QCD Aspects of Radiative Leptonic B Decays (with Subleading Power Corrections)

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Why power corrections?

- Understanding the general properties power expansion in EFTs (HQET, SCET, NRQCD).

- Interesting to understand the strong interaction dynamics of heavy quark decays.
  - Factorization properties of the subleading-power amplitudes.
  - Renormalization and asymptotic properties of higher-twist B-meson DAs.
  - Interplay of different QCD techniques.

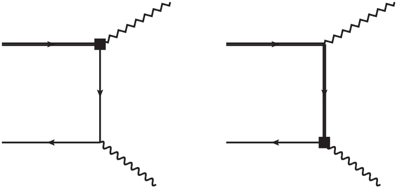
- Precision determinations of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$.
  Power corrections, QED corrections, BSM physics.

- Crucial to understand the CP violation in B-meson decays.
  Strong phase of $\mathscr{A}(B \to M_1 M_2)$ @ m_b scale in the leading power.

- Indispensable for understanding the flavour puzzles.
  - P_5' anomaly in $B \to K^* \ell^+ \ell^-$.
  - Color suppressed hadronic B-meson decays.
  - Polarization fractions of penguin dominated $B_{(s)} \to VV$ decays.

General aspects of $B \to \gamma \ell \nu$

- Tree diagrams:



Kinematics:

$$p_B \equiv p + q = m_B\, v\,, \qquad p = \frac{n\cdot p}{2}\,\bar{n}\,, \qquad q = \frac{n\cdot q}{2}\,\bar{n} + \frac{\bar{n}\cdot q}{2}\,n\,.$$

- Decay amplitude:

$$\mathscr{M}(B^- \to \gamma \ell \nu) = \frac{G_F V_{ub}}{\sqrt{2}}\,(i\, g_{em}\, \varepsilon_\nu^*) \left\{ T^{\nu\mu}(p,q)\, \bar{\ell}\, \gamma_\mu\,(1-\gamma_5)\nu + Q_\ell\, f_B\, \bar{\ell}\, \gamma^\nu\,(1-\gamma_5)\nu \right\}.$$

Hadronic tensor:

$$\begin{aligned} T_{\nu\mu}(p,q) &\equiv \int d^4x\, e^{ip\cdot x}\, \langle 0|\mathrm{T}\{j_{\nu,em}(x), \left[\bar{u}\gamma_\mu(1-\gamma_5)b\right](0)\}|B^-(p+q)\rangle\,, \\ &= v\cdot p\left[-i\,\varepsilon_{\mu\nu\rho\sigma}\, n^\rho\, v^\sigma\, F_V(n\cdot p) + g_{\mu\nu}\,\hat{F}_A(n\cdot p)\right] + v_\nu\, p_\mu\, F_1(n\cdot p) \\ &\quad + v_\mu\, p_\nu\, F_2(n\cdot p) + v\cdot p\; v_\mu\, v_\nu\, F_3(n\cdot p) + \frac{p_\mu\, p_\nu}{v\cdot p}\, F_4(n\cdot p)\,. \end{aligned}$$

General aspects of $B \to \gamma \ell \, \nu$

- Ward identity [Grinstein and Pirjol, 2000; Khodjamirian and Wyler, 2001]:

$$p_\nu \, T^{\nu\mu}(p,q) = -(Q_b - Q_u) f_B \, p_B^\mu \, .$$

$$\Downarrow$$

$$\hat{F}_A(v \cdot p) = -F_1(v \cdot p) \, , \qquad F_3(v \cdot p) = -\frac{(Q_b - Q_u) f_B \, m_B}{(v \cdot p)^2} \, .$$

- Reduced parametrization:

$$T_{\nu\mu}(p,q) = -i \, v \cdot p \, \varepsilon_{\mu\nu\rho\sigma} \, n^\rho \, v^\sigma \, F_V(n \cdot p) + \left[g_{\mu\nu} \, v \cdot p - v_\nu \, p_\mu \right] \hat{F}_A(n \cdot p) \underbrace{- \frac{(Q_b - Q_u) f_B \, m_B}{v \cdot p} v_\mu \, v_\nu \, .}_{\text{contact term}}$$

- Absorb the photon emission off the lepton [Beneke and Rohrwild, 2011]:

$$\left[g_{\mu\nu} \, v \cdot p - v_\nu \, p_\mu \right] \hat{F}_A(n \cdot p) = -Q_\ell f_B \, g_{\mu\nu} + \left[g_{\mu\nu} \, v \cdot p - v_\nu \, p_\mu \right] \underbrace{\left[\hat{F}_A(n \cdot p) + \frac{Q_\ell f_B}{v \cdot p} \right]}_{F_A(n \cdot p) \, .} \underbrace{+ \frac{v_\nu p_\mu}{v \cdot p} \, Q_\ell f_B}_{\text{irrelevant after the contraction with } \varepsilon_\nu^*}$$

Current status of $B \to \gamma \ell \nu$

- Factorization properties at leading power [Korchemsky, Pirjol and Yan, 2000; Descotes-Genon and Sachrajda, 2002; Lunghi, Pirjol and Wyler, 2003; Bosch, Hill, Lange and Neubert, 2003].

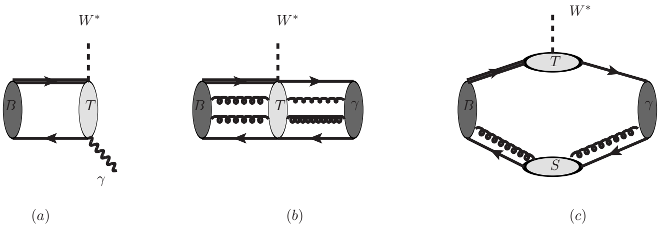
- Leading power contributions at NLL and (partial)-subleading power corrections at tree level [Beneke and Rohrwild, 2011].

- Subleading power corrections from the dispersion technique:
  - Soft two-particle correction at tree level [Braun and Khodjamirian, 2013].
  - Soft two-particle correction at one loop [Wang, 2016].
  - Three-particle B-meson DA's contribution at tree level [Wang, 2016; Beneke et al, 2018].
  - Subleading effective current and twist-5 and 6 corrections at tree level. [Beneke et al, 2018].

- Subleading power corrections from the direct QCD approach:
  - Hadronic photon corrections at tree level up to the twist-4 accuracy [Khodjamirian, Stoll, Wyler, 1995; Ali, Braun, 1995; Eilam, Halperin, Mendel, 1995].
  - Hadronic photon corrections of twist-two at one loop and of higher-twist at tree level [Ball and Kou, 2003; Wang and Shen, 2018].

The general picture

- Schematic structure of the distinct mechanisms:



A: hard subgraph that includes both photon and W^* vertices $\left(\frac{\Lambda}{m_b}\right)^{1/2} + \left(\frac{\Lambda}{m_b}\right)^{3/2} + \ldots$

B: real photon emission at large distances $\left(\frac{\Lambda}{m_b}\right)^{3/2} + \ldots$

C: Feynman mechanism: soft quark spectator $\left(\frac{\Lambda}{m_b}\right)^{3/2} + \ldots$

- Operator definitions of different terms needed for an unambiguous classification.

Method I: Dispersion technique

- Basic idea [Khodjamirian, 1999]:

$$\tilde{T}_{\nu\mu}(p,q) \equiv \int d^4x\, e^{ip\cdot x} \langle 0|\mathrm{T}\{j_{\nu,em}(x), [\bar{u}\gamma_\mu(1-\gamma_5)b](0)\}|B^-(p+q)\rangle\Big|_{p^2<0},$$
$$= v\cdot p\left[-i\,\varepsilon_{\mu\nu\rho\sigma}\,n^\rho\,v^\sigma\,F_V^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p) + g_{\mu\nu}^{\perp}\,\hat{F}_A^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p)\right] + \ldots$$

- Power counting: $n\cdot p \sim \mathscr{O}(m_b)$, $\bar{n}\cdot p \sim \mathscr{O}(\Lambda)$.

- Dispersion relations [Braun and Khodjamirian, 2013]:

$$F_V^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p) = \frac{2}{3}\,\frac{f_\rho\,m_\rho}{m_\rho^2-p^2-i0}\,\frac{2\,m_B}{m_B+m_\rho}\,V(q^2) + \frac{1}{\pi}\int_{\omega_s}^{\infty} d\omega'\,\frac{\mathrm{Im}_{\omega'}\,F_V^{B\to\gamma^*}(n\cdot p,\omega')}{\omega'-\bar{n}\cdot p-i0},$$
$$\hat{F}_A^{B\to\gamma^*}(n\cdot p,\bar{n}\cdot p) = \frac{2}{3}\,\frac{f_\rho\,m_\rho}{m_\rho^2-p^2-i0}\,\frac{2\,(m_B+m_\rho)}{n\cdot p}\,A_1(q^2) + \frac{1}{\pi}\int_{\omega_s}^{\infty} d\omega'\,\frac{\mathrm{Im}_{\omega'}\,\hat{F}_A^{B\to\gamma^*}(n\cdot p,\omega')}{\omega'-\bar{n}\cdot p-i0}.$$

- LCSR for the $B\to\rho$ form factors:

$$\frac{2}{3}\,\frac{f_\rho\,m_\rho}{n\cdot p}\,\mathrm{Exp}\left[-\frac{m_\rho^2}{n\cdot p\,\omega_M}\right]\frac{2\,m_B}{m_B+m_\rho}\,V(q^2) = \frac{1}{\pi}\int_0^{\omega_s} d\omega'\,e^{-\omega'/\omega_M}\left[\mathrm{Im}_{\omega'}\,F_V^{B\to\gamma^*}(n\cdot p,\omega')\right],$$
$$\frac{2}{3}\,\frac{f_\rho\,m_\rho}{n\cdot p}\,\mathrm{Exp}\left[-\frac{m_\rho^2}{n\cdot p\,\omega_M}\right]\frac{2\,(m_B+m_\rho)}{n\cdot p}\,A_1(q^2) = \frac{1}{\pi}\int_0^{\omega_s} d\omega'\,e^{-\omega'/\omega_M}\left[\mathrm{Im}_{\omega'}\,\hat{F}_A^{B\to\gamma^*}(n\cdot p,\omega')\right].$$

Method I: Dispersion technique

- Improved dispersion relations (setting $\bar{n} \cdot p = 0$) [Master formula I]:

$$F_V(n\cdot p) = \underbrace{\frac{1}{\pi}\int_0^{\omega_s} d\omega' \, \frac{n\cdot p}{m_\rho^2} \, \mathrm{Exp}\left[\frac{m_\rho^2 - \omega' \, n\cdot p}{n\cdot p \, \omega_M}\right] \left[\mathrm{Im}_{\omega'} \, F_V^{B\to\gamma^*}(n\cdot p, \omega')\right]}_{\text{nonperturbative modification}}, \\ + \frac{1}{\pi}\int_{\omega_s}^{\infty} d\omega' \, \frac{1}{\omega'} \left[\mathrm{Im}_{\omega'} \, F_V^{B\to\gamma^*}(n\cdot p, \omega')\right].$$

- Comparison with the HQE result [Master formula II]:

$$F_V(n\cdot p) = \underbrace{\frac{1}{\pi}\int_0^{\infty} d\omega' \, \frac{1}{\omega'} \left[\mathrm{Im}_{\omega'} \, F_V^{B\to\gamma^*}(n\cdot p, \omega')\right]}_{\text{HQE expression, not always well defined}} \\ + \underbrace{\frac{1}{\pi}\int_0^{\omega_s} d\omega' \left\{\frac{n\cdot p}{m_\rho^2} \, \mathrm{Exp}\left[\frac{m_\rho^2 - \omega' \, n\cdot p}{n\cdot p \, \omega_M}\right] - \frac{1}{\omega'}\right\}}_{\mathscr{O}(1)} \left[\mathrm{Im}_{\omega'} \, F_V^{B\to\gamma^*}(n\cdot p, \omega')\right].$$

- Spectral density at tree level:

$$\frac{1}{\pi}\mathrm{Im}_{\omega'} \, F_V^{B\to\gamma^*}(n\cdot p, \omega') = \frac{Q_u f_B m_B}{n\cdot p} \, \underbrace{\phi_B^+(\omega', \mu)} + \mathscr{O}(\alpha_s, \Lambda/m_b).$$

of $\mathscr{O}(1/\Lambda)[\mathscr{O}(1/m_b)]$ for $\omega' \sim \mathscr{O}(\Lambda)[\omega' \sim \mathscr{O}(\Lambda^2/m_b)]$

Power suppressed soft contribution!

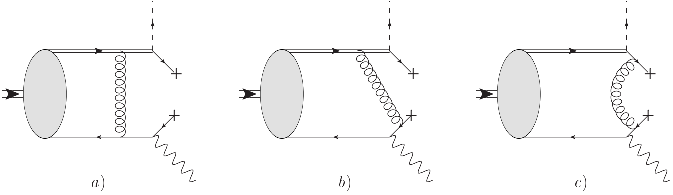
Method I: Dispersion technique

- New Improvement I [Wang, 2016]:
  - Soft two-particle correction at NLO:

$$
\begin{aligned}
F_{V,2P}^{\rm soft, NLL}(n \cdot p) &\propto \int_0^{\omega_s} d\omega' \left\{ \frac{n \cdot p}{m_\rho^2} \, {\rm Exp}\left[\frac{m_\rho^2 - \omega' \, n \cdot p}{n \cdot p \, \omega_M} \right] - \frac{1}{\omega'} \right\} \phi_{B,\rm eff}^{+}(\omega', \mu) \,, \\
\phi_{B,\rm eff}^{+}(\omega', \mu) &= \phi_B^{+}(\omega', \mu) + \frac{\alpha_s(\mu) \, C_F}{4\pi} \, [\text{New}] \,.
\end{aligned}
$$

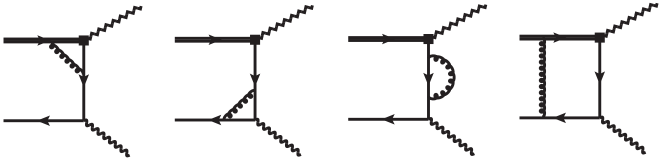
  - Three-particle correction with KKQT parametrization at tree level.

- New Improvement II [Beneke et al, 2018]:
  - Compute the matrix element of the subleading effective current $\bar{q} \, \gamma_\mu \, \frac{i \overrightarrow{\not{D}}}{2 m_b} \, h_v$ at LO.
  - Three-particle correction with BJM parametrization at tree level.
  - Twist-five and -six four-particle corrections in factorization approximation.



Method II: Direct QCD approach

- Leading power contribution at NLO:



- Perturbative QCD factorization formula:

$$F_{V,\mathrm{LP}}(n\cdot p) = F_{A,\mathrm{LP}}(n\cdot p) = \frac{Q_u\, m_B}{n\cdot p}\tilde{f}_B(\mu)\, C_\perp(n\cdot p,\mu) \int_0^\infty d\omega\, \frac{\phi_B^+(\omega,\mu)}{\omega} J_\perp(n\cdot p,\omega,\mu)\,.$$

- Both the hard function and the jet function can be determined with two different methods.

$$\begin{aligned} C_\perp &= 1 - \frac{\alpha_s\, C_F}{4\,\pi}\left[2\,\ln^2\frac{\mu}{n\cdot p} + 5\,\ln\frac{\mu}{m_b} - 2\,\mathrm{Li}_2\left(1-\frac{1}{r}\right) - \ln^2 r \right. \\ &\qquad \left. + \frac{3r-2}{1-r}\,\ln r + \frac{\pi^2}{12} + 6\right], \\ J_\perp &= 1 + \frac{\alpha_s\, C_F}{4\,\pi}\left[\ln^2\frac{\mu^2}{n\cdot p\,(\omega - \bar{n}\cdot p)} - \frac{\pi^2}{6} - 1\right] + \mathcal{O}(\alpha_s^2)\,. \end{aligned}$$

QCD resummation for the leading power contribution

- RG evolution of the hard functions at NLL:

$$\frac{d}{d\ln\mu} C_\perp(n\cdot p,\mu) = \Big[-\underbrace{\Gamma_{\rm cusp}(\mu)}_{\text{three loops}} \ln\frac{\mu}{n\cdot p} + \underbrace{\gamma_h(\mu)}_{\text{two loops}}\Big] C_\perp(n\cdot p,\mu),$$

$$\frac{d}{d\ln\mu}\tilde{f}_B(\mu) = \underbrace{\tilde{\gamma}(\mu)}_{\text{two loops}}\tilde{f}_B(\mu).$$

- RG evolution of the twist-two B-meson LCDA at one loop [Lange and Neubert, 2003]:

$$\frac{d\phi_B^+(\omega,\mu)}{d\ln\mu} = -\Big[\Gamma_{\rm cusp}(\alpha_s)\ln\frac{\mu}{\omega} + \gamma_+(\alpha_s)\Big]\phi_B^+(\omega,\mu) - \omega\int_0^\infty d\omega'\,\Gamma_+(\omega,\omega',\mu)\,\phi_B^+(\omega',\mu).$$

  - Renormalization of $[\bar{q}_s(t\bar{n})\Gamma b_v(0)]$ does not commute with the short-distance expansion [Braun, Ivanov and Korchemsky, 2004].

$$[(\bar{q}_s Y_s)(t\bar{n})\not{\bar{n}}\Gamma(Y_s^\dagger b_v)(0)]_R \neq \sum_{p=0} \frac{t^p}{p!}\Big[\bar{q}_s(0)\,(n\cdot\overleftarrow{D})^p\,\not{\bar{n}}\Gamma b_v(0)\Big]_R.$$

  - Eigenfunctions of the Lange-Neubert renormalization kernel [Bell, Feldmann, YMW and Yip, 2013].
  - Two-loop evolution of $\phi_B^+(\omega,\mu)$ essential to the NLL resummation.

QCD resummation for the leading power contribution

- (Partial)-NLL resummation improved factorization formula:

$$
\begin{aligned}
F_{V,\mathrm{LP}}(n\cdot p) &= F_{A,\mathrm{LP}}(n\cdot p) \\
&= \frac{Q_u m_B}{n\cdot p} \left[U_2(n\cdot p, \mu_{h2}, \mu) \tilde{f}_B(\mu_{h2})\right] \left[U_1(n\cdot p, \mu_{h1}, \mu)\, C_\perp(n\cdot p, \mu_{h1})\right] \\
&\quad \times \underbrace{\int_0^\infty d\omega \, \frac{\phi_B^+(\omega,\mu)}{\omega} J_\perp(n\cdot p, \omega, \mu)}\,. \\
&= \lambda_B^{-1}(\mu) \left\{ 1 + \frac{\alpha_s(\mu)\, C_F}{4\pi} \left[\ln^2 \frac{\mu^2}{n\cdot p\, \mu_0} + 2 \ln \frac{\mu^2}{n\cdot p\, \mu_0}\, \sigma_1(\mu) + \sigma_2(\mu) - \frac{\pi^2}{6} - 1 \right] \right\}.
\end{aligned}
$$

- Inverse-logarithmic moments [Beneke and Rohrwild, 2011]:

$$
\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\, \phi_B^+(\omega,\mu)\,, \qquad \sigma_B^{(n)}(\mu) = \lambda_B(\mu) \int_0^\infty \frac{d\omega}{\omega}\, \ln^n \frac{\mu_0}{\omega}\, \phi_B^+(\omega,\mu)\,.
$$

$$
\frac{\lambda_B(\mu_0)}{\lambda_B(\mu)} = 1 + \frac{\alpha_s(\mu_0)\, C_F}{4\pi} \ln \frac{\mu}{\mu_0} \left[2 - 2 \ln \frac{\mu}{\mu_0} - 4\, \sigma_B^{(1)}(\mu_0) \right] + \mathcal{O}(\alpha_s^2)\,.
$$

$$
\frac{d}{d\ln\mu} \sigma_B^{(n)}(\mu) = \mathcal{O}(\alpha_s) \Rightarrow \text{No } [\alpha_s(\mu)]^0 \ln(\mu/\mu_0) \text{ due to the evolution}\,.
$$

- Not aiming at resummation of $\ln^k(\mu/\mu_0)$ given the fact that $\mu \sim 1.5$GeV and $\mu_0 \sim 1.0$GeV.
  Resummation in the “dual" momentum space [Bell, Feldmann, YMW and Yip, 2013].

Sub-leading power corrections in QCD

- Subleading-power local contributions [Beneke and Rohrwild, 2011]:

$$F^{\rm LC}_{V,NLP}(n\cdot p) = -F^{\rm LC}_{A,NLP}(n\cdot p) = \frac{Q_u f_B m_B}{(n\cdot p)^2} + \frac{Q_b f_B m_B}{n\cdot p\, m_b}\,.$$

  - Q_u term: Subleading correction from the hard-collinear u-quark propagator.
  - Q_b term: Subleading correction from the photon radiation off the b-quark.
  - Power suppressed local contributions violate the large-recoil symmetry relation.

- Subleading-power non-local contribution:

$$F^{\rm NLC}_{V,NLP}(n\cdot p) = \xi(v\cdot p) \stackrel{?}{=} -\frac{i Q_u}{(n\cdot p)^2}\, \overline{C}_{\perp,NLP}(n\cdot p,\mu)$$
$$\times \int ds \left\langle 0 \left| [\bar{u}\, Y_s](s\bar{n})\, \frac{i n\cdot \overleftarrow{\partial}}{i\bar{n}\cdot \overleftarrow{\partial}}\, \not{\bar{n}}\, (1-\gamma_5)\, [Y_s^{\dagger}\, b_v](0) \right| B^{-}(v) \right\rangle \tilde{J}_{NLP}\left(\frac{\mu^2 s}{v\cdot p}\right).$$

  A systematic SCET treatment of power corrections to $B\to\gamma\ell\nu$ in demand.

- $B\to\gamma\ell\nu$ with scalar fields [Beneke and Feldmann, 2003]:

$$\langle\gamma|J|\bar{q}\,b\rangle = (C_0+C_1)\star\phi_{q\bar{b}} + \left[C_2\star\phi_{q\bar{b}g}\right]_{\nu} + \left[\phi^{\gamma}_{q\bar{q}}\star C_3\right]_{\nu}\star\phi_{q\bar{b}}\,.$$

  Communication between the soft and collinear systems.

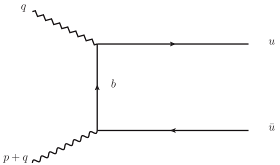
Twist-two hadronic photon correction from LCSR

- Main task: QCD factorization for the vacuum-to-photon correlation function.

$$\Pi_\mu(p,q) = \int d^4x\, e^{iq\cdot x} \langle \gamma(p)| T\{\underbrace{\bar{u}(x)\,\gamma_{\mu\perp}\,(1-\gamma_5)\,b(x)}_{\text{weak current}}, \bar{b}(0)\,\gamma_5\,u(0)\}|0\rangle\,.$$

- Power counting scheme:

$$n\cdot p \sim \mathcal{O}(m_b)\,,$$
$$|(p+q)^2 - m_b^2| \sim \mathcal{O}(m_b^2)\,.$$



- QCD matrix element at LO:

$$F_\mu(p,q) = \int d^4x\, e^{iq\cdot x} \langle q(zp)\,\bar{q}(\bar{z}p)| T\{\bar{u}(x)\,\gamma_{\mu\perp}\,(1-\gamma_5)\,b(x), \bar{b}(0)\,\gamma_5\,u(0)\}|0\rangle\,,$$
$$= \frac{i}{2}\,\frac{\bar{n}\cdot q}{z(p+q)^2 + \bar{z}\,q^2 - m_b^2 + i0}\,\bar{u}(zp)\,\gamma_{\mu\perp}\not{n}\,(1+\gamma_5)\,v(\bar{z}p)\,.$$

Hard-collinear factorization for the considered correlation function (pion-photon form factor).

Twist-two factorization at tree level

- Operator matching automatically:

$$F_\mu^{(0)}(p,q) = \frac{i}{2}\,\frac{\bar{n}\cdot q}{z'(p+q)^2 + \bar{z}'\,q^2 - m_b^2 + i0} * \langle O_{A,\mu}(z,z')\rangle^{(0)}\,.$$

- Collinear operator in the momentum space:

$$\begin{aligned} O_{j,\mu}(z') &= \frac{n\cdot p}{2\pi}\int d\tau\, e^{-iz'\tau n\cdot p}\,\bar{\xi}(\tau n)\,W_c(\tau n,0)\,\Gamma_j\,\xi(0)\,, \\ \Gamma_A &= \gamma_{\mu\perp}\not{n}\,(1+\gamma_5)\,. \end{aligned}$$

- Matrix element of the collinear operator:

$$\langle O_{A,\mu}(z,z')\rangle = \langle q(zp)\,\bar{q}(\bar{z}p)|O_{A,\mu}(z')|0\rangle = \bar{\xi}(zp)\,\gamma_{\mu\perp}\not{n}\,(1+\gamma_5)\,\xi(zp)\,\delta(z-z') + \mathcal{O}(\alpha_s)\,.$$

- Operator matching with the collinear operator defining the standard photon DAs:

$$\begin{aligned} O_{A,\mu} &= O_{1,\mu} + O_{2,\mu} + O_{E,\mu}\,, \\ \Gamma_1 &= \gamma_{\mu\perp}\not{n}\,, \qquad \Gamma_2 = \frac{n^\nu}{2}\,\varepsilon_{\mu\nu\alpha\beta}\,\sigma^{\alpha\beta}\,, \qquad \underbrace{\Gamma_E}_{\text{evanescent operator}} = \gamma_{\mu\perp}\not{n}\,\gamma_5 - \frac{n^\nu}{2}\,\varepsilon_{\mu\nu\alpha\beta}\,\sigma^{\alpha\beta}\,. \end{aligned}$$

Twist-two factorization at tree level

- Operator matching with the evanescent operator:

$$F_\mu(p,q) = \sum_i C_i(z', (p+q)^2, q^2) * \langle O_{i,\mu}(z,z') \rangle .$$

$$\text{Expansion} \quad \Downarrow \quad \text{at} \quad \text{tree} \quad \text{level}$$

$$C_1^{(0)} = C_2^{(0)} = C_E^{(0)} = \frac{i}{2} \, \frac{\bar{n} \cdot q}{z'(p+q)^2 + \bar{z}' \, q^2 - m_b^2 + i0} .$$

- Hard-collinear factorization at tree level:

$$\Pi_\mu(p,q) = \frac{i}{2} g_{\rm em} \, Q_u \, \chi(\mu) \, \langle \bar{q} q \rangle(\mu) \, \varepsilon^{*\alpha}(p) \left[g_{\mu\alpha}^{\perp} - i \, \varepsilon_{\mu\alpha\nu\beta} \, n^\nu \, v^\beta \right] \times \int_0^1 dz \, \phi_\gamma(z,\mu) \, \frac{n \cdot p \, \bar{n} \cdot q}{z(p+q)^2 + \bar{z} q^2 - m_b^2 + i0} + \mathcal{O}(\alpha_s) .$$

Evanescent operator does not mix into the physical operators at LO.

- Twist-two photon DA:

$$\langle \gamma(p) | \bar{\xi}(x) \, W_c(x,0) \, \sigma_{\alpha\beta} \, \xi(0) | 0 \rangle = i \, g_{\rm em} \, Q_q \, \chi(\mu) \, \langle \bar{q} q \rangle(\mu) \, (p_\beta \, \varepsilon^*_\alpha - p_\alpha \, \varepsilon^*_\beta) \int_0^1 dz \, e^{i z p \cdot x} \, \phi_\gamma(z,\mu) .$$

Twist-two LCSR at tree level

- Hadronic dispersion relation:

$$\Pi_\mu(p,q) = \frac{i}{2}\, g_{\rm em}\, \frac{f_B\, m_B^2}{m_b + m_u}\, \varepsilon^{*\,\alpha}(p) \left[g^{\perp}_{\mu\alpha}\, F^{\rm 2PLT}_{A,\,{\rm photon}}(n\cdot p) - i\, \varepsilon_{\mu\alpha\nu\beta}\, n^\nu\, v^\beta\, F^{\rm 2PLT}_{V,\,{\rm photon}}(n\cdot p) \right]$$

$$\times \frac{n\cdot p}{(p+q)^2 - m_B^2 + i0} + \int_{s_0}^{\infty} ds\, \frac{\rho^h_\mu(s,q^2)}{s-(p+q)^2 - i0}\,.$$

- Tree-level sum rules:

$$\frac{f_B\, m_B}{m_b + m_u}\, F^{\rm 2PLT}_{V(A),\,{\rm photon}}(n\cdot p) = Q_u\, \chi(\mu)\, \langle \bar{q} q \rangle(\mu) \int_{z_0}^{1} \frac{dz}{z} \exp\left[-\frac{m_b^2 - \bar{z}\, q^2}{z\, M^2} + \frac{m_B^2}{M^2} \right] \phi_\gamma(z,\mu) + \mathcal{O}(\alpha_s)\,.$$

- Power counting scheme:

$$\left(s_0 - m_b^2\right) \sim M^2 \sim \mathcal{O}(m_b\, \Lambda)\,, \qquad \bar{z}_0 = \frac{s_0 - m_b^2}{s_0 - q^2} \sim \Lambda/m_b\,.$$

$$\Downarrow$$

$$F^{\rm 2PLT}_{V,\,{\rm photon}} \sim F^{\rm 2PLT}_{A,\,{\rm photon}} \sim \mathcal{O}\left(\frac{\Lambda}{m_b}\right)^{3/2}.$$

Hadronic photon correction is power suppressed in the heavy quark expansion.

Twist-two factorization at NLO

- QCD matrix element at NLO:

- Extracting the hard contribution with the method of regions:

$$F_\mu^{(1)}(p,q) = \sum_{i=1,2} T_{i,\rm hard}^{(1)}(z',(p+q)^2,q^2) * \langle O_{i,\mu}(z,z')\rangle^{(0)} + \ldots .$$

One-loop QCD amplitude in NDR scheme:

$$\begin{aligned} T_{i,\rm hard}^{(1)}\Big|_{\rm NDR} = & \frac{\alpha_s C_F}{4\pi} \Bigg\{ (-2) \left[\frac{1-r_2}{r_1-r_2} \ln \frac{1-r_1}{1-r_2} + \frac{1-r_3}{r_1-r_3} \ln \frac{1-r_1}{1-r_3} + \frac{3}{1-r_1} \right] \left(\frac{1}{\varepsilon} + \ln \frac{\mu^2}{m_b^2} \right) \\ & + \frac{2(1-r_2)}{r_1-r_2} {\rm Li}_2 \left(1 - \frac{1-r_1}{1-r_2} \right) + \frac{2(1-r_3)}{r_1-r_3} {\rm Li}_2 \left(1 - \frac{1-r_1}{1-r_3} \right) \\ & + \left(\frac{1-r_2}{r_1-r_2} + \frac{1-r_3}{r_1-r_3} \right) \ln^2(1-r_1) - \frac{1-r_2}{r_1-r_2} \ln^2(1-r_2) - \frac{1-r_3}{r_1-r_3} \ln^2(1-r_3) \\ & + \left[\frac{2}{r_1(r_3-r_1)} + \frac{2(r_3-2)}{r_3-r_1} + \frac{6}{1-r_1} - \frac{2-r_2}{r_1-r_2} + \frac{4}{r_1} - 4 \right] \ln(1-r_1) + \ldots \Bigg\} C_{i,\rm hard}^{(0)} . \end{aligned}$$

Twist-two factorization at NLO

- Expanding the matching equation at NLO:

$$\sum_i T_i^{(1)}(z', (p+q)^2, q^2) * \langle O_{i,\mu}(z,z')\rangle^{(0)}$$
$$= \sum_i \left[C_i^{(1)}(z', (p+q)^2, q^2) * \langle O_{i,\mu}(z,z')\rangle^{(0)} + C_i^{(0)}(z', (p+q)^2, q^2) * \langle O_{i,\mu}(z,z')\rangle^{(1)} \right].$$

- One-loop renormalized matrix elements of collinear operators:

$$\langle O_{i,\mu}\rangle^{(1)} = \sum_j \left[M_{ij,\,\mathrm{bare}}^{(1),R} + Z_{ij}^{(1)} \right] \langle O_{j,\mu}\rangle^{(0)}.$$

Vanishing bare matrix element $M_{ij,\,\mathrm{bare}}^{(1),R}$ in dimensional regularization $\Rightarrow$

$$C_i^{(1)} = T_i^{(1)} - \sum_{j=1,2,E} C_j^{(0)} * Z_{ji}^{(1)} = \underbrace{T_i^{(1)} - C_i^{(0)} * Z_{ii}^{(1)}}_{T_{i,\,\mathrm{hard}}^{(1),\,\mathrm{reg}}} - C_E^{(0)} * \underbrace{Z_{Ei}^{(1)}}_{\text{operator mixing}}.$$

- The IR finite matrix element of the evanescent operator vanishes (Dugan and Grinstein, 1991).

$$Z_{Ei}^{(1)} = -M_{Ei,\,\mathrm{bare}}^{(1),\,\mathrm{off}}.$$

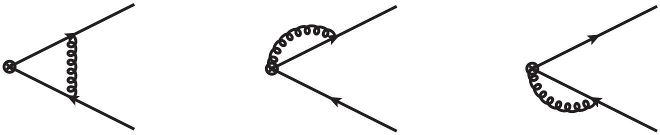
IR singularities regularized by any parameter other than the dimensions of spacetime.

Twist-two factorization at NLO

- Master formula for the hard function:

$$C_i^{(1)} = T_i^{(1)} - C_i^{(0)} * Z_{ii}^{(1)} + C_E^{(0)} * M_{Ei,\,\mathrm{bare}}^{(1),\,\mathrm{off}} = T_{i,\,\mathrm{hard}}^{(1),\,\mathrm{reg}} + C_E^{(0)} * M_{Ei,\,\mathrm{bare}}^{(1),\,\mathrm{off}} \,.$$

- The IR subtraction:



Only the first diagram can potentially generate the operator mixing.

$$C_E^{(0)} * M_{Ei,\,\mathrm{bare}}^{(1),\,\mathrm{off}} = 0 \,, \text{ with } i = 1,2 \,.$$

γ_5-scheme independent subtraction term!

$$C_i^{(1)} = T_{i,\,\mathrm{hard}}^{(1),\,\mathrm{reg}} \,.$$

Twist-two factorization at NLO

- The NLO factorization formula:

$$\Pi_\mu(p,q) = g_{\rm em}\, Q_u\, n\cdot p\, \chi(\mu)\, \langle \bar{q}q\rangle(\mu)\ \varepsilon^{*\alpha}(p) \left[g^{\perp}_{\mu\alpha} - i\,\varepsilon_{\mu\alpha\nu\beta}\, n^\nu\, v^\beta \right]$$

$$\int_0^1 dz\, \phi_\gamma(z,\mu)\ \left[C_i^{(0)}(z,(p+q)^2,q^2) + C_i^{(1)}(z,(p+q)^2,q^2) \right] + \mathcal{O}(\alpha_s^2)\,.$$

- RG evolution of the twist-two photon LCDA [Lepage and Brodsky,1979]:

$$\mu^2 \frac{d}{d\mu^2} \left[\chi(\mu)\, \langle \bar{q}q\rangle(\mu)\, \phi_\gamma(z,\mu) \right] = \int_0^1 dz'\, \widetilde{V}(z,z')\, \left[\chi(\mu)\, \langle \bar{q}q\rangle(\mu)\, \phi_\gamma(z',\mu) \right],$$

$$\widetilde{V}_0(z,z') = 2\,C_F \left[\frac{\bar{z}}{\bar{z}'}\, \frac{1}{z-z'}\, \theta(z-z') + \frac{z}{z'}\, \frac{1}{z'-z}\, \theta(z'-z) \right]_+ \underbrace{-C_F\, \delta(z-z')}_{\text{New term!}}\,.$$

Multiplicative renormalization at LO:

$$\chi(\mu)\, \langle \bar{q}q\rangle(\mu)\, \phi_\gamma(z,\mu) = 6\,z\,\bar{z} \sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma^{(0)}_{T,n}/(2\beta_0)} \chi(\mu_0)\, \langle \bar{q}q\rangle(\mu_0)\, a_n(\mu_0)\, C_n^{3/2}(2z-1)\,.$$

Twist-two factorization at NLL

- NLL resummation needs two-loop evolution kernel [Belitsky, Müller and Freund, 1999; etc]:

$$\widetilde{V}_1(z,z') = \frac{N_f}{2}\, C_F\, \widetilde{V}_N(z,z') + C_F\, C_A\, \widetilde{V}_G(z,z') + C_F^2\, \widetilde{V}_F(z,z')\,.$$

  Two-loop evolution of the Gegenbauer moment [Müller, 1994/1995]:

$$\chi(\mu)\,\langle \bar{q} q\rangle(\mu)\, a_n(\mu) = E_{T,n}^{\rm NLO}(\mu,\mu_0)\,\chi(\mu_0)\,\langle \bar{q} q\rangle(\mu_0)\, a_n(\mu_0)$$
$$+\frac{\alpha_s(\mu)}{4\pi}\sum_{k=0}^{n-2} E_{T,n}^{\rm LO}(\mu,\mu_0)\, d_{T,n}^{k}(\mu,\mu_0)\,\chi(\mu_0)\,\langle \bar{q} q\rangle(\mu_0)\, a_n(\mu_0)\,.$$

  Construction from the forward anomalous dimensions and the special conformal anomaly matrix.

- NLL resummation improved factorization formula:

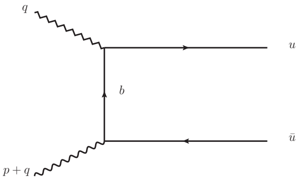
$$\Pi_\mu(p,q) = g_{\rm em}\, Q_u\, n\cdot p\, \chi(\mu)\,\langle \bar{q} q\rangle(\mu)\,\varepsilon^{*\,\alpha}(p)\left[g^{\perp}_{\mu\alpha} - i\,\varepsilon_{\mu\alpha\nu\beta}\, n^\nu\, v^\beta\right]$$
$$\times \underbrace{\sum_{n=0} a_n(\mu)\, K_n((p+q)^2, q^2)}_{\text{dispersion form}} + \mathcal{O}(\alpha_s^2)\,,$$

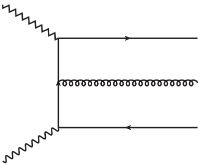
$$\int_0^\infty \frac{ds}{s-(p+q)^2 - i0}\left[\rho^{(0)}(s,q^2) + \frac{\alpha_s\, C_F}{4\,\pi}\,\rho^{(1)}(s,q^2)\right].$$

- NLL twist-two LCSR of $F_{V,\,\rm photon}^{\rm 2PLT}(n\cdot p)$ can be constructed immediately.

Higher-twist hadronic photon corrections

- General picture:





Higher twist corrections from both two-particle and three-particle photon LCDAs.

- Systematic studies of the photon LCDAs [Ball, Braun and Kivel, 2002]:

$$\langle \gamma(p)|\bar{q}(x)\, W_c(x,0)\, \sigma_{\alpha\beta}\, q(0)|0\rangle$$

$$= i\, g_{\rm em}\, Q_q\, \langle \bar{q} q\rangle(\mu)\, (p_\beta\, \varepsilon^*_\alpha - p_\alpha\, \varepsilon^*_\beta) \int_0^1 dz\, e^{i z p\cdot x} \left[\chi(\mu)\, \overbrace{\phi_\gamma(z,\mu)}^{\text{twist 2}} + \frac{x^2}{16}\, \overbrace{\mathbb{A}(z,\mu)}^{\text{twist 4}} \right]$$

$$+ \frac{i}{2}\, g_{\rm em}\, Q_q\, \frac{\langle \bar{q} q\rangle(\mu)}{q\cdot x}\, (x_\beta\, \varepsilon^*_\alpha - x_\alpha\, \varepsilon^*_\beta) \int_0^1 dz\, e^{i z p\cdot x}\, \underbrace{h_\gamma(z,\mu)}_{\text{twist 4}} .$$

$$\langle \gamma(p)|\bar{q}(x)\, W_c(x,0)\, g_s\, G_{\alpha\beta}(v x)\, i\, \gamma_\rho\, q(0)|0\rangle$$

$$= g_{\rm em}\, Q_q f_{3\gamma}(\mu)\, p_\rho\, (p_\beta\, \varepsilon^*_\alpha - p_\alpha\, \varepsilon^*_\beta) \int [\mathscr{D}\alpha_i]\, e^{i(\alpha_q + v\,\alpha_g) p\cdot x}\, \overbrace{V(\alpha_i,\mu)}^{\text{twist 3}} .$$

Can be rewritten as collinear matrix elements in SCET [Hardmeier et al, 2006].

Two-particle higher-twist hadronic photon corrections

- Two-particle higher twist corrections:

$$\Pi_\mu(p,q) \supset \frac{i}{4}\, g_{\rm em}\, Q_q\, (p\cdot q) \int_0^1 dz \left\{ \varepsilon_\mu^* \left[\frac{\rho_{A,2}^{\rm 2PHT}((p+q)^2,q^2,z)}{[(zp+q)^2-m_b^2+i0]^2} + \frac{\rho_{A,3}^{\rm 2PHT}((p+q)^2,q^2,z)}{[(zp+q)^2-m_b^2+i0]^3} \right] \right.$$
$$\left. -i\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\nu}\, n^\alpha\, v^\beta \left[\frac{\rho_{V,2}^{\rm 2PHT}((p+q)^2,q^2,z)}{[(zp+q)^2-m_b^2+i0]^2} + \frac{\rho_{V,3}^{\rm 2PHT}((p+q)^2,q^2,z)}{[(zp+q)^2-m_b^2+i0]^3} \right] \right\}.$$

  - Two-particle higher twist corrections violate the symmetry relations of $B \to \gamma$ form factors.

$$\rho_{A,2} \neq \rho_{V,2}\,, \qquad \rho_{A,3} \neq \rho_{V,3}\,.$$

  - Heavy-quark scaling of the two-particle corrections:

$$F_{V,\,\rm photon}^{\rm 2PHT,\,LL}(n\cdot p) \sim F_{A,\,\rm photon}^{\rm 2PHT,\,LL}(n\cdot p) \sim \left(\frac{\Lambda}{m_b}\right)^{3/2}.$$

  - Only suppressed by one factor of Λ/m_b compared with the “direct" photon contribution.
  - No correspondence between the heavy-quark expansion and the twist expansion [see also $B \to \pi$ form factors, pion-photon form factor].

Three-particle higher-twist hadronic photon corrections

- Quark propagator in the background gluon field [Balitsky and Braun, 1988]:

$$\langle 0|T\{\bar{b}(x), b(0)\}|0\rangle \supset i g_s \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \int_0^1 dv \left[\frac{v x_\mu}{k^2 - m_b^2} G^{\mu\nu}(vx)\gamma_\nu - \frac{\not{k} + m_b}{2(k^2-m_b^2)^2} G^{\mu\nu}(vx)\sigma_{\mu\nu} \right] + \ldots$$

- Three particle corrections at tree level:

$$\Pi_\mu(p,q) \supset i g_{\rm em} Q_q (p\cdot q) \int_0^1 dv \int [\mathcal{D}\alpha_i] \left\{ \varepsilon_\mu^* \left[\frac{\rho_{A,2}^{\rm 3P}((p+q)^2, q^2, \alpha_i, v)}{[((\alpha_q + v\,\alpha_g)p+q)^2 - m_b^2 + i0]^2} + \frac{\rho_{A,3}^{\rm 3P}((p+q)^2, q^2, \alpha_i, v)}{[((\alpha_q + v\,\alpha_g)p+q)^2 - m_b^2 + i0]^3} \right] - i\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^{*\,\nu} n^\alpha v^\beta \times \left[\frac{\rho_{V,2}^{\rm 3P}((p+q)^2, q^2, \alpha_i, v)}{[((\alpha_q + v\,\alpha_g)p+q)^2 - m_b^2 + i0]^2} + \frac{\rho_{V,3}^{\rm 3P}((p+q)^2, q^2, \alpha_i, v)}{[((\alpha_q + v\,\alpha_g)p+q)^2 - m_b^2 + i0]^3} \right] \right\}.$$

- Heavy-quark scaling of the three-particle corrections:

$$F_{V,\,\rm photon}^{\rm 3P,\,LL}(n\cdot p) \sim F_{A,\,\rm photon}^{\rm 3P,\,LL}(n\cdot p) \sim \left(\frac{\Lambda}{m_b}\right)^{5/2}.$$

Double suppression compared with the “direct" photon contribution.

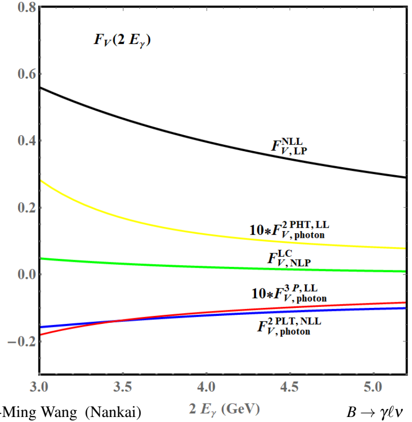
Final expressions of the $B \to \gamma$ form factors

- Adding up different pieces together:

$$F_V(n\cdot p) = F_{V,\rm LP}(n\cdot p) + F_{V,\rm NLP}^{\rm LC}(n\cdot p) + F_{V,\rm photon}^{\rm 2PLT}(n\cdot p) + F_{V,\rm photon}^{\rm 2PHT,\,LL}(n\cdot p) + F_{V,\rm photon}^{\rm 3P,\,LL}(n\cdot p)\,,$$

$$F_A(n\cdot p) = F_{A,\rm LP}(n\cdot p) + F_{A,\rm NLP}^{\rm LC}(n\cdot p) + F_{A,\rm photon}^{\rm 2PLT}(n\cdot p) + F_{A,\rm photon}^{\rm 2PHT,\,LL}(n\cdot p) + F_{A,\rm photon}^{\rm 3P,\,LL}(n\cdot p) + \frac{Q_\ell f_B}{v\cdot p}\,.$$

- Breakdown of various contributions [$\lambda_B = 354\,\mathrm{MeV}$]:



Numerics with central inputs:

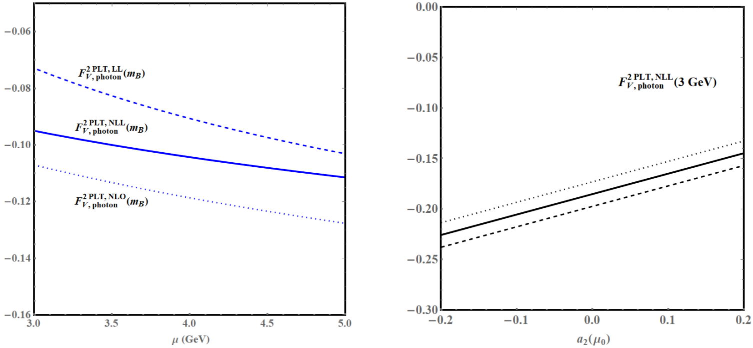
$$F_{V,2P}^{\rm LP,NLL}(m_B) = 0.28\,,$$
$$F_{V,\rm photon}^{\rm 2PLT,NLL}(m_B) = -0.10\,,$$
$$F_{V,\rm photon}^{\rm 2PHT,LL}(m_B) = 0.008\,,$$
$$F_{V,\rm photon}^{\rm 3P,LL}(m_B) = -0.008\,,$$
$$F_{V,NLP}^{\rm LC}(m_B) = 0.009\,,$$
$$F_{A,NLP}^{\rm LC}(m_B) = 0.048\,.$$

Twist-two hadronic photon effect yields $\mathcal{O}(30\%)$ correction.

Strong cancellation between $F_{V,\rm photon}^{\rm 2PHT,LL}$ and $F_{V,\rm photon}^{\rm 3P,LL}$.

Numerical impacts of higher-order corrections

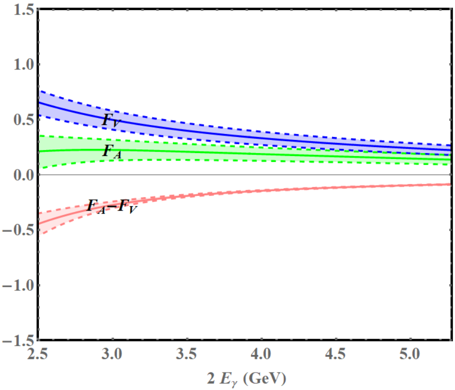
- Radiative correction and a_2 dependence of the twist-two hadronic photon correction:



  - NLO QCD correction shifts the LO twist-two hadronic photon correction $\mathcal{O}(20-40)\%$.
  - NLL resummation yields $\mathcal{O}(10-20)\%$ correction to the NLO prediction.
  - Variation of $a_2(\mu_0) \in [-0.2, 0.2]$ leads to $\mathcal{O}(35\%)$ uncertainty at $n \cdot p = 3\,\mathrm{GeV}$.
    $\Rightarrow$ More information about $a_2(\mu_0)$ necessary.
  - Variation of $a_4(\mu_0) \in [-0.2, 0.2]$ yields minor uncertainty.
    $\Rightarrow$ Yet higher conformal spin contributions might not be sizeable numerically.

Photon-energy dependence of the $B \to \gamma$ form factors

Including power suppressed hadronic photon corrections and the local contribution:



- Dominant uncertainties from $\lambda_B(\mu_0)$, $a_2(\mu_0)$ and μ.
- $F_A - F_V$ only comes from the subleading power contributions:

  (a) Power suppressed local corrections.

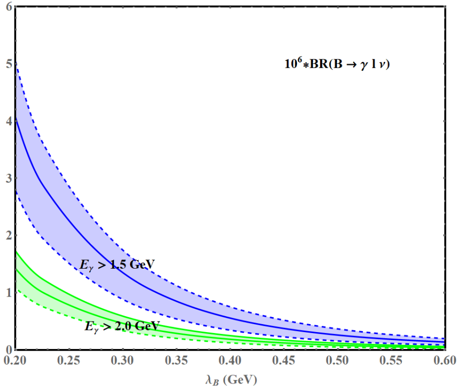
  (b) Higher-twist hadronic photon corrections.
- Faster growing F_V than F_A with the decrease of E_γ.

Partial branching fractions of $B \to \gamma \ell \nu$

- Integrated decay rate $\Delta BR(E_{\rm cut})$:

$$\Delta BR(E_{\rm cut}) = \tau_B \int_{E_{\rm cut}}^{m_B/2} dE_\gamma \, \frac{d\Gamma}{dE_\gamma} (B \to \gamma \ell \nu) \, .$$

- $\lambda_B(\mu_0)$ dependence of $\Delta BR(E_{\rm cut})$:



  - Belle 2015 data: $\Delta BR(1\,{\rm GeV}) < 3.5 \times 10^{-6}$.
  - Belle-II 2018 data [Talk by Bernlochner]: $\Delta BR(1\,{\rm GeV}) = (1.4 \pm 1.0 \pm 0.4) \times 10^{-6}$.
  - Expected statistical error for $\Delta BR(1\,{\rm GeV})$ with $50\,{\rm ab}^{-1}$ of Belle-II data: ${}^{+0.18}_{-0.17} \times 10^{-6}$.
  - The photon-energy cut not sufficiently large. Power corrections numerically important for $E_\gamma < 1.5\,{\rm GeV}$.

Concluding Remarks

- Understanding power corrections in $B \to \gamma \ell \nu$ important for precision flavour physics.

- Subleading power contributions from hadronic photon corrections.
    - NLL twist-two hadronic photon effect yields $\mathscr{O}(30\,\%)$ correction at $\lambda_B = 354\,\text{MeV}$.
    - Strong cancellation between $F_{V,\text{photon}}^{\text{2PHT,LL}}$ and $F_{V,\text{photon}}^{\text{3P,LL}}$.
    - Large-recoil symmetry violation due to $F_{V(A),NLP}^{\text{LC}}$, $F_{V,\text{photon}}^{\text{2PHT,LL}}$ and $F_{V,\text{photon}}^{\text{3P,LL}}$.

- The inverse moment $\lambda_B(\mu_0)$ not sufficient to describe $B \to \gamma \ell \nu$ in general.

- Connection between the dispersion technique and the direct QCD approach?
  Can be also investigated in a simpler process $\gamma^* \gamma \to \pi$.

- A systematic treatment of subleading power corrections in SCET in demand.
    - SCET representation of the QCD $b \to u$ current at NLP including QED interaction.
    - QCD factorization for the subleading power SCET matrix elements.
    - QCD resummation for the parametrically large logarithms in the heavy quark expansion.