Revisiting the unitary constraints on dimension six operators in SMEFT

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in collaboration with Jue Zhang and Qing-Hong Cao
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Already good analysis exist

YITP-SB-14-46

Unitarity Constraints on Dimension-Six Operators

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YITP-SB-17-19

Unitarity Constraints on Dimension-six Operators II: Including Fermionic Operators

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Why
one more
paper?

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Motivation

Motivation

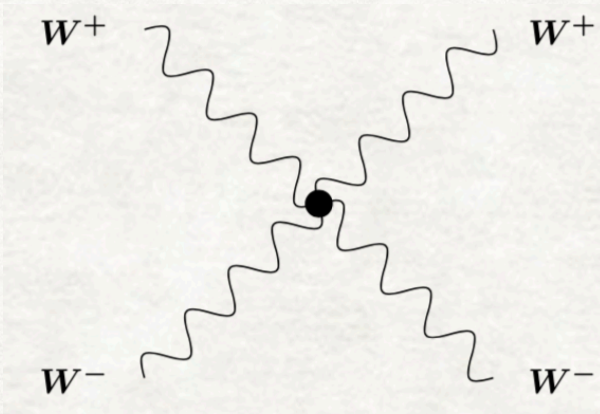
Operator combination

Unitary limitation

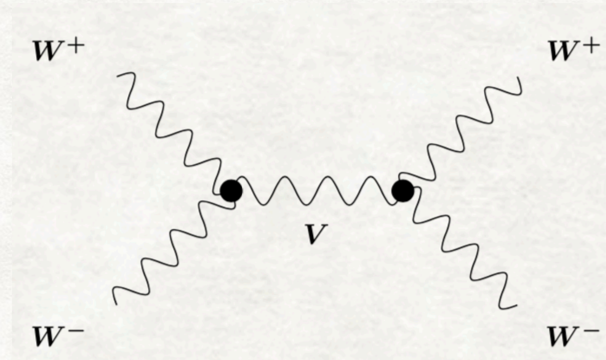
(Possible) cancellation

Specific NP structure?

Long before Higgs discovery, unitary already told us very useful information about *new physics* scale m_H



$\propto \mathcal{O}(E^4)$



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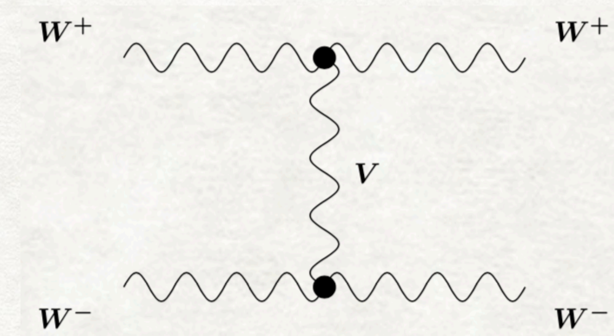

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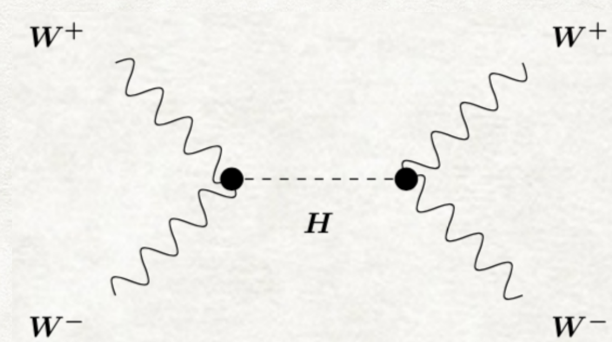
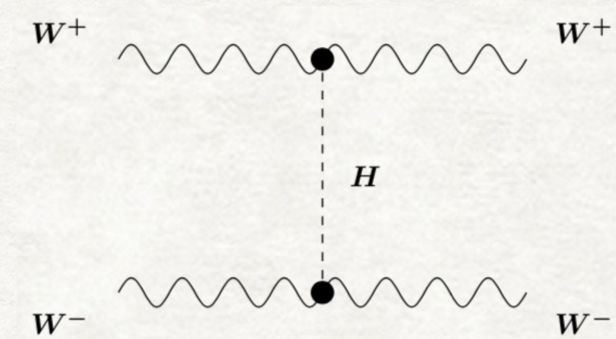


$\propto \mathcal{O}(E^2)$



$\propto \mathcal{O}(E^2)$

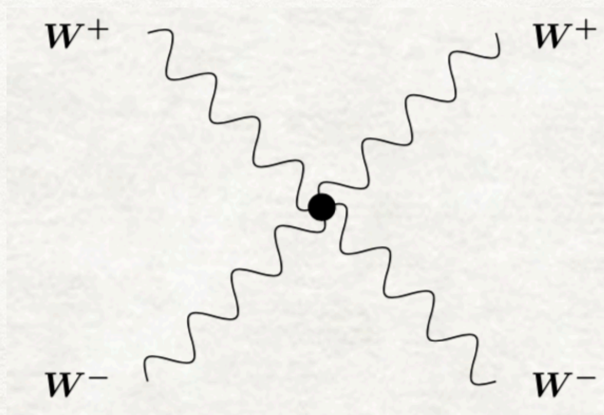
$$a^J = A(\frac{E}{M_W})^4 + B(\frac{E}{M_W})^2 + C,$$

Gauge principle $\Rightarrow A = 0$;

Higgs Involved $\Rightarrow (B = 0)$ & $(C < \#)$

Non-trivial cancellation occurs to save unitarity

Long before Higgs discovery, unitary already told us very useful information about *new physics* scale m_H



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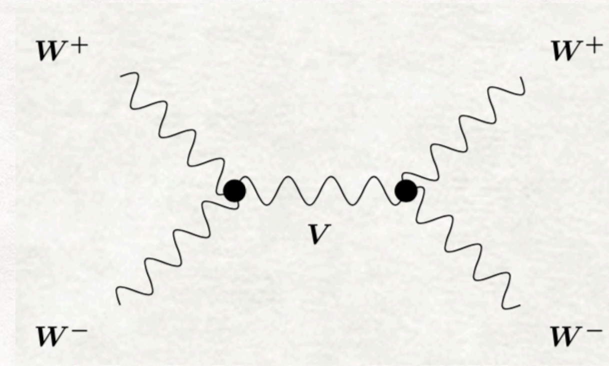

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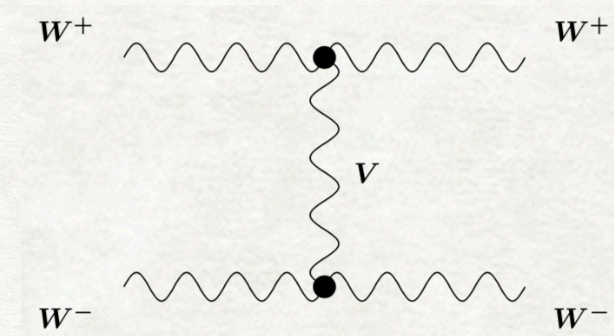

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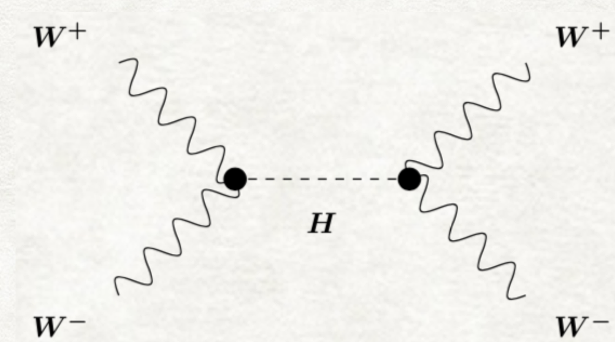
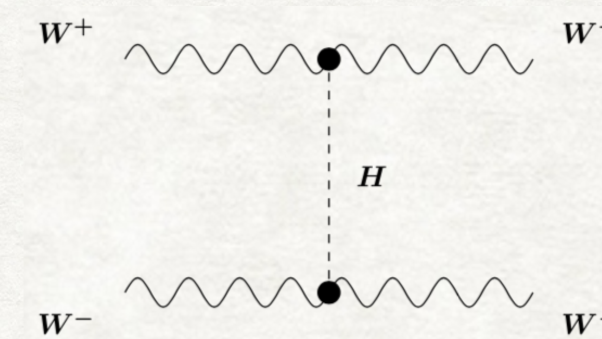


$\propto \mathcal{O}(E^2)$



$\propto \mathcal{O}(E^2)$

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$\boxed{\text{Unitarity constrain the mass of Higgs}}$

Without any new states discovery on LHC (up to now), SMEFT is a suitable tools to describe NP

Without any new states discovery on LHC (up to now), SMEFT is a suitable tools to describe NP

$E \sim \Lambda_{NP}$ $\mathscr{L}_{UV}$

UV complete theory,
probably containing new particles

$E \sim \langle v_{EM} \rangle$ $$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_n^{\infty} \sum_i \frac{C_i}{\Lambda^n} O_i$$

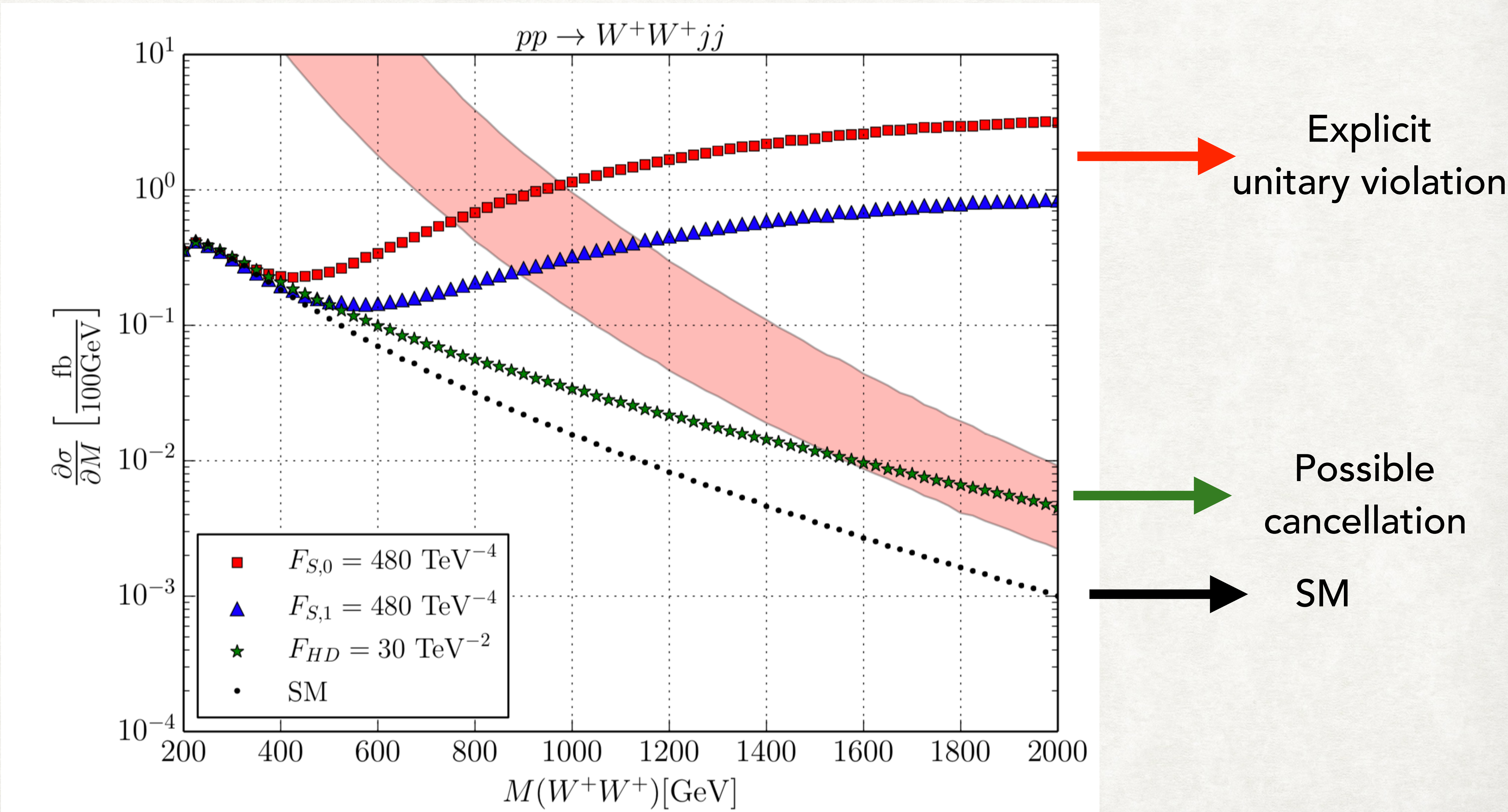
All SM particles presented,
with $SU(3)_c \times SU(2)_L \times U(1)_Y$ imposed

$E \sim m_b$ $$\mathscr{L}_W = \mathscr{L}_{\text{QCD}} + \mathscr{L}_{\text{QED}} + \mathscr{L}_{4f} + \dots$$

Only some SM particles presented,
with $SU(3)_c \times U(1)_Q$ imposed

Without any new states discovery on LHC (up to now), SMEFT is a suitable tools to describe NP

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_{n}^{\infty} \sum_{i} \frac{C_i}{\Lambda^n} O_i$$



Phys.Rev. D91 (2015) 096007

Without any new states discovery on LHC (up to now), SMEFT is a suitable tools to describe NP

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_{n}^{\infty} \sum_{i} \frac{C_i}{\Lambda^n} O_i$$

Question:

$\boxed{\text{Is it still a valid tool for the anticipated c.m.s energy } \sqrt{s}_{eff} \sim \text{ 4 - 5 TeV ?}}$

If no unitary violation were found in the future, possible cancellation?

Operator Basis
Warsaw basis v.s. HISZ basis

Operators constrained by unitary condition (Warsaw basis)

$\mathcal{O}_W$	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$\mathcal{O}_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$\mathcal{O}_{\varphi D}$	$(\varphi^\dagger D^\mu\varphi)^*(\varphi^\dagger D_\mu\varphi)$
$\mathcal{O}_{\varphi W}$	$\varphi^\dagger\varphi W^I_{\mu\nu} W^{I\mu\nu}$	$\mathcal{O}_{\varphi B}$	$\varphi^\dagger\varphi B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi W^I_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}^{(1)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$	$\mathcal{O}^{(3)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$	$\mathcal{O}_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}^{(1)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_p\gamma^\mu q_r)$	$\mathcal{O}^{(3)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$	$\mathcal{O}_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}_p\gamma^\mu d_r)$	$\mathcal{O}_{\varphi ud}$	$(\widetilde{\varphi}^\dagger iD_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$		
$\mathcal{O}_{eW}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}W^I_{\mu\nu}$
$\mathcal{O}_{uB}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}B_{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi W^I_{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi B_{\mu\nu}$

Four fermion operator: they play with themselves, almost no combination occurs.

With universal flavor assumption

1008.4884
hep-ph/9612268

Operator Basis
Warsaw basis v.s. HISZ basis

Operators constrained by unitary condition (HISZ basis)

Using EOM

$$2O_W^{HISZ} - \frac{1}{4}g^2 O_{\varphi W} - \frac{1}{4}g_1 g O_{\varphi WB} + \frac{3}{4}g^2 O_{\varphi\Box} = -\frac{g^2}{4}\left[O_{\varphi l}^{(3)} + O_{\varphi q}^{(3)}\right]$$

$$2O_B^{HISZ} - \frac{1}{4}g_1^2 O_{\varphi B} - \frac{1}{4}g_1 g O_{\varphi WB} + g_1^2\left(O_{\varphi D} + \frac{1}{4}O_{\varphi\Box}\right) = -\frac{g_1^2}{2}\left[-\frac{1}{2}O_{\varphi l}^{(1)} + \frac{1}{6}O_{\varphi q}^{(1)} - O_{\varphi e} + \frac{2}{3}O_{\varphi u} - \frac{1}{3}O_{\varphi d}\right]$$

Trade $\{O_{\varphi l}^{(1)}, O_{\varphi l}^{(3)}\}$ for $\{O_W^{HISZ}, O_B^{HISZ}\}$

$$O_W^{HISZ} \equiv \left(D_\mu \phi\right)^\dagger \left(ig\frac{\tau^i}{2}W^{i;\mu\nu}\right)\left(D_\nu \phi\right)$$

$$O_B^{HISZ} \equiv \left(D_\mu \phi\right)^\dagger \left(i\frac{g_1}{2}B^{\mu\nu}\right)\left(D_\nu \phi\right)$$

With universal flavor assumption

1008.4884
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Operator Basis
Warsaw basis v.s. HISZ basis

Question:

$$\boxed{\text{Theoretically equivalent; How about phenomenology?}}$$

Unitary condition & SMEFT

For scattering matrix *S*,

$$1 = S^\dagger S = (1 + iT)^\dagger (1 + iT) \Rightarrow -i(T - T^\dagger) = T^\dagger T$$

Partial wave unitarity

$$\langle i | T | f \rangle \sim \sum_J (2J+1) d^J(\theta) a^J \Rightarrow |Re(a^J)| \leq \frac{1}{2}$$

SM & SMEFT interference terms for $\langle v_{EM} \rangle \ll \sqrt{s} \leq \Lambda$:

$$a^J \propto s\frac{C_i}{\Lambda^2} + o(s^{1/2}) \Rightarrow |s\frac{C_i}{\Lambda^2}| \leq f_i$$

Unitary condition

Procedure:

1. Calculate all relevant partial wave helicity amplitudes with condition

$$\langle v_{EM} \rangle \ll \sqrt{s} \leq \Lambda$$

2. Using couple channel method to strength the unitary constraint

3. Analytically or numerically solve the couple channel system, require each eigenvalue smaller than some constants (e.g. 1/2 for $VV \to VV$ scattering) then marginalize the unitary bounds to individual operators.

Dealing with Identical Particles

$$\langle i|T|f\rangle = \langle A_{\lambda_1}B_{\lambda_2}|T|C_{\lambda_3}D_{\lambda_4}\rangle \sim I_{iden}\sum_J (2J+1)d^J_{\alpha\beta}(\theta)a^J_{\alpha\beta}$$

Where the identical-particle factor reads

$$I_{iden} = \sqrt{\delta_{A\lambda_1,B\lambda_2}+1}\sqrt{\delta_{C\lambda_3,D\lambda_4}+1}$$

Dealing with Identical Particles

$$\langle i | T | f \rangle = \langle A_{\lambda_1} B_{\lambda_2} | T | C_{\lambda_3} D_{\lambda_4} \rangle \sim I_{iden} \sum_J (2J+1) d^J_{\alpha\beta}(\theta) a^J_{\alpha\beta}$$

Where the identical-particle factor reads

$$I_{iden} = \sqrt{\delta_{A,B} + 1}\sqrt{\delta_{C,D} + 1}$$

It has nothing to do with helicity of particles

For those identical particles with different helicities, a factor $\sqrt{2}$ is missing.

I_{iden} stems from two different sources:
one is to account for the phase space double counting for the case $\lambda_3 = \lambda_4$,
the other is to account for the wrong normalization of the identical two particle states for the case $\lambda_3 \neq \lambda_4$

Verified also by e.g. 1610.08420

Unitary condition

Procedure:

1. Calculate all relevant partial wave helicity amplitudes with condition

$$\langle v_{EM} \rangle \ll \sqrt{s} \leq \Lambda$$

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3. Analytically or numerically solve the couple channel system, require each eigenvalue smaller than some constants (e.g. 1/2 for $VV \to VV$ scattering) then marginalize the unitary bounds to individual operators.

Couple Channel Analysis —— $VV \to VV$ process

$$W^+W^- \to W^+W^-,\ W^+W^- \to ZZ,\ W^+W^- \to Z\gamma,\ W^+W^- \to Zh$$
$$W^+W^- \to \gamma\gamma,\ W^+W^- \to \gamma h,\ W^+W^- \to hh$$

Combine these scattering processes and classified by (Q,J)

$e.g.\ Q = 0$

$$\begin{array}{cc} & (W^+W^-\ ZZ\ Z\gamma\ Zh\ \gamma\gamma\ \gamma h\ hh) \\ \begin{pmatrix} W^+W^- \\ Z\,Z \\ Z\gamma \\ Zh \\ \gamma\gamma \\ \gamma h \\ hh \end{pmatrix} & \begin{pmatrix} A_{11} & A_{12} & A_{13} & \cdots \\ A_{21} & \cdots & & \\ & \vdots & & \\ A_{71} & \cdots & & A_{77} \end{pmatrix} \end{array}$$

A_{ij}: 9×9 matrix (helicity partial wave amplitudes)

Benjamin W. Lee, et al. Phys.Rev. D16 (1977) 1519

Couple Channel Analysis —— $f\bar{f} \to VV$ process

$$e^+e^- \to W^+W^-,\ e^+e^- \to ZZ,\ u\bar{u} \to W^+W^-,\ u\bar{u} \to ZZ,\ \ldots$$

Combine initial fermion pair and classified by (Q,J)

$e.g.\ Q = 0$

$$\frac{1}{\sqrt{2N_g}} | \sum_{i=1}^{N_g} (e^-_{-,i}e^+_{+,i} + \nu_{-,i}\bar{\nu}_{+i})\rangle \to W^+_0 W^-_0$$

$$\frac{1}{\sqrt{2N_g(1+N_c)}} \left| \sum_{i=1}^{N_g} \left(-e^-_{-,i}e^+_{+,i} + \nu_{-,i}\bar{\nu}_{+i} + \sum_{a=1}^{N_c} (-d^a_{-,i}\bar{d}^a_{+,i} + u^a_{-,i}\bar{u}^a_{+,i}) \right) \right\rangle \to W^+_+ W^-_+$$

.

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N_g : generations

U. Baur and D. Zeppenfeld, Phys. Lett. B201, 383 (1988)

Couple Channel Analysis —— $f\bar{f} \to VV$ process

$$e^+e^- \to W^+W^-,\ e^+e^- \to ZZ,\ u\bar{u} \to W^+W^-,\ u\bar{u} \to ZZ,\ \ldots$$

$e.g.\ Q = 0$

$$\begin{array}{cc} & (W^+W^- \;\; ZZ \;\; Z\gamma \;\; Zh \;\; \ldots) \\ \begin{pmatrix} e^+e^- \\ \bar{\nu}\,\nu \\ u\bar{u} \\ d\bar{d} \end{pmatrix} & \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & \ldots \\ A_{21} & A_{22} & A_{23} & A_{24} & \ldots \\ A_{31} & A_{32} & A_{33} & A_{34} & \ldots \\ A_{41} & A_{42} & A_{43} & A_{44} & \ldots \end{pmatrix} \end{array}$$

… : not contribute at order of $\mathcal{O}(s)$

A_{ij}: $n\times 9$ matrix (helicity partial wave amplitudes)
n: particle dependent

$\boxed{\text{Combination of } VV \text{ states; Singular Value Decomposition}}$

A factor of $\sqrt{m}$ is missing, with m being the number of different final states

Helicity amplitudes: combination of operators

Process	Helicity	Warsaw basis	HISZ basis - Warsaw basis
wwww	$---+$	$6gC_W$	0
wwww	$-0+0$	$-C_{\varphi W}+\frac{9gC_W}{4}+\left(C_{\varphi W}+\frac{3gC_W}{4}\right)\cos(\theta)$	$-\frac{1}{4}g^2C_W^{HISZ}\sin^2\left(\frac{\theta}{2}\right)$
wwww	0000	$C_{\varphi D}+2C_{\varphi\Box}$	$-\frac{3}{4}(C_W^{HISZ}g^2+C_B^{HISZ}g_1^2)$
wzwz	$---+$	$-\frac{3g^3C_W(\cos(\theta)-1)}{g^2+g_1^2}$	0
wzwz	$--00$	$\frac{3g^2C_W\cos(\theta)-2C_{\varphi WB}g_1}{2\sqrt{g^2+g_1^2}}$	$-\frac{g(C_B^{HISZ}+C_W^{HISZ})g_1^2}{8\sqrt{g^2+g_1^2}}$
wzwz	$-0+0$	$C_{\varphi W}(\cos(\theta)-1)$	$-\frac{1}{4}g^2C_W^{HISZ}\sin^2\left(\frac{\theta}{2}\right)$
wzwz	$-00+$	$\frac{3(\cos(\theta)-3)C_Wg^2+2(\cos(\theta)+1)C_{\varphi WB}g_1}{4\sqrt{g^2+g_1^2}}$	$\frac{(\cos(\theta)+1)g(C_B^{HISZ}+C_W^{HISZ})g_1^2}{16\sqrt{g^2+g_1^2}}$
wzwz	$0-0+$	$\frac{(\cos(\theta)-1)\left(C_{\varphi W}g^2-C_{\varphi WB}g_1g+C_{\varphi B}g_1^2\right)}{g^2+g_1^2}$	$\frac{(C_B^{HISZ}-C_W^{HISZ})\sin^2\left(\frac{\theta}{2}\right)g_1^2\left(g^2-g_1^2\right)}{4\left(g^2+g_1^2\right)}$
wzwz	0000	$(\frac{1}{4}C_{\varphi D}-C_{\varphi\Box})(\cos(\theta)-1)$	$\frac{-3}{4}g^2C_W^{HISZ}\sin^2\left(\frac{\theta}{2}\right)$
wzwa	$00--$	$\frac{g(2C_{\varphi WB}+3C_W\cos(\theta)g_1)}{2\sqrt{g^2+g_1^2}}$	$\frac{(C_B^{HISZ}+C_W^{HISZ})g_1g^2}{8\sqrt{g^2+g_1^2}}$
wzwa	$-00+$	$-\frac{g(2C_{\varphi WB}(\cos(\theta)+1)-3C_W(\cos(\theta)-3)g_1)}{4\sqrt{g^2+g_1^2}}$	$-\frac{g^2(C_B^{HISZ}+C_W^{HISZ})g_1\cos^2\left(\frac{\theta}{2}\right)}{8\sqrt{g^2+g_1^2}}$
wzwa	$0-0+$	$\frac{\sin^2\left(\frac{\theta}{2}\right)\left(-C_{\varphi WB}g^2+2(C_{\varphi B}-C_{\varphi W})g_1g+C_{\varphi WB}g_1^2\right)}{g^2+g_1^2}$	$\frac{gg_1\sin^2\left(\frac{\theta}{2}\right)}{8\left(g^2+g_1^2\right)}\left(\left(3g_1^2-g^2\right)C_B^{HISZ}+\left(g_1^2-3g^2\right)C_W^{HISZ}\right)$
wzwh	$-0+0$	$\frac{-3}{4}gC_W(\cos(\theta)+3)$	0
wzwh	0000	$\frac{1}{4}C_{\varphi D}(\cos(\theta)+3)$	$-\frac{1}{8}C_B^{HISZ}(\cos(\theta)+3)g_1^2$
wawa	$0-0+$	$\frac{(\cos(\theta)-1)\left(C_{\varphi B}g^2+C_{\varphi WB}g_1g+C_{\varphi W}g_1^2\right)}{g^2+g_1^2}$	$-\frac{g^2\sin^2\left(\frac{\theta}{2}\right)g_1^2}{2\left(g^2+g_1^2\right)}(C_B^{HISZ}+C_W^{HISZ})$
wwww	$--00$	$\frac{3}{2}gC_W\cos(\theta)-2C_{\varphi W}$	$-\frac{g^2}{4}C_W^{HISZ}$
wwww	$-0+0$	$-C_{\varphi W}+\left(C_{\varphi W}-\frac{3gC_W}{4}\right)\cos(\theta)-\frac{9gC_W}{4}$	$-\frac{1}{4}g^2C_W^{HISZ}\sin^2\left(\frac{\theta}{2}\right)$
wwww	0000	$-(\frac{1}{2}C_{\varphi D}+C_{\varphi\Box})(\cos(\theta)+1)$	$\frac{3}{4}(C_B^{HISZ}g_1^2+C_W^{HISZ}g^2)\cos^2\left(\frac{\theta}{2}\right)$
wwzz	$--00$	$-2C_{\varphi W}$	$-\frac{g^2}{4}C_W^{HISZ}$
wwzz	$00--$	$-\frac{2\left(C_{\varphi W}g^2-C_{\varphi WB}g_1g+C_{\varphi B}g_1^2\right)}{g^2+g_1^2}$	$\frac{\left(g^2-g_1^2\right)}{4\left(g^2+g_1^2\right)}(C_B^{HISZ}g_1^2-C_W^{HISZ}g^2)$
wwzz	$-0+0$	$\frac{2C_{\varphi WB}(\cos(\theta)-1)g_1-3g^2C_W(\cos(\theta)+3)}{4\sqrt{g^2+g_1^2}}$	$-\frac{g\sin^2\left(\frac{\theta}{2}\right)g_1^2}{8\sqrt{g^2+g_1^2}}(C_B^{HISZ}+C_W^{HISZ})$
wwzz	0000	$\frac{1}{2}(C_{\varphi D}-4C_{\varphi\Box})$	$\frac{3g^2}{4}C_W^{HISZ}$
wwza	$00--$	$\frac{-C_{\varphi WB}g^2+2(C_{\varphi B}-C_{\varphi W})g_1g+C_{\varphi WB}g_1^2}{g^2+g_1^2}$	$\frac{gg_1}{8\left(g^2+g_1^2\right)}\left(\left(3g_1^2-g^2\right)C_B^{HISZ}+\left(g_1^2-3g^2\right)C_W^{HISZ}\right)$

Why do $O(s^n)\,(n > 1)$ terms vanish?

1. Field & parameter redefinition

e.g. $O_{\varphi W}:\ (\varphi^\dagger\varphi)W^I_{\mu\nu}W^{I\mu\nu} \Rightarrow \frac{v^2}{2}W^I_{\mu\nu}W^{I\mu\nu}$

$$\Rightarrow W^I_\mu \to (1 + v^2 O_{\varphi W})W^{I,r}_\mu,\ g \to (1 - v^2 O_{\varphi W})g^r$$

2. Gauge symmetry protection (inferred)

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$$\Rightarrow W^I_\mu \to (1 + v^2 O_{\varphi W}) W^{I,r}_\mu, g \to (1 - v^2 O_{\varphi W}) g^r$$

2.Gauge symmetry protection (inferred)

Wrong example: w/o redefinition (and using original propagators & vertices):

$$\mathscr{M}(W^+W^- \to W^+W^-):\ \frac{s^2}{\Lambda^2} \frac{O_{\varphi W}(12\cos(\theta) + \cos(2\theta) - 5)}{4\mathrm{mw}^2}$$

Why do $O(s^n)\,(n>1)$ terms vanish?

1.Field & parameter redefinition

e.g. $\mathscr{O}_{\varphi W}:\ (\varphi^\dagger\varphi)W^I_{\mu\nu}W^{I\mu\nu} \Rightarrow \dfrac{v^2}{2}W^I_{\mu\nu}W^{I\mu\nu}$

$$\Rightarrow W^I_\mu \to (1+v^2\mathscr{O}_{\varphi W})W^{I,r}_\mu,\ g \to (1-v^2\mathscr{O}_{\varphi W})g^r$$

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Wrong example: w/o redefinition (and using original propagators & vertices):

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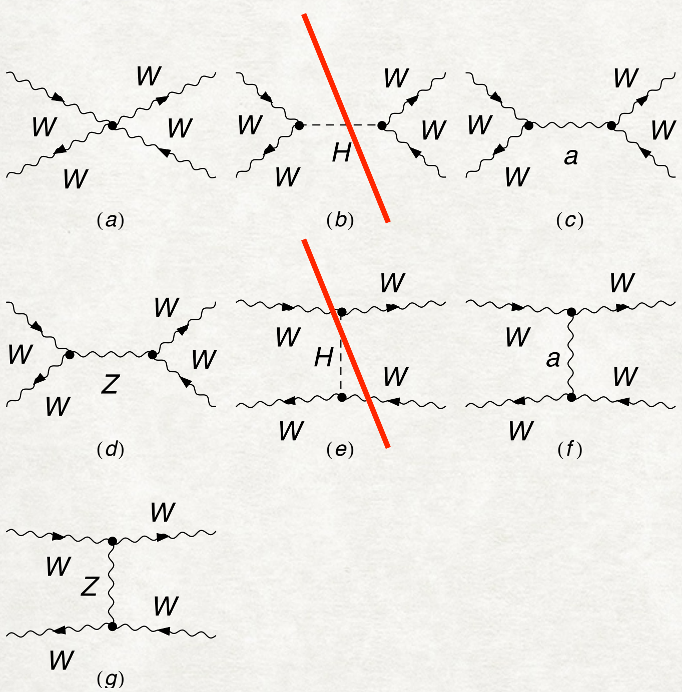


New terms in SM results, induced by redefinition ($O(s^2)$):

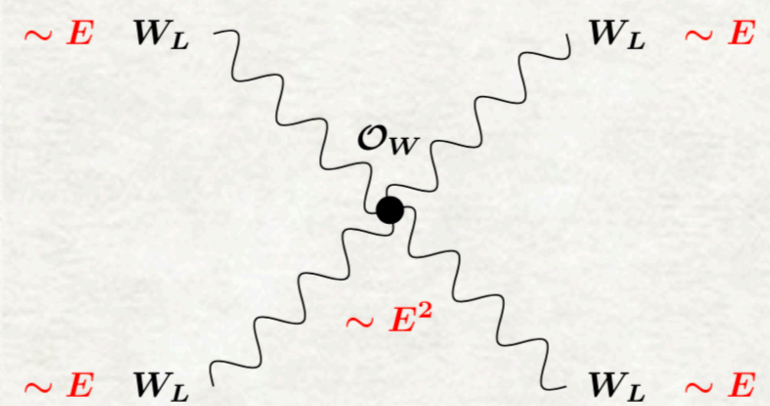
$$\begin{bmatrix} \frac{g^2s^2(12\cos(\theta)+\cos(2\theta)-5)}{32\text{mw}^4} \\ -\frac{g^2g_1^2s^2\cos(\theta)}{4\text{mw}^4\left(g^2+g_1^2\right)} \\ -\frac{g^4s^2\cos(\theta)}{4\text{mw}^4\left(g^2+g_1^2\right)} \\ \frac{g^2g_1^2s^2\sin^2\left(\frac{\theta}{2}\right)(\cos(\theta)+3)}{8\text{mw}^4\left(g^2+g_1^2\right)} \\ \frac{g^4s^2\sin^2\left(\frac{\theta}{2}\right)(\cos(\theta)+3)}{8\text{mw}^4\left(g^2+g_1^2\right)} \\ \text{(added up)}\Downarrow \\ 0 \end{bmatrix} \to \begin{bmatrix} (1+2v^2\mathcal{O}_{\varphi W})\left[\frac{g^2s^2(12\cos(\theta)+\cos(2\theta)-5)}{32\text{mw}^4}\right] \\ (1+4v^2\mathcal{O}_{\varphi W}+2\frac{g}{g_1}\frac{g^2}{g_1^2+g^2}v^2\mathcal{O}_{\varphi WB})\left[-\frac{g^2g_1^2s^2\cos(\theta)}{4\text{mw}^4\left(g^2+g_1^2\right)}\right] \\ (1+4v^2\mathcal{O}_{\varphi W}-2\frac{g_1g}{g_1^2+g^2}v^2\mathcal{O}_{\varphi WB})\left[-\frac{g^4s^2\cos(\theta)}{4\text{mw}^4\left(g^2+g_1^2\right)}\right] \\ (1+4v^2\mathcal{O}_{\varphi W}+2\frac{g}{g_1}\frac{g^2}{g_1^2+g^2}v^2\mathcal{O}_{\varphi WB})\left[\frac{g^2g_1^2s^2\sin^2\left(\frac{\theta}{2}\right)(\cos(\theta)+3)}{8\text{mw}^4\left(g^2+g_1^2\right)}\right] \\ (1+4v^2\mathcal{O}_{\varphi W}-2\frac{g_1g}{g_1^2+g^2}v^2\mathcal{O}_{\varphi WB})\left[\frac{g^4s^2\sin^2\left(\frac{\theta}{2}\right)(\cos(\theta)+3)}{8\text{mw}^4\left(g^2+g_1^2\right)}\right] \\ \text{(added up)}\Downarrow \\ -\frac{s^2\mathcal{O}_{\varphi W}(12\cos(\theta)+\cos(2\theta)-5)}{4\text{mw}^2} \end{bmatrix}$$

Why do $O(s^n)\,(n>1)$ terms vanish?

1. Field & parameter redefinition
2. Gauge symmetry protection (inferred)

Take $\epsilon_{abc}W^a_{\mu\nu}W^b_{\nu\rho}W^c_{\rho\mu}$ as an example:

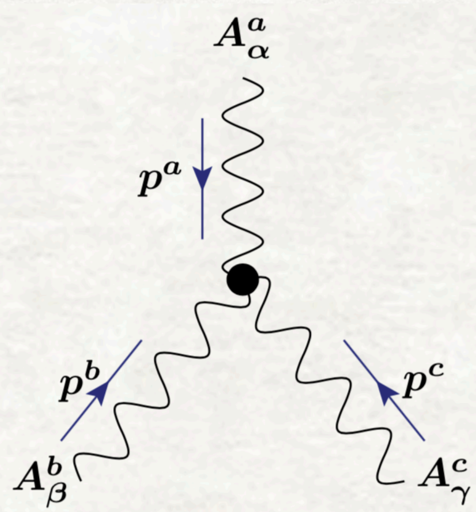
Naive counting:



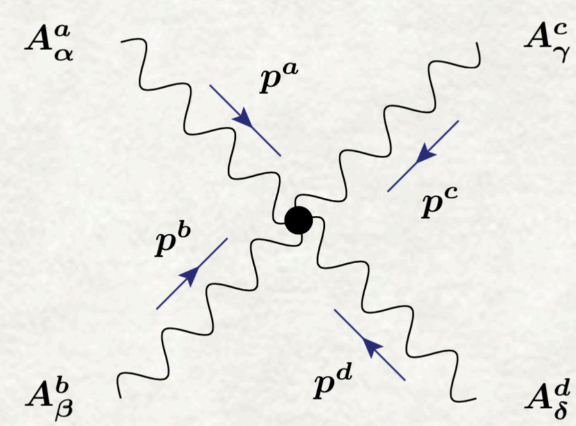
$\propto s^3$

$O_W \Rightarrow c_{32} = -c_{31} = c_{41} = c_{42} = c_{43}$

What if c_i are arbitrary?



$\frac{igC_W f_{abc}}{\Lambda^2}\left(c_{31}\mathcal{T}_{31} + c_{32}\mathcal{T}_{32}\right)$



$\frac{ig^2C_W}{\Lambda^2}\left[f_{abe}f_{cde}\left(c_{41}\mathcal{T}_{41} + c_{42}\mathcal{T}_{42} + c_{43}\mathcal{T}_{43}\right)\right.$
$\left. +(b\leftrightarrow c) + (b\to d, c\to b, d\to c)\right]$

Why do $O(s^n)\,(n>1)$ terms vanish?

1.Field & parameter redefinition

2.Gauge symmetry protection (inferred)

Take $\epsilon_{abc}W^a_{\mu\nu}W^b_{\nu\rho}W^c_{\rho\mu}$ as an example:

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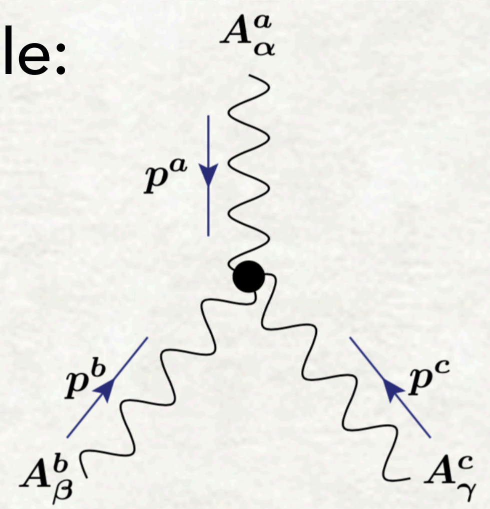
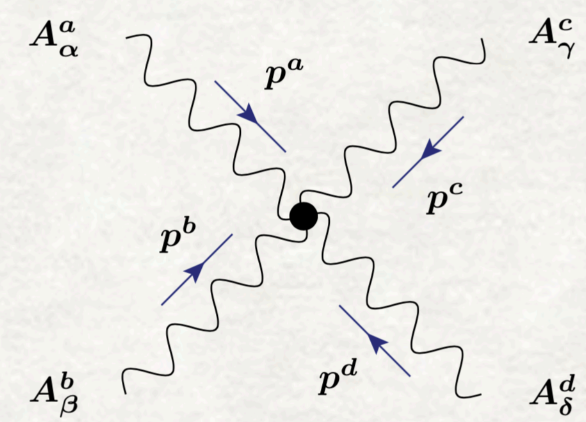


$\frac{igC_W f_{abc}}{\Lambda^2}\left(c_{31}\mathcal{T}_{31} + c_{32}\mathcal{T}_{32}\right)$



$\frac{ig^2C_W}{\Lambda^2}\left[f_{abe}f_{cde}\left(c_{41}\mathcal{T}_{41} + c_{42}\mathcal{T}_{42} + c_{43}\mathcal{T}_{43}\right)\right.$
$\left. +(b\leftrightarrow c) + (b\to d, c\to b, d\to c)\right]$

For $W^+W^- \to W^+W^-$, the helicity amplitudes read as

$$\mathcal{M}_{(++,++),(++,+0),(++,+-)} = \underset{\substack{\| \\ 0}}{(\#)\,s^3} + \underset{\substack{\| \\ 0}}{(\#)\,s^{5/2}} + \underset{\substack{\| \\ 0}}{(\#)\,s^2} + \underset{\substack{\| \\ 0}}{(\#)\,s^{3/2}} + (\#)s + \ldots$$

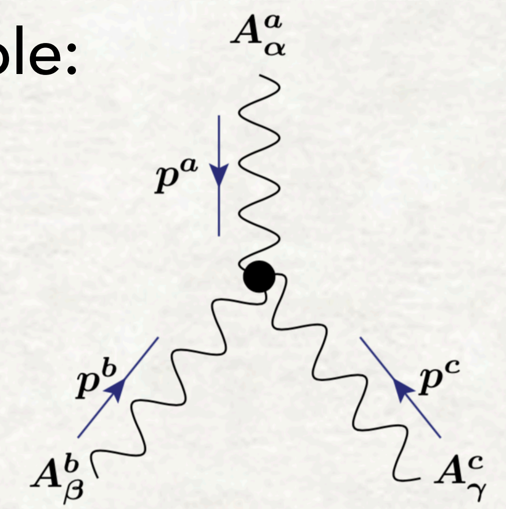
Why do $O(s^n)\,(n>1)$ terms vanish?

1. Field & parameter redefinition
2. Gauge symmetry protection (inferred)

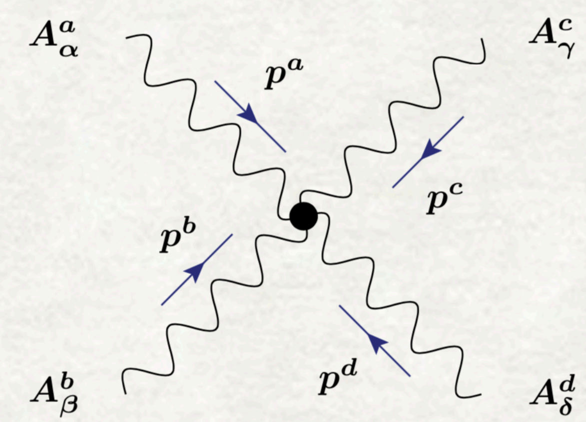
Take $\epsilon_{abc} W^a_{\mu\nu} W^b_{\nu\rho} W^c_{\rho\mu}$ as an example:

$$O_W \Rightarrow c_{32} = -c_{31} = c_{41} = c_{42} = c_{43}$$

What if c_i are arbitrary?



$\frac{igC_W f_{abc}}{\Lambda^2}\left(c_{31}\mathcal{T}_{31} + c_{32}\mathcal{T}_{32}\right)$



$\frac{ig^2C_W}{\Lambda^2}\left[f_{abe}f_{cde}\left(c_{41}\mathcal{T}_{41} + c_{42}\mathcal{T}_{42} + c_{43}\mathcal{T}_{43}\right)\right.$
$\left.+(b\leftrightarrow c) + (b\to d, c\to b, d\to c)\right]$

For $W^+W^- \to W^+W^-$, the helicity amplitudes read as

$$\mathscr{M}_{(++,++),(++,+0),(++,+-)} = \underbrace{(\#)\, s^3}_{=0} + \underbrace{(\#)\, s^{5/2}}_{=0} + \underbrace{(\#)\, s^2}_{=0} + \underbrace{(\#)\, s^{3/2}}_{=0} + (\#)s + \ldots$$

$\boxed{\text{Gauge structure restored! Spurious high energy terms disappear!}}$

Hard to get analytical bounds for *all* operators

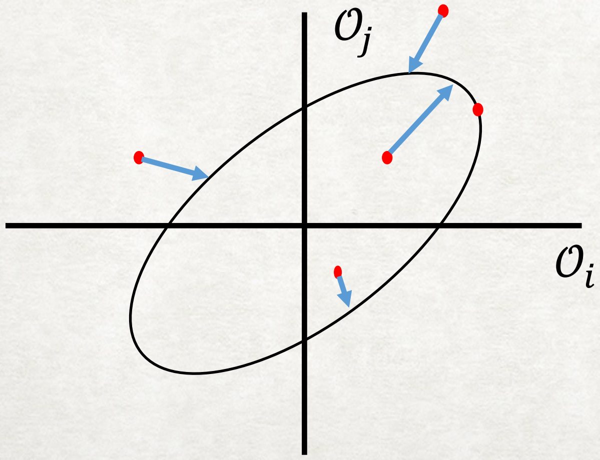
Numerical method

MultiNest program ⇒ numerically get the unitary bounds for each operators (with other operators non-vanishing)

However, very unlikely to make random test points sit just onto the boundary

⇒ **very inefficient**

We have to turn on 20 operators **simultaneously**



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Numerical method

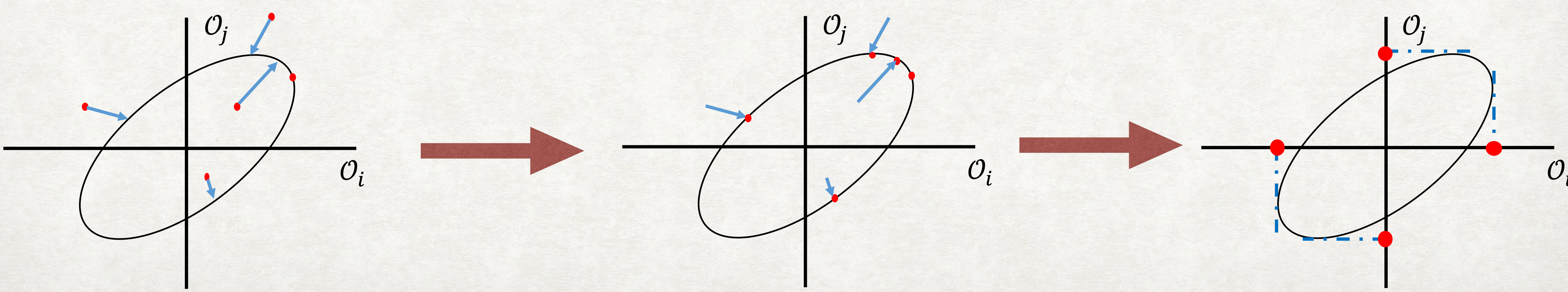
MultiNest program ⇒ numerically get the unitary bounds for each operators (with other operators non-vanishing)

However, very unlikely to make random test points sit just onto the boundary

⇒ **very inefficient**

Eigen values $\sim \dfrac{s}{\Lambda^2} f(c_i)$**, with** $f(c_i)$ **being homogeneous linear functions**

⇒ rescale all operators to make the test point just sit onto the unitary boundary.



Main results

Numerical results on unitary bounds

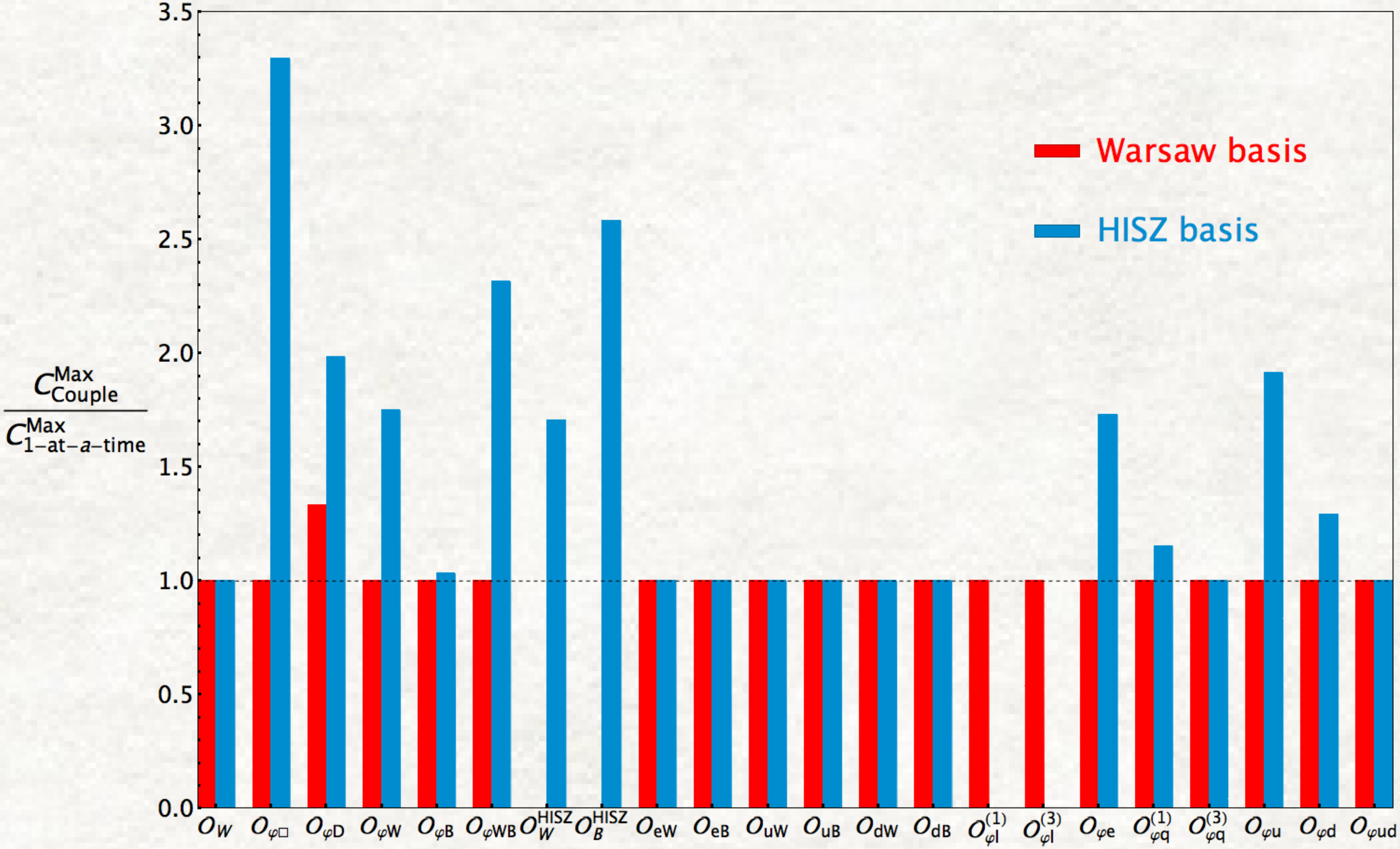
$$|s\frac{C_i}{\Lambda^2}| \leq f_i$$

Operator Name	One-at-a-time Bounds on Wilson coefficients C_i	Couple Channel Bound on Wilson coefficients C_i	
		Numerical results (Warsaw)	Numerical results (HISZ)
$\mathcal{O}_W$	6.27	$6.27\ (\frac{4\pi}{3g})$	$6.27\ (\frac{4\pi}{3g})$
$\mathcal{O}_{\varphi\Box}$	16.76	16.76	55.22
$\mathcal{O}_{\varphi D}$	50.27	67.02	99.74
$\mathcal{O}_{\varphi W}$	10.26	10.26	17.95
$\mathcal{O}_{\varphi B}$	17.77	17.77	18.35
$\mathcal{O}_{\varphi WB}$	25.13	25.13	58.18
$\mathcal{O}_W^{\mathrm{HISZ}}$	80.80	/	$137.96(\frac{8\sqrt{6}\pi}{g^2})$
$\mathcal{O}_B^{\mathrm{HISZ}}$	378.87	/	$978.23(\frac{16\sqrt{6}\pi}{g_1^2})$
$\mathcal{O}_{eW}$	12.57	$12.57(4\pi)$	$12.57(4\pi)$
$\mathcal{O}_{eB}$	21.77	$21.77(4\sqrt{3}\pi)$	$21.767(4\sqrt{3}\pi)$
$\mathcal{O}_{uW}$	7.26	$7.26(\frac{4\pi}{\sqrt{3}})$	$7.26(\frac{4\pi}{\sqrt{3}})$
$\mathcal{O}_{uB}$	12.57	$12.57(4\pi)$	$12.57(4\pi)$
$\mathcal{O}_{dW}$	7.26	$7.26(\frac{4\pi}{\sqrt{3}})$	$7.26(\frac{4\pi}{\sqrt{3}})$
$\mathcal{O}_{dB}$	12.57	$12.57(4\pi)$	$12.57(4\pi)$
$\mathcal{O}_{\varphi l}^{(1)}$	15.39	$15.39(2\sqrt{6}\pi)$	0[a]
$\mathcal{O}_{\varphi l}^{(3)}$	15.39	$15.39(2\sqrt{6}\pi)$	0[b]
$\mathcal{O}_{\varphi e}$	21.77	$21.76(4\sqrt{3}\pi)$	$37.70(12\pi)$
$\mathcal{O}_{\varphi q}^{(1)}$	8.89	$8.88(2\sqrt{2}\pi)$	$10.26(4\sqrt{\frac{2}{3}}\pi)$
$\mathcal{O}_{\varphi q}^{(3)}$	8.89	$8.88(2\sqrt{2}\pi)$	$8.88(2\sqrt{2}\pi)$
$\mathcal{O}_{\varphi u}$	12.57	$12.56(4\pi)$	$24.06(4\sqrt{\frac{11}{3}}\pi)$
$\mathcal{O}_{\varphi d}$	12.57	$12.56(4\pi)$	$16.22(4\sqrt{\frac{5}{3}}\pi)$
$\mathcal{O}_{\varphi ud}$	17.77	$17.77(4\sqrt{2}\pi)$	$17.77(4\sqrt{2}\pi)$

[a,b] flavor universality is assumed

parentheses: partial analytical results.

w/ v.s. w/o couple-channel analysis
In two operator basis



The Warsaw basis seems more "independent" than HISZ basis

Operator Basis
Warsaw basis v.s. HISZ basis

Question:

Theoretically equivalent;
How about phenomenology?

Combine with global fit results

Unitary bounds: $|s\frac{C_i}{\Lambda^2}| \leq f_i$

Global fit: $|\frac{C_i}{\Lambda^2}| \leq g_i$

↓

If C_i takes global fit boundary value: $\sqrt{s} \leq \frac{f_i}{g_i}$

Combine with global fit results — HISZ basis

Operator Names	$\sqrt{s}_{max}$ (TeV)	Operator Names	$\sqrt{s}_{max}$ (TeV)	Operator Names	$\sqrt{s}_{max}$ (TeV)
$\mathcal{O}_W$	5.7	$\mathcal{O}_{\varphi\Box}$	1.4	$\mathcal{O}_{\varphi D}$	14.6
$\mathcal{O}_{\varphi W}$	3.7	$\mathcal{O}_{\varphi B}$	6.6	$\mathcal{O}_{\varphi WB}$	13.2
$\mathcal{O}_W^{\mathrm{HISZ}}$	2.6	$\mathcal{O}_B^{\mathrm{HISZ}}$	5.9	$\mathcal{O}_{eW}$	/
$\mathcal{O}_{eB}$	/	$\mathcal{O}_{uW}$	2.4	$\mathcal{O}_{uB}$	/
$\mathcal{O}_{dW}$	/	$\mathcal{O}_{dB}$	/	$\mathcal{O}_{\varphi l}^{(1)}$	/
$\mathcal{O}_{\varphi l}^{(3)}$	/	$\mathcal{O}_{\varphi e}$	17.7	$\mathcal{O}_{\varphi q}^{(1)}$	7.1
$\mathcal{O}_{\varphi q}^{(3)}$	5.3	$\mathcal{O}_{\varphi u}$	7.7	$\mathcal{O}_{\varphi d}$	2.9
$\mathcal{O}_{\varphi ud}$	29.2				

Possible unitary violation on LHC (with anticipated $\sqrt{s}_{\mathrm{eff}}$ go up to 4-5 TeV)

$O_{\varphi\Box}$, $\sqrt{s}_{max} = 1.46$ TeV old results 2.1 TeV

$O_{\varphi W}$, $\sqrt{s}_{max} = 3.7$ TeV old results 5.2 TeV

O_W^{HISZ}, $\sqrt{s}_{max} = 2.64$ TeV old results 4.7 TeV

O_{uW}, $\sqrt{s}_{max} = 2.4$ TeV old results 2.7 TeV

$O_{\varphi d}$, $\sqrt{s}_{max} = 2.9$ TeV old results 3.5 TeV

EWPD constrains from 1705.09294

Combine with global fit results — HISZ basis

Operator Names	$\sqrt{s}_{max}$ (TeV)	Operator Names	$\sqrt{s}_{max}$ (TeV)	Operator Names	$\sqrt{s}_{max}$ (TeV)
$\mathcal{O}_W$	5.7	$\mathcal{O}_{\varphi\Box}$	1.4	$\mathcal{O}_{\varphi D}$	14.6
$\mathcal{O}_{\varphi W}$	3.7	$\mathcal{O}_{\varphi B}$	6.6	$\mathcal{O}_{\varphi WB}$	13.2
$\mathcal{O}_W^{\mathrm{HISZ}}$	2.6	$\mathcal{O}_B^{\mathrm{HISZ}}$	5.9	$\mathcal{O}_{eW}$	/
$\mathcal{O}_{eB}$	/	$\mathcal{O}_{uW}$	2.4	$\mathcal{O}_{uB}$	/
$\mathcal{O}_{dW}$	/	$\mathcal{O}_{dB}$	/	$\mathcal{O}_{\varphi l}^{(1)}$	/
$\mathcal{O}_{\varphi l}^{(3)}$	/	$\mathcal{O}_{\varphi e}$	17.7	$\mathcal{O}_{\varphi q}^{(1)}$	7.1
$\mathcal{O}_{\varphi q}^{(3)}$	5.3	$\mathcal{O}_{\varphi u}$	7.7	$\mathcal{O}_{\varphi d}$	2.9
$\mathcal{O}_{\varphi ud}$	29.2				

Possible unitary violation on LHC (with anticipated $\sqrt{s}_{\mathrm{eff}}$ go up to 4-5 TeV)

Take care of these operators on LHC future analysis in SMEFT framework

$O_{\varphi\Box}$, $\sqrt{s}_{max} = 1.46$ TeV old results 2.1 TeV

$O_{\varphi W}$, $\sqrt{s}_{max} = 3.7$ TeV old results 5.2 TeV

O_W^{HISZ}, $\sqrt{s}_{max} = 2.64$ TeV old results 4.7 TeV

O_{uW}, $\sqrt{s}_{max} = 2.4$ TeV old results 2.7 TeV

$O_{\varphi d}$, $\sqrt{s}_{max} = 2.9$ TeV old results 3.5 TeV

EWPD constrains from 1705.09294

Combine with global fit results — Warsaw basis

13 operators in common

Operator Name	Unitary Bound $\lvert\frac{\mathcal{O}_i s}{\Lambda^2}\rvert \leq C$	Global Fit within 95% CL, $\frac{\mathcal{O}_i \langle v \rangle^2}{\Lambda^2} \sim C''$	Global Fit In Our Convention $\frac{\mathcal{O}_i}{\Lambda^2} \sim C'$	The upper bound on $\sqrt{s}$(TeV)
$\mathcal{O}_W$	6.270	-0.05 ± 0.06	-0.826 ± 0.991	1.86
$\mathcal{O}_{\varphi\Box}$	16.76	0.50 ± 0.27	8.26 ± 4.46	1.15
$\mathcal{O}_{\varphi D}$	67.02	-0.001 ± 0.014	-0.0165 ± 0.231	16.46
$\mathcal{O}_{\varphi W}$	10.26	-0.002 ± 0.014	-0.0330 ± 0.231	6.23
$\mathcal{O}_{\varphi B}$	17.77	0.003 ± 0.005	0.0496 ± 0.0826	11.59
$\mathcal{O}_{\varphi WB}$	25.13	0.006 ± 0.007	0.0991 ± 0.116	10.81
$\mathcal{O}_{\varphi l}^{(1)}$	15.39	0.002 ± 0.003	0.0330 ± 0.0496	13.65
$\mathcal{O}_{\varphi l}^{(3)}$	15.39	-0.015 ± 0.011	-0.248 ± 0.182	5.98
$\mathcal{O}_{\varphi e}$	21.76	0.002 ± 0.007	0.0330 ± 0.116	12.08
$\mathcal{O}_{\varphi q}^{(1)}$	8.88	-0.002 ± 0.003	-0.0330 ± 0.0496	10.37
$\mathcal{O}_{\varphi q}^{(3)}$	8.88	-0.017 ± 0.013	-0.281 ± 0.215	4.23
$\mathcal{O}_{\varphi u}$	12.56	0.000 ± 0.011	0.000 ± 0.182	8.31
$\mathcal{O}_{\varphi d}$	12.56	-0.036 ± 0.017	0.595 ± 0.281	3.79

Possible unitary violation on LHC (with anticipated $\sqrt{s}_{\mathrm{eff}}$ go up to 4-5 TeV)

O_W, $\sqrt{s}_{max} = 1.86$ TeV $O_{\varphi q}^{(3)}$, $\sqrt{s}_{max} = 4.23$ TeV

$O_{\varphi\Box}$, $\sqrt{s}_{max} = 1.15$ TeV $O_{\varphi d}$, $\sqrt{s}_{max} = 3.79$ TeV

Global fits from 1803.03252

With LHC Run2 data used

Without any new states discovery on LHC (up to now), SMEFT is a suitable tools to describe NP

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_{n}^{\infty} \sum_{i} \frac{C_i}{\Lambda^n} O_i$$

Question:

$\boxed{\text{Is it still a valid tool for the anticipated c.m.s energy } \sqrt{s}_{eff} \sim \text{ 4 - 5 TeV ?}}$ ✓

$\boxed{\text{If no unitary violation were found in the future, possible cancellation?}}$

Ongoing…

Summary

- Unitary constraints on 20 dim-6 operators in SMEFT is *re*-obtained
  - With identical particle issues corrected
  - Couple-channel analysis for $f\bar{f} \to VV$ are presented
  - The results are reached for 20 operators simultaneously
  - **The Wilson coefficients are constrained tighter now**
- **The Warsaw basis is less sensitive to operator combinations comparing with HISZ basis**
- Global fit results are tight, thus still safe to use SMEFT in LHC scenario for most operators
- **Operator combinations may be our new blind direction in boosted region**

Outlooks

- The tail of distribution is sensible to high dimensional operators, helpful to discriminate combinations ⇒ phenomenology study (ongoing);
- The low energy experiments constrain flavor-dependent operators very tight, needed to taken into account with RGE
- The CPV operators may also be constrained by unitary condition

Thanks for your attention !

Backups0

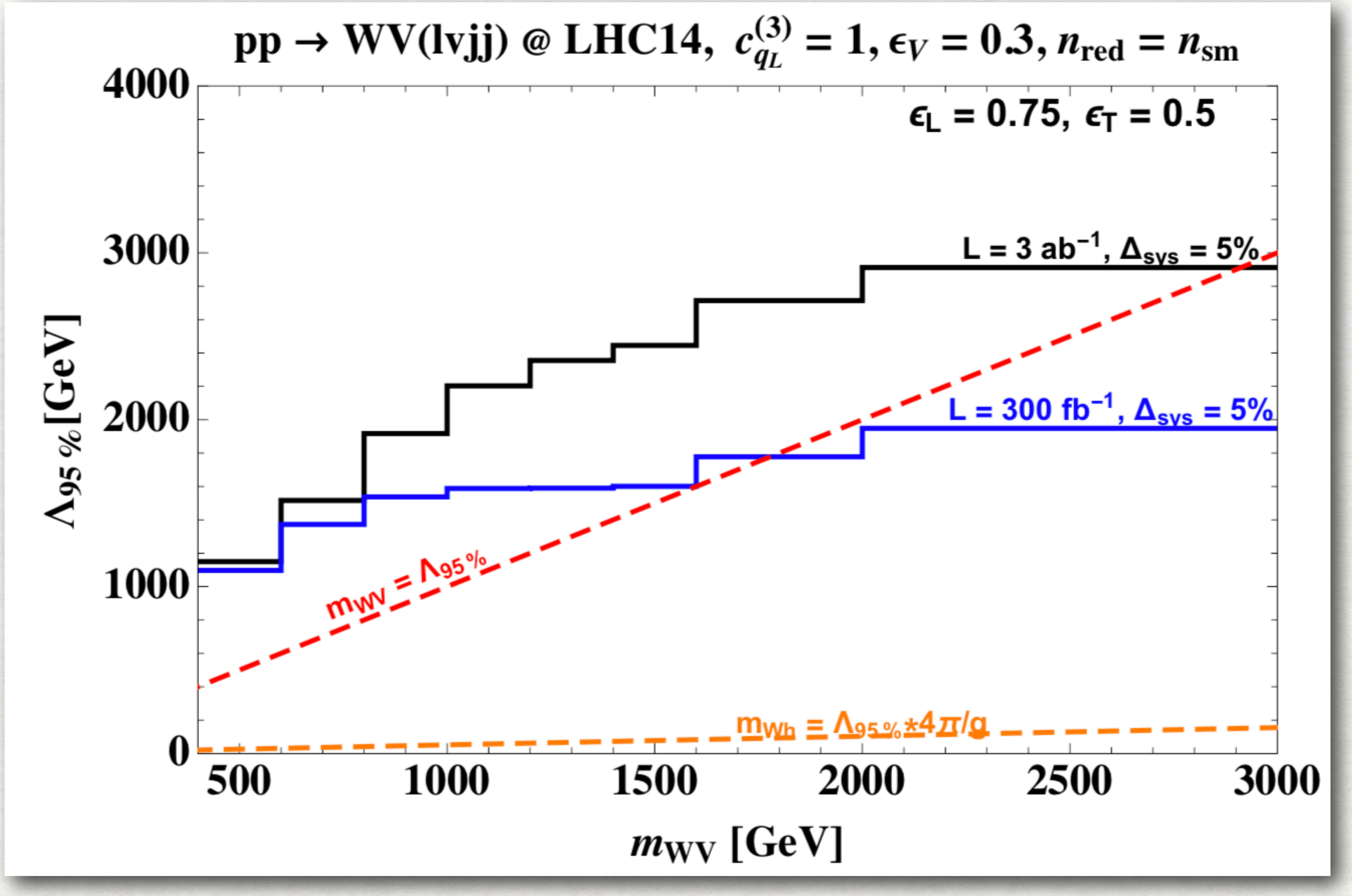
Where to find?
—Tail effects

Operators contribute to observables in combinations, e.g.

Observable	$\delta O/O_{\mathrm{SM}}$
$W_L^+W_L^-$	$\left[(c_W+c_{HW}-c_{2W})T_f^3+(c_B+c_{HB}-c_{2B})Y_f t_w^2\right]\frac{E^2}{\Lambda^2},\ c_f\frac{E^2}{\Lambda^2}$
$W_T^+W_T^-$	$c_{3W}\frac{m_W^2}{\Lambda^2}+c_{3W}^2\frac{E^4}{\Lambda^4},\ c_{TWW}\frac{E^4}{\Lambda^4}$
$W_L^\pm Z_L$	$\left(c_W+c_{HW}-c_{2W}+4c_L^{(3)q}\right)\frac{E^2}{\Lambda^2}$
$W_T^\pm Z_T(\gamma)$	$c_{3W}\frac{m_W^2}{\Lambda^2}+c_{3W}^2\frac{E^4}{\Lambda^4},\ c_{TWB}\frac{E^4}{\Lambda^4}$
$W_L^\pm h$	$\left(c_W+c_{HW}-c_{2W}+4c_L^{(3)q}\right)\frac{E^2}{\Lambda^2}$
Zh	$\left[(c_W+c_{HW}-c_{2W})T_f^3-(c_B+c_{HB}-c_{2B})Y_f t_w^2\right]\frac{E^2}{\Lambda^2},\ c_f\frac{E^2}{\Lambda^2}$
$Z_T Z_T$	$(c_{TWW}+t_w^4 c_{TBB}-2T_f^3 t_w^2 c_{TWB})\frac{E^4}{\Lambda^4}$
$\gamma\gamma$	$(c_{TWW}+c_{TBB}+2T_f^3 c_{TWB})\frac{E^4}{\Lambda^4}$
$\hat{S}$	$(c_W+c_B)\frac{m_W^2}{\Lambda^2}$
$h\to Z\gamma$	$(c_{HW}-c_{HB})\frac{(4\pi v)^2}{\Lambda^2}$
$h\to W^+W^-$	$(c_W+c_{HW})\frac{m_W^2}{\Lambda^2}$

1804.08688

Where to find?
—Tail effects



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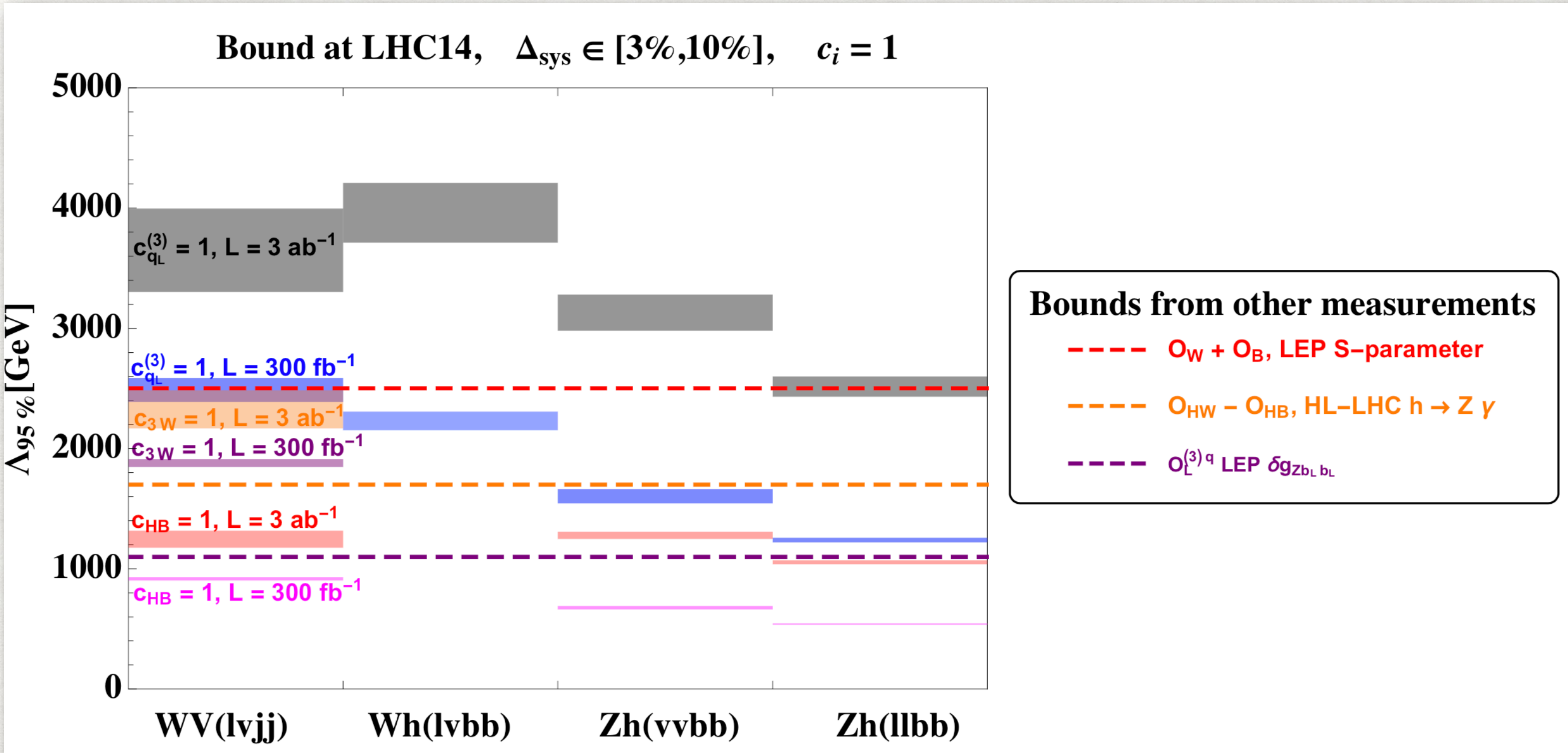
Where to find?
—Tail effects



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Backups1

Identical particle issues

*ref: Part. Phys. Nucl. Phys. Cosmol. 15, pp.76 (2011)

Symmetrized two particle states* for identical particle pair:

$$|\theta\phi; C_{\lambda_3}(k_3)D_{\lambda_4}(k_4)\rangle \Rightarrow \left(\frac{1}{\sqrt{2}}\right)^{1+\delta_{\lambda_3\lambda_4}} \left[|\theta\phi; C_{\lambda_3}(k_3)D_{\lambda_4}(k_4)\rangle + (-1)^{\lambda_3-\lambda_4}|(\pi-\theta)\phi; C_{\lambda_3}(k'_3)D_{\lambda_4}(k'_4)\rangle\right] (C=D)$$

Thus the symmetrized helicity amplitudes are $(A \neq B, C = D)$

$$\boxed{\left(\frac{1}{\sqrt{2}}\right)^{1+\delta_{\lambda_3\lambda_4}} \left[1+(-1)^{2s}P_{C\leftrightarrow D}\right] f_{cd,ab}(\theta,\phi) = \left(\frac{1}{\sqrt{2}}\right)^{1+\delta_{\lambda_3\lambda_4}} \left[f_{cd,ab}(\theta,\phi) + (-1)^{-\lambda} f_{dc,ab}(\pi-\theta,\phi)\right]}$$

Where

$$\begin{aligned}(2\pi)^4\delta(k_1+k_2-k_3-k_4)f_{cd,ab}(\theta,\phi) &\equiv \frac{2\pi}{|p_C|}(2\pi)^4\delta(k_1+k_2-k_3-k_4)\langle\theta\phi; cd\,|\,T\,|\,00; ab\rangle \\ &= \frac{2\pi}{|p_C|}\langle V_{C,\lambda_3}(k_3)V_{D,\lambda_4}(k_4)\,|\,T\,|\,V_{A,\lambda_1}(k_1)V_{B,\lambda_2}(k_2)\rangle\end{aligned}$$

$(\lambda_1 = a, \lambda_2 = b\ldots)$

and following equation is used

$$P_{C\leftrightarrow D} f_{cd,ab}(\theta,\phi) = (-1)^{-2s-\lambda} e^{-2i\phi\lambda'} f_{dc,ab}(\pi-\theta,\phi)$$

s: spin of particle C

$\lambda = c - d, \lambda' = a - b$

Identical particle issues

Thus the symmetrized helicity amplitudes are $(A \neq B, C = D)$

$$\boxed{\left(\frac{1}{\sqrt{2}}\right)^{1+\delta_{\lambda_3\lambda_4}} \left[1 + (-1)^{2s} P_{C\leftrightarrow D}\right] f_{cd,ab}(\theta,\phi) = \left(\frac{1}{\sqrt{2}}\right)^{1+\delta_{\lambda_3\lambda_4}} \left[f_{cd,ab}(\theta,\phi) + (-1)^{-\lambda} f_{dc,ab}(\pi-\theta,\phi)\right]}$$

Then

$$\int |M|^2 \mathrm{d}\Omega \supset \int \mathrm{d}\Omega \left| \left(\frac{1}{\sqrt{2}}\right)^{1+\delta_{\lambda_3\lambda_4}} \left[f_{cd,ab}(\theta,\phi) + (-1)^{-\lambda} f_{dc,ab}(\pi-\theta,\phi)\right] \right|^2 \sim$$

$$\int \mathrm{d}\Omega \left(\frac{1}{2}\right)^{1+\delta_{\lambda_3\lambda_4}} \left| \langle\theta\phi; cd| \, T \, |00; ab\rangle + (-1)^{b-a} \langle(\pi-\theta)\phi; dc| \, T \, |00; ab\rangle \right|^2$$

$$= \int \mathrm{d}\Omega \left(\frac{1}{2}\right)^{1+\delta_{\lambda_3\lambda_4}} \left| \left(\langle\theta\phi; cd| + (-1)^{b-a} \langle(\pi-\theta)\phi; dc|\right) T \, |00; ab\rangle \right|^2$$

Note that $\langle\theta\phi; cd|$ and $\langle(\phi-\theta)\phi; dc|$ do not interference if $\lambda_3 \neq \lambda_4$

We have to discuss separately

Identical particle issues

$1. c = d,$ i.e. $\lambda_3 = \lambda_4$

$$\frac{1}{2}\int |M|^2 \mathrm{d}\Omega \supset \frac{1}{8}\int \mathrm{d}\Omega \Big[|\langle\theta\phi; cd| T |00; ab\rangle|^2 + \left|(-1)^{b-a} \langle(\pi-\theta)\phi; dc| T |00; ab\rangle\right|^2$$

$$+2\mathrm{Re}((-1)^{b-a} \langle\theta\phi; cd| T |00; ab\rangle^* \langle(\pi-\theta)\phi; dc| T |00; ab\rangle)\Big]$$

$$= \frac{1}{2}\int \mathrm{d}\Omega |\langle\theta\phi; cd| T |00; ab\rangle|^2 \sim \frac{1}{2}\int \mathrm{d}\Omega |f_{cd,ab}(\theta,\phi)|^2 .$$

$2. c \neq d,$ i.e. $\lambda_3 \neq \lambda_4$

$$\int |M|^2 \mathrm{d}\Omega \supset \frac{1}{2}\int \mathrm{d}\Omega \left[|\langle\theta\phi; cd| T |00; ab\rangle|^2 + \left|(-1)^{b-a} \langle(\pi-\theta)\phi; dc| T |00; ab\rangle\right|^2 \right]$$

$$= \frac{1}{2}\int \mathrm{d}\Omega \left[|\langle\theta\phi; cd| T |00; ab\rangle|^2 + |\langle\theta\phi; dc| T |00; ab\rangle|^2 \right] .$$

$$= \frac{1}{2}\int \mathrm{d}\Omega \left[|f_{cd,ab}(\theta,\phi)|^2 + |f_{dc,ab}(\theta,\phi)|^2 \right] .$$

Identical particle issues

$1. c = d,$ i.e. $\lambda_3 = \lambda_4$

$$\frac{1}{2}\int \mathrm{d}\Omega\, |f_{cd,ab}(\theta,\phi)|^2$$

$2. c \neq d,$ i.e. $\lambda_3 \neq \lambda_4$

$$\frac{1}{2}\int \mathrm{d}\Omega \left[\, |f_{cd,ab}(\theta,\phi)| + |f_{dc,ab}(\theta,\phi)| \,\right]$$

While in our convention,

i.e. the symmetry factor I_{iden} should have nothing to do with helicities:

$$\left(\frac{1}{2}\right)^{\delta_{CD}} \int |M|^2 \supset \left(\frac{1}{2}\right)^{\delta_{CD}} \int \mathrm{d}\Omega \left[\sum_{cd} |f_{cd,ab}(\theta,\phi)|^2\right]$$
$$= \delta_{CD}\delta_{cd}\frac{1}{2}\int \mathrm{d}\Omega |f_{cd,ab}(\theta,\phi)|^2 + \delta_{CD}(1-\delta_{cd})\frac{1}{2}\int \mathrm{d}\Omega \left[|f_{cd,ab}(\theta,\phi)|^2 + |f_{dc,ab}(\theta,\phi)|^2\right]$$
$$+(1-\delta_{CD})\int \mathrm{d}\Omega \left[|f_{cd,ab}(\theta,\phi)|^2 + |f_{dc,ab}(\theta,\phi)|^2\right].$$

Consist

Backups2

Using EOM to transform from one basis to another

EOM: $\sum_i a_i \mathcal{O}_i = \mathbf{0}$

$$0 = \langle in | \sum_i a_i \mathcal{O}_i | fi \rangle = \sum_i a_i \langle in | \mathcal{O}_i | fi \rangle = \sum_i a_i m_i$$

For a seriese of operators $\sum_i C_i \mathcal{O}_i$

$$\langle in | \sum_i C_i \mathcal{O}_i | fi \rangle = \sum_i C_i m_i \qquad \text{with } \sum_i a_i m_i = 0$$

Thus we can use $\sum_i a_i m_i = 0$ to transform from one basis to another

Backups3

Helicity amplitudes

Process	Helicity	Warsaw basis	HISZ basis - Warsaw basis
wwww	$---+$	$6gC_W$	0
wwww	$-0+0$	$-C_{\varphi W}+\frac{9gC_W}{4}+\left(C_{\varphi W}+\frac{3gC_W}{4}\right)\cos(\theta)$	$-\frac{1}{4}g^2C_W^{HISZ}\sin^2\left(\frac{\theta}{2}\right)$
wwww	0000	$C_{\varphi D}+2C_{\varphi\Box}$	$-\frac{3}{4}(C_W^{HISZ}g^2+C_B^{HISZ}g_1^2)$
wzwz	$---+$	$-\frac{3g^3C_W(\cos(\theta)-1)}{g^2+g_1^2}$	0
wzwz	$--00$	$\frac{3g^2C_W\cos(\theta)-2C_{\varphi WB}g_1}{2\sqrt{g^2+g_1^2}}$	$-\frac{g(C_B^{HISZ}+C_W^{HISZ})g_1^2}{8\sqrt{g^2+g_1^2}}$
wzwz	$-0+0$	$C_{\varphi W}(\cos(\theta)-1)$	$-\frac{1}{4}g^2C_W^{HISZ}\sin^2\left(\frac{\theta}{2}\right)$
wzwz	$-00+$	$\frac{3(\cos(\theta)-3)C_Wg^2+2(\cos(\theta)+1)C_{\varphi WB}g_1}{4\sqrt{g^2+g_1^2}}$	$\frac{(\cos(\theta)+1)g(C_B^{HISZ}+C_W^{HISZ})g_1^2}{16\sqrt{g^2+g_1^2}}$
wzwz	$0-0+$	$\frac{(\cos(\theta)-1)\left(C_{\varphi W}g^2-C_{\varphi WB}g_1g+C_{\varphi B}g_1^2\right)}{g^2+g_1^2}$	$\frac{(C_B^{HISZ}-C_W^{HISZ})\sin^2\left(\frac{\theta}{2}\right)g_1^2\left(g^2-g_1^2\right)}{4\left(g^2+g_1^2\right)}$
wzwz	0000	$(\frac{1}{4}C_{\varphi D}-C_{\varphi\Box})(\cos(\theta)-1)$	$\frac{-3}{4}g^2C_W^{HISZ}\sin^2\left(\frac{\theta}{2}\right)$
wzwa	$00--$	$\frac{g(2C_{\varphi WB}+3C_W\cos(\theta)g_1)}{2\sqrt{g^2+g_1^2}}$	$\frac{(C_B^{HISZ}+C_W^{HISZ})g_1g^2}{8\sqrt{g^2+g_1^2}}$
wzwa	$-00+$	$-\frac{g(2C_{\varphi WB}(\cos(\theta)+1)-3C_W(\cos(\theta)-3)g_1)}{4\sqrt{g^2+g_1^2}}$	$-\frac{g^2(C_B^{HISZ}+C_W^{HISZ})g_1\cos^2\left(\frac{\theta}{2}\right)}{8\sqrt{g^2+g_1^2}}$
wzwa	$0-0+$	$\frac{\sin^2\left(\frac{\theta}{2}\right)\left(-C_{\varphi WB}g^2+2(C_{\varphi B}-C_{\varphi W})g_1g+C_{\varphi WB}g_1^2\right)}{g^2+g_1^2}$	$\frac{gg_1\sin^2\left(\frac{\theta}{2}\right)}{8\left(g^2+g_1^2\right)}\left(\left(3g_1^2-g^2\right)C_B^{HISZ}+\left(g_1^2-3g^2\right)C_W^{HISZ}\right)$
wzwh	$-0+0$	$\frac{-3}{4}gC_W(\cos(\theta)+3)$	0
wzwh	0000	$\frac{1}{4}C_{\varphi D}(\cos(\theta)+3)$	$-\frac{1}{8}C_B^{HISZ}(\cos(\theta)+3)g_1^2$
wawa	$0-0+$	$\frac{(\cos(\theta)-1)\left(C_{\varphi B}g^2+C_{\varphi WB}g_1g+C_{\varphi W}g_1^2\right)}{g^2+g_1^2}$	$-\frac{g^2\sin^2\left(\frac{\theta}{2}\right)g_1^2}{2\left(g^2+g_1^2\right)}(C_B^{HISZ}+C_W^{HISZ})$
wwww	$--00$	$\frac{3}{2}gC_W\cos(\theta)-2C_{\varphi W}$	$-\frac{g^2}{4}C_W^{HISZ}$
wwww	$-0+0$	$-C_{\varphi W}+\left(C_{\varphi W}-\frac{3gC_W}{4}\right)\cos(\theta)-\frac{9gC_W}{4}$	$-\frac{1}{4}g^2C_W^{HISZ}\sin^2\left(\frac{\theta}{2}\right)$
wwww	0000	$-(\frac{1}{2}C_{\varphi D}+C_{\varphi\Box})(\cos(\theta)+1)$	$\frac{3}{4}(C_B^{HISZ}g_1^2+C_W^{HISZ}g^2)\cos^2\left(\frac{\theta}{2}\right)$
wwzz	$--00$	$-2C_{\varphi W}$	$-\frac{g^2}{4}C_W^{HISZ}$
wwzz	$00--$	$-\frac{2\left(C_{\varphi W}g^2-C_{\varphi WB}g_1g+C_{\varphi B}g_1^2\right)}{g^2+g_1^2}$	$\frac{\left(g^2-g_1^2\right)}{4\left(g^2+g_1^2\right)}(C_B^{HISZ}g_1^2-C_W^{HISZ}g^2)$
wwzz	$-0+0$	$\frac{2C_{\varphi WB}(\cos(\theta)-1)g_1-3g^2C_W(\cos(\theta)+3)}{4\sqrt{g^2+g_1^2}}$	$-\frac{g\sin^2\left(\frac{\theta}{2}\right)g_1^2}{8\sqrt{g^2+g_1^2}}(C_B^{HISZ}+C_W^{HISZ})$
wwzz	0000	$\frac{1}{2}(C_{\varphi D}-4C_{\varphi\Box})$	$\frac{3g^2}{4}C_W^{HISZ}$
wwza	$00--$	$\frac{-C_{\varphi WB}g^2+2(C_{\varphi B}-C_{\varphi W})g_1g+C_{\varphi WB}g_1^2}{g^2+g_1^2}$	$\frac{gg_1}{8\left(g^2+g_1^2\right)}\left(\left(3g_1^2-g^2\right)C_B^{HISZ}+\left(g_1^2-3g^2\right)C_W^{HISZ}\right)$

Helicity amplitudes

wwza	$-00+$	$-\frac{g(2(\cos(\theta)+1)C_{\varphi WB}+3(\cos(\theta)-3)C_W g_1)}{4\sqrt{g^2+g_1^2}}$	$-\frac{g^2 g_1 \cos^2\left(\frac{\theta}{2}\right)}{8\sqrt{g^2+g_1^2}}(C_B^{HISZ}+C_W^{HISZ})$
wwzh	$--00$	$\frac{3}{2}gC_W\cos(\theta)$	0
wwzh	0000	$\frac{1}{2}C_{\varphi D}\cos(\theta)$	$-\frac{1}{4}C_B^{HISZ}\cos(\theta)g_1^2$
wwaa	$00--$	$-\frac{2\left(C_{\varphi B}g^2+C_{\varphi WB}g_1 g+C_{\varphi W}g_1^2\right)}{g^2+g_1^2}$	$-\frac{g^2 g_1^2}{2\left(g^2+g_1^2\right)}(C_B^{HISZ}+C_W^{HISZ})$
zzzz	$--00$	$-\frac{2\left(C_{\varphi W}g^2+C_{\varphi WB}g_1 g+C_{\varphi B}g_1^2\right)}{g^2+g_1^2}$	$-\frac{(C_W^{HISZ}+C_B^{HISZ})g^2}{4}$
zzzz	$-0+0$	$\frac{(\cos(\theta)-1)\left(C_{\varphi W}g^2+C_{\varphi WB}g_1 g+C_{\varphi B}g_1^2\right)}{g^2+g_1^2}$	$\frac{1}{8}(\cos(\theta)-1)(C_W^{HISZ}g^2+C_B^{HISZ}g_1^2)$
zzza	$00--$	$\frac{C_{\varphi WB}g^2+2(C_{\varphi B}-C_{\varphi W})g_1 g-C_{\varphi WB}g_1^2}{g^2+g_1^2}$	$\frac{1}{8}gg_1(C_B^{HISZ}-C_W^{HISZ})$
zzza	$-00+$	$\frac{(\cos(\theta)+1)\left(-C_{\varphi WB}g^2+2(C_{\varphi W}-C_{\varphi B})g_1 g+C_{\varphi WB}g_1^2\right)}{2\left(g^2+g_1^2\right)}$	$\frac{1}{8}gg_1\cos^2\left(\frac{\theta}{2}\right)(C_W^{HISZ}-C_B^{HISZ})$
zzaa	$00--$	$-\frac{2\left(C_{\varphi B}g^2-C_{\varphi WB}g_1 g+C_{\varphi W}g_1^2\right)}{g^2+g_1^2}$	0
zaza	$0-0+$	$\frac{(\cos(\theta)-1)\left(C_{\varphi B}g^2-C_{\varphi WB}g_1 g+C_{\varphi W}g_1^2\right)}{g^2+g_1^2}$	0

Helicity amplitudes

Process	Helicity	Warsaw basis	HISZ basis
enwz	$-+--$	$-\frac{3g^2 C_W \sin(\theta)}{\sqrt{2}\sqrt{g^2+g_1^2}}$	$-\frac{3g^2 C_W \sin(\theta)}{\sqrt{2}\sqrt{g^2+g_1^2}}$
enwz	$-+00$	$\sqrt{2}\sin(\theta)C_{\varphi l}^{(3)}$	$-\frac{g^2 C_W^{HISZ}\sin(\theta)}{4\sqrt{2}}$
enwz	$+++0$	$\sqrt{2}\sin(\theta)C_{eW}$	$\sqrt{2}\sin(\theta)C_{eW}$
enwz	$++0+$	$\frac{\sqrt{2}\sin(\theta)(g_1 C_{eB}-gC_{eW})}{\sqrt{g^2+g_1^2}}$	$\frac{\sqrt{2}\sin(\theta)(g_1 C_{eB}-gC_{eW})}{\sqrt{g^2+g_1^2}}$
enwa	$++0+$	$-\frac{\sqrt{2}\sin(\theta)(gC_{eB}+g_1 C_{eW})}{\sqrt{g^2+g_1^2}}$	$-\frac{\sqrt{2}\sin(\theta)(gC_{eB}+g_1 C_{eW})}{\sqrt{g^2+g_1^2}}$
duwz	$---0$	$3\sqrt{2}\sin(\theta)C_{uW}$	$3\sqrt{2}\sin(\theta)C_{uW}$
duwz	$--0-$	$-\frac{3\sqrt{2}\sin(\theta)(g_1 C_{uB}+gC_{uW})}{\sqrt{g^2+g_1^2}}$	$-\frac{3\sqrt{2}\sin(\theta)(g_1 C_{uB}+gC_{uW})}{\sqrt{g^2+g_1^2}}$
duwz	$-+00$	$3\sqrt{2}\sin(\theta)C_{\varphi q}^{(3)}$	$3\sqrt{2}\sin(\theta)C_{\varphi q}^{(3)} - \frac{3g^2 C_W^{HISZ}\sin(\theta)}{4\sqrt{2}}$
duwz	$+++0$	$3\sqrt{2}\sin(\theta)C_{dW}$	$3\sqrt{2}\sin(\theta)C_{dW}$
duwz	$++0+$	$\frac{3\sqrt{2}\sin(\theta)(g_1 C_{dB}-gC_{dW})}{\sqrt{g^2+g_1^2}}$	$\frac{3\sqrt{2}\sin(\theta)(g_1 C_{dB}-gC_{dW})}{\sqrt{g^2+g_1^2}}$
duwz	$+-00$	$-\frac{3C_{\varphi ud}\sin(\theta)}{\sqrt{2}}$	$-\frac{3C_{\varphi ud}\sin(\theta)}{\sqrt{2}}$
duwa	$--0-$	$\frac{3\sqrt{2}\sin(\theta)(gC_{uB}-g_1 C_{uW})}{\sqrt{g^2+g_1^2}}$	$\frac{3\sqrt{2}\sin(\theta)(gC_{uB}-g_1 C_{uW})}{\sqrt{g^2+g_1^2}}$
duwa	$++0+$	$-\frac{3\sqrt{2}\sin(\theta)(gC_{dB}+g_1 C_{dW})}{\sqrt{g^2+g_1^2}}$	$-\frac{3\sqrt{2}\sin(\theta)(gC_{dB}+g_1 C_{dW})}{\sqrt{g^2+g_1^2}}$
eeww	$-+00$	$\sin(\theta)(C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)})$	$\frac{1}{8}\sin(\theta)(C_W^{HISZ}g^2 + C_B^{HISZ}g_1^2)$
eeww	$+-00$	$\sin(\theta)C_{\varphi e}$	$\frac{1}{4}C_B^{HISZ}\sin(\theta)g_1^2 + \sin(\theta)C_{\varphi e}$
eezz	$---0$	$\frac{\sin(\theta)(g_1 C_{eB}+gC_{eW})}{\sqrt{g^2+g_1^2}}$	$\frac{\sin(\theta)(g_1 C_{eB}+gC_{eW})}{\sqrt{g^2+g_1^2}}$
eeza	$--0-$	$\frac{\sin(\theta)(g_1 C_{eW}-gC_{eB})}{\sqrt{g^2+g_1^2}}$	$\frac{\sin(\theta)(g_1 C_{eW}-gC_{eB})}{\sqrt{g^2+g_1^2}}$

Helicity amplitudes

$$
\begin{pmatrix}
\text{nnww} & -+00 & \sin(\theta)(C_{\varphi l}^{(1)} - C_{\varphi l}^{(3)}) & \frac{1}{8}\sin(\theta)(C_B^{HISZ} g_1^2 - C_W^{HISZ} g^2) \\
\text{uuww} & -+00 & 3\sin(\theta)(C_{\varphi q}^{(1)} - C_{\varphi q}^{(3)}) & 3\sin(\theta)(C_{\varphi q}^{(1)} - C_{\varphi q}^{(3)}) - \frac{1}{8}\sin(\theta)(3C_W^{HISZ} g^2 + C_B^{HISZ} g_1^2) \\
\text{uuww} & +-00 & 3\sin(\theta)C_{\varphi u} & 3\sin(\theta)C_{\varphi u} - \frac{1}{2}C_B^{HISZ}\sin(\theta)g_1^2 \\
\text{uuzz} & ---0 & \frac{3\sin(\theta)(gC_{uW} - g_1 C_{uB})}{\sqrt{g^2+g_1^2}} & \frac{3\sin(\theta)(gC_{uW} - g_1 C_{uB})}{\sqrt{g^2+g_1^2}} \\
\text{uuza} & --0- & \frac{3\sin(\theta)(gC_{uB} + g_1 C_{uW})}{\sqrt{g^2+g_1^2}} & \frac{3\sin(\theta)(gC_{uB} + g_1 C_{uW})}{\sqrt{g^2+g_1^2}} \\
\text{ddww} & -+00 & 3\sin(\theta)(C_{\varphi q}^{(1)} + C_{\varphi q}^{(3)}) & 3\sin(\theta)(C_{\varphi q}^{(1)} + C_{\varphi q}^{(3)}) + \frac{1}{8}\sin(\theta)(3C_W^{HISZ} g^2 - C_B^{HISZ} g_1^2) \\
\text{ddww} & +-00 & 3\sin(\theta)C_{\varphi d} & \frac{1}{4}C_B^{HISZ}\sin(\theta)g_1^2 + 3\sin(\theta)C_{\varphi d} \\
\text{ddzz} & ---0 & \frac{3\sin(\theta)(g_1 C_{dB} + gC_{dW})}{\sqrt{g^2+g_1^2}} & \frac{3\sin(\theta)(g_1 C_{dB} + gC_{dW})}{\sqrt{g^2+g_1^2}} \\
\text{ddza} & --0- & \frac{3\sin(\theta)(g_1 C_{dW} - gC_{dB})}{\sqrt{g^2+g_1^2}} & \frac{3\sin(\theta)(g_1 C_{dW} - gC_{dB})}{\sqrt{g^2+g_1^2}} \\
\text{eezh} & -+00 & -\sin(\theta)(C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)}) & \frac{1}{8}\sin(\theta)\left(g^2 C_W^{HISZ} - C_B^{HISZ} g_1^2\right) \\
\text{nnzh} & -+00 & -\sin(\theta)(C_{\varphi l}^{(1)} - C_{\varphi l}^{(3)}) & -\frac{1}{8}\sin(\theta)\left(C_W^{HISZ} g^2 + C_B^{HISZ} g_1^2\right) \\
\text{uuzh} & -+00 & -3\sin(\theta)(C_{\varphi q}^{(1)} - C_{\varphi q}^{(3)}) & -3\sin(\theta)(C_{\varphi q}^{(1)} - C_{\varphi q}^{(3)}) + \frac{1}{8}\sin(\theta)\left(C_B^{HISZ} g_1^2 - 3C_W^{HISZ} g^2\right) \\
\text{ddzh} & -+00 & -3\sin(\theta)(C_{\varphi q}^{(1)} + C_{\varphi q}^{(3)}) & -3\sin(\theta)(C_{\varphi q}^{(1)} + C_{\varphi q}^{(3)}) + \frac{1}{8}\sin(\theta)\left(C_B^{HISZ} g_1^2 + 3g^2 C_W^{HISZ}\right)
\end{pmatrix}
$$

Backups4

Tensor Structure

$$
\begin{aligned}
\mathcal{T}_{31} &= p^a_\gamma p^b_\alpha p^c_\beta - p^a_\beta p^b_\gamma p^c_\alpha, \\
\mathcal{T}_{32} &= g_{\alpha\beta}\left[p^a_\gamma (p^b \cdot p^c) - p^b_\gamma (p^a \cdot p^c)\right] - g_{\alpha\gamma}\left[p^a_\beta (p^b \cdot p^c) - p^c_\beta (p^a \cdot p^b)\right] \\
&\quad + g_{\beta\gamma}\left[p^b_\alpha (p^a \cdot p^c) - p^c_\alpha (p^a \cdot p^b)\right], \\
\mathcal{T}_{41} &= g_{\alpha\delta}\left[p^a_\beta p^b_\gamma + p^c_\beta p^d_\gamma\right] - g_{\alpha\gamma}\left[p^a_\beta p^b_\delta + p^c_\delta p^d_\beta\right] + g_{\beta\gamma}\left[p^a_\delta p^b_\alpha + p^c_\delta p^d_\alpha\right] - g_{\beta\delta}\left[p^a_\gamma p^b_\alpha - p^c_\alpha p^d_\gamma\right], \\
\mathcal{T}_{42} &= g_{\alpha\beta}\left[p^a_\gamma p^b_\delta - p^a_\delta p^b_\gamma\right] + g_{\gamma\delta}\left[p^c_\alpha p^d_\beta - p^c_\beta p^d_\alpha\right], \\
\mathcal{T}_{43} &= \left(g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma}\right)\left(p^a \cdot p^b + p^c \cdot p^d\right),
\end{aligned}
$$

Helicity amplitudes

$$
\begin{aligned}
\mathcal{M}^{\rm int}|_{s^3} \propto\ & (x^3+3x)(f_{ade}f_{bce}-f_{ace}f_{bde})(c_{31}+c_{32}-2c_{41}+c_{42}+c_{43}) \\
& + (x^2-3)(f_{ade}f_{bce}+f_{ace}f_{bde})(c_{31}+c_{32}-2c_{41}+c_{42}+c_{43}), \\
\mathcal{M}^{\rm int}|_{s^2} \propto\ & 2x^3(f_{ade}f_{bce}-f_{ace}f_{bde})(c_{31}+c_{32}-2c_{41}+c_{42}+c_{43}) \\
& + x^2(f_{ade}f_{bce}+f_{ace}f_{bde})(7c_{31}+7c_{32}-8c_{41}+8c_{43}) \\
& + 2x(f_{ade}f_{bce}-f_{ace}f_{bde})(2c_{31}+2c_{32}-10c_{41}+5c_{42}+5c_{43}) \\
& - (f_{ade}f_{bce}+f_{ace}f_{bde})(7c_{31}+7c_{32}-16c_{41}+12c_{42}+4c_{43}),
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}^{\rm int}|_{s^{5/2}} \propto\ & x^2(f_{ade}f_{bce}-f_{ace}f_{bde})(c_{31}+c_{32}-2c_{41}+c_{42}+c_{43}) \\
& + x(f_{ade}f_{bce}+f_{ace}f_{bde})(c_{41}-c_{42}) \\
& + 3(f_{ade}f_{bce}-f_{ace}f_{bde})(c_{31}+c_{32}-c_{41}+c_{43}), \\
\mathcal{M}^{\rm int}|_{s^{3/2}} \propto\ & 2x^2(f_{ade}f_{bce}-f_{ace}f_{bde})(c_{31}+c_{32}-2c_{41}+c_{42}+c_{43}) \\
& + x(f_{ade}f_{bce}+f_{ace}f_{bde})(5c_{31}+4c_{32}-4c_{41}+c_{42}+4c_{43}) \\
& + (f_{ade}f_{bce}-f_{ace}f_{bde})(13c_{31}+10c_{32}-8c_{41}+c_{42}+10c_{43}).
\end{aligned}
$$