Parton Shower

Qi-Shu Yan

2018-05-14/18

Outline

- Why Parton Shower?
- What's Parton Shower?
- How to describe Parton Shower at LO?

Main References:

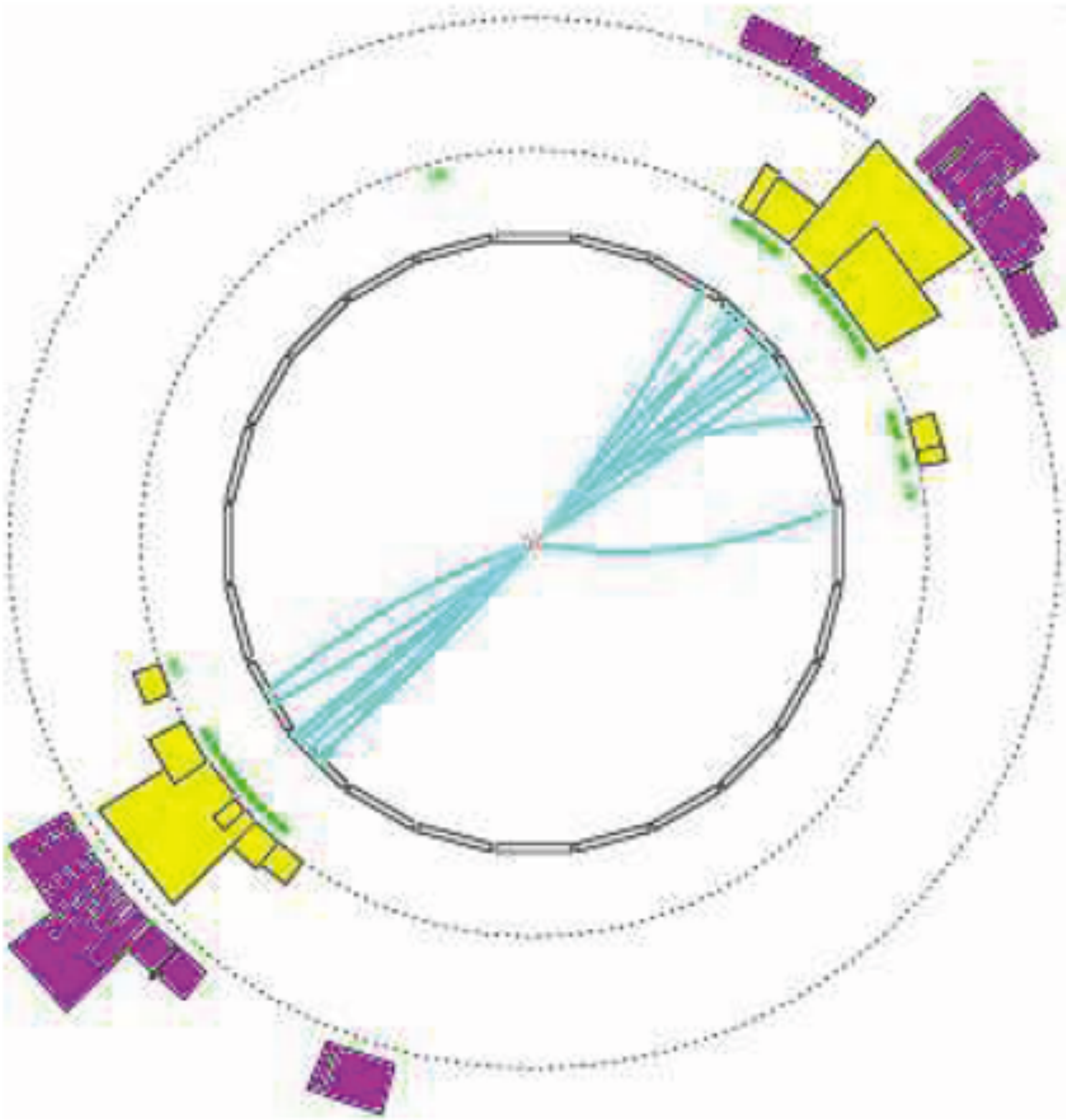
QCD and Collider Physics, *R.K.Ellis, W.J.Stirling and B.R. Webber*

Parton Shower, *Oliver Mattelaer, 2015, Beijing*

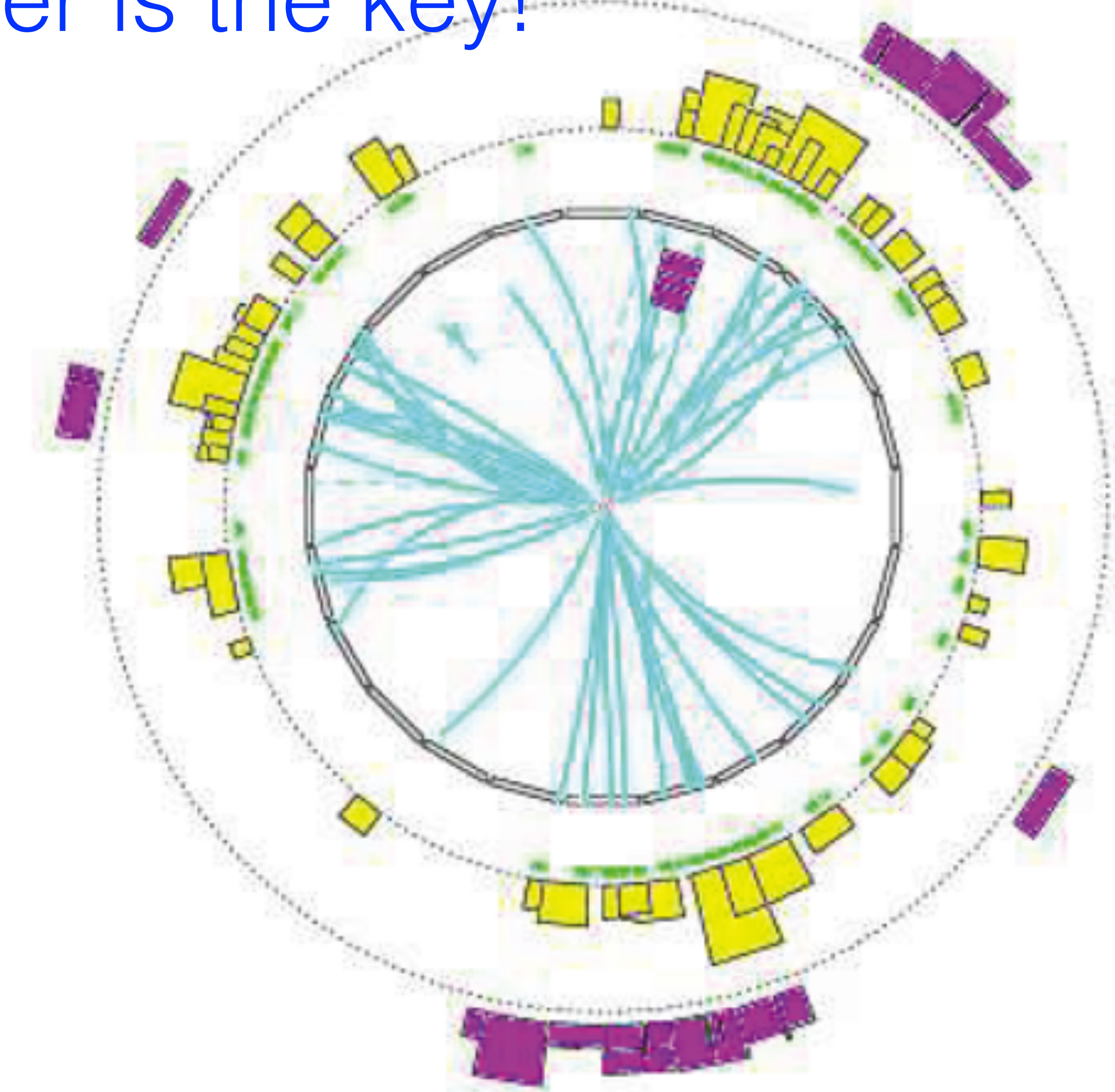
Introduction to parton-shower event generators, *Stefan Hoche, 1411.4085*

Understand Exp. by Th.?

Parton shower is the key!



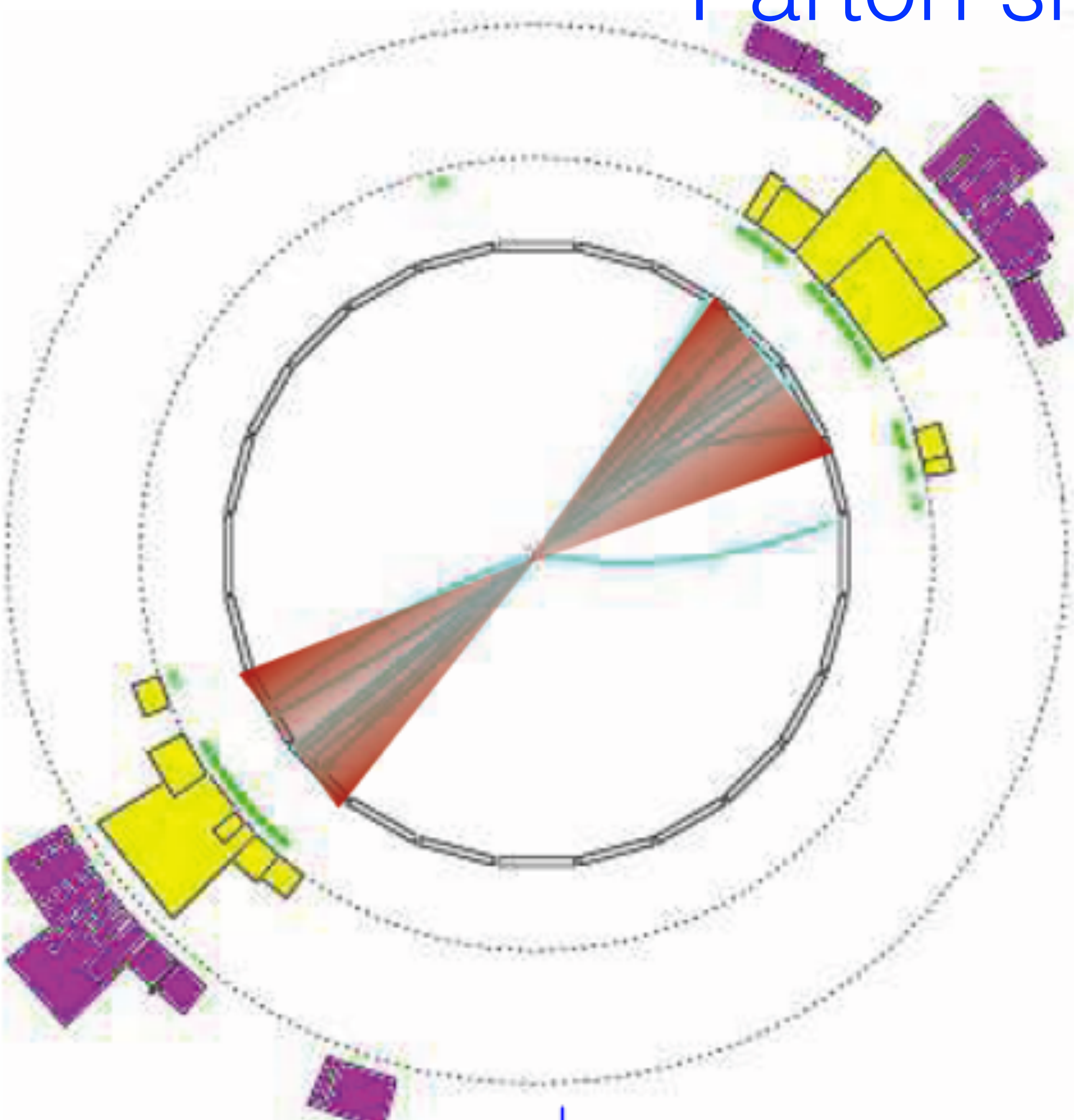
?2 → 16?



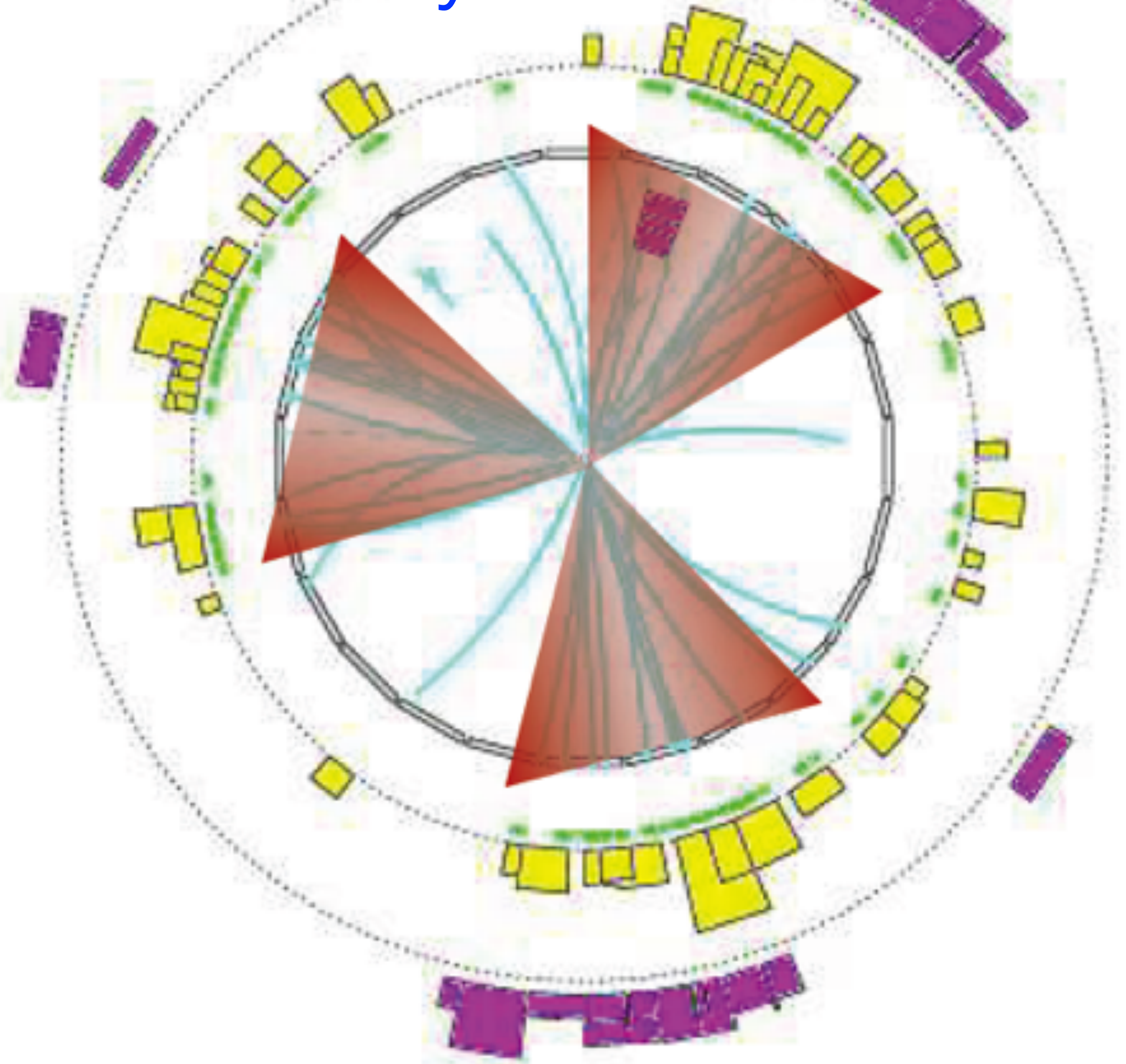
?2 → 40?

Understand Exp. by Th.?

Parton shower is the key!



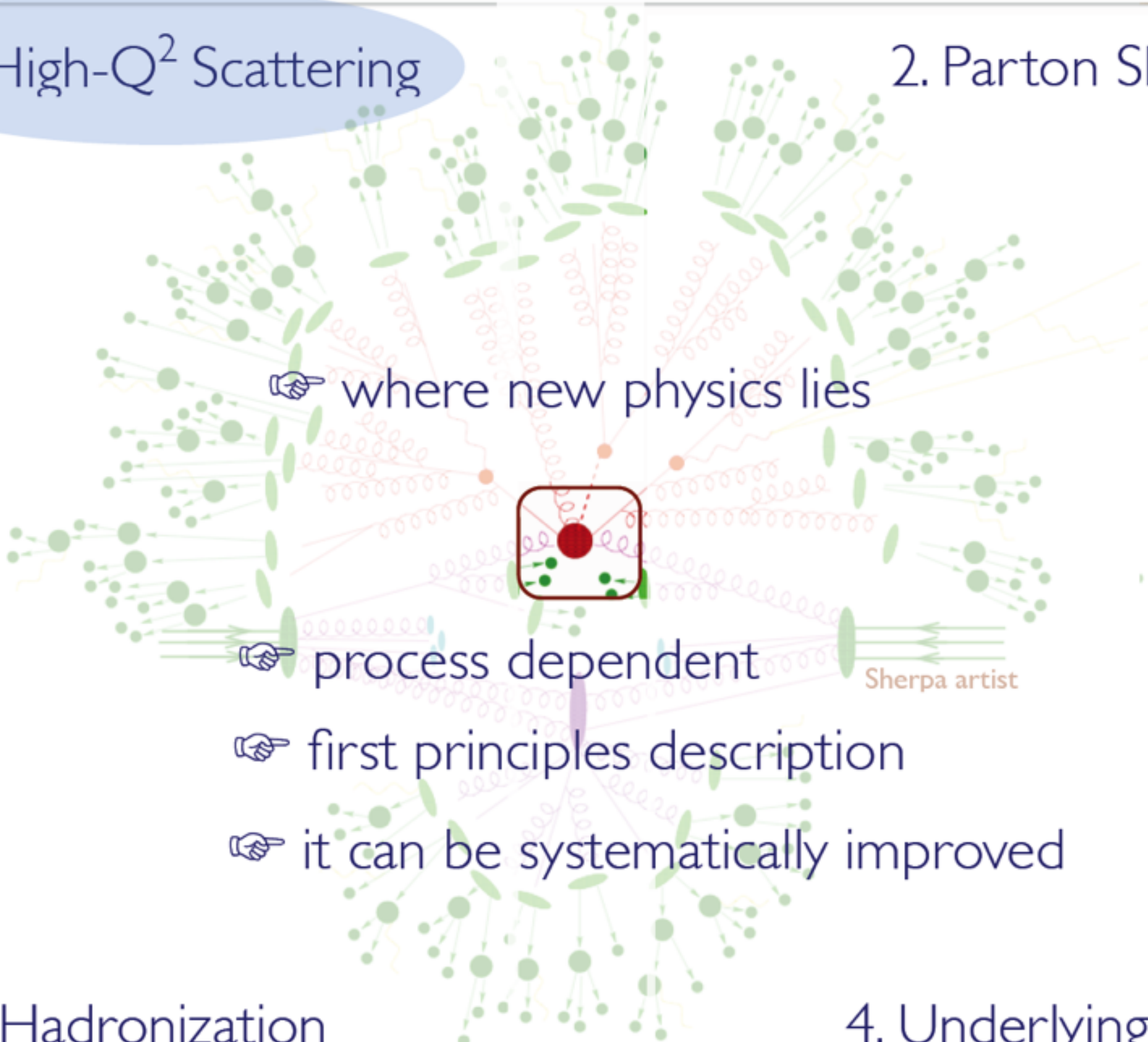
$e^+e^- \to q\bar{q}$



$e^+e^- \to q\bar{q}g$

1. High-Q^2 Scattering

2. Parton Shower



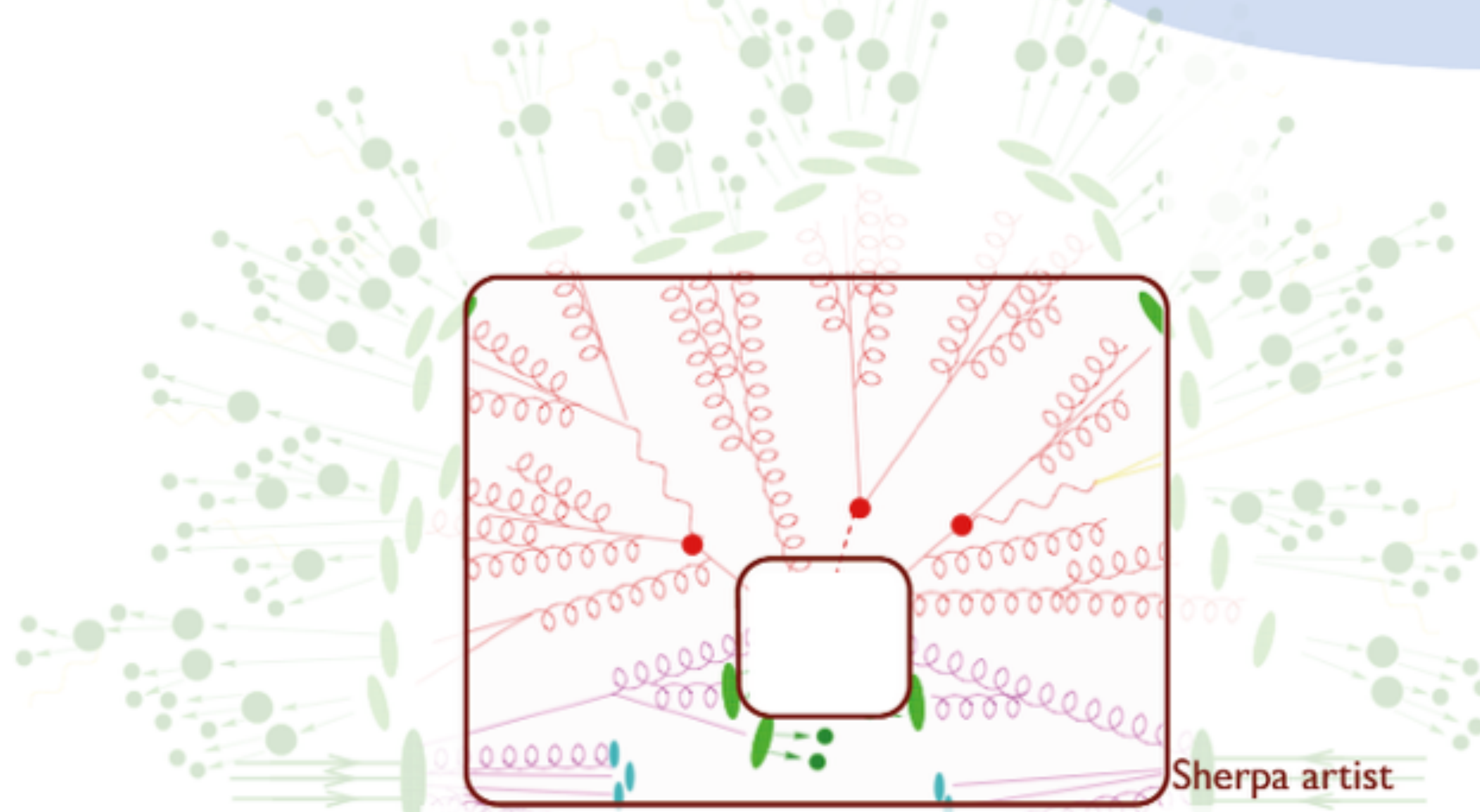
- ☞ where new physics lies
- ☞ process dependent
- ☞ first principles description
- ☞ it can be systematically improved

3. Hadronization

4. Underlying Event

1. High-Q^2 Scattering

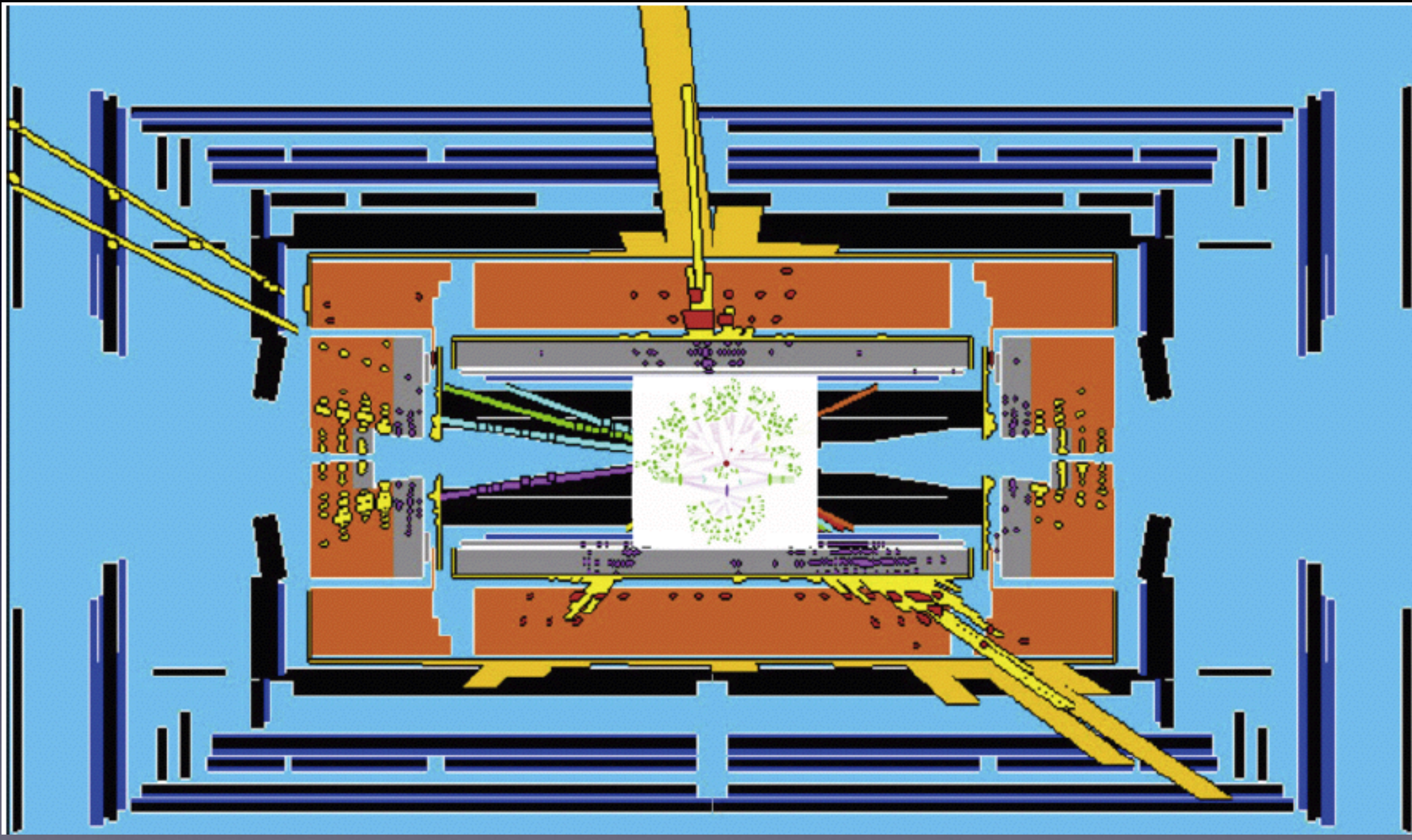
2. Parton Shower



- ☞ QCD -"known physics"
- ☞ universal/ process independent
- ☞ first principles description

3. Hadronization

4. Underlying Event



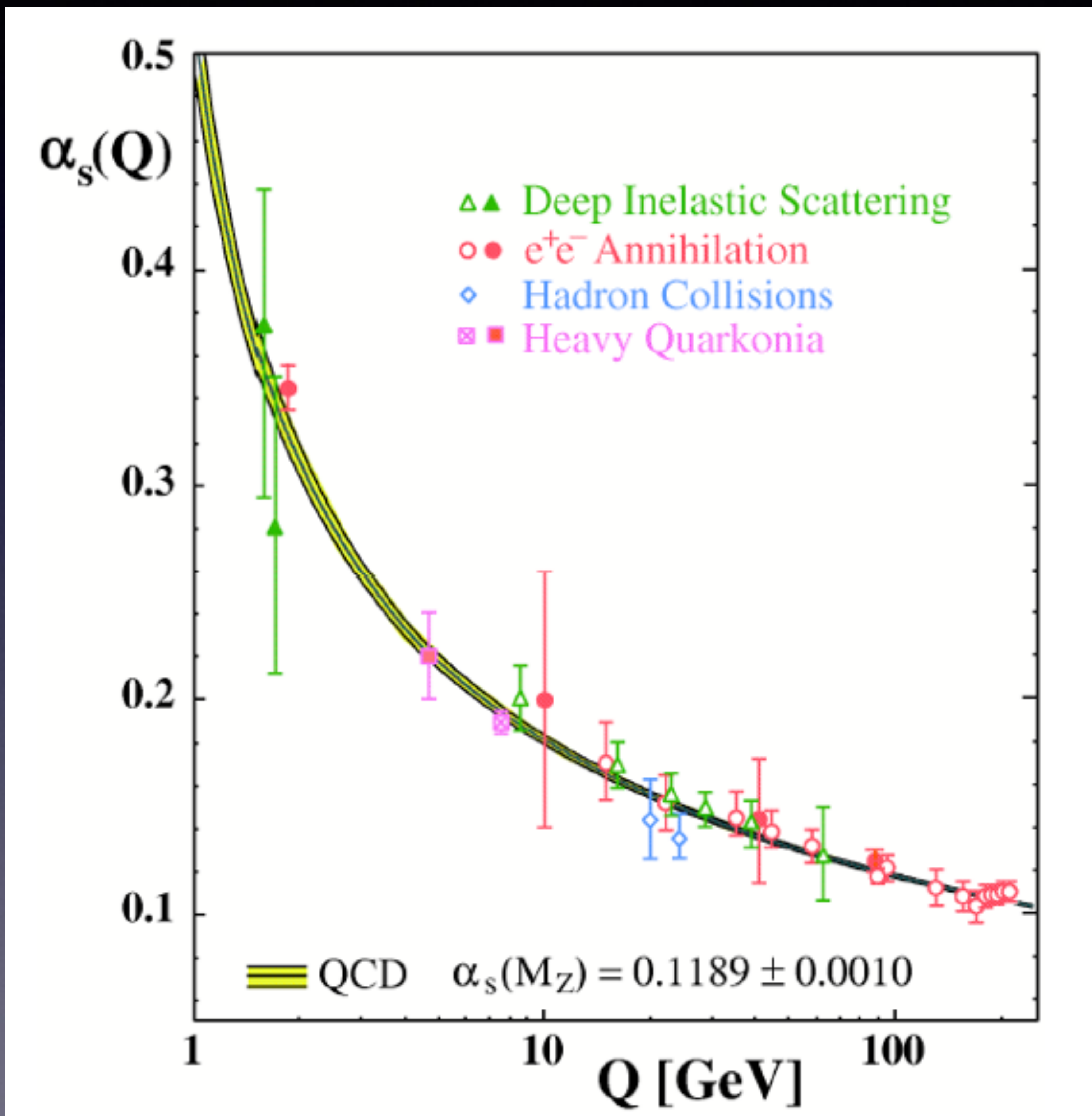
QCD

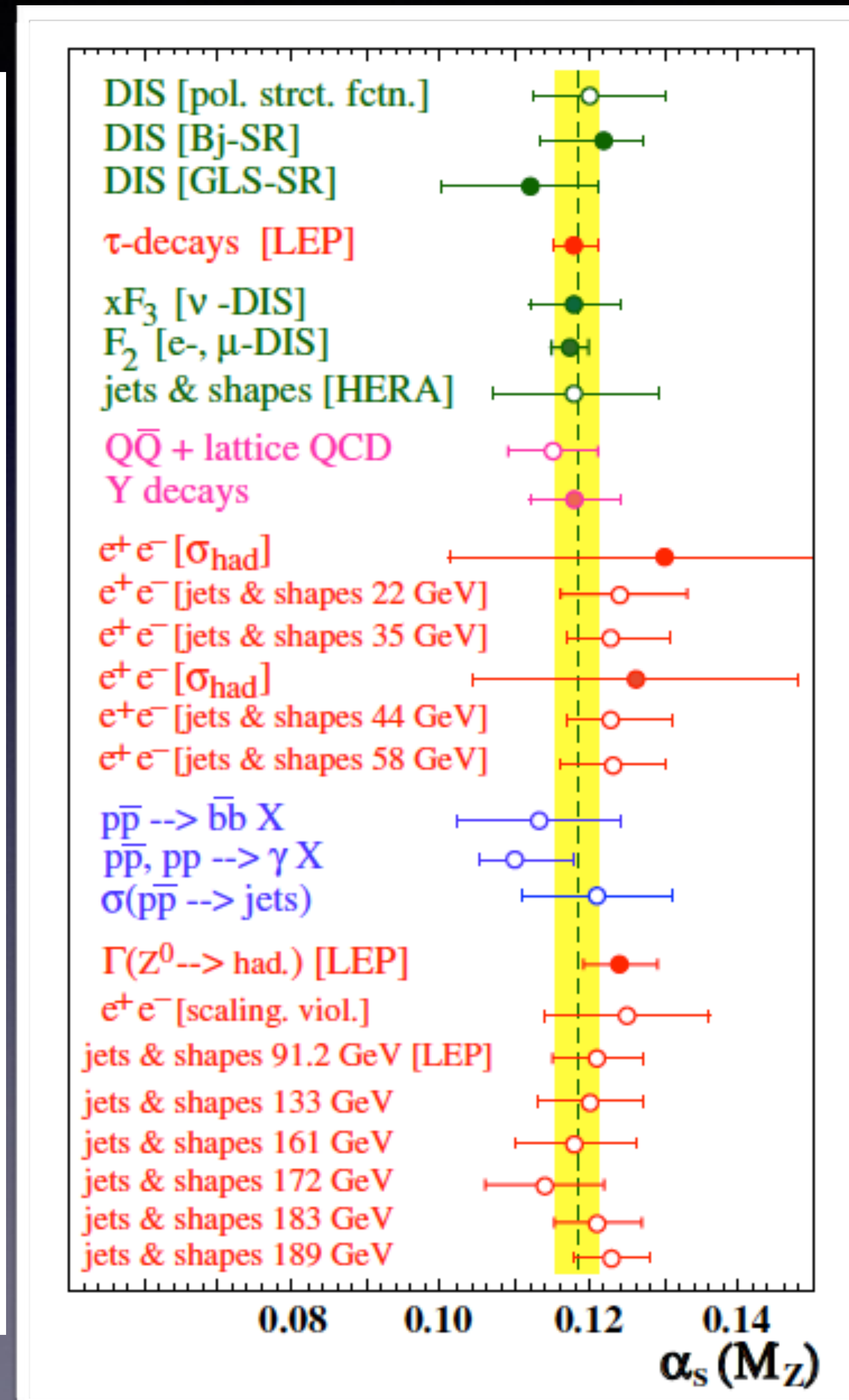
$$\mathcal{L}_{\text{classical}} = -\frac{1}{4} F^A_{\alpha\beta} F_A^{\alpha\beta} + \sum_{\text{flavours}} \bar{q}_a (i \not{D} - m)_{ab} q_b \,.$$

$$F^A_{\alpha\beta} = \left[\partial_\alpha \mathcal{A}^A_\beta - \partial_\beta \mathcal{A}^A_\alpha - g f^{ABC} \mathcal{A}^B_\alpha \mathcal{A}^C_\beta\right]$$

There is no asymptotic quarks or gluons.
Only color singlet is observed.

$$q^2\frac{\partial a_s}{\partial q^2} = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + O(a_s^6)$$



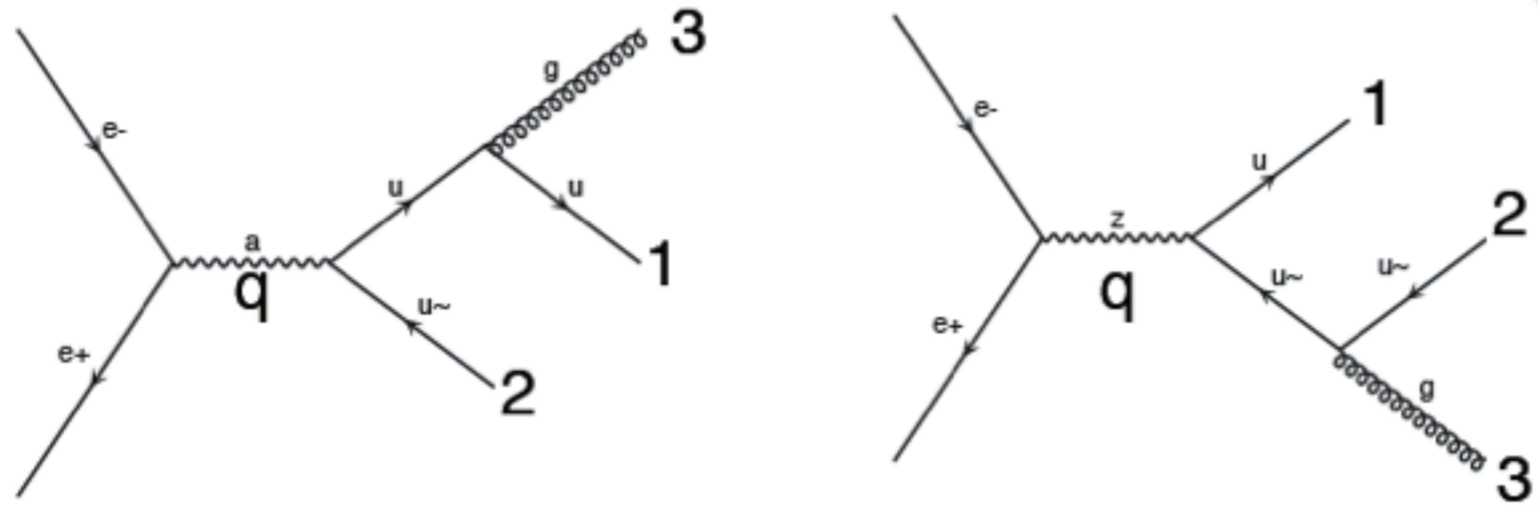


Description of Parton Shower

- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation
- This effect should be **unitary:** the inclusive cross section shouldn't change when extra radiation is added
- Remember that parton-level cross sections for a hard process are inclusive in anything else.
  E.g. for LO Drell-Yan production **all** radiation is included via PDFs (apart from non-perturbative power corrections)
- And finally we want to turn partons into hadrons (hadronization)....

The first Example

$$e^+e^- \to q\bar{q}g$$



$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

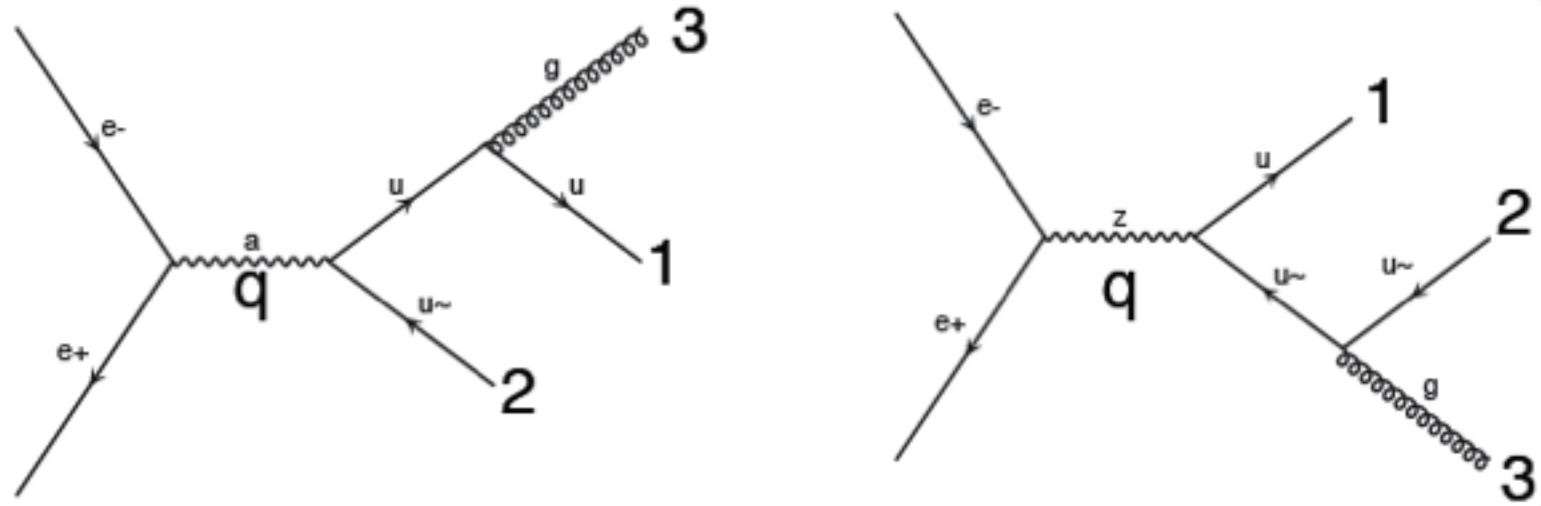
- Divergent at $x_1 = 1$ and $x_2 = 1$
  - Soft Divergencies
  - Collinear Divergencies

$$(1-x_1) = \frac{x_2 x_3}{2}(1 - cos\theta_{23})$$

$$(1-x_2) = \frac{x_1 x_3}{2}(1 - cos\theta_{13})$$

The first Example

$$e^+e^- \to q\bar{q}g$$



$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

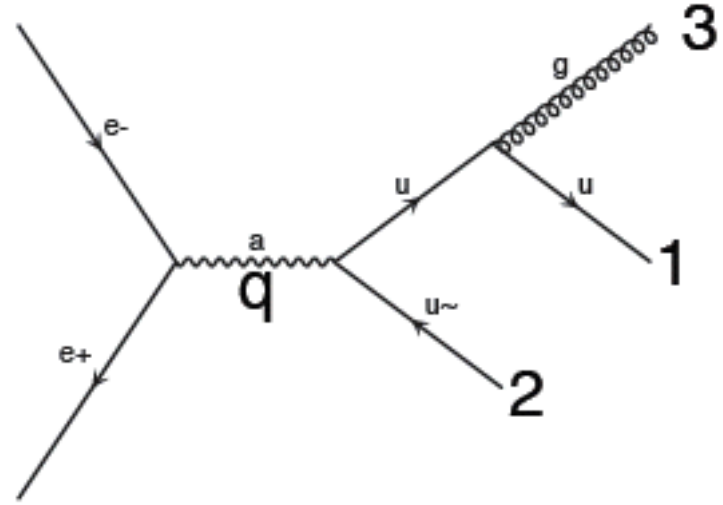
- Divergent at $x_1 = 1$ and $x_2 = 1$
  - Soft Divergencies
  - Collinear Divergencies

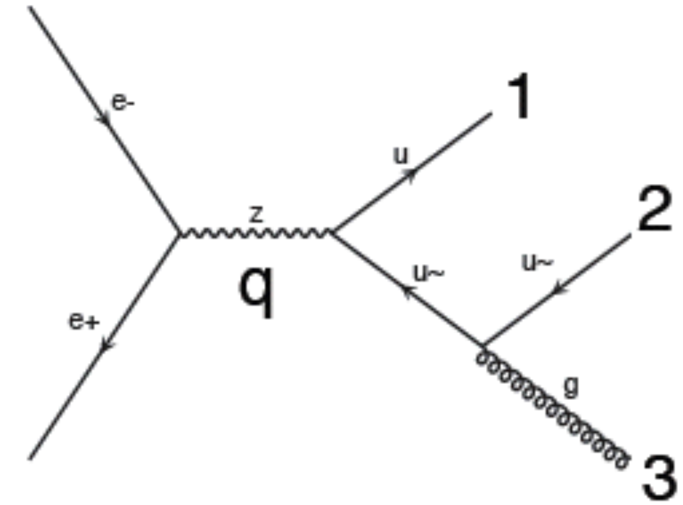
$$(1-x_1) = \frac{x_2 x_3}{2}(1 - cos\theta_{23})$$

$$(1-x_2) = \frac{x_1 x_3}{2}(1 - cos\theta_{13})$$

The first Example

$$e^+e^- \to q\bar{q}g$$





$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$

$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$

$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$

$$x_1 + x_2 + x_3 = 2$$

- Divergent at $x_1 = 1$ and $x_2 = 1$
  - Soft Divergencies
  - Collinear Divergencies

$$(1 - x_1) = \frac{x_2 x_3}{2}(1 - cos\theta_{23})$$

$$(1 - x_2) = \frac{x_1 x_3}{2}(1 - cos\theta_{13})$$

The first Example

$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

$$x_1 = 2k_1 \cdot q/q^2 = 2E_q/\sqrt{S}$$
$$x_2 = 2k_2 \cdot q/q^2 = 2E_{\bar{q}}/\sqrt{S}$$
$$x_3 = 2k_3 \cdot q/q^2 = 2E_g/\sqrt{S}$$
$$x_1 + x_2 + x_3 = 2$$

- Change the variable to x_3 and $\cos\theta_{13}$

$$\frac{d\sigma}{dx_3 d\cos\theta_{13}} = \sigma_0 C_F \frac{\alpha_s}{2\pi} \left(\frac{2}{\sin^2\theta_{13}} \frac{1+(1-x_3)^2}{x_3} - x_3 \right)$$

- Collinear limit
  - Split our integral in two

$$\frac{2\, dcos\theta_{13}}{sin^2\theta_{13}} = \frac{dcos\theta_{13}}{1-cos\theta_{13}} + \frac{dcos\theta_{13}}{1+cos\theta_{13}}$$
$$\approx \frac{d\cos\theta_{13}}{(1-\cos\theta_{13})} + \frac{d\cos\theta_{23}}{(1-\cos\theta_{23})}$$
$$\approx \frac{d\theta_{13}^2}{\theta_{13}^2} + \frac{d\theta_{23}^2}{\theta_{23}^2}$$

The first Example

- Change the variable to x_3 and $\cos\theta_{13}$

$$\frac{d\sigma}{dx_3 d\cos\theta_{13}} = \sigma_0 C_F \frac{\alpha_s}{2\pi}\left(\frac{2}{\sin^2\theta_{13}}\frac{1+(1-x_3)^2}{x_3} - x_3\right)$$

- Collinear limit
  - Split our integral in two

$$\frac{2\,dcos\theta_{13}}{sin^2\theta_{13}} = \frac{dcos\theta_{13}}{1-cos\theta_{13}} + \frac{dcos\theta_{13}}{1+cos\theta_{13}}$$

$$\approx \frac{d\cos\theta_{13}}{(1-\cos\theta_{13})} + \frac{d\cos\theta_{23}}{(1-\cos\theta_{23})}$$

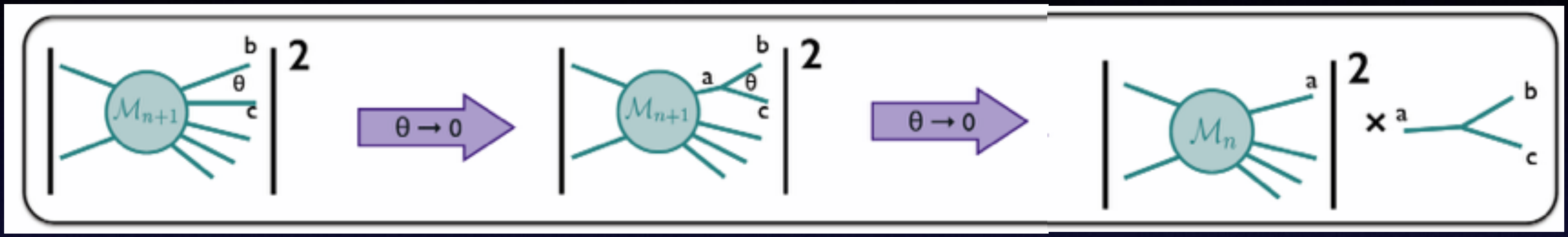
$$\approx \frac{d\theta_{13}^2}{\theta_{13}^2} + \frac{d\theta_{23}^2}{\theta_{23}^2}$$

$$d\sigma = \sigma_0 \sum_{\text{jets}} C_F \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2}\, dz \frac{1+(1-z)^2}{z}$$

☞ z fraction of energy

☞ **Generic Formula**

Collinear Factorization



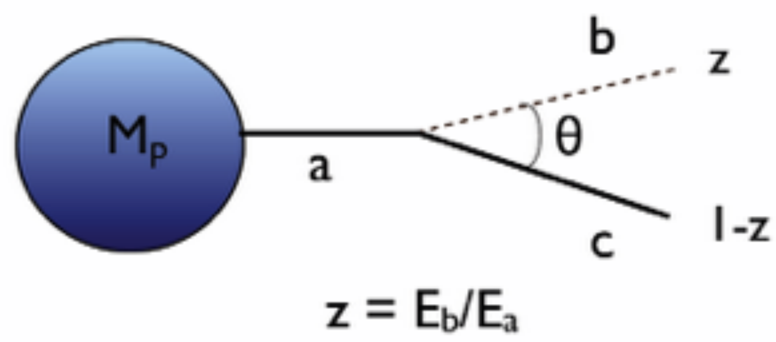
- Consider a process for which two particles are separated by a small angle θ.
- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.
- The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability.

Collinear Factorization

- The process factorizes in the collinear limit. This procedure it universal!

$$\frac{1}{(p_b+p_c)^2} \simeq \frac{1}{2E_bE_c(1-\cos\theta)} = \frac{1}{t}$$

$t > 0$



soft and collinear

divergencies

Collinear factorization:

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

when θ is small.

Collinear Factorization

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

- **t** can be called the 'evolution variable' (will become clearer later): it can be the virtuality $\mathbf{m^2}$ of particle a or its $\mathbf{p_T^2}$ or $\mathbf{E^2\theta^2}$ …

$$d\theta^2/\theta^2 = dm^2/m^2 = dp_T^2/p_T^2$$

$$m^2 \simeq z(1-z)\theta^2 E_a^2$$

$$p_T^2 \simeq z m^2$$

  - It represents the hardness of the branching and tends to 0 in the collinear limit.
  - Different choice of 'evolution parameter' in different Parton-shower code

Collinear Factorization

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

- **z** is the "energy variable": it is defined to be the energy fraction taken by parton **b** from parton **a.** It represents the energy sharing between **b** and **c** and tends to 1 in the soft limit (parton c going soft)

- **Φ** is the azimuthal angle. It can be chosen to be the angle between the polarization of a and the plane of the branching.

Collinear Factorization

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

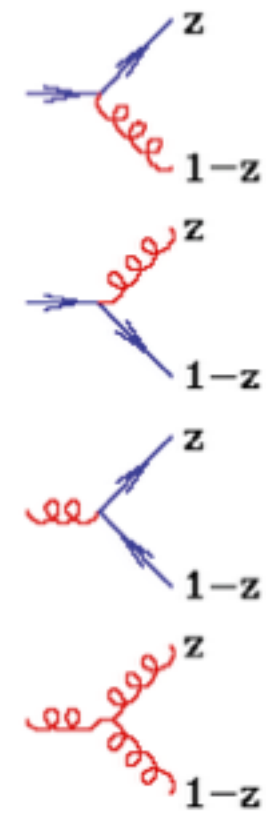
The spin averaged (unregulated) splitting functions for the various types of branching are (Altarelli-Parisi):

$$\hat{P}_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)}\right],$$

$$\hat{P}_{gq}(z) = C_F \left[\frac{1+(1-z)^2}{z}\right],$$

$$\hat{P}_{qg}(z) = T_R \left[z^2+(1-z)^2\right],$$

$$\hat{P}_{gg}(z) = C_A \left[\frac{z}{(1-z)} + \frac{1-z}{z} + z(1-z)\right].$$



$C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}.$

Encode the difference between quark/gluon jets!

Collinear Factorization

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

- Each choice of argument for α_S is equally acceptable at the leading-logarithmic accuracy. However, there is a choice that allows one to resum certain classes of subleading logarithms.

- The higher order corrections to the partons splittings imply that the splitting kernels should be modified: $P_{a\longrightarrow bc}(z) \longrightarrow P_{a\longrightarrow bc}(z) + \alpha_s P'_{a\longrightarrow bc}(z)$

For $g \longrightarrow gg$ branchings $P'_{a\longrightarrow bc}(z)$ diverges as $-b_0 \log[z(1-z)]\ P_{a\longrightarrow bc}(z)$ (just z or 1-z if quark is present)

- Recall the one-loop running of the strong coupling:

$$\alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1 + \alpha_S(\mu^2) b_0 \log \frac{Q^2}{\mu^2}} \sim \alpha_S(\mu^2)\left(1 - \alpha_S(\mu^2) b_0 \log \frac{Q^2}{\mu^2}\right)$$

- We can therefore include the $P'(z)$ terms by choosing $p_T^2 \sim z(1-z)Q^2$ as argument of α_S:

$$\begin{aligned}\alpha_S(Q^2)\left(P_{a\to bc}(z) + \alpha_S(Q^2) P'_{a\to bc}\right) &= \alpha_S(Q^2)\left(1 - \alpha_S(Q^2) b \log z(1-z)\right) P_{a\to bc}(z) \\ &\sim \alpha_S(z(1-z)Q^2) P_{a\to bc}(z)\end{aligned}$$

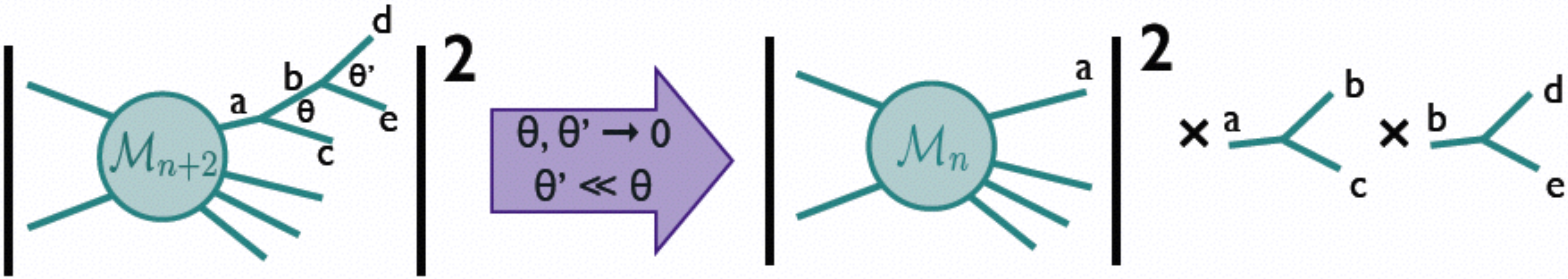
Emphasize:

Collinear Limit

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

- **t** is the evolution parameter (control the collinear behaviour)
- **z** is the energy sharing variable
- **alpha_s** need to be evaluated at the scale t
- **P** is the splitting Kernel (control the soft behaviour)

Multiple Emission

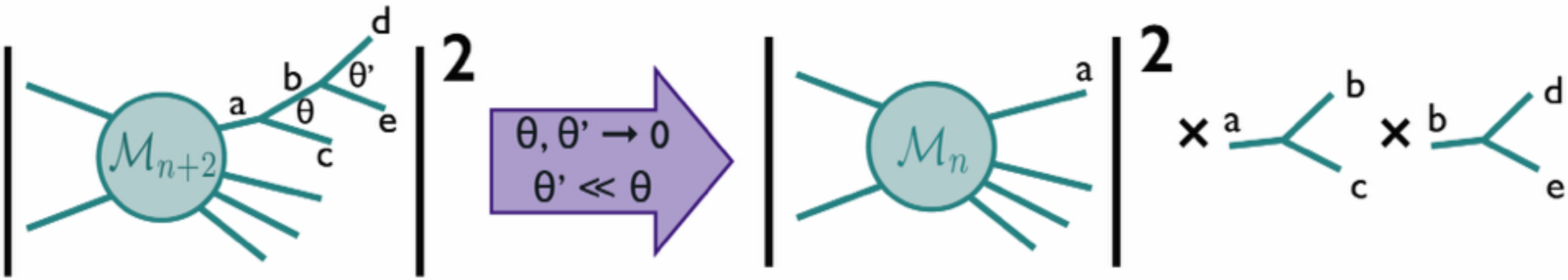


- Now consider $\mathbf{M_{n+1}}$ as the new core process and use the recipe we used for the first emission in order to get the dominant contribution to the (n+2)-body cross section: add a new branching at angle much smaller than the previous one:

$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z) \times \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_s}{2\pi} P_{b\to de}(z')$$

- This can be done for an arbitrary number of emissions.The recipe to get the leading collinear singularity is thus cast in the form of an iterative sequence of emissions whose probability does not depend on the past history of the system: a 'Markov chain'. No interference!!!

Multiple Emission



- The dominant contribution comes from the region where the subsequently emitted partons satisfy the strong ordering requirement: $\theta \gg \theta' \gg \theta''$...

  For the rate for multiple emission we get

$$\sigma_{n+k} \propto \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \cdots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

  where Q is a typical hard scale and Q_0 is a small infrared cutoff that separates perturbative from non perturbative regimes.

- Each power of α_s comes with a logarithm. The logarithm can be easily large, and therefore it can lead to a breakdown of perturbation theory.

Sudakov Factor

- What is the probability of no emission?

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int dz\hat{P}(z)$$

- So the probability of no emission between two scales:

$$P_{no-branching}(Q^2,t) = \lim_{N\to\infty}\prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)$$

$$\simeq \lim_{N\to\infty} e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_S}{2\pi}\int dz\hat{P}(z)\right)}$$

Sudakov form factor

$$\Delta(Q^2,t) \simeq e^{-\int_t^{Q^2}\frac{dt'}{t'}\int dz\frac{\alpha_S}{2\pi}\hat{P}(z)} \equiv e^{-\int_t^{Q^2}dp(t')}$$

➡Property: Δ(A,B) = Δ(A,C) Δ(C,B)

Parton Shower

- The Sudakov form factor is the heart of the parton shower. It gives the probability that a parton does not branch between two scales
- Using this no-emission probability the branching tree of a parton is generated.
- Define **dP_k** as the probability for k ordered splittings from leg a at given scales

$$
\begin{aligned}
dP_1(t_1) &= \Delta(Q^2, t_1)\, dp(t_1)\Delta(t_1, Q_0^2), \\
dP_2(t_1, t_2) &= \Delta(Q^2, t_1)\, dp(t_1)\, \Delta(t_1, t_2)\, dp(t_2)\, \Delta(t_2, Q_0^2)\Theta(t_1 - t_2), \\
\ldots &= \ldots \\
dP_k(t_1, ..., t_k) &= \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)
\end{aligned}
$$

- **Q_0^2** is the hadronization scale (~1 GeV). Below this scale we do not trust the perturbative description for parton splitting anymore.

Unitarity

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l) \Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, ...$$

- Summing over all number of emissions

$$\sum_{k=0}^{\infty} P_k = \Delta(Q^2, Q_0^2) \sum_{k=0}^{\infty} \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k = \Delta(Q^2, Q_0^2) \exp \left[\int_{Q_0^2}^{Q^2} dp(t) \right] = 1$$

- Hence, the total probability is conserved

Singularity

- We have shown that the showers is unitary. However, how are the IR divergences cancelled explicitly? Let's show this for the first emission:
  Consider the contributions from (exactly) 0 and 1 emissions from leg a:

$$\frac{d\sigma}{\sigma_n} = \Delta(Q^2, Q_0^2) + \Delta(Q^2, Q_0^2) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

- Expanding to first order in α_s gives

$$\frac{d\sigma}{\sigma_n} \simeq 1 - \sum_{bc} \int_{Q_0^2}^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z) + \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)$$

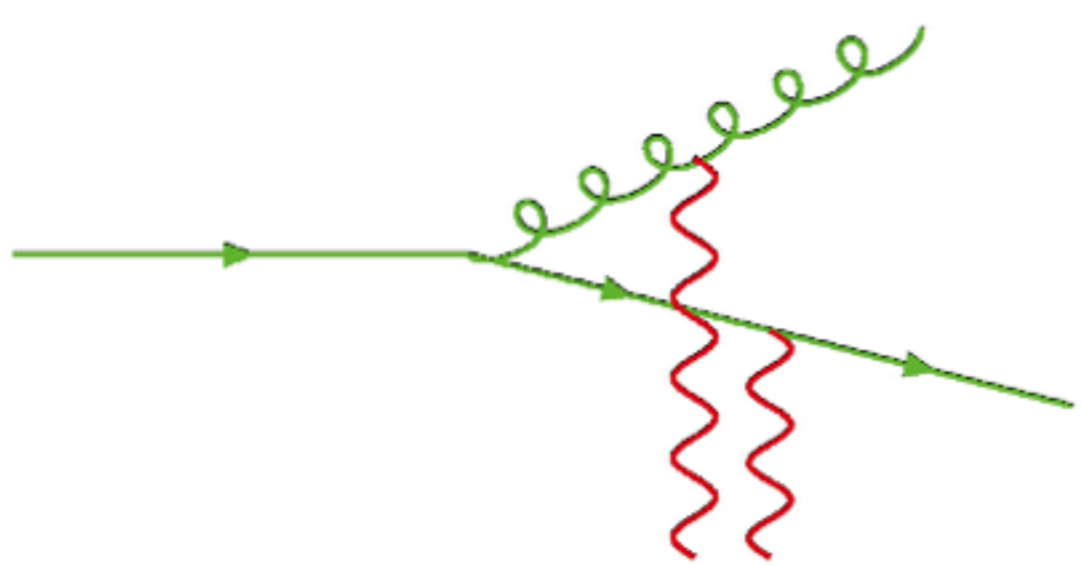
- Same structure of the two latter terms, with opposite signs: cancellation of divergences between the approximate virtual and approximate real emission cross sections.

- The probabilistic interpretation of the shower ensures that infrared divergences will cancel for each emission.

Coherent Splitting

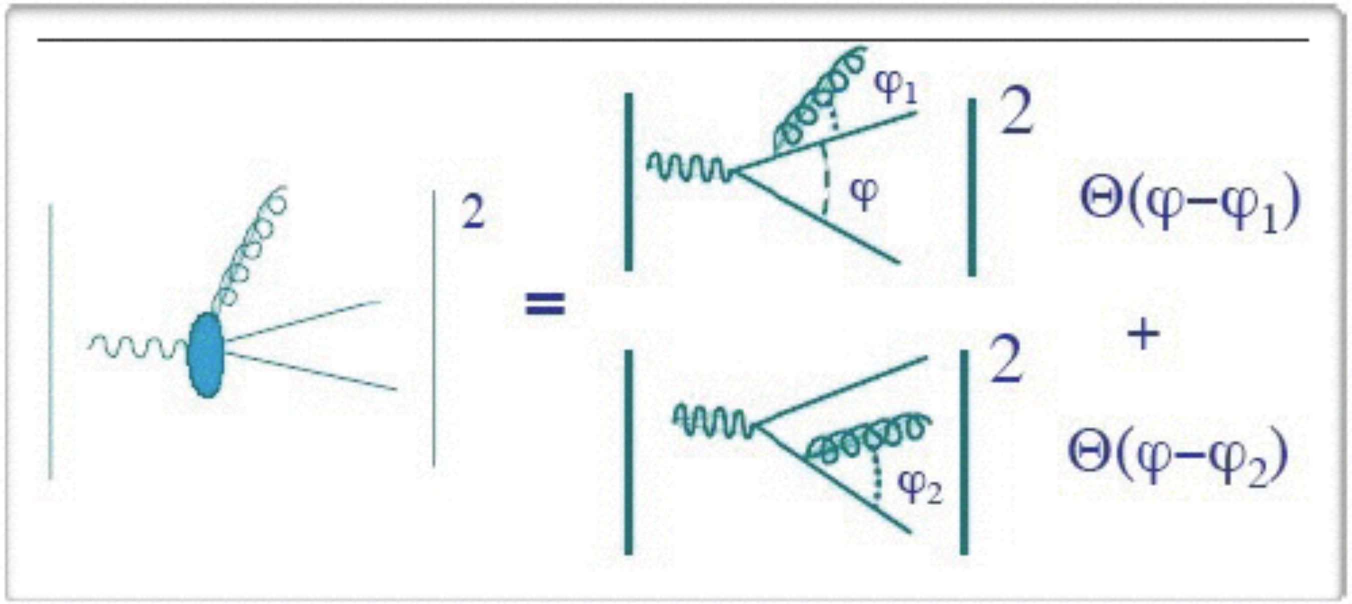
$$\Delta(Q^2,t) = \exp\left[-\sum_{bc}\int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi}\frac{\alpha_s}{2\pi} P_{a\to bc}(z)\right]$$

- There is a lot of freedom in the choice of evolution parameter t. It can be the virtuality m^2 of particle a or its p_T^2 or $E^2\theta^2$ ... For the collinear limit they are all equivalent
- However, in the soft limit ($z \longrightarrow 0,1$) they behave differently
- Can we chose it such that we get the correct soft limit?
- Soft gluon comes from the full event!



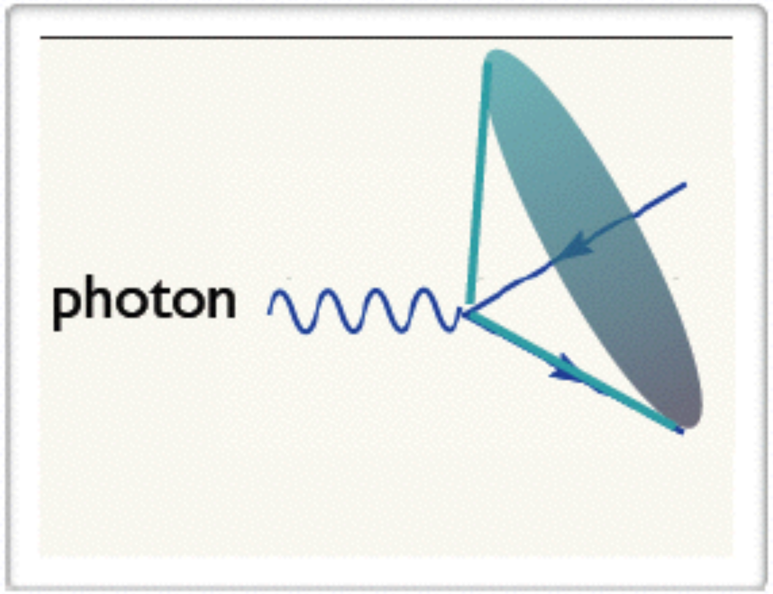
- Quantum Interference

Coherent Splitting

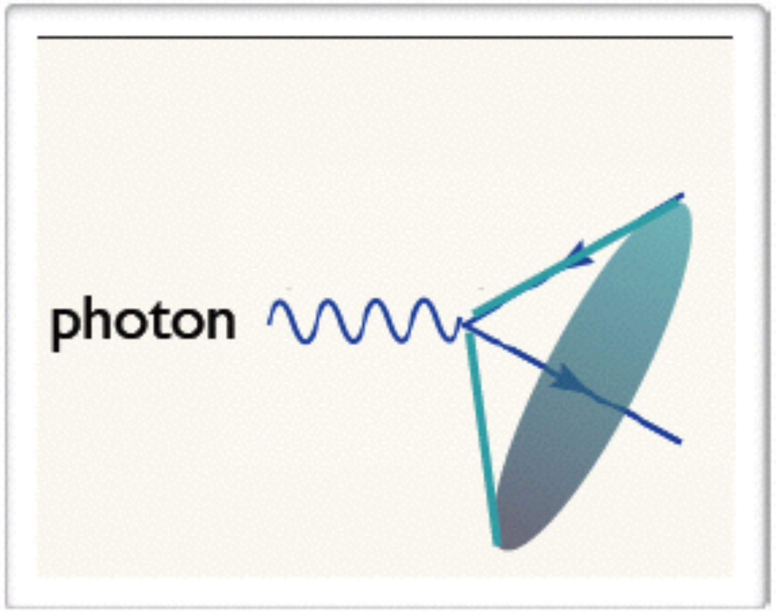


Chudakov effect

Radiation inside cones around the original partons is allowed (and described by the eikonal approximation), outside the cones it is zero (after averaging over the azimuthal angle)



+



Coherent Splitting

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega} \frac{d\Omega}{2\pi} \frac{\alpha_S}{2\pi} \sum_{i,j} C_{ij} W_{ij}$$

$$W_{ij} = \frac{\omega^2 p_i \cdot p_j}{p_i \cdot q \, p_j \cdot q} = \frac{1 - v_i v_j \cos\theta_{ij}}{(1 - v_i \cos\theta_{iq})(1 - v_j \cos\theta_{jq})}$$

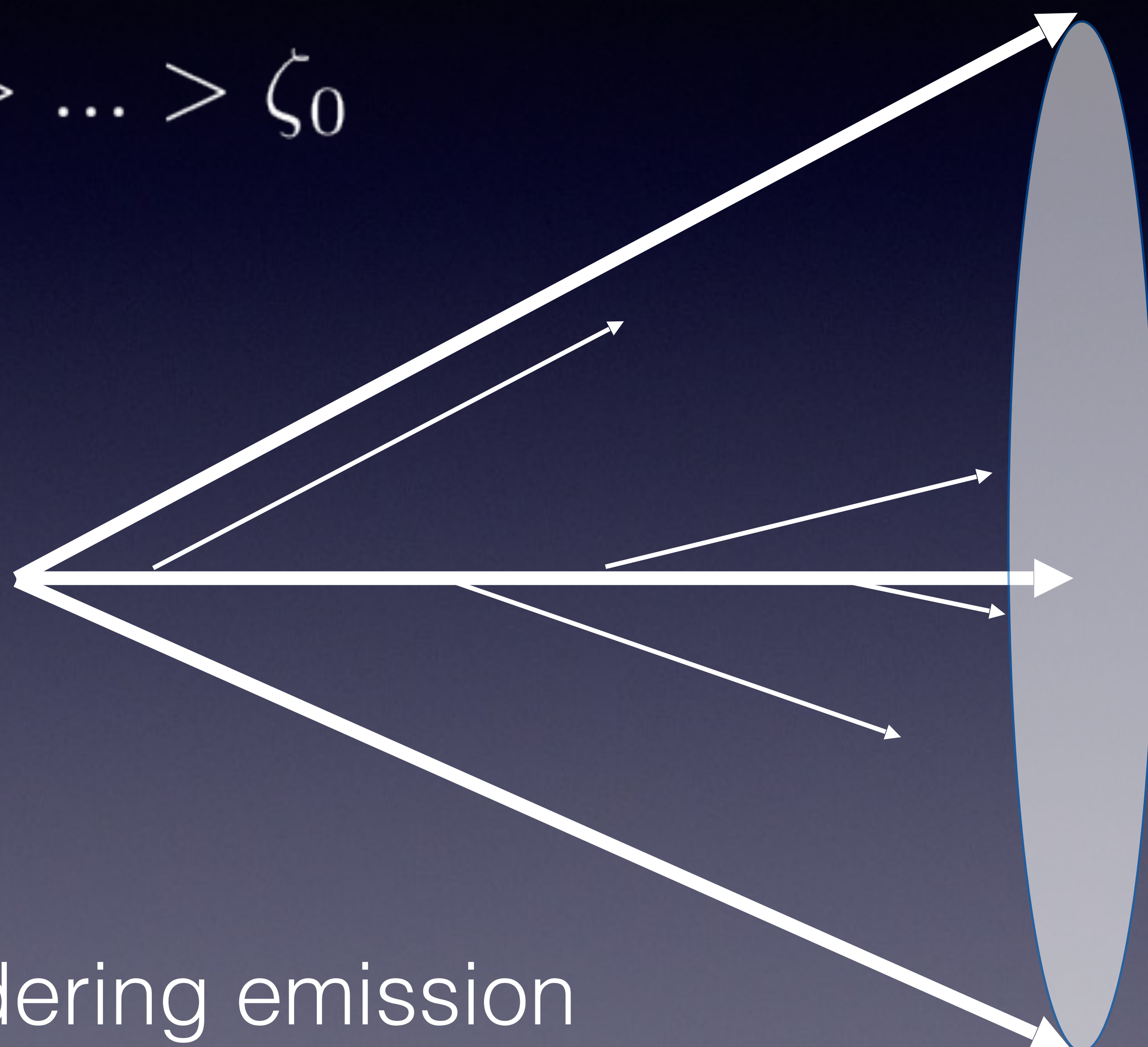
$$W_{ij} = W_{ij}^{[i]} + W_{ij}^{[j]}$$

$$W_{ij}^{[i]} = \frac{1}{2}\left(W_{ij} + \frac{1}{1 - \cos\theta_{iq}} - \frac{1}{1 - \cos\theta_{jq}}\right)$$

$$\begin{aligned} \int_0^{2\pi} \frac{d\phi_{iq}}{2\pi} W_{ij}^{[i]} &= \frac{1}{1 - \cos\theta_{iq}} \quad \text{if } \theta_{iq} < \theta_{ij} \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

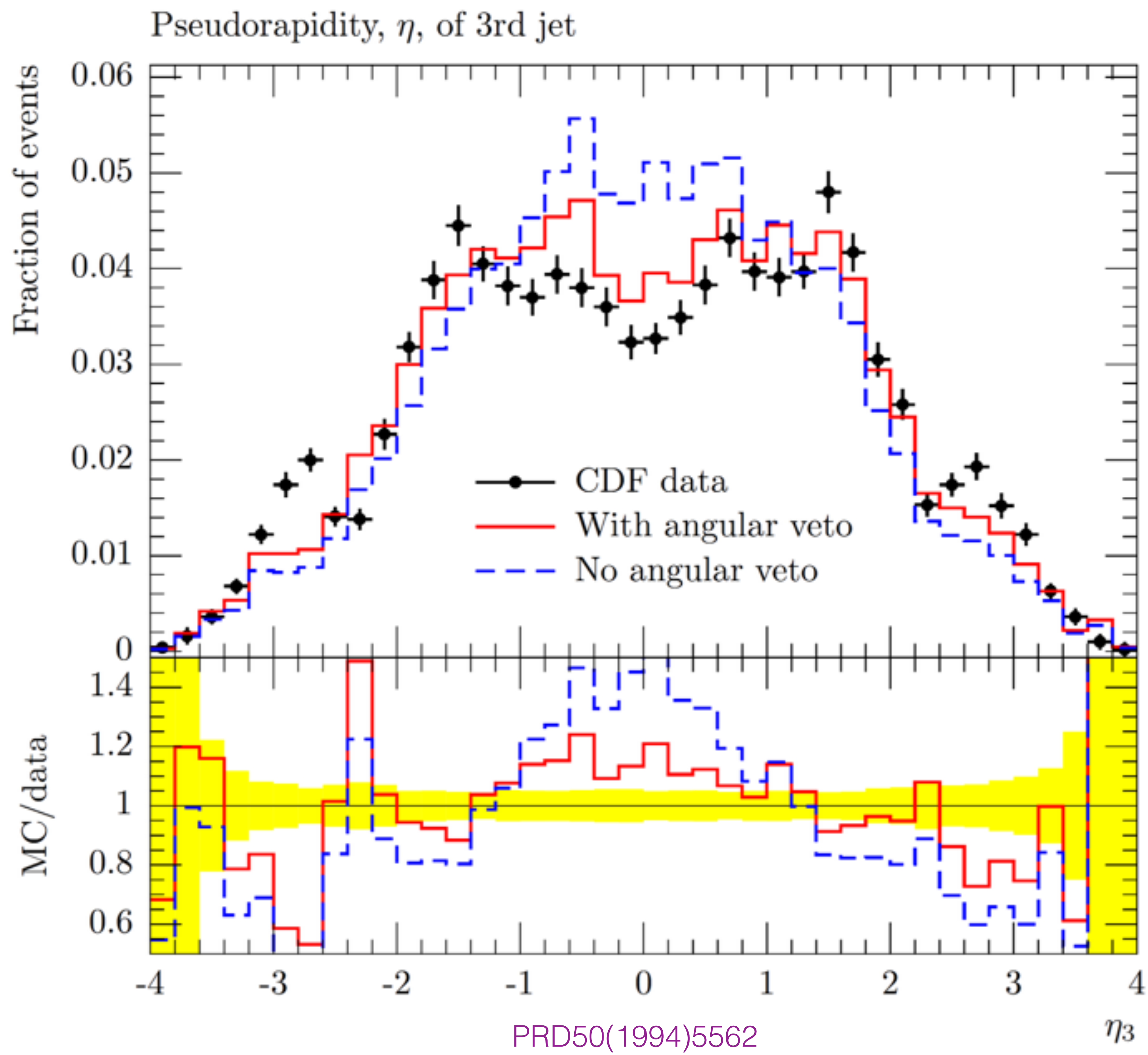
$$\zeta_0 = t_0/E^2$$

$$\zeta_n > \zeta_{n-1} > \ldots > \zeta_0$$



Angular ordering emission

Angular Ordering



PRD50(1994)5562

ME

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

Shower MC

1. Resums logs to all orders
2. Computationally cheap
3. No limit on particle multiplicity
4. Valid when partons are collinear and/or soft
5. Partial interference through angular ordering
6. Needed for hadronization

Summary

- Parton showering is controlled by the QCD
- In the collinear limit, the cross section can be factorisable
- Sudakov form factor is the heart of power shower
- Radiations are angular ordered due to the quantum interference