KKMC at BESIII

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Out Line

- Introduction to KKMC
- Formula in KKMC
- Modification to current KKMC code in BESIII to solve the problem in generating ISR/FSR MC
- Comparison with ConExc
- Summary

A note with detailed information has been uploaded to DocDB

http://docbes3.ihep.ac.cn/DocDB/0007/000717/002/kkmc_V2.pdf

Introduction

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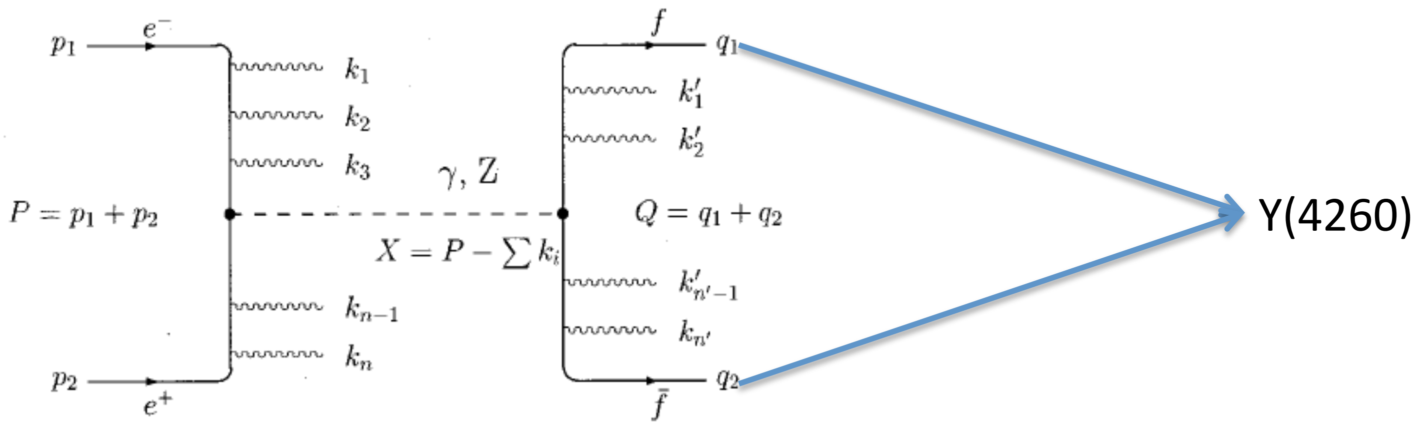
- KK Monte Carlo(KKMC) for two fermion final states in e+e- collision.
- e+e- →ff+nγ, f=μ,τ,d,u,s,c,b.   Here the γ can be ISR, FSR.
- For BESIII simulation, we usually use Y(4260) as parent particle. So only ff=ccbar channel is open.
- Other channels can be used to generate continuum events, like

ee→μμ,ττ



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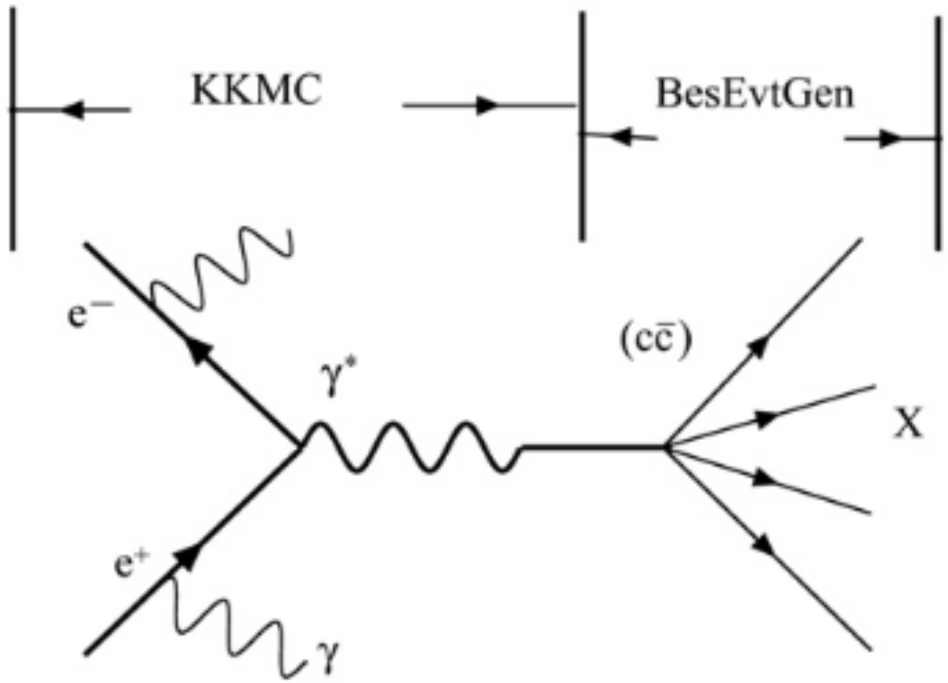
Introduction



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The KKMC is used to simulate the e+e– annihilation till c ¯c production including ISR effects,

Then the charmonium decays are generated with BesEvtGen models

Cross section formula
Exclusive Exponentiation（EEX)

$$\sigma_{EEX}^{(r)}=\sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\frac{1}{n!}\frac{1}{n'!}\int d^{n+n'+2}\mathrm{Lips}(p_1+p_2;q_1,q_2,k_1\ldots,k_n,k'_1\ldots,k'_{n'})\rho_{EEX}^{(r)},\quad r|=0,1,2,3,$$

PRD63,113009

$$\rho_{EEX}^{(r)}(p_1,p_2,q_1,q_2,k_1\ldots,k_n,k'_1\ldots,k'_n)$$

$$=\exp[Y_e(\Omega_I;p_1,p_2)+Y_f(\Omega_F;q_1,q_2)]\prod_{j=1}^{n}\tilde{S}_I(k_j)\bar{\Theta}(\Omega_I;k_j)\prod_{l=1}^{n'}\tilde{S}_F(k'_l)\bar{\Theta}(\Omega_F;k'_l)\Bigg\{\bar{\beta}_0^{(r)}(X,p_1,p_2,q_1,q_2)$$

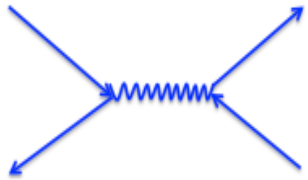
$$+\sum_{j=1}^{n}\frac{\bar{\beta}_{1I}^{(r)}(X,p_1,p_2,q_1,q_2,k_j)}{\tilde{S}_I(k_j)}+\sum_{l=1}^{n'}\frac{\bar{\beta}_{1F}^{(r)}(X,p_1,p_2,q_1,q_2,k'_l)}{\tilde{S}_F(k'_l)}+\sum_{n\geqslant j>k\geqslant 1}\frac{\bar{\beta}_{2II}^{(r)}(X,p_1,p_2,q_1,q_2,k_j,k_k)}{\tilde{S}_I(k_j)\tilde{S}_I(k_k)}$$

$$+\sum_{n'\geqslant l>m\geqslant 1}\frac{\bar{\beta}_{2FF}^{(r)}(X,p_1,p_2,q_1,q_2,k'_l,k'_m)}{\tilde{S}_F(k'_l)\tilde{S}_F(k'_m)}+\sum_{j=1}^{n}\sum_{l=1}^{n'}\frac{\bar{\beta}_{2IF}^{(r)}(X,p_1,p_2,q_1,q_2,k_j,k'_l)}{\tilde{S}_I(k_j)\tilde{S}_F(k'_l)}$$

$$+\sum_{n\geqslant j>k>l\geqslant 1}\frac{\bar{\beta}_{3III}^{(r)}(X,p_1,p_2,q_1,q_2,k_j,k_k,k_l)}{\tilde{S}_I(k_j)\tilde{S}_I(k_k)\tilde{S}_I(k_l)}\Bigg\}.$$

EEX formula

- The perturbative QED matrix element is located in the β function. The lower index is the number of real photons, the upper index is the correction order.
- KKMC calculated up to $O(\alpha^3)$ correction when using EEX.
- $\beta_0^{(0)}$: born process



$$\bar{\beta}_0^{(0)}(X, p_1, p_2, q_1, q_2) = \frac{1}{4} \sum_{k,l=1,2} \frac{d\sigma^{\mathrm{Born}}}{d\Omega}(X^2, \vartheta_{kl}),$$

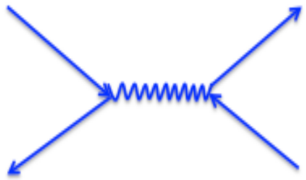
$$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

EEX formula

- $\beta_0^{(r)}$: born process+o(α^r) virtual photon correction



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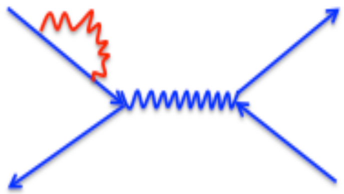
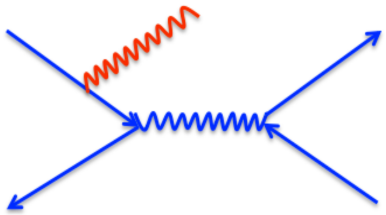
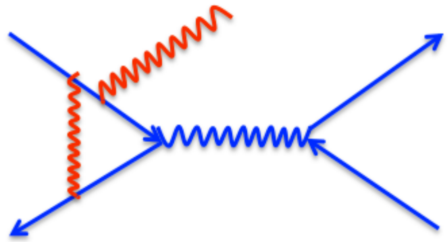
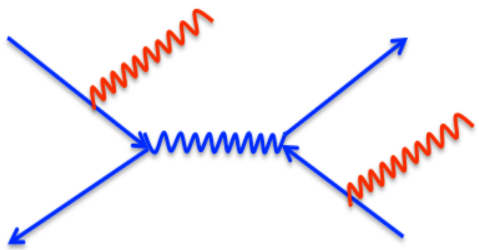


$\beta_{1I}^{(r)}, \beta_{1F}^{(r)}$ The o(α^r) correction with 1 real photon+ virtual photons



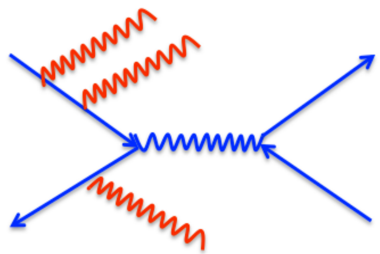


$\beta_{2II}^{(r)}, \beta_{2IF}^{(r)}, \beta_{2FF}^{(r)}$ The o(α^r) correction with 2 real photon (no virtual photons)



EEX formula

$\beta_{3III}^{(r)}$ The o(α^r) correction with 3 real photon (only ISR photons)



Although the formula only calculated up to 3 photons, the code allow the generation of more than 3 photons, then one photon is regarded as soft photon? For example, if 4 ISR is generated, the formula would be $\tilde{S}_I(K_i) \times \beta_{3III}^{(r)}(K_j, K_k, K_l) \propto \alpha \cdot \beta_{3III}^{(r)}(K_j, K_k, K_l)$
The correction might be not precious, but at least the order is right

$\Theta(\Omega; k)$ Is a sign function to veto the IR divergence points

EEX formula

- Soft factor for real photons emitted from initial and final states fermions

$$\tilde{S}_I(k_j) = -Q_e^2 \frac{\alpha}{4\pi^2}\left(\frac{p_1}{k_j p_1} - \frac{p_2}{k_j p_2}\right)^2,$$

$$\tilde{S}_F(k'_l) = -Q_f^2 \frac{\alpha}{4\pi^2}\left(\frac{q_1}{k'_l q_1} - \frac{q_2}{k'_l q_2}\right)^2,$$

- YFS factor is used to represent the integral over all the very soft photons that's not generated in MC.

$$Y_e(\Omega_I; p_1, p_2) = \gamma_e ln \frac{2E_{min}}{\sqrt{2p_1p_2}} + \frac{1}{4}\gamma_e + Q_e^2 \frac{\alpha}{\pi}(-\frac{1}{2} + \frac{\pi^2}{3})$$

$$Y_f(\Omega_F; q_1, q_2) = \gamma_f ln \frac{2E_{min}}{\sqrt{2q_1q_2}} + \frac{1}{4}\gamma_f + Q_f^2 \frac{\alpha}{\pi}(-\frac{1}{2} + \frac{\pi^2}{3})$$

Coherent Exclusive Exponentiation（CEEX）

$$\sigma^{(r)} = \frac{1}{\mathrm{flux}(s)} \sum_{n=0}^{\infty} \int \mathrm{dLIPS}_{n+2}(p_a + p_b; p_c, p_d, k_1, \ldots, k_n)\, \rho^{(r)}_{\mathrm{CEEX}}(p_a, p_b, p_c, p_d, k_1, \ldots, k_n),$$

$$\begin{aligned}
&\rho^{(r)}_{\mathrm{CEEX}}(p_a, p_b, p_c, p_d, k_1, k_2, \ldots, k_n) \\
&= \frac{1}{n!} e^{Y(\Omega; p_a, \ldots, p_d)} \overline{\Theta}(\Omega) \sum_{\sigma_1, \ldots, \sigma_n = \mp 1} \; \sum_{\lambda_A, \bar{\lambda}_A = \mp 1} \; \sum_{i,j,l,m=0}^{3} \hat{\varepsilon}_1^i \hat{\varepsilon}_2^j \sigma^i_{\lambda_a \bar{\lambda}_a} \sigma^j_{\lambda_b \bar{\lambda}_b} \\
&\quad \times \mathfrak{M}_n^{(r)} \begin{pmatrix} p\, k_1\, k_2 \ldots k_n \\ \lambda\, \sigma_1 \sigma_2 \ldots \sigma_n \end{pmatrix} \left[\mathfrak{M}_n^{(r)} \begin{pmatrix} p\, k_1\, k_2 \ldots k_n \\ \bar{\lambda}\, \sigma_1 \sigma_2 \ldots \sigma_n \end{pmatrix} \right]^{\star} \sigma^l_{\bar{\lambda}_c \lambda_c} \sigma^m_{\bar{\lambda}_d \lambda_d} \hat{h}_3^l \hat{h}_4^m, \qquad (4)
\end{aligned}$$

Spin vector for initial fermions

Pauli Matrix

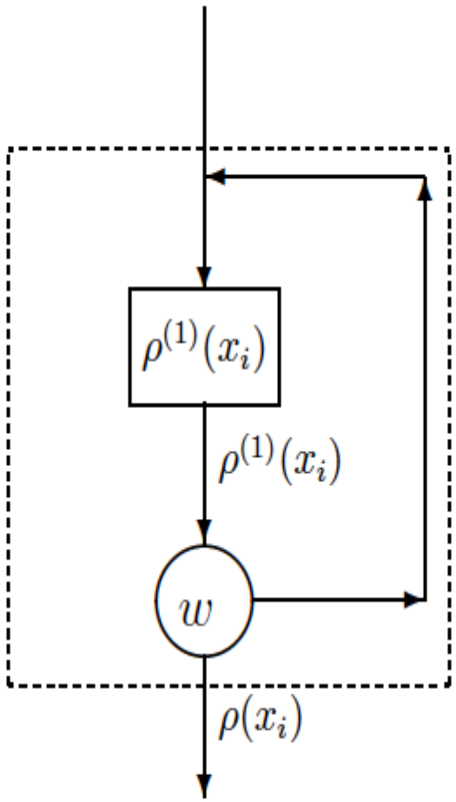
Spin vector for final fermions

EEX has sumed over all the spins,   but CEEX are calculating them coherently
In KKMC,   use CEEX only when Ecms>12GeV by default,
so we are using EEX all the time in BESIII.

Sampling method

arXiv:physics/9906056v1

- The cross section is very complex, how to do sampling.



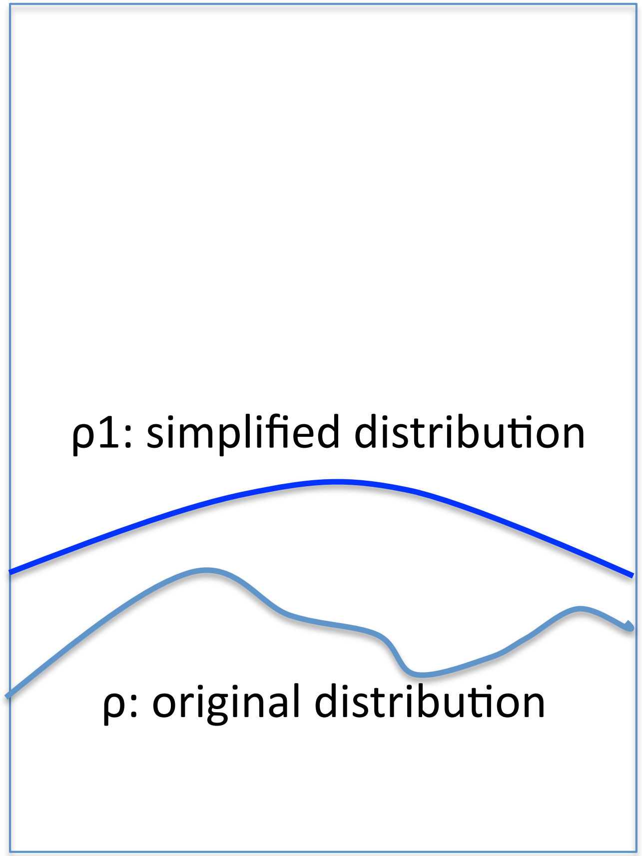
•ρ is the completed distribution after all mechanism Is considered, which is the distribution we need.

•ρ1 is a distributon easy to generate and calculate Which is called Crude distribution in the code.

•ρ1 is most simplified model, without considering most of the mechanism, such as ISR, FSR…

$$w(x) = \frac{\rho(x_i)}{\rho^{(1)}(x_i)}$$

Sampling method



❑Frist generate the event according to a simplified distribution ρ1, and we can also calculate the cross section σ 1

❑Then we calculate the weight for each event

$$w(x) = \frac{\rho(x_i)}{\rho^{(1)}(x_i)}$$

❑The pick and throw using this weight

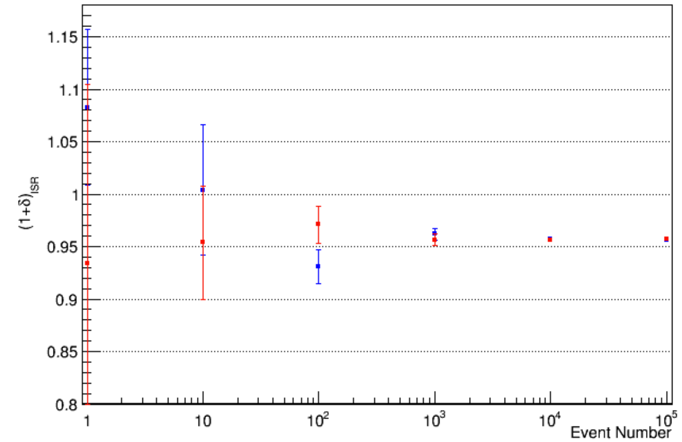
•W=ω_max
•Generat random number r between（0， 1）

•If rW<ω (x), keep the event, else throw

❑The left events would satisfy ρ (x) distribution.
❑And the integral cross section σ for ϱ can be calculated as the multiplication of σ1 and average weight

$$\sigma = \sigma^{(1)} \cdot < \omega >$$

Sampling method



- This plot shows the $1+\delta_{ISR}$ vs. generated MC event number, red and blue dots are from two sets of mc with different random seed.
- As the calculation of integrated cross section σ need the average weight. So to get an accurate enough $1+\delta_{ISR}$, we need to generate enough events. (>10000 is suggested).

A code flow of generate MC with KKMC

- The original EEX formula can be simplified and transformed to a simple distribution
- Then sample according to the input cross section to get Sx(the invariant mass after ISR)
- Generating ISR multiplicity（ISR photon number) according a Poisson distribution with theoretical average multiplicity
- Generating the energy and angular distribution of each ISR
- Generating FSR multiplicity according to the other Poisson distributionGenerating the energy and angular distribution of each ISR
- Calculating the weight using the generated ISR/FSR momentum

$$w(x) = \frac{\rho(x_i)}{\rho^{(1)}(x_i)}$$

- Pick and throw according the weigh
- The left events will satisfy EEX formula, the momentum sum of ccbar is assigned to Ψ (or Y) and will be passed to EvtGen to decay

The modification to KKMC used in BESIII

The out put $1+\delta_{ISR}$ when ISR is turned ON/OFF

When ISR is turned off

$$1+\delta_{ISR} = \frac{m\_xSecPb}{Input(s)} = B(1,S) * const.$$

When ISR is turned on

$$1+\delta_{ISR} = \frac{m\_xSecPb}{Xborn} = \frac{const * \int B(1,s') * (1+\delta_{ISR}) * Input(s') ds'}{Input(s)}$$

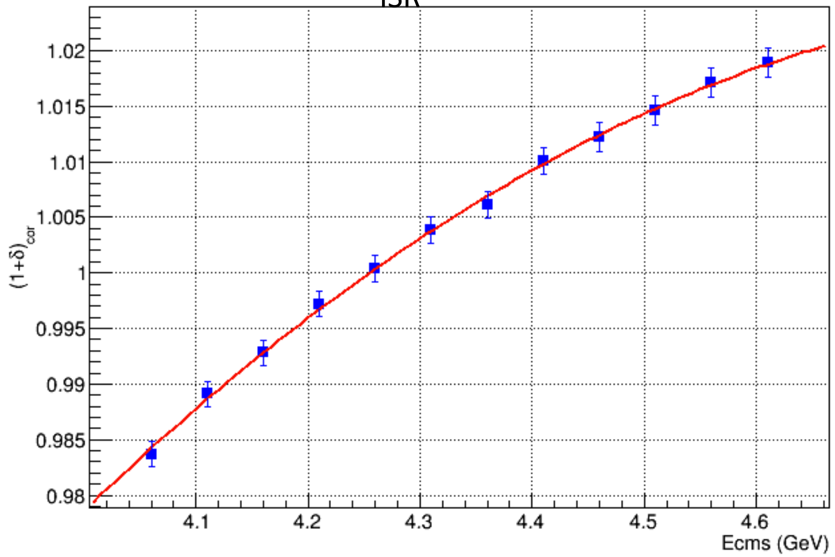
B(1,S) is a short for BornV_Differential() function in the code. Which is calculating the born Differential cross section of $e^+e^- \rightarrow$ ccbar.

Input(S) is the input cross section.

$1+\delta_{ISR}$ is represent all the radiation corrections in the formula. It's not a constant, and very complex according to EEX formula, I just use this symbol here to represent the radiation Correction.

The problem in original code

The non-unit $1+\delta_{ISR}$ value when ISR is off



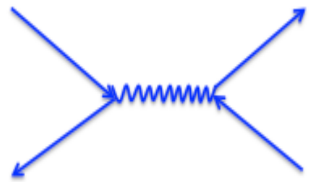
The non-unit $1+\delta_{ISR}$ value when ISR is off,

The old method is to turn On/Off radiation and make the divide to get $1+\delta_{ISR}$,
Now we can see it's because the B(1,S) function is not flat

$$1 + \delta_{ISR} = \frac{m\_xSecPb}{Input(s)} = B(1, S) * const.$$

B(1,s)=BornV_Differential() function

- B(1,s) is calculating the differential cross section of $e^+e^- \rightarrow$ccbar



- The thoeretical differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{\vec{p_f}}{\vec{p_i}}|M_{fi}|^2$$
$$= \frac{1}{64\pi^2 s}\sqrt{1-\frac{4M^2}{s}} \times \frac{4}{4}e^2 Q_f^2[1+cos^2\theta + \frac{4M^2}{s}sin^2\theta]$$
$$= \frac{e^2 Q_f^2}{64\pi^2 s}\sqrt{1-\frac{4M^2}{s}}[1+cos^2\theta + \frac{4M^2}{s}sin^2\theta]$$

Propagator

Phase space term

Angular distribution part

Where Qf is the charge of c quark, M is the mass of c quark. θis the out going angle Of c quark (assuming input e+e- along z-axis).

BornV_Differential() function

- The formula in the original code

$$B(1,s) = \left(\frac{1}{s}\frac{1}{1-xfem}\right)^2 \sqrt{1-\frac{4M^2}{s}}\left[1+cos^2\theta+\frac{4M^2}{s}sin^2\theta\right]\times s^2$$

- We have neglected the Weak correction in the original code as it's contribution is negligible in BESIII range(<0.05%)

$\left(\frac{1}{1-xfem}\right)^2$ is the vacuum polarization factor is from a lookup table

Modification to B(1,S)

- The idea is all the invariant mass s–dependent

part in B(1,S) can be and should be moved to the input cross section Input(s).

$$1+\delta_{ISR} = \frac{m\_xSecPb}{Xborn} = \frac{const * \int B(1,s') * (1+\delta_{ISR}) * Input(s')ds'}{Input(s)}$$

- So we neglected the phase space term in B(1,S)

Modification to B(1,S)

- And for the angular distribution part in B(1,s), we want to keep the θ −dependent, but remove the s-dependent, as the θ will be used in calculating the ISR correction. So the angular part is integrated over θ , and the result is divided from B(1,S)

$$\int [1 + cos^2\theta + \frac{4M^2}{s} sin^2\theta] d\theta$$

$$= \int_{-1}^{1} (1 + \frac{4M^2}{s} + (1 - \frac{4M^2}{s}) \cdot x^2] dx$$

$$= \frac{8}{3} + \frac{16M^2}{3s}$$

The modification of B(1,s)

- At last the B(1,s) function becomes

$$B(1,s) = (\frac{1}{1-xfem})^2[1+cos^2\theta+\frac{4M^2}{s}sin^2\theta]/(\frac{8}{3}+\frac{16M^2}{3s})$$

Which should be flat along s, after vacuum polarization is turned off.

So when ISR is turned off, $1+\delta_{ISR} = \frac{m\_xSecPb}{Input(s)} = B(1,S)*const.$

the $1+\delta_{ISR}$ should be equal to 1 at all energy point, as long as we normalize it at one energy point.

Compare the formula with ConExc when ISR is on

ConExc

$$\sigma_{e^+e^-\to\gamma X_i}(s) = \int dm \frac{2m}{s} W(s,x) \frac{\sigma_0(m)}{[1+\Pi(m)]^2}$$

$$= \int dx W(s,x) \frac{\sigma_0(m)}{[1+\Pi(m)]^2}$$

$$1+\delta_{ISR} = \frac{\sigma_{e^+e^-\to\gamma X_i}(s)}{\sigma_0(s)}$$

W(s,x) is the radiator function,
σ_0(m) is the input cross section
$1/(1+\Pi)^2$ is vacuum polariztion factor

ConExc sample the Xs(mass after ISR) using this formula, and assign all the lost energy to one single ISR photon

KKMC

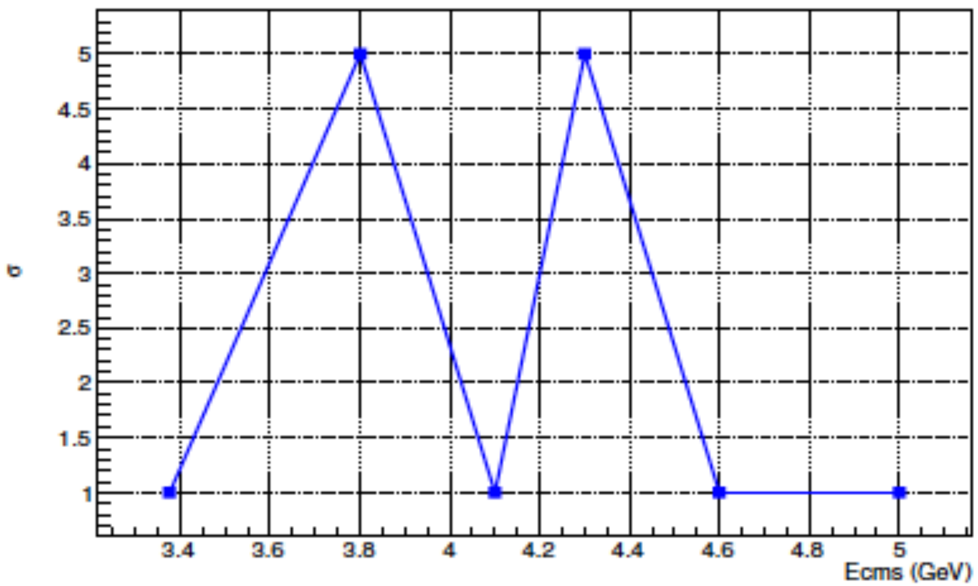
$$1+\delta_{ISR} = \frac{m\_xSecPb}{Xborn} = \frac{const * \int B(1,s') * (1+\delta_{ISR}) * Input(s') ds'}{Input(s)}$$

$$B(1,s) \bullet Input(s) \Leftrightarrow \frac{\sigma_0(s)}{[1+\Pi(s)]^2}$$

$1+\delta_{ISR}$ vs. W(s,X)

Compare between KKMC & ConExc with MC

- The process is ee→π+π-J/ψ, and the input cross section is the two triangle distribution as below



Comparison of $1+\delta_{ISR}$

- The modified KKMC and ConExc

The difference Of the δ ISR

	KKMC				ConExc		
E_{cms} GeV	ISR/VP off	$1+\delta_{ISR}$	$1+\delta_{VP}$	ConExc$^{\to}_{-}{}^{\to}$	$1+\delta_{ISR}$	$1+\delta_{VP}$	δ_{isr} diff.
3.80	1.00026	0.8797 ± 0.0003	1.0575		0.8776	1.05715	0.0021
4.10	1.00030	1.3119 ± 0.0003	1.0516		1.3099	1.05165	0.0020
4.26	1.00020	0.8824 ± 0.0003	1.0535		0.8795	1.05334	0.0029
4.30	1.00007	0.8756 ± 0.0003	1.0522		0.8733	1.05216	0.0023
4.36	1.00022	0.9359 ± 0.0003	1.0511		0.9338	1.0511	0.0021
4.42	1.00030	0.9842 ± 0.0003	1.0525		0.9816	1.05244	0.0026
4.60	1.00034	1.3753 ± 0.0003	1.0546		1.3743	1.05465	0.0010

- The original KKMC and ConExc

E_{cms} GeV	ISR/VP off	$1+\delta_{ISR}$	$1+\delta_{VP}$	ConExc$^{\to}_{-}{}^{\to}$	$1+\delta_{ISR}$	$1+\delta_{VP}$	δ_{isr} diff.
3.80	0.9575 ± 0.0003	0.8403 ± 0.0005	1.0575		0.8776	1.05715	-0.0373
4.10	0.9890 ± 0.0004	1.2839 ± 0.0005	1.0516		1.3099	1.05165	-0.0260
4.26	1.0019 ± 0.0004	0.8795 ± 0.0005	1.0535		0.8795	1.05334	0.0000
4.30	1.0041 ± 0.0004	0.8762 ± 0.0005	1.0522		0.8733	1.05216	0.0029
4.36	1.0078 ± 0.0004	0.9402 ± 0.0005	1.0511		0.9338	1.0511	0.0064
4.42	1.0117 ± 0.0004	0.9909 ± 0.0005	1.0525		0.9816	1.05244	0.0093
4.60	1.0213 ± 0.0004	1.3912 ± 0.0005	1.0546		1.3743	1.05465	0.0169

Comparison of $1+\delta_{ISR}$

- Clearly the modified KKMC agree with ConExc much better and steady.
- The difference between original KKMC and ConExc is much larger and varied when Ecms changed.

Comparison of M(X) between MC generated with KKMC and ConExc

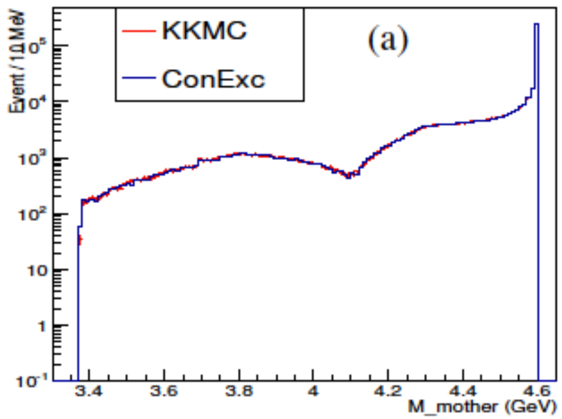
- M(X) is the invariant mass of the system after radiation, or it's the mass of ψ or Y generated

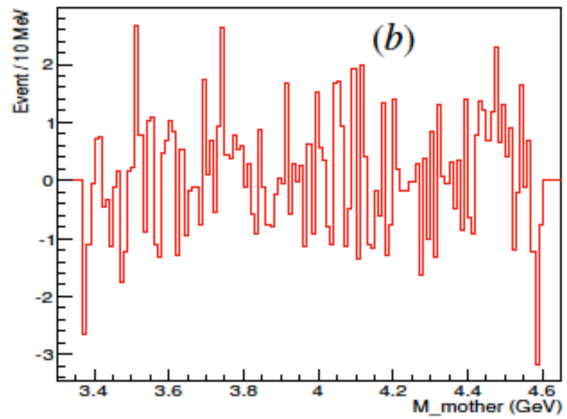
$$\chi_i = \frac{N_{i1} - N_{i2}}{\sqrt{N_{i1} + N_{i2}}}$$

$$\chi^2 = \sum_i \chi_i^2$$

Modified KKMC
And ConExc

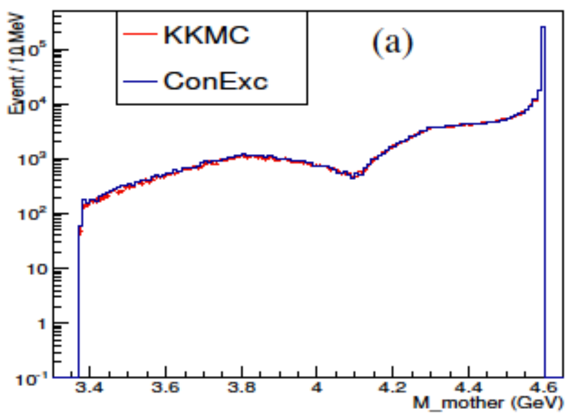
$$\frac{\chi^2}{N_{bin}-1} = 1.09.$$

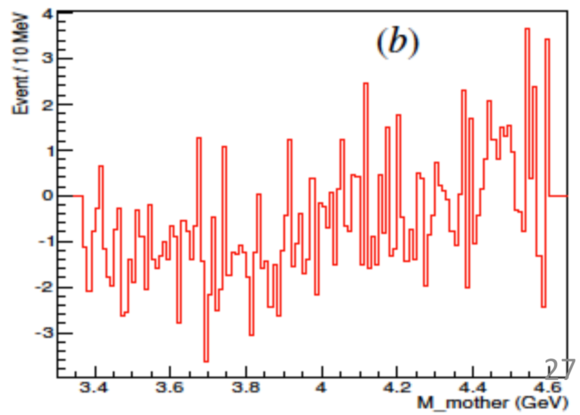




Original KKMC
And ConExc

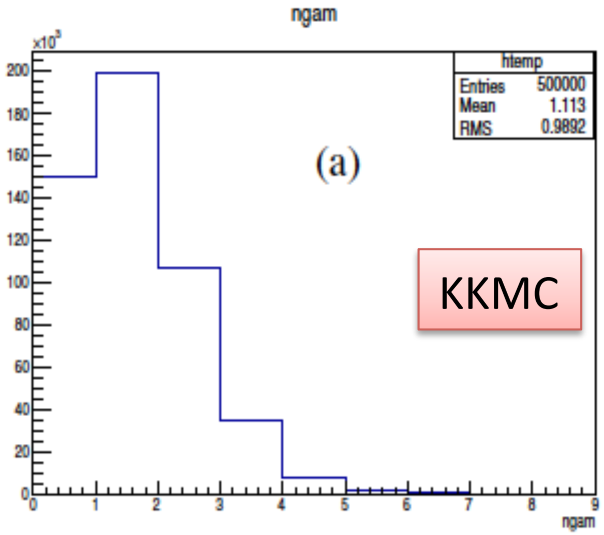
$$\frac{\chi^2}{N_{bin}-1} = 2.15.$$

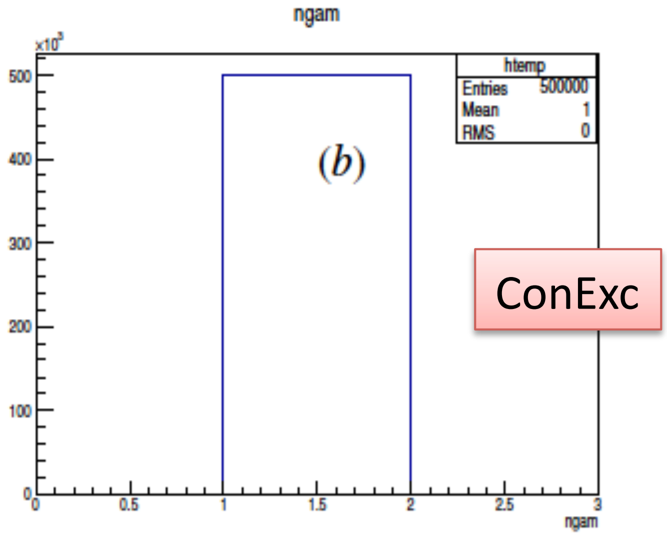


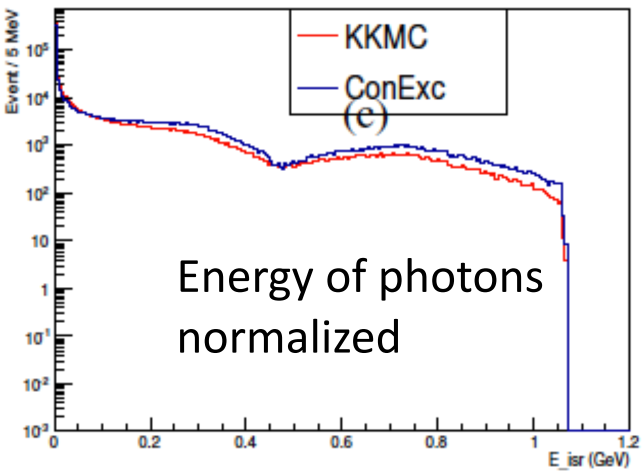


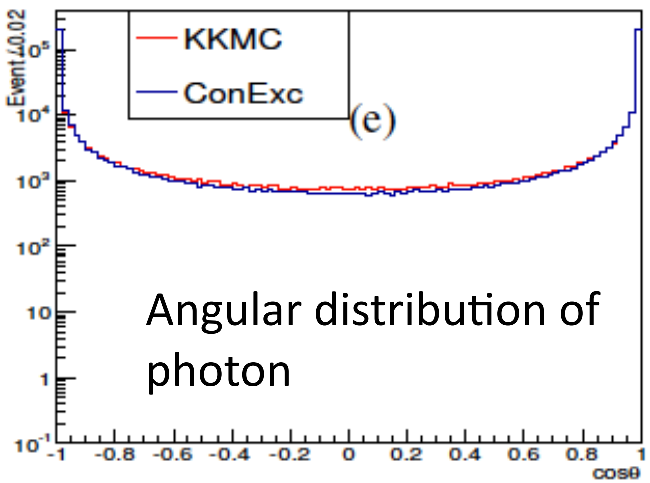
Comparison of other distributions

- The multiplicity of radiated photons

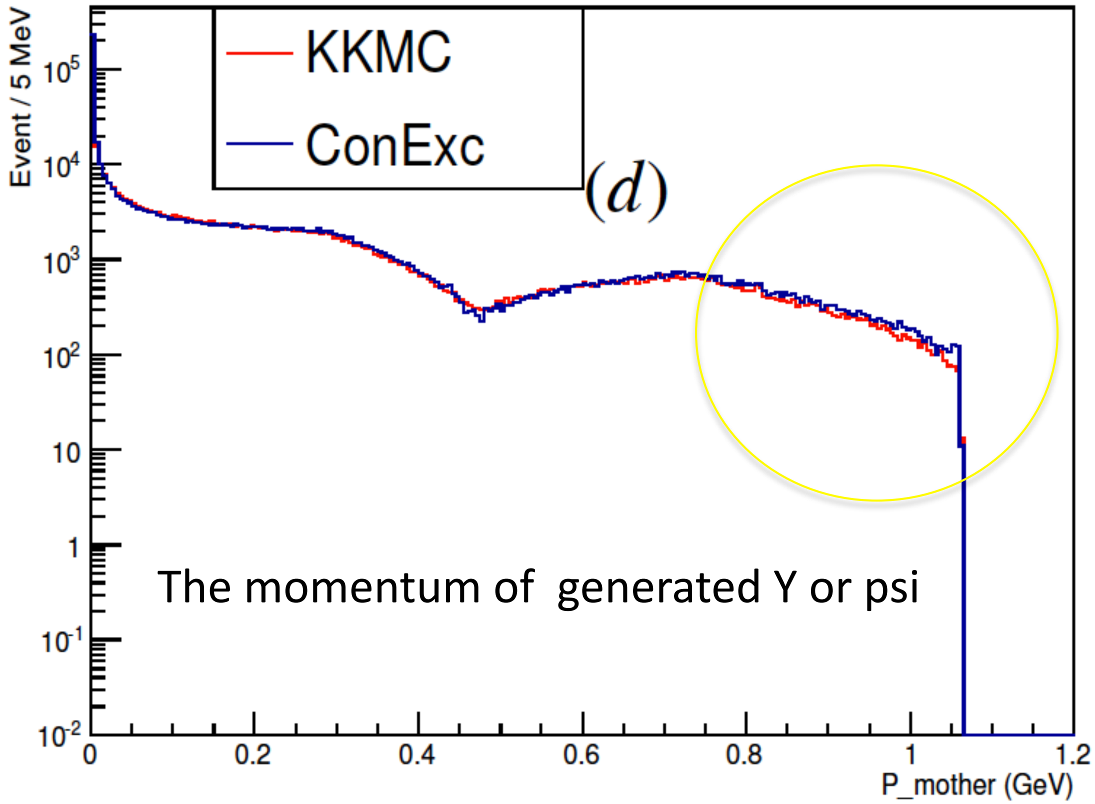








Comparison of other distributions



As most of the photons are along the incoming e+e- direction, and their direction can be opposite, so in KKMC, the momentum of generated Y/psi is slightly smaller than that in ConExc. As in ConExc, all the lost energy are assigned to one single photon. But since most of the photon is very soft, the influence is small for most of the analysis.

Compare the efficiency of MC generate with KKMC and ConExc

- ee→π+π-J/ψ at 4.6GeV is generated and analyzed with 4C kinematic fit
- When calculating the cross section, it's the multiplication of efficiency and $(1+\delta_{ISR})$ that determine the result

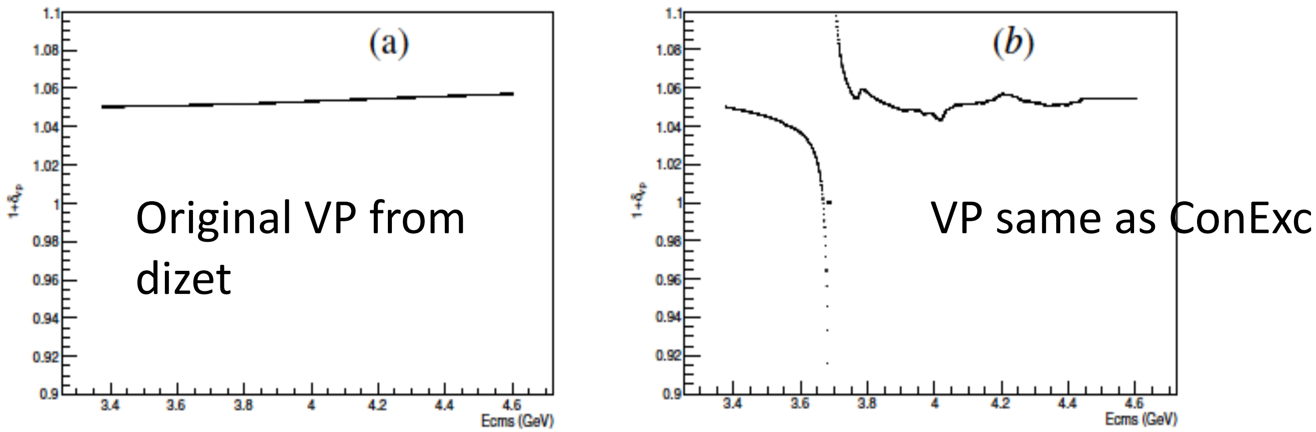
$$\sigma = \frac{N}{Lum \cdot \epsilon \cdot (1 + \delta_{isr})}$$

MC	$1+\delta_{ISR}$	efficiency $\chi^2_{4c} < 200$	$\epsilon * (1+\delta_{sir})$	efficiency $\chi^2_{4c} < 40$	$\epsilon * (1+\delta_{sir})$
KKMC Original	1.3912	147596/500000	0.4107	123330/500000	0.3432
KKMC Modified	1.3753	146040/500000	0.4017	122233/500000	0.3362
ConExc	1.3743	145868/500000	0.4009	122328/500000	0.3362

Modified KKMC agree with ConExc, much better

Summary: What's changed in KKMC?

- The function BornV_Differential() is modified, the weak correction is neglected. All the s-dependent components are thrown.
- The modification is controlled with a mode flag: KKMC.ModeIndexExpXS. These modification is only performed for mode=-2, or mode>=0. Which means only works for the situation when you need to input your own cross section or the existing mode listed in KKMC by Professor Ping RongGang.
- Besides, the vacuum polarization is changed to the same method used in ConExc by searching a lookup table.



How to use KKMC now

```
//**************job options for generator (KKMC)****************
#include "$KKMCROOT/share/jobOptions_KKMC.txt"
KKMC.CMSEnergy = 4.6;
KKMC.BeamEnergySpread=0.00;
KKMC.NumberOfEventPrinted=1;
KKMC.GeneratePsi4260=true;
KKMC.ParticleDecayThroughEvtGen = true;
KKMC.ThresholdCut =  3.37604;
KKMC.RadiationCorrection = 1;
KKMC.TagISR = 1;
KKMC.TagFSR = 1;
KKMC.ModeIndexExpXS = -2;
KKMC.IHVP = 1;
```

ConExc hasn't consider the beam energy spread, you can use the correct Beam spread here.

Must use 4260 as mother particle as usual

Must be smaller than the frist bin of your Input cross section, explained in note

Switch to on/off ISR and FSR

When using mode -2, you need to provide the Input cross section with xs_user.dat file

=0 to turn off vacuum polarization correction, other wise turn on

The jobOption usage is exactly same as original KKMC, nothing changed.

The log file

```
****************************************************************************
*                       KK2f_Finalize  printouts                           *
*       4.60000000                  cms energy total        cmsene    a0 *
*           100026                  total no of events      nevgen    a1 *
*                 ** principal info on x-section **                         *
*       0.00024932  +- 0.00000008  xs_tot MC R-units       xsmc      a1 *
*       1.02337716                  xs_tot     picob.       xSecPb    a3 *
*       0.00033857                  error      picob.       xErrPb    a4 *
*       1.02337716                       (1+del_ISR)*VP      fact     xx *
*       1.05464459                           VP factor       fact     xx *
*       0.97035263                           1+del_ISR       fact     xx *
*       0.00033083                  relative error          erel      a5 *
*       0.11934539                  WTsup, largest WT       WTsup    a10 *
*                        ** some auxiliary info **                          *
*       1.00000000                  xs_born    picobarns     xborn   a11 *
*       0.84715974                  Raw phot. multipl.               === *
*       7.00000000                  Highest phot. mult.              === *
*                          End of KK2f  Finalize                           *
****************************************************************************
```

Now, you can directly read out the $1+\delta_{ISR}$, and vacuum polarization factor and their Multiplication from the simulation job's log file.

No need to turn off ISR to make the divide any more.

Summary

- Generally, the difference between KKMC and ConExc is very small, for most of the analysis, their result are both accurate enough.
- Be careful when the lost energy from radiation is very huge, where the ConExc's single ISR treatment might cause some problem.
- A new KKMC version KKMC-00-00-59 has been checked into CVS
- A note with detailed information has been uploaded to DocDB

http://docbes3.ihep.ac.cn/DocDB/0007/000717/002/kkmc_V2.pdf