Mixed Electroweak-QCD Corrections to $e^+e^- \to H\mu^+\mu^-$

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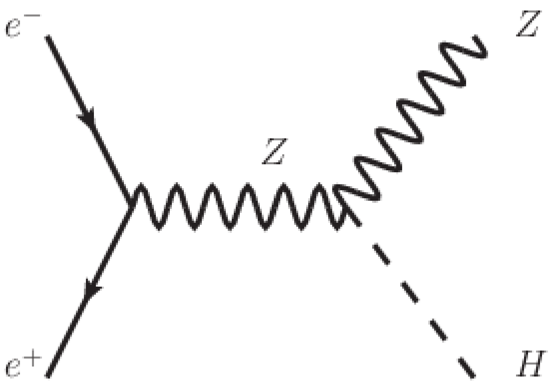
Outline

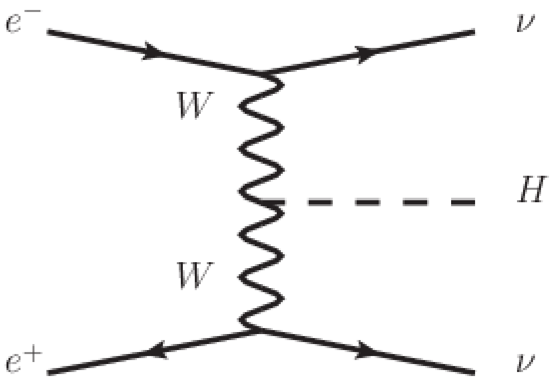
- Motivations
- Renormalization of Electroweak Theory
- Finite Width Effects
- Calculations
- Numerical Results

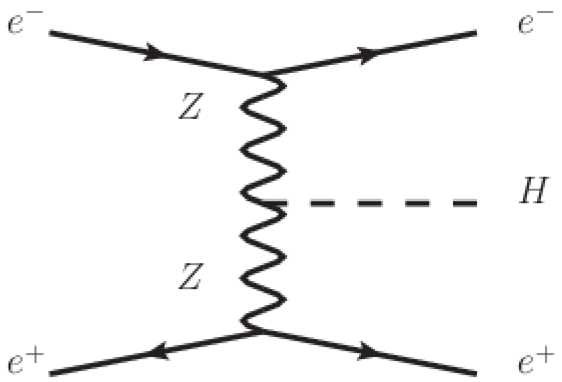
Motivations

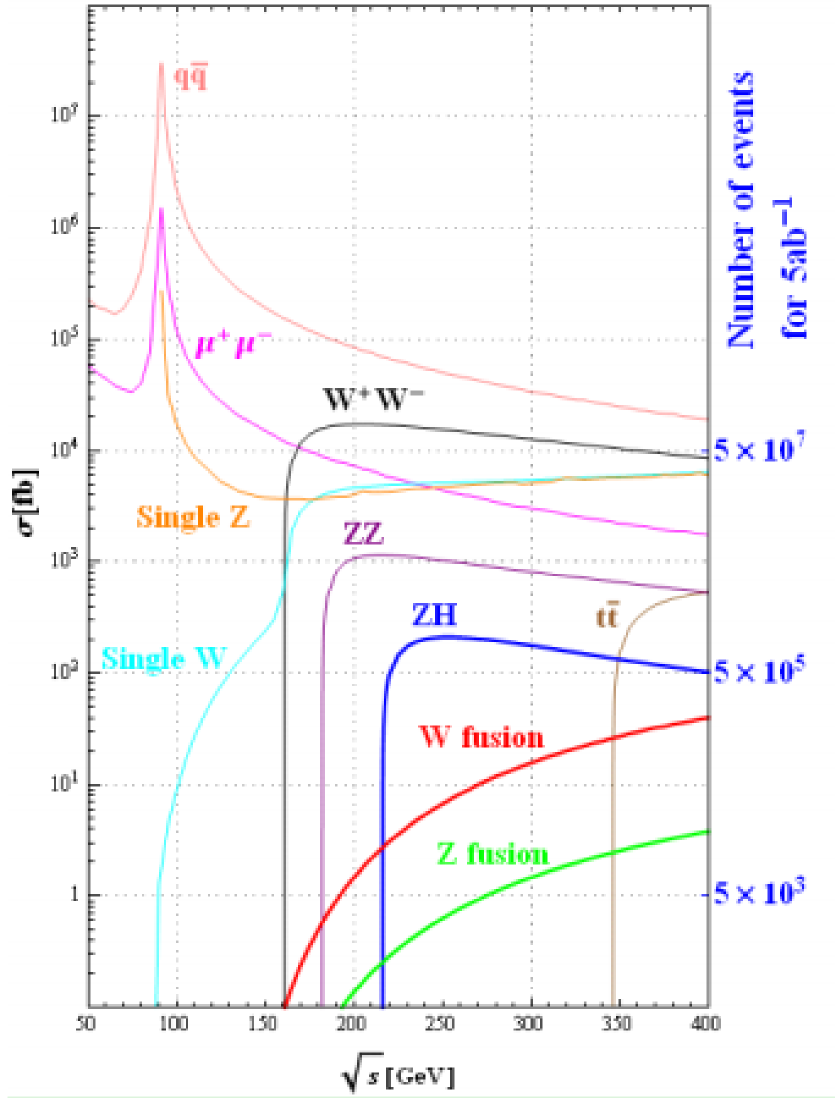
Higgs productions in electron-positron colliders



dominant at CEPC

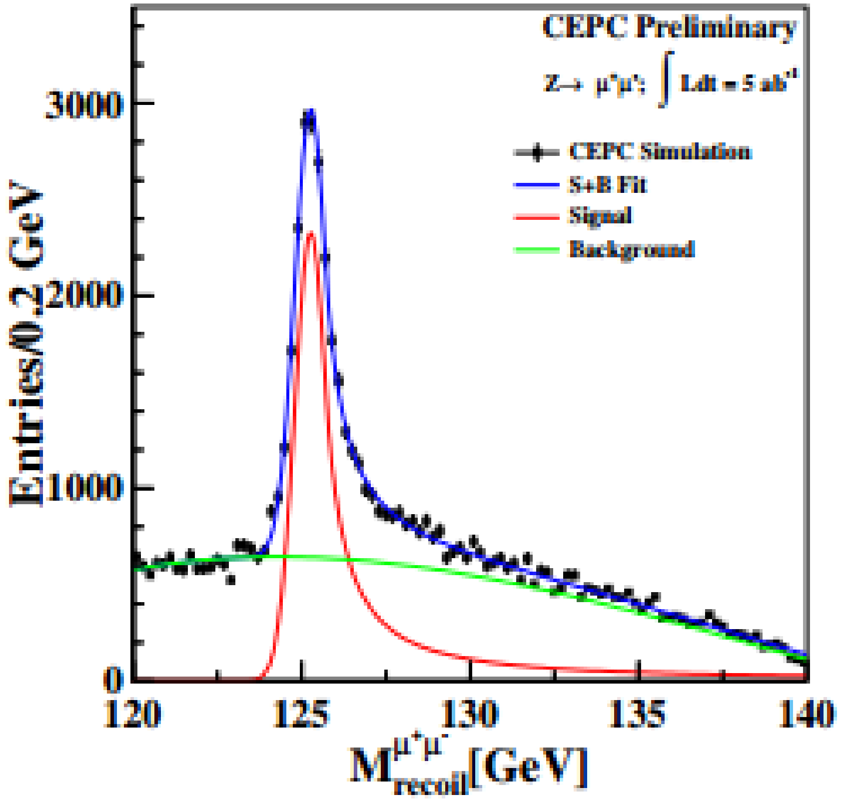






Previous works on $e^+e^- \to H\mu^+\mu^-$

LO	Jones, and Petcov PLB (1979)
ISR	Berends, Kleiss NPB (1985)
NLO $(e^+e^- \to He^+e)$	*Boudjema, et al. PLB (2004)*



(precision at CEPC: 0.9%)

Renormalization of Electroweak Theory

Bohm, and Spiesberger (1986)
Hollik (1990)
Denner (1993)

classical Lagrangian

$$\mathscr{L}_C = \mathscr{L}_F + \mathscr{L}_H + \mathscr{L}_{YM},$$

$$q_- \equiv \begin{pmatrix} u_- \\ d_- \end{pmatrix}$$

$$u_\pm \equiv \omega_\pm u$$

$$d_\pm \equiv \omega_\pm d$$

$$\omega_\pm \equiv \frac{1}{2}(1 \pm \gamma^5)$$

$$\mathscr{L}_F = i\bar{q}\not{D}q - \left(\lambda_u \bar{q}_- \Phi u_+ + \lambda_d \bar{q}_- i\sigma^2 \Phi^* d_+ + h.c.\right),$$

$$\mathscr{L}_H = (D_\mu \Phi)^\dagger D^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{4}(\Phi^\dagger \Phi)^2,$$

$$\mathscr{L}_{YM} = -\frac{1}{4}\left(\partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c\right)^2 - \frac{1}{4}\left(\partial_\mu B_\nu^a - \partial_\nu B_\mu^a\right)^2,$$

where

$$D_\mu \equiv \partial_\mu + i g_1 \frac{Y_W}{2} B_\mu - i g_2 I_W^a W_\mu^a$$

spontaneous symmetry breaking

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \qquad \Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\chi) \end{pmatrix}.$$

fermion mass $m_{u/d} = \frac{1}{\sqrt{2}}\lambda_{u/d} v;$

Higgs mass $M_H = \sqrt{2}\mu.$

unbroken symmetry $Q \equiv I_W^3 + \frac{Y_W}{2}$

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix}\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right)$$

$$s_w = \frac{g_1}{\sqrt{g_1^2 + g_2^2}};$$
$$c_w = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}.$$
$$M_W = \frac{1}{2} g_2 v;$$
$$M_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2} v.$$

free parameters $e, \ \theta_w, \ m_q, \ M_H, \ M_W, \ M_Z$

gauge fixing (`t Hooft gauge)

$$\mathscr{L}_{\text{fix}} = -\frac{1}{2}\left((F^\gamma)^2 + (F^Z)^2 + 2F^+F^-\right),$$

where

$$F^\pm \equiv \frac{1}{\sqrt{\xi^W}}\partial^\mu W^\pm_\mu \mp iM_W\sqrt{\xi^W}\phi^\pm;$$

$$F^Z \equiv \frac{1}{\sqrt{\xi^Z}}\partial^\mu Z_\mu - M_Z\sqrt{\xi^Z}\chi;$$

$$F^A \equiv \frac{1}{\sqrt{\xi^A}}\partial^\mu A_\mu.$$

Feynman gauge:
$\xi = 1$

ghosts

$$\mathscr{L}_{FP} = \sum_{\alpha=A,Z,\pm} \bar{u}^\alpha \frac{\delta F^\alpha}{\delta\theta^\beta} u^\beta,$$

full Lagrangian

$$\mathscr{L} = \mathscr{L}_C + \mathscr{L}_{\text{fix}} + \mathscr{L}_{FP}.$$

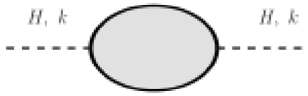
renormalization of couplings

field renormalizations

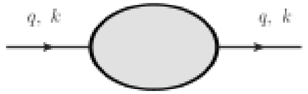
$$e_0 = Z_e e = (1 + \delta Z_e)e,$$
$$M^2_{W,0} = M^2_W + \delta M^2_W,$$
$$M^2_{Z,0} = M^2_Z + \delta M^2_Z,$$
$$M^2_{H,0} = M^2_H + \delta M^2_H,$$
$$m_{q,0} = m_q + \delta m_q,$$
$$s_{w,0} = s_w + \delta s_w,$$
$$c_{w,0} = c_w + \delta c_w.$$

$$W_0^\pm = Z_W^{1/2} W^\pm = (1 + \frac{1}{2}\delta Z_W)W^\pm,$$
$$\begin{pmatrix} A_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} Z^{1/2}_{AA} & Z^{1/2}_{AZ} \\ Z^{1/2}_{ZA} & Z^{1/2}_{ZZ} \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{ZZ} & \frac{1}{2}\delta Z_{ZA} \\ \frac{1}{2}\delta Z_{AZ} & 1 + \frac{1}{2}\delta Z_{AA} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}$$
$$H_0 = Z_H^{1/2} H = (1 + \frac{1}{2}\delta Z_H)H,$$
$$q_{\pm,0} = Z^{1/2}_{q_\pm} q_\pm = (1 + \frac{1}{2}\delta Z_{q_\pm})q_\pm.$$

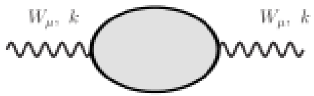
one-particle irreducible diagrams



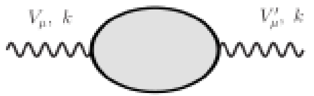
$$=i\Sigma^{H}(k^2),$$



$$=i\left[\not{k}\omega_{-}\Sigma^{q^{-}}(k^2)+\not{k}\omega_{+}\Sigma^{q^{+}}(k^2)+m_{q,0}\Sigma^{q,S}(k^2)\right],$$

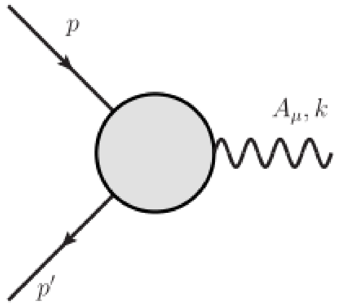


$$=-i\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\Sigma_T^{W}(k^2)-i\frac{k_\mu k_\nu}{k^2}\Sigma_L^{W}(k^2),$$



$$=-i\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\Sigma_T^{VV'}(k^2)-i\frac{k_\mu k_\nu}{k^2}\Sigma_L^{VV'}(k^2),$$

$V=A,Z$



$$=-iQ_q\bar{u}(p')\Gamma^{q}_{0,\mu}(p,p')u(p).$$

full propagators

$$D_0^H(k) \equiv \int d^4x e^{-ik\cdot x} \langle 0| T\{H_0(0)H_0(x)\}|0\rangle$$
$$= \frac{i}{k^2 - M_{H,0}^2 + \Sigma^H(k^2)},$$

$$D_{0,\mu\nu}^W(k) \equiv \int d^4x e^{-ik\cdot x} \langle 0| T\{W_{0,\mu}(0)W_{0,\nu}(x)\}|0\rangle$$
$$= \frac{-ig_{\mu\nu}}{k^2 - M_{W,0}^2 + \Sigma_T^W(k^2)} + \frac{i}{k^2 - M_{W,0}^2 + \Sigma_T^W(k^2)} \frac{k_\mu k_\nu}{k^2} \times \frac{k^2 - \xi^W\left(k^2 + \Sigma_T^W(k^2) - \Sigma_L^W(k^2)\right)}{k^2 - \xi^W\left(M_{W,0}^2 - \Sigma_L^W(k^2)\right)},$$

$$D_0^{q_\pm q_\pm}(k) \equiv \int d^4x e^{-ik\cdot x} \langle 0| T\{q_{\pm,0}(0)\bar{q}_{\pm,0}(x)\}|0\rangle$$
$$= \frac{i}{\left(1 + \Sigma^{q_\pm}(k^2)\right) \not{k} - \frac{m_{q,0}^2\left(1 - \Sigma^{q,S}(k^2)\right)^2}{\left(1 + \Sigma^{q_\mp}(k^2)\right)\not{k}}},$$

$$D_{0,\mu\nu}^{VV'}(k) \equiv \int d^4x e^{-ik\cdot x} \langle 0| T\left\{V_{0,\mu}(0)V'_{0,\nu}(x)\right\}|0\rangle$$
$$= -ig_{\mu\nu}\begin{pmatrix} k^2 + \Sigma_T^{AA}(k^2) & \Sigma_T^{AZ}(k^2) \\ \Sigma_T^{AZ}(k^2) & k^2 - M_{Z,0} + \Sigma_T^{ZZ}(k^2) \end{pmatrix}^{-1}$$

+logitudinal parts

renormalization conditions:

a. Physical masses equal to the real parts of the poles of full propagators.
b. Residues of renormalized propagators equal to one.
c. The renormalized charge is defined as the full $ee\gamma$-coupling in the Thomson limit.
d. $c_w = M_W/M_Z$.

counter terms:

$$\delta M_W^2 = \mathrm{Re}\Sigma_T^W(M_W^2), \qquad \delta Z_W = -\mathrm{Re}\left.\frac{\partial \Sigma_T^W(k^2)}{\partial k^2}\right|_{k^2=M_W^2},$$

$$\delta M_H^2 = \mathrm{Re}\Sigma^H(M_H^2), \qquad \delta Z_H = -\mathrm{Re}\left.\frac{\partial \Sigma^H(k^2)}{\partial k^2}\right|_{k^2=M_H^2},$$

$$\delta m_q = \frac{m_q}{2}\mathrm{Re}\left[\Sigma^{q-}(m_q^2) + \Sigma^{q+}(m_q^2) + 2\Sigma^{q,S}(m_q^2)\right],$$

$$\delta Z^{q^\pm} = -\mathrm{Re}\Sigma^{q\pm}(m_q^2) - m_q^2\frac{\partial}{\partial k^2}\mathrm{Re}\left.\left[\Sigma^{q-}(k^2) + \Sigma^{q+}(k^2) + 2\Sigma^{q,S}(k^2)\right]\right|_{k^2=m_q^2}.$$

$$\delta M_Z^2 = \mathrm{Re}\Sigma_T^{ZZ}(M_Z^2), \qquad \delta Z_Z = -\mathrm{Re}\left.\frac{\partial \Sigma_T^{ZZ}(k^2)}{\partial k^2}\right|_{k^2=M_Z^2},$$

$$\delta Z_{AZ} = -2\mathrm{Re}\frac{\Sigma_T^{AZ}(M_Z^2)}{M_Z^2}, \qquad \delta Z_{ZA} = 2\frac{\Sigma_T^{AZ}(0)}{M_Z^2},$$

$$\delta Z_{AA} = -\left.\frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2}\right|_{k^2=0}.$$

$$\delta c_w = \frac{c_2}{2}\left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2}\right),$$

$$\delta s_w = -\frac{c_w}{s_w}\delta c_w.$$

$$\delta Z_e \approx -\frac{1}{2}\left(\delta Z_{AA} + \frac{s_w}{c_w}\delta Z_{ZA}\right).$$

resummation of leading logarithms

light fermions

$$\alpha \to \alpha(s) \equiv \frac{\alpha}{1-(\Delta\alpha(s))_l}$$

$$\Delta\alpha_l(s) \equiv \Pi_l^{AA}(0) - \Pi_l^{AA}(s)$$

$$\Pi^{VV'}(k^2) \equiv \frac{\Sigma_T^{VV'}(k^2)}{k^2}.$$

$\alpha(M_Z^2)$ scheme:
$s = M_Z^2$

top quark

$$\alpha \to \alpha_{G_\mu} \equiv \frac{\sqrt{2}}{\pi} G_\mu M_W^2 s_w^2 = \frac{\alpha}{1-\Delta r}$$

Finite Width Effects

problems of finite width

double counting
unitarity
gauge invariance

gauge invariant schemes

pole scheme Veltman (1963), Stuart (1991), Sirlin (1991)
factorization scheme Kurihara, et al. (1995), Baur, et al. (1992)
fermion-loop scheme Beenakker et al. (1996), (1997)
unstable-particle effective theory Beneke, et al. (2004), Beneke (2015)
complex mass scheme Denner, et al. (1999), Denner, and Dittmaier (2006)

Calculations

kinematics

$$e^+(p_+) + e^-(p_-) \to \mu^+(k_1) + \mu^-(k_2) + H(k_3)$$

amplitude

$$\mathcal{M} = \mathcal{M}_{0,0} + \alpha\mathcal{M}^{(1,0)} + \alpha\alpha_s\mathcal{M}^{(1,1)} + \dots$$

cross section

$$\sigma = \frac{1}{8s_0}\int [dk_1][dk_2][dk_3]\sum_{\text{pol}}|\mathcal{M}|^2(2\pi)^4\delta^{(4)}(P - \sum_{i=1}^{3}k_i)$$

$$= \frac{1}{16\pi s_0}\int ds_{12}\int d\Pi_1\int d\Pi_2\sum_{\text{pol}}|\mathcal{M}|^2$$

$$d\Pi_1 \equiv [dQ][dk_3](2\pi)^4\delta^{(4)}(P - Q - k_3) = \frac{x}{32\pi^2}\times d\cos(\theta)d\phi,$$

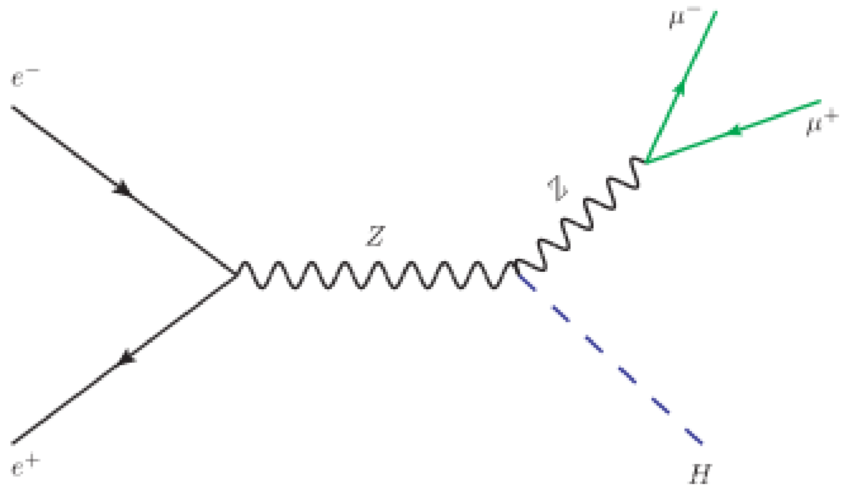
$$d\Pi_2 \equiv [dk_1][dk_2](2\pi)^4\delta^{(4)}(Q - k_1 - k_2) = \frac{1}{32\pi^2}d\cos(\theta')d\phi',$$

$$[dk_i] \equiv \frac{d^3k_i}{(2\pi)^3}\frac{1}{2k_i^0}$$

$$s_{ij} = (k_i + k_j)^2$$

$$y = \frac{s_{12}}{s_0}$$

$$x = \sqrt{\left(1 - y\frac{M_H^2}{s}\right)^2 - 4y\frac{M_H^2}{s}}$$



LO amplitude

$$\mathcal{M}^{(0,0)} = -\frac{e^3 M_Z}{c_w s_w} \frac{1}{(s_0 - M_Z^2)(s_{12} - M_Z^2)} \bar{v}_e \Gamma_{Z,\nu} u_e \bar{u}_\mu \Gamma_Z^\nu v_\mu,$$

$$\Gamma_V^\mu = g_V^- \gamma^\mu \frac{1-\gamma^5}{2} + g_V^+ \gamma^\mu \frac{1+\gamma^5}{2}$$

$$g_Z^- = \frac{s_w}{c_w} - \frac{1}{2 s_w c_w}, \quad g_Z^+ = \frac{s_w}{c_w}, \quad g_\gamma^- = g_\gamma^+ = 1$$

factorization scheme

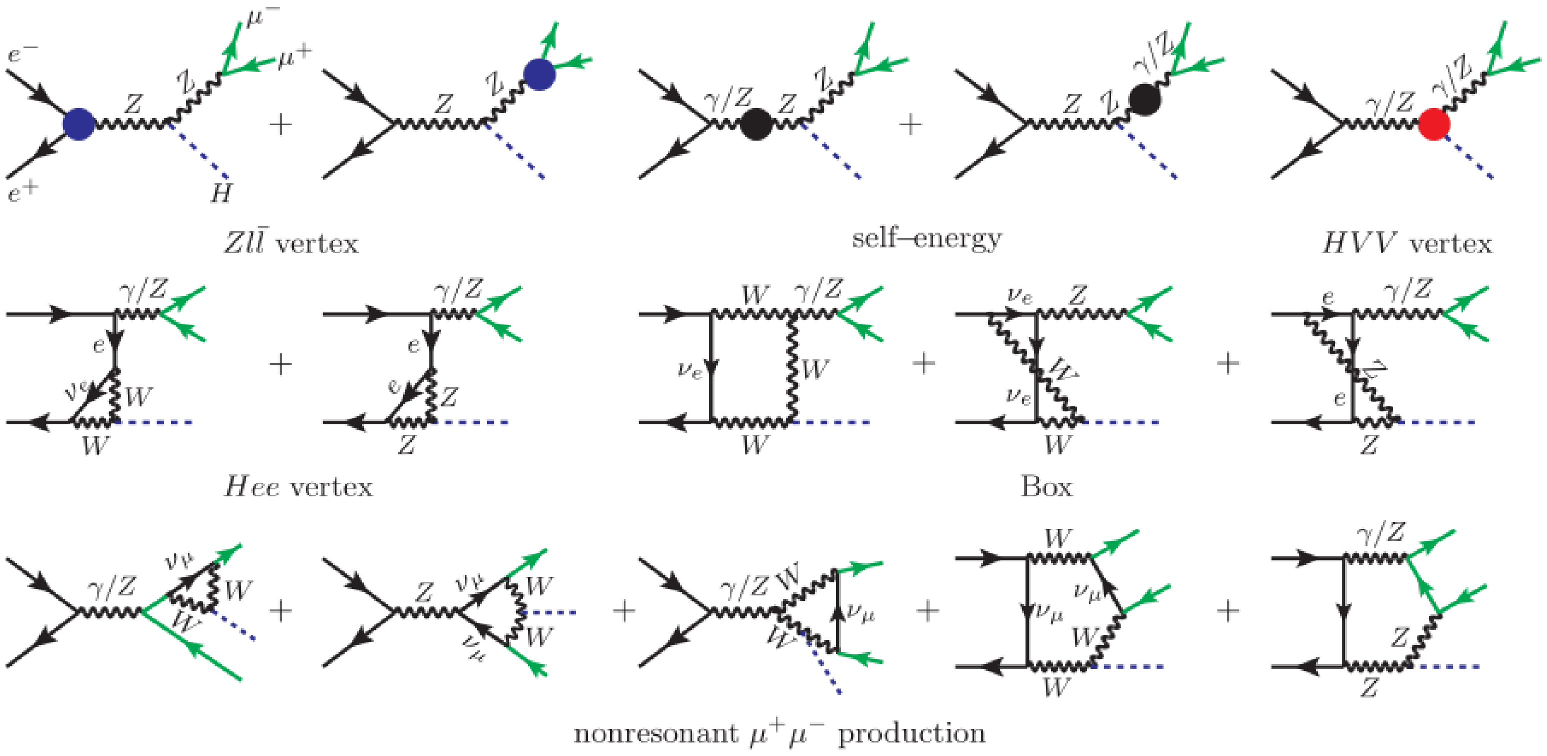
$$\mathcal{M}'^{(0,0)} = \frac{s_{12} - M_Z^2}{s_{12} - M_Z^2 + iM_Z\Gamma_Z}\mathcal{M}^{(0,0)} \equiv \mathcal{F}\mathcal{M}^{(0,0)}$$

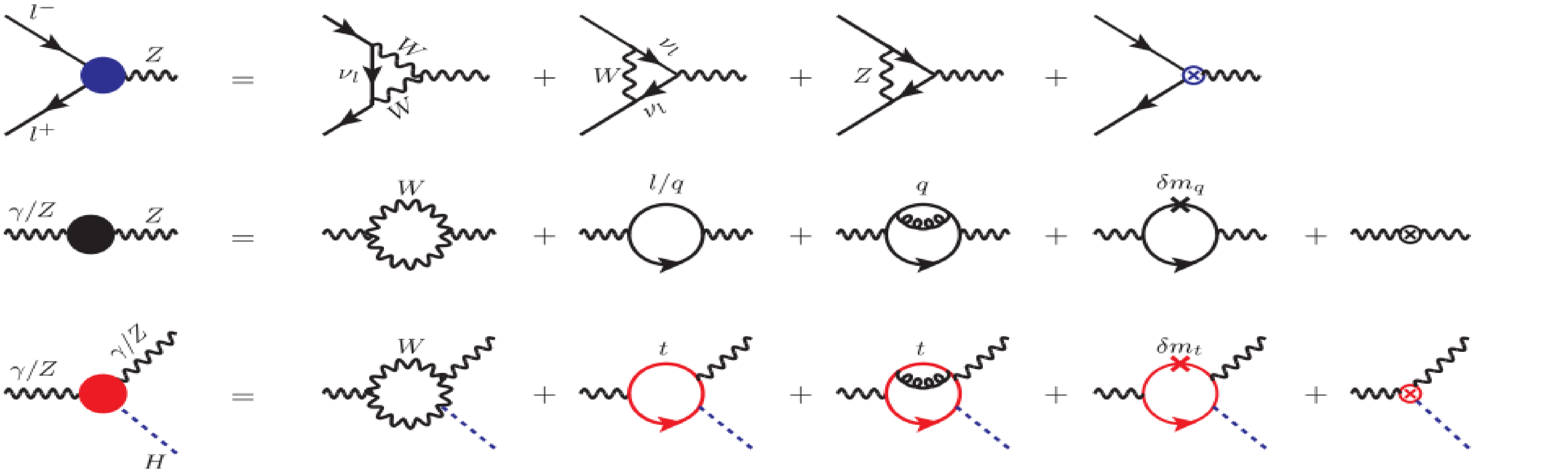
LO cross section

$$\sigma^{(0,0)} = \kappa \int_0^{y_0} dy d\cos(\theta) \frac{x\left(x^2 \sin(\theta)^2 + 8y\right)|\mathcal{F}|^2}{\left(y - M_Z^2/s\right)^2}$$
$$= \frac{4}{3}\kappa \int_0^{y_0} dy \frac{x\left(x^2 + 12y\right)|\mathcal{F}|^2}{\left(y - M_Z^2/s\right)^2},$$

$$\kappa \equiv \frac{\alpha^3 M_Z^2}{192 c_w^2 s_w^2} \frac{\left({g_Z^-}^2 + {g_Z^+}^2\right)^2}{\left(s_0 - M_Z^2\right)^2}, \quad y_0 = \left(1 - \frac{m_H}{\sqrt{s}}\right)^2$$

representative diagrams





NLO amplitude

$$\mathcal{M}'^{(1,0)} = \mathcal{F}\mathcal{M}^{(1,0)} + i\frac{\mathrm{Im}\left\{\hat{\Sigma}_{ZZ}^{(1,0)}(M_Z)\right\}}{\alpha\left(m_Q^2 - M_Z^2\right)}\mathcal{M}'^{(0,0)}.$$

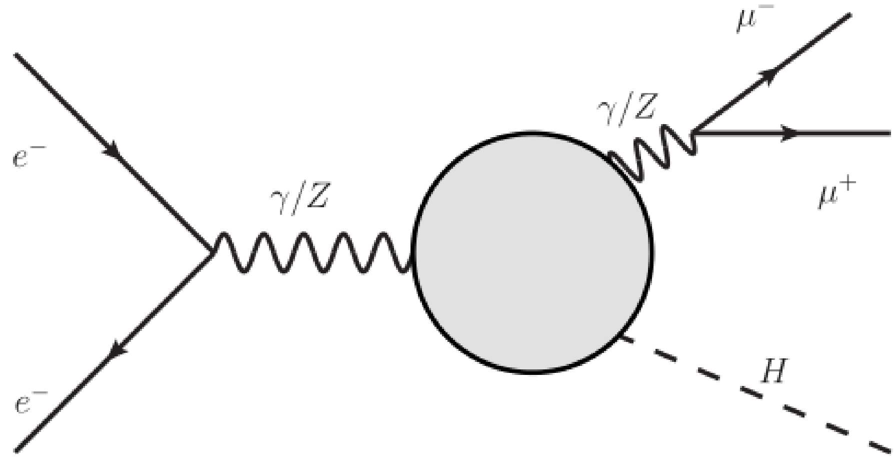
purely imaginary

NNLO amplitude

$$\mathcal{M}_{V_1V_2}^{(1,1)} = -e^2\frac{1}{(s^2 - m_{V_1}^2)(ys - m_{V_2}^2)}\bar{v}_e\Gamma_{V_1,\mu}u_e T_{V_1V_2H}^{\mu\nu}(P,Q)\,\bar{u}_\mu\Gamma_{V_2,\nu}v_\mu.$$

$$\mathcal{M}'^{(1,1)} = \mathcal{F}\mathcal{M}^{(1,1)} + i\frac{\mathrm{Im}\left\{\Sigma_{ZZ}^{(1,1)}(M_Z)\right\}}{\alpha\alpha_s\left(ys - M_Z^2\right)}\mathcal{M}^{(0,0)}.$$

$$T_{V_1V_2H}^{\mu\nu} = \frac{T_{V_1V_2H,1}}{s}P^\mu P^\nu + \frac{T_{V_1V_2H,2}}{s}Q^\mu Q^\nu + \frac{T_{V_1V_2H,3}}{s}P^\mu Q^\nu + \frac{T_{V_1V_2H,4}}{s}Q^\mu P^\nu + T_{V_1V_2H,5}g^{\mu\nu} + \frac{T_{V_1V_2H,6}}{s}\epsilon^{\mu\nu\alpha\beta}P^\alpha Q^\beta.$$



NNLO cross section

$$\sigma^{(1,1)} = \frac{\alpha^3 s^2 M_Z}{18c_w s_w}\sum_{V_1,V_2=Z,\gamma}\frac{\left(g_{V_1}^- g_Z^- + g_{V_1}^+ g_Z^+\right)\left(g_{V_2}^- g_Z^- + g_{V_2}^+ g_Z^+\right)}{\left(s - M_Z^2\right)\left(s - m_{V_1}^2\right)} \times \int dy\frac{xy\left|\mathcal{F}\right|^2}{\left(ys - M_Z^2\right)\left(ys - m_{V_2}^2\right)}\mathcal{T}_{V_1V_2}(y),$$

$$\mathcal{T}_{V_1V_2}(y) = \frac{1}{8e}\left(\frac{1}{y} - \frac{M_H^2}{ys} + 1\right)x^2 T_{V_1V_2H,4} + \frac{1}{e}\left(\frac{x^2}{4y} + D - 1\right)T_{V_1V_2H,5}.$$

numerical results

inputs

$$\alpha = 7.2973525664(17) \times 10^{-3};$$

$$\alpha_s(M_Z) = 0.1182;$$

$$G_\mu = 1.1663787(6) \times 10^{-5}\text{GeV}^{-2};$$

$$m_t = 174.2 \pm 1.4\text{GeV};$$

$$M_Z = 91.1876(21)\text{GeV};$$

$$M_W = 80.385(15)\text{GeV};$$

$$\Gamma_Z = 2.4952 \pm 0.0023\text{GeV};$$

$$\left(\Delta\alpha(M_Z^2)\right)_l = 0.05906.$$

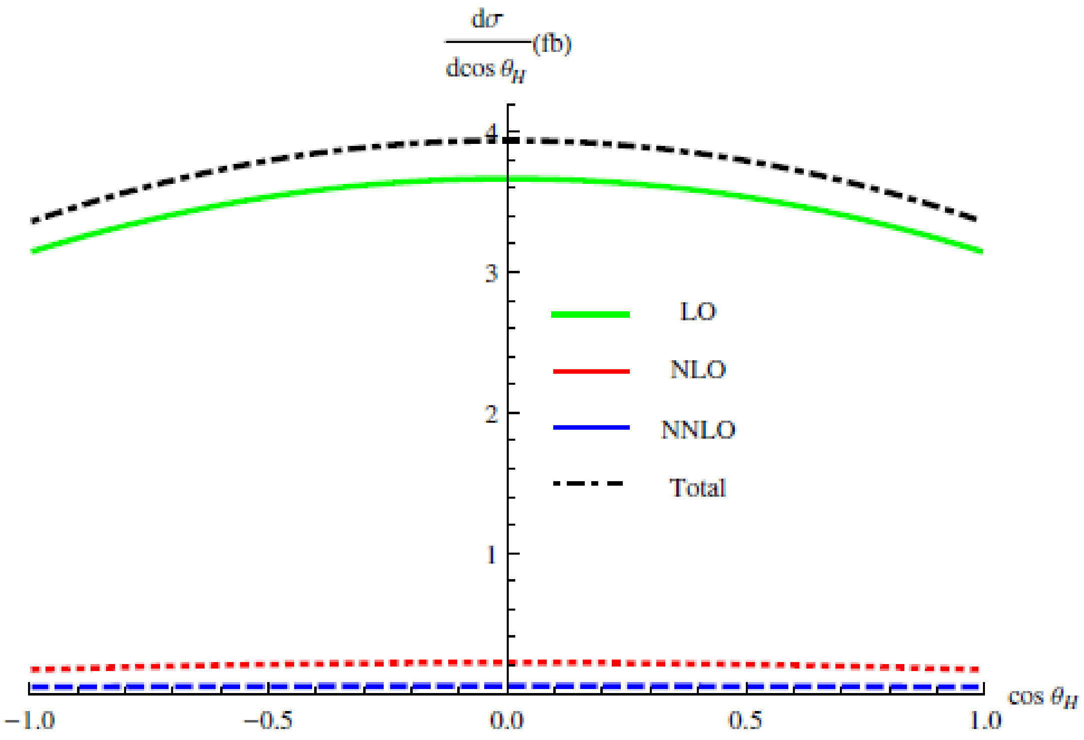


FIG. 5: Differential cross section at $\sqrt{s} = 240\text{GeV}$

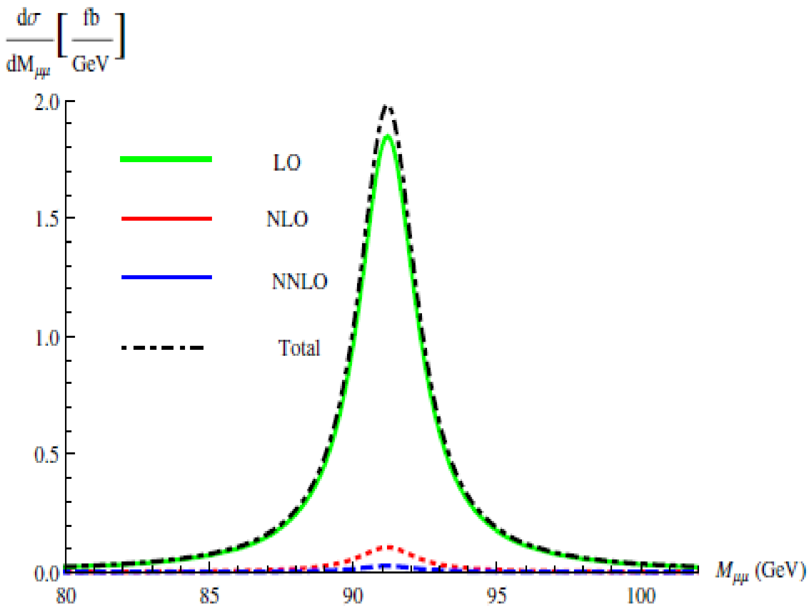


FIG. 6: Spectrum of $\mu^+\mu^-$ at $\sqrt{s} = 240\text{GeV}$, where $M_{\mu\mu}$ is the inavariant mass of $\mu^+\mu^-$.

$d\sigma/dy$(fb) \ m(GeV)		50	70	80	85	90	91	92	95	100	110	σ
LO		0.66	2.39	8.03	24.45	309.02	570.98	407.45	53.27	9.66	1.31	6.9828 (fb)
NLO	resonant	0.04	0.14	0.47	1.42	17.78	32.82	23.39	3.05	0.55	0.07	0.4015 (fb)
	nonresonant	65	39	22	12	1	0	-0	-7	-16	-24	8.5 (10^{-4}fb)
NNLO		0.01	0.04	0.13	0.35	4.54	8.37	5.97	0.79	0.15	0.02	0.103 (fb)

TABLE I: Differential cross section over $\mu^+\mu^-$ invariant mass m, and the total cross sections of $e^+e^- \to \mu^+\mu^- H$ at $\sqrt{s} =$ 240GeV. The strong coupling constant $\alpha_s(\sqrt{s}/2) = 0.1135$ and $m_+ = \sqrt{s} - M_H$ represents the upper bound for m.

$\sqrt{s}$	schemes	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)
240	$\alpha(0)$	$6.983^{+0.023}_{-0.023}$	$7.385^{+0.037}_{-0.037}$	$7.488^{+0.036+0.004}_{-0.036-0.009}$
	$\alpha(M_Z^2)$	$8.382^{+0.028}_{-0.027}$	$7.317^{+0.037}_{-0.036}$	$7.448^{+0.036+0.005}_{-0.035-0.011}$
	G_μ	$7.772^{+0.004}_{-0.004}$	$7.527^{+0.016}_{-0.017}$	$7.554^{+0.017+0.001}_{-0.017-0.002}$
250	$\alpha(0)$	$7.036^{+0.023}_{-0.023}$	$7.424^{+0.037}_{-0.037}$	$7.527^{+0.037+0.005}_{-0.037-0.009}$
	$\alpha(M_Z^2)$	$8.446^{+0.028}_{-0.028}$	$7.350^{+0.037}_{-0.036}$	$7.481^{+0.037+0.006}_{-0.037-0.011}$
	G_μ	$7.831^{+0.004}_{-0.004}$	$7.564^{+0.017}_{-0.017}$	$7.591^{+0.017+0.001}_{-0.016-0.002}$

TABLE II: Total cross section at 240GeV and 250GeV.

	LO (fb)	$\mathcal{O}(\alpha)$(fb)	$\mathcal{O}(\alpha\alpha_s)$ (fb)
$\sigma(\mu^+\mu^- H)$	6.983	0.402	0.103
$\tilde{\sigma}(\mu^+\mu^- H)$	7.241	0.416	0.103

TABLE III: Narrow-width approximation for $\sigma(\mu^+\mu^- H)$ at $\sqrt{s} = 240$GeV in $\alpha(0)$ scheme.