Heavy Quarkonium Associated Production and Multi Parton Interaction

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Outline

Introduction

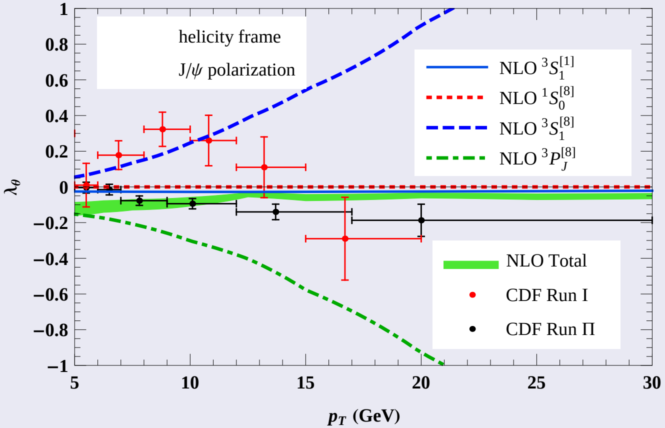
Quarkonium productions

Quarkonium production have been studied by

1. Kuang-Ta Chao group
2. Yu Jia group
3. B. A. Kniehl group
4. Cong-Feng Qiao group
5. Jian-Xiong Wang group
6. ...

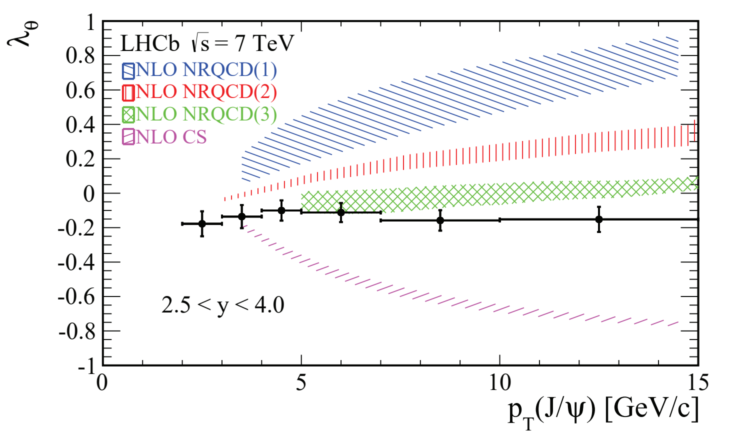
Quarkonium productions

NLO J/ψ polarization at CDF, arXiv:1201.2675



Quarkonium productions

NLO J/ψ at LHCb, Chao/Wang/Kniehl, 1506.03981

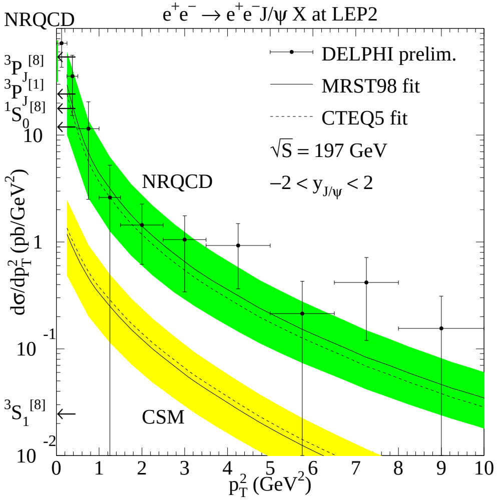


Long distance matrix elements (LDMEs)

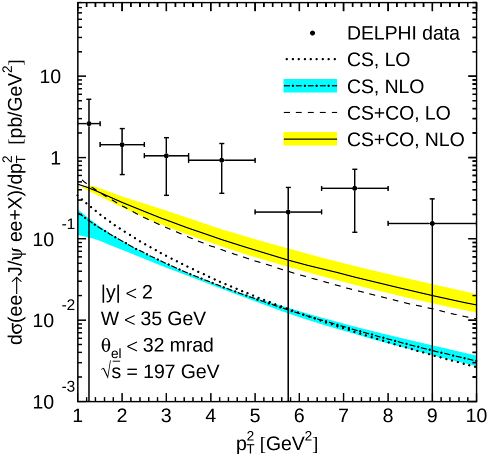
The LDMEs of J/ψ extracted from the experimental $J/\psi/\eta_c$ hadronic production by five theory groups in unit of 10^{-2} GeV^3 (1105.0820 , 1201.2675, 1205.6682, 1403.3612, 1412.0508)

	$\langle 0\vert O^{J/\psi}(^1S_0^8)\vert 0\rangle$	$\frac{\langle 0\vert O^{J/\psi}(^3P_0^8)\vert 0\rangle}{m_c^2}$	$M^{J/\psi}_{3.9\pm 0.8}$
Kniehl	4.97 ± 0.44	-0.716 ± 0.089	2.2 ± 0.8
Chao, set1	8.9 ± 0.98	0.56 ± 0.21	11.1 ± 0.4
set2	0	2.4	9.4 ± 1.9
set3	11	0	11
Wang	9.7 ± 0.9	-0.95 ± 0.25	6.0 ± 1.5
Bodwin	9.9 ± 2.2	0.49 ± 0.45	11.8 ± 2.8
Zhang	$0.44 \sim 1.13$	1.7 ± 0.5	7.4 ± 2.4

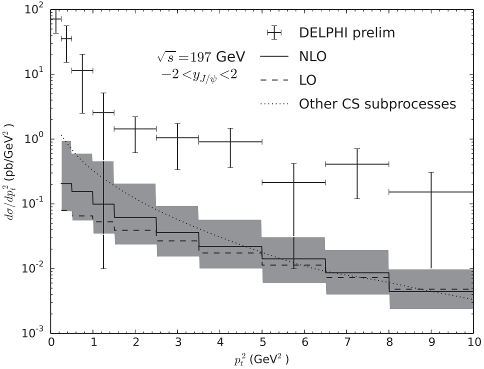
LO $e^+e^- \to e^+e^- J/\psi + X$ at LEP, hep-ph/0112259



NLO $e^+e^- \to e^+e^- J/\psi + X$ at LEP, 1105.0820



NLO $e^+e^- \to e^+e^- J/\psi + c\bar{c} + X$ at LEP, 1608.06231



CO LDMEs, 1212.2037

	Butenschoen, Kniehl[18]	Gong, Wang, Wan, Zhang[53]	Chao, Ma, Shao, Wang, Zhang[52] default set	set 2	set 3
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle$	1.32 GeV^3	1.16 GeV^3	1.16 GeV^3	1.16 GeV^3	1.16 GeV^3
$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$	0.0497 GeV^3	0.097 GeV^3	0.089 GeV^3	0	0.11 GeV^3
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$	0.0022 GeV^3	-0.0046 GeV^3	0.0030 GeV^3	0.014 GeV^3	0
$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$	-0.0161 GeV^5	-0.0214 GeV^5	0.0126 GeV^5	0.054 GeV^5	0
$\langle \mathcal{O}^{\psi'}(^3S_1^{[1]}) \rangle$		0.758 GeV^3			
$\langle \mathcal{O}^{\psi'}(^1S_0^{[8]}) \rangle$		-0.0001 GeV^3			
$\langle \mathcal{O}^{\psi'}(^3S_1^{[8]}) \rangle$		0.0034 GeV^3			
$\langle \mathcal{O}^{\psi'}(^3P_0^{[8]}) \rangle$		0.0095 GeV^5			
$\langle \mathcal{O}^{\chi_0}(^3P_0^{[1]}) \rangle$		0.107 GeV^5			
$\langle \mathcal{O}^{\chi_0}(^3S_1^{[8]}) \rangle$		0.0022 GeV^3			

A constraint of LDMEs from e^+e^- at B factories

A constraint of LDMEs can be get through $e^+e^- \to J/\psi + X$ at $\mathcal{O}(\alpha_s + v^2)$, 1409.2293 (EPJC77 (2017), 597)

1. $M_k^{J/\psi} = \langle 0|\mathcal{O}(^1S_0^8)|0\rangle + k\langle 0|\mathcal{O}(^3P_0^8)|0\rangle/m_c^2)$

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3. $\sqrt{s} = 4.6 - 5.6$ GeV: If $\sigma[J/\psi^{CO}] = \sigma[J/\psi\pi^+\pi^-] \sim 10$ pb, $M_{11\pm3}^{J/\psi} = (2 \pm 1) \times 10^{-2}\ \mathrm{GeV}^3$

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4. $\langle 0|\mathcal{O}(^3P_0^8)|0\rangle/m_c^2 \sim (-0.1 \pm 0.2) \times 10^{-2}\ \mathrm{GeV}^3$ and $\langle 0|\mathcal{O}(^1S_0^8)|0\rangle \sim (3 \pm 2) \times 10^{-2}\ \mathrm{GeV}^3$

LDMEs from e^+e^- and pp

NLO Color octet mechanism can not explain J/ψ production and polarization at e^+e^- and pp colliders with a set of universal LDMEs

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2. Kniehl's and Wang's LDMEs will give $\sigma[e^+e^- \to J/\psi^{CO}] < 0$ at $\sqrt{s} = 4.6 - 5.6$ GeV.
3. Chao's and Bodwin's LDMEs will give $\sigma[e^+e^- \to J/\psi^{CO}] \sim 2$ pb, which is about a factor of 5 larger than $\sigma[J/\psi + LH] = 0.43$ pb at $\sqrt{s} = 10.6$ GeV (0901.2775).

Quarkonium production and double parton scattering

Many quarkonium associated production processes seems to be dominant by Double-Parton Scattering (DPS).

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2. $J/\psi + charm$ and $\Upsilon + charm$ (LHCb, arXiv:1205.0975, 1510.05949)

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3. $J/\psi + J/\psi$ (D0, arXiv:1406.2380; CMS, arXiv:1406.0484)

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3. $J/\psi + J/\psi$ (D0, arXiv:1406.2380; CMS, arXiv:1406.0484)
4. $\Upsilon + J/\psi$ (D0, arXiv:1511.02428)

Multi parton scattering

The inclusive cross section to produce *n* hard particles in hadronic colliders is a convolution of generalized *n*-parton distribution functions (PDF) and elementary partonic cross sections summed over all involved partons,

$$\begin{aligned}
&\sigma^{\mathrm{NPS}}_{hh'\to a_1\ldots a_n} = \\
&\left(\frac{m}{n!}\right) \sum_{i_1,..,i_n,i'_1,..,i'_n} \int \Gamma^{i_1\ldots i_n}_{h}(x_1,..,x_n;\mathbf{b_1},..,\mathbf{b_n};Q_1^2,..,Q_n^2) \\
&\times \hat{\sigma}^{i_1 i'_1}_{a_1}(x_1,x'_1,Q_1^2)\cdots\hat{\sigma}^{i_n i'_n}_{a_n}(x_n,x'_n,Q_n^2) \\
&\times \Gamma^{i'_1\ldots i'_n}_{h'}(x'_1,\ldots,x'_n;\mathbf{b_1}-\mathbf{b},\ldots,\mathbf{b_n}-\mathbf{b};Q_1^2,\ldots,Q_n^2) \\
&\times dx_1\ldots dx_n\, dx'_1,\ldots,dx'_n\, d^2b_1,\ldots,d^2b_n\, d^2b.
\end{aligned} \tag{1}$$

Double parton scattering and Single parton scattering

SPS and DPS

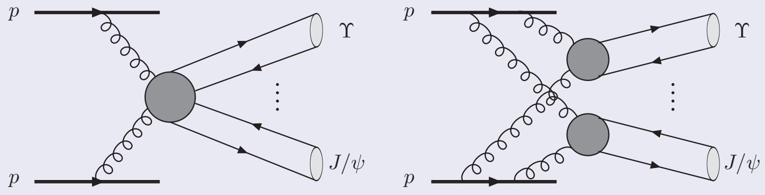


Figure: SPS and DPS of $pp \to J/\psi + \Upsilon + X$.

Triple parton scattering

TPS

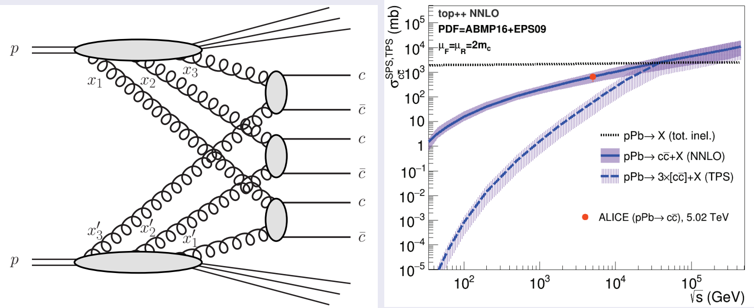


Figure: TPS of $pp \to c\bar{c} + c\bar{c} + c\bar{c}$ (PRL118, 122001).

The n-parton distribution function (1708.07519)

It encodes all the 3D structure information of the hadron.

1. Assumption 1: the n-PDF are factored in terms of longitudinal and transverse components,

$$\Gamma_h^{i_1 \ldots i_n} = D_h^{i_1 \ldots i_n}(x_1, .., x_n; Q_1^2, .., Q_n^2) f(\mathbf{b_1}) \ldots f(\mathbf{b_n}) \quad (2)$$

2. We can get hadron-hadron overlap function $T(\mathbf{b}) = \int f(\mathbf{b_1}) f(\mathbf{b_1} - \mathbf{b}) d^2 b_1$, where $1 = \int T(\mathbf{b}) d^2 b$.
3. Assumption 2: the longitudinal components reduce to the product of independent single PDF

$$D_h^{i_1 \ldots i_n}(x_1, .., x_n; Q_1^2, .., Q_n^2) = D_h^{i_1}(x_1; Q_1^2) \cdots D_h^{i_n}(x_n; Q_n^2) \quad (3)$$

The cross sections and σ_{eff}^{nPS} (1708.07519)

The cross sections of *n*-particle associated production

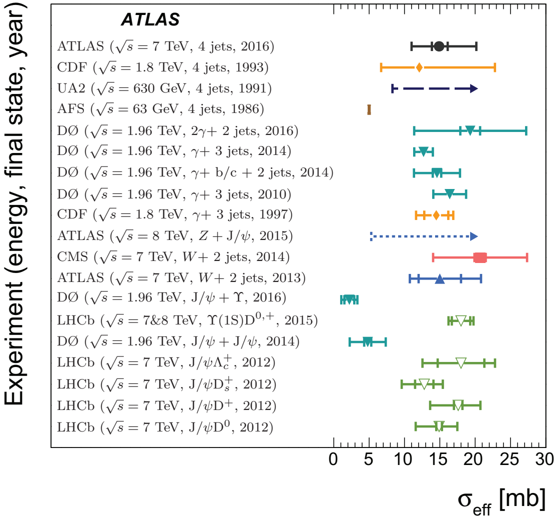
Then we can get

$$\sigma_{hh' \to a_1 \ldots a_n}^{nPS} = \binom{m}{n!} \frac{\sigma_{hh' \to a_1}^{SPS} \cdots \sigma_{hh' \to a_n}^{SPS}}{\left(\sigma_{eff}^{nPS}\right)^{n-1}}, \qquad (4)$$

σ_{eff}^{nPS}

$$\left(\frac{1}{\sigma_{eff}^{nPS}}\right)^{n-1} = \int d^2b\, T^n(\mathbf{b}) \qquad (5)$$

σ_{eff}^{DPS} (arXiv:1608.01857)



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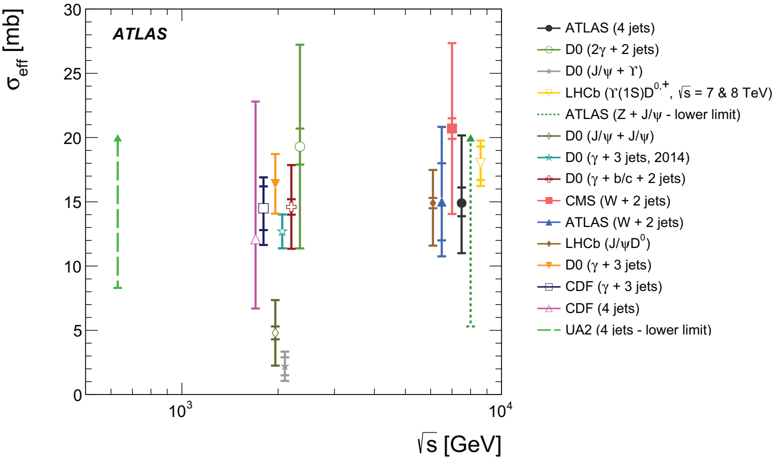


Prompt $J/\psi + \Upsilon$ @ D0

Prompt $J/\psi + \Upsilon(1S, 2S, 3S)$ @ D0 (arXiv:1511.02428)

$$\sigma_{D0}^{J/\psi+\Upsilon} = 27 \pm 9 \pm 7 \text{ fb} \tag{6}$$

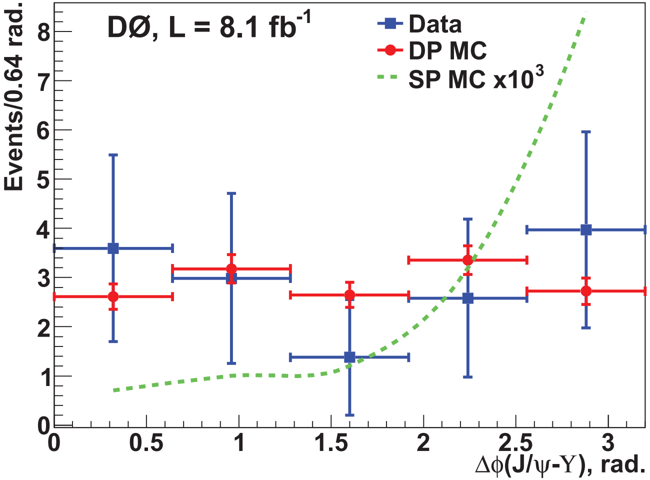
Ignore the SPS contribution

$$\sigma_{DPS}^{J/\psi+\Upsilon} = \sigma_{D0}^{J/\psi+\Upsilon} = \frac{\sigma^{J/\psi}\sigma^{\Upsilon}}{\sigma_{eff}} \tag{7}$$

σ_{eff}

$$\sigma_{eff} = 2.2 \pm 0.7 \pm 0.9 \text{ mb} \tag{8}$$

The distribution of the azimuthal angle between the $J/\psi + \Upsilon$



Color-Singlet contributions of $J/\psi + \Upsilon$

Color-Singlet contributions

Unlike J/ψ-pair or Υ-pair production, neither $\mathcal{O}(\alpha_S^4)$ nor $\mathcal{O}(\alpha_S^5)$ contributions survive in Color-Singlet Model (CSM).

The approximated Loop-Induced (LI) contribution

The approximated Loop-Induced (LI) contribution in CSM at $\mathcal{O}(\alpha_S^6)$ was estimated in Ref. (arXiv:1503.00246) with in the specific limit $\hat{s} \gg |\hat{t}| \gg m^2_{\psi,\Upsilon}$, where $\hat{s}$ and $\hat{t}$ are the Mandelstam variables.

Color-Octet contributions of $J/\psi + \Upsilon$

Color-Octet contributions

The process is a golden observable to probe the so-called Color-Octet Mechanism (COM) (arXiv:1007.3095)

Color-Octet contributions at $\sqrt{s} = 115$ GeV

The Color Octet (CO) contribution were predicted for AFTER@LHC energies $\sqrt{s} = 115$ GeV (arXiv:1504.06531) with HELAC-Onia (arXiv:1212.5293, 1507.03435).

Hadroproduction of $\Upsilon + J/\psi$

SPS contributions were absence

However, the exact calculations of the complete SPS contributions were absence in the literature.

First complete study of $\Upsilon + J/\psi$

We present the first complete study of the simultaneous production of prompt ψ and Υ mesons by including all leading contributions, at order $\mathcal{O}(\alpha_S^6)$ or equivalent.

The frame of Calculation

Cross sections

Hadron and Parton level cross sections

$$\sigma(h_1 h_2 \to \mathcal{C} + \mathcal{B} + X) = \sum_{a,b} f_{a/h_1} \otimes f_{b/h_2} \otimes \hat{\sigma}(ab \to \mathcal{C} + \mathcal{B} + X). \quad (9)$$

Parton level cross section

$$d\hat{\sigma}(ab \to \mathcal{C} + \mathcal{B} + X) = \sum_{n_1,n_2} \hat{\sigma}(ab \to c\bar{c}[n_1] + b\bar{b}[n_2] + X) \langle O^{\mathcal{C}}(n_1)\rangle \langle O^{\mathcal{B}}(n_2)\rangle \quad (10)$$

Long distance matrix elements

Fock states Of J/ψ

$$
\begin{aligned}
|J/\psi\rangle &= \mathcal{O}(1)|c\bar{c}(^3S_1^{[1]})\rangle + \mathcal{O}(v_c^2)|c\bar{c}(^3S_1^{[8]})gg\rangle \\
&+ \mathcal{O}(v_c^2)|c\bar{c}(^3P_J^{[1,8]})g\rangle + \mathcal{O}(v_c^2)|c\bar{c}(^1S_0^{[8]})g\rangle + \ldots .
\end{aligned}
$$

v^2

$$
\begin{aligned}
v_b^2 &\sim v_c^2 \sim 0.1 - 0.3 \\
\alpha_S &\sim 0.2 \\
\alpha_S &\sim v_c^2 \sim v_b^2
\end{aligned} \tag{11}
$$

Amplitude

$$\begin{aligned} & \mathcal{M}(A+B \to H_{c\bar{c}}(^{2S+1}L_J)(2p_1)+D) \\ = & \sum_{L_z S_z} \sum_{s_1 s_2} \sum_{jk} \int \mathrm{d}^3\vec{q}\,\Phi_{c\bar{c}}(\vec{q}) \langle s_1; s_2 \mid SS_z \rangle \langle 3j; \bar{3}k \mid 1 \rangle \\ & \times \mathcal{M}\left[A+B \to c_j^{s_1}(p_1+q)+\bar{c}_k^{s_2}(p_1-q)+D\right], \quad (12) \end{aligned}$$

where $\langle 3j; \bar{3}k \mid 1 \rangle = \delta_{jk}/\sqrt{N_c}$, $\langle s_1; s_2 \mid SS_z \rangle$ is the color CG coefficient for $c\bar{c}$ pairs projecting out appropriate bound states, and $\langle s_1; s_2 \mid SS_z \rangle$ is the spin CG coefficient.
$\mathcal{M}\,[A+B \to c+\bar{c}+D]$ is the quark level scattering amplitude.

QED

J^{PC} Of J/ψ or Υ are 1^{--}

QED contributions may be important too.

α

$$\begin{aligned} \alpha &\sim 0.008 \\ \alpha_S &\sim \sqrt{\alpha} \end{aligned} \tag{13}$$

$\mathcal{O}(\alpha_S^6)$

Color Singlet

The $\mathcal{O}(\alpha_S^4)$ and $\mathcal{O}(\alpha_S^5)$ contributions to $\Upsilon + \psi$ direct production in CSM vanish because of P-parity and C-parity conservation.

Color Octet

$\mathcal{O}(\alpha_S^4 v_c^i v_b^j) \leq \mathcal{O}(\alpha_S^6)$ with $i + j \geq 4$

EW

$\mathcal{O}(\alpha_S^2 \alpha^2) \leq \mathcal{O}(\alpha_S^6)$ with $i + j \geq 4$

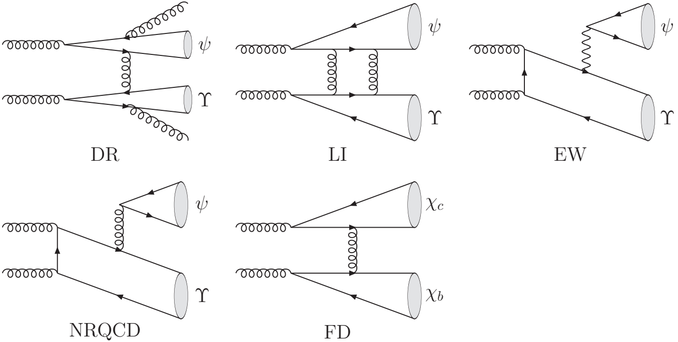
Feeddown for $\chi_{c,b}$

$\mathcal{O}(\alpha_S^4 v_c^i v_b^j) \leq \mathcal{O}(\alpha_S^6)$ with $i + j \geq 4$

Order of SPS

Label	HELAC-Onia 2.0 syntax	First order
DR	`g g > cc~(3S11) bb~(3S11) g g`	$\mathcal{O}(\alpha_S^6)$
LI	`addon 8`	$\mathcal{O}(\alpha_S^6)$
EW	`p p > cc~(3S11) bb~(3S11)`	$\mathcal{O}(\alpha_S^2\alpha^2)$
INTER	`addon 8`	$\mathcal{O}(\alpha_S^4\alpha)$
COM	`g g > jpsi y(1s)`	$\mathcal{O}(\alpha_S^4 v_c^i v_b^j), i+j \geq 4$

Feynman Diagram of SPS



Loop Induced Contributions

The program of Amplitude

Exit

Create Amplitude AA 2 Psi Upsilon

Load FeynCalc, FeynArts and Tarcer

ColorSinglet COddTop COddIns JpsiProject

reduction Associated FAD Separate FeynCalc Function

Generate Feynman diagrams

Input of Calculation AA 2 Psi Upsilon

loop AMP and Export

The program of load FeynArts

Create Amplitude AA 2 Psi Upsilon

Load FeynCalc, FeynArts and Tarcer

```
If[ $FrontEnd === Null,
        $FeynCalcStartupMessages = False;
        Print["Computation of the AA 2 Psi Upsilon"];
];
$LoadFeynArts = $LoadTARCER  = True;
<<FeynCalc`
$FAVerbose=0;
```

FeynCalc 9.2.0. For help, use the
documentation center, check out the wiki or write to the mailing list.

See also the supplied examples. If you use FeynCalc in your research, please cite

- V. Shtabovenko, R. Mertig and F. Orellana,
  Comput. Phys. Commun., 207C, 432–444, 2016, arXiv:1601.01167
- R. Mertig, M. Böhm, and A. Denner, Comput. Phys. Commun., 64, 345–359, 1991.

FeynArts 3.1 patched for use with FeynCalc, for documentation use the
manual or visit www.feynarts.de.

TARCER 2.0, for description see the preprint hep-ph/9801383 at arxiv.org

The program of create amplitude

Generate Feynman diagrams

```
process = {V[1], V[1]} -> {F[3, {2}], -F[3, {2}], F[4, {3}], -F[4, {3}]}
```

$\{V(1), V(1)\} \to \{F(3, \{2\}), -F(3, \{2\}), F(4, \{3\}), -F(4, \{3\})\}$

```
zeroList = {};

tops = CreateTopologies[1, 2 -> 4,
   ExcludeTopologies -> {Tadpoles, SelfEnergies, Triangles}];
tops = DiagramSelect[tops, ColorSinglet[3, 4], ColorSinglet[5, 6],
   COddTop[3, 4], COddTop[5, 6]];
ins = InsertFields[tops, process, InsertionLevel → {Particles},
   GenericModel -> "Lorentz", Model -> "SMQCD",
   ExcludeParticles -> {S[1], S[2], S[3], V[1 | 2 | 3 | 4], U[1], U[2], U[3], F[1], U[4]},
   ExcludeFieldPoints -> {FieldPoint[V[5], V[5], V[5]],
FieldPoint[V[5], V[5], V[5], V[5]]}];
ins = DiagramSelect[ins, COddIns[3, 4], COddIns[5, 6]];
```

The program of diagram selection

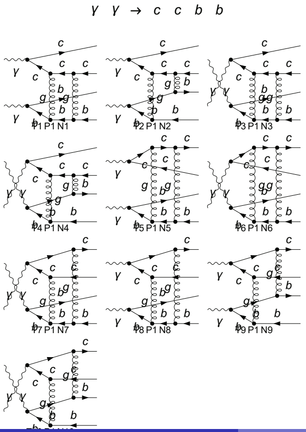
```
subGraphGroup[1] = {1, 2, 3, 4};
subGraphGroup[2] = {5, 6, 17, 18};
subGraphGroup[3] = {7, 8, 9, 10};
subGraphGroup[4] = {11, 12, 23, 24};
subGraphGroup[5] = {13, 14, 15, 16};
subGraphGroup[6] = {19, 20, 21, 22};
subGraphGroup[7] = {25, 26, 29, 30};
subGraphGroup[8] = {27, 28, 31, 32};
subGraphGroup[9] = {33, 34};
subGraphGroup[10] = {35, 36};

selectedDiagram = Table[subGraphGroup[ii][[1]], {ii, 1, 10}]
```

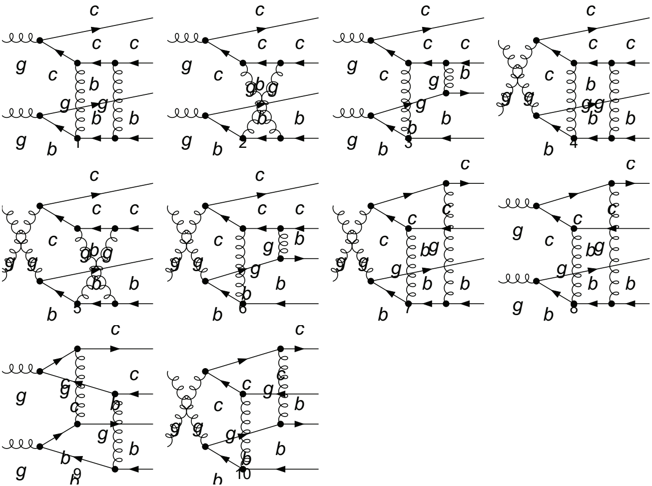
The program of Feynman diagram

```
insExtract = DiagramExtract[ins, selectedDiagram];
Paint[insExtract, ColumnsXRows -> {3, 4}]
```

Shape: Starting Java and the topology editor. This may take a moment.



Feynman diagram



The program of create amplitude

```
amps = FCFAConvert[CreateFeynAmp[ins], IncomingMomenta -> {k1, k2, k3},
     OutgoingMomenta -> {p1, p1, p4, p4, p5}, LoopMomenta -> {q}, DropSumOver -> True,
     ChangeDimension → 4, UndoChiralSplittings → True, SMP → False] /.
    FCGV[xx_] :→ ToExpression[xx] //. SMP[xx_] :→ gStrong;

amps // FCE // StandardForm
```

```
{ 1/(16 π^4) i Spinor[Momentum[p1], MC, 1] . (-2/3 i EL GA[Lor1]) . (MC + GS[k2 - p1 - 2 p4]) .
    (-i gStrong GA[Lor5] SUNTF[{Glu8}, Col3, Col7]) . (MC + GS[-p1 - q]) .
    (-i gStrong GA[Lor3] SUNTF[{Glu7}, Col7, Col4]) .Spinor[-Momentum[p1], MC, 1]
  Spinor[Momentum[p4], MB, 1] . (1/3 i EL GA[Lor2]) . (MB + GS[-k2 + p4]) .
    (-i gStrong GA[Lor6] SUNTF[{Glu8}, Col5, Col8]) . (MB + GS[-p4 + q]) .
    (-i gStrong GA[Lor4] SUNTF[{Glu7}, Col8, Col6]) .Spinor[-Momentum[p4], MB, 1]
  FAD[{k2 - p4, MB}, Dimension → 4] FAD[{-k2 + p1 + 2 p4, MC}, Dimension → 4]
  FAD[q, {p1 + q, MC}, {-p4 + q, MB}, k2 - 2 p4 + q, Dimension → 4]
  FV[Polarization[k1, i], Lor1]
  FV[Polarization[k2, i], Lor2] MT[Lor3, Lor4] MT[Lor5, Lor6],
```

Loop induced contributions of $J/\psi + \Upsilon$

Tree contributions is 0

Unlike J/ψ-pair or Υ-pair production, neither $\mathcal{O}(\alpha_S^4)$ nor $\mathcal{O}(\alpha_S^5)$ contributions survive in Color-Singlet Model (CSM).

The Loop-Induced (LI) contributions are UV and IR finite

1. The amplitude can be calculated in $D = 4$ directly.
2. The gluon mass is introduced to test IR divergence.
3. The momentum and polarization vector can be written in $D = 4$ directly.
4. ...

Momentum for IR and UV finite amplitude

Momentum of $g(k_1)g(k_2) \to J/\psi(p_1) + \Upsilon(p_2)$

$$
\begin{aligned}
k_1 &= \{\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2}\} \\
k_2 &= \{\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2}\} \\
p_1 &= \{E_1, 0, p \times \sin\theta, p \times \cos\theta\} \\
p_2 &= \{E_2, 0, -p \times \sin\theta, -p \times \cos\theta\} \qquad (14)
\end{aligned}
$$

Amplitude Calculation

Create Amplitude AA 2 Psi Upsilon

Input of Calculation AA 2 Psi Upsilon

Load FeynCalc, amps

amps QED

amps

OneLoop4QQDijkl OneLoop3QQDijk OneLoop2QQDij OneLoop1QQDi

fadNOqTerm[9] fadqqqTerm[9] fadNOqTerm[10] fadqqqTerm[10] shiftNoQQFAD[9] shiftNoQQFAD[10]

Momentum and polarization vector of $g(k_1)g(k_2)$

Momentum of $g(k_1)g(k_2)$

$$\begin{aligned} k_1 &= \{\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2}\} \\ k_2 &= \{\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2}\} \end{aligned} \quad (15)$$

Polarization of $g(k_1)g(k_2)$

$$\begin{aligned} \epsilon_1(k_1) &= \epsilon_2(k_2) = \{0, 1, 0, 0\} \\ \epsilon_2(k_1) &= \epsilon_1(k_2) = \{0, 0, 1, 0\} \end{aligned} \quad (16)$$

Momentum and polarization of $J/\psi(p_1) + \Upsilon(p_2)$

Momentum of $J/\psi(p_1) + \Upsilon(p_2)$

$$
\begin{aligned}
p_1 &= \{E_1, 0, p \times \sin\theta, p \times \cos\theta\} \\
p_2 &= \{E_2, 0, -p \times \sin\theta, -p \times \cos\theta\}
\end{aligned} \tag{17}
$$

Polarization of $J/\psi(p_1) + \Upsilon(p_2)$

$$
\begin{aligned}
\epsilon_L(p_1) &= 1/m_J\{p, 0, E_1\sin\theta, E_1\cos\theta\} \\
\epsilon_L(p_2) &= 1/m_\Upsilon\{p, 0, -E_2\sin\theta, -E_2\cos\theta\} \\
\epsilon_{T1}(p_1) &= \epsilon_{T1}(p_2) = \{0, 1, 0, 0\} \\
\epsilon_{T2}(p_1) &= \epsilon_{T2}(p_2) = \{0, 0, -\cos\theta, \sin\theta\}
\end{aligned} \tag{18}
$$

Vector define

```
mom[k1] = {ss/2, 0, 0, ss/2};
mom[k2] = {ss/2, 0, 0, -ss/2};
mom[p1] = {ec, pc3*Sin[th], 0, pc3*Cos[th]};
mom[p3] = {eb, -pc3*Sin[th], 0, -pc3*Cos[th]};

mom[g1pp[1]] = {0, 1, 0, 0};
mom[g1pp[2]] = {0, 0, 1, 0};

mom[g2pp[1]] = {0, 1, 0, 0};
mom[g2pp[2]] = mom[g1pp[2]];


mom[Jpp[3]] = {pc3, ec*Sin[th], 0, ec*Cos[th]}/MC;
mom[Jpp[1]] = {0, -Cos[th], 0, Sin[th]};
mom[Jpp[2]] = mom[g1pp[2]];

mom[Upp[3]] = {-pc3, eb*Sin[th], 0, eb*Cos[th]}/MB;
mom[Upp[1]] = {0, -Cos[th], 0, Sin[th]};
mom[Upp[2]] = mom[g1pp[2]];
```

Amplitude

Amplitude of $g(k_1, 1)g(k_2, 1) \to J/\psi(p_1, L) + \Upsilon(p_2, L)$

- Cololor factor can be calculated diagram by diagram. It can be considered as a global factor.

Amplitude

Amplitude of $g(k_1, 1)g(k_2, 1) \to J/\psi(p_1, L) + \Upsilon(p_2, L)$

- Cololor factor can be calculated diagram by diagram. It can be considered as a global factor.
- Spin projector operator can be used directly.

Amplitude

Amplitude of $g(k_1, 1)g(k_2, 1) \to J/\psi(p_1, L) + \Upsilon(p_2, L)$

- Cololor factor can be calculated diagram by diagram. It can be considered as a global factor.
- Spin projector operator can be used directly.
- The scalar product of $k_1,\ k_2,\ p_1,\ p_2$ and polarization vector can be expressed by $s,\ m_J,\ m_\Upsilon,\ E_1,\ p,\ \theta$.

Amplitude

Amplitude of $g(k_1, 1)g(k_2, 1) \to J/\psi(p_1, L) + \Upsilon(p_2, L)$

- Cololor factor can be calculated diagram by diagram. It can be considered as a global factor.
- Spin projector operator can be used directly.
- The scalar product of $k_1,\ k_2,\ p_1,\ p_2$ and polarization vector can be expressed by $s,\ m_J,\ m_\Upsilon,\ E_1,\ p,\ \theta$.
- Loop integrate.

Amplitude

Amplitude of $g(k_1, 1)g(k_2, 1) \to J/\psi(p_1, L) + \Upsilon(p_2, L)$

- Cololor factor can be calculated diagram by diagram. It can be considered as a global factor.
- Spin projector operator can be used directly.
- The scalar product of $k_1,\ k_2,\ p_1,\ p_2$ and polarization vector can be expressed by $s,\ m_J,\ m_\Upsilon,\ E_1,\ p,\ \theta$.
- Loop integrate.
- Amplitude can be expressed by $s,\ m_J,\ m_\Upsilon,\ E_1,\ p,\ \theta$.

Amplitude

Amplitude of $g(k_1, 1)g(k_2, 1) \to J/\psi(p_1, L) + \Upsilon(p_2, L)$

- Cololor factor can be calculated diagram by diagram. It can be considered as a global factor.
- Spin projector operator can be used directly.
- The scalar product of $k_1,\ k_2,\ p_1,\ p_2$ and polarization vector can be expressed by $s,\ m_J,\ m_\Upsilon,\ E_1,\ p,\ \theta$.
- Loop integrate.
- Amplitude can be expressed by $s,\ m_J,\ m_\Upsilon,\ E_1,\ p,\ \theta$.
- Simplify the amplitude

FAD reduce

fadNOqTerm[9] fadqqqTerm[9] fadNOqTerm[10] fadqqqTerm[10] shiftNoQQFAD[9] shiftNoQQFAD[10]

```
fadNOqTerm[9] = 1/(2 (MB2 - MC2)) (
  1/(-2 MC2 - t/2) FAD[{p1 + q, MC}, {-p3 + q, MB}, k2 - 2 p3 + q, {k2 - p1 - 2 p3 + q, MC}] +
  1/(2 MB^2 + t/2) FAD[{p1 + q, MC}, {-p3 + q, MB}, {k2 - p3 + q, MB}, k2 - 2 p3 + q] +
  1/(-2 MC2 - t/2) FAD[{p1 + q, MC}, {k2 - p3 + q, MB}, k2 - 2 p3 + q, {k2 - p1 - 2 p3 + q, MC}] +
  1/(2 MB^2 + t/2) FAD[{-p3 + q, MB}, {k2 - p3 + q, MB}, k2 - 2 p3 + q, {k2 - p1 - 2 p3 + q, MC}] );
```

Simplify Input

Input of Calculation AA 2 Psi Upsilon

loop AMP and Export

mc2mb2Rep simplifyForm

$$\text{mc2mb2Rep} = \Big\{ eb + ec \to \frac{ss}{2},\ MB^2 \to MB2,\ MB^4 \to MB2^2,\ MC^2 \to MC2,\ MC^4 \to MC2^2,$$
$$eb\,ec + pc3^2 \to \frac{1}{8}(-t-u),\ 4\,MB^2 + 4\,MC^2 - s \to t+u,\ 4\,MB2 + 4\,MC2 - s \to t+u,$$
$$ss^2 \to s,\ 4\,MB^2 + 4\,MC^2 - s - 2\,t \to -t+u,\ 4\,MB2 + 4\,MC2 - s - 2\,t \to -t+u,$$
$$4\,MC^2 - s - t \to -4\,MB^2 + u,\ 4\,MB^2 + 4\,MC^2 - s - t \to u,\ 8\,MB^2 + 4\,MC^2 - s - t \to 4\,MB^2 + u,$$
$$16\,MB^2 + 4\,MC^2 - s - t \to 12\,MB^2 + u,\ 4\,MC2 - s - t \to -4\,MB2 + u,\ 4\,MB2 + 4\,MC2 - s - t \to u,$$
$$8\,MB2 + 4\,MC2 - s - t \to 4\,MB2 + u,\ 16\,MB2 + 4\,MC2 - s - t \to 12\,MB2 + u,$$
$$4\,MB^2 + 8\,MC^2 - s - u \to 4\,MC^2 + t,\ 4\,MB2 + 8\,MC2 - s - u \to 4\,MC2 + t,\ 4\,MB2 + 4\,MC2 - t - u \to s,$$
$$8\,MB2 - s - t - u \to 4\,(MB2 - MC2),\ 8\,MC^2 - s - t - u \to -4\,MB^2 + 4\,MC^2,$$
$$8\,MC2 - s - t - u \to -4\,MB2 + 4\,MC2,\ -2\,MB^2 - 2\,MC^2 + \frac{s}{4} + \frac{t}{2} + \frac{u}{2} \to -\frac{s}{4},$$

Polarization Simplify

mc2mb2Rep simplifyForm

Diagram 1 - 8

```
For[iiDiag = 1,        ≤ 8,       ++,
 ampTR[iiDiag] = ampNUMShift[iiDiag] /.           → Tr // C
 P    ["Le              [",           ]=",      [iiDiag] // Le        ];

 ampRED[iiDiag] = ampTR[iiDiag] /.
   {P    [L            [eJ], M        [p1]] → 0, P    [L            [e ], M        [p3]] → 0,
    P    [L            [L    ], M        [k1]] → 0,
    P    [L            [L    ], M        [k2]] → 0,
    P    [L            [L    ], M        [k1]] → 0,
    P    [L            [L    ], M        [k2]] → 0,
    P    [L            [L    ], M        [p3]] → -P    [L            [L    ], M        [p1]],
    P    [L            [L    ], M        [p3]] →
     -P    [L            [L    ], M        [p1]]};
```

Loop Calculation

```
loopNoq4Ranks[iiDiag] = OneLoop[q,                [iiDiag] * ampDENShift[iiDiag]] /.
    {-2 MB^2 - 2 MC^2 + s/4 + t/2 + u/2 → -s/4, -2 MB^2 - 3 MC^2 + s/2 + t + u/2 → 1/2 (-2 MC^2 + t),
     -4 MB^2 - 4 MC^2 + s + t + u → 0} //. mc2mb2Rep //. s

loopNoq4RanksAddMG[iiDiag] = loopNoq4Ranks[iiDiag] //.
  {B0[a1_, 0, m1_] :> B0[a1, mg2, m1], B0[a1_, m1_, 0] :> B0[a1, m1, mg2],
   C0[a1_,    _,    _, m1___, 0, m2___] :> C0[a1, a2, a3, m1, mg2, m2],
   D0[a1_,    _,    _,    _,    _,    _, m1___, 0, m2___] :>
    D0[a1, a2, a3, a4, a5, a6, m1, mg2, m2], P    [xx___, {m1___, 0, m2___},
     P              → True, PaVeAutoReduce → True] :> P    [xx, {m1, m      }],
   P    [xx___, {m1___, 0, m2___}] :> P    [xx, {m1, m      }]};

P    ["                  [",         ] is finished"];
```

Helicity insert

```
For[h  = 1, h  ≤ 3, h  ++,
  For[hU = 1, hU ≤ 3, hU++,
    heAmp        [iiDiag, hg1, hg2, h     ] = loop              [iiDiag] /.
        {Lorent      [Lor1] → Momentum[g1pp[hg1]], Lorent      [Lor2] ->
          Momentum[g2pp[hg2]], Lorent       [e ] -> Momentum[Jpp[h ]],
         Lorent      [eU] -> Momentum[Upp[hU]]} //. mc2mb2Rep //. simpli

    heAmp[iiDiag, hg1, hg2, h     ] =
     DE          [[iiDiag]] * (heAmp         [iiDiag, hg1, hg2, h     ] +
           loopq4RanksAddMG[iiDiag, hg1, hg2, h     ] /. {ss   → s,
           -2 MB^2 - 2 MC^2 + s/4 + t/2 + u/2 → -s/4, -2 MB^2 - 3 MC^2 + s/2 + t + u/2 → 1/2 (-2 MC^2 + t),
           -4 MB^2 - 4 MC^2 + s + t + u → 0}) /. PaVe[xxx___, PaVeAutoOrder → True,
         PaVeAutoReduce → True] -> PaVe[xxx] //. mc2mb2Rep //. simpli      ];
 ];
```

Helicity insert

```
For[h = 1, h ≤ 3, h ++,
  For[hU = 1, hU ≤ 3, hU++,
    heAmp [iiDiag, hg1, hg2, h ] = loop [iiDiag] /.
        {Lorent [Lor1] → Momentum[g1pp[hg1]], Lorent [Lor2] ->
          Momentum[g2pp[hg2]], Lorent [e ] -> Momentum[Jpp[h ]],
         Lorent [eU] -> Momentum[Upp[hU]]} //. mc2mb2Rep //. simpli

    heAmp[iiDiag, hg1, hg2, h ] =
     DE [[iiDiag]] * (heAmp [iiDiag, hg1, hg2, h ] +
           loopq4RanksAddMG[iiDiag, hg1, hg2, h ] /. {ss → s,
           -2 MB^2 - 2 MC^2 + s/4 + t/2 + u/2 → -s/4, -2 MB^2 - 3 MC^2 + s/2 + t + u/2 → 1/2 (-2 MC^2 + t),
           -4 MB^2 - 4 MC^2 + s + t + u → 0}) /. PaVe[xxx___, PaVeAutoOrder → True,
          PaVeAutoReduce → True] -> PaVe[xxx] //. mc2mb2Rep //. simpli ];
 ];
```

Amplitude Export

List of PaVe

$$A = (List\ of\ PaVe).(List\ of\ Coefficients) \tag{19}$$

Export

```
am         = Table[he   [i                    ] // Variab
    {i         }, {hg1, 1, 2}, {hg2, 1, 2}, {hJ, 1, 3}, {hU, 1, 3}] // Variab

SF       = Union[Cases[am          _PaVe], Cases[am          _D0i],
   Cases[am          _C0i], Cases[am          _B0i], Cases[am          _A0],
   Cases[am          _B0], Cases[am          _C0], Cases[am          _D0]];

otherSF       =
 DeleteCases[DeleteCases[DeleteCases[DeleteCases[DeleteCases[DeleteCases[DeleteCases[
        DeleteCases[am          _PaVe], _D0i], _C0i], _B0i], _A0], _B0], _C0], _D0]
```

PaVe Export

```
Export["SFLoopToolsVar.txt", {SFPaVeVar //. {PaVe[aa1_, {bpp___}, {cm1_, cm2_, cm3_}] :>
      C  [String   ["cc", ToString[aa1]] // ToExpression, bpp, cm1, cm2, cm3],
     PaVe[aa1_, aa2_, {bpp___}, {cm1_, cm2_, cm3_}] :>
      C  [String   ["cc", ToString[aa1], ToString[aa2]] // ToExpression,
       bpp, cm1, cm2, cm3], PaVe[aa1_, {bpp___}, {cm1_, cm2_, cm3_, cm4_}] :>
      D  [String   ["dd", ToString[aa1]] // ToExpression, bpp, cm1, cm2, cm3, cm4],
     PaVe[aa1_, aa2_, {bpp___}, {cm1_, cm2_, cm3_, cm4_}] :>
      D  [String   ["dd", ToString[aa1], ToString[aa2]] // ToExpression, bpp, cm1,
       cm2, cm3, cm4], PaVe[aa1_, aa2_, aa3_, {bpp___}, {cm1_, cm2_, cm3_, cm4_}] :>
      D  [String   ["dd", ToString[aa1], ToString[aa2], ToString[aa3]] //
        ToExpression, bpp, cm1, cm2, cm3, cm4],
     PaVe[aa1_, aa2_, aa3_, aa4_, {bpp___}, {cm1_, cm2_, cm3_, cm4_}] :>
      D  [String   ["dd", ToString[aa1], ToString[aa2], ToString[aa3],
         ToString[aa4]] // ToExpression, bpp, cm1, cm2, cm3, cm4]}},       ];
```

Coefficients Export

```
For hg1 = 1, hg1 ≤ 2, hg1++,
 For[hg2 = 1, hg2 ≤ 2, hg2++,

  For[hJ = 1, hJ ≤ 3, hJ++,
   For[hU = 1, hU ≤ 3, hU++,
    For[iiSFPaVe = 1, iiSFPaVe ≤ Length[SFPaVeVar], iiSFPaVe++,
     coef[iiSFPaVe, hg1, hg2, hJ, hU] =
      Plus @@ Table[(Coefficient[resTotpartPaVe[hg1, hg2, hJ, hU][[iiDiag]], spvv[
              iiSFPaVe]] // Simplify) //. mc2mb2Rep //. simplifyForm, {iiDiag, 1, 1}]
    ];


    Export[StringJoin["spvvCoefList[", ToString[hg1], ",",
      ToString[hg2], ",", ToString[hJ], ",", ToString[hU], "].txt"], {Table[
       coef[iiSFPaVe, hg1, hg2, hJ, hU], {iiSFPaVe, 1, Length[SFPaVeVar]}]}, "List"];
```

Numerical Result of $\Upsilon + J/\psi$

Direct SPS cross sections @ D0 in fb

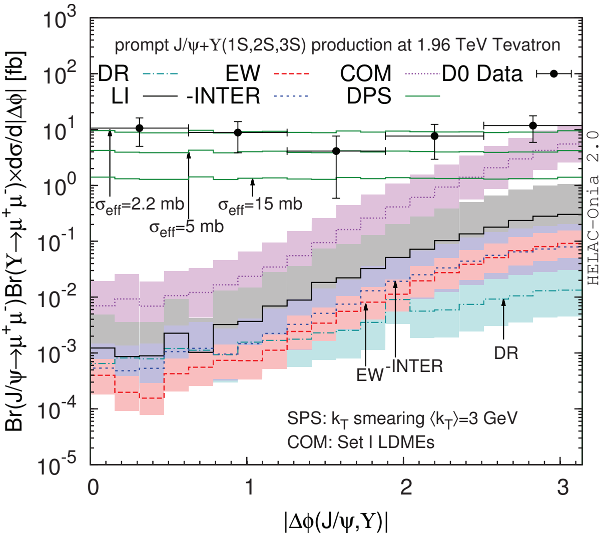
		J/ψ	$\psi(2S)$
DR	$\Upsilon(1S)$	$3.58^{+233\%}_{-66.4\%} \pm 4.4\%$	$2.34^{+233\%}_{-66.4\%} \pm 4.4\%$
	$\Upsilon(2S)$	$1.78^{+233\%}_{-66.4\%} \pm 4.4\%$	$1.17^{+233\%}_{-66.4\%} \pm 4.4\%$
	$\Upsilon(3S)$	$1.36^{+233\%}_{-66.4\%} \pm 4.4\%$	$0.894^{+233\%}_{-66.4\%} \pm 4.4\%$
LI	$\Upsilon(1S)$	$56.2^{+264\%}_{-70.2\%} \pm 4.7\%$	$36.8^{+264\%}_{-70.2\%} \pm 4.7\%$
	$\Upsilon(2S)$	$28.0^{+264\%}_{-70.2\%} \pm 4.7\%$	$18.4^{+264\%}_{-70.2\%} \pm 4.7\%$
	$\Upsilon(3S)$	$21.4^{+264\%}_{-70.2\%} \pm 4.7\%$	$14.0^{+264\%}_{-70.2\%} \pm 4.7\%$
EW	$\Upsilon(1S)$	$15.8^{+75.4\%}_{-46.4\%} \pm 4.6\%$	$10.4^{+75.4\%}_{-46.4\%} \pm 4.6\%$
	$\Upsilon(2S)$	$7.90^{+75.4\%}_{-46.4\%} \pm 4.6\%$	$5.18^{+75.4\%}_{-46.4\%} \pm 4.6\%$
	$\Upsilon(3S)$	$6.04^{+75.4\%}_{-46.4\%} \pm 4.6\%$	$3.96^{+75.4\%}_{-46.4\%} \pm 4.6\%$
INTER	$\Upsilon(1S)$	$-16.6^{+162\%}_{-62.0\%} \pm 4.8\%$	$-10.9^{+162\%}_{-62.0\%} \pm 4.8\%$
	$\Upsilon(2S)$	$-8.29^{+162\%}_{-62.0\%} \pm 4.8\%$	$-5.43^{+162\%}_{-62.0\%} \pm 4.8\%$
	$\Upsilon(3S)$	$-6.34^{+162\%}_{-62.0\%} \pm 4.8\%$	$-4.15^{+162\%}_{-62.0\%} \pm 4.8\%$
COM	$\Upsilon(1S)$	$409^{+138\%}_{-56.7\%} \pm 4.4\%$	$174^{+138\%}_{-56.8\%} \pm 4.4\%$
	$\Upsilon(2S)$	$135^{+139\%}_{-57.0\%} \pm 4.4\%$	$57.6^{+139\%}_{-57.1\%} \pm 4.4\%$
	$\Upsilon(3S)$	$197^{+137\%}_{-56.6\%} \pm 4.4\%$	$84.1^{+138\%}_{-56.7\%} \pm 4.4\%$

SPS cross sections @ D0 & LHCb

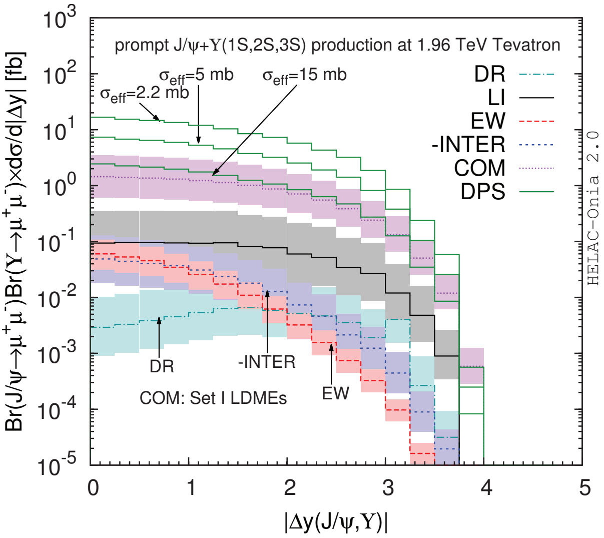
Experiment	CSM				COM			
	DR	LI	EW	INTER	Set I	Set II	Set III	Set IV
D0: $27 \pm 42.2\%$	$0.0146^{+233\%}_{-66.6\%}$	$0.229^{+264\%}_{-70.4\%}$	$0.065^{+75.5\%}_{-46.6\%}$	$-0.068^{+162\%}_{-62.2\%}$	$2.96^{+135\%}_{-56.2\%}$	$1.41^{+160\%}_{-77.6\%}$	$1.80^{+143\%}_{-58.0\%}$	$0.418^{+144\%}_{-58.3\%}$
LHCb	$0.255^{+391\%}_{-79.7\%}$	$6.05^{+436\%}_{-82.2\%}$	$1.71^{+135\%}_{-65.2\%}$	$-3.23^{+262\%}_{-75.9\%}$	$38.8^{+238\%}_{-73.0\%}$	$21.2^{+243\%}_{-73.6\%}$	$28.1^{+243\%}_{-73.8\%}$	$6.57^{+243\%}_{-73.9\%}$

TABLE III: Cross sections $\sigma(pp(\bar{p}) \to J/\psi\Upsilon) \times \mathrm{Br}(J/\psi \to \mu^+\mu^-)\mathrm{Br}(\Upsilon \to \mu^+\mu^-)$ (in units of fb) of prompt J/ψ and $\Upsilon(1S, 2S, 3S)$ simultaneous production at the Tevatron in the D0 fiducial region [10] and at $\sqrt{s} = 13$ TeV LHC in the LHCb acceptance $2 < y_{J/\psi,\Upsilon} < 4.5$, where we have also included feeddown contributions from higher-excited quarkonia decay.

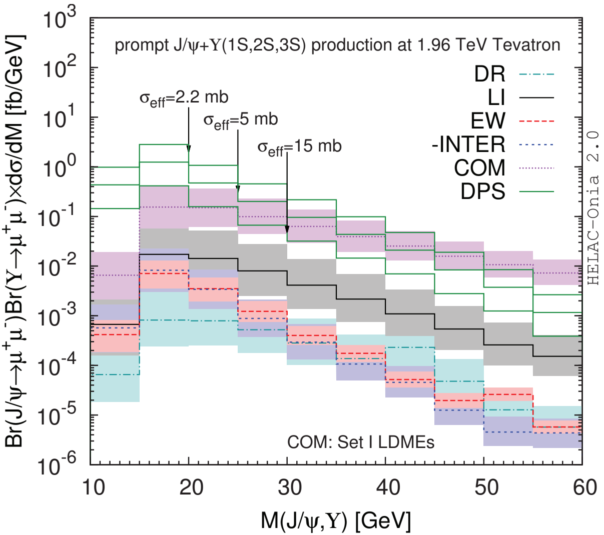
dphi @ D0


prompt J/ψ+Υ(1S,2S,3S) production at 1.96 TeV Tevatron
DR
EW
COM
D0 Data
LI
-INTER
DPS
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
EW
-INTER
DR
SPS: kT smearing ⟨kT⟩=3 GeV
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/d|Δϕ| [fb]
|Δϕ(J/ψ,Υ)|
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
0
1
2
3

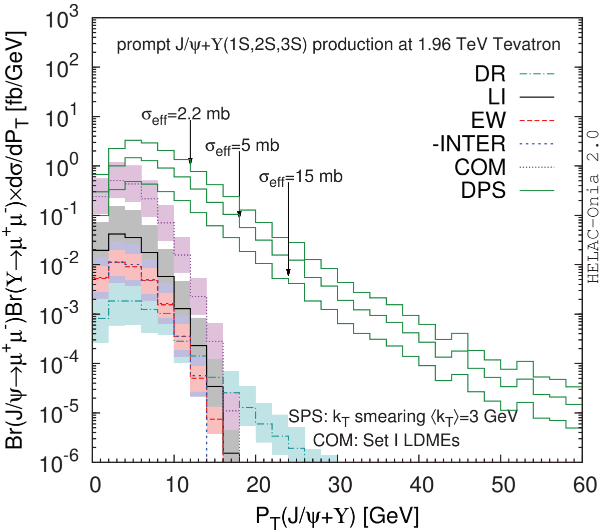
dy @ D0


prompt J/ψ+Υ(1S,2S,3S) production at 1.96 TeV Tevatron
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
DR
LI
EW
-INTER
COM
DPS
DR
-INTER
EW
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/d|Δy| [fb]
|Δy(J/ψ,Υ)|
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
0
1
2
3
4
5

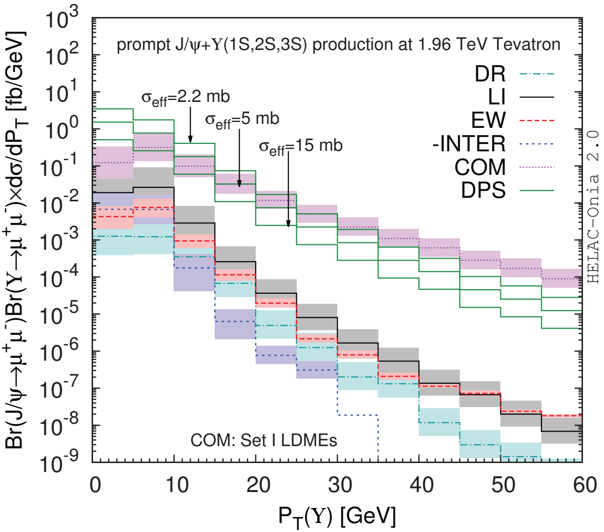
dM @ D0


prompt J/ψ+Υ(1S,2S,3S) production at 1.96 TeV Tevatron
DR
LI
EW
-INTER
COM
DPS
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/dM [fb/GeV]
M(J/ψ,Υ) [GeV]
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10
20
30
40
50
60

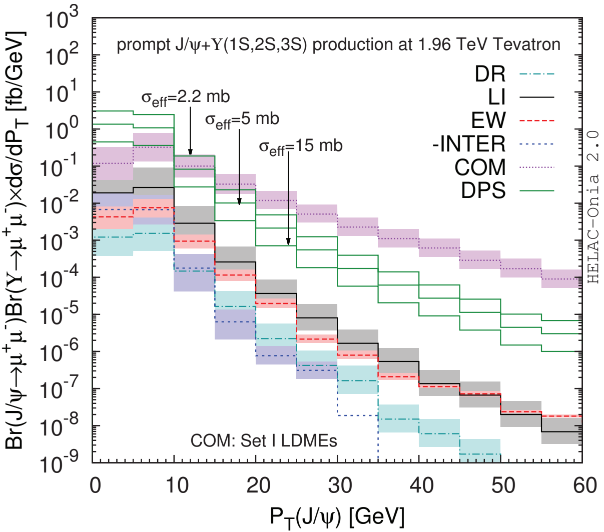
dPt @ D0


prompt J/ψ+Υ(1S,2S,3S) production at 1.96 TeV Tevatron
DR
LI
EW
-INTER
COM
DPS
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
SPS: kT smearing ⟨kT⟩=3 GeV
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/dPT [fb/GeV]
PT(J/ψ+Υ) [GeV]
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
0
10
20
30
40
50
60

dptY @ D0

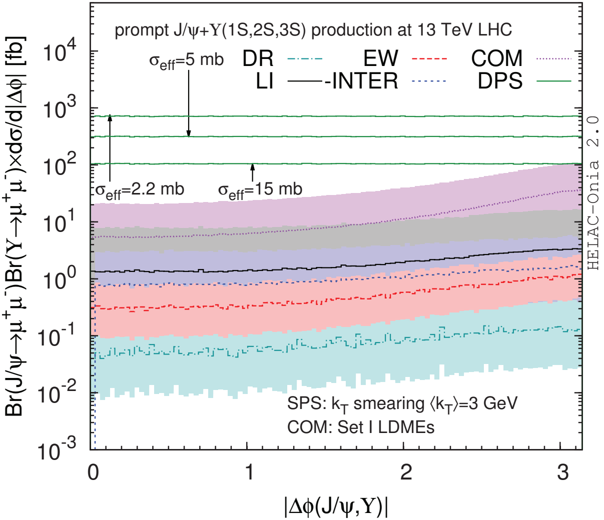

prompt J/ψ+Υ(1S,2S,3S) production at 1.96 TeV Tevatron
DR
LI
EW
-INTER
COM
DPS
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/dPT [fb/GeV]
PT(Υ) [GeV]
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
0
10
20
30
40
50
60

dptpsi @ D0

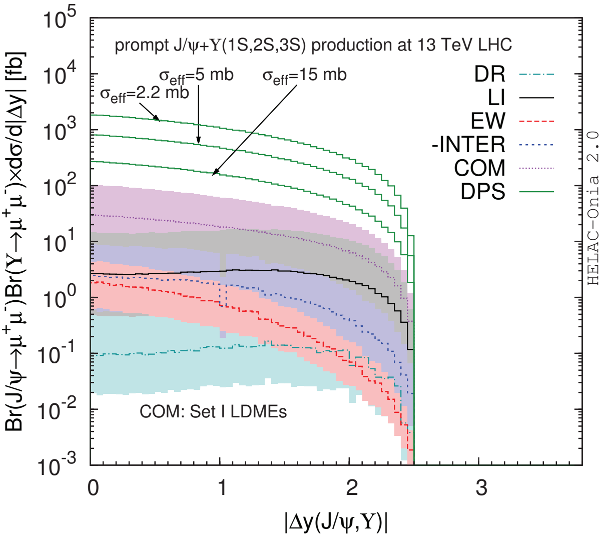


prompt J/ψ+Υ(1S,2S,3S) production at 1.96 TeV Tevatron
DR
LI
EW
-INTER
COM
DPS
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/dPT [fb/GeV]
PT(J/ψ) [GeV]

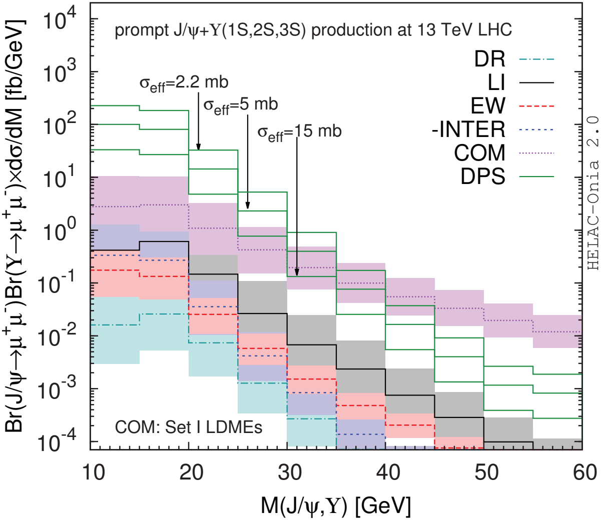
dphi @ LHCB


prompt J/ψ+Υ(1S,2S,3S) production at 13 TeV LHC
σeff=5 mb
DR
EW
COM
LI
-INTER
DPS
σeff=2.2 mb
σeff=15 mb
SPS: kT smearing ⟨kT⟩=3 GeV
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/d|Δφ| [fb]
|Δφ(J/ψ,Υ)|
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
0
1
2
3

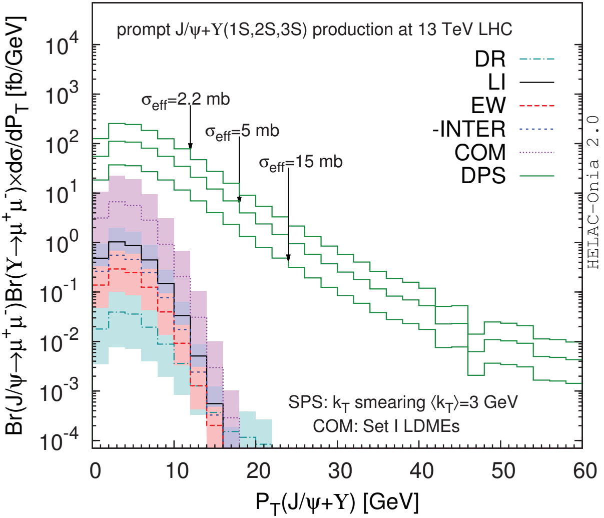
dy @ LHCB


prompt J/ψ+Υ(1S,2S,3S) production at 13 TeV LHC
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
DR
LI
EW
-INTER
COM
DPS
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/d|Δy| [fb]
|Δy(J/ψ,Υ)|
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
0
1
2
3

dM @ LHCB

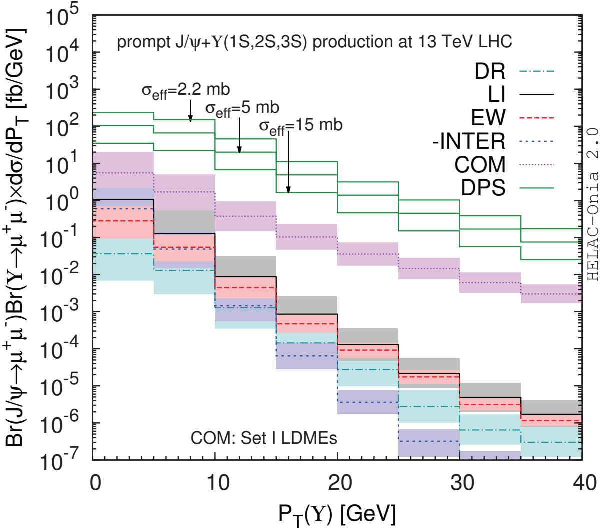

prompt J/ψ+Υ(1S,2S,3S) production at 13 TeV LHC
DR
LI
EW
-INTER
COM
DPS
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/dM [fb/GeV]
M(J/ψ,Υ) [GeV]
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10
20
30
40
50
60

dPt @ LHCB

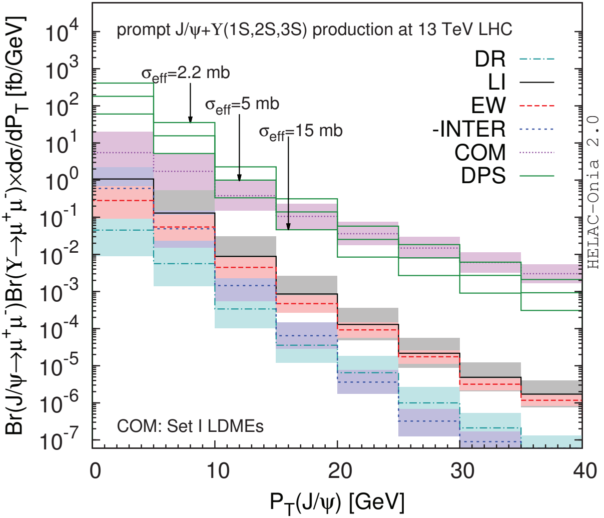


prompt J/ψ+Υ(1S,2S,3S) production at 13 TeV LHC
DR
LI
EW
-INTER
COM
DPS
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
SPS: kT smearing ⟨kT⟩=3 GeV
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/dPT [fb/GeV]
PT(J/ψ+Υ) [GeV]

dptY @ LHCB


prompt J/ψ+Υ(1S,2S,3S) production at 13 TeV LHC
DR
LI
EW
-INTER
COM
DPS
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/dPT [fb/GeV]
PT(Υ) [GeV]
0
10
20
30
40
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7

dptpsi @ LHCB


prompt J/ψ+Υ(1S,2S,3S) production at 13 TeV LHC
σeff=2.2 mb
σeff=5 mb
σeff=15 mb
DR
LI
EW
-INTER
COM
DPS
COM: Set I LDMEs
HELAC-Onia 2.0
Br(J/ψ→μ+μ-)Br(Υ→μ+μ-)×dσ/dPT [fb/GeV]
PT(J/ψ) [GeV]
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
0
10
20
30
40

Numerical Result of $\Upsilon + J/\psi + \phi$

Numerical Result for $\Upsilon + J/\psi + \phi$

SPS cross section of $\Upsilon,\ J/\psi,\ \phi$ at LHCb

1. We can get the inclusive cross sections of $\Upsilon,\ J/\psi,\ \phi$ at $\sqrt{s} = 13$ TeV at LHCb, it 0.2(15) μb for $\Upsilon(J/\psi)$, and the cross sections is 0.6 mb for $p_T(\phi) > 2$ GeV.

Numerical Result for $\Upsilon + J/\psi + \phi$

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2. With the NLO(LO) LDMEs, $\sigma^{SPS}[\Upsilon + J/\psi] \sim 8(24)$ pb, then $\sigma^{SPS}[\Upsilon + J/\psi] \sim \sqrt{\sigma[\Upsilon]\sigma[J/\psi]} \times \alpha_s^3$ and $\alpha_s \sim 0.16(0.23)$ (1605.03061).

Numerical Result for $\Upsilon + J/\psi + \phi$

SPS cross section of $\Upsilon,\ J/\psi,\ \phi$ at LHCb

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3. SPS $\Upsilon + J/\psi + \phi$: $\mathcal{O}(\alpha_s^9)$, very small.

Numerical Result for $\Upsilon + J/\psi + \phi$

SPS cross section of $\Upsilon,\ J/\psi,\ \phi$ at LHCb

1. We can get the inclusive cross sections of $\Upsilon,\ J/\psi,\ \phi$ at $\sqrt{s} = 13$ TeV at LHCb, it 0.2(15) μb for $\Upsilon(J/\psi)$, and the cross sections is 0.6 mb for $p_T(\phi) > 2$ GeV.
2. With the NLO(LO) LDMEs, $\sigma^{SPS}[\Upsilon + J/\psi] \sim 8(24)$ pb, then $\sigma^{SPS}[\Upsilon + J/\psi] \sim \sqrt{\sigma[\Upsilon]\sigma[J/\psi]} \times \alpha_s^3$ and $\alpha_s \sim 0.16(0.23)$ (1605.03061).
3. SPS $\Upsilon + J/\psi + \phi$: $\mathcal{O}(\alpha_s^9)$, very small.
4. DPS $\Upsilon + J/\psi + \phi$: about $3 \times \sigma^{SPS}[\Upsilon + J/\psi]\frac{\sigma[\phi]}{\sigma_{eff}^{DPS}} \sim 1.4\ \mathrm{pb}$ for $p_T(\phi) > 2$ GeV and $\sigma_{eff}^{DPS} \sim 10$ mb.

Numerical Result for $\Upsilon + J/\psi + \phi$

SPS cross section of Υ, J/ψ, ϕ at LHCb

1. We can get the inclusive cross sections of Υ, J/ψ, ϕ at $\sqrt{s} = 13$ TeV at LHCb, it 0.2(15) μb for $\Upsilon(J/\psi)$, and the cross sections is 0.6 mb for $p_T(\phi) > 2$ GeV.
2. With the NLO(LO) LDMEs, $\sigma^{SPS}[\Upsilon + J/\psi] \sim 8(24)$ pb, then $\sigma^{SPS}[\Upsilon + J/\psi] \sim \sqrt{\sigma[\Upsilon]\sigma[J/\psi]} \times \alpha_s^3$ and $\alpha_s \sim 0.16(0.23)$ (1605.03061).
3. SPS $\Upsilon + J/\psi + \phi$: $\mathcal{O}(\alpha_s^9)$, very small.
4. DPS $\Upsilon + J/\psi + \phi$: about $3 \times \sigma^{SPS}[\Upsilon + J/\psi] \frac{\sigma[\phi]}{\sigma_{eff}^{DPS}} \sim$ 1.4 pb for $p_T(\phi) > 2$ GeV and $\sigma_{eff}^{DPS} \sim 10$ mb.
5. TPS $\Upsilon + J/\psi + \phi$: about $\frac{\sigma[\Upsilon]\sigma[J/\psi]\sigma[\phi]}{(\sigma_{eff}^{TPS})^2} \sim$ 28 pb for $p_T(\phi) > 2$ GeV and $\sigma_{eff}^{TPS} \sim 8$ mb.

Search for TPS in $\Upsilon + J/\psi + \phi$ at LHCb

Estimate the number of events

1. $Br[\Upsilon(J/\psi) \to \mu^+\mu^-] = 0.024(0.06)$ and $Br[\phi \to K^+K^-] = 0.5$.

Search for TPS in $\Upsilon + J/\psi + \phi$ at LHCb

Estimate the number of events

1. $Br[\Upsilon(J/\psi) \to \mu^+\mu^-] = 0.024(0.06)$ and $Br[\phi \to K^+K^-] = 0.5$.
2. Integrated luminosity of LHCb is about 4 fb^{-1} at $\sqrt{s} = 13$ TeV.

Search for TPS in $\Upsilon + J/\psi + \phi$ at LHCb

Estimate the number of events

1. $Br[\Upsilon(J/\psi) \to \mu^+\mu^-] = 0.024(0.06)$ and $Br[\phi \to K^+K^-] = 0.5$.
2. Integrated luminosity of LHCb is about 4 fb^{-1} at $\sqrt{s} = 13$ TeV.
3. Number of events for $\Upsilon(\mu^+\mu^-) + J/\psi(\mu^+\mu^-) + \phi(K^+K^-)$ with $p_T(\phi) > 2\ \mathrm{GeV}$ is about 80.

Search for TPS in $\Upsilon + J/\psi + \phi$ at LHCb

Estimate the number of events

1. $Br[\Upsilon(J/\psi) \to \mu^+\mu^-] = 0.024(0.06)$ and $Br[\phi \to K^+K^-] = 0.5$.
2. Integrated luminosity of LHCb is about $4\ \mathrm{fb}^{-1}$ at $\sqrt{s} = 13$ TeV.
3. Number of events for $\Upsilon(\mu^+\mu^-) + J/\psi(\mu^+\mu^-) + \phi(K^+K^-)$ with $p_T(\phi) > 2\ \mathrm{GeV}$ is about 80.
4. We can introduce cut to suppress SPS and DPS contributions.

Numerical Result for $\Upsilon + J/\psi + \phi$ at CMS/Atlas

$\Upsilon,\ J/\psi,\ \phi$ at CMS/Atlas

1. We can get the inclusive cross sections of $\Upsilon,\ J/\psi,\ \phi$ at $\sqrt{s} = 13$ TeV at CMS/Atlas, it $0.4,\ 30,\ 1200\ \mu$b for $p_T(\phi) > 2$ GeV.

Numerical Result for $\Upsilon + J/\psi + \phi$ at CMS/Atlas

$\Upsilon,\ J/\psi,\ \phi$ at CMS/Atlas

1. We can get the inclusive cross sections of $\Upsilon,\ J/\psi,\ \phi$ at $\sqrt{s} = 13$ TeV at CMS/Atlas, it $0.4,\ 30,\ 1200$ μb for $p_T(\phi) > 2$ GeV.
2. SPS and DPS are very small.

Numerical Result for $\Upsilon + J/\psi + \phi$ at CMS/Atlas

$\Upsilon, J/\psi, \phi$ at CMS/Atlas

1. We can get the inclusive cross sections of $\Upsilon, J/\psi, \phi$ at $\sqrt{s} = 13$ TeV at CMS/Atlas, it $0.4, 30, 1200\ \mu$b for $p_T(\phi) > 2$ GeV.
2. SPS and DPS are very small.
3. TPS $\Upsilon + J/\psi + \phi$: about $\frac{\sigma[\Upsilon]\sigma[J/\psi]\sigma[\phi]}{(\sigma_{eff}^{TPS})^2} \sim 200\ \text{pb}$ for $p_T(\phi) > 2$ GeV and $\sigma_{eff}^{TPS} \sim 8$ mb.

Numerical Result for $\Upsilon + J/\psi + \phi$ at CMS/Atlas

$\Upsilon,\ J/\psi,\ \phi$ at CMS/Atlas

1. We can get the inclusive cross sections of $\Upsilon,\ J/\psi,\ \phi$ at $\sqrt{s} = 13$ TeV at CMS/Atlas, it $0.4,\ 30,\ 1200\ \mu$b for $p_T(\phi) > 2$ GeV.
2. SPS and DPS are very small.
3. TPS $\Upsilon + J/\psi + \phi$: about $\frac{\sigma[\Upsilon]\sigma[J/\psi]\sigma[\phi]}{(\sigma_{eff}^{TPS})^2} \sim$ 200 pb for $p_T(\phi) > 2$ GeV and $\sigma_{eff}^{TPS} \sim 8$ mb.
4. Integrated luminosity of LHCb is about 40 fb^{-1} at $\sqrt{s} = 13$ TeV.

Numerical Result for $\Upsilon + J/\psi + \phi$ at CMS/Atlas

$\Upsilon,\ J/\psi,\ \phi$ at CMS/Atlas

1. We can get the inclusive cross sections of $\Upsilon,\ J/\psi,\ \phi$ at $\sqrt{s} = 13$ TeV at CMS/Atlas, it $0.4,\ 30,\ 1200\ \mu$b for $p_T(\phi) > 2$ GeV.
2. SPS and DPS are very small.
3. TPS $\Upsilon + J/\psi + \phi$: about $\frac{\sigma[\Upsilon]\sigma[J/\psi]\sigma[\phi]}{(\sigma_{eff}^{TPS})^2} \sim$ 200 pb for $p_T(\phi) > 2$ GeV and $\sigma_{eff}^{TPS} \sim 8$ mb.
4. Integrated luminosity of LHCb is about 40 fb^{-1} at $\sqrt{s} = 13$ TeV.
5. Number of events for $\Upsilon(\mu^+\mu^-) + J/\psi(\mu^+\mu^-) + \phi(K^+K^-)$ with $p_T(\phi) > 2\ \mathrm{GeV}$ is about 6000.

Numerical Result for $\Upsilon + J/\psi + \phi$ at CMS/Atlas

$\Upsilon,\ J/\psi,\ \phi$ at CMS/Atlas

1. We can get the inclusive cross sections of $\Upsilon,\ J/\psi,\ \phi$ at $\sqrt{s} = 13$ TeV at CMS/Atlas, it $0.4,\ 30,\ 1200\ \mu$b for $p_T(\phi) > 2$ GeV.
2. SPS and DPS are very small.
3. TPS $\Upsilon + J/\psi + \phi$: about $\frac{\sigma[\Upsilon]\sigma[J/\psi]\sigma[\phi]}{(\sigma_{eff}^{TPS})^2} \sim$ 200 pb for $p_T(\phi) > 2$ GeV and $\sigma_{eff}^{TPS} \sim 8$ mb.
4. Integrated luminosity of LHCb is about 40 fb^{-1} at $\sqrt{s} = 13$ TeV.
5. Number of events for $\Upsilon(\mu^+\mu^-) + J/\psi(\mu^+\mu^-) + \phi(K^+K^-)$ with $p_T(\phi) > 2$ GeV is about 6000.
6. We can introduce cut to suppress SPS and DPS contributions.

Summary

We have performed the first complete analysis of simultaneous production of prompt ψ and Υ mesons including all leading SPS contributions.

Our work shows that it is in fact most probably dominated by DPS contributions for D0 data.

Finally, we show that $\Upsilon + J/\psi + \phi$ at LHC is dominated by TPS. It may be studied by experimenters.