D^0–$\overline{D}^0$ mixing parameter y in FAT approach

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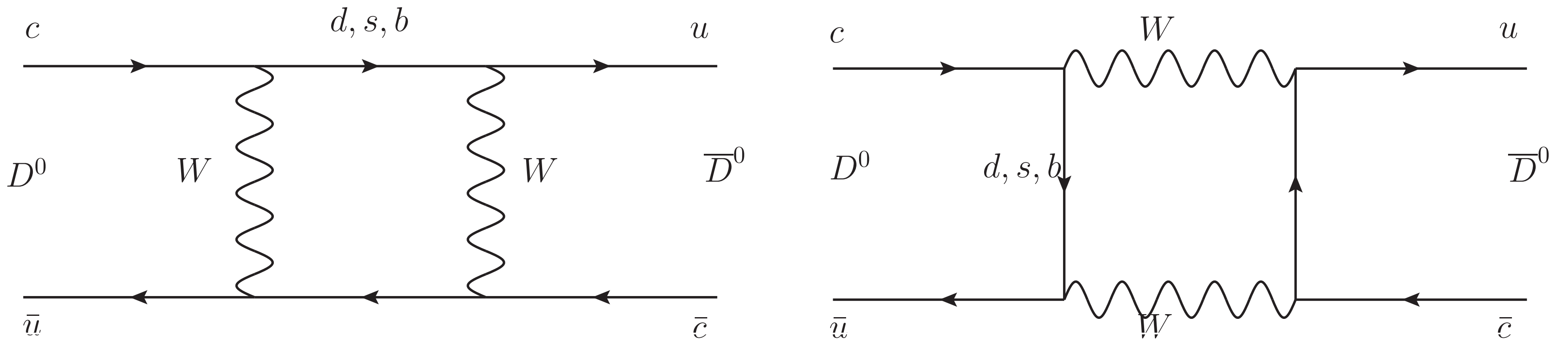
OUTLINE

1. $D^0 - \overline{D}^0$ mixing and Motivation
2. Topology and FAT approach
3. Numerical results and Discussion
4. Summary

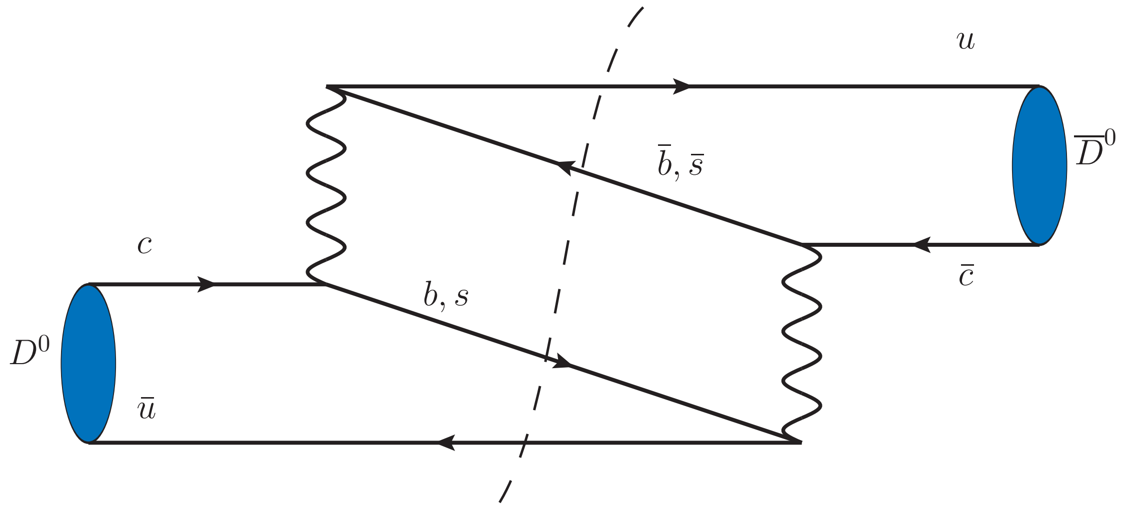
OUTLINE

D^0-$\overline{D}^0$ mixing

1. The box diagram contributions



2. The long distance contributions



D^0-$\overline{D}^0$ mixing: The description of Quantum Mechanics

1. Evolution Equation

$$i\frac{d}{dt}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = H \begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix}$$

CPT invariance imposes $M_{11} = M_{22}, \Gamma_{11} = \Gamma_{22}$.

2. The mass eigenstates of H

$$\begin{cases} |D_1\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}(p|D^0\rangle + q|\overline{D}^0\rangle) \\ |D_2\rangle = \frac{1}{\sqrt{|p|^2+|q|^2}}(p|D^0\rangle - q|\overline{D}^0\rangle) \end{cases}, \qquad \frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

with differences in mass and width

$$\begin{cases} \Delta M_D \equiv M_2 - M_1 = -2Re\left[\frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12})\right] \\ \Delta\Gamma_D \equiv \Gamma_1 - \Gamma_2 \quad = -2Im\left[\frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12})\right] \end{cases}, \quad and \quad x_D = \frac{\Delta M_D}{\Gamma_D},\ y_D = \frac{\Delta\Gamma_D}{2\Gamma_D}$$

where $\Gamma_D = (\Gamma_1 + \Gamma_2)/2$

3. In the limit of CP conservation $\frac{q}{p} = 1$, $|D_1\rangle = |D_+\rangle$, $|D_2\rangle = |D_-\rangle$, and $\mathcal{CP}|D^0\rangle = +|\overline{D}^0\rangle$

$$|D_\pm\rangle = (|D^0\rangle \pm |\overline{D}^0\rangle)/\sqrt{2}$$

Motivation

1. The current world averages of charm mixing parameter

$$x_D = (0.46^{+0.14}_{-0.15})\%, \quad y_D = (0.62 \pm 0.08)\%$$

[Y. Amhis et al.(Heavy Flavor Averaging Group(HFAG))(2014),1412.7515]

2. The calculation of box diagram, with internal quark d, s and b [Bigi,Uraltsev,'01].

$$\begin{cases} \Delta M_D^{b\bar{b}} \sim Q \times |V_{cb}^* V_{ub}|^2 \\ \Delta M_D^{s,d} \sim Q' \times |V_{cs}^* V_{us}|^2 \dfrac{(m_s^2 - m_d^2)^2}{m_c^4} \end{cases} \Rightarrow \quad y_D^{box} \sim x_D^{box} \sim few \times 10^{-5}$$

3. For inclusive approach, with short-distance contributions calculated based on the heavy quark expansion, both NLO in α_s and leading $1/m_c$ corrections [Bobrowskia, Lenz,Riedla,Rohrwilda,'10; Bobrowski,Lenz,Rauh,'13] were taken into account

$$x_D \sim O(10^{-5}),\ y_D \sim O(10^{-6}).$$

4. In an exclusive method, y_D was computed by [H.Y.Cheng,C.W.Chiang,'10]

$$\begin{cases} y_{PP+VP} = (0.36 \pm 0.26)\%, & (A,\ A1) \\ y_{PP+VP} = (0.24 \pm 0.22)\%, & (S,\ S1) \end{cases}$$

y_D in exclusive approach

1. The parameter y can be computed via [Falk,Grossman,Ligeti,Petrov,'02]

$$
\begin{aligned}
y_D &= \frac{1}{2\Gamma}\sum_n \rho_n \eta_{CP}(n)\big(\langle D^0|H_W|n\rangle\langle\bar{n}|H_W|D^0\rangle + \langle D^0|H_W|\bar{n}\rangle\langle n|H_W|D^0\rangle\big) \\
&= \frac{1}{2\Gamma}\sum_n \rho_n \left(|\mathcal{A}(D_+\to n)|^2 - |\mathcal{A}(D_-\to n)|^2\right) \\
&= \frac{1}{\Gamma}\sum_n \eta_{\rm CP}(n)\rho_n \mathcal{R}e\left[\mathcal{A}(D^0\to n)\mathcal{A}^*(D^0\to\bar{n})\right] \\
&= \sum_n \eta_{\rm CKM}(n)\eta_{\rm CP}(n)\cos\delta_n\sqrt{\mathcal{B}(D^0\to n)\mathcal{B}(D^0\to\bar{n})}.
\end{aligned}
$$

where ρ_n is the phase-space factor and $\delta_n = \delta_{D^0\to\bar{n}} - \delta_{D^0\to n}$=Arg$\left[\frac{A(D^0\to\bar{n})}{A(D^0\to n)}\right]$.
And with $\mathcal{CP}|n\rangle = \eta_{\rm CP}|\bar{n}\rangle$, $\eta_{\rm CP} = \begin{cases} +1, & for\ PP,\ PV\ modes, \\ (-1)^L, & for\ VV\ modes. \end{cases}$
$\eta_{\rm CKM} = (-1)^{n_s}$ with n_s being the number of the s or $\bar{s}$ quarks in the final state.

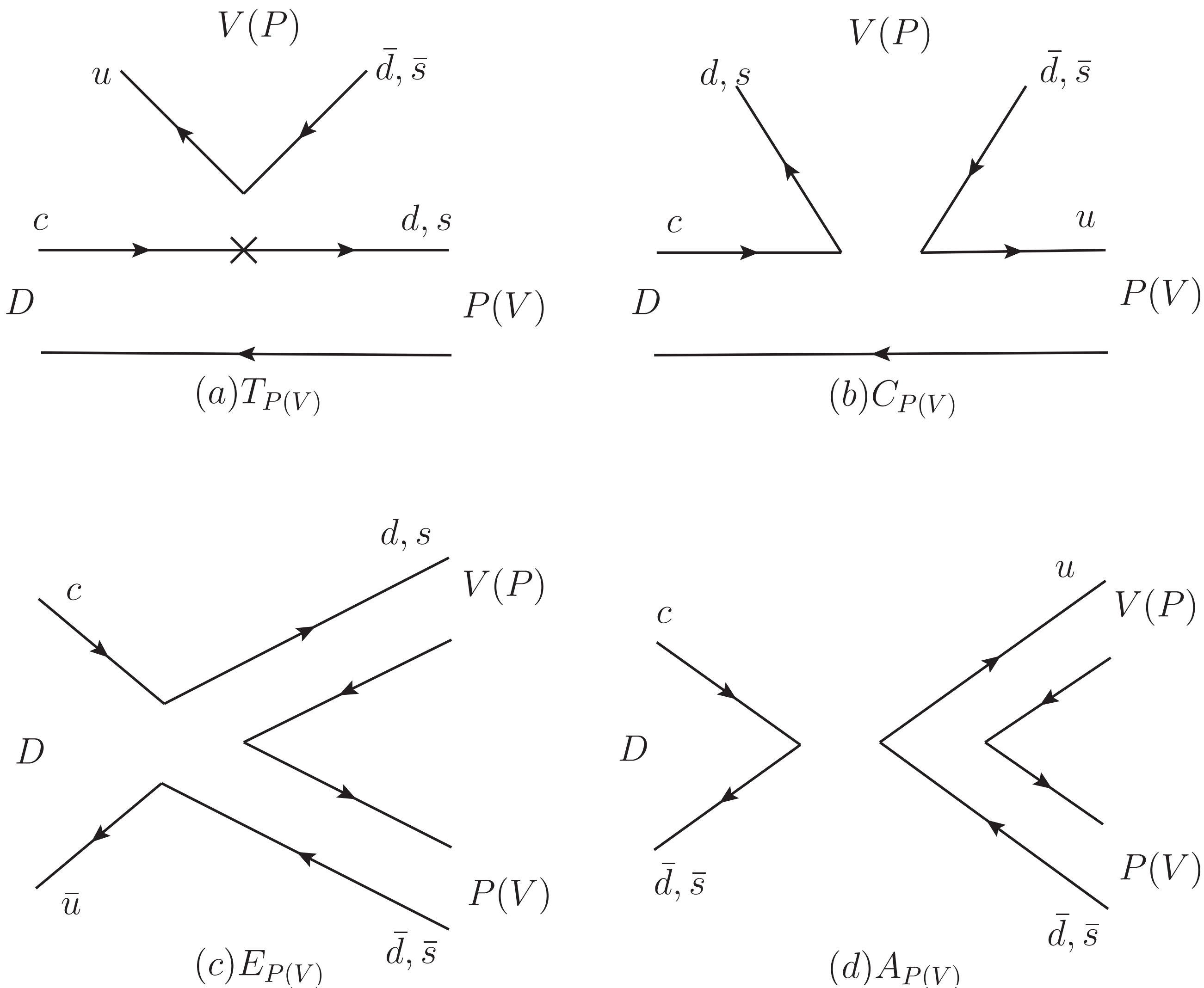
2. The total contributions for y_D

$$y_D = y_{PP} + y_{PV} + y_{VV} + \cdots .$$

OUTLINE

Topology diagram

According to current structure of weak decays, there are four topology at tree level.



T: the color allowed external W-emission diagram

C: the color suppressed internal W-emission diagram

E: the W-exchange diagram

A: the W-annihilation diagram

[H.n.Li,C.D.Lu,F.S.Yu,'12; H.n.Li,C.D.Lu,Q.Qin,F.S.Yu,'14]

FAT approach

1. The topology amplitude [H.n.Li,C.D.Lu,F.S.Yu,'12; H.n.Li,C.D.Lu,Q.Qin,F.S.Yu,'14]

$$T_P(C_P) = \frac{G_F}{\sqrt{2}} V_{CKM} a_1(a_2^P) f_V m_V F_1^{DP}(m_V^2) 2(\epsilon_V \cdot p_D)$$
$$T_V(C_V) = \frac{G_F}{\sqrt{2}} V_{CKM} a_1(a_2^V) f_P m_V A_0^{DV}(m_P^2) 2(\epsilon_V \cdot p_D)$$
$$E_{P,V}^{nf} = \frac{G_F}{\sqrt{2}} V_{CKM} C_2 \chi_{q(s)}^E e^{i\phi_{q(s)}^E} f_D m_V \frac{f_P f_V}{f_\pi f_\rho} (\epsilon_V \cdot p_D)$$
$$A_{P,V}^{nf} = \frac{G_F}{\sqrt{2}} V_{CKM} C_1 \chi_{q(s)}^A e^{i\phi_{q(s)}^A} f_D m_V \frac{f_P f_V}{f_\pi f_\rho} (\epsilon_V \cdot p_D)$$

with Glauber phase $Exp(iS_\pi)$ for final state π and the effective Wilson coefficients:

$$a_1(\mu) = C_2(\mu) + \frac{C_1(\mu)}{N_C}, \quad a_2^{P(V)}(\mu) = C_1(\mu) + C_2(\mu)\Big(\frac{1}{N_C} + \chi_{P(V)}^C e^{i\phi_{P(V)}^C}\Big).$$

2. The advantage of FAT approach:
(1) The branching ratios of $D \to PP$ and PV are best described.
(2) Successfully predict the difference of CP violation:

$$\Delta a_{CP}^{dir} = a_{CP}^{dir}(K^+K^-) - a_{CP}^{dir}(\pi^+\pi^-) = (-0.6 \sim -1.9) \times 10^{-3}.$$

was confirmed by the LHCb data, $\Delta a_{CP}^{dir} = (-0.61 \pm 0.76) \times 10^{-3}$[R. Aaij et al.(LHCb),'16].

OUTLINE

The fitting results

1. We focus on the D^0 decay modes, then those irrelevant about annihilation amplitudes will not be considered. The updated parameters [**HYJ**,F.S.Yu,Q.Qin,H.n.Li,C.D.Lu,'18]

$$\chi^C_{\rm nf} = -0.81 \pm 0.01,\ \phi^C_{\rm nf} = 0.22 \pm 0.14,$$
$$\chi^E_q = 0.056 \pm 0.002,\ \phi^E_q = 5.03 \pm 0.06,$$
$$\chi^E_s = 0.130 \pm 0.008,\ \phi^E_s = 4.37 \pm 0.10,$$
$$S_\pi = -0.92 \pm 0.07,$$

for the $D^0 \to PP$ decays, and

$$\chi^C_P = 0.63 \pm 0.03,\ \phi^C_P = 1.57 \pm 0.11,$$
$$\chi^C_V = 0.71 \pm 0.03,\ \phi^C_V = 2.77 \pm 0.10,$$
$$\chi^E_q = 0.49 \pm 0.03,\ \phi^E_q = 1.61 \pm 0.07,$$
$$\chi^E_s = 0.54 \pm 0.03,\ \phi^E_s = 2.23 \pm 0.08,$$
$$S_\pi = -1.88 \pm 0.12,$$

for the $D^0 \to PV$ decays.

2. The branching ratios are listed in Table 1, and

$$\chi^2/dof = \begin{cases} 1.1, & \text{for the PP modes with 13 data} \\ 1.8, & \text{for the PV modes with 19 data} \end{cases}$$

The fitting results

Branching fractions in units of 10^{-3}. [**HYJ**,F.S.Yu,Q.Qin,H.n.Li,C.D.Lu,'18]

Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)	$\mathcal{B}$(FAT)	Modes	$\mathcal{B}$(exp)[PDG]	$\mathcal{B}$(FAT)
$\pi^0\overline{K}^0$	24.0 ± 0.8	24.2 ± 0.8	$\pi^0\overline{K}^{*0}$	37.5 ± 2.9	35.9 ± 2.2	$\overline{K}^0\rho^0$	$12.8^{+1.4}_{-1.6}$	13.5 ± 1.4
π^+K^-	39.3 ± 0.4	39.2 ± 0.4	π^+K^{*-}	54.3 ± 4.4	62.5 ± 2.7	$K^-\rho^+$	111.0 ± 9.0	105.0 ± 5.2
$\eta\overline{K}^0$	9.70 ± 0.6	9.6 ± 0.6	$\eta\overline{K}^{*0}$	9.6 ± 3.0	6.1 ± 1.0	$\overline{K}^0\omega$	22.2 ± 1.2	22.3 ± 1.1
$\eta'\overline{K}^0$	19.0 ± 1.0	19.5 ± 1.0	$\eta'\overline{K}^{*0}$	<1.10	0.19 ± 0.01	$\overline{K}^0\phi$	$8.47^{+0.66}_{-0.34}$	8.2 ± 0.6
$\pi^+\pi^-$	1.421 ± 0.025	1.44 ± 0.02	$\pi^+\rho^-$	5.09 ± 0.34	4.5 ± 0.2	$\pi^-\rho^+$	10.0 ± 0.6	9.2 ± 0.3
K^+K^-	4.01 ± 0.07	4.05 ± 0.07	K^+K^{*-}	1.62 ± 0.15	1.8 ± 0.1	K^-K^{*+}	4.50 ± 0.30	4.3 ± 0.2
$K^0\overline{K}^0$	0.36 ± 0.08	0.29 ± 0.07	$K^0\overline{K}^{*0}$	0.18 ± 0.04	0.19 ± 0.03	$\overline{K}^0K^{*0}$	0.21 ± 0.04	0.19 ± 0.03
$\pi^0\eta$	0.69 ± 0.07	0.74 ± 0.03	$\eta\rho^0$		1.4 ± 0.2	$\pi^0\omega$	0.117 ± 0.035	0.10 ± 0.03
$\pi^0\eta'$	0.91 ± 0.14	1.08 ± 0.05	$\eta'\rho^0$		0.25 ± 0.01	$\pi^0\phi$	1.35 ± 0.10	1.4 ± 0.1
$\eta\eta$	1.70 ± 0.20	1.86 ± 0.06	$\eta\omega$	2.21 ± 0.23	2.0 ± 0.1	$\eta\phi$	0.14 ± 0.05	0.18 ± 0.04
$\eta\eta'$	1.07 ± 0.26	1.05 ± 0.08	$\eta'\omega$		0.044 ± 0.004			
$\pi^0\pi^0$	0.826 ± 0.035	0.78 ± 0.03	$\pi^0\rho^0$	3.82 ± 0.29	4.1 ± 0.2			
π^0K^0		0.069 ± 0.002	π^0K^{*0}		0.103 ± 0.006	$K^0\rho^0$		0.039 ± 0.004
π^-K^+	0.133 ± 0.009	0.133 ± 0.001	π^-K^{*+}	$0.345^{+0.180}_{-0.102}$	0.40 ± 0.02	$K^+\rho^-$		0.144 ± 0.009
ηK^0		0.027 ± 0.002	ηK^{*0}		0.017 ± 0.003	$K^0\omega$		0.064 ± 0.003
$\eta'K^0$		0.056 ± 0.003	$\eta'K^{*0}$		0.00055 ± 0.00004	$K^0\phi$		0.024 ± 0.002

y_{PP}

vanish in the SU(3) symmetry limit

$$\mathcal{B}(\pi^+\pi^-) + \mathcal{B}(K^+K^-) - 2\cos\delta_{K^+\pi^-}\sqrt{\mathcal{B}(K^-\pi^+)\mathcal{B}(K^+\pi^-)}$$

$$+\mathcal{B}(\pi^0\pi^0) + \mathcal{B}(K^0\bar{K}^0) - 2\cos\delta_{K^0\pi^0}\sqrt{\mathcal{B}(\bar{K}^0\pi^0)\mathcal{B}(K^0\pi^0)}$$

$$+\mathcal{B}(\pi^0\eta) + \mathcal{B}(\pi^0\eta') + \mathcal{B}(\eta\eta) + \mathcal{B}(\eta\eta')$$

$$-2\cos\delta_{K^0\eta}\sqrt{\mathcal{B}(\bar{K}^0\eta)\mathcal{B}(K^0\eta)} - 2\cos\delta_{K^0\eta'}\sqrt{\mathcal{B}(\bar{K}^0\eta')\mathcal{B}(K^0\eta')}$$

$$\Rightarrow \qquad y_{PP} = (0.10 \pm 0.02)\%$$

[**HYJ**,F.S.Yu,Q.Qin,H.n.Li,C.D.Lu,'18]

y_{PV}

$$
\begin{aligned}
&Br(\pi^0\rho^0)+Br(\pi^0\omega)+Br(\pi^0\phi)+Br(\eta\omega)+Br(\eta'\omega)+Br(\eta\phi)+Br(\eta\rho^0)+Br(\eta'\rho^0)\\
&-2\cos\delta_{K^{*-}\pi^+}\sqrt{Br(K^{*-}\pi^+)Br(K^{*+}\pi^-)}-2\cos\delta_{K^{*0}\pi^0}\sqrt{Br(K^{*0}\pi^0)Br(\bar{K}^{*0}\pi^0)}\\
&-2\cos\delta_{K^-\rho^+}\sqrt{Br(K^-\rho^+)Br(K^+\rho^-)}-2\cos\delta_{K^0\rho^0}\sqrt{Br(K^0\rho^0)Br(\bar{K}^0\rho^0)}\\
&-2\cos\delta_{K^{*0}\eta}\sqrt{Br(K^{*0}\eta)Br(\bar{K}^{*0}\eta)}-2\cos\delta_{K^{*0}\eta'}\sqrt{Br(K^{*0}\eta')Br(\bar{K}^{*0}\eta')}\\
&-2\cos\delta_{K^0\omega}\sqrt{Br(K^0\omega)Br(\bar{K}^0\omega)}-2\cos\delta_{K^0\phi}\sqrt{Br(K^0\phi)Br(\bar{K}^0\phi)}\\
&+2\cos\delta_{K^+K^{*-}}\sqrt{Br(K^+K^{*-})Br(K^-K^{*+})}+2\cos\delta_{K^0\bar{K}^{*0}}\sqrt{Br(K^0\bar{K}^{*0})Br(\bar{K}^0K^{*0})}\\
&+2\cos\delta_{\pi^+\rho^-}\sqrt{Br(\pi^+\rho^-)Br(\pi^-\rho^+)}
\end{aligned}
$$

$$\Rightarrow \qquad y_{PV}=(0.11\pm0.07)\%$$

[**HYJ**,F.S.Yu,Q.Qin,H.n.Li,C.D.Lu,'18]

y_{PV}

$$y_{PV} = 0.32 \pm 0.07 \quad \Longrightarrow \quad y_{PV} = 0.11 \pm 0.07$$

	Before 2016		After 2016	
	$\mathscr{B}_{\rm exp}$	$\mathscr{B}_{\rm th}$	$\mathscr{B}_{\rm exp}$	$\mathscr{B}_{\rm th}$ 10^{-3}
$D^0 \to \overline{K}^{*0} K^0$	< 1	1.1	0.18 ± 0.04	0.19 ± 0.03
$D^0 \to K^{*0} \overline{K}^0$	< 0.56	1.1	0.21 ± 0.04	0.19 ± 0.03
	PDG16		LHCb,'16	

Studies of the resonance structure in $D^0 \to K_S^0 K^{\pm} \pi^{\mp}$ decays
LHCb Collaboration (Roel Aaij (CERN) *et al.*) Show all 726 authors
Sep 22, 2015 - 35 pages
Phys.Rev. D93 (2016) no.5, 052018

[H.n.Li,C.D.Lu,Q.Qin,F.S.Yu,'14] [**HYJ**,F.S.Yu,Q.Qin,H.n.Li,C.D.Lu,'18)]

y_{PV}

$$
\begin{aligned}
&Br(\pi^0\rho^0) + Br(\pi^0\omega) + Br(\pi^0\phi) + Br(\eta\omega) + Br(\eta'\omega) + Br(\eta\phi) + Br(\eta\rho^0) + Br(\eta'\rho^0) \\
&-2\cos\delta_{K^{*-}\pi^+}\sqrt{Br(K^{*-}\pi^+)Br(K^{*+}\pi^-)} - 2\cos\delta_{K^{*0}\pi^0}\sqrt{Br(K^{*0}\pi^0)Br(\bar{K}^{*0}\pi^0)} \\
&-2\cos\delta_{K^-\rho^+}\sqrt{Br(K^-\rho^+)Br(K^+\rho^-)} - 2\cos\delta_{K^0\rho^0}\sqrt{Br(K^0\rho^0)Br(\bar{K}^0\rho^0)} \\
&-2\cos\delta_{K^{*0}\eta}\sqrt{Br(K^{*0}\eta)Br(\bar{K}^{*0}\eta)} - 2\cos\delta_{K^{*0}\eta'}\sqrt{Br(K^{*0}\eta')Br(\bar{K}^{*0}\eta')} \\
&-2\cos\delta_{K^0\omega}\sqrt{Br(K^0\omega)Br(\bar{K}^0\omega)} - 2\cos\delta_{K^0\phi}\sqrt{Br(K^0\phi)Br(\bar{K}^0\phi)} \\
&+2\cos\delta_{K^+K^{*-}}\sqrt{Br(K^+K^{*-})Br(K^-K^{*+})} + 2\cos\delta_{K^0\bar{K}^{*0}}\sqrt{Br(K^0\bar{K}^{*0})Br(\bar{K}^0K^{*0})} \\
&+2\cos\delta_{\pi^+\rho^-}\sqrt{Br(\pi^+\rho^-)Br(\pi^-\rho^+)}
\end{aligned}
$$

$\cos\delta \sim 1$

10^{-3}	Before 2016 $\mathscr{B}_{\rm exp}$	Before 2016 $\mathscr{B}_{\rm th}$	After 2016 $\mathscr{B}_{\rm exp}$	After 2016 $\mathscr{B}_{\rm th}$
$D^0 \to \overline{K}^{*0}K^0$	< 1	1.1	0.18 ± 0.04	0.19 ± 0.03
$D^0 \to K^{*0}\overline{K}^0$	< 0.56	1.1	0.21 ± 0.04	0.19 ± 0.03

Before2016 : 2.2×10^{-3}

After2016 : 0.38×10^{-3}

Calculating formula for $D^0 \to VV$

1. The emission-type amplitudes

$$
\begin{aligned}
T(C) \;=\; & \frac{G_F}{\sqrt{2}} V_{CKM} a_1(\mu) \left(a_2^C(\mu)\right) f_{V_1} m_1 \\
& \times \left[-ix(m_D+m_2) A_1^{DV_2}(m_1^2) + i\frac{2m_D^2 p_c^2}{(m_D+m_2)m_1 m_2} A_2^{DV_2}(m_1^2) \right],
\end{aligned}
$$

in which the Wilson coefficients and the kinetic quantities are given by

$$
a_1(\mu) = \frac{C_1(\mu)}{N_c} + C_2(\mu), \qquad a_2^C(\mu) = C_1(\mu) + C_2(\mu)\left(\frac{1}{N_c} + \chi_V^C e^{i\phi_V^C}\right),
$$

$$
x = \frac{m_D^2 - m_1^2 - m_2^2}{2m_1 m_2}, \qquad p_c^2 = \frac{m_1^2 m_2^2 (x^2-1)}{m_D^2}.
$$

2. The annihilation-type amplitudes

$$
E = -i\frac{G_F}{\sqrt{2}} V_{CKM} C_2(\mu) \chi_{q(s)}^E e^{i\phi_{q(s)}^E} f_D \frac{f_{V_1} f_{V_2}}{f_\rho^2} m_D^2 \frac{|p_c|}{\sqrt{m_1 m_2}}.
$$

[**HYJ**,F.S.Yu,Q.Qin,H.n.Li,C.D.Lu,'18]

Calculating results and y_{VV}

Branching ratios for the $D^0 \to VV$ decays in units of 10^{-3}.

Modes	$\mathcal{B}_{\rm tot}$(exp)	$\mathcal{B}_{\rm long}$(exp)	$\mathcal{B}_{\rm long}$(FAT)
$\rho^0\overline{K}^{*0}$	15.9 ± 3.5		14.3±1.6
ρ^+K^{*-}	65.0 ± 25.0		41.8±2.4
$\overline{K}^{*0}\omega$	11.0 ± 5.0		37.7±2.7
$\rho^+\rho^-$			4.1±0.3
$K^{*+}K^{*-}$			1.18±0.06
$K^{*0}\overline{K}^{*0}$			0.043±0.006
$\rho^0\rho^0$	1.83 ± 0.13	1.25 ± 0.13	1.4±0.2
$\rho^0\omega$			1.37±0.08
$\rho^0\phi$			0.65±0.04
$\omega\omega$			0.53±0.08
$\omega\phi$			1.4±0.1
ρ^0K^{*0}			0.041±0.005
ρ^-K^{*+}			0.143±0.008
$K^{*0}\omega$			0.108±0.008

1. A longitudinal amplitude A_0 is a linear combination of the partial waves S and D, namely, of the $L = 0$ and 2 final states, leading to $\eta_{\rm CP}(n) = +1$.

2. We obtain the longitudinal VV contribution

$$y_{VV} = (-0.042 \pm 0.034)\%.$$

[**HYJ**,F.S.Yu,Q.Qin,H.n.Li, C.D.Lu,18]

OUTLINE

Summary

1. Based on FAT approach, we obtain the charm mixing parameter y from the contributions of PP, PV and VV modes:

$$\begin{cases} y_{PP} = (0.10 \pm 0.02)\%, \\ y_{PV} = (0.11 \pm 0.07)\%, \\ y_{VV} = (-0.042 \pm 0.034)\%, \quad \text{negligible} \end{cases} \Rightarrow y_{PP+PV} = (0.21 \pm 0.07)\%$$

   which far below the data $y_{\rm exp} = (0.61 \pm 0.08)\%$. And, it is much more precise than those in [H.Y.Cheng,C.W.Chiang,'10].

2. In conclusion, we need new consideration and other decay modes, such as VA, AP, or multi-particle final states should be considered in calculation.

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Thank you for your attention!

Cabibbo-Kabayashi-Maskawa Matrix

1. V_{CKM} represent the element of CKM matrix:

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} = \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

   where (d, s, b) and (d', s', b') are respectively mass eigenstates and weak interaction eigenstates. The mixing parameter $\lambda \sim 0.02$.

2. The non-leptonic two body decays of charm meson can be classified

CF	$V_{ud}V_{cs} \sim 1$	Cabibbo-favored
SCS	$V_{ud}V_{cd}, V_{us}V_{cs} \sim 10^{-1}$	singly Cabibbo suppressed
DCS	$V_{cd}V_{us} \sim 10^{-2}$	doubly Cabibbo suppressed