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# Measurement of $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ in the vicinity of the 

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#### Abstract

We report the measurement of the cross section of the process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ close to the $X(3872)$ mass in search for the direct formation $e^{+} e^{-} \rightarrow X(3872)$. Due to the axialvector properties of the $X(3872)$ state, at least two virtual photons are required. No signal is observed and an upper limit on the product of electronic width and branching fraction of $X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi$ is determined: $\Gamma_{e e} \times \mathcal{B}\left(X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi\right)<9 \mathrm{meV}$ at the $90 \%$ confidence level. This is an improvement of a factor of about 14 compared to existing limits.


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## 1 Introduction

The observation of the $X$ (3872) resonance by the Belle Collaboration in 2003 [1] and its quick confirmation by other experiments [2-6] mark the beginning of $X Y Z$ spectroscopy. Being inconsistent with the predictions of the $q \bar{q}$ meson model, the $X Y Z$ states are candidates for tetraquarks, meson molecules, hybrid mesons and more [7]. The $X(3872)$ state is probably the best known representative of this class. It has been observed in $B$ decays, in radiative transitions of the $Y(4260)$ resonance, as well as in inclusive $p p$ and $p \bar{p}$ collisions. Up to now, decays into five different final states are established [7, 8]. Its mass is very close to the $D \bar{D}^{*}$ threshold, which is also a decay channel. This supports the meson molecule model [9]. However, one important disrcriminant between the different models is the width, which is still unknown and only an upper limit of 1.2 MeV at the $90 \%$ confidence level exists [10].

The quantum numbers $1^{++}$of the $X(3872)$ particle [11] allow only a suppressed formation in $e^{+} e^{-}$ collisions via two virtual photons. Nevertheless, modern $e^{+} e^{-}$colliders, such as BEPCII, may have sufficient luminosity to observe direct $X(3872)$ formation. A scan of the lineshape would provide important information about the width. The electronic width $\Gamma_{e e}$, which itself may help to reveal the $X(3872)$ 's nature, serves as a measure of the feasability of such a scan. The current upper limit is $\Gamma_{e e} \times \mathcal{B}\left(X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi\right)<130 \mathrm{meV}$ at $90 \%$ confidence level, determined by BESIII using the ISR technique [12]. A theoretical prediction using Vector Meson Dominance yields $\Gamma_{e e} \gtrsim 30 \mathrm{meV}$ [13]. Combined with the lower limit on $\mathcal{B}\left(X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi\right)$ of $2.6 \%$ [8], this results in a lower bound of $\Gamma_{e e} \times \mathcal{B}\left(X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi\right) \gtrsim 0.78 \mathrm{meV}$.

The aim of this analysis is to measure the cross section $\sigma\left(e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi\right)$ around the the $X(3872)$
mass and the subsequent determination of the product $\Gamma_{e e} \times \mathcal{B}\left(X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi\right)$ or an improved limit on this quantity. Although the production of a hadronic non-vector final state has been observed in $e^{+} e^{-}$collisions [14], it would be the first observation of the formation of a non-vector resonance in $e^{+} e^{-}$ annihilations.

The next sections describe the used data sets and the Monte Carlo Simulations of the $e^{+} e^{-} \rightarrow$ $\pi^{+} \pi^{-} J / \psi$ process. The basic event selection is outlined in Section 4. In Section 5, the further analysis strategy based on a background study is illustrated. The cross section $\sigma\left(e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi\right)$ is determined in Section 6 and the upper limit calculation is explained in Section 7. The analysis is concluded in Section 8.

## 2 Data Samples

This analysis aims for measuring (or setting an improved upper limit on) the product $\Gamma_{e e}^{X(3872)} \times \mathcal{B}(X(3872) \rightarrow$ $\left.\pi^{+} \pi^{-} J / \psi\right)$ of the $X(3872)$. Therefore, two dedicated data sets were recorded in June 2017 in the vicinity of the $X(3872)$ mass $(3871.69 \mathrm{MeV})$. The intented center-of-mass for one data set were directly the central $X(3872)$ mass (refered to as on-resonance sample) and ca. 4 MeV below that value for the other data set (refered to as off-resonance sample). During data taking the Beam Energy Measurement System (BEMS) was running for a precise realtime ${ }^{1}$ measurment of $\sqrt{s}$. During the first two runs, the beam energies were slightly too low. In order to correct for this, they were increased slightly. As a result, the off-resonance sample contains two runs with slightly different $\sqrt{s}$. Additionally, the $\sqrt{s}$ obtained from the BEMS is more precise than result of the usual analysis of dimuon events and is thus used for the offline analysis as well. The BEMS is also capable of measuring the energy spread of $\sqrt{s}$. See Section A for a detailed description of the BEMS measurement.

Furthermore, the two data sets from the 2013 XYZ scan, that have $\sqrt{s}$ closest to the $X(3872)$ mass, are used in this analysis. For these data sets, the center-of-mass has been determined via the analysis of dimuon events [15].

All four data sets are reconstructed with the BOSS version 7.0.3. Hence, this version is used for the whole analysis. The luminosity of all data sets have been determined by the analysis of Bhabha events. For the 2013 data the values are taken from [16]. A description of the luminosity determination of the 2017 data can be found in Section B. An overview on the data sets is given in Table 1.

[^0]Table 1: Overview of the data sets. The center-of-mass energy $\sqrt{s}$ and its spread $\delta \sqrt{s}$ together with the method, how these values are obtained, are listed. In addition, the integrated luminosity $\int \mathcal{L} \mathrm{d} t$, the year of data taking and the run numbers are shown.

| $\sqrt{s} / \mathrm{MeV}$ | $\delta \sqrt{s} / \mathrm{MeV}$ | $\sqrt{s}$ Determination | $\int \mathcal{L} \mathrm{d} t / \mathrm{pb}^{-1}$ | Year | Run numbers |
| :--- | :---: | :---: | :---: | ---: | ---: |
| $3807.7 \pm 0.6$ | not measured | Dimuon | $50.5 \pm 0.5$ | 2013 | $33490-33556$ |
| $3866.32 \pm 0.10$ | $1.45 \pm 0.09$ |  |  |  | $52108-52109$ |
| $3867.410 \pm 0.031$ | $1.406 \pm 0.025$ | BEMS | $108.9 \pm 1.3$ | 2017 | $52110-52206$ |
| $3871.31 \pm 0.06$ | $1.73 \pm 0.06$ | BEMS | $110.3 \pm 0.8$ | 2017 | $52207-52297$ |
| $3896.2 \pm 0.8$ | not measured | Dimuon | $52.6 \pm 0.5$ | 2013 | $33572-33657$ |

## 3 Monte Carlo Simulations of $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$

The event selection is optimized and the backgrounds are estimated by the use of Monte Carlo (MC) simulations. As already mentioned in the previous section, all steps of the analysis are performed with the BOSS version 7.0.3. The goal of this analysis is the measurement of (or an improved upper limit on) the electronic width $\Gamma_{e e}^{X(3872)}$ of the $X(3872)$ in the reaction $e^{+} e^{-} \rightarrow X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi$ and the subsequent $J / \psi \rightarrow \ell^{+} \ell^{-}$decay, where $\ell=e, \mu$. This resonant signal process is accompanied by the nonresonant continuum process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ (c.f. Figure 1). The corresponding Feynman diagrams are shown in Figure 2.

### 3.1 Signal Process $e^{+} e^{-} \rightarrow X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi$

The $X(3872)$ decay into $\pi^{+} \pi^{-} J / \psi$ is known to proceed via the intermediate $\rho^{0} J / \psi$ state $[6,10,11]$ and the $J / \psi$ is reconstructed via its dilepton decay. Thus, the signal process is simulated within EvtGen [17] according to


The acronyms in parantheses indicate the EvtGen decay model used for the sub-decay in the corresponding line. The ISR is simulated with KKMC [18] with a cross section lineshape assumed to be flat, i.e. constant. Final state radiation (FSR) is simulated with PHOTOS [19]. For each of the four different data samples, $5 \times 10^{5}$ events with $J / \psi \rightarrow e^{+} e^{-}$and $5 \times 10^{5}$ events with $J / \psi \rightarrow \mu^{+} \mu^{-}$are simulated.
expected cross section


Figure 1: Expected cross section of $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ assuming a constant continuum of 17 pb , an electronic witdth of $\Gamma_{e e}^{X(3872)}=0.13 \mathrm{eV}$, a total width of $\Gamma_{\text {tot }}^{X(3872)}=1.0 \mathrm{MeV}$ and a spread of $\sqrt{s}$ of 1.5 MeV . The black markers indicate the center-of-mass energies of the used data sets. The error bars indicate an estimation of the expected uncertainty.


Figure 2: Feynman diagrams of the non-resonant continuum process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ (left) and the resonant signal process $e^{+} e^{-} \rightarrow X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi$ (right), of which it is known, that the $\pi^{+} \pi^{-}$pair forms a $\rho^{0}$ meson [ $6,10,11$ ].

### 3.2 Continuum Process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$

The non-resonant $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ reaction proceeds via a single virtual photon and the final state is therefore constrained to the $1^{--}$quantum numbers. The relative small phasespace available for the $\pi^{+} \pi^{-}$ pair suggests that it exists in an $S$-wave. The different possibilities to model this within EvtGen are presented in the following. Again, ISR is simulated with KKMC and FSR is simulated with PHOTOS.

- In the EvtGen version within BOSS, there is the JPIPI model which models the decay of a $1^{--}$ state to $\pi^{+} \pi^{-} J / \psi$ and is tuned to reproduce the distributions of the $\psi^{\prime} \rightarrow \pi^{+} \pi^{-} J / \psi$ decay observed in real data. The mass of the $\psi^{\prime}$ is $120-210 \mathrm{MeV}$ below the $\sqrt{s}$ values of the data sets used in this analysis making this model a good candidate for the continuum process. However, during simulation the warning message "prob > maxprob" was printed several times implying that the distributions are not sampled correctly. The full decay chain is modelled according to

$$
\begin{array}{rll}
e^{+} e^{-} \longrightarrow \pi^{+} \pi^{-} J / \psi & (\mathrm{JPIPI}) \\
& \longmapsto e^{+} e^{-} & (\mathrm{VLL}) \\
& \mu^{+} \mu^{-} & (\mathrm{VLL})
\end{array}
$$

- Very similar to the JPIPI model, there is the VVPIPI model, designed to model the decay of a $1^{--}$state to $\pi^{+} \pi^{-}$and a $1^{--}$state, where the $\pi^{+} \pi^{-}$system is dominated by an $S$-wave. This is a natural choice for the continuum process. Unfortunately, the same warning message as with the JPIPI model was shown. The implication is the same, i.e. that the distributions are not sampled correctly. The full chain is

- The $S$-wave character of the $\pi^{+} \pi^{-}$system suggests that they form the intermediate state of a $\sigma$ meson. This corresponds to

(PHSP)
- Another possibility to incorporate the $\sigma$ meson is the VVS_PWAVE model. It models the decay of a $1^{--}$state to another $1^{--}$state and a (pseudo-) scalar meson. The amplitudes and phases for the $S$-, $P$-, and $D$-wave contributions can be specified, although only a pure $P$-wave configuration has been tested by the developers. Here, only the $S$-wave contributes. The full chain is

- The simplest (and probably the least realistic) way to simulate the reaction is the PHSP model:


For all the different models and data sets, $5 \times 10^{5}$ events are simulated for each of the two $J / \psi$ decay modes. The difference between the models is best visualized in the $m\left(\pi^{+} \pi^{-}\right)$distributions (c.f. Figures 3 and 4).

The limited statistics of the used data sets doesn't allow a reliable judgement about which of the models reflects reality most (c.f. Figure 10). Therefore, another data set of BESIII taken at $\sqrt{s}=$ 4007.6 MeV in 2011 is analyzed. This set has $482 \mathrm{pb}^{-1}$ (compared to the total integrated luminosity of $322.3 \mathrm{pb}^{-1}$ of the used four sets) and is only 10 MeV above the highest energy point of this analysis. The $m\left(\pi^{+} \pi^{-}\right)$distribution of this data set is shown in Figure 5 . The majority of events is clustered towards higher dipion masses. This favours the VVPIPI and JPIPI model over the models with the $\sigma$ resonance and the PHSP model. In this analysis uses the VVPIPI model for the continuum, but the difference to the $\sigma$ model with PHSP decay is used as a systematic uncertainty (c.f. section 6.2).

## 4 Event Selection

The final state $\pi^{+} \pi^{-} \ell^{+} \ell^{-}(\ell=e, \mu)$ consists of four charged tracks with zero net charge. Events with the following criteria are selected:

- Each charged track is required to to fulfill the standard tracking cuts of BESIII. They need to originate from a confined volume around the interaction point and lie within the detectors acceptance:

$$
-\left|z_{p o c a}\right|<10 \mathrm{~cm}
$$



Figure 3: Simulated distribution of $m\left(\pi^{+} \pi^{-}\right)$for the different data sets and models of the reaction $e^{+} e^{-} \rightarrow$ $\pi^{+} \pi^{-} J / \psi$ described in Sections 3.1 and 3.2 with $J / \psi \rightarrow e^{+} e^{-}$.


Figure 4: Simulated distribution of $m\left(\pi^{+} \pi^{-}\right)$for the different data sets and models of the reaction $e^{+} e^{-} \rightarrow$ $\pi^{+} \pi^{-} J / \psi$ described in Sections 3.1 and 3.2 with $J / \psi \rightarrow \mu^{+} \mu^{-}$.


Figure 5: Invariant dipion mass distribution for the data at $\sqrt{s}=4007.6 \mathrm{MeV}$. The shown events correspond to the $J / \psi$ peak region as defind in section 5.2 and Figure 11. The accumulation of events in the low $m\left(\pi^{+} \pi^{-}\right)$region in the $e^{+} e^{-}$mode is background from $e^{+} e^{-} \rightarrow \gamma e^{+} e^{-}$as discussed in Section 5.
$-r_{p o c a}<1 \mathrm{~cm}$

- $\cos \theta<0.93$

Here, $z_{\text {poca }}$ is the $z$-component of the point of closest approach of the track with respect to the interaction point, which in turn is determined for easch run independently. The radial component of the point of closest approach is $r_{p o c a}=\sqrt{x_{p o c a}^{2}+y_{p o c a}^{2}}$. For the third requirement, $\theta$ is the polar angle of the momentum vector of the track at the point of closest approach. Charged tracks fulfilling these requirements are labeled "good tracks".

- Each candidate event needs to have a total number of good tracks with net zero charge.
- The lepton tracks coming from the $J / \psi$ decay have a large momentum (in the lab frame), while the pion tracks have a relative low momentum (c.f. Figure 6a). Tracks with lower momentum than $0.6 \mathrm{GeV} / c$ are identified as pion candidates and lepton candidates are required to have larger momentum than $1.0 \mathrm{GeV} / c$. Each candidate event needs to have two pion candidates with opposite charge and two lepton candidates with opposite charge.
- The two $J / \psi$ decay modes can be distinguished by the energy deposition in the EMC associated to the lepton tracks. Electrons deposit a large fraction of their energy, while the muons pass the EMC almost undisturbed leaving only a small energy deposition (c.f. Figure 6b). Each event must have either two electron candidates or two muon candidates.


Figure 6: MC simulation of the signal process at $\sqrt{s}=3871.31 \mathrm{MeV}$.

## 5 Background Study

### 5.1 Monte Carlo Simulations of Backgrounds

In order to estimate the background contamination from non $-\pi^{+} \pi^{-} J / \psi$ events, the following backgrounds were simulated.

- The most dominant background for the events, in which the $J / \psi$ candidate is reconstructed in the $e^{+} e^{-}$mode, is the radiative Bhabha process $e^{+} e^{-} \rightarrow \gamma e^{+} e^{-}$with subsequent conversion of the photon in the detector material. Here, the resulting $e^{+} e^{-}$pair might be reconstructed as $\pi^{ \pm}$ candidates. This background is simulated with the Babayaga 3.5 event generator [20].
- A similar background appears in the $\mu^{+} \mu^{-}$reconstruction mode of the $J / \psi$. Here, ther photon from radiative dimuon events ( $e^{+} e^{-} \rightarrow \gamma \mu^{+} \mu^{-}$) converts and the created $e^{+} e^{-}$pair is again misidentified as a $\pi^{-} \pi^{-}$pair. Compared to the radiative Bhabha process, this reaction has a much lower cross section. This process is simulated with the Phokhara event generator [21].
- Another QED process with four charged tracks in the final state ist the reaction $e^{+} e^{-} \rightarrow e^{+} e^{-} e^{+} e^{-}$. Here, one of the $e^{+} e^{-}$pairs might be misidentified as a pion pair and the total event is accepted as a $\pi^{+} \pi^{-} e^{+} e^{-}$event. This process is simulated with the BesTwogam event generator ref.
- Similar to the previous process the reaction $e^{+} e^{-} \rightarrow e^{+} e^{-} \mu^{+} \mu^{-}$could contribute to the overall background. Here, either the $e^{+} e^{-}$or the $\mu^{+} \mu^{-}$pair might be misidentified as a pion pair and the total event is accepted as a $\pi^{+} \pi^{-} e^{+} e^{-}$or a $\pi^{+} \pi^{-} \mu^{+} \mu^{-}$event. This process is also simulated with the BesTwogam event generator.
- Similar to the previous two processes the reaction $e^{+} e^{-} \rightarrow e^{+} e^{-} q \bar{q}$ could contribute to the overall background. The $q \bar{q}$ stands for an $u \bar{u}$ or a $d \bar{d}$ quark pair. Among others, they can hadronise into a $\pi^{+} \pi^{-}$pair. The final state is identical to the signal in the $J / \psi \rightarrow e^{+} e^{-}$mode. This process is simulated with the BesTwogam event generator as well.
- The most dominant background for the $\mu^{+} \mu^{-}$reconstruction mode is the reaction $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-}$. Pions have a similar mass as muons and deposit a similar amount of energy in the EMC. They are easily misidentified as muons. This reaction is simulated with the ConExc event generator ref, which also simulates ISR.
- Another hadronic background with four charged tracks is the reaction $e^{+} e^{-} \rightarrow K_{S}^{0} K^{ \pm} \pi^{\mp}$ with the subsequent $K_{S}^{0}$ decay into two charged pions. Again, these events might be identified as $\pi^{+} \pi^{-} \mu^{+} \mu^{-}$ events. This reaction is also simulated with ConExc.
- Another hadronic background with four charged tracks and a non negligible cross section is the reaction $e^{+} e^{-} \rightarrow K^{+} K^{-} \pi^{+} \pi^{-}$. Again, these events might be identified as $\pi^{+} \pi^{-} \mu^{+} \mu^{-}$events. This reaction is also simulated with ConExc.
- The $\psi^{\prime}$ state is only $120-210 \mathrm{MeV}$ below the $\sqrt{s}$ of the used data sets. This implies a nonnegligible cross section for its production in ISR and its subsequent decay to the signal final state $e^{+} e^{-} \rightarrow \gamma_{I S R} \psi^{\prime} \rightarrow \gamma_{I S R} \pi^{+} \pi^{-} J / \psi \rightarrow \gamma_{I S R} \pi^{+} \pi^{-} \ell^{+} \ell^{-}$. Without the detection of the ISR photon, this process has the same signature as the signal process. This background is simulated in EvtGen using the model VECTORISR for the emission of the ISR photon, the model JPIPI for the $\psi^{\prime}$ decay and the VLL model for the $J / \psi \rightarrow \ell^{+} \ell^{-}$decay.

An overview of the background MC samples is given in Table 2. In the following, the background MC samples are scaled to the integrated luminosity of data and added up to one cocktail MC sample per energy point.

### 5.2 Background Rejection Cuts

Each event passing the basic selection criteria (c.f. Section 4) is subjected to a kinematic fit. The kinematic constraints are the conservation of total four momentum, i.e. the four momenta of all tracks should add up to $(\sqrt{s}, 0,0,0)$ in the center-of-mass frame. Additionally, the tracks are constrained to originate from a common vertex. For the 2017 data sets, the measured beam energy spread is included in the fit. For the 2013 data, there is no information about the beam spread, so it is not included. Due to inconsitencies between the estimated error of the track parameters in data and in MC, the resulting $\chi^{2}$

Table 2: Overview of the generated background MC samples. For each process and center-of-mass energy, the cross section $\sigma$ as calculated by the event generator is shown. From the cross section, the integrated luminosity for each $\sqrt{s}$, and the number of simulated events $N_{s i m}$, the ratio of $N_{\text {sim }}$ to the number of expected events is calculated. The expected number of entries within the $J / \psi$ peak region as defined in Figure 11 after all cuts listed in the $N_{\text {exp }}^{\text {peak, } \ell \ell}$ columns, each standing for one $J / \psi$ mode. For the process $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) \pi^{+} \pi^{-} \pi^{+} \pi^{-}$, ConExc gives a cross section which is ca. a factor of 10 too low compared to the measurement by BABAR [22]. For this analysis, the cross section from ConExc is magnified by a factor of 10 . The process $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) K_{S}^{0} K^{ \pm} \pi^{\mp}$ is generated with the exclusive decay $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$, so the corresponding branching fraction is included in the cross section. The process $e^{+} e^{-} \rightarrow \gamma_{I S R} \psi^{\prime}$ is generated with the exclusive decays of $\psi^{\prime}->\pi^{+} \pi^{-} J / \psi$ and $J / \psi \rightarrow \ell^{+} \ell^{-}$. Its cross section is calculated by hand.

| Process | $\sqrt{s} / \mathrm{MeV}$ | $\sigma / \mathrm{nb}$ | $N_{\text {sim }}$ | $N_{\text {sim }} / N_{\text {exp }}$ | $N_{\text {exp }}^{\text {peak,ee }}$ | $N_{\text {exp }}^{\text {peak, } \mu \mu}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $e^{+} e^{-} \rightarrow \gamma e^{+} e^{-}$ | 3807.7 | 26.026 | 10 M | 7.60 | $0.26 \pm 0.19$ | $0.00 \pm 0.13$ |
| $e^{+} e^{-} \rightarrow \gamma e^{+} e^{-}$ | 3867.4 | 26.026 | 10.9 M | 3.85 | $2.60 \pm 0.82$ | $0.26 \pm 0.26$ |
| $e^{+} e^{-} \rightarrow \gamma e^{+} e^{-}$ | 3871.3 | 26.026 | 10.9 M | 3.80 | $0.79 \pm 0.46$ | $0.26 \pm 0.26$ |
| $e^{+} e^{-} \rightarrow \gamma e^{+} e^{-}$ | 3896.2 | 26.026 | 10 M | 7.30 | $0.68 \pm 0.31$ | $0.00 \pm 0.14$ |
| $e^{+} e^{-} \rightarrow \gamma \mu^{+} \mu^{-}$ | 3807.7 | 0.0501 | 880 k | 348 | $0.00 \pm 0.00$ | $0.01 \pm 0.00$ |
| $e^{+} e^{-} \rightarrow \gamma \mu^{+} \mu^{-}$ | 3867.4 | 0.0460 | 970 k | 194 | $0.00 \pm 0.01$ | $0.01 \pm 0.01$ |
| $e^{+} e^{-} \rightarrow \gamma \mu^{+} \mu^{-}$ | 3871.3 | 0.0457 | 940 k | 186 | $0.00 \pm 0.01$ | $0.01 \pm 0.01$ |
| $e^{+} e^{-} \rightarrow \gamma \mu^{+} \mu^{-}$ | 3896.2 | 0.0441 | 950 k | 410 | $0.00 \pm 0.00$ | $0.01 \pm 0.01$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} e^{+} e^{-}$ | 3807.7 | 16.94 | 11 M | 12.8 | $0.31 \pm 0.16$ | $0.00 \pm 0.08$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} e^{+} e^{-}$ | 3867.4 | 17.28 | 11 M | 5.85 | $1.37 \pm 0.48$ | $0.00 \pm 0.17$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} e^{+} e^{-}$ | 3871.3 | 17.30 | 11 M | 5.76 | $1.21 \pm 0.46$ | $0.00 \pm 0.17$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} e^{+} e^{-}$ | 3896.2 | 17.44 | 11 M | 12.0 | $1.00 \pm 0.29$ | $0.00 \pm 0.08$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} \mu^{+} \mu^{-}$ | 3807.7 | 7.997 | 11 M | 27.2 | $0.11 \pm 0.06$ | $0.04 \pm 0.04$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} \mu^{+} \mu^{-}$ | 3867.4 | 8.144 | 11 M | 12.4 | $0.00 \pm 0.08$ | $0.32 \pm 0.16$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} \mu^{+} \mu^{-}$ | 3871.3 | 8.153 | 11 M | 12.2 | $0.00 \pm 0.08$ | $0.16 \pm 0.12$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} \mu^{+} \mu^{-}$ | 3896.2 | 8.213 | 11 M | 25.5 | $0.12 \pm 0.07$ | $0.12 \pm 0.07$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} q \bar{q}$ | 3807.7 | 1.358 | 1 M | 14.6 | $0.00 \pm 0.07$ | $0.00 \pm 0.07$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} q \bar{q}$ | 3867.4 | 1.398 | 990 k | 6.50 | $0.00 \pm 0.15$ | $0.00 \pm 0.15$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} q \bar{q}$ | 3871.3 | 1.400 | 1 M | 6.48 | $0.00 \pm 0.15$ | $0.00 \pm 0.15$ |
| $e^{+} e^{-} \rightarrow e^{+} e^{-} q \bar{q}$ | 3896.2 | 1.427 | 1 M | 13.3 | $0.00 \pm 0.08$ | $0.00 \pm 0.08$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) \pi^{+} \pi^{-} \pi^{+} \pi^{-}$ | 3807.7 | 1.690 | 1 M | 11.8 | $0.00 \pm 0.01$ | $1.20 \pm 0.10$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) \pi^{+} \pi^{-} \pi^{+} \pi^{-}$ | 3867.4 | 1.569 | 1 M | 5.85 | $0.02 \pm 0.02$ | $2.60 \pm 0.21$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) \pi^{+} \pi^{-} \pi^{+} \pi^{-}$ | 3871.3 | 1.562 | 1 M | 5.80 | $0.00 \pm 0.02$ | $2.26 \pm 0.20$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) \pi^{+} \pi^{-} \pi^{+} \pi^{-}$ | 3896.2 | 1.516 | 1 M | 12.5 | $0.00 \pm 0.01$ | $1.36 \pm 0.10$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) K_{S}^{0} K^{ \pm} \pi^{\mp}$ | 3807.7 | 0.0269 | 1 M | 368 | $0.00 \pm 0,00$ | $0.25 \pm 0.01$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) K_{S}^{0} K^{ \pm} \pi^{\mp}$ | 3867.4 | 0.0254 | 1 M | 181 | $0.00 \pm 0,00$ | $0.40 \pm 0.02$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) K_{S}^{0} K^{ \pm} \pi^{\mp}$ | 3871.3 | 0.0253 | 1 M | 179 | $0.00 \pm 0,00$ | $0.48 \pm 0.03$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{\text {ISR }}\right) K_{S}^{0} K^{ \pm} \pi^{\mp}$ | 3896.2 | 0.0247 | 1 M | 384 | $0.00 \pm 0,00$ | $0.19 \pm 0.01$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) K^{+} K^{-} \pi^{+} \pi^{-}$ | 3807.7 | 0.2044 | 500 k | 48.4 | $0.00 \pm 0.02$ | $0.00 \pm 0.02$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) K^{+} K^{-} \pi^{+} \pi^{-}$ | 3867.4 | 0.1932 | 500 k | 23.8 | $0.00 \pm 0.04$ | $0.00 \pm 0.04$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) K^{+} K^{-} \pi^{+} \pi^{-}$ | 3871.3 | 0.1925 | 500 k | 23.5 | $0.00 \pm 0.04$ | $0.00 \pm 0.04$ |
| $e^{+} e^{-} \rightarrow\left(\gamma_{I S R}\right) K^{+} K^{-} \pi^{+} \pi^{-}$ | 3896.2 | 0.1882 | 500 k | 50.5 | $0.00 \pm 0.02$ | $0.00 \pm 0.02$ |
| $e^{+} e^{-} \rightarrow \gamma_{I S R} \psi^{\prime}$ | 3807.7 | 0.0892 | 1 M | 222 | $0.01 \pm 0.01$ | $0.00 \pm 0.00$ |
| $e^{+} e^{-} \rightarrow \gamma_{I S R} \psi^{\prime}$ | 3867.4 | 0.0577 | 1 M | 159 | $0.07 \pm 0.02$ | $0.10 \pm 0.03$ |
| $e^{+} e^{-} \rightarrow \gamma_{I S R} \psi^{\prime}$ | 3871.3 | 0.0564 | 1 M | 161 | $0.09 \pm 0.02$ | $0.11 \pm 0.03$ |
| $e^{+} e^{-} \rightarrow \gamma_{I S R} \psi^{\prime}$ | 3896.2 | 0.0490 | 1 M | 388 | $0.04 \pm 0.01$ | $0.07 \pm 0.01$ |



Figure 7: Comparisson between data, signal MC, and background MC distributions of $\cos \theta_{\pi^{+} \pi^{-}}$. All four data sets are combined. The cut $\cos \theta_{\pi^{+} \pi^{-}}<0.95$ is indicated by the arrow. The other cuts on $\chi^{2}$ and $\cos _{\pi^{ \pm} e^{\mp}}$ are applied as well as a cut on the fitting range of the invariant dilepton mass ( $3.0 \mathrm{GeV} / c^{2}<m\left(\ell^{+} \ell^{-}\right)<3.2 \mathrm{GeV} / c^{2}$, c.f. Figure11). The MC distributions are scaled to match the integrated luminosity of the data.
distributions differ, too. This effect is compensated by a correction of the track parameter errors of the MC events, as described in [23].

The background contamination is reduced by the following cuts:

- The background from gamma conversion can be reduced a lot by requiring $\cos \theta_{\pi^{+} \pi^{-}}<0.95$, when $\theta_{\pi^{+} \pi^{-}}$is the opening angle between the two pion candidates. The as pion misidentified electrons from gamma conversion carry the boost of the initial photon and are therefore almost parallel and peak at $\cos \theta_{\pi^{+} \pi^{-}}=1$ (c.f. Figure 7). This cut is applied to both $J / \psi$ reconstruction modes. It reduces almost all of the radiative dimuon background, while the large abundancy of radiative Bhabha events requires an additional cut.
- The electrons from gamma conversion can also be identified as a $\pi^{ \pm} \ell^{\mp}$ pair. The cut $\cos \theta_{\pi^{ \pm} e^{\mp}}<$ 0.98 is applied only to the electron mode (c.f. Figure 8).
- All events must have a $\chi^{2}$ of the kinematic fit lower than 60 . The corresponding distribution is shown in Figure 9.

Figure 10 show the distributions of the invariant dipion mass. The peak at the lower edge of the spectrum in the $e^{+} e^{-}$reconstruction mode is caused by the gamma conversion background. The position and shape of this peak is reproduced in MC. However, the absolute number of events is underestimated by a factor of ca. 5 . In principle at cut on $m\left(\pi^{+} \pi^{-}\right)$would remove this background. Unfortunately, the different MC models of the continuum process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ differ significantly in the lower region


Figure 8: Comparisson between data, signal MC, and background MC distributions of $\cos \theta_{\pi^{ \pm} \ell^{\mp}}$. All four data sets are combined. The cut $\cos \theta_{\pi^{ \pm} e^{\mp}}<0.98$ is indicated by the arrow. The other cuts on $\chi^{2}$ and $\cos _{\pi^{+} \pi-}$ are applied as well as a cut on the fitting range of the invariant dilepton mass $\left(3.0 \mathrm{GeV} / c^{2}<m\left(\ell^{+} \ell^{-}\right)<3.2 \mathrm{GeV} / c^{2}\right.$, c.f. Figure11). The MC distributions are scaled to match the integrated luminosity of the data.


Figure 9: Comparisson between data, signal MC, and background MC distributions of $\chi^{2}$. All four data sets are combined. The cut $\chi^{2}<60$ is indicated by the arrow. The other cuts on $\cos _{\pi^{+} \pi-}$ and $\cos \theta_{\pi^{ \pm} e^{\mp}}$ are applied as well as a cut on the fitting range of the invariant dilepton mass $\left(3.0 \mathrm{GeV} / c^{2}<m\left(\ell^{+} \ell^{-}\right)<\right.$ $3.2 \mathrm{GeV} / c^{2}$, c.f. Figure11). The MC distributions are scaled to match the integrated luminosity of the data.


Figure 10: Comparisson between data, continuum MC, and background MC distributions of $m\left(\pi^{+} \pi^{-}\right)$. All four data sets are combined. All cuts as well as a cut on the fitting range of the invariant dilepton mass $\left(3.0 \mathrm{GeV} / c^{2}<m\left(\ell^{+} \ell^{-}\right)<3.2 \mathrm{GeV} / c^{2}\right.$, c.f. Figure11) are applied. The MC distributions are scaled to match the integrated luminosity of the data.
of the distribution (c.f. Figure 3). Due to this model ambiguity, a cut would introduce a large systematic uncertainty.

The invariant mass distributions of the dilepton pairs are shown in Figure 11. The $J / \psi$ peak is clearly visible and the peak region $3.08 \mathrm{GeV} / c^{2}<m\left(\ell^{+} \ell^{-}\right)<3.12 \mathrm{GeV} / c^{2}$ is indicated by arrows.

Figures 12-15 show the same distributions as Figures $7-10$, but only with the events in the $J / \psi$ peak region instead of the whole fit range.

## 6 Cross Section Measurent of $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$

The cross section of the process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ is determined using

$$
\begin{equation*}
\sigma\left(e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi\right)=\frac{N_{\text {obs }}}{\int \mathcal{L} \mathrm{d} t \cdot \epsilon \cdot(1+\delta) \cdot \mathcal{B}\left(J / \psi \rightarrow \ell^{+} \ell^{-}\right)} \tag{1}
\end{equation*}
$$


#### Abstract

with:


- $N_{\text {obs }}$ is the number of observed events, extracted from a fit to the $\ell^{+} \ell^{-}$mass spectrum, c.f. next section. The number of events in the $J / \psi$ peak is equal to $N_{o b s}$.
- $\int \mathcal{L} \mathrm{d} t$ is the integrated luminosity (c.f. Table 1 ).
- The efficiency $\epsilon$ is determined from the analysis of the signal and continuum MC samples. In the beginning it is assumed, that there is no $X(3872)$ signal and the efficiency is solely determined from the continuum MC. After the observation of an enhancement at the $X(3872)$ mass, the efficiency


Figure 11: Comparisson between data, signal MC, and background MC distributions of $m\left(\ell^{+} \ell^{-}\right)$. All four data sets are combined. All cuts are applied. The MC distributions are scaled to match the integrated luminosity of the data. The arrows indicate the peak region.


Figure 12: The same as Figure 7, but only containing events in the $J / \psi$ peak region, c.f. Figure 11.


Figure 13: The same as Figure 8 , but only containing events in the $J / \psi$ peak region, c.f. Figure 11.


Figure 14: The same as Figure 9, but only containing events in the $J / \psi$ peak region, c.f. Figure 11.


Figure 15: The same as Figure 10, but only containing events in the $J / \psi$ peak region, c.f. Figure11.
for the 3871.3 MeV data set would be the weighted average of the efficiencies of the continuum and the signal model according to the cross section fraction attributed to continuum and the $X / 3872$ ) signal. However, there is no such enhancement at the $X(3872)$ mass and the efficiency is just the one obtained from the continuum MC.

- $(1+\delta)$ is the radiative correction factor to account for ISR. It is calculated from the KKMC event generator assuming a constant lineshape of the cross section. After the measurement of the true lineshape, the new one would be used as input to KKMC and a new value of $(1+\delta)$ would be obtained to calculate a new value for the cross section. This procedure would be iterated until the values of $(1+\delta)$ and $\sigma$ converge. However, the measured lineshape cannot be distinguished from a constant, so the iterative procedure doesn't need to be done.
- Finally, the exclusive reconstruction of the $J / \psi$ in its dilepton decays needs to be incorporated by the branching fraction for this decay $\mathcal{B}\left(J / \psi \rightarrow \ell^{+} \ell^{-}\right)$. The values are taken from the PDG [8].


### 6.1 Fit to the $m\left(\ell^{+} \ell^{-}\right)$Distribution

As mentioned above, the number of signal events is extracted from a maximum likelihood fit to the invariant dilepton mass spectrum. This is done independently for both $J / \psi$ decay modes in the range of $3.0 \mathrm{GeV} / c^{2}<m\left(\ell^{+} \ell^{-}\right)<3.2 \mathrm{GeV} / c^{2}$. The MC lineshape is used as the signal pdf, while the background is modeled as a linear function. The complete model $p$ and the likelihood function $L$ are

$$
\begin{align*}
p(m, a, f) & =f \cdot s(m)+(1-f) \cdot b(m, a)  \tag{2}\\
L(a, f) & =\prod_{i=1}^{N} p\left(m_{i}, a, f\right) \tag{3}
\end{align*}
$$

where $m=m\left(\ell^{+} \ell^{-}\right)$is the invariant dilepton mass, $f$ is the signal fraction and $a$ is a parameter of the linear function. $N$ is the total number of events included in the fit and the invariant dilepton mass of event $i$ is $m_{i}$.

Instead of maximizing the likelihood funtion directly, the negative log-likelihood function $N L L=$ $-\log N$ is minimized, which is equivalent but numerically much more stable. The fit is performed in the ROOT framework [24] using the RooFit [25] library and the Minuit minimizer [26]. The number of signal events is $N_{o b s}=\hat{f} \cdot N$ when $\hat{f}$ is the value of $f$ at the global minimum of $N L L$. Its error is estimated from the second derivative of $\left.N L L\right|_{\text {min }}$ with respect to the fit parameters.

Alternatively, an extended maximum likelihood fit could have been performed. Here, the expected number of events $v$ is included and $v_{s}=v f$ and $v_{b}=v(1-f)$ directly substituted. The extended likelihood function contains the additional Poissonian term:


Figure 16: Fit of the $m\left(e^{+} e^{-}\right)$distribution for each data set. The markers with error bars is the data distribution. The red line represents the full fit pdf, while the dashed gray line is the background contribution to the pdf.

$$
\begin{align*}
L\left(a, v_{s}, v_{b}\right) & =\mathrm{e}^{-v} \frac{v^{N}}{N!} \prod_{i=1}^{N} p\left(m_{i}, a, v_{s}, v_{b}\right)  \tag{4}\\
& \propto \mathrm{e}^{-v_{s}-v_{b}} \prod_{i=1}^{N}\left[v_{s} s(m)+v_{b} b(m, a)\right] \tag{5}
\end{align*}
$$

1 As long as the model has no explicit dependency on $v$, the minimization of the corresponding NLL will
2 yield the same result as the standard maximum likelihood fit.
$3 \quad$ Figures 16 and 17 show the $m\left(\ell^{+} \ell^{-}\right)$distribution with the fit pdf overlayed. The result of the fit is 4 summarized in Table 3.

## 5 6.2 Systematic Uncertainties

6 In the following, the systematic uncertainties affecting the cross section measurement are discussed.

7 - The uncertainty of the integrated luminosity for each data set is listed in Table 1 (also in Table 3).

- For the measurement, four charged tracks are analyzed. It was shown in [? ], that the uncertainty of the tracking efficiency is $1 \%$ per track yielding a $4 \%$ uncertainty.


Figure 17: Fit of the $m\left(\mu^{+} \mu^{-}\right)$distribution for each data set. The markers with error bars is the data distribution. The red line represents the full fit pdf, while the dashed gray line is the background contribution to the pdf.

Table 3: Result of the fit to the dilepton mass distribution. Shown are the results of the two independent $J / \psi$ modes and a combined value.

| $\sqrt{s} / \mathrm{MeV}$ | $3807.7 \pm 0.6$ | $3867.410 \pm 0.031$ | $3871.31 \pm 0.06$ | $3896.2 \pm 0.8$ |
| :--- | :---: | :---: | :---: | :---: |
| $\int \mathcal{L} \mathrm{~d} t / \mathrm{pb}^{-1}$ | $50.5 \pm 0.5$ | $108.9 \pm 1.3$ | $110.3 \pm 0.8$ | $52.6 \pm 0.5$ |
| $(1+\delta)$ | $0.895 \pm 0.007$ | $0.895 \pm 0.007$ | $0.895 \pm 0.007$ | $0.895 \pm 0.007$ |
| $\mathcal{B}\left(J / \psi \rightarrow e^{+} e^{-}\right) / \%$ | $5.971 \pm 0.032$ | $5.971 \pm 0.032$ | $5.971 \pm 0.032$ | $5.971 \pm 0.032$ |
| $N_{o+e^{+} e^{-}}$ | $20 \pm 5$ | $31 \pm 6$ | $24 \pm 6$ | $15 \pm 4$ |
| $\epsilon^{e^{+} e^{-}} / \%$ | $31.789 \pm 0.008$ | $31.344 \pm 0.008$ | $31.291 \pm 0.008$ | $31.683 \pm 0.008$ |
| $\sigma^{e^{+} e^{-}} / \mathrm{pb}$ | $23 \pm 6$ | $17.5 \pm 3.3$ | $13.5 \pm 3.1$ | $18 \pm 5$ |
| $\mathcal{B}\left(J / \psi \rightarrow \mu^{+} \mu^{-}\right) / \%$ | $5.961 \pm 0.033$ | $5.961 \pm 0.033$ | $5.961 \pm 0.033$ | $5.961 \pm 0.033$ |
| $N_{o b s}^{\mu^{+} \mu^{-}}$ | $18 \pm 4$ | $43 \pm 7$ | $29 \pm 6$ | $19 \pm 5$ |
| $\epsilon^{\mu^{+} \mu^{-}} / \%$ | $45.39 \pm 0.010$ | $44.91 \pm 0.010$ | $44.73 \pm 0.010$ | $45.14 \pm 0.010$ |
| $\sigma^{\mu^{+} \mu^{-}} / \mathrm{pb}$ | $14.6 \pm 3.5$ | $16.3 \pm 2.7$ | $11.0 \pm 2.2$ | $14.9 \pm 4.0$ |
| $\sigma^{\ell^{+} e^{-}} / \mathrm{pb}$ | $16.9 \pm 3.0$ | $16.8 \pm 2.1$ | $11.8 \pm 1.8$ | $16.1 \pm 3.0$ |

- The radiative correction factor $(1+\delta)$ is calculated by KKMC. The MC samples were generated in 100 sub samples for each $\sqrt{s}$ and $J / \psi$ mode. Thus, there are 800 different values for $(1+\delta)$. The mean value is used in the determination of the cross section (Equation (1)). The standard deviation is used as the systematic uncertainty of $(1+\delta)$.
- The values for $\mathcal{B}\left(J / \psi \rightarrow \ell^{+} \ell^{-}\right)$together with their uncertainties are taken from the PDG [8]. The values are given in Table 3.
- In [23] it was shown, that the uncertainty associated to the kinematic fit can be estimated by half the efficiency difference of the MC analysis with and without the helix parameter correction (c.f. section 5.2).
- In the fit to the $m\left(\ell^{+} \ell^{-}\right)$distribution, the background is modeled as a linear function. This choice has a certain ambiguity and its impact on the extracted cross section is acompanied by a systematic uncertainty. It is estimated by the difference in $N_{s i g}$ when the fit is performed with a quadratic background parameterization.
- The ambiguity of the MC modeling of the continuum process $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ is already mentioned in Section 3.2 and indicated in Figures 3 and 4. The systematic uncertainty associated to this ambiguity is estimated by the efficiency difference when using the $\sigma$ PHSP model instead of the VVPIPI model.
- The efficiency $\epsilon$ is determined by the analysis of MC events. The statistical limited MC set attaches an error on the efficiency. The relative large size of each signal and continuum MC set of 500 k implies a relative small errror which can be seen in Table 3. It is less than $0.1 \%$ and can be neglected.

The contributions of each source of systematic uncertainty are listed in Table 4.

### 6.3 Result

The final result of the cross section measuremnt is listed in Table 5 and shown in Figure 18. The statistical uncertainty clearly dominates the overall uncertainty. There is no enhancement at the $X(3872)$ visible. In fact, there is a small dip in the cross section. However, this cannot be explained by a destructive interference between the $X(3872)$ and the continuum amplitudes, since they carry different quantum numbers (c.f. Figure 1). One might argue that the process $e^{+} e^{-} \rightarrow \gamma^{*} \gamma^{*} \rightarrow \rho^{0} J / \psi$ could happen via double vector meson dominance. It is noteworthy, that the threshold for this reaction is very close to the $X(3872)$ mass. A simplified calculation according to [27] yields a global maximum of the $e^{+} e^{-} \rightarrow \rho^{0} J / \psi$

Table 4: Relative systematic uncertainties (in \%) affecting the measured cross section $\sigma\left(e^{+} e^{-} \rightarrow\right.$ $\left.\pi^{+} \pi^{-} J / \psi\right)$. The total uncertainty is the quadratic sum of the individual errors.

| Source | 3807.7 MeV |  | 3867.4 MeV |  | 3871.3 MeV |  | 3896.2 MeV |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $e^{+} e^{-}$ | $\mu^{+} \mu^{-}$ | $e^{+} e^{-}$ | $\mu^{+} \mu^{-}$ | $e^{+} e^{-}$ | $\mu^{+} \mu^{-}$ |  | $\mu^{+} \mu^{-}$ |
| $\int \mathcal{L} \mathrm{d} t$ | 1.0 |  | 1.2 |  | 0.7 |  | 1.0 |  |
| Tracking | 4.0 |  | 4.0 |  | 4.0 |  | 4.0 |  |
| ( $1+\delta$ ) | 0.7 |  | 0.7 |  | 0.7 |  | 0.7 |  |
| $\mathcal{B}\left(J / \psi \rightarrow \ell^{+} \ell^{-}\right)$ | 0.5 | 0.6 | 0.5 | 0.6 | 0.5 | 0.6 | 0.5 | 0.6 |
| Kinematic fit | 0.9 | 0.7 | 0.8 | 0.7 | 0.9 | 0.7 | 0.9 | 0.7 |
| $m\left(\ell^{+} \ell^{-}\right)$fit | 4.9 | 0.7 | 3.9 | 2.0 | 5.1 | 8.1 | 10.4 | 1.4 |
| Decay model | 2.2 | 3.6 | 2.7 | 4.0 | 2.2 | 3.7 | 2.4 | 4.0 |
| Total | 6.9 | 5.6 | 6.4 | 6.2 | 7.0 | 9.9 | 11 | 6.0 |

Table 5: Cross section of $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ as determined in this analysis. The first error is the statistical and the second one is the systematic uncertainty.

|  |  | $\sigma\left(e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi\right)$ |  |
| :--- | :---: | :---: | :---: |
| $\sqrt{s} / \mathrm{MeV}$ | $e^{+} e^{-}$mode | $\mu^{+} \mu^{-}$mode | both modes combined |
| 3907.7 | $14.6 \pm 3.5 \pm 1.6$ | $16.9 \pm 3.0 \pm 0.8$ | $16.9 \pm 3.0 \pm 0.7$ |
| 3867.4 | $16.3 \pm 2.7 \pm 1.1$ | $16.8 \pm 2.1 \pm 1.0$ | $16.8 \pm 2.1 \pm 0.8$ |
| 3871.3 | $11.0 \pm 2.2 \pm 0.9$ | $11.8 \pm 1.8 \pm 1.1$ | $11.8 \pm 1.8 \pm 0.7$ |
| 3896.2 | $14.9 \pm 4.0 \pm 2.0$ | $16.1 \pm 3.0 \pm 0.9$ | $16.1 \pm 3.0 \pm 0.8$ |

1 cross section of less than 0.24 pb , which in addition is dominated by the $0^{++}$and $2^{++}$states. Furthermore, 2 the cross section is strongly peaked towards $\cos \theta \pm 1$, which is outside the detectors acceptance. In total, there is little room for a $1^{++}$amplitude intefering with the $X(3872)$. The measured cross section is in good agreement with a constant cross section. ${ }^{2}$ Since ther is no sign of direct $X(3872)$ production, an upper limit on $\Gamma_{e e}^{X(3872)}$ is determined in the next section.

[^1]

Figure 18: Cross section of $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$. The results of both $J / \psi$ decay modes are combined. The nominal mass of the $X(3872)$ as listed in the PDG [8] is indicated by the vertical line. The error bars represent the statistical errors and the small horizontal lines above and below the error bars represent the total uncertainties, i.e. the quadratic sum of the statistical and systematic errors.

## 7 Upper Limit on $\Gamma_{e e}^{X 3872}$

### 7.1 Lineshape

Due to the different quantum numbers of the continuum process and the resonant $X(3872)$ formation, the total $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ cross section is modeled as its incoherent sum. The continuum is assumed to be flat, i.e. a constant. The $X(3872)$ is modeled as a reletivistic Breit-Wigner resonance. Only the $\pi^{+} \pi^{-} J / \psi$ decay mode is taken into account, so the corresponding branching fraction needs to be included in the lineshape parameterization (using $c=\hbar=1$ ):

$$
\begin{equation*}
\sigma_{e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi}(\sqrt{s})=\sigma_{c o n t}+12 \pi \frac{\Gamma_{t o t} \Gamma_{e e} \times \mathcal{B}\left(X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi\right)}{\left(s-m_{0}^{2}\right)^{2}+m_{0}^{2} \Gamma_{t o t}^{2}} \tag{6}
\end{equation*}
$$

where $\sigma_{c o n t}, \Gamma_{t o t}$, and $\Gamma_{e e}$ are the constant continuum, the total width and the electronic width of the $X(3872)$, respectively. The $X(3872)$ mass is $m_{0}$ and $s$ is the Mandelstam variable. Of the $X(3872)$ parameters, only the mass is known $\left(m(X(3872))=(3871.69 \pm 0.17) \mathrm{MeV} / c^{2}\right.$ [8]). Since the branching ratio $\mathcal{B}\left(X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi\right)$ has not been determined yet, this analysis treats the product $\Gamma_{e e} \times \mathcal{B}(X(3872) \rightarrow$ $\left.\pi^{+} \pi^{-} J / \psi\right)$ as one parameter and an upper limit on this product is set instead of a limit on $\Gamma_{e e}$ alone. In total, there are three unknown parameters.

In each data set, $\sqrt{s}$ is Gaussian distributed according to the energy spread in Table 1. The resulting measured cross section for data set $i$ is

$$
\begin{align*}
\sigma_{i} & =\sigma_{e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi}\left(\sqrt{s_{i}}\right) \otimes N\left(\sqrt{s_{i}} \mid 0, \delta \sqrt{s_{i}}\right)  \tag{7}\\
& =\int \sigma_{e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi}(x) \times N\left(\sqrt{s_{i}}-x \mid 0, \delta \sqrt{s_{i}}\right) \mathrm{d} x  \tag{8}\\
& =\sigma_{\text {cont }}+12 \pi \Gamma_{e e} \times \mathcal{B}\left(X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi\right) \int \frac{\Gamma_{\text {tot }}}{\left(x^{2}-m_{0}^{2}\right)^{2}+m_{0}^{2} \Gamma_{\text {tot }}^{2}} \times N\left(\sqrt{s_{i}}-x \mid 0, \delta \sqrt{s_{i}}\right) \mathrm{d} x \tag{9}
\end{align*}
$$

when $N(x \mid \mu, \sigma)$ is the normal distribution with mean $\mu$ and variance $\sigma^{2}$. For the 2017 data, the BEMS information can be used, while the two 2013 data points are far away from the $X(3872)$ mass. As a result, they are only sensitive to the constant term $\sigma_{\text {const }}$ in the lineshape and a convolution with a moderate energy spread won't alter the cross section value. Nevertheless, a $\sqrt{s}$ spread of 1.5 MeV is assumed.

With the lineshape parameterization, the model aquires an explicit dependency on the total number of events, so here, the extended likelihood function as defined in (5) is used. The expected number of signal events is substituted by the cross section with the relation given in (1). As a result, the likelihood function for each data set $i$ and $J / \psi$ mode $j$ is now depending on the cross section:

$$
\begin{equation*}
L_{i}^{j}\left(a_{i}^{j}, v_{s, i}^{j}, v_{b, i}^{j}\right) \longrightarrow L_{i}^{j}\left(\sigma_{i}, a_{i}^{j}, v_{b, i}^{j}\right) \tag{10}
\end{equation*}
$$

The overall likelihood function is

$$
\begin{equation*}
L=\prod_{i=1}^{4} \prod_{j=e e, \mu \mu} L_{i}^{j}\left(\sigma=\sigma_{i}, a_{i}^{j}, v_{b, i}^{j}\right) \tag{11}
\end{equation*}
$$

### 7.2 Bayesian Formalism

In Bayesian formalism, the likelihood function $L(x, \theta)$ is interpreted as the conditional $\operatorname{pdf} f(x \mid \theta)$, i.e. the likelihood of observing data $x$ given the parameter $\theta$ is true [28]. This is not restricted to single parameter models, but $\theta \in \mathbb{R}^{N}$ and of course the data $x \in \mathbb{R}^{N}$ consists of many measurements. Using Bayes' Theorem [29], this pdf can be turned into a conditional pdf giving the likelihood of the parameters $\theta$ giving the data:

$$
\begin{align*}
f(\theta \mid x) & =\frac{f(x \mid \theta) \pi(\theta)}{\int f(x \mid \theta) \pi(\theta) \mathrm{d} \theta}  \tag{12}\\
& \propto f(x \mid \theta) \pi(\theta) \tag{13}
\end{align*}
$$

The prior pdf $\pi(\theta)$ gives the likelihood of the parameters before the measurement $x$ was obtained. In (12) the denominator is independent of $\theta$ and can be viewed as a normalization constant. In most cases, the prior pdf is unknown and very often a flat, i.e. constant one, is assumed. Although this is strictly speaking not a pdf and not in all cases the optimal choice, ${ }^{3}$ but the mode of the resulting posterior pdf $(\theta \mid x)$ coincides with the maximum likelihood fit. In this analysis, the priors are constant in the physical region, i.e. $\Gamma_{t o t}>0$ and $\Gamma_{e e} \times \mathcal{B}\left(X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi\right)>0$, and are set to zero in the unphysical region.

Most of the time, there are parameters in a model, that are not of interest, e.g. the coefficients of a polynomial describing the background. Consider the parameter set $\theta=\left(\tilde{\theta}, \theta_{n}\right)$ where $\tilde{\theta}$ is the interesting parameter set and $\theta_{n}$ are the others, so-called nuissance parameters. The posterior pdf concerning only the parameters of interest is obtained by the marginalization of the nuissance parameters[28]:

$$
\begin{equation*}
f(\tilde{\theta} \mid x)=\int f\left(\left(\tilde{\theta}, \theta_{n}\right) \mid x\right) \mathrm{d} \theta_{n} \tag{14}
\end{equation*}
$$

[^2]

In this analysis, the two parameters of interest are $\Gamma_{e e} \times \mathcal{B}:=\Gamma_{e e} \times \mathcal{B}\left(X(3872) \rightarrow \pi^{+} \pi^{-} J / \psi\right)$ and $\Gamma_{\text {tot }}$, so the marginalized likelihood is

$$
\begin{equation*}
\hat{\mathfrak{L}}\left(\Gamma_{e e} \times \mathcal{B}, \Gamma_{t o t}\right)=\int \prod_{i=1}^{4} \prod_{j=e e, \mu \mu} L_{i}^{j}\left(\sigma=\sigma_{i}, a_{i}^{j}, v_{b, i}^{j}\right) \mathrm{d} a_{i}^{j} \mathrm{~d} v_{b, i}^{j} \mathrm{~d} \sigma_{\text {cont }} \tag{15}
\end{equation*}
$$

(15) can be written as

$$
\begin{equation*}
\hat{\mathfrak{L}}\left(\Gamma_{e e} \times \mathcal{B}, \Gamma_{t o t}\right)=\int \prod_{i=1}^{4} \prod_{j=e e, \mu \mu} \hat{\mathfrak{\imath}}_{i}^{j}\left(\sigma=\sigma_{i}\right) \mathrm{d} \sigma_{c o n t} \tag{17}
\end{equation*}
$$

All the integrations are performed numerically within ROOT [24].
A $90 \%$ credible interval in Bayesian inference can be defined by the following two properties [28]:

- For each point in the interval, the posterior pdf is larger than for every other point outside the interval.
- The integral of the (normalized) posterior over the interval must be $90 \%$.

This definition can be extended to the multi-parameter case, where the interval becomes a multi-dimensional region. An upper limit is the upper interval boundary, when the lower interval boundary coincides with the border to the unphysical region.

### 7.3 Incorporation of Systematic Uncertainties

There are two different kinds of systematic uncertainties which have to be treated differently. The first kind affects the cross section measurement and is described in Section 6.2. The second kind affects the lineshape parameterization and includes the uncertainties associated with the $X(3872)$ mass, $\sqrt{s}$, and the beam spread.


Figure 19: The marginalized likelihoods before and after the concolution with the systematic uncertainties $\hat{\mathfrak{L}}_{i}^{j}(\sigma)$ and $\mathfrak{L}_{i}^{j}(\sigma)$ as well as the normalized product of both modes for all four data sets. The systematic uncertainties have only little effect as can also be seen in Figure 18.

## 1 7.3.1 Uncertainty of the Cross Section

2 The systematic uncertainties of the cross section are determined in Section 6.2 and listed in Tables 4 3 and 5. They are incorporated by the convolution of (16) with a Gaussian with the corresponding variance:

$$
\begin{equation*}
\mathfrak{Q}_{i}^{j}(\sigma)=\hat{\mathfrak{R}}_{i}^{j}(\sigma) \otimes N\left(\sigma \mid 0, \Delta_{s y s, i}^{j} \sigma\right) \tag{18}
\end{equation*}
$$

${ }^{4}$ The marginalized likelihoods $\hat{\mathfrak{L}}_{i}^{j}(\sigma)$ and $\mathfrak{L}_{i}^{j}(\sigma)$ as well as the product $\mathfrak{L}_{i}(\sigma):=\mathfrak{L}_{i}^{e e}(\sigma) \times \mathfrak{L}_{i}^{\mu \mu}(\sigma)$ are plotted 5 in Figure 19.

## 6 7.3.2 Uncertainty of the Lineshape Parameterization

7 The uncertainties affecting the lineshape are the following.

- The mass of the $X(3872)$ is taken from the PDG together with its uncertainty $m_{0}=(3871.69 \pm$ $0.17) \mathrm{MeV} / c^{2}$ [8].
- The measured center-of-mass energies have uncertainties which have to be taken into account. They are given in Table 1.
- The consideration of the beam energy spread in the cross section lineshape makes the overall result sensitive to the corresponding uncertainty. The BEMS provided the uncertainies on the energy spread for the 2017 data (also given in Table 1). For the 2013 data there is no such information, but Section 7.1 discussed the negligible impact of the energy spread for those data points. Nevertheless, uncertainties of 0.1 MeV are assumed which is approximately twice as large as the uncertainty for the on-resonance and four times as large as the one for the off-resonance data set.

A common approach to include these systematic uncertainties is the extension of the likelihood function by Gaussian prior pdfs for the systematically uncertain parameters and the subsequent marginalization over them [28]. This turns the one-dimensional integral of (17) into a ten-dimensional one making it computational very expensive. This issue is resolved by the application of a MC integration technique. The following procedure is repeated several times.

1. The values for the parameters with a systematic uncertainty are sampled from the corresponding normal distributions.
2. The marginalized likelihood $\hat{\mathfrak{R}}\left(\Gamma_{e e} \times \mathcal{B}, \Gamma_{\text {tot }}\right)$ is calculated (only one-dimensional integral, c.f. (17)).

Finally, the likelihoods of each repetition are summed up. This is equivalent to an averaging, because the likelihoods aren't normalized. Effectively, a nine-dimensional integral is converted into a sum:

$$
\begin{equation*}
\mathfrak{Q}\left(\Gamma_{e e} \times \mathcal{B}, \Gamma_{\text {tot }}\right)=\sum_{i} \hat{\mathfrak{Q}}\left(\Gamma_{e e} \times \mathcal{B}, \Gamma_{\text {tot }}\right)_{\theta=\theta_{i}} \tag{19}
\end{equation*}
$$

where $\theta$ stands for the parameters with an uncertainty and $\theta_{i}$ are the sampled parameters in iteration $i$.
In Figure 20 the obtained upper limit of $\Gamma_{e e} \times \mathcal{B}$ assuming $\Gamma_{\text {tot }} \approx 1.2 \mathrm{MeV}$ depending on the number of iterations in the MC integration is shown. After already 1000 iterations a stable value is obtained. Of course, all iterations are used for the result.

### 7.4 Result

The likelihood function (19) is shown in Figure 21. Since the total width $\Gamma_{\text {tot }}$ is unknown, the upper limit on $\Gamma_{e e} \times \mathcal{B}$ can be determined as a function of $\Gamma_{t o t}$. For each value of $\Gamma_{t o t}$, the likelihood is integrated over $\Gamma_{e e} \times \mathcal{B}$ until the intgral becomes $90 \%$. This is shown in Figure 22 for a fixed $\Gamma_{\text {tot }}=1$. The upper limit depending on the total width is shown in Figure 23.


Figure 20: Development of the obtained upper limit on $\Gamma_{e e} \times \mathcal{B}$ at $90 \%$ C.L. depending on the number of iterations in the MC integration. A total width of $\Gamma_{t o t} \approx 1.2 \mathrm{MeV}$ is assumed, the current upper limit.


Figure 21: Unnormalized marginalized likelihood function $\mathfrak{L}\left(\Gamma_{e e} \times \mathcal{B}, \Gamma_{t o t}\right)$. The gray line indicates the current $90 \%$ upper limit on $\Gamma_{\text {tot }}$.

$$
\Gamma_{\text {tot }}=1.207500 \mathrm{MeV}
$$



Figure 22: Determination of the upper limit on $\Gamma_{e e} \times \mathcal{B}$ for an assumed total width of approximately 1.2 MeV , its current upper limit. The gray area indicates the $90 \%$ integral. The value of $\Gamma_{e e} \times \mathcal{B}$ at the right edge of that area is the upper limit on $\gamma_{e e} \times \mathcal{B}$ at the $90 \%$ confidence level.


Figure 23: The upper limit on $\Gamma_{e e} \times \mathcal{B}$ at the $90 \%$ confidence level as a function of the total width. The gray line indicates the current upper limit on $\Gamma_{t o t}$.

Marginalized Likelihood


Figure 24: Likelihood function of $\Gamma_{e e} \times \mathcal{B}$ and $\Gamma_{\text {tot }}$ with the non-uniform prior for $\Gamma_{\text {tot }}$ reflecting its known upper limit.

A unique value for the upper limit of $\Gamma_{e e} \times \mathcal{B}$ could be obtained by a marginalization of the likelihood over $\Gamma_{t o t}$. However, s significant fraction of the likelihood stretches far beyond the current upper limit of $\Gamma_{t o t}$ (c.f. Figure 21). Consequently, this marginalization is not feasible. A possibility to incorporate the current knowledge about $\Gamma_{\text {tot }}$, namely the upper limit of 1.2 MeV , is to use a non-uniform prior for it. In the determination of the limit, the likelihood function of $\Gamma_{t o t}$ is a zero-mean Gaussian [10]. A standard deviation of 0.7295 MeV ensures the $90 \%$ limit at 1.2 MeV . This Gaussian shape is used as the prior pdf of $\Gamma_{\text {tot }}$. The resulting likelihood function is shown in Figure 24.

After the marginalization of the obtained likelihood function over $\Gamma_{t o t}$, it depends only on $\Gamma_{e e} \times \mathcal{B}$. It is plotted in Figure 25 and the $90 \%$ limit is determined. After rounding, it is $\Gamma_{e e} \times \mathcal{B}<9 \mathrm{meV}$ at $90 \%$ confidence level.

A two-dimensional credible region for $\left(\Gamma_{t o t}, \Gamma_{e e} \times \mathcal{B}\right)$ can also be constructed when the non-uniform prior is used. It is shown in Figure 26.

## 8 Conclusion

In this analysis, the cross section of $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-} J / \psi$ was measured at four different $\sqrt{s}$ close to the $X(3872)$ mass. The result of this measurement is listed in Table 5. Since there is no enhancement at the $X(3872)$ mass, an upper limit for $\Gamma_{e e} \times \mathcal{B}$ is determined. The $90 \%$ upper limit depending on the total width is shown in Figure 23. For an assumed total width of 1.2 MeV , the upper limit on $\Gamma_{e e} \times \mathcal{B}$ is 11 meV


Figure 25: Likelihood function of $\Gamma_{e e} \times \mathcal{B}$ after the marginalization over $\Gamma_{t o t}$ with the non-uniform prior. The $90 \%$ integral is indicated as the gray area and its right edge corresponds to the upper limit.

90 \% Credible Region


Figure 26: Two-dimensional credible region for $\left(\Gamma_{t o t}, \Gamma_{e e} \times \mathcal{B}\right)$ at the $90 \%$ confidence level.

1 at the $90 \%$ confidence level. When the current knowledge about the total width is incorporated, a two-
2 dimensional $90 \%$ credible region of $\left(\Gamma_{t o t}, \Gamma_{e e} \times \mathcal{B}\right)$ can be constructed. Furthermore, a $\Gamma_{\text {tot }}$ independent
3 upper limit on $\Gamma_{e e} \times \mathcal{B}$ of 9 meV is obtained, an improvement of a factor 14 compared to the previous
4 limit [12]. It is in no conflict to the theoretical prediction of $\Gamma_{e e} \times \mathcal{B} \gtrsim 0.78 \mathrm{meV}$ [13].

## Appendix

## A Determination of the Center-of-Mass Energy of the 2017 Data

## A. 1 BEMS Result

In order to have precise $\sqrt{s}$ values around the $X(3872)$ mass, the beam energies were monitored by the beam energy measurement system (BEMS) during data taking for the two data sets of 2017. The BEMS measures the beam energy as well as the beam energy spread of both beams independently. Each measurement point is the result of the analysis of the energy spectrum of Compton back scattered photons collected over a certain time period. They are associated to the BESIII data via a time stamp.

The BEMS result is shown in Figure 27. The three different beam energy configurations are clearly visible. Although the BEMS recorded data for ca. 17 hours at the first beam energy, there were almost no collisions in this period, and only two runs were recorded by BESIII. After these two runs, the beam energies were increased in order to run at a center-of-mass energy 4 MeV below the $X(3872)$ mass.

For the on-resonance data set, the BEMS was only able to measure the positron beam at the beginning. The next section shows, that these BEMS values can still be used for the whole data set.

For each of the three different beam configurations, the values of the beam energies and beam energy spreads are averaged. The center-of-mass energy is obtained under consideration of the finite beam crossing angle $\theta=22 \mathrm{mrad}$ :

$$
\begin{equation*}
\sqrt{s}=2 \cos (\theta / 2) \cdot \sqrt{E_{e^{+}} E_{e^{-}}} \tag{20}
\end{equation*}
$$

The corresponding error of this quantity is:

$$
\begin{equation*}
\Delta \sqrt{s}=\cos (\theta / 2) \cdot \sqrt{\frac{E_{e^{-}}}{E_{e^{+}}}\left(\Delta E_{e^{+}}\right)^{2}+\frac{E_{e^{+}}}{E_{e^{-}}}\left(\Delta E_{e^{-}}\right)^{2}} \tag{21}
\end{equation*}
$$

The spread of $\sqrt{s}$ is obtained from the same relation:

$$
\begin{equation*}
\delta \sqrt{s}=\cos (\theta / 2) \cdot \sqrt{\frac{E_{e^{-}}}{E_{e^{+}}}\left(\delta E_{e^{+}}\right)^{2}+\frac{E_{e^{+}}}{E_{e^{-}}}\left(\delta E_{e^{-}}\right)^{2}} \tag{22}
\end{equation*}
$$

The uncertainty of $\delta \sqrt{s}$ is calculated via Gaussian error propagation:

$$
\begin{equation*}
\Delta(\delta \sqrt{s})=\frac{\cos ^{2}(\theta / 2)}{\delta \sqrt{s}} \cdot \sqrt{\left(\delta E_{e^{+}} \cdot \Delta\left(\delta E_{e^{+}}\right)\right)^{2}+\left(\delta E_{e^{-}} \cdot \Delta\left(\delta E_{e^{-}}\right)\right)^{2}} \tag{23}
\end{equation*}
$$

The result of this calculation is summarized in tables 6 and 7. The errors are only statistical.


Figure 27: BEMS result. For each of the three different beam configurations, the values are averaged and indicated by the horizontal lines.

Table 6: BEMS information for the three energy points.

| Run Numbers | $E_{e^{-}} / \mathrm{MeV}$ | $E_{e^{+}} / \mathrm{MeV}$ | $\sqrt{s} / \mathrm{MeV}$ |
| :--- | :---: | :---: | :---: |
| $52108-52109$ | $1932.83 \pm 0.05$ | $1933.72 \pm 0.09$ | $3866.32 \pm 0.10$ |
| $52110-52206$ | $1933.505 \pm 0.016$ | $1934.139 \pm 0.027$ | $3867.410 \pm 0.031$ |
| $52207-52297$ | $1935.376 \pm 0.018$ | $1936.17 \pm 0.06$ | $3871.31 \pm 0.06$ |

Table 7: Beam energy spread for the three energy points.

| Run Numbers | $\delta E_{e^{-}} / \mathrm{MeV}$ | $\delta E_{e^{+}} / \mathrm{MeV}$ | $\delta \sqrt{s} / \mathrm{MeV}$ |
| :--- | :---: | :---: | :---: |
| $52108-52109$ | $1.04 \pm 0.06$ | $1.01 \pm 0.11$ | $1.45 \pm 0.09$ |
| $52110-52206$ | $1.136 \pm 0.020$ | $0.830 \pm 0.032$ | $1.406 \pm 0.025$ |
| $52207-52297$ | $1.189 \pm 0.024$ | $1.26 \pm 0.08$ | $1.73 \pm 0.06$ |

## A. 2 Cross Check via the Analysis of the Dimuon Process

To verify the BEMS result and in particular to check the stability of the center-of-mass energy after the BEMS could no longer provide information on the positron beam, the center-of-mass energy is determined by the analysis of the reaction $e^{+} e^{-} \rightarrow\left(\gamma_{I S R / F S R}\right) \mu^{+} \mu^{-}$. This study is guided by BAM00165 [15,32], the $\sqrt{s}$ determination of the 2013 data. The center-of-mass energy is given by

$$
\begin{equation*}
\sqrt{s}_{\mu^{+} \mu^{-}}=m\left(\mu^{+} \mu^{-}\right)+\Delta m_{\text {rad }}+\Delta m_{\text {calib }} \tag{24}
\end{equation*}
$$

where $m\left(\mu^{+} \mu^{-}\right)$is the invariant dimuon mass, $\Delta m_{\text {rad }}$ is the correction due to ISR/FSR, and $\Delta m_{\text {calib }}$ is the correction due to the momentum calibration.

## A.2.1 MC Samples

For the center-of-mass determination with the dimuon process, several MC samples of each 200 k events have been generated:

- For both data samples (on/off-resonance), the process $e^{+} e^{-} \rightarrow\left(\gamma_{I S R / F S R}\right) \mu^{+} \mu^{-}$is simulated with Babayaga 3.5 [20] including ISR and FSR.
- In order to study the effect of radiative corrections, the $e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}$reaction is simulated with Babayaga 3.5 without ISR and FSR. Again, MC samples for both center-of-mass energies are generated.
- For the momentum calibration, the process $e^{+} e^{-} \rightarrow \gamma_{I S R}\left(\gamma_{F S R}\right) J / \psi\left(\rightarrow \mu^{+} \mu^{-}\right)$is simulated with EvtGen and the VECTORISR model [17]. For the on-resonance $\sqrt{s}$, two MC sets are generated: One with FSR and one without FSR.


## A.2.2 Invariant Dimuon Mass

The $e^{+} e^{-} \rightarrow(\gamma) \mu^{+} \mu^{-}$events are selected by the following criteria:

- Exactly one positively and one negatively charged track are required.
- These have to fulfill the standard vertex requirements: $\left|z_{p o c a}\right|<10 \mathrm{~cm}$ and $r_{p o c a}<1 \mathrm{~cm}$.
- The tracks are constrained to the barrel region: $\cos \theta<0.8$.
- The tracks have to be back-to-back, i.e. the cosine of the opening angle between the two tracks needs to less than -0.9997 . This corresponds to a minimum opnening angle of $178.6^{\circ}$.


Figure 28: Comparisson of the energy deposition distributions of data and MC. The MC histograms are normalized to the data histograms. The top (bottom) row shows the positively (negatively) charged track and the left (right) hand column shows the off resonance (on resonance) data set. The arrows indicate the cut value. A slight shift between data and MC is noticable.

- The muons are identified by a maximum energy deposition in the EMC: $E_{E M C}<0.4 \mathrm{GeV}$
- Background from cosmic muons is suppressed by requiring that the difference of the timing signals in the TOF associated with the two tracks needs to be less than 2 ns .

Figures 28-32 show the comparisson between data and MC in various distributions. In general, there is good agreement between data and MC. However, there is a slight shift in the distributions of the energy deposition in the EMC and the opening angle between the two tracks.

Figure 33 shows the invariant dimuon mass distributions for the on- and off-resonance data sets. A fit with a Gaussian is superimposed. The fit range is defined by $[\mu-\sigma, \mu+1.5 \sigma]$. The results are ( $3865.52 \pm 0.08$ ) MeV and $(3869.48 \pm 0.08) \mathrm{MeV}$ for the off- and on-resonance data sets respectively.

## A.2.3 Radiative Correction

The effect of ISR and FSR is determined by the analysis of MC events with and without ISR/FSR. Therefore, the event selection criteria are the same as above. Figure 34 shows the fits to the corresponding dimuon mass distributions. The correction due to ISR/FSR $\Delta m_{r a d}$ from (24) is given by

$$
\begin{equation*}
\Delta m_{\mathrm{rad}}=m_{0}^{M C}\left(\mu^{+} \mu^{-}\right)-m_{I S R / F S R}^{M C}\left(\mu^{+} \mu^{-}\right) \tag{25}
\end{equation*}
$$



Figure 29: Comparisson of the track $\cos \theta$ distributions of data and MC. The MC histograms are normalized to the data histograms. The top (bottom) row shows the positively (negatively) charged track and the left (right) hand column shows the off resonance (on resonance) data set. The cut on the barrel region is already applied.


Figure 30: Comparisson of the distributions of the opening angle between the two tracks of data and MC. The MC histograms are normalized to the data histograms. The left (right) hand histogram shows the off resonance (on resonance) data set. The arrows indicate the cut value.


Figure 31: Comparisson of the time difference of both tracks in the tof of data and MC. The MC histograms are normalized to the data histograms. The left (right) hand histogram shows the off resonance (on resonance) data set. The arrows indicate the cut values. The accumulations in data at $\left|\Delta t_{T O F}\right|>5 \mathrm{~ns}$ are due to cosmic muons.


Figure 32: Comparisson of the invariant $\mu^{+} \mu^{-}$mass distributions of data and MC. The MC histograms are normalized to the data histograms. The left (right) hand histogram shows the off resonance (on resonance) data set.


Figure 33: Fit to the observed invariant $\mu^{+} \mu^{-}$mass distribution. The left (right) hand plot shows the off resonance (on resonance) data set. The obtained values are ( $3865.52 \pm 0.08$ ) MeV and ( $3869.48 \pm$ $0.08) \mathrm{MeV}$ for the off- and on-resonance data sets respectively.


Figure 34: Fit to the invariant $\mu^{+} \mu^{-}$mass distribution of the MC set with and without ISR/FSR. The top (bottom) row shows the radiative corrections switched on (off) and the left (right) hand column shows the off resonance (on resonance) MC set.

Table 8: Correction to the invariant dimuon mass due to radiative effects.

| Data Set | $m_{0}^{M C}\left(\mu^{+} \mu^{-}\right) / \mathrm{MeV}$ | $m_{I S R / F S R}^{M C}\left(\mu^{+} \mu^{-}\right) / \mathrm{MeV}$ | $\Delta m_{\text {rad }} / \mathrm{MeV}$ |
| :--- | :---: | :---: | :---: |
| Off Resonance | $3869.44 \pm 0.16$ | $3866.28 \pm 0.15$ | $3.16 \pm 0.22$ |
| On Resonance | $3873.39 \pm 0.16$ | $3870.50 \pm 0.15$ | $2.89 \pm 0.22$ |

when $m_{0}^{M C}\left(\mu^{+} \mu^{-}\right)$is the fitted mass of the MC set without ISR/FSR and $m_{I S R / F S R}^{M C}\left(\mu^{+} \mu^{-}\right)$is the fitted mass of the MC set with ISR/FSR. The resulting values are summarized in Table 8. Since both center-of-mass energies are very close together, the effect of radiative corrections are expected to be the same for both data sets. This is taken into account by avergaing both values and apply it to both data sets. The final value is $\Delta m_{\text {rad }}=(3.03 \pm 0.16) \mathrm{MeV}$.

## A.2.4 Momentum Callibration

The momentum calibration is checked with the determination of the $J / \psi$ mass in the reaction $e^{+} e^{-} \rightarrow$ $\gamma_{I S R}\left(\gamma_{F S R}\right) J / \psi\left(\rightarrow \mu^{+} \mu^{-}\right)$. Therefore, the event selection criteria are the same as above with the exeption that the requirement of the opening angle between the two tracks is dropped. Figure 35a shows the fit to data (both data samples combined) in the $J / \psi$ region. The signal is modelled by a Crystal Ball function and the background is modelled as a quadratic funciton. The obtained mass $m_{F S R}^{\text {data }}(J / \psi)$ needs to be corrected for FSR effects, which are determined using MC with and without FSR. The fits with the same fit models are shown in Figures 35b and 35c. The resulting value for the $/ J / \psi$ mass and its deviation

(a) Fit of the $J / \psi$ peak in data.


Figure 35: Fit to the $J / \psi$ peak in $e^{+} e^{-} \rightarrow \gamma_{I S R}\left(\gamma_{F S R}\right) J / \psi\left(\rightarrow \mu^{+} \mu^{-}\right)$.
from the PDG value are listed in Table 9. For a center-of-mass energy corresponding to the $J / \psi$ mass, the correction to the dimuon mass due to momentum calibration is $\Delta m_{\text {calib }}^{J / \psi}=(-1.1 \pm 0.4) \mathrm{MeV} / c^{2}$. However, this correction is not independent from $\sqrt{s}$. In [33], it was shown that this correction can be described by a linear function with a slope of $(5.44 \pm 0.33) \times 10^{4} / \mathrm{MeV}$. Using this slope, $\Delta m_{\text {calib }}$ is extrapolated from the $J / \psi$ mass to the $X(3872)$ mass region and yields $\Delta m_{\text {calib }}=(-1.5 \pm 0.3) \mathrm{MeV}$

## A.2.5 Result

The combination of the above intermediate results is shown in Table 10. The final result of the dimuon analysis agrees within the statistical error bars with the BEMS measurement. Since this only serves as a cross check, the systematic uncertainty is not determined.

## A.2.6 Run Dependency of the Center-of-Mass Energy

The center-of-mass energy has also been determined for each run independently. The result is shown in Figure 36. In particular, it shows that $\sqrt{s}$ is stable during the on-resonance data taking. This justifies the usage of the BEMS result of this period, which is missing a large part of measurements.

Table 9: Summary of the Fits to the $J / \psi$ peak in $e^{+} e^{-} \rightarrow \gamma_{I S R}\left(\gamma_{F S R}\right) J / \psi\left(\rightarrow \mu^{+} \mu^{-}\right)$and the resulting corrections. $m_{F S R}^{d a t a}(J / \psi)$ is the fitted mass in data. $m_{F S R}^{M C}(J / \psi)$ and $m_{n o F S R}^{M C}(J / \psi)$ are the fitted masses in MC with and without FSR. The difference, of the two, i.e. the effect of FSR at the $J / \psi$ mass, is $\Delta m_{F S R}^{J / \psi}$. The fitted data mass after the FSR correction is $m_{0}^{\text {data }}(J / \psi)$ and its difference to the PDG value is $\Delta m_{\text {calib }}^{J / \psi}$, which is then also the mass correction due to momentum calibration for $\sqrt{s}=m(J / \psi)$.

| $m_{F S R}^{\text {data }}(J / \psi)$ | $(3097.48 \pm 0.28) \mathrm{MeV} / c^{2}$ |
| :--- | ---: |
| $m_{F S R}^{M C}(J / \psi)$ | $(3098.35 \pm 0.05) \mathrm{MeV} / c^{2}$ |
| $m_{n o}^{M C}(J / \psi R$ |  |
| $\Delta m_{F S R}^{J / \psi}$ | $(3098.83 \pm 0.05) \mathrm{MeV} / c^{2}$ |
| $m_{0}^{\text {data }}(J / \psi)$ | $(0.48 \pm 0.07) \mathrm{MeV} / c^{2}$ |
| $\Delta m_{\text {calib }}^{J / \psi}$ | $(-1.1 \pm 0.3) \mathrm{MeV} / c^{2}$ |

Table 10: Result of the center-of-mass determination via the analysis of dimuon events. $m\left(\mu^{+} \mu^{-}\right)$is the invariant dimuon mass, $\Delta m_{\text {rad }}$ is the correction due to ISR/FSR, and $\Delta m_{\text {calib }}$ is the correction due to the momentum calibration. The result $\sqrt{s}_{\mu^{+} \mu^{-}}$is compared to the BEMS result $\sqrt{s}_{B E M S}$. Both results agree within the error bars.

| Data Set | Off Resonance | On Resonance |
| :--- | :---: | :---: |
| $m\left(\mu^{+} \mu^{-}\right) / \mathrm{MeV}$ | $(3865.52 \pm 0.08)$ | $(3869.48 \pm 0.08)$ |
| $\Delta m_{\text {rad }} / \mathrm{MeV}$ | $3.03 \pm 0.16$ | $3.03 \pm 0.16$ |
| $\Delta m_{\text {calib }} / \mathrm{MeV}$ | $-1.5 \pm 0.3$ | $-1.5 \pm 0.3$ |
| $\sqrt{s}_{\mu^{+} \mu^{-}} / \mathrm{MeV}$ | $3867.05 \pm 0.35$ | $3871.01 \pm 0.35$ |
| $\sqrt{s}_{\text {BEMS }} / \mathrm{MeV}$ | $3867.410 \pm 0.031$ | $3871.31 \pm 0.06$ |

Run Dependent $\sqrt{s}$ from $\mathrm{e}^{+} \mathrm{e}^{-} \rightarrow \mu^{+} \mu^{-}$


Figure 36: The center-of-mass energy as determined for each run via the analysis of dimuon events. The two different energies for the on- and off-resonance data samples are clearly visible. The energy doesn't show large fluctuations within the two separate samples.

## B Determination of the Integrated Luminosity of the 2017 Data

Since this is the first analysis of the two 2017 data sets, its integrated luminosity needs to be determined. This is done by the analysis of (radiative) Bhabha events. The reaction $e^{+} e^{-} \rightarrow(\gamma) e^{+} e^{-}$has a large cross section, which can be calculated by theory with an accuracy at the sub-percent level. The integrated luminosity is then given by the relation

$$
\begin{equation*}
\int \mathcal{L} \mathrm{d} t=\frac{N_{o b s}}{\sigma \cdot \epsilon} \tag{26}
\end{equation*}
$$

where $\sigma$ is calculated by the event generator, $N_{o b s}$ is the number of observed Bhabha events and the efficiency $\epsilon$ is determined by the analysis of MC events.

The analysis strategy follows the one from the luminosity determination of the $X Y Z$ scan data in BAM-00110 [16, 34]. BOSS 7.0.3 is used.

## B. 1 Monte Carlo Data Set

The MC events are generated with Babayaga 3.5 [20]. For both energy points, a MC data set of each 200 k events is generated with the following generator settings:

- Ebeam $=\sqrt{s} / 2$.
- MinThetaAngle $=20^{\circ}$.
- MaxThetaAngle $=160^{\circ}$.
- MinimumEnergy $=0.01 \mathrm{GeV}$.
- RunningAlpha $=1$.
- $F S R_{-}$switch $=1$.


## B. 2 Event Selection

The $e^{+} e^{-} \rightarrow(\gamma) e^{+} e^{-}$events are selected by the following criteria:

- Exactly one positively and one negatively charged track are required.
- These have to fulfill the standard vertex requirements: $\left|z_{p o c a}\right|<10 \mathrm{~cm}$ and $r_{p o c a}<1 \mathrm{~cm}$.
- The tracks are constrained to the barrel region: $\cos \theta<0.8$.


Figure 37: Comparisson of the track $\cos \theta$ distributions of data and MC. The MC histograms are normalized to the data histograms. The top (bottom) row shows the positively (negatively) charged track and the left (right) hand column shows the off resonance (on resonance) data set. The cut on the barrel region is already applied.

Table 11: Determination of the integrated luminosity. For both center-of-mass energies, the Bhabha cross section as calculated by the event generator, the efficiency, the number of observed events as well as the integrated luminosity are listed. The shown error is statistical only.

| $\sqrt{s} / \mathrm{MeV}$ | $\sigma / \mathrm{nb}$ | $\epsilon / \%$ | $N_{\text {obs }}$ | $\int \mathcal{L} \mathrm{d} t / \mathrm{pb}^{-1}$ |
| :--- | :---: | :---: | :---: | :---: |
| 3867.4 | $490.22 \pm 0.35$ | $14.4 \pm 0.1$ | 7681324 | $108.87 \pm 0.04$ |
| 3871.3 | $489.54 \pm 0.35$ | $14.4 \pm 0.1$ | 7768498 | $110.31 \pm 0.04$ |

- The tracks have to have a minimum momentum: $p>p_{c u t}$. The cut value is proportional to $\sqrt{s}$ :

$$
\begin{equation*}
p_{\text {cut }}=\frac{\sqrt{s}}{4.26 \mathrm{GeV}} \times 2.0 \mathrm{GeV} / c \tag{27}
\end{equation*}
$$

This relation as well as the following was optimized in BAM-00110 [16, 34] to the data set at $\sqrt{s}=4.26 \mathrm{GeV}$

- The eletrons are identified by minimum energy deposition in the EMC, which is also $\sqrt{s}$ dependend:

$$
\begin{equation*}
E_{E M C}>\frac{\sqrt{s}}{4.26 \mathrm{GeV}} \times 1.55 \mathrm{GeV} \tag{28}
\end{equation*}
$$

Figures 37-40 show the good agreement between various distributions of the data and MC samples.
The determined luminosity is summarized in Table 11.


Figure 38: Comparisson of the track momentum distributions of data and MC. The MC histograms are normalized to the data histograms. The top (bottom) row shows the positively (negatively) charged track and the left (right) hand column shows the off resonance (on resonance) data set. The arrows indicate the cut values.


Figure 39: Comparisson of the energy deposition distributions of data and MC. The MC histograms are normalized to the data histograms. The top (bottom) row shows the positively (negatively) charged track and the left (right) hand column shows the off resonance (on resonance) data set. The arrows indicate the cut values.


Figure 40: Comparisson of the track azimuthal angle distributions of data and MC. The MC histograms are normalized to the data histograms. The top (bottom) row shows the positively (negatively) charged track and the left (right) hand column shows the off resonance (on resonance) data set.

## B. 3 Systematic Uncertainties

The systematic uncertainties affecting the luminosity are the following. They are determined using the same strategy as in BAM-00110 [16, 34], where the justification for the specific values of the cut variations can be found as well.

- The tracking uncertainty is determined by the reconstuction of Bhabha events with MDC and EMC and the reconstruction with the EMC only. Therefore, some event selection criteria are changed or added:
$-p_{e^{ \pm}}>\frac{2}{4.26} \cdot \sqrt{s}$. This cut is only applied for the method using the MDC and the EMC.
$-E_{e^{ \pm}}^{E M C}>\frac{1.8}{4.26} \cdot \sqrt{s}$
$-5^{\circ}<|\Delta \phi|<40^{\circ}$, with $\Delta \phi=\left|\phi_{e^{+}}^{E M C}-\phi_{e^{-}}^{E M C}\right|-180^{\circ}$ and $\phi_{e^{ \pm}}^{E M C}$ being the azimuthal angle of the $e^{ \pm}$cluster in the EMC.
- The constraint to the barrel region is applied to the EMC clusters instead of the momenum vectors: $\left|\cos \theta_{e^{ \pm}}^{E M C}\right|<0.8$.

The change of integrated luminosity is taken as systematic uncertainty.

- The cut on $\cos \left(\theta_{e^{ \pm}}\right)$has been varied from 0.8 to 0.7 and the difference in the resulting luminosity is taken as the systematic uncertainty of this cut.

Table 12: Relative systematic uncertainties (in \%) of the integrated luminosity. The total error is the quadratic sum.

| Source | 3867.4 MeV | 3871.3 MeV |
| :--- | :---: | :---: |
| Tracking | 0.80 | 0.48 |
| $\cos \left(\theta_{e^{ \pm}}\right)$cut | 0.09 | 0.12 |
| $E_{e^{ \pm} C}^{E c}$ cut | 0.09 | 0.06 |
| $p_{e^{ \pm}}$cut | 0.09 | 0.16 |
| $\sqrt{s}$ | 0.65 | 0.02 |
| $\sigma$ | 0.50 | 0.50 |
| Trigger | 0.10 | 0.10 |
| Total | 1.16 | 0.71 |

Table 13: Integrated luminosity of th two 2017 data sets. The first error is statistical and the second one is systematic.

| $\sqrt{s} / \mathrm{MeV}$ | $\int \mathcal{L} \mathrm{d} t / \mathrm{pb}^{-1}$ |
| :--- | ---: |
| 3867.4 | $108.87 \pm 0.04 \pm 1.26$ |
| 3871.3 | $110.31 \pm 0.04 \pm 0.78$ |

- The cut value of $E_{e^{ \pm}}^{E M C}$ is increased by $10 \%$ and the change in luminosity is taken as the uncertainty.
- The cut value of $p_{e^{ \pm}}$is increased by $3 \%$ and the change in luminosity is taken as the uncertainty.
- To be conservative, the $\sqrt{s}$ uncertainty is estimated by a 2 MeV shift of the MC sample. As a result, the selection efficiency and the calculated cross section are altered and the change in luminosity is taken as the systematic uncertainty. This variation is larger than the difference to the $\sqrt{s}$ of the first two runs of the off-resonance data set.
- The uncertainty of the cross section calculation is quoted from Babayaga 3.5 [20].
- The trigger efficiency uncertainty has been determined in [35].

Their values and the total systematic uncertainties are listed in Table 12.

## B. 4 Result

The result is summarized in Table 13.

## B. 5 Cross Check: Luminosity Determination via Dimuon Process

The analysis of the dimuon process for the center-of-mass energy determination can also be used to cross check the result of the luminosity determination. The corresponding MC samples and event selection criteria are described in Section A.2. The data MC agreement is demonstrated by Figures 28-32. The cross

Table 14: Determination of the integrated luminosity via $e^{+} e^{-} \rightarrow(\gamma) \mu^{+} \mu^{-}$. For both center-of-mass energies, the cross section as calculated by the event generator, the efficiency, the number of observed events as well as the integrated luminosity are listed. The shown error is statistical only.

| $\sqrt{s} / \mathrm{MeV}$ | $\sigma / \mathrm{nb}$ | $\epsilon / \%$ | $N_{\text {obs }}$ | $\int \mathcal{L} \mathrm{d} t / \mathrm{pb}^{-1}$ |
| :--- | :---: | :---: | :---: | :---: |
| 3867.4 | $6.096 \pm 0.005$ | $51.86 \pm 0.16$ | 349512 | $110.56 \pm 0.19$ |
| 3871.3 | $6.025 \pm 0.005$ | $51.85 \pm 0.16$ | 361745 | $115.79 \pm 0.19$ |

section for $e^{+} e^{-} \rightarrow(\gamma) \mu^{+} \mu^{-}$is obtained from the MC generator Babayaga 3.5 and the efficiency is determined by the analysis of MC events. The result is summarized in Table 14. The result is signifi-

3 cantly larger than the values obtained from Bhabha events. This is probably caused by $e^{+} e^{-} \rightarrow(\gamma) \pi^{+} \pi^{-}$ 4 contamination.

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[^0]:    ${ }^{1}$ The BEMS result is delayed by ca. 20 min , thus it is not a strict realtime measurement.

[^1]:    ${ }^{2} \mathrm{~A}$ simple fit of a constant to the measured values gives $\chi^{2} / N D F=4.28 / 3$ or alternatively a $p$-value of 0.23 .

[^2]:    ${ }^{3} \mathrm{~A}$ constant cannot be normalized and is therefore no pdf. Nevertheless, it can result in a proper, i.e. normalizable, posterior pdf $(\theta \mid x)$. There are different approaches to construct so-called noninformative or objective priors, which are favourable, but very difficult to obtain in multi-parameter models [30, 31].

