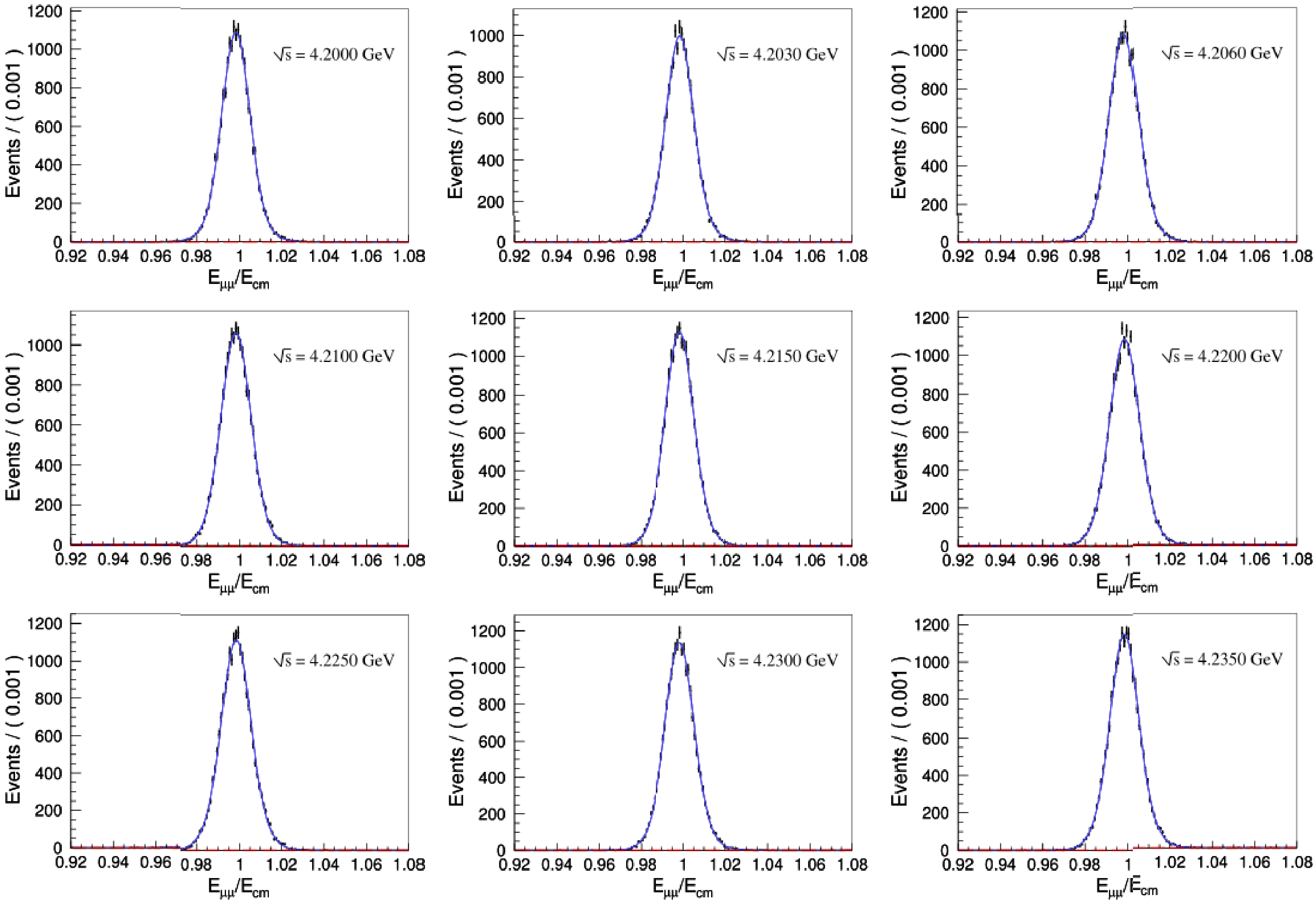
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



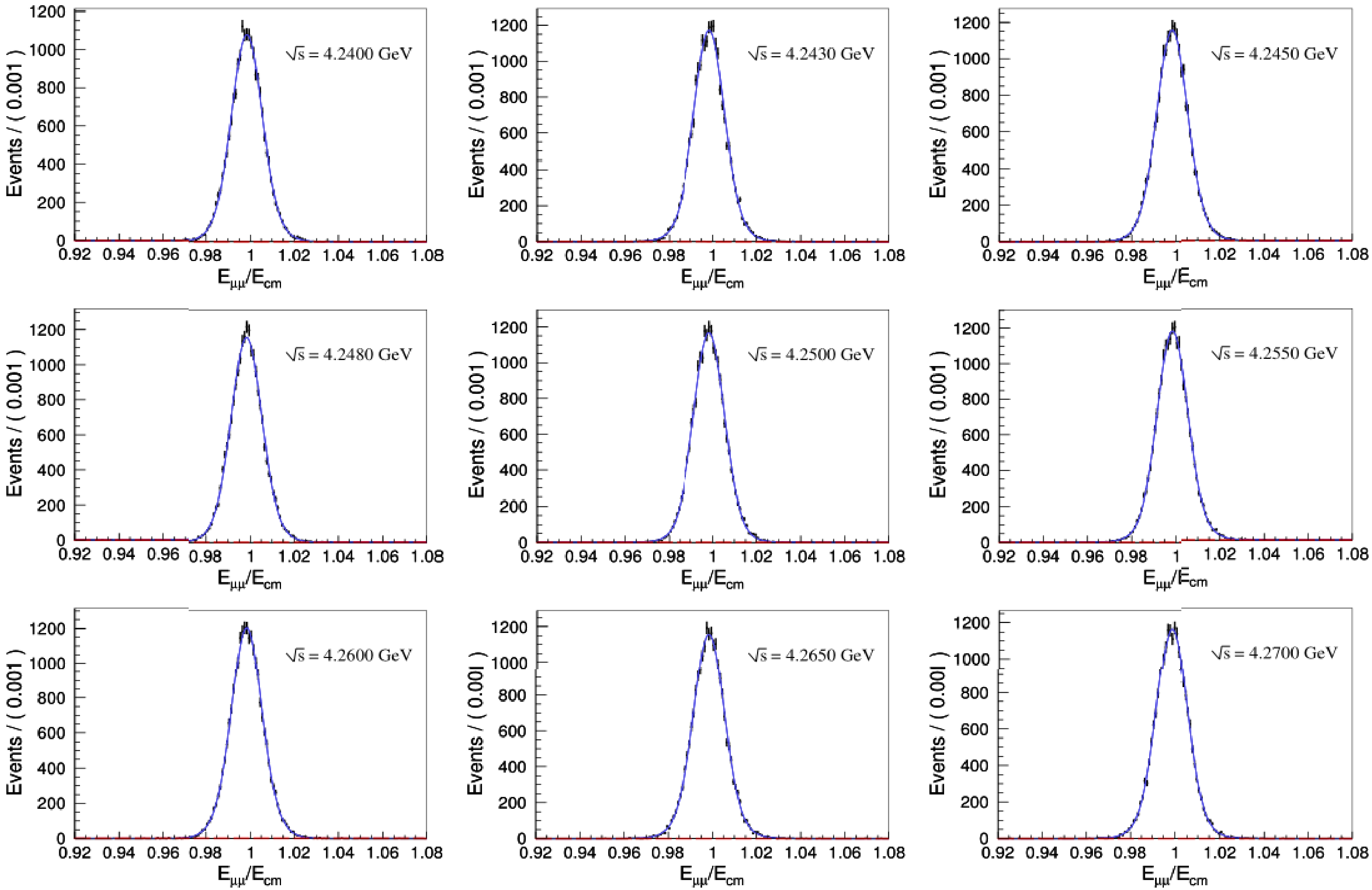
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



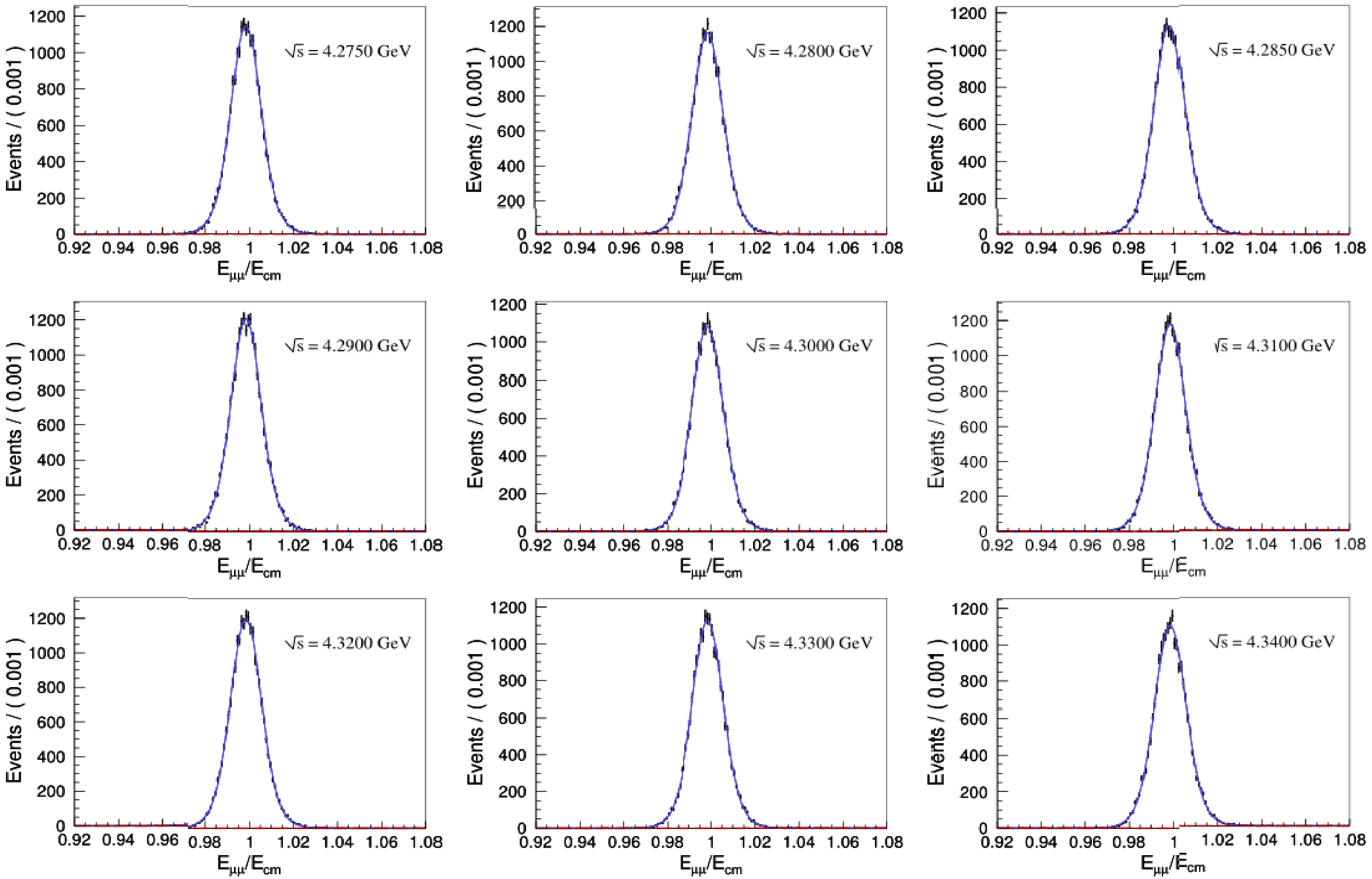
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



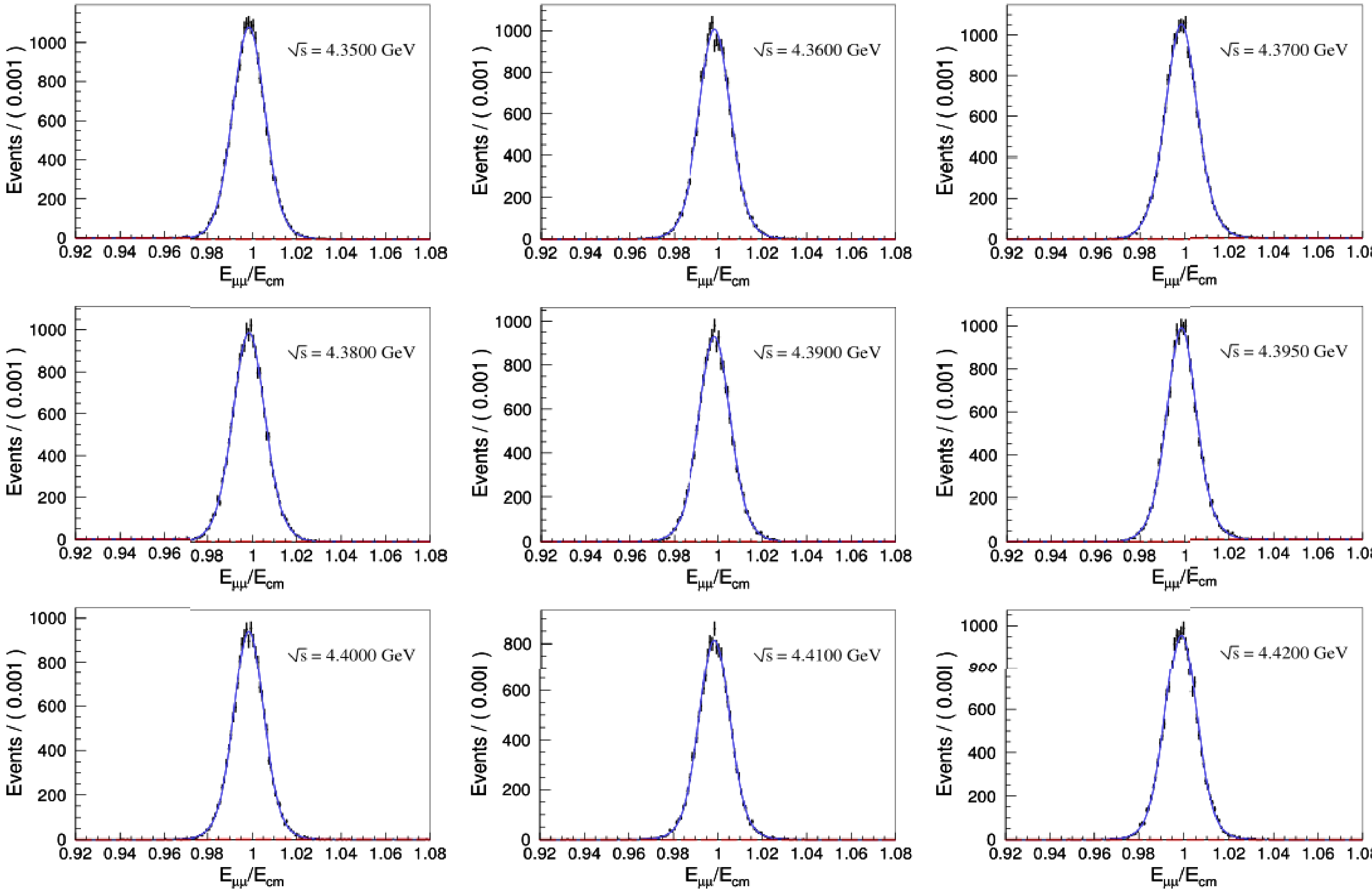
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



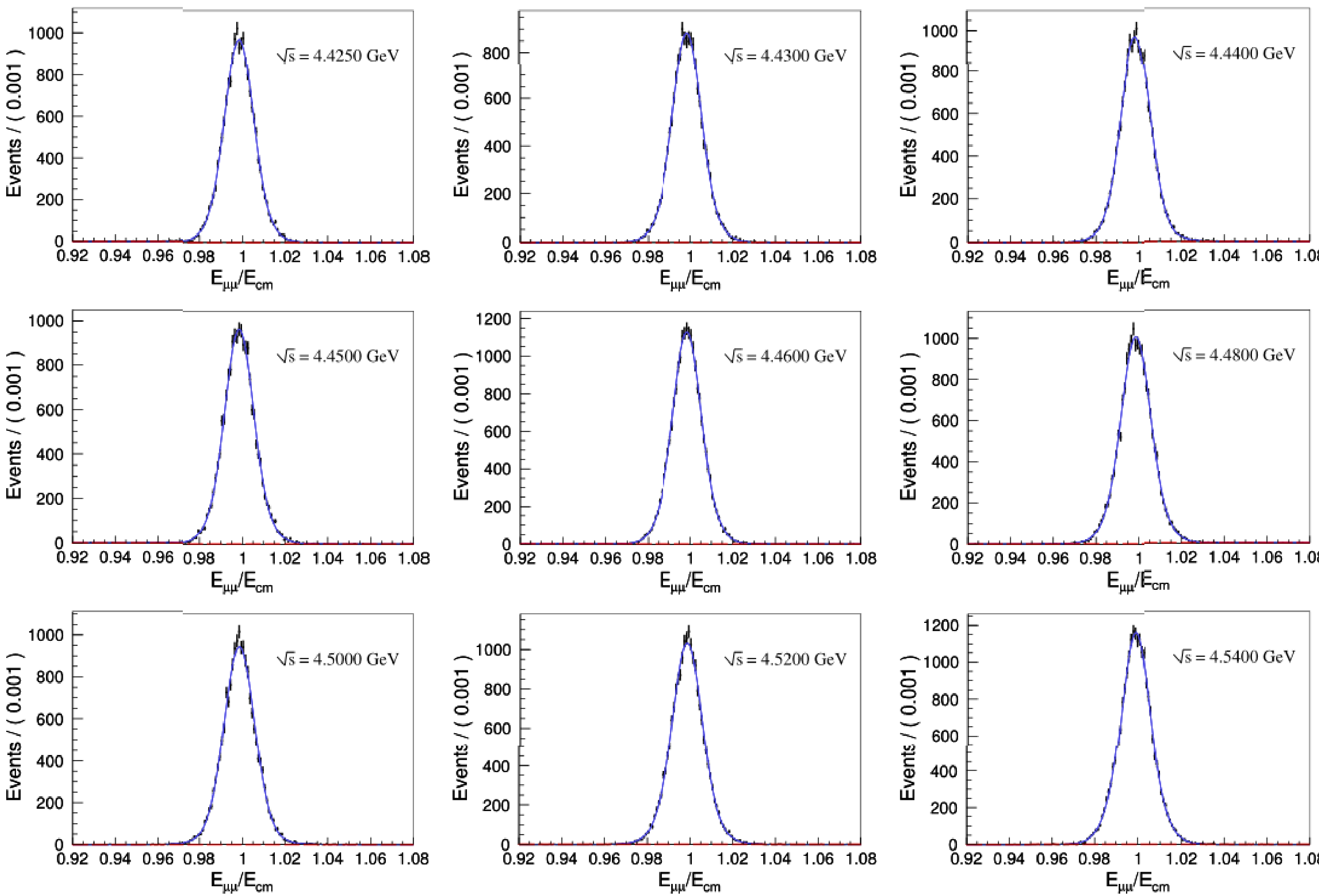
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



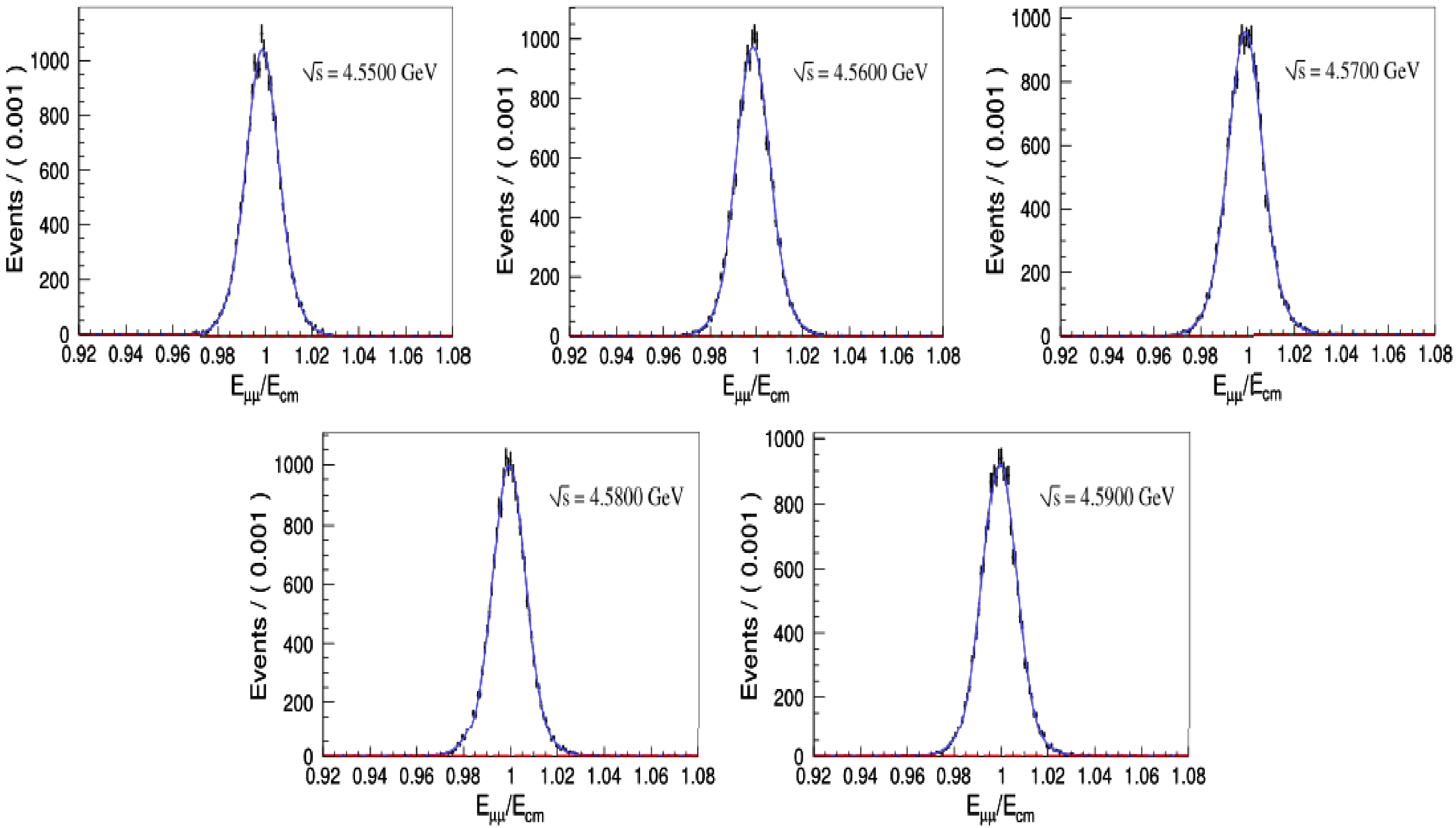
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



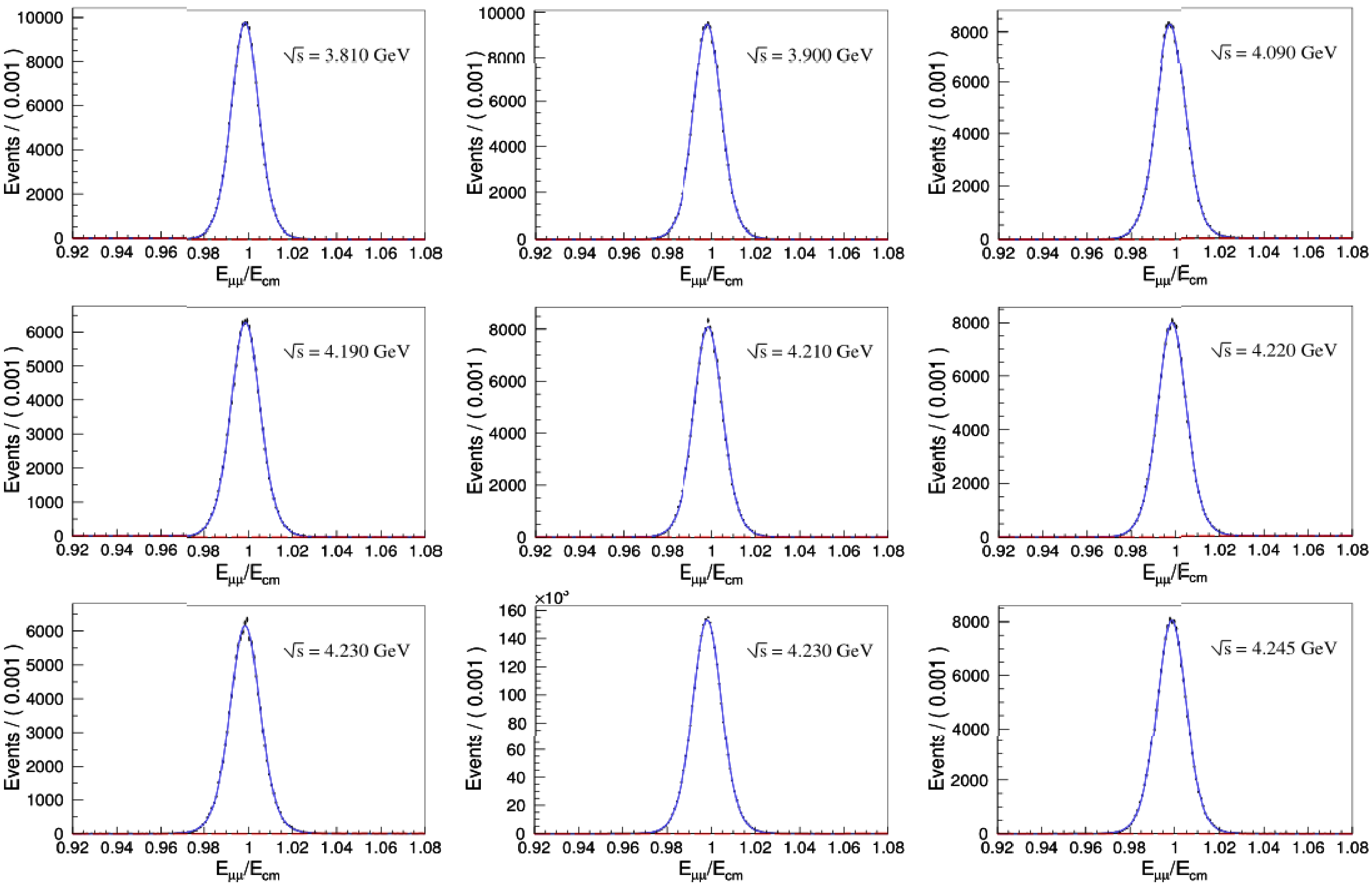
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



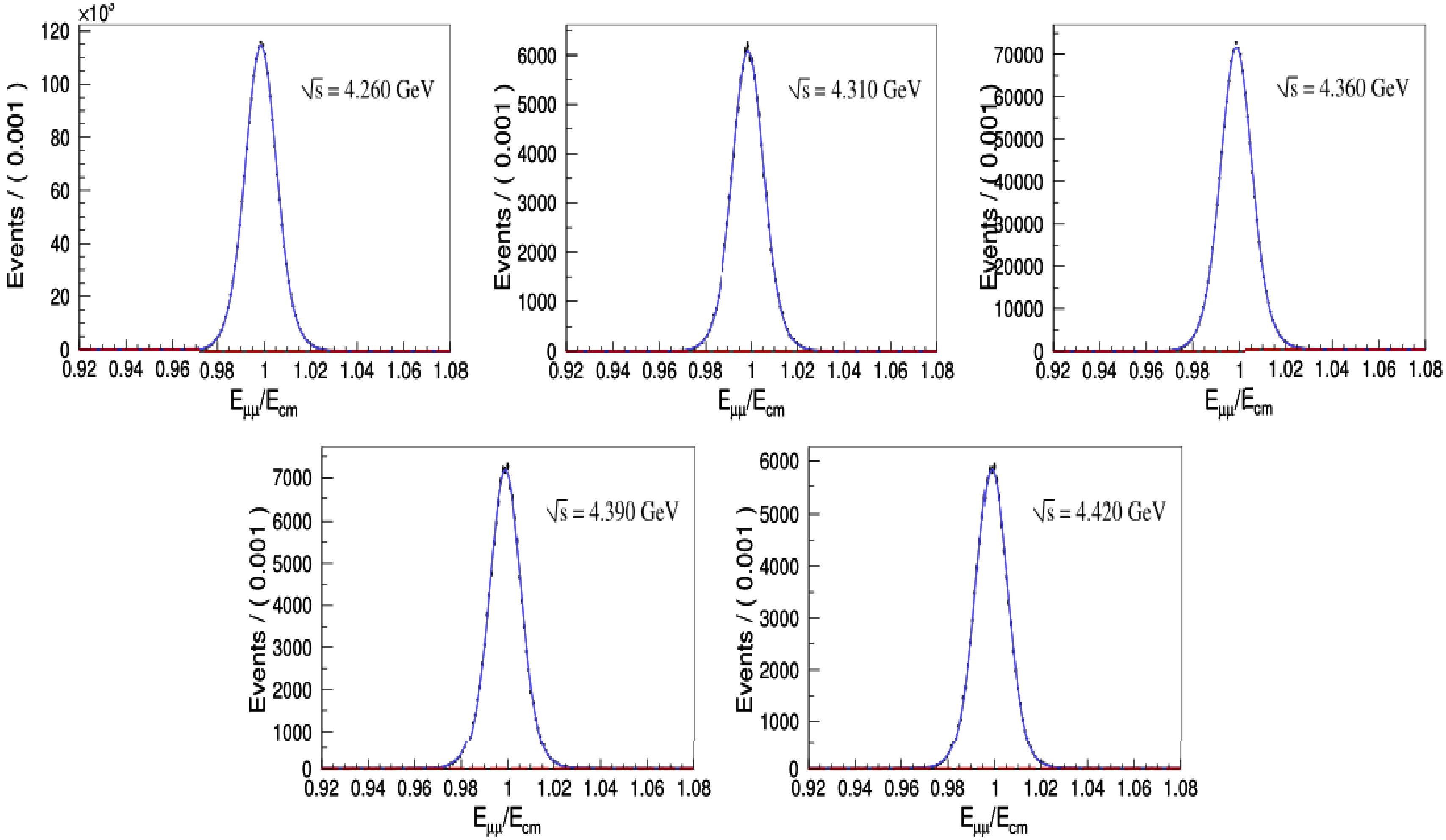
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



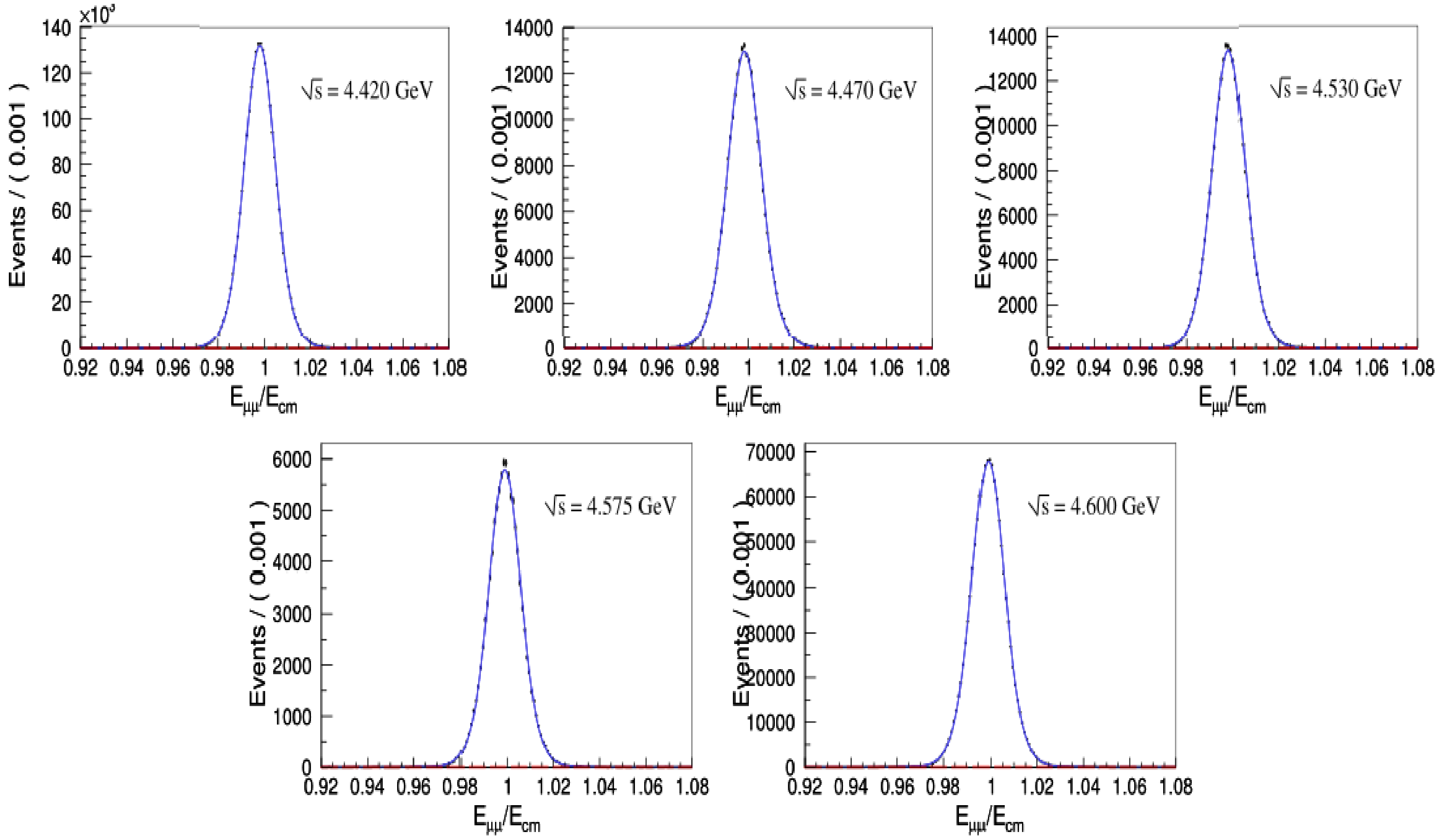
Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



Distributions of $E_{measured}/E_{cm}$ for $e^+e^- \rightarrow \mu^+\mu^-$



Correlations

The systematic error sources are correlated among the different energy points, so these cannot be considered directly in the fit. But their effects on the final results can be estimated by the "offset method [see M. Botje, J. Phys. G 28 779(2002)]"

- **When constructing the χ^2 to fit the cross section, we have considered the correlation between different energy points.**
- **At first, we fit the cross sections only considering the statistical uncertainties. At this stage, the uncertainties of the parameter values from the fits are only due to the statistical.**
- **To estimate the systematic uncertainties of the parameter values of the structures, we shifted the observed cross sections by $+1\Delta_{sys}$ or $-1\Delta_{sys}$, where Δ_{sys} is the correlated systematic error, then we fitted these cross sections again.**

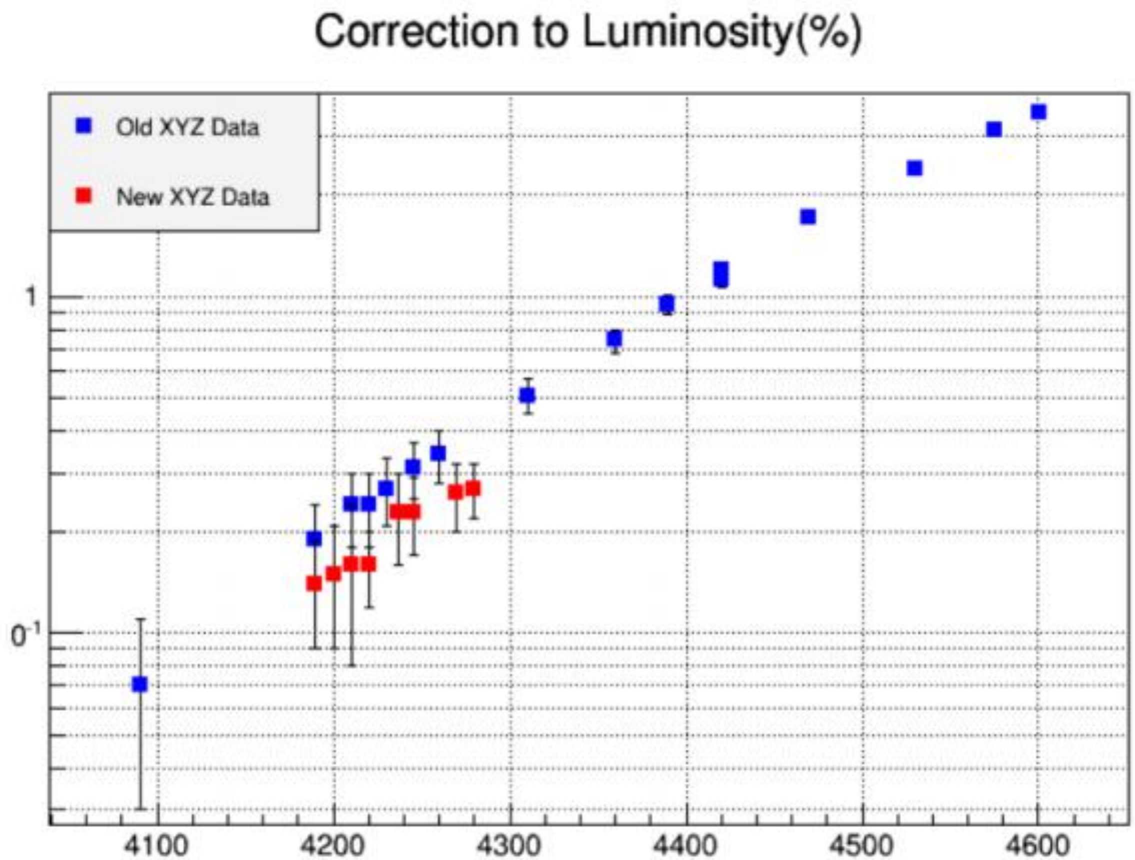
The effects of the systematic uncertainty (correlation between energy points) on the parameters are still in progress.

Result of old XYZ data

Table 6: A summary of the update on the luminosity of the old XYZ data with only statistical errors.

Energy Point(MeV)	Correction Factor(%)	New Result(pb^{-1})	Previous Result(pb^{-1})	Difference(%)
3810	-	-	50.54 ± 0.03	-
3900	-	-	52.61 ± 0.03	-
4009	-	-	481.96 ± 0.10	-
4090	0.07	52.86 ± 0.03	52.63 ± 0.03	+0.43
4190	0.19	43.33 ± 0.03	43.09 ± 0.03	+0.56
4210	0.24	54.95 ± 0.03	54.55 ± 0.03	+0.73
4220	0.24	54.60 ± 0.03	54.13 ± 0.03	+0.86
4230_1	0.27	44.54 ± 0.03	44.40 ± 0.03	+0.32
4230_2	0.27	1056.37 ± 0.13	1047.34 ± 0.14	+0.86
4245	0.31	55.88 ± 0.03	55.59 ± 0.04	+0.53
$4260_{1,2}$	0.34	828.36 ± 0.12	523.74 ± 0.10 301.93 ± 0.08	+0.32
4310	0.51	45.08 ± 0.03	44.90 ± 0.03	+0.40
4360	0.74	543.94 ± 0.10	539.84 ± 0.10	+0.76
4390	0.95	$55.57 \pm \pm 0.04$	55.18 ± 0.04	+0.70
4420_1	1.13	46.80 ± 0.03	44.67 ± 0.03	+4.77
4420_2	1.20	1043.86 ± 0.13	1028.89 ± 0.13	+1.45
4470	1.71	111.09 ± 0.04	109.94 ± 0.04	+1.05
4530	2.38	112.12 ± 0.04	109.98 ± 0.04	+1.95
4575	3.13	48.93 ± 0.03	47.67 ± 0.03	+2.64
4600	3.51	586.89 ± 0.11	566.93 ± 0.11	+3.52

Size of EMC Correction



More serious the problem if BES is to go beyond 4.6 GeV.