Matching the Quasi Meson Distribution Amplitude in RI/MOM scheme

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OUTLINE

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3. One loop matching coefficient

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Introduction

The study of heavy flavor physics is important and active!

1. Finding new physics.
2. Testing QCD and factorizations.
3. CP violation.

《Rare B decays at LHCb》李一鸣
《CPV in B decays at LHCb》谢跃红
《Theoretical Overview of B Physics》李新强

《Recent Belle & Belle-II Results》沈成平
《BESIII Recent Results》王大勇
《Highlights of theories in charm physics》于福升

Introduction

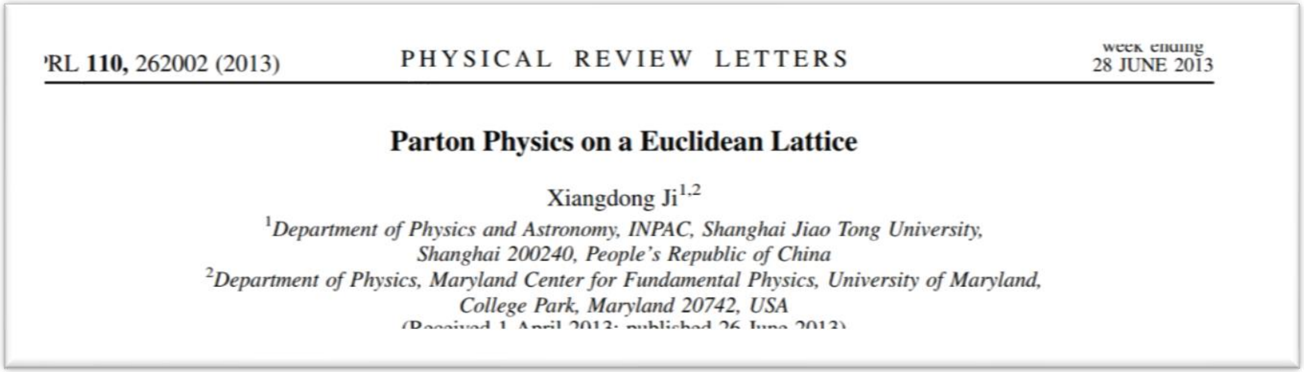
$$\mathcal{M}[H] = \int_0^1 dx T_H(Q, \mu; x) \phi_H(x; \mu)$$

- The LCDAs are essential for the studying of exclusive processes and hadron structures.

- LCDAs cannot be evaluated in perturbation theory.

- Lattice QCD can be utilized to calculate only the lowest moments of LCDAs.

LaMET



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Parton Physics on a Euclidean Lattice

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Large-momentum effective field theory: LaMET

LaMET is a theory allowing ab initio computation of light-cone physics on a Euclidean lattice!

Step 1: Constructing lattice operators and evaluate the ME

Step 2: Lattice calculations

Step 3: Extracting the light-cone physics from the lattice ME

LaMET

- Calculate the equal-time correlators (quasi quantities) instead of the light-cone ones.

- The matrix elements defined by these equal-time correlators can be simulated on the lattice.

- The quasi observables can be factorized as the convolution of a matching coefficient and the corresponding light-cone observable.

LaMET

1. Quark PDFs.

X.Xiong, X. Ji, J.-H. Zhang, Y. Zhao Phys.Rev.D 2014
Y.-Q. Ma and J.-W. Qiu 2014
……

2. Transverse momentum dependent (TMD) PDFs.

X. Ji, P.Sun, X. Xiong and F. Yuan, Pys. Rev. D 2015
X.Ji,L.-C. Jin, F.Yuan, J.-H. Zhang, Y. Zhao Phys.Rev.D 2015
……

3. Generalized parton distributions (GPDs).

X. Ji, A. Schafer, X. Xiong and J.-H. Zhang Phys. Rev. D 2015
X. Xiong and J.-H. Zhang Phys. Rev. D 2015
……

4. Light-cone distribution amplitudes (LCDAs).

J.-H. Zhang, J.-W. C, X. Ji, Lu. J, H.-W. L Phys.Rev. D 2017
J. Xu, Q.-A. Zhang and S. Zhao, Phys. Rev. D 2018
……

5. Gluon PDFs.

W. Wang, S. Zhao and R.Zhu, Eur.Phys.J. C 2018
W. Wang and S. Zhao, JHEP 2018
……

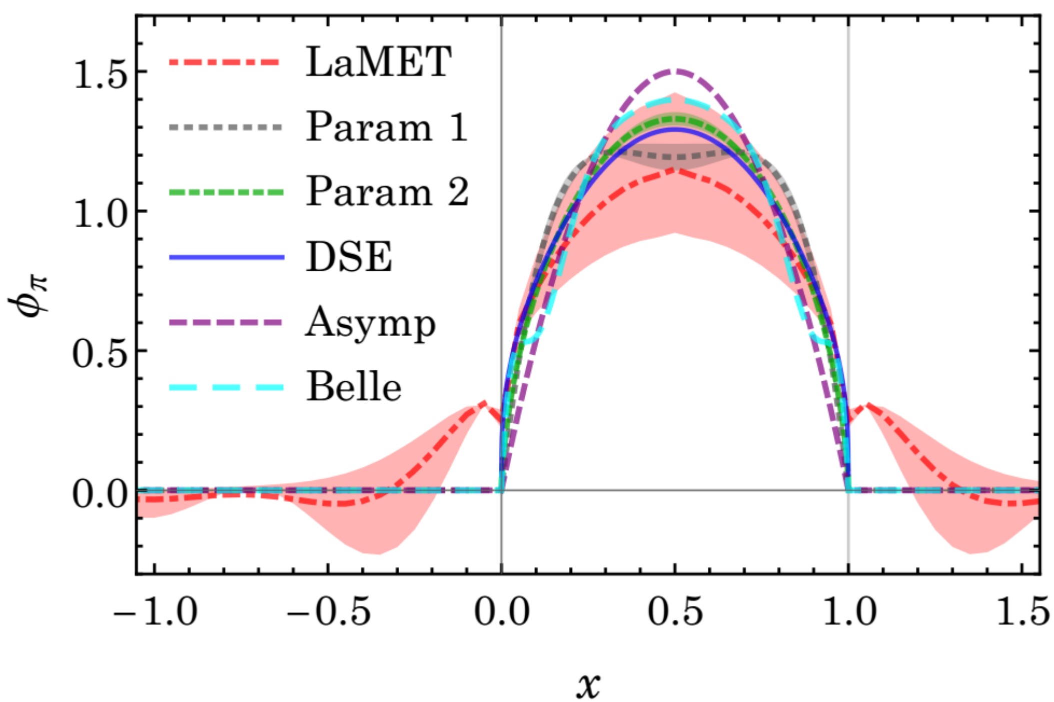
LCDA

Lattice calculation → Quasi-DA → Normal-LCDA

LQCD　　　　Matching

The matching coefficients have been calculated in dimensional regularization and transverse momentum cutoff schemes.

J.-H. Zhang, J.-W. C, X. Ji, Lu. J, H.-W. L Phys.Rev. D 2017



LCDA

These schemes are not suitable for a nonperturbative renormalization of the quasi-PDF on the lattice.

The RI/MOM (regularization independent momentum subtraction) scheme was proposed to serve this purpose.

The perturbative matching coefficient that converts quasi-DA in the RI/MOM scheme to LCDA in $\overline{MS}$ scheme is still not available yet.

Distribution amplitudes

The LCDAs are defined by the matrix elements of non-local gauge invariant quark bilinear operators, in which the two fermion fields are separated in the n direction.

$$\mathcal{O}_V^\Gamma(\xi^-) = \bar{\psi}(\xi^-)\Gamma W(\xi^-, 0)\psi(0),$$

Fourier transformation

$$O_V^\Gamma(x) = \int \frac{d\xi^-}{2\pi} e^{-ix\xi^- P^+} \mathcal{O}_V^\Gamma(\xi^-),$$

Take longitudinally polarized vector for instance,

$$f_V \frac{m_V}{P^+} \epsilon^{*+} \phi_V^{\parallel}(x, \mu) = \langle V, P, \epsilon^* | O_V^{\parallel}(x) | 0 \rangle,$$

$$f_V \frac{m_V}{P^+} \epsilon^{*+} = \langle V, P, \epsilon^* | \mathcal{O}_V^{\parallel}(0) | 0 \rangle$$

So,

$$\phi_V^\Gamma(x, \mu) = \frac{\langle V, P, \epsilon^* | O_V^\Gamma(x) | 0 \rangle}{\langle V, P, \epsilon^* | \mathcal{O}_V^\Gamma(0) | 0 \rangle}.$$

Distribution amplitudes

Similarly, for Quasi-DAs

$$\widetilde{\mathcal{O}}^{\Gamma}(z) = \bar{\psi}(z)\Gamma W(z,0)\psi(0),$$

$$f_V \epsilon_z^* \frac{m_V}{P_z} \widetilde{\phi}_V^{\parallel}(x, P_z) = \langle V, P, \epsilon^* | \widetilde{O}_V^{\parallel}(x) | 0 \rangle,$$

$$f_V \epsilon_z^* \frac{m_V}{P_z} = \langle V, P, \epsilon^* | \widetilde{\mathcal{O}}_V^{\parallel}(0) | 0 \rangle$$

So, we have

$$\widetilde{\phi}_V^{\Gamma}(x, \mu) = \frac{\langle V, P, \epsilon^* | \widetilde{\mathcal{O}}_V^{\Gamma}(x) | 0 \rangle}{\langle V, P, \epsilon^* | \widetilde{\mathcal{O}}_V^{\Gamma}(0) | 0 \rangle}.$$

The factorization formula,

$$\begin{aligned}
&\widetilde{\phi}_R(\Gamma, x, P^z, \mu_R, p_R^z) \\
&= \int_0^1 dy\, C_\Gamma \left(x, y, r, \frac{P^z}{\mu}, \frac{P^z}{p_R^z} \right) \phi(\Gamma, y, \mu) \\
&\quad + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(P^z)^2} \right),
\end{aligned}$$

Renormalization

The RI/MOM renormalization factor Z is calculated nonperturbatively on the lattice by imposing the condition that the renormalized loop corrections in the matrix element of the spatial correlator in an off-shell quark state vanish at subtraction $\{\widetilde{\mu}\}$ scales,

$$Z(\Gamma, z, a, \{\widetilde{\mu}\}) = \left. \frac{\langle p'|\widetilde{O}(\Gamma, z, a)|p''\rangle}{\langle p'|\widetilde{O}(\Gamma, z, a)|p''\rangle_{\text{tree}}} \right|_{\{\widetilde{\mu}\}}$$

where $\{\widetilde{\mu}\} = \{p^2 = -\mu_R^2, p^z = p_R^z\}$.

The UV divergence of the quasi-DA depends on the operator itself, not the external states.

We have the freedom to choose external states as long as the Z calculated from lattice can remove the UV divergent part of the bare matrix element.

One simple choice: renormalization factor for the quasi-PDF.

$$Z_s(\Gamma, z, a, \mu_R, p_R^z) = \left. \frac{\langle p|\widetilde{O}(\Gamma, z, a)|p\rangle}{\langle p|\widetilde{O}(\Gamma, z, a)|p\rangle_{\text{tree}}} \right|_{\{\widetilde{\mu}\}}$$

Renormalization

The bare correlator for the meson on the lattice,

$$\widetilde{h}(\Gamma, z, P^z, a) = \langle P, \epsilon | \widetilde{O}(\Gamma, z, a) | 0 \rangle$$

which is renormalized as

$$\begin{aligned} &\widetilde{h}_R(\Gamma, z, P^z, \mu_R, p^z_R) \\ &= \lim_{a \to 0} Z_s^{-1}(\Gamma, z, a, \mu_R, p^z_R) \widetilde{h}(\Gamma, z, P^z, a) \,, \end{aligned}$$

The renormalized quasi-DA,

$$\begin{aligned} &\widetilde{\phi}_R(\Gamma, x, P^z, \mu_R, p^z_R) \\ =&P^z \int \frac{dz}{2\pi} e^{ixzP^z} \frac{\widetilde{h}_R(\Gamma, z, P^z, \mu_R, p^z_R)}{\widetilde{h}_R(\Gamma, 0, \mu_R)} \,. \end{aligned}$$

Similarly, the renormalized LCDA,

$$\begin{aligned} &\phi_R(\Gamma, y, \mu) \\ =&P^+ \int \frac{d\xi^-}{2\pi} e^{-iy\xi^- P^+} \frac{h_R(\Gamma, \xi^-, \mu)}{h_R(\Gamma, 0, \mu)} \,. \end{aligned}$$

One loop matching coefficient

The renormalized quasi-DA in the RI/MOM scheme can be matched to LCDA through the factorization formula,

$$\begin{aligned}&\widetilde{\phi}_R(\Gamma, x, P^z, \mu_R, p_R^z)\\&= \int_0^1 dy\, C_\Gamma\left(x, y, r, \frac{P^z}{\mu}, \frac{P^z}{p_R^z}\right)\phi(\Gamma, y, \mu)\\&\quad + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(P^z)^2}\right),\end{aligned}$$

$$C_\Gamma(x, y)\,|_{tree} = \delta(x - y)$$

where $r = \mu_R^2/(p_R^z)^2$.

The bare matching coefficient

$$C_B^{(1)}\left(\Gamma, x, y, \frac{P^z}{\mu}\right) = \widetilde{\phi}_B^{(1)}(\Gamma, x, y, P^z) - \phi^{(1)}(\Gamma, x, y, \mu)$$

One loop matching coefficient

We have calculated $\Gamma = \gamma^+\gamma_5,\ \gamma^+,\ \gamma^+\gamma_\perp$ for pseudoscalar, longitudinally polarized vector and transversely polarized vector meson LCDAs;
$\Gamma = \gamma^z\gamma_5,\ \gamma^t,\ \gamma^z\gamma_\perp$ for pseudoscalar, longitudinally polarized vector and transversely polarized vector meson quasi-DAs, respectively.

Since we take the on-shell limit to obtain the bare matching coefficient, $C_B^{(1)}$.

$$C_B^{(1)}\left(\Gamma, x, y, \frac{P_z}{\mu}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases} [H_1(\Gamma,x,y)]_{+(y)} & x<0<y \\ [H_2(\Gamma,x,y,P^z/\mu)]_{+(y)} & 0<x<y \\ [H_2(\Gamma,1-x,1-y,P^z/\mu)]_{+(y)} & y<x<1 \\ [H_1(\Gamma,1-x,1-y)]_{+(y)} & y<1<x \end{cases}$$

where

$$H_1(\Gamma,x,y) = \begin{cases} \frac{1+x-y}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x} + \frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{y-x}{-x} & \Gamma=\gamma^z\gamma_5 \text{ and } \gamma^t \\ \frac{1}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x} + \frac{1}{y-x}\frac{x}{y}\ln\frac{y-x}{-x} & \Gamma=\gamma^z\gamma_\perp \end{cases},$$

$$H_2\left(\Gamma,x,y,\frac{P_z}{\mu}\right) = \begin{cases} \frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{4x(y-x)(P^z)^2}{\mu^2} + \frac{1+x-y}{y-x}\left(\frac{1-x}{1-y}\ln\frac{y-x}{1-x} - \frac{x}{y}\right) & \Gamma=\gamma^z\gamma_5 \\ \frac{1+y-x}{y-x}\frac{x}{y}\left(\ln\frac{4x(y-x)(P^z)^2}{\mu^2} - 1\right) + \frac{1+x-y}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x} & \Gamma=\gamma^t \\ \frac{1}{y-x}\frac{x}{y}\ln\frac{4x(y-x)(P^z)^2}{\mu^2} + \frac{1}{y-x}\left(\frac{1-x}{1-y}\ln\frac{y-x}{1-x} - \frac{x}{y}\right) & \Gamma=\gamma^z\gamma_\perp \end{cases}.$$

One loop matching coefficient

Determine the counter term of the quasi-DA in RI/MOM scheme.

Using the renormalization factor for the quasi-PDF to renormalize the quais-DA

$$C_{CT}^{(1)}\left(\Gamma, x, y, r, \frac{P^z}{p_R^z}\right) = \left|\frac{P^z}{p_R^z}\right| \widetilde{q}^{(1)}\left(\Gamma, \frac{P^z}{p_R^z}(x-y)+1, r\right)_{+(y)} .$$

The $\tilde{q}_r$ has been calculated in arXiv:1807.06566 [hep-lat]

Finally, we have the one-loop matching coefficient C in factorization formula,

$$C_\Gamma\left(x, y, r, \frac{P^z}{\mu}, \frac{P^z}{p_R^z}\right) = \delta(x-y) + C_B^{(1)}\left(\Gamma, x, y, \frac{P_z}{\mu}\right) - C_{CT}^{(1)}\left(\Gamma, x, y, r, \frac{P^z}{p_R^z}\right) + \mathcal{O}(\alpha_s^2).$$

Summary

- The quasi-DA can be renormalized in the RI/MOM scheme with the same renormalization factor that has already been calculated for the quasi-PDF case.

- Derive the one-loop matching coefficient that matches RI/MOM quasi-DA in the Landau gauge to $\overline{MS}$ LCDA within the framework of LaMET.

- Our results include the matching coefficients for pseudoscalar, longitudinally polarized vector, and transversely polarized vector DAs.

- These matching coefficients are ready to be applied to extracting the LCDAs from the lattice matrix elements of quasi-DAs.

Thank you !

Back Up

$$\langle P|O(\gamma^\mu\gamma_5,0)|0\rangle = if_P P^\mu\,,$$
$$\langle P,\epsilon_{||}|O(\gamma^\mu,0)|0\rangle = f_V^{||} M_V \epsilon_{||}^\mu\,,$$
$$\langle P,\epsilon_\perp|O(\sigma^{\mu\nu},0)|0\rangle = if_V^\perp(\epsilon_\perp^\mu P^\nu - \epsilon_\perp^\nu P^\mu)$$

$$\mathcal{O}_V^\Gamma(\xi^-) = \bar\psi(\xi^-)\Gamma W(\xi^-,0)\psi(0), \tag{1}$$

where $\Gamma = \gamma^+\gamma_\perp^\alpha$ for transversely polarized vector meson, and $\Gamma = \gamma^+$ for longitudinally polarized vector meson. $W(\xi^-,0)$ is the Wilson line with the end points $(0,\xi^-,0_\perp)$ and $(0,0,0_\perp)$. In LCDAs the Wilson line is light-like

$$W(\xi^-,0) = P\exp\left[-ig_s\int_0^{\xi^-} n\cdot A(\lambda n)d\lambda\right], \tag{2}$$

where P denotes that the exponential is path ordered. We also need the Fourier transformation of these operators, which are denoted by $O_V^\Gamma(x)$

$$O_V^\Gamma(x) = \int\frac{d\xi^-}{2\pi}e^{-ix\xi^-P^+}\mathcal{O}_V^\Gamma(\xi^-), \tag{3}$$

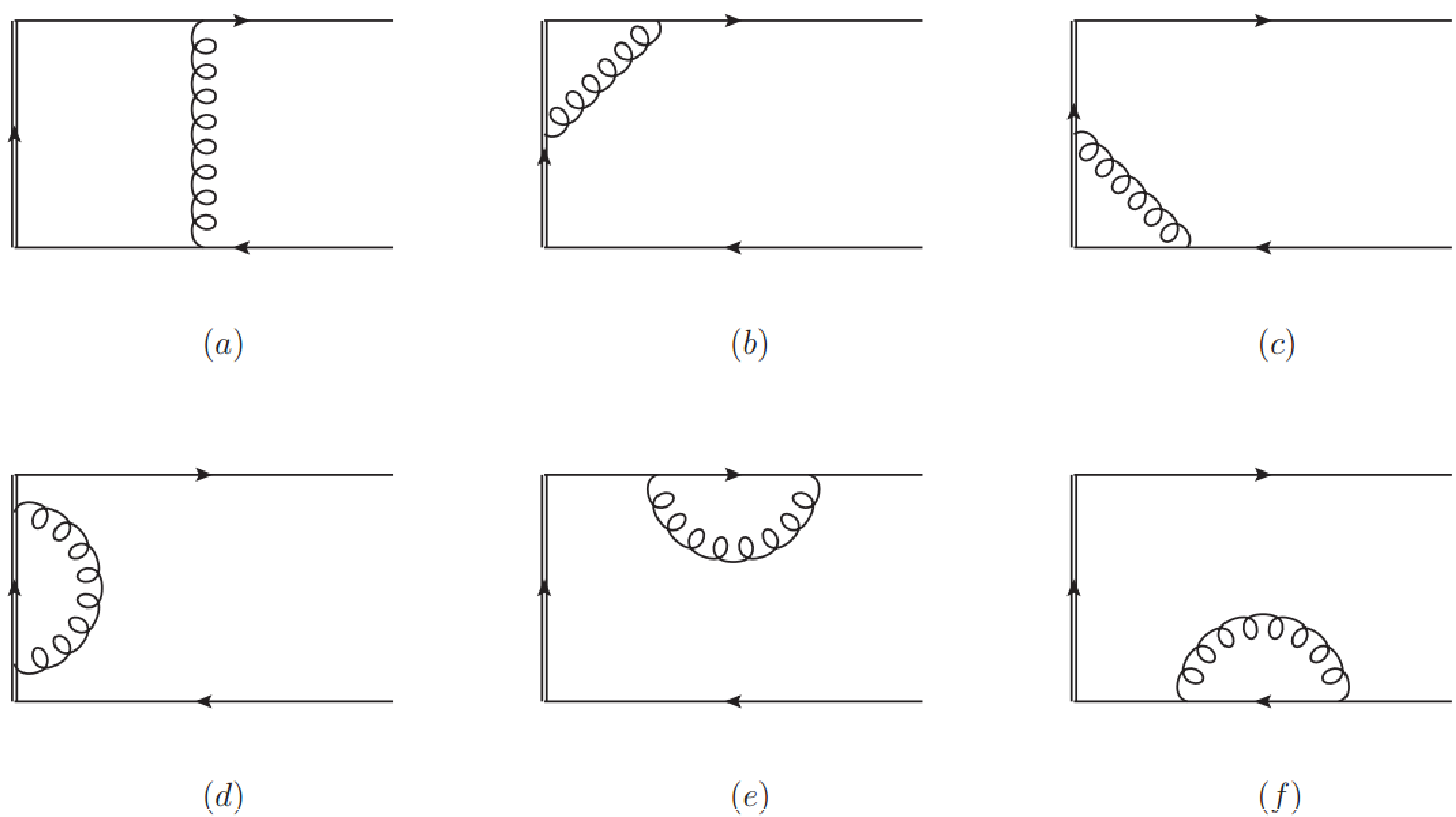


FIG. 1: Feynman diagrams for LCDAs and quasi-DAs at one loop level. The double line denotes the Wilson line.

to study the inverse moment. The inverse moment of the LCDAs is defined by

$$\left\langle \frac{1}{x} \right\rangle_\Gamma \equiv \int_0^1 dx \frac{\hat{\phi}_\Gamma(x;\mu)}{x} \,. \tag{4.1}$$

The Gegenbauer moments are also commonly used, which are defined by

$$(a_n)_\Gamma \equiv \frac{2(2n+3)}{3(2+n)(1+n)} \int_0^1 dx \hat{\phi}_\Gamma(x) C_n^{(3/2)}(2x-1) \,.$$

In order to combine the "real" and "virtual" contributions (defined in Ref. [?]) in a compact form at one-loop level, we introduce a plus function $[h(x,y)]_{+(y)}$ which is defined as

$$\int dx [h(x,y)]_{+(y)} g(x) = \int dx \; h(x,y)[g(x) - g(y)] \tag{22}$$

Quark Generalized Parton Distributions

$$p^{\mu} = \frac{p''^{\mu} + p'^{\mu}}{2}, \quad \Delta^{\mu} = p''^{\mu} - p'^{\mu}, \quad t = \Delta^2, \quad \xi = \frac{p''^{+} - p'^{+}}{p''^{+} + p'^{+}}.$$

- Depend on **quark momentum fraction** x, **skewness** ξ and **nucleon momentum transfer** t.

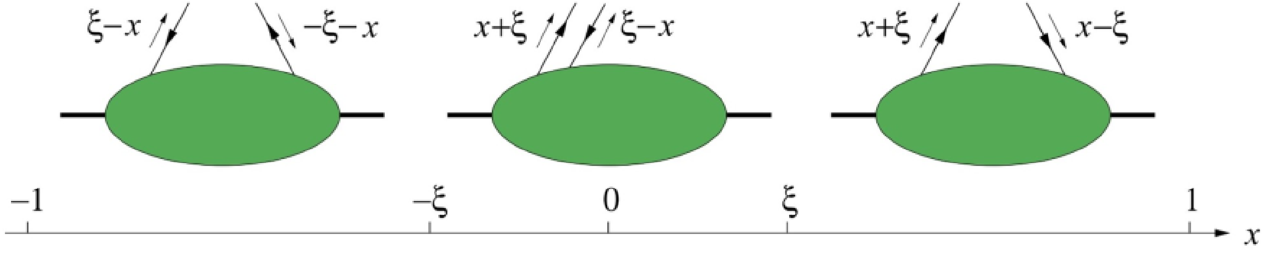


Figure: The parton interpretation of GPDs in the three x-intervals $[-1,-\xi]$, $[-\xi,\xi]$ and $[\xi,1]$

Quark Generalized Parton Distributions

The momentum fraction $x \in [-1, 1]$, which falls into the following three regions:

- $x \in [-1, -\xi]$, both momentum fractions $x + \xi$ and $x - \xi$ are negative: emission and reabsorption of antiquarks with respective momentum fractions $\xi - x$ and $-\xi - x$.
- $x \in [-\xi, \xi]$, one has $x + \xi > 0$ but $x - \xi < 0$: a quark with momentum fraction $x + \xi$ and an antiquark with $\xi - x$ emitted from the initial proton.
- $x \in [\xi, 1]$ both $x + \xi$ and $x - \xi$ are positive: emission and reabsorption of a quark.

The first and third case are commonly referred to as **DGLAP regions** and the second as **ERBL region**.

Large Momentum Effective Theory (LaMET)

Relating *parton physics observables* to *equal-time correlators in a large momentum nucleon states* (quasi-observables).

- Light-cone observables: $p_z \to \infty$, then $\Lambda \to \infty$.
  Quasi observables: $\Lambda \to \infty$, then $p_z \to \infty$.
  These two limits **do not commute**!
- They have **same IR** but different UV behaviours, while the UV difference is **controllable** and **calculable**.

Factorization formula between light-cone and quasi GPDs:

$$\mathcal{H}(x,\xi,t,p_z) = \int_{-1}^{1} \frac{dy}{|y|} Z_H\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{p_z}\right) H(y,\xi,t,\mu) + \mathcal{O}\left(\frac{M^2}{p_z^2},\frac{\Lambda_{QCD}^2}{p_z^2}\right).$$

$$\widetilde{\mathcal{F}}(\Gamma, x, P^z, \widetilde{\mu}) = \int_0^1 dy\, \widetilde{C}_\Gamma\left(x, y, \frac{\tilde{\mu}}{\mu}, \frac{P^z}{\mu}\right) \mathcal{F}(\bar{\Gamma}, y, \mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\right) \tag{13}$$

where $\mathcal{O}\left(M^2/(P^z)^2, \Lambda^2_{\rm QCD}/(P^z)^2\right)$ are mass and higher-twist corrections. Since the choice of Γ corresponds to a unique $\bar{\Gamma}$, we suppress the label $\bar{\Gamma}$ of the matching coefficient $\widetilde{C}_\Gamma$. On the other hand, the renormalized local operators in Eqs. (3) and (11) are related by

$$\mathcal{V}(\bar{\Gamma}, \mu) = \widetilde{Z}(\bar{\Gamma}, \Gamma, \mu, \widetilde{\mu})\mathcal{V}(\Gamma, \widetilde{\mu}) \tag{14}$$

where $\widetilde{Z}(\bar{\Gamma}, \Gamma, \mu, \widetilde{\mu})$ contains kinematic factors in Eq. (5) and the scheme conversion factor when LCDA and quasi-DA are renormalized in different schemes. Combining Eqs. (13) and (14), we have the matching formula between quasi-DA and LCDA [21, 23]

$$\widetilde{\phi}(\Gamma, x, P^z, \widetilde{\mu}) = \int_0^1 dy\, C_\Gamma\left(x, y, \frac{\tilde{\mu}}{\mu}, \frac{P^z}{\mu}\right) \phi(\bar{\Gamma}, y, \mu) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(P^z)^2}\right) \tag{15}$$

where $C_\Gamma = \widetilde{Z}\,\widetilde{C}_\Gamma$ is still perturbatively calculable.

Order of limits

- Thus the difference between the matrix elements o and O is the order of limits:

  o: $P \to \infty$, followed by UV cut-off

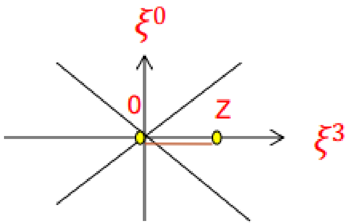
  O: UV cut-off imposed first, followed by $P \to \infty$

- This is the starndard set-up for effective field theory, such as HQET. The generic argument for factorization follow through. Hence we have large-momentum effective field theory: LaMET.
- Perturbative proof case by case.

A Euclidean quasi-distribution

- Consider space correlation in a large momentum P in the z-direction.

$$\tilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$



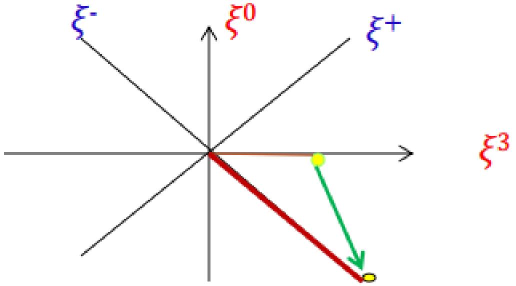
- Quark fields separated along the z-direction
- The gauge-link along the z-direction
- The matrix element depends on the momentum P.

Taking the limit P-> ∞ first

- After renormalizing all the UV divergences, one has the standard quark distribution!
  - One can prove this using the standard OPE
  - One can also see this by writing

    |P> = U(Λ(p)) |p=0>

  and applying the boost operator on the gauge link.



Step 1: Constructing lattice operators and evaluate the ME

- Construct a *frame-dependent, Euclidean* quasi-operator "O".
- In the IMF limit, O becomes a light-cone (light-front, parton) operator *o*.

$$O_1 = A^0 \;\rightarrow\; \mathrm{o} = \Lambda\, A^+$$

There are many operators leading to the same light-cone operator.

$$O_2 = A^3 \;\rightarrow\; \mathrm{o} = \Lambda$$

$$O_3 = \alpha A^0 + (1-\alpha) A^3 \;\rightarrow\; \mathrm{o} = \Lambda\, A^+$$

Step 2: lattice calculations

- Compute the matrix element of O on a lattice
- It will depend on the momentum of the hadron P, O(P,a).
- It also depends on the details of the lattice actions (UV specifics).

Step 3: Extracting the light-cone physics from the lattice ME

- Extract light-front physics o(μ) from O(P,a) at large P through a EFT matching condition or factorization theorem,

$$O(P,a) = Z(\frac{\mu}{P})o(\mu) + \frac{c_2}{P^2} + \frac{c_4}{P^4} + \cdots$$

Where Z is perturbatively calculable.

- Infrared physics of O(P,a) is entirely captured by the parton physics o(μ). In particular, it contains all the collinear divergence when P gets large.