Investigation of light tetraquark states with $J^{PC} = 0^{+-}$

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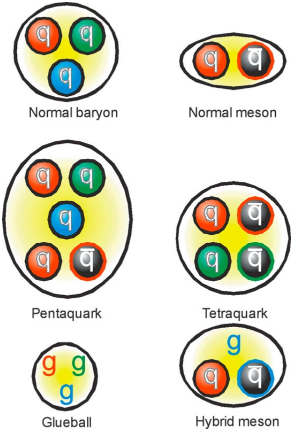
Outline

- Introduction to tetraquark states
- Introduction to QCD Sum Rules
- Our analysis to light tetraquark states with $J^{PC} = 0^{+-}$
- Decay behavior
- Summary

Tetraquark States

- Exotic states can be reached in other configurations: hybrids, tetraquarks ...
- Bound states of diquarks and antidiquarks.



Tetraquark States

- $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ are not allowed in quark model.

For a system made of fermion and antifermion,

$$J = L + S,\ P = (-1)^{L+1},\ C = (-1)^{L+S}$$

L	S	Quantum Number
0	0	0^{-+}
0	1	1^{--}
1	0	1^{+-}
1	1	$0^{++}, 1^{++}, 2^{++}$

- $J^{PC} = 1^{-+}$ light tetraquark states:

H.-X. Chen, A. Hosaka, and S.-L. Zhu, Phys.Rev. D78, 117502 (2008), 0808.2344.

S. Narison, Phys.Lett.B675 (2009) 319-325

- $J^{PC} = 1^{-+}$ hybrid states:

  lightest exotic hybrid, experiments

C. A. Meyer and E. S. Swanson, Prog. Part. Nucl. Phys. 82, 21 (2015), 1502.07276.

Z.-R. Huang, H.-Y, Jin, T. G. Steele, Z.-F. Zhang, Nucl.Part.Phys.Proc

294-296(2018)113-118.

- $J^{PC} = 0^{--}$ light tetraquark states :

C.-K. Jiao, W. Chen, H.-X. Chen, and S.-L. Zhu,Phys. Rev. D79, 114034 (2009), 0905.0774.

Z.-R. Huang, W. Chen, T. G. Steele, Z.-F. Zhang, and H.-Y. Jin, Phys. Rev. D95, 076017 (2017), 1610.02081.

- $J^{PC} = 0^{+-}$ light tetraquark states:

M.-L. Du, W. Chen, X.-L. Chen, and S.-L. Zhu, Chin. Phys. C37, 033104 (2013), 1203.5199.

Authors discussed 0^{+-} light tetraquark states using currents with covariant derivatives, but gave no results due to bad Operator Product Expansion behaviors.

Therefore we construct vector currents without derivatives for 1^{--} quantum number which can also couple to 0^{+-} tetraquark states.

Introduction to QCD Sum Rules

- Two-point correlation function(Quark-Gluon level)

$$\Pi_{\mu\nu}(q) = i\int d^4 e^{iqx}\langle 0|T\{j_\mu(x)j_\nu^\dagger(0)\}|0\rangle$$
$$= -\Pi_1(q^2)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + \Pi_0(q^2)\frac{q_\mu q_\nu}{q^2} ,$$

where $\Pi_0(q^2)$ is the correlator that couples to scalar channel. $\Pi_1(q^2)$ is the correlator that couples to vector channel.

- Dispersion Relation(Hadron level)

$$\Pi(q^2) = \int_0^\infty ds \frac{\frac{1}{\pi} Im\Pi(s)}{s - q^2 - i\epsilon}$$

- Operator Product Expansion(OPE)

$$\Pi(q^2) = \sum_D C_D(q^2)\langle 0|O_D|0\rangle \,,$$

where O_D is a local operator with dimension D.

- Spectral Density

$$\rho(s) = \frac{1}{\pi} Im\Pi(s) = f_0^2 \delta(s - m_0^2) + \ldots$$

Laplace Sum Rules

- Borel Transformation

$$B_{M_B^2}\Pi(q^2) = \lim_{\substack{-q^2,n\to\infty \\ -q^2/n=M_B^2}} \frac{(-q^2)^{n+1}}{n!}(\frac{d}{dq^2})^n\Pi(q^2)\,,$$

where M_B^2 is Borel paremeter, and $M_B^2 = \frac{1}{\tau}$.
Applying borel transformation :

$$f_0^2 e^{-m_0^2\tau} = \int_0^{s_0} e^{-s\tau} Im\Pi(s)ds \to LSR\ moment$$

- Mass Prediction for LSR

$$m_{0(LSR)}^2 = -\frac{d}{d\tau}\ln(\int_0^{s_0} e^{-s\tau}\rho(s)ds)$$

Finite Energy Sum Rules

- FESR Moment(local duality)

$$\int_0^{s_0} s^n \rho(s) ds$$

- Mass Prediction for FESR

$$m^2_{0(FESR)} = \frac{\int_0^{s_0} s^{n+1}\rho(s)ds}{\int_0^{s_0} s^n \rho(s)ds}$$

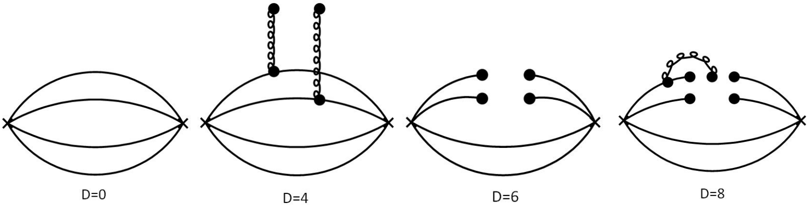
Our analyses to light tetraquark states with $J^{PC} = 0^{+-}$

We first give 8 interpolating vector currents which can couple to 1^{--} and 0^{+-} tetraquark states. We also investigate both $ud\bar{u}\bar{d}$ and $us\bar{u}\bar{s}$ systems.

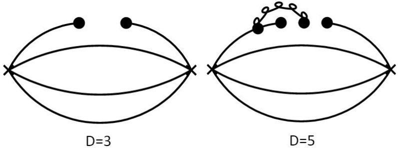
$$
\begin{aligned}
J_{1\mu} &= u_a^T C\gamma_5 d_b(\bar{u}_a\gamma_\mu\gamma_5 C\bar{d}_b^T + \bar{u}_b\gamma_\mu\gamma_5 C\bar{d}_a^T) - u_a^T C\gamma_\mu\gamma_5 d_b(\bar{u}_a\gamma_5 C\bar{d}_b^T + \bar{u}_b\gamma_5 C\bar{d}_a^T)\,,\\
J_{2\mu} &= u_a^T C\gamma^\nu d_b(\bar{u}_a\sigma_{\mu\nu} C\bar{d}_b^T - \bar{u}_b\sigma_{\mu\nu} C\bar{d}_a^T) - u_a^T C\sigma_{\mu\nu} d_b(\bar{u}_a\gamma^\nu C\bar{d}_b^T - \bar{u}_b\gamma^\nu C\bar{d}_a^T)\,,\\
J_{3\mu} &= u_a^T C\gamma_5 d_b(\bar{u}_a\gamma_\mu\gamma_5 C\bar{d}_b^T - \bar{u}_b\gamma_\mu\gamma_5 C\bar{d}_a^T) - u_a^T C\gamma_\mu\gamma_5 d_b(\bar{u}_a\gamma_5 C\bar{d}_b^T - \bar{u}_b\gamma_5 C\bar{d}_a^T)\,,\\
J_{4\mu} &= u_a^T C\gamma^\nu d_b(\bar{u}_a\sigma_{\mu\nu} C\bar{d}_b^T + \bar{u}_b\sigma_{\mu\nu} C\bar{d}_a^T) - u_a^T C\sigma_{\mu\nu} d_b(\bar{u}_a\gamma^\nu C\bar{d}_b^T + \bar{u}_b\gamma^\nu C\bar{d}_a^T)\,,\\
J_{5\mu} &= u_a^T C d_b(\bar{u}_a\gamma_\mu C\bar{d}_b^T + \bar{u}_b\gamma_\mu C\bar{d}_a^T) - u_a^T C\gamma_\mu d_b(\bar{u}_a C\bar{d}_b^T + \bar{u}_b C\bar{d}_a^T)\,,\\
J_{6\mu} &= u_a^T C\gamma^\nu\gamma_5 d_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5 C\bar{d}_b^T + \bar{u}_b\sigma_{\mu\nu}\gamma_5 C\bar{d}_a^T) - u_a^T C\sigma_{\mu\nu}\gamma_5 d_b(\bar{u}_a\gamma^\nu C\bar{d}_b^T\\
&\quad + \bar{u}_b\gamma^\nu C\bar{d}_a^T)\,,\\
J_{7\mu} &= u_a^T C d_b(\bar{u}_a\gamma_\mu C\bar{d}_b^T - \bar{u}_b\gamma_\mu C\bar{d}_a^T) - u_a^T C\gamma_\mu d_b(\bar{u}_a C\bar{d}_b^T - \bar{u}_b C\bar{d}_a^T)\,,\\
J_{8\mu} &= u_a^T C\gamma^\nu\gamma_5 d_b(\bar{u}_a\sigma_{\mu\nu}\gamma_5 C\bar{d}_b^T - \bar{u}_b\sigma_{\mu\nu}\gamma_5 C\bar{d}_a^T) - u_a^T C\sigma_{\mu\nu}\gamma_5 d_b(\bar{u}_a\gamma^\nu\gamma_5 C\bar{d}_b^T\\
&\quad - \bar{u}_b\gamma^\nu\gamma_5 C\bar{d}_a^T)\,.
\end{aligned}
$$

- Take $ud\bar{u}\bar{d}$ system as an example

After analyses of nonperturbative condensates through OPE method, only those with even dimensions survive, such as the following diagrams that we calculate:



But for diagrams with odd dimensional condensates, they vanish due to the chiral limit we take: $m_u = m_d = 0$



- For $ud\bar{u}\bar{d}$ system, we give the LSR moment results after calculation:

$$
\begin{aligned}
M_0^{ud\bar{u}\bar{d}}(\tau, s_0) &= \int_0^{s_0} \rho_0^{ud\bar{u}\bar{d}}(s) e^{-\tau s_0} ds \\
&= a_i \frac{1}{\pi^6} \frac{e^{-s_0\tau}\{-s_0\tau[s_0\tau(s_0\tau+3)+6]-6\}+6}{\tau^4} \\
&+ b_i \frac{\langle \alpha_s G^2 \rangle}{\pi^5} \frac{1-e^{-s_0\tau}(s_0\tau+1)}{\tau^2} - c_i \frac{\langle \bar{q}q \rangle^2}{\pi^2} \frac{1-e^{-s_0\tau}}{\tau} \\
&+ d_i \frac{\langle \bar{q}Gq \rangle \langle \bar{q}q \rangle}{\pi^2} [2\gamma_E - \ln(\pi) - 2\ln(2) - \ln(\frac{1}{\tau}) + \Gamma(0, s_0\tau)],
\end{aligned}
$$

	i							
	1	2	3	4	5	6	7	8
a_i	1/30720	1/20480	1/61440	1/10240	1/30720	1/10240	1/61440	1/20480
b_i	-1/1536	1/1536	1/1536	11/1536	-1/1536	11/1536	1/1536	1/1536
c_i	1/6	1/4	1/12	1/2	1/6	1/2	1/12	1/4
d_i	1/24	1/16	1/48	1/8	1/24	1/8	1/48	1/16

- For $us\bar{u}\bar{s}$ system, we give the LSR moment results after calculation for J_1 as an example:

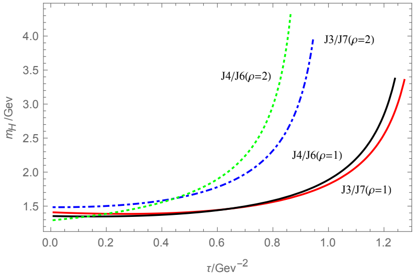
$$M_0^{us\bar{u}\bar{s}}(\tau, s_0) = \int_0^{s_0} \rho_0^{us\bar{u}\bar{s}}(s) e^{-\tau s_0} ds = -a_i' \frac{e^{-s_0\tau}\{-s_0\tau[s_0\tau(s_0\tau+3)+6]-6\}+6}{\tau^4}$$
$$- b_i' \frac{e^{-s_0\tau}[-s_0\tau(s_0\tau+2)-2]+2}{\tau^3} - c_i' \frac{1-e^{-s_0\tau}(s_0\tau+1)}{\tau^2}$$
$$- d_i' \frac{1-e^{-s_0\tau}}{\tau} - e_i' + f_i'[\gamma_E - \ln(\frac{1}{\tau}) + \Gamma(0, s_0\tau)].$$

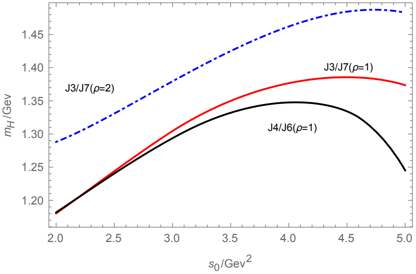
$$a_1' = -\frac{1}{30720\pi^6}, b_1' = \frac{5m_s^2}{1536\pi^6}, c_1' = \frac{\langle\alpha_s G^2\rangle}{1536\pi^5} - \frac{m_s^4}{256\pi^6} - \frac{m_s\langle\bar{s}s\rangle}{32\pi^4}, d_1' = -\frac{\langle\alpha_s G^2\rangle m_s^2}{384\pi^5} - \frac{m_s\langle\bar{s}Gs\rangle}{32\pi^4} + \frac{\langle\bar{q}q\rangle^2}{12\pi^2} + \frac{\langle\bar{s}s\rangle^2}{12\pi^2},$$
$$e_1' = -\frac{m_s^2\langle\bar{q}q\rangle^2}{2\pi^2} + \frac{m_s^2\langle\bar{s}s\rangle^2}{24\pi^2} - \frac{\langle\alpha_s G^2\rangle m_s\langle\bar{q}q\rangle}{64\pi^3} + \frac{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}{24\pi^2} + \frac{\langle\bar{s}Gs\rangle\langle\bar{s}s\rangle}{24\pi^2},$$
$$f_1' = -\frac{m_s^2\langle\bar{q}q\rangle^2}{3\pi^2} - \frac{\gamma_E m_s^2\langle\bar{q}q\rangle^2}{6\pi^2} + \frac{m_s^2\langle\bar{q}q\rangle^2\ln(2)}{3\pi^2} + \frac{m_s^2\langle\bar{q}q\rangle^2\ln(\pi)}{6\pi^2} - \frac{m_s^2\langle\bar{s}s\rangle^2}{12\pi^2} - \frac{\gamma_E m_s^2\langle\bar{s}s\rangle^2}{24\pi^2} + \frac{m_s^2\langle\bar{s}s\rangle^2\ln(2)}{12\pi^2}$$
$$+ \frac{m_s^2\langle\bar{s}s\rangle^2\ln(\pi)}{24\pi^2} - \frac{\langle\alpha_s G^2\rangle m_s\langle\bar{q}q\rangle}{288\pi^3} - \frac{\gamma_E\langle\alpha_s G^2\rangle m_s\langle\bar{q}q\rangle}{576\pi^3} + \frac{\langle\alpha_s G^2\rangle m_s\langle\bar{q}q\rangle\ln(2)}{288\pi^3} + \frac{\langle\alpha_s G^2\rangle m_s\langle\bar{q}q\rangle\ln(\pi)}{576\pi^3}$$
$$+ \frac{\langle\alpha_s G^2\rangle m_s\langle\bar{s}s\rangle}{144\pi^3} + \frac{\gamma_E\langle\alpha_s G^2\rangle m_s\langle\bar{s}s\rangle}{288\pi^3} - \frac{\langle\alpha_s G^2\rangle m_s\langle\bar{s}s\rangle\ln(2)}{144\pi^3} - \frac{\langle\alpha_s G^2\rangle m_s\langle\bar{s}s\rangle\ln(\pi)}{288\pi^3} + \frac{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}{12\pi^2}$$
$$+ \frac{\gamma_E\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}{24\pi^2} - \frac{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle\ln(2)}{12\pi^2} - \frac{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle\ln(\pi)}{24\pi^2} + \frac{\langle\bar{s}Gs\rangle\langle\bar{s}s\rangle}{12\pi^2} + \frac{\gamma_E\langle\bar{s}Gs\rangle\langle\bar{s}s\rangle}{24\pi^2} - \frac{\langle\bar{s}Gs\rangle\langle\bar{s}s\rangle\ln(2)}{12\pi^2}$$
$$- \frac{\langle\bar{s}Gs\rangle\langle\bar{s}s\rangle\ln(\pi)}{24\pi^2}$$

Our analyses to light tetraquark states with $J^{PC} = 0^{+-}$

Analysis Principles for LSR:

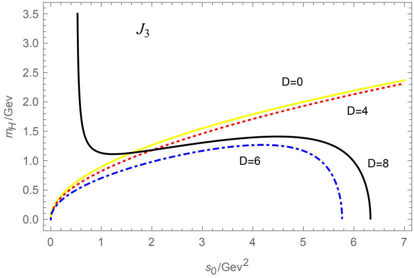
- Dimension 8 contributions $<$ 10% overall contributions
- Convergence behaviors for all dimensional OPE terms
- τ and s_0 stabilities are required:

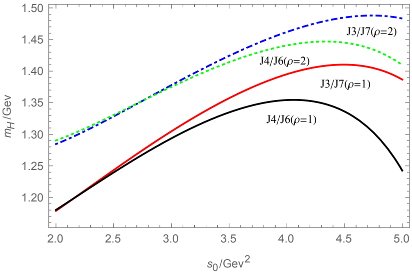




Analysis Principles for FESR:

- Convergence shown from $m_H - s_0$ curves when adding higher dimension OPE terms
- s_0 stability is required





Our analyses to light tetraquark states with $J^{PC} = 0^{+-}$

- Mass prediction for $ud\bar{u}\bar{d}$ system:

$$m_0^{ud\bar{u}\bar{d}} = 1.43 \pm 0.09 \;\; GeV$$

- Mass prediction for $us\bar{u}\bar{s}$ system:

$$m_0^{us\bar{u}\bar{s}} = 1.54 \pm 0.12 \;\; GeV$$

Decay Behavior

Due to the limitation of I , G , J , P , C , we can give the following possible decay patterns:

- $ud\bar{u}\bar{d}(0^-0^{+-}) \to \pi^0 b_1\,, \omega\sigma$ (p-wave)
- $ud\bar{u}\bar{d}(1^+0^{+-}) \to h_1\pi^0\,, \rho\sigma$ (p-wave)
- $ud\bar{u}\bar{d}(2^-0^{+-}) \to h_1\pi^0$ (p-wave)

There is no any s-wave decay channel allowed for $ud\bar{u}\bar{d}$.

- The strong decays are totally forbidden for neutral $us\bar{u}\bar{s}$ tetraquark states with $I = 0, 1$.
- Weakly decay: $us\bar{u}\bar{s} \to K\pi\pi$,...
- Electromagnetic decay: $us\bar{u}\bar{s} \to K^*K\gamma$,...

Summary

- Consider all the possible colored diquark-antidiquark vector currents.

- Reliable LSR and FESR analyses with standard stability criterion.

- Mass predictions:

  $m_0^{ud\bar{u}\bar{d}} = 1.43 \pm 0.09 Gev$ and $m_0^{us\bar{u}\bar{s}} = 1.54 \pm 0.12 Gev$.

- Roughly give decay patterns