# Strong Decays of the Excited $D$ and $D_s$ Mesons

$\left(D_0^*(2400),\ D_J^*(3000),\ D_J(3000),\ D_{sJ}(3040)\right)$

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Base on

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# Outline

# Motivation

In 2004, the FOCUS Collaboration and the Belle Collaboration observed the $D_0^*$ $(1P(0^+)$ [1], has been studied widely and carefully).

In 2009, $D_{sJ}(3040)$ was reported by the BABAR Collaboration in the $D^*K$ channel[2].

In 2013, the LHCb Collaboration announced $D_J(3000)$ and $D_J^*(3000)$ in $D^*\pi$ and $D\pi$ mass spectrum respectively[3].

$$M_{D_0^*(2400)^0} = (2308 \pm 17 \pm 15 \pm 28)\text{MeV}, \qquad M_{D_J^*(3000)} = 3008.1 \pm 4.0 \text{ MeV},$$

$$\Gamma_{D_0^*(2400)^0} = (276 \pm 21 \pm 18 \pm 60)\text{MeV}, \qquad \Gamma_{D_J^*(3000)} = 110.5 \pm 11.5 \text{ MeV}.$$

$$M_{D_{sJ}(3040)^+} = \left(3044 \pm 8^{+30}_{-5}\right) \text{ MeV}, \qquad M_{D_J(3000)^0} = (2971.8 \pm 8.7) \text{ MeV},$$

$$\Gamma_{D_{sJ}(3040)^+} = \left(239 \pm 35^{+46}_{-42}\right) \text{ MeV}, \qquad \Gamma_{D_J(3000)^0} = (188.1 \pm 44.8) \text{ MeV}.$$

[1] Phys. Lett. B 586, 11 (2004), Phys. Rev. D 69, 112002 (2004)

[2] Phys. Rev. D 80, 092003 (2009).

[3] JHEP 2013, 145 (2013)

Spin analysis indicates that $D_{sJ}(3040)$ and $D_J(3000)$ has an unnatural parity. Their masses are lower than the $3^1S_0$ and higher than the $1^1D_2$ and $1^3D_2$ states in theoretical predictions, located in the mass region of $2P(1^+)$ states.

The parity of $D_J^*(3000)$ is still uncertain, but most work treate it as a natural parity particle. The assignments of $0^+$, $1^-$, $2^+$, $3^-$ and $4^+$ are possible.

**Table 1-1:** Several natural parity candidates of $D_J^*(3000)^0$ (MeV)

| $J^P$ | $n^{2S+1}J_L$ | Godfrey1985 | Pierro2001 | Ebert2009 | Sun2013 | Godfrey2016 |
|---|---|---|---|---|---|---|
| $0^+$ | $1^3P_0$ | 2400 | 2377 | 2466 | 2398 | 2399 |
| | $2^3P_0$ | - | 2949 | 2919 | 2932 | 2931 |
| $1^-$ | $3^3S_1$ | - | 3226 | 3096 | 3111 | 3110 |
| $2^+$ | $2^3P_2$ | - | 3035 | 3012 | 2957 | 2957 |
| $3^-$ | $1^3D_3$ | 2830 | 2799 | 2863 | 2833 | 2833 |
| | $2^3D_3$ | - | - | 3335 | 3226 | 3226 |
| $4^+$ | $1^3F_4$ | 3110 | 3091 | 3187 | 3113 | 3113 |

Phys. Rev. D 32, 189 (1985), Phys. Rev. D 64, 114004 (2001), Eur. Phys. J. C 66, 197 (2009),

Phys. Rev .D 88, 094020 (2013), Phys. Rev. D 93, 034035 (2016)

**Table 1-2:** Decay widths of $D_J^*(3000)^0$ with different assignments (MeV)

| $n^{2S+1}L_J$ | Mode | Sun2013 | Yu2015 | Lü2014 | Song2015 | Godfrey2016 |
|---|---|---|---|---|---|---|
| $3^3S_1$ | $D\pi$ | 0.91 | 5.45 | 14.0 | 13.5 | 3.21 |
| | $D^*\pi$ | 3.5 | 4.85 | 19.4 | 25.7 | 5.6 |
| | Total | 18.0 | 87.2 | 158.0 | 103.0 | 80.4 |
| $2^3P_0$ | $D\pi$ | 49 | 35.9 | 83.5 | 72.5 | 25.4 |
| | $D^*\pi$ | - | - | - | - | - |
| | Total | 194 | 224.5 | 639.3 | 298.4 | 190 |
| $2^3P_2$ | $D\pi$ | 1.8 | 5.0 | 1.92 | 1.46 | 5.0 |
| | $D^*\pi$ | $8.1 \times 10^{-3}$ | 17.8 | 11.89 | 0.12 | 17.1 |
| | Total | 47.0 | 174.5 | 110.5 | 68.9 | 114 |
| $1^3F_2$ | $D\pi$ | 16 | 18.8 | 28.6 | 26.1 | 23.1 |
| | $D^*\pi$ | 13 | 15.7 | 21.0 | 18.8 | 18.5 |
| | Total | 136 | 116.4 | 342.9 | 222.0 | 243 |
| $1^3F_4$ | $D\pi$ | 1.2 | 21.3 | 9.96 | 4.97 | 15.8 |
| | $D^*\pi$ | 1.8 | 14.1 | 9.41 | 5.31 | 15.2 |
| | Total | 39 | 102.3 | 103.9 | 94.5 | 129 |

Phys. Rev .D 88, 094020 (2013), Chin. Phys. C 39, 063101 (2015), Phys. Rev. D 90, 054024 (2014), Phys. Rev. D 92, 074011 (2015), Phys. Rev. D 93, 034035 (2016).

In 2016, the LHCb announced another natural parity $D$ meson ($3214 \pm 29$ MeV)[4], and confirmed its quantum number is $2^3P_2$. Thus $D_J^*(3000)$ cannot be the $2^+$ state.

Otherwise, $D_J(3000)$ was only found in $D^*\pi$ spectrum, while $D_J^*(3000)$ only in $D\pi$ spectrum in the LHCb experiment.

$$D(^3P_0) \nrightarrow D^*\pi$$
$$D(1^3F_4,\ 3^3S_1,\ 2^3P_2,\ 2^3D_3) \rightarrow D^*\pi$$

Thus, the assignment of $2^3P_0$ ($2P\ 0^+$) for $D_J^*(3000)$ is more reasonable.

We try to calculate the two-body strong decays of the excited $D$ and $D_s$ mesons $D_0^*(2400)$, $D_J^*(3000)$, $D_J(3000)$, $D_{sJ}(3040)$ by using the relativistic Bethe-Salpeter (BS) method.

[4] Phys. Rev. D 94, 072001 (2016).

# B-S(Bethe-Salpeter) method

The BS equation of two-body bound state can read in momentum space as

$$S_1^{-1}\chi_P(q)S_2^{-1} = \mathrm{i}\int \frac{\mathrm{d}^4 k}{(2\pi)^4} I(P;q,k)\chi_P(k), \tag{2-1}$$

We follow Salpeter to take the instantaneous approximation $I(P;q,k) \approx I(q_\perp - k_\perp)$
The three-dimensional salpeter wave function $\psi(q_\perp)$ is defined by

$$\psi(q_\perp) = \mathrm{i}\int \frac{\mathrm{d}q_P}{2\pi}\chi_P(q), \quad \chi_P(q) = S_1(p_1)\int \frac{\mathrm{d}^3 k}{(2\pi)^3} I(q_\perp - k_\perp)\psi_P(k_\perp)S_2(p_2) \tag{2-2}$$

We adopt the Cornell potential as the interaction kernel $I(r)$

$$I(r) = V_s(r) + V_0 + \gamma_0 \otimes \gamma^0 V_v(r) = \frac{\lambda}{\alpha}(1 - \mathrm{e}^{-\alpha r}) + V_0 - \frac{4}{3}\frac{\alpha_s}{r}\mathrm{e}^{-\alpha r}, \tag{2-3}$$

# B-S wave founction

For example, we express the relativistic wave function of a scalar meson as

$$\varphi_{0^+}(q_\perp) = M\left[\frac{\not{q}_\perp}{M}f_{a1}(q_\perp) + \frac{\not{P}\not{q}_\perp}{M^2}f_{a2}(q_\perp) + f_{a3}(q_\perp) + \frac{\not{P}}{M}f_{a4}(q_\perp)\right], \tag{2-4}$$

Within BS framwork, the four wave functions $f_{ai}$ are independent, they have the following relations

$$\begin{aligned} f_{a3} &= \frac{q_\perp^2(\omega_1 + \omega_2)}{M(m_1\omega_2 + m_2\omega_1)}f_{a1}, \\ f_{a4} &= \frac{q_\perp^2(\omega_1 - \omega_2)}{M(m_1\omega_2 + m_2\omega_1)}f_{a2}, \end{aligned} \tag{2-5}$$

In our calculation, we only keep the positive energy parts $\varphi^{++}_{P_i}(q_{i\perp})$ of the relativistic wave functions because the negative energy part contributes too small.

$$\varphi^{++}_{0^+}(q_\perp) = A_1 + A_2\frac{\not{P}}{M} + A_3\frac{\not{q}_\perp}{M} + A_4\frac{\not{P}\not{q}_\perp}{M^2}. \tag{2-6}$$

${}^1P_1$ and ${}^3P_1$ are

$$\varphi^{++}_{1^{+-}}(q_\perp) = q_\perp \cdot \epsilon\left[B_1 + \frac{\not{P}}{M}B_2 + \frac{\not{q}_\perp}{M}B_3 + \frac{\not{P}\not{q}_\perp}{M^2}B_4\right]\gamma_5, \tag{2-7}$$

$$\varphi^{++}_{1^{++}}(q_\perp) = i\varepsilon_{\mu\nu\alpha\beta}\frac{P^\nu}{M}q^\alpha_\perp\epsilon^\beta\gamma^\mu\left[C_1 + \frac{\not{P}}{M}C_2 + \frac{\not{q}_\perp}{M}C_3 + \frac{\not{P}\not{q}_\perp}{M^2}C_4\right]. \tag{2-8}$$

# Formalism of strong decay

We take the channel $D_0^*(2400)^0 \to D^+\pi^-$ as an example. The Feynman diagram of this process is shown in Fig. 3-1.

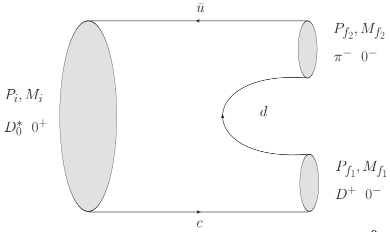



**Figure 3-1:** Feynman diagram for $D_0^*(2400)^0 \to D^+\pi^-$.

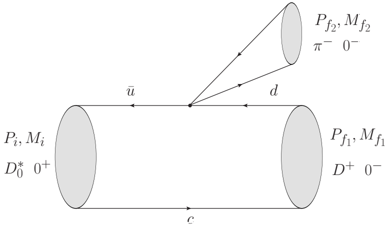



**Figure 3-2:** Feynman diagram with the low-energy approximation.

By using the reduction formula, the transition matrix element can be written as

$$
\begin{aligned}
T =& \langle D^+(P_{f1})\pi^-(P_{f2}) \mid D_0^*(P_i)\rangle \\
=& \int \mathrm{d}^4x e^{iP_{f2}\cdot x}(M_{f2}^2 - P_{f2}^2)\langle D^+(P_{f1}) \,|\phi_\pi(x)|\, D_0^*(P_i)\rangle.
\end{aligned}
\tag{3-1}
$$

By using the PCAC relation, the field can be expressed as

$$\phi_\pi(x) = \frac{1}{M_{f2}^2 f_\pi}\partial^\mu(\overline{u}\gamma_\mu\gamma_5 d), \tag{3-2}$$

According to the low energy theorem the Feynman diagram turns to Fig. 3-2 and the amplitude can be written as

$$\begin{aligned} T =& \frac{M_{f2}^2 - P_{f2}^2}{M_{f2}^2 f_\pi}\int d^4x e^{iP_{f2}\cdot x}\langle D^+(P_{f1})\left|\partial^\mu(\overline{u}\gamma_\mu\gamma_5 d)\right| D_0^*(P_i)\rangle \\ \approx& - i\frac{P_{f2}^\mu}{f_\pi}\int d^4x e^{iP_{f2}\cdot x}\langle D^+(P_{f1})\left|\overline{u}\gamma_\mu\gamma_5 d\right| D_0^*(P_i)\rangle \\ =& - i\frac{P_{f2}^\mu}{f_\pi}(2\pi)^4\delta^4(P_i - P_{f1} - P_{f2})\langle D^+(P_{f1})\left|\overline{u}\gamma_\mu\gamma_5 d\right| D_0^*(P_i)\rangle. \end{aligned} \tag{3-3}$$

Within Mandelstam formalism, we can write the hadronic transition amplitude as

$$\begin{aligned} \mathcal{M} =& - \mathrm{i}\frac{P_{f2}^\mu}{f_\pi}\langle D^+(P_{f1})\left|\overline{u}\gamma_\mu\gamma_5 d\right| D_0^*(P_i)\rangle \\ =& - \mathrm{i}\frac{P_{f2}^\mu}{f_\pi}\int\frac{\mathrm{d}^3q}{(2\pi)^3}\mathrm{Tr}\left[\overline{\varphi}_{P_{f1}}^{++}(q_{f1\perp})\frac{\not{P}_i}{M_i}\varphi_{P_i}^{++}(q_\perp)\gamma_\mu\gamma_5\right]. \end{aligned} \tag{3-4}$$

# Mixing of the $1^+$ state

For heavy-light $1^+$ state, we do not use the $S$-$L$ coupling, but $j$-$j$ coupling. The orbital angular momentum $\vec{L}$ couples with the light quark spin $\vec{s}_q$.

Then $1^+$ state can be grouped into a doublet by the total angular momentum of the light quark($|j_l = 1/2\rangle$ and $|j_l = 3/2\rangle$).

$$\begin{pmatrix} |J^P = 1^+, j_l = 3/2\rangle \\ |J^P = 1^{+'}, j_l = 1/2\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |^1P_1\rangle \\ |^3P_1\rangle \end{pmatrix}. \tag{3-5}$$

We solve the Salpeter equations for $^3P_1$ and $^1P_1$ states individually, and use these mixing relations to calculate the contributions of two physical $1^+$ states.

We choose the ideal mixing angle $\theta = 35.3^\circ$ within the heavy quark limit.

# The effective Lagrangian

The PCAC rule is not valid for the light vector mesons. When $\rho$ or $\omega$ meson appears in the final states, we choose the effective Lagrangian method to calculate the transition amplitude. The Lagrangian of quark-meson coupling can be expressed as

$$\mathcal{L}_{qqV} = \bar{q}_i(a\gamma_\mu + \frac{\mathrm{i}b}{2M_{P_{f2}}}\sigma_{\mu\nu}P^\nu_{f2})V^\mu_{ij}q_j. \tag{3-6}$$

where $V^\mu_{ij}$ is the field of the light vector meson; $q_i$ and $\bar{q}_j$ are its constitute quarks. And we choose the parameters $a = -3$ and $b = 2$ which represent the vector and tensor coupling strength, respectively. Then we use Eq. (3-6) to derive the light-vector meson's vertex and get the transition amplitude

$$\mathcal{M} = -\mathrm{i}\int \frac{\mathrm{d}^3q}{(2\pi)^3}\mathrm{Tr}\left[\overline{\varphi}^{++}_{P_{f1}}(q_{f1\perp})\frac{\not{P}_i}{M_i}\varphi^{++}_{P_i}(q_\perp)(a\gamma_\mu + \frac{\mathrm{i}b}{2M_{f2}}\sigma_{\mu\nu}P^\nu_{f2})\varepsilon^\mu_2\right]. \tag{3-7}$$

The two-body decay width can be expressed as

$$\Gamma = \frac{1}{2J+1}\frac{|\vec{P}_{f1}|}{8\pi M_i^2}\sum_\lambda |\mathcal{M}|^2. \tag{3-8}$$

# $D_J(3000)$ Numerical Results (as $2P(1^+)$)

**Table 4-3:** The decay widths of $D_J(3000)^0$ as the 2P($1^+$) state(MeV).

| | Final state | Ours | QPC Model | $^3P_0$ Model | Godfrey-Isgur | $^3P_0$ Model$_2$ |
|---|---|---|---|---|---|---|
| $1^+ \to 1^-0^-$ | $D^*(2007)^0\pi^0$ | 0.97 | 1.3 | 11.85 | 37.9 | 15.64 |
| | $D^*(2010)^+\pi^-$ | 1.83 | | 23.62 | | 31.25 |
| | $D^*(2007)^0\eta$ | 0.10 | 0.49 | 2.48 | 5.0 | 6.88 |
| | $D^*(2007)^0\eta'$ | 0.08 | 0.00026 | 18.72 | □ | 0.95 |
| | $D^*(2600)^0\pi^0$ | 4.78 | □ | □ | 1.3 | 5.93 |
| | $D^*(2600)^+\pi^-$ | 9.56 | □ | □ | | 11.84 |
| | $D^*(2650)^0\pi^0$ | 0.26 | □ | □ | □ | 0.02 |
| | $D^*(2650)^+\pi^-$ | 0.52 | □ | □ | □ | 0.04 |
| $1^+ \to 0^-1^-$ | $D^0\rho^0$ | 1.90 | 4.7 | 17.27 | 3.4 | 1.16 |
| | $D^+\rho^-$ | 4.31 | | 34.52 | | 1.98 |
| | $D^0\omega$ | 1.89 | 1.5 | 17.30 | 1.1 | 0.95 |
| $1^+ \to 1^-1^-$ | $D^*(2007)^0\rho^0$ | 11.30 | 14 | 18.46 | 24.4 | 32.84 |
| | $D^*(2010)^+\rho^-$ | 21.29 | | 36.26 | | 62.68 |
| | $D^*(2007)^0\omega$ | 10.90 | 4.6 | 17.53 | 8.2 | 31.31 |
| $1^+ \to 0^+0^-$ | $D_0^*(2400)^0\pi^0$ | 0.46 | 11 | 0.17 | 4.9 | 0.94 |
| | $D_0^*(2400)^+\pi^-$ | 1.40 | | □ | | 1.98 |
| | $D_0^*(2400)^0\eta$ | 0.23 | 0.14 | 0.30 | □ | 0.4 |
| $1^+ \to 1^+0^-$ | $D_1(2420)^0\pi^0$ | 2.34 | 8.8 | 0.024 | 5.2 | 11.32 |
| | $D_1(2420)^+\pi^-$ | 4.70 | | □ | | 22.62 |
| | $D_1(2420)\eta$ | 0.0029 | 0.0023 | 0.0061 | □ | 0.03 |
| | $D_1(2430)^0\pi^0$ | 0.15 | 5.3 | 0.0081 | 2.5 | 0.15 |
| | $D_1(2430)^+\pi^-$ | 0.30 | | □ | | 0.29 |
| | $D_1(2430)^0\eta$ | – | – | 0.003 | – | – |
| $1^+ \to 2^+0^-$ | $D_2^*(2460)^0\pi^0$ | 2.89 | 3.3 | 28.05 | 7.4 | 6.98 |
| | $D_2^*(2460)^+\pi^-$ | 5.68 | | 56.21 | | 13.70 |
| | $D_2^*(2460)^0\eta$ | – | – | 0.56 | – | – |
| $1^+ \to 0^-1^-$ | $D_s^+K^{*-}$ | 0.41 | 0.7 | 3.82 | 14.3 | 10.41 |
| $1^+ \to 1^-0^-$ | $D_s^{*+}K^-$ | 0.055 | 0.099 | 1.22 | 9.0 | 4.14 |
| $1^+ \to 0^+0^-$ | $D_{s0}^*(2317)^+K^-$ | 5.29 | 1.2 | 0.52 | □ | 0.74 |
| $1^+ \to 1^-1^-$ | $D_s^{*+}K^*$ | – | – | 4.08 | – | – |
| $1^+ \to 1^+0^-$ | $D_{s1}(2460)^+K^-$ | 0.043 | 0.045 | 0.024 | □ | 0.01 |
| | $D_{s1}(2536)^+K^-$ | – | – | 0.049 | – | – |
| Total | $Exp: 188.1 \pm 44.8$ | 93.6 | 57.1 | 293.1 | 124.6 | 277.2 |

Phys.Rev.D88,094020(2013).

Chin.Phys. C 39, 063101 (2015).

Phys. Rev. D 93, 034035 (2016).

Phys. Rev. D 90, 054024 (2014).

# $D_J(3000)$**Numerical Results(**$2P(1^{+'})$**)**

**Table 4-4:** The decay widths of $D_J(3000)^0$ as the 2P($1^{+'}$) state(MeV).

| | Final state | Ours | QPC Model | $^3P_0$ Model | Godfrey-Isgur | $^3P_0$ Model$_2$ |
|---|---|---|---|---|---|---|
| $1^+ \to 1^-0^-$ | $D^*(2007)^0\pi^0$ | 13.37 | 38 | 10.03 | 21.6 | 18.79 |
| | $D^*(2010)^+\pi^-$ | 25.40 | | 20.32 | | 36.92 |
| | $D^*(2007)^0\eta$ | 5.03 | 5.2 | 4.92 | □ | 4.39 |
| | $D^*(2007)^0\eta'$ | 4.15 | 0.023 | 2.71 | □ | 3.80 |
| | $D^*(2600)^0\pi^0$ | 13.14 | □ | □ | 20.9 | 20.90 |
| | $D^*(2600)^+\pi^-$ | 26.28 | □ | □ | | 42.04 |
| | $D^*(2650)^0\pi^0$ | 1.01 | □ | □ | □ | 0.02 |
| | $D^*(2650)^+\pi^-$ | 2.02 | □ | □ | □ | 0.32 |
| $1^+ \to 0^-1^-$ | $D^0\rho^0$ | 2.00 | 7.6 | 5.61 | 18.8 | 26.99 |
| | $D^+\rho^-$ | 4.26 | | 10.59 | | 53.14 |
| | $D^0\omega$ | 2.15 | 2.5 | 4.99 | 6.11 | 26.55 |
| $1^+ \to 1^-1^-$ | $D^*(2007)^0\rho^0$ | 5.51 | 15 | 21.07 | 23.3 | 29.47 |
| | $D^*(2010)^+\rho^-$ | 10.38 | | 41.34 | | 57.33 |
| | $D^*(2007)^0\omega$ | 5.41 | 4.9 | 19.93 | 7.3 | 28.70 |
| $1^+ \to 0^+0^-$ | $D_0^*(2400)^0\pi^0$ | 1.90 | 6 | 0.24 | □ | 1.93 |
| | $D_0^*(2400)^+\pi^-$ | 4.09 | | □ | □ | 4.06 |
| | $D_0^*(2400)^0\eta$ | 0.53 | 0.068 | 0.27 | □ | 0.84 |
| $1^+ \to 1^+0^-$ | $D_1(2420)^0\pi^0$ | 2.33 | 14 | 0.0081 | 15.9 | 2.77 |
| | $D_1(2420)^+\pi^-$ | 4.69 | | □ | | 5.53 |
| | $D_1(2420)\eta$ | 0.0023 | 0.0042 | 0.003 | □ | 0.0072 |
| | $D_1(2430)^0\pi^0$ | 1.84 | 11 | 0.0099 | 5.3 | 0.11 |
| | $D_1(2430)^+\pi^-$ | 3.64 | | □ | | 0.21 |
| | $D_1(2430)^0\eta$ | – | – | 0.0015 | – | – |
| $1^+ \to 2^+0^-$ | $D_2^*(2460)^0\pi^0$ | 30.69 | 38 | 5.39 | 82.3 | 40.40 |
| | $D_2^*(2460)^+\pi^-$ | 58.01 | | 10.52 | | 80.53 |
| | $D_2^*(2460)^0\eta$ | – | – | 0.024 | – | – |
| $1^+ \to 0^-1^-$ | $D_s^+K^{*-}$ | 0.12 | 0.12 | 7.13 | 4.0 | 1.48 |
| $1^+ \to 1^-0^-$ | $D_s^{*+}K^-$ | 1.14 | 3.7 | 9.45 | 4.4 | 0.95 |
| $1^+ \to 0^+0^-$ | $D_{s0}^*(2317)^+K^-$ | 0.42 | 0.67 | 0.83 | □ | 1.19 |
| $1^+ \to 1^-1^-$ | $D_s^{*+}K^*$ | – | – | 2.05 | – | – |
| $1^+ \to 1^+0^-$ | $D_{s1}(2460)^+K^-$ | 0.049 | 0.082 | 0.0081 | □ | 0.00021 |
| | $D_{s1}(2536)^+K^-$ | – | – | 0.024 | – | – |
| Total | $Exp$ : 188.1 ± 44.8 | 229.6 | 146.8 | 177.5 | 209.9 | 489.3 |


Phys.Rev.D88,094020(2013).

Chin.Phys. C 39, 063101 (2015).

Phys. Rev. D 93, 034035 (2016).

Phys. Rev. D 90, 054024 (2014).

# $D_{sJ}(3040)$Numerical Results($2P(1^+)$)

**Table 4-5:** The decay widths of $D_{sJ}(3040)^+$ as the 2P($1^+$) state(MeV).

| | Final state | Ours | Goldfrey-Isgur | Constituent quark model |
|---|---|---|---|---|
| $1^+ \to 1^-0^-$ | $D^*(2007)^0K^+$ | 0.02 | 61.3 | 7.99 |
| | $D^*(2010)^+K^0$ | 0.02 | | 7.79 |
| $1^+ \to 0^+0^-$ | $D_0^*(2400)^0K^+$ | 3.46 | 4.95 | 6.86 |
| | $D_0^*(2400)^+K^0$ | 3.86 | | 6.43 |
| $1^+ \to 2^+0^-$ | $D_2^*(2460)^0K^+$ | 1.05 | 0.67 | 3.00 |
| | $D_2^*(2460)^+K^0$ | 1.96 | | 2.89 |
| $1^+ \to 1^-1^-$ | $D^*(2007)^0K^{*+}$ | 17.06 | 38.9 | 39.84 |
| | $D^*(2010)^+K^{*0}$ | 15.81 | | 37.36 |
| $1^+ \to 0^-1^-$ | $D^0K^{*+}$ | 4.83 | 6.54 | 12.74 |
| | $D^+K^{*0}$ | 4.47 | | 13.27 |
| $1^+ \to 1^+0^-$ | $D_1(2420)^0K^+$ | 2.75 | 3.52 | 4.99 |
| | $D_1(2420)^+K^0$ | 2.7 | | 5.01 |
| | $D_1(2430)^0K^+$ | 0.08 | 1.29 | 1.59 |
| | $D_1(2430)^+K^0$ | 0.05 | | 1.52 |
| $1^+ \to 1^-0^-$ | $D_s^{*+}\eta$ | 3.77 | 9.65 | 1.10 |
| $1^+ \to 0^+0^-$ | $D_{s0}^*(2317)^+\eta$ | 1.56 | □ | 1.19 |
| $1^+ \to 1^+0^-$ | $D_{s1}(2460)^+\eta$ | 0.03 | □ | 0.10 |
| $1^+ \to 1^+1^-$ | $D_s^+\phi$ | □ | 16.2 | 0.40 |
| Total | $Exp: 239 \pm 35^{+46}_{-42}$ | 63.5 | 143.0 | 154.1 |


Phys. Rev. D 93, 034035 (2016).

Eur. Phys. J. C 71, 1582 (2011).

## $D_{sJ}(3040)$Numerical Results($2P(1^{+'})$)

**Table 4-6:** The decay widths of $D_{sJ}(3040)^+$ as the 2P($1^{+'}$) state(MeV).

| | Final state | Ours | Goldfrey-Isgur | Constituent quark model |
|---|---|---|---|---|
| $1^+ \to 1^-0^-$ | $D^*(2007)^0K^+$ | 48.06 | 36.5 | 34.35 |
| | $D^*(2010)^+K^0$ | 47.00 | | 34.84 |
| $1^+ \to 0^+0^-$ | $D_0^*(2400)^0K^+$ | 3.71 | 1.14 | 19.07 |
| | $D_0^*(2400)^+K^0$ | 3.74 | | 14.39 |
| $1^+ \to 2^+0^-$ | $D_2^*(2460)^0K^+$ | 2.83 | 28.4 | 39.68 |
| | $D_2^*(2460)^+K^0$ | 4.87 | | 38.97 |
| $1^+ \to 1^-1^-$ | $D^*(2007)^0K^{*+}$ | 12.67 | 29.7 | 34.59 |
| | $D^*(2010)^+K^{*0}$ | 11.77 | | 32.24 |
| $1^+ \to 0^-1^-$ | $D^0K^{*+}$ | 5.05 | 32.1 | 31.85 |
| | $D^+K^{*0}$ | 4.78 | | 30.31 |
| $1^+ \to 1^+0^-$ | $D_1(2420)^0K^+$ | 2.73 | 12.2 | 1.76 |
| | $D_1(2420)^+K^0$ | 2.67 | | 1.77 |
| | $D_1(2430)^0K^+$ | 1.58 | 3.38 | 0.5 |
| | $D_1(2430)^+K^0$ | 1.24 | | 0.48 |
| $1^+ \to 1^-0^-$ | $D_s^{*+}\eta$ | 4.22 | 0.153 | 6.20 |
| $1^+ \to 0^+0^-$ | $D_{s0}^*(2317)^+\eta$ | 0.37 | □ | 3.12 |
| $1^+ \to 1^+0^-$ | $D_{s1}(2460)^+\eta$ | 0.07 | □ | 0.03 |
| $1^+ \to 1^+1^-$ | $D_s^+\phi$ | □ | 4.15 | 0.39 |
| Total | $Exp: 239 \pm 35^{+46}_{-42}$ | 157.4 | 147.6 | 324.5 |

Phys. Rev. D 93, 034035 (2016).

Eur. Phys. J. C 71, 1582 (2011).

# $D_0^*$ Results (as $0^+$)

**Table 4-7:** $D_0^*(2400)$ strong decay widths (MeV)

| Mass | Chanel | | Decay Width(MeV) | Exp. Value[5] |
|---|---|---|---|---|
| $2318 \pm 29$ | $D_0^*(2400)^0 \rightarrow$ | $D^+\pi^-$ | 151.5 | $267 \pm 40$ |
| | | $D^0\pi^0$ | 74.8 | |
| $2351 \pm 7$ | $D_0^*(2400)^+ \rightarrow$ | $D^+\pi^0$ | 81.6 | $230 \pm 17$ |
| | | $D^0\pi^+$ | 164.3 | |

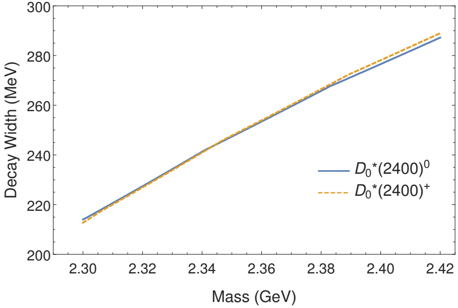



**Figure 4-3:** $\Gamma_{D_0^*(2400)^{(0,+)}}$ versus the mass.

[5]Paticle Data Group 2016

# $D_J^*(3000)$ Results (as $2P(0^+)$)

**Table 4-8:** Two-body strong decay widths (MeV) of $D_J^*(3000)^0$ as the $2P(0^+)$ state. "-" means the channel is forbidden, "□" means the channel is not included by this method.

| Chanel | Final States | Ours | $^3P_0$ Model | QPC Model | Relativistic quark model | Effective Lagrangian |
|---|---|---|---|---|---|---|
| $D(^1S_0)\pi$ | $D^+\pi^-$ | 11.6 | 23.94 | 49 | 25.4 | 66.2 |
| | $D^0\pi^0$ | 6.1 | 11.97 | | | 33.3 |
| $D(2^1S_0)\pi$ | $D(2550)^+\pi^-$ | 6.9 | □ | □ | 18.6 | □ |
| | $D(2550)^0\pi^0$ | 3.3 | | | | |
| $D\eta$ | $D^0\eta^0$ | 0.51 | 4.26 | 8.8 | 1.53 | 10.8 |
| $D\eta'$ | $D^0\eta'^0$ | 6.0 | 1.07 | 2.7 | 4.94 | □ |
| $D_sK$ | $D_s^+K^-$ | ~$10^{-3}$ | 2.85 | 6.6 | 0.76 | 54.2 |
| $D_1(2420)\pi$ | $D_1(2420)^0\pi^0$ | 18.7 | 26.20 | 38 | 96.1($^1P_1$) | □ |
| | $D_1(2420)^+\pi^-$ | 36.8 | □ | | | |
| $D_1(2420)\eta$ | $D_1(2420)^0\eta^0$ | 0.85 | 1.37 | 1.1 | □ | □ |
| $D_1(2430)\pi$ | $D_1(2430)^0\pi^0$ | 2.1 | 6.69 | 30 | □ | □ |
| | $D_1(2430)^+\pi^-$ | 4.1 | □ | | | |
| $D_1(2430)\eta$ | $D_1(2430)^0\eta^0$ | 0.12 | 0.35 | 0.91 | □ | □ |
| $D_s(2460)K$ | $D_{s1}(2460)^+K^-$ | 1.2 | 12.81 | 1.5 | □ | □ |
| $D^*\rho$ | $D^*(2007)^0\rho^0$ | 7.0 | 31.60 | 41 | 32 | □ |
| | $D^*(2010)^+\rho^-$ | 13.3 | 62.01 | | | |
| $D^*\omega$ | $D^*(2007)^0\omega^0$ | 7.5 | 29.91 | 13 | 10.2 | □ |
| $D_s^*K^*$ | $D_s^{*+}K^*(892)^-$ | 4.1 | 3.06 | 1.0 | □ | □ |
| $D_s(2536)K^-$ | $D_{s1}(2536)^+K^-$ | - | 6.40 | - | - | - |
| Total | | 130.2 | 224.5 | 193.6 | 189.5 | 164.5 |
| Experimental value | | | | 110.5 ± 11.5 | | |

Chin. Phys. C 39, 063101 (2015). Phys.Rev.D88,094020(2013). Phys. Rev. D 93, 034035 (2016). Phys. Rev. D 97, 014015 (2018).

**Table 4-9:** Two-body strong decay widths (MeV) of $D_J^*(3000)^+$ as the $2P(0^+)$ state.

| Chanel | Final States | Width | Chanel | Final States | Width |
|---|---|---|---|---|---|
| $D(^1S_0)\pi$ | $D^+\pi^0$ | 6.5 | $D(2^1S_0)\pi$ | $D^0\pi^+$ | 3.8 |
| | $D^0\pi^+$ | 13.5 | | $D^0\pi^+$ | 7.7 |
| $D\eta$ | $D^0\eta^0$ | 0.56 | $D\eta'$ | $D^0\eta'^0$ | 5.7 |
| $D(2420)\pi$ | $D_1(2420)^+\pi^0$ | 18.3 | $D(2430)\pi$ | $D_1(2430)^+\pi^0$ | 2.1 |
| | $D_1(2420)^0\pi^+$ | 37.4 | | $D_1(2430)^0\pi^+$ | 4.3 |
| $D(2420)\eta$ | $D_1(2420)^+\eta^0$ | 0.77 | $D(2430)\eta$ | $D_1(2430)^+\eta^0$ | 0.11 |
| $D^*\rho$ | $D^*(2010)^+\rho^0$ | 6.1 | $D^*\omega$ | $D^*(2010)^+\omega^0$ | 6.5 |
| | $D^*(2007)^0\rho^-$ | 12.9 | $D_s(2460)K$ | $D_{s1}(2460)^+K^0$ | 1.2 |
| $D_sK$ | $D_s^+K^0$ | 0.05 | $D_s^*K^*$ | $D_s^{*+}K^*(892)^0$ | 3.8 |
| | Total | | | 131.3 | |

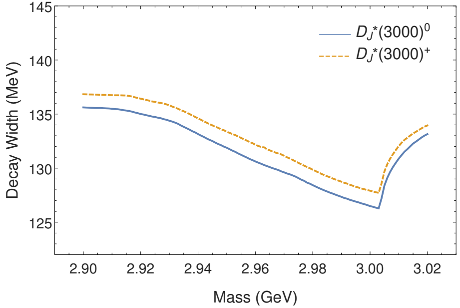



**Figure 4-4:** $\Gamma_{D_J^*(3000)^{(0,+)}}$ versus the mass.

# Discussion

*Question1:*

Which one of the $2P(1^+)$ doublet should be assigned to $D_J(3000)$ and $D_{sJ}(3040)$?

*Thoughts:*

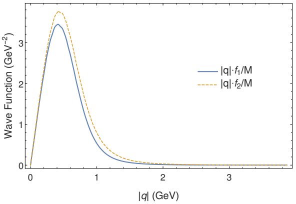
From the total width and the result of $D^*\pi$ and $D^*K$ channel, we favour the broad $1^{+'}$ state.

*Question2:*

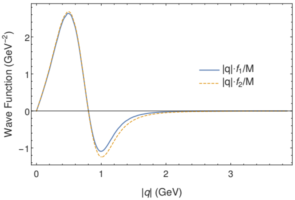
The $2P$ state $D_J^*(3000)^0$ has larger phase space and more decay channels than those of $1P$ state $D_0^*(2400)^0$, why we get a narrower full width?

*Thoughts:*

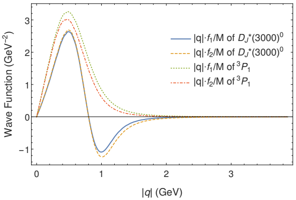
We consider the reason is the different structures of wave functions.



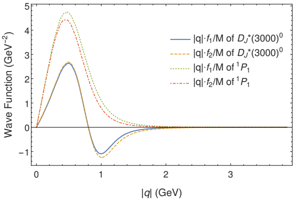

a) $0^+(1P)$ state $D_0^*(2400)^0$



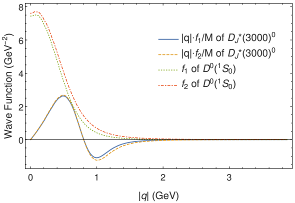

b) $0^+(2P)$ state $D_J^*(3000)^0$



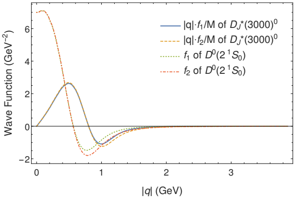

c) $D_J^*(3000)^0$ and $D_1(2420)(^3P_1$ state)




d) $D_J^*(3000)^0$ and $D_1(2420)(^1P_1$ state)




e) $D_J^*(3000)^0$ and $D^0(^1S_0$ state)




f) $D_J^*(3000)^0$ and $D^0(2^1S_0$ state)

**Figure 4-5:** Several examples of wave functions for some states

# Summary

- By using BS method, calculate the strong decay width of $D_0^*(2400)$(as $0^+(1P)$), $D_J^*(3000)$(as $0^+(2P)$), $D_{sJ}(3040)$ and $D_J(3000)$(as $1^+(2P)$), close to the present experiment results;

- For $D_J(3000)$, $D^*\pi$, $D_2^*(2460)\pi$, and $D_2^*(2600)\pi$ are dominant; For $D_{sJ}(3040)$, $D^*K$, $D^*K^*$, $DK^*$ are dominant. The broad $2P(1^{+'})$ assignment is more possible.

- For $D_J^*(3000)$, $D\pi$, $D\rho$ and $D_1(2420)\pi$ channels contribute much. And try to explain"why the $2P$ state get narrower width than the $1P$ state" with the structure of BS wave functions.

*Thanks !*

*More details* → *Phys. Rev. D 97, 054002 (2018).* & *Eur. Phys. J. C 78, 583 (2018).*

# Backup

# Backup-BS equation

The BS equation of two-body bound state can read in momentum space as

$$S_1^{-1}\chi_P(q)S_2^{-1} = \mathrm{i}\int \frac{\mathrm{d}^4k}{(2\pi)^4} I(P;q,k)\chi_P(k), \tag{6-1}$$

We follow Salpeter to take the instantaneous approximation $I(P;q,k) \approx I(q_\perp - k_\perp)$ The three-dimensional salpeter wave function $\psi(q_\perp)$ is defined by

$$\psi(q_\perp) = \mathrm{i}\int \frac{\mathrm{d}q_P}{2\pi}\chi_P(q), \quad \chi_P(q) = S_1(p_1)\int \frac{\mathrm{d}^3k}{(2\pi)^3} I(q_\perp - k_\perp)\psi_P(k_\perp)S_2(p_2) \tag{6-2}$$

In this work, we adopt the Cornell potential as the interaction kernel $I(r)$ as follow form

$$I(r) = V_s(r) + V_0 + \gamma_0 \otimes \gamma^0 V_v(r) = \frac{\lambda}{\alpha}(1 - \mathrm{e}^{-\alpha r}) + V_0 - \frac{4}{3}\frac{\alpha_s}{r}\mathrm{e}^{-\alpha r}, \tag{6-3}$$

where $\lambda$ is the string constant, $\alpha_s(r)$ is the running strong coupling constant and $V_0$ is an adjustable parameter fixed by the meson's mass. In momentum space, the potential can read as

$$I(\vec{q}) = -\left(\frac{\lambda}{\alpha} + V_0\right)(2\pi)^3\delta^3(\vec{q}) + \frac{\lambda}{\pi^2}\frac{1}{(\vec{q}^2 + \alpha^2)^2} - \frac{2}{3\pi^2}\frac{\alpha_s(\vec{q})}{(\vec{q}^2) + \alpha^2}, \tag{6-4}$$

where the coupling constant $\alpha_s(\vec{q})$ is defined by:

# Mass predictions of $1^+$ state

**Table 6-10:** Mass spectrum of the 2P states in the $D$ and $D_s$ families (in units of MeV).

| State | ours | Ref. [6] | Ref. [7] | Ref. [8] | Ref. [9] |
|---|---|---|---|---|---|
| $D(2^1P_1)$ | 2933 | 2940 | 2932 | 3045 | |
| $D(2^3P_1)$ | 2952 | 2960 | 3021 | 2995 | |
| $D_s(2^1P_1)$ | 3029 | 3040 | 3067 | 3165 | 2959.0 |
| $D_s(2^3P_1)$ | 3036 | 3020 | 3154 | 3114 | 2986.4 |

[6] Phys. Rev. D 84, 034006 (2011)
[7] Eur. Phys. J. C 66, 197 (2010).
[8] Phys. Rev. D 64, 114004 (2001).
[9] Phys. Rev. D 90, 014009 (2014).

# Parameters backup

In this paper, the masses of constituent quarks that we adopt are listed as follows: $m_u = 0.305$ GeV, $m_d = 0.311$ GeV, $m_s = 0.50$ GeV, and $m_c = 1.62$ GeV . Other parameters are $\alpha = 0.060$ GeV, $\lambda = 0.210$ GeV$^2$, $\Lambda_{QCD} = 0.270$ GeV, $f_\pi = 0.1304$ GeV, $f_K = 0.1562$ GeV , $f_{\eta_1} = 1.07 f_\pi$, $f_{\eta_8} = 1.26 f_\pi$, $M_{\eta_1} = 0.923$ GeV, and $M_{\eta_8} = 0.604$ GeV. The masses of other involved mesons are shown in Table 6-11.

**Table 6-11:** Masses of involved mesons (GeV).

| | | | |
|---|---|---|---|
| $m_{D_0^*(2400)^0} = 2.318$ | $m_{D_0^*(2400)^+} = 2.351$ | $m_{D_J^*(3000)^{(0,+)}} = 3.008$ | $m_{D_s^+} = 1.968$ |
| $m_{D_1(2420)^0} = 2.421$ | $m_{D_1(2420)^+} = 2.423$ | $m_{D_1(2430)^{(0,+)}} = 2.427$ | $m_{D_s^{*+}} = 2.112$ |

**Table 6-12:** $D_0^*(2400)^{0,+}$ strong decay widths (MeV). Ref. adopts Chiral Quark Model, Ref. adopts the $^3P_0$ Model and Ref. adopts the Pseudoscalar Emission Model.

| Chanel | | Ours | Ref. zhaoqiang2008 | Ref. Close2005 | Ref. Godfrey2005 | Exp. PDG2016 |
|---|---|---|---|---|---|---|
| $D_0^*(2400)^0 \rightarrow$ | $D^+\pi^-$ | 151.5 | 266 | 283 | 277 | $267 \pm 40$ |
| | $D^0\pi^0$ | 74.8 | | | | |
| $D_0^*(2400)^+ \rightarrow$ | $D^+\pi^0$ | 81.6 | □ | □ | □ | $230 \pm 17$ |
| | $D^0\pi^+$ | 164.3 | | | | |

# Experiment

## Belle Collaboration results

$D_0^*$ life time is short，more experiment through $B^- \to D_0^*(2400)^0\pi^- \to D^+\pi^-\pi^-$ channel. In 2004, the Belle collaboration confirmed $D_0^*$ 和 $D_1'$ in $B$ meson decay：

$$M_{D_0^*(2400)^0} = (2308 \pm 17 \pm 15 \pm 28)\text{MeV}$$

$$\Gamma_{D_0^*(2400)^0} = (276 \pm 21 \pm 18 \pm 60)\text{MeV}$$

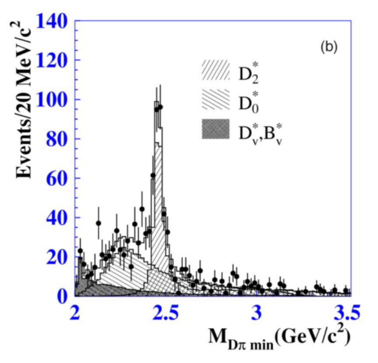



**Figure 6-6:** $D_0^*$2400 fitting graph [a]

[a] Belle Collaboration. Physical Review D, 2004, 69(112002)

# Experiment

## LHCb experiment results

In 2013, the LHCb observed several excited state of $D$ mesons, including several new resonances around 3 GeV. $D_J(3000)$ and $D_J^*(3000)$ was considered to be the $2P$ excited state of $P$−wave $J^P = 1^+$ and $J^P = 0^+$ :

$$M_{D_J(3000)} = (2971.8 \pm 8.7)\text{MeV}$$

$$\Gamma_{D_J(3000)} = (188.1 \pm 44.8)\text{MeV}$$

$$M_{D_J^*(3000)} = (3008.1 \pm 4.0)\text{MeV}$$

$$\Gamma_{D_J^*(3000)} = (110.5 \pm 11.5)\text{MeV}$$

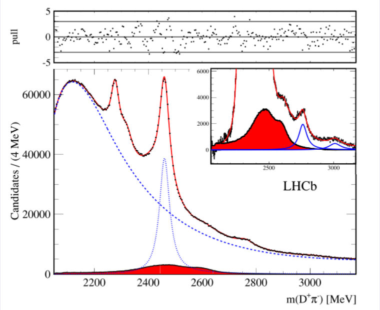



**Figure 6-7:** LHCb fitting graph of $D_0^*3000$[a]

[a] LHCb Collaboration. Journal of High Energy Physics, 2013:145

## Pseudo-vector $D$ meson positive wave function

$$\varphi^{++}_{1^{++}}(q_\perp) = \mathrm{i}\varepsilon_{\mu\nu\alpha\beta}\frac{P''^{\nu}}{M''}q_\perp^{\alpha}\epsilon^{\beta}\gamma^{\mu}\left[C_1(q_\perp) + \frac{\not{P}''}{M''}C_2(q_\perp) + \frac{q_\perp}{M''}C_3(q_\perp) + \frac{\not{P}''\not{q}_\perp}{M''^2}C_4(q_\perp)\right] \quad (6\text{-}6)$$

$$\varphi^{++}_{1^{+-}}(q_\perp) = q_\perp\cdot\epsilon\left[D_1(q_\perp) + \frac{\not{P}''}{M''}D_2(q_\perp) + \frac{\not{q}_\perp}{M''}D_3(q_\perp) + \frac{\not{P}''\not{q}_\perp}{M''^2}D_4(q_\perp)\right]\gamma_5 \quad (6\text{-}7)$$

In the case when heavy-light $1^+$ state is involved, if we use the $S$-$L$ coupling, the $^3P_1$ and $^1P_1$ states cannot describe the physical states. Within the heavy quark limit($m_Q \to \infty$), its spin decouples and the properties of the heavy-light $1^+$ state are determined by those of the light quarks. So $j$-$j$ coupling should be used instead. The orbital angular momentum $\vec{L}$ couples with the light quark spin $\vec{s}_q$, which is $\vec{j}_l = \vec{L} + \vec{s}_q$. Then $1^+$ state can be grouped into a doublet by the total angular momentum of the light quark($|j_l = 1/2\rangle$ and $|j_l = 3/2\rangle$).[a b]:

$$\begin{pmatrix} |J^P = 1^+, j_l = 1/2\rangle \\ |J^P = 1^+, j_l = 3/2\rangle \end{pmatrix} = \begin{pmatrix} -\sin\theta & \cos\theta \\ \cos\theta & \sin\theta \end{pmatrix}\begin{pmatrix} |^1P_1\rangle \\ |^3P_1\rangle \end{pmatrix} \quad (6\text{-}8)$$

Mixing angle(in heavy quark limit):

$$\theta = \arctan\sqrt{1/2} \approx 35.3^\circ$$

[a] Matsuki, Progress of Theoretical Physics 2010,(124.285)

[b] Barnes, Physical Review D 2005, 72(054026)