$Z \to \pi\pi$, KK, a touchstone of PQCD

Shan Cheng

HNU 湖南大学

In collaboration with Qin Qin, arXiv:1810.10524

2018 年 10 月 26 日 @ HAUT，全国第十六届重味物理与 CP 破坏研讨会 (HFCPV-2018)

Overview

Motivation

† Tera-Z factory like CEPC and/or FCC-ee,
Higgs and Z properties, as well as QCD.

† Wave function (momentum distribution)
and further for form factor (redistribution).

† the controversy in small-x region,
k_T factorization and PQCD approach *VS* other approaches,
Resummed to a Sudakov factor *VS* leading power of $1/m_b$.

† Why $\pi\pi$, KK channel ? flavour conserving, tree level.

A touchstone of PQCD approach

† Form factor can be calculated in three different ways,
PQCD is the most powerful.

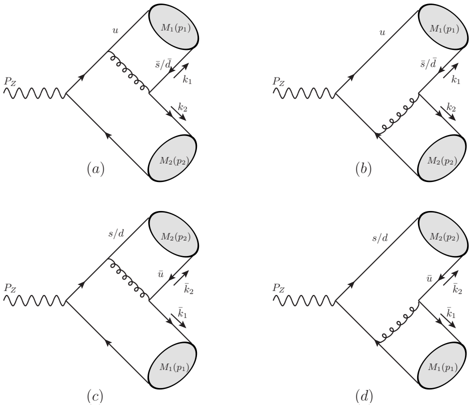
† In Z decay, energetic final states, leading power is enough,
a clean channel with the total uncertainty $\in 10\%$.

Decay width of $Z \to \pi\pi$

- $J_\mu^{(Z)} = \frac{g}{2\cos\theta_w} \sum_q \left[(T_q - 2Q_q \sin^2\theta_w) \bar{q}\gamma_\mu q - T_q \bar{q}\gamma_\mu\gamma_5 q \right]$,
  gauge coupling, weak angle, hypercharge, electrocharge.
- Decay amplitude $i\mathcal{M}(Z \to \pi^+\pi^-) = \epsilon_Z^\mu \langle \pi^+\pi^- | J_\mu^{(Z)} | 0 \rangle$,
  Form factor $\langle \pi^+\pi^- | \bar{u}\gamma^\mu u | 0 \rangle = (p_1^\mu - p_2^\mu)\mathcal{G}(Q^2)$.
- Partial decay width

  $$\Gamma(Z \to \pi^+\pi^-) = \frac{1}{48\pi M_Z} \sum_s |\mathcal{M}|^2 = \frac{M_Z}{48\pi} (g_V^u - g_V^d)^2 \left| \mathcal{G}(M_Z^2) \right|^2.$$
- $Z\bar{q}q$ coupling $g_V^q = g/(2\cos\theta_w) \times (T_q - 2Q_q \sin^2\theta_w)$.
- The mass effect have been neglected.
- $Z \to \pi^0\pi^0$, $\bar{K}^0 K^0$ are forbidden.

Kinematic



- $p_Z = \frac{m_Z}{\sqrt{2}}(1, 1, 0)$, $p_1 = \frac{m_Z}{\sqrt{2}}(1, 0, 0)$, $p_2 = \frac{m_Z}{\sqrt{2}}(0, 1, 0)$,
- $k_1 = (x_1 \frac{m_Z}{\sqrt{2}}, 0, \mathrm{k}_{1\mathrm{T}})$, $k_2 = (0, x_2 \frac{m_Z}{\sqrt{2}}, \mathrm{k}_{2\mathrm{T}})$, $\bar{k}_1 = p_1 - k_1$, $\bar{k}_2 = p_2 - k_2$,
- $\alpha_1 \equiv x_2 Q^2$, $\alpha_2 \equiv x_1 Q^2$, $\beta \equiv x_1 x_2 Q^2$.

Form factor at LO

$$\mathcal{G}_{\rm II}(q^2)_{\rm LO} = -16\pi C_F Q^2 \int_0^1 dx_1 dx_2 \int db_1 db_2 b_1 b_2\, \alpha_s(\mu) x_2 \phi_\pi(x_1)\phi_\pi(x_2) \times h_{\rm II}(x_1, b_1, x_2, b_2)\,{\rm Exp}[-S_{\rm II}(x_1, b_1, x_2, b_2, \mu)]\,, \quad (1)$$

- $\mu = \mathrm{Max}[\sqrt{\alpha_i}, 1/b_i]$, $\phi(x) = f_\pi 6x(1-x)/(2\sqrt{2N_C})$.
- Sudokov factor S_{II}, double and single log in the vertex correction + single log in the quark self correction. [J. Botts, eta, 1989 ; H.N. Li, eta, 1992]
- Hard kernel h_{II}:

$$\int \frac{k_{1T}^2}{(2\pi)^2}\frac{k_{2T}^2}{(2\pi)^2}\frac{1}{[\beta - (k_T^2) + i\epsilon]}\frac{1}{[\alpha_1 - k_{1T}^2 + i\epsilon]} \longrightarrow \int db_1 db_2 b_1 b_2\, h_{\rm II}(x_i, b_i)$$

$$h_{\rm II}(x_i, b_i) = K_0(i\sqrt{\beta}b_2)\left[K_0(i\sqrt{\alpha_1}b_1)J_0(i\sqrt{\alpha_1 b_2})\Theta(b_1 - b_2) + \{b_1 \leftrightarrow b_2\}\right] \quad (2)$$

† Bessel function of the first kind and the modified Bessel function.
† The integral in Eq (1) holds well in the moderate region, $1 - 50\,\mathrm{GeV}^2$.
† Oscillates violently when Q^2 goes higher.
† No physical meaning, large hierarchy between Q^2 and k_T^2.

The hard function is not "hard"

- The integral of the hard function in Eq (1) is not HARD.
- How to evade the oscillation ?
- Hierarchy ansatz $x_i Q^2 \gg x_1 x_2 Q^2 \gg k_T^2$ in PQCD @ NLO.
  - † Drop the sub-leading term in the quark propagator, retain it in the gluon, when Q^2 is enough large.
  - † Form factor is mainly determined by the hard gluon [H.C. Hu, etc, 2013 ; S. Cheng, etc, 2015].
  - † The hard function reduces to

$$\frac{1}{\alpha_1} \int \frac{k_T^2}{(2\pi)^2} \frac{1}{[\beta - (k_T^2) + i\epsilon]} \longrightarrow \frac{1}{\alpha_1} \int dbb\, h_{\rm I}(x_i, b_i)$$

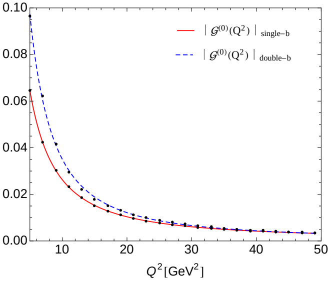
$$h_{\rm I}(x_i, b) = 1/\alpha_1 \times K_0(i\sqrt{\beta} b_2) \tag{2}$$

- The modified form factor at LO, $b = b_1 = b_2$,

$$\mathcal{G}_{\rm I}(q^2)_{\rm LO} = 16\pi C_F \int_0^1 dx_1 dx_2 \int dbb\, \alpha_s(\mu) \phi_\pi(x_1) \phi_\pi(x_2)\, K_0(i\sqrt{\beta} b_2)\, {\rm Exp}[-S_{\rm II}(x_i, b, \mu)]\,, \tag{3}$$

- Extend the valid Q^2 from dozens- to thousands GeV^2.

Form fator at LO



- We suggest a parameterization, reciprocal of square polynomial

$$Abs[\mathcal{G}(Q^2)] = \frac{A + Q^2 B}{Q^4 + Q^2 C + A}, \quad [A.Khodjamirian, 1999] \tag{4}$$

† @ LO, $A^{(0)} = 0.0879\,, \quad B^{(0)} = 46.1\,, \quad C^{(0)} = 10.9\,.$

The NLO correction

- The NLO hard correction is avalible for single-b convoluted formula

$$\mathcal{G}_{\mathrm{I}}(q^2)_{\mathrm{NLO}} = \quad 16\pi C_F \int_0^1 dx_1 dx_2 \int dbb\alpha_s(\mu)\phi_\pi(x_1)\phi_\pi(x_2)\mathrm{Exp}[-S_{\mathrm{II}}(x_i, b, \mu)] \,, \\ \times \frac{\alpha_s C_F}{4\pi}\left[\tilde{h}(x_i, b, Q, \mu)K_0(\sqrt{i\beta}b) + \frac{i\pi}{2} H_0^{(1)\prime\prime}(i\sqrt{\beta}b)\right] \,, \tag{5}$$

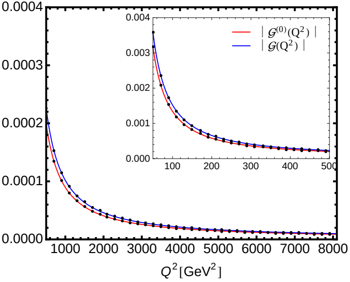
† $\tilde{h}(x_i, b, Q)$ in [H.C. Hu, etc, 2013]

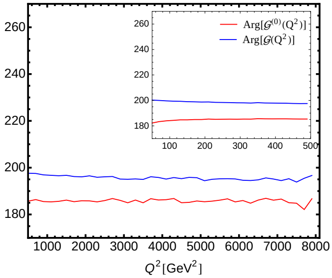
† For $H_0^{(1)\prime\prime}(x) \equiv \left[d^2 H_\alpha^{(1)}(x)/d^2\alpha\right]_{\alpha=0}$, we parameterize,

$$\begin{aligned} & \mathrm{Re}[H_0^{(1)\prime\prime}(x)] \\ = \quad & \mathrm{Which}[\ x \geqslant 10, \\ & \frac{\alpha_s C_F}{4\pi}\Big[0.798 + 0.454x - 0.0603x^2 + 0.00590x^3 - 0.00021x^4 - 1.35\log x \\ & \quad + J_0(x)\left(-0.581 + 1.48\log x - 0.497\log^2 x\right) \\ & \quad + Y_0(x)\left(-3.62 - 0.194x + 0.665\log x + 0.331\log^2 x\right)\Big] \,, \\ & x < 10, \\ & \cdots\ ] \,, \end{aligned} \tag{6}$$

† For the small and large argument, reproduce the result obtained by the asymptotic expansion of Hankel function.

NLO correction





- The NLO correction mostly comes from $\tilde{h}$ term,
  $\sim 11\%$ for Abs in the whole energy region,
  $< 8\%$ for Arg deviated from the LO π.
- @ NLO, Eq (4), $A^{(1)} = 0.0996\,,\;\; B^{(1)} = 48.2\,,\;\; C^{(1)} = 12.6\,.$

Branching ratio of $Z \to \pi\pi$

- $\sin^2\theta_W(\overline{\mathrm{MS}}) = 0.231\,,\;\; \alpha_s(m_Z^2) = 0.1182\,,\;\; \alpha(m_Z)^{-1} = 127.950\,.$
- Eqs (3, 5)

$$\begin{aligned}\mathcal{G}(M_Z^2)\times 10^6 &= (-8.29 - i0.771)|_{LO} \\ &+(-0.975 - i1.59)|_{NLO1} \\ &+(0.211 + i0.00760)|_{NLO2}\,,\end{aligned} \tag{7}$$

- Branching ratios
  $\mathcal{B}(Z \to \pi^+\pi^-) = 0.83 \pm 0.02 \pm 0.02 \pm 0.04 \times 10^{-12}$,
  $\mathcal{B}(Z \to K^+K^-) = 1.74 \pm 0.04 \pm 0.04 \pm 0.02 \times 10^{-12}$,
  Quite hopeful to be measured at a Tera-Z factory.
- A good touchstone of PQCD.

Outlook

- $Z \to B_{(c)} B_{(c)}, D_{(s)} D_{(s)}$
  - † Factorization (m_b and/or m_c to λ_{QCD}),
  - † Wave function of fast moving $B_{(c)}$ (spin effect, HQET ?).
  - † Sudakov factor of B_c (quark mass effect).
- FCNC, $Z \to BK$ (penguin and new loop effect, mass effect).

The End, Thank you.