Exclusive Quarkonium Electroproduction

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Introduction

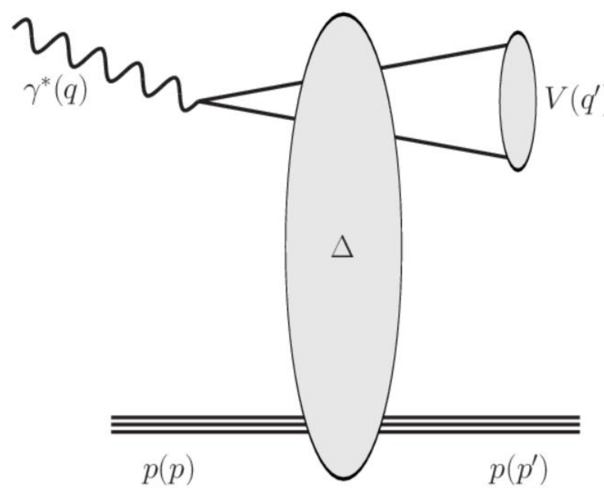
$$\gamma^*(q)p(p) \to V(q')p(p')$$

Kinematic region: $z = \frac{q' \cdot p}{q \cdot p} \approx 1 (\text{i.e.}, t = (p' - p)^2 \approx 0)$.

Also be termed as "elastic" or "diffractive" process.

$q^2 = 0$: photoproduction

$q^2 \neq 0$: electroproduction



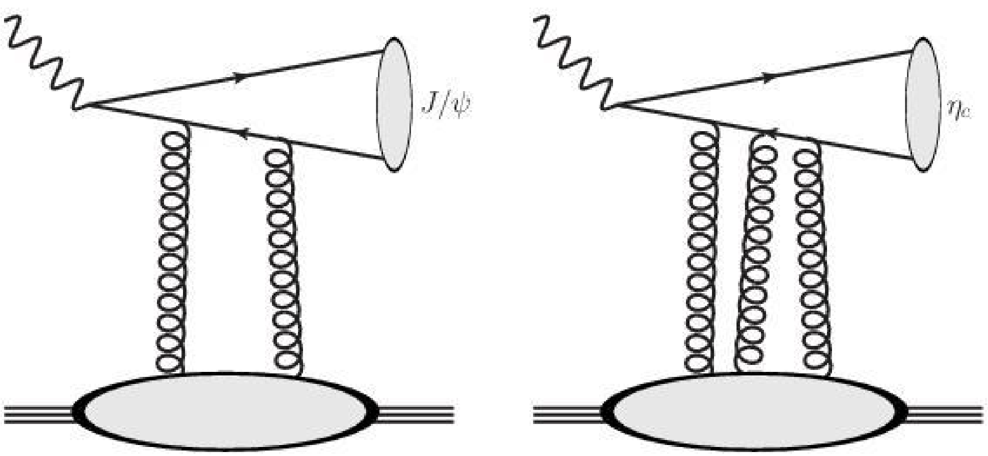
Introduction

Regge theory:

J/ψ production: exchange Pomeron($c = +1, I = 0$);

η_c production: exchange Odderon($c = -1, I = 0$).

QCD:

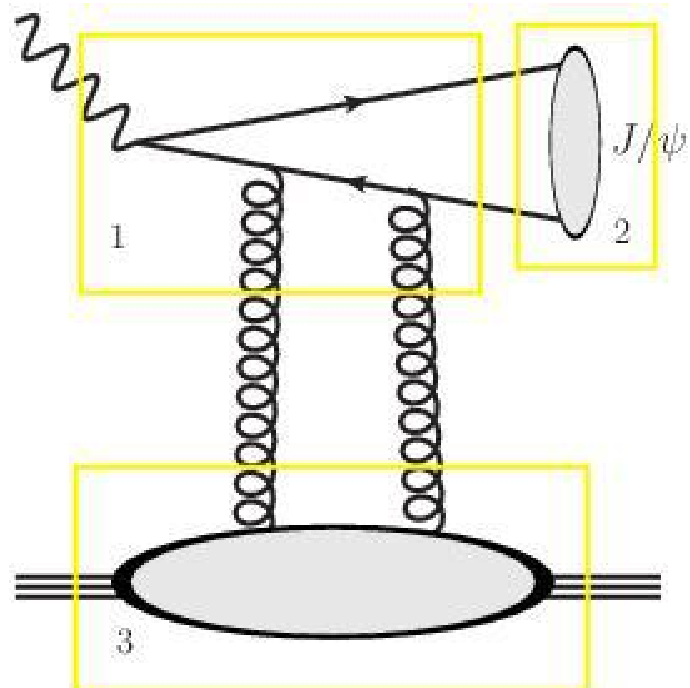


Factorization

1. Partonic process. Hard scale provided by M_V or Q . Perturbative QCD.

2. $q\bar{q}$ pair to quarkonium state. NRQCD long-distanse matrix element.

3. Parton distribution. k_t factorization or collinear factorization.

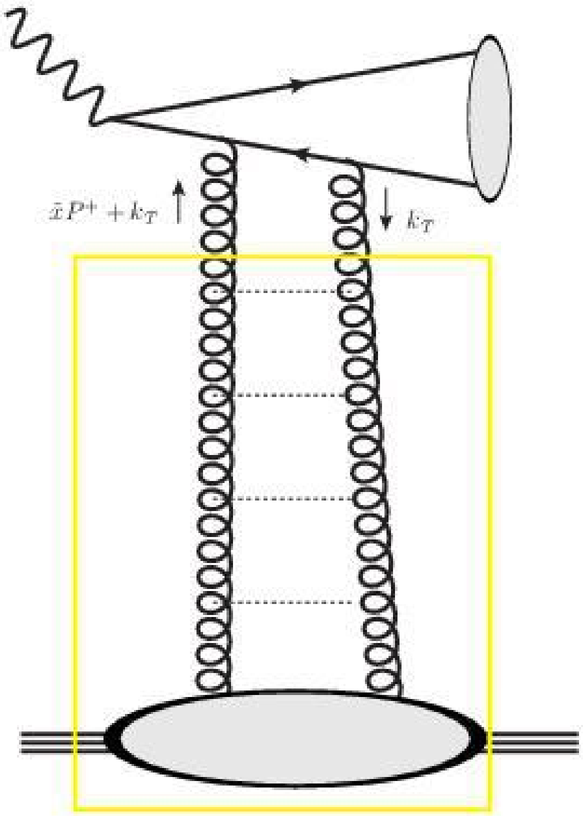


k_t fatorization

- Based BFKL method and resummed all leading-log terms.
- Introduce the "unintegrated" pdf: $f(x, k_T)$

$$xg(x, \mu^2) = \int^{\mu^2} \frac{d^2 k_T}{\pi k_T^2} f(x, k_T)$$

- The amplitude[1]:
  $\mathrm{Im}\mathcal{M} \propto xg(x, \mu^2)$
  where
  $\mu^2 \sim (Q^2 + M_{J/\psi}^2)/4$
  $x = (Q^2 + M_{J/\psi}^2)/(W^2 + Q^2)$
- Strong dependence on scale.



[1] M. G. Ryskin, Z. Phys. C 57, 89 (1993)

k_t fatorization

- Improvement[1]:

$$\left[\frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4}xg(x,\bar{Q}^2)\right] \longrightarrow \int_{Q_0^2}^{(W^2-M_{J/\psi}^2)/4} \frac{\mathrm{d}k_T^2\,\alpha_s(\mu^2)}{\bar{Q}^2(\bar{Q}^2+k_T^2)} \frac{\partial\left[xg(x,k_T^2)\sqrt{T(k_T^2,\mu^2)}\right]}{\partial k_T^2} + \ln\left(\frac{\bar{Q}^2+Q_0^2}{\bar{Q}^2}\right)\frac{\alpha_s(\mu_{\mathrm{IR}}^2)}{\bar{Q}^2Q_0^2}xg(x,Q_0^2)\sqrt{T(Q_0^2,\mu_{IR}^2)}\,.$$

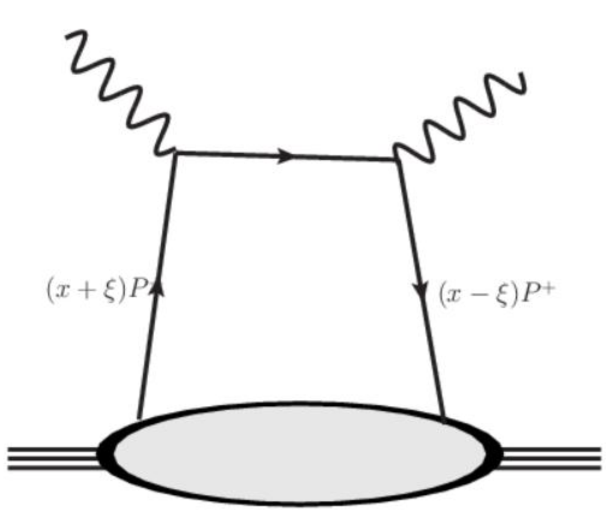
- Shortage:

1. Effect beyond leading-log.
2. Difficult to NLO.
3. η_c?

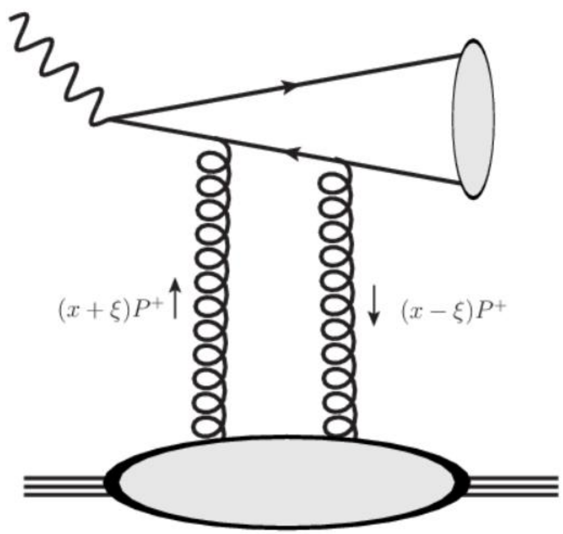
[1] S. P. Jones, A. D. Martin, M. G. Ryskin, J. Phys. G 44, 3 (2017).

Collinear factorization

Collinear factorization is commonly used in DVCS and exclusive VM production, and lead to the concept of generalized parton distributions (GPDs) .



DVCS



exclusive VM production

$$\mathcal{M} \sim \int_{-1}^{1} dx C(x,\mu_F)\mathrm{GPD}(x,\mu_F)$$

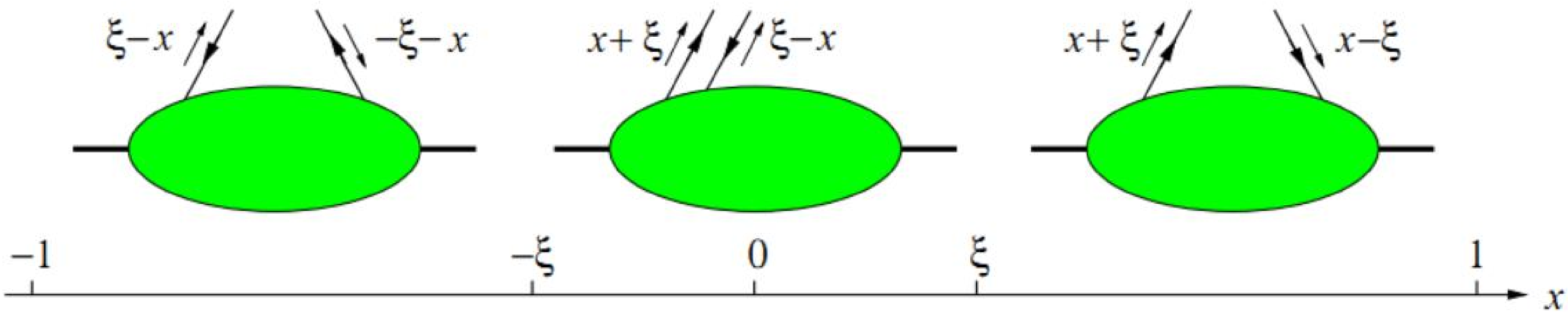
skewness: $\xi = \dfrac{M_V^2 + Q^2}{2s - M_V^2 + Q^2}$

GPDs

twist-2 GPDs: $H^g(x,\xi,t), H^q(x,\xi,t)...$ with the skewness ξ

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \overline{\psi}(-\lambda n/2)\gamma^\mu \psi(\lambda n/2) | P \rangle = H(x,\xi,\Delta^2)\overline{U}(P')\gamma^\mu U(P) + E(x,\xi\Delta^2)\overline{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P) + \cdots,$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \overline{\psi}(-\lambda n/2)\gamma^\mu \gamma_5 \psi(\lambda n/2) | P \rangle = \widetilde{H}(x,\xi,\Delta^2)\overline{U}(P')\gamma^\mu \gamma_5 U(P) + \widetilde{E}(x,\xi,\Delta^2)\overline{U}(P')\frac{\gamma_5\Delta^\mu}{2M}U(P) + \cdots,$$



$|x| > \xi$: DGLAP region

$|x| < \xi$: ERBL region

GPDs

- Forward limit:

$$H^g(x,\xi,t) \xrightarrow{\xi=0} xg(x) \quad H^q(x,\xi,t) \xrightarrow{\xi=0} q(x)$$

- extrapolate from PDFs:

1) foward model
for $x >> \xi$, $H(x,\xi) \approx H(x,0)$;
for other region ...

2) Shuvaev transform (poor for large ξ)

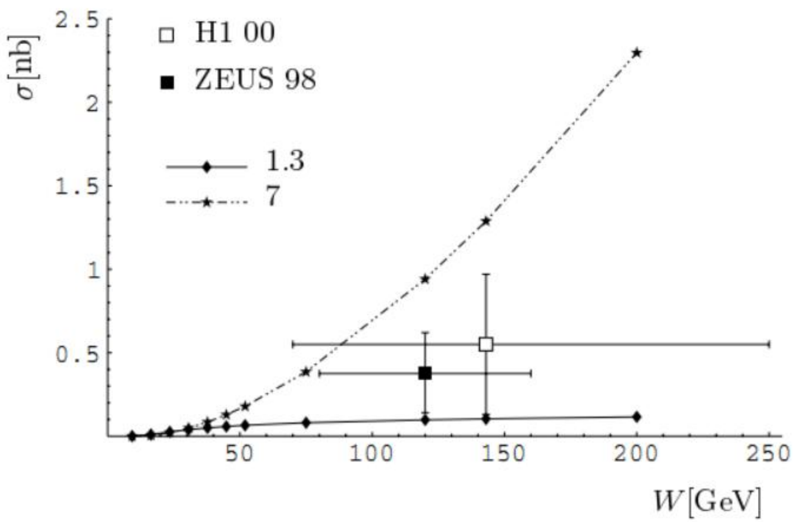
3) Double distribution model (singular)

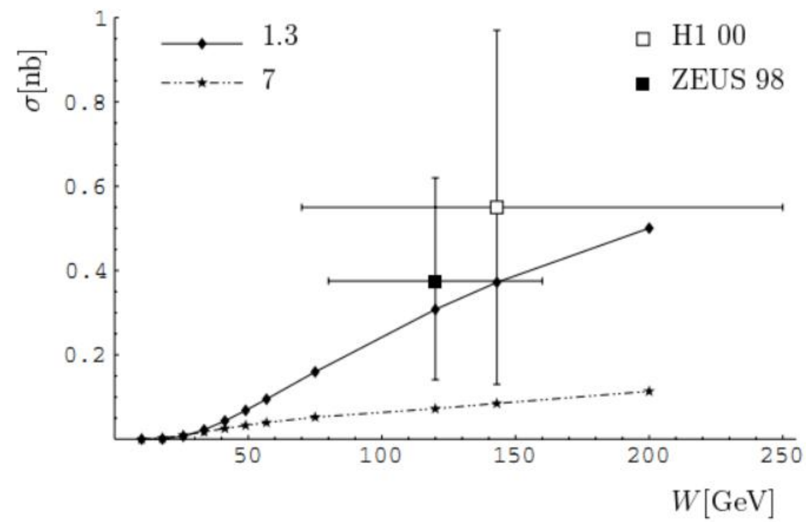
Fraught with uncertainties!

For review on GPDs:
M. Diehl, Phys, Rept, 388 41 (2003),
A. V. Belitskly, A. V. Radyushkin, Phys, Rept 418, 1 (2005).

Photoproduction

Υ photoproduction at NLO[1]:

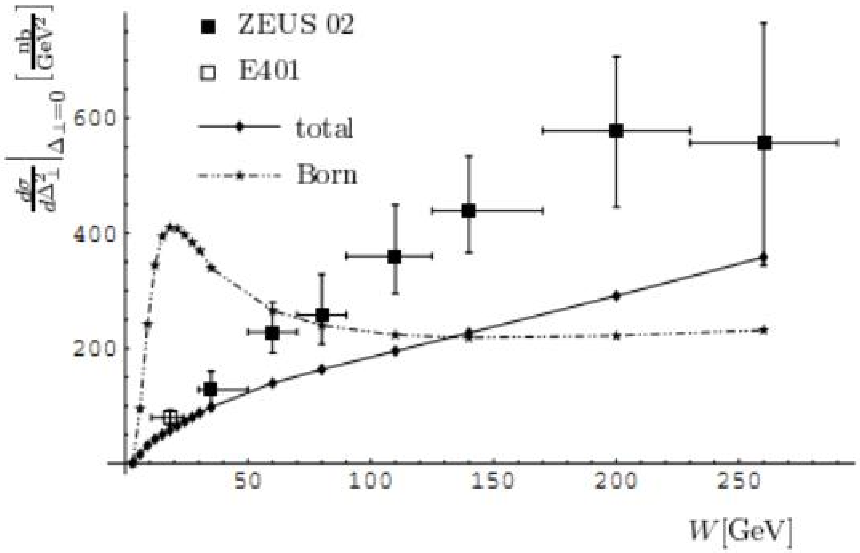


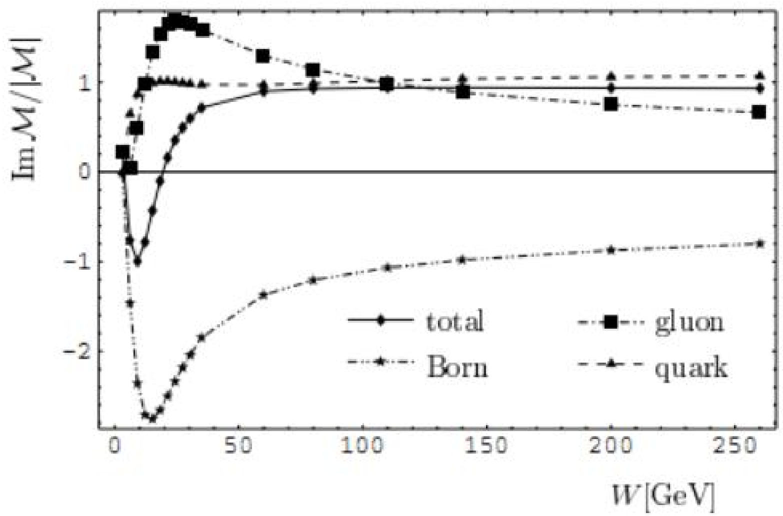


[1] D. Yu. Ivanov, A. Schafer, L. Szymanowski, G. Krasnikov, Eur. Phys. J. C 34, 297 (2004); 75, 75 (E) (2015).

Photoproduction

J/ψ photoproduction at NLO:





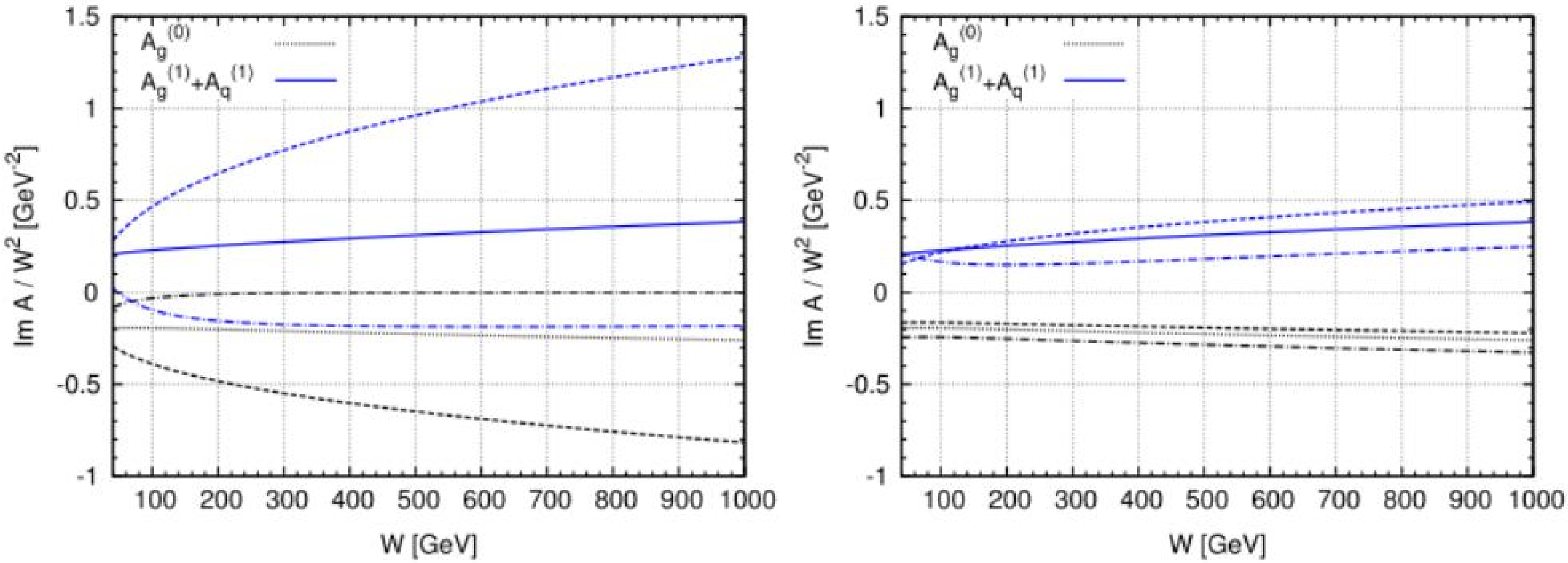
small value of m_c lead to :

1) large $\alpha_s(m_c)$;

2) small ξ.

Photoproduction

Improve the convergence of perturbative series by resumming $\ln\frac{1}{\xi}$ terms[1].



[1] S. P. Jones, A. D. Martin, M. G. Ryskin, T. Teubner, J. Phys. G: Nucl. Part. Phys. 43 035002 (2016).

Electroproduction

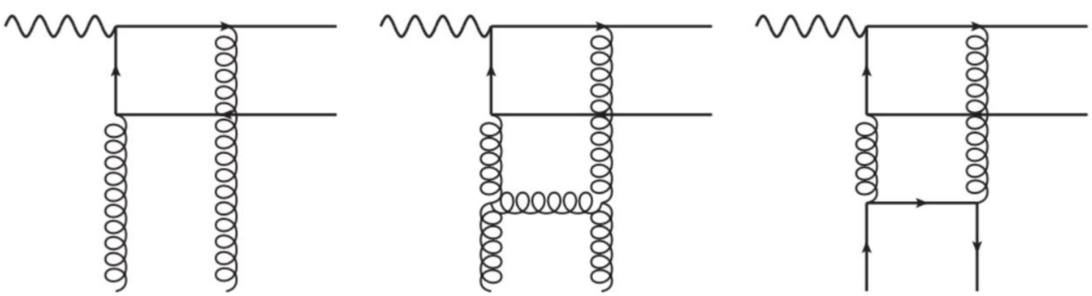
hard scale: Q^2, M_V^2

skewness: $\xi = \dfrac{Q^2 + M_V^2}{2s + Q^2 - M_V^2}$

expect a better convergence than photoproduction

$$\mathcal{M} \sim \int_{-1}^{1} dx \big[C^{(0)}(x, \mu_F)\mathrm{GPD}(x, \mu_F) + C^{(1)}(x, \mu_R, \mu_F)\mathrm{GPD}(x, \mu_F)\big]$$

partonic process:



(a) (b) (c)

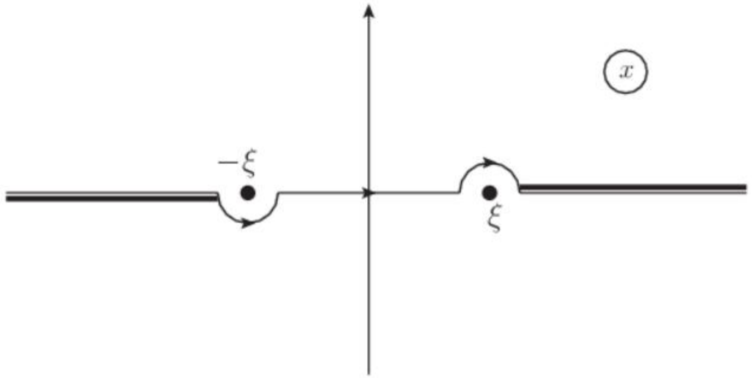
Electroproduction

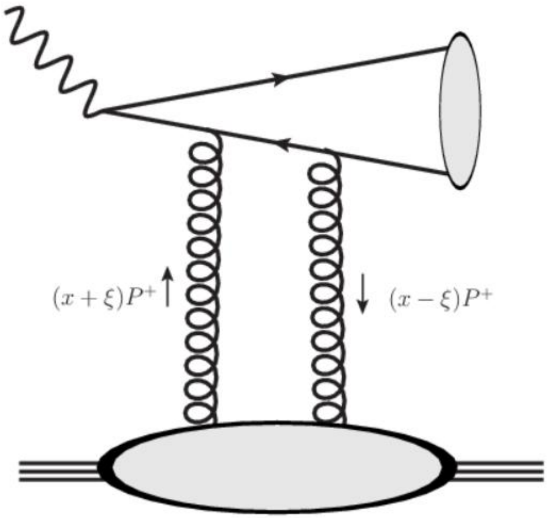
- Ultraviolet singularities. Removed by $\overline{\text{MS}}$ renomalization.
- Infrared singularities. Parts cancelled each other, remaining absorbed by GPDs.
- Singularities at $x = \pm\xi$

$x > \xi, x \to x + i\varepsilon;$

$x < -\xi, x \to x - i\varepsilon;$

$|x| < \xi,$ arbitrary sign. Amplitude is pure real.





Electroproduction

$$A^{\lambda\lambda'} = c_0^{\lambda\lambda'} + \sum_{i=1}^{11} c_i^{\lambda\lambda'} f_i \pm \{y_1 \to 1 - y_1 - y_2\},$$

$$d_1 = (1+y_2)^{-1}, d_2 = (1+2y_2)^{-1}, d_3 = (1-2y_1)^{-1}, d_4 = (2y_1+2y_2-1)^{-1},$$

$$d_5 = (2y_1+y_2-1)^{-1}, d_6 = (6y_1+y_2-5)^{-1}, d_7 = (6y_1+5y_2-1)^{-1}.$$

$$f_1 = \sqrt{\tfrac{-y_1}{1-y_1}}\tfrac{\ln(\sqrt{1-y_1}+\sqrt{-y_1})}{-y_1}, f_2 = f_1^2, f_3 = \sqrt{\tfrac{1+y_2}{y_2}}\tfrac{\ln(\sqrt{1+y_2}+\sqrt{y_2})}{1+y_2}, f_4 = f_3^2,$$

$$f_5 = d_1 d_3 d_5 \ln(\tfrac{1-y_1}{1+y_2}), f_6 = d_1 d_3 d_4 d_5 \ln(2+2y_2),$$

$$f_7 = d_1 \ln(1+y_2)^2, f_8 = d_1^2(\ln(\tfrac{4m^2}{\mu_F^2})\ln(\tfrac{1-y_1}{1+y_2}) + \ln(1-y_1)^2),$$

$$f_9 = d_1^2 \mathrm{Li}_2(-1+2y_1), f_{10} = d_1^2 \mathrm{Li}_2(-1-2y_2),$$

$$f_{11} = d_1^2 d_5 C_0(-4y_2, -1+2y_1, 1-2y_1-2y_2, 1, 1, 0), \quad (13)$$

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[1] For f_{11}, $C_0(-4y_2, -1+2y_1, 1-2y_1-2y_2, 1, 1, 0) = \frac{1}{2(2y_1+y_2-1)}\{\mathrm{Li}_2[\frac{2(1-y_1)}{1+y_2}] - \mathrm{Li}_2[\frac{(1-2y_1-2y_2)(1+y_2)}{1+y_2-4(1-y_1)(y_1+y_2)}] + \mathrm{Li}_2[\frac{2(1-y_1)(1-2y_1-2y_2)}{1+y_2-4(1-y_1)(y_1+y_2)}] - \mathrm{Li}_2[\frac{2(1-y_1)y_2}{(1+y_2)y_2-\sqrt{(1+y_2)y_2}(2y_1+y_2-1)}] - \mathrm{Li}_2[\frac{2(1-y_1)y_2}{(1+y_2)y_2+\sqrt{(1+y_2)y_2}(2y_1+y_2-1)}]\} + \{y_1 \to 1 - y_1 - y_2\}$

Electroproduction

For A_g^{++} and A_g^{--}, $c_i^{++} = c_i^{--} = c_i$,

$$c_0 = d_1^2 d_2 d_3 d_4 \Big(\frac{13\pi^2 a^4}{4320 b^2} - \frac{31\pi^2 a^4}{2160} + \frac{\pi^2 a^4}{5} + \frac{2a^2}{9} - \frac{\pi^2 a^3}{180 b^2}$$
$$+ \frac{88 a^2 b}{9} + \frac{38}{27}\pi^2 a^3 b + \frac{163\pi^2 a^2}{4320} - \frac{82\pi^2 a^2}{72} - \frac{38 a^2}{9} + \frac{22 a^2 b^2}{3} + \frac{67}{54}\pi^2 a^2 b^2$$
$$+ \frac{19\pi^2 a^2}{5760 b^2} - \frac{133 a^2 b}{9} - \frac{275}{108}\pi^2 a^2 b - \frac{29\pi^2 a^2}{2160} + \frac{323\pi^2 a^2}{432} + \frac{11 a^2}{3} - \frac{11\pi^2 a}{1728 b^2}$$
$$+ \frac{11ab}{3} + \frac{25}{144}\pi^2 ab - \frac{32\pi^2 a}{288 b} + \frac{35\pi^2 a}{72} - \frac{8a}{9} + \frac{71\pi^2}{1728 a} - \frac{313\pi^2}{1728} \Big) - d_1 \frac{5}{24} \ln\frac{\mu^2}{\mu_F^2},$$

$$c_1 = y_1^{-1}\Big(-\frac{2a^2}{45} + \frac{23a}{135} - \frac{1}{36ab} - \frac{2a}{9} - \frac{1}{18a} - \frac{37}{135} + \frac{11}{36}\Big),$$

$$c_2 = y_1^{-1}\Big(-\frac{11a}{135} + \frac{1}{72ab} + \frac{1}{36a} - \frac{1}{45} - \frac{1}{9}\Big),$$

$$c_3 = d_1^2 d_5^2 y_2\Big(-\frac{2a^4}{45} + \frac{11a^3}{135} - \frac{2a^3}{9} + \frac{4a^2 b}{9} + \frac{4a^2}{45} - \frac{8a^2}{3} - \frac{8ab}{135} + \frac{4a}{9}\Big),$$

$$c_4 = d_1^3 y_2\Big(-\frac{8a^2}{144 b^2} - \frac{11a^2}{135} + \frac{8a}{48 b^2} - \frac{8a}{45} - \frac{a}{36} - \frac{11}{1440} - \frac{47}{144}\Big),$$

$$c_5 = d_3 d_5\Big(-\frac{4a^6}{45} + \frac{19a^5}{35} + \frac{8a^4 b}{9} - \frac{73a^4}{45} - \frac{37a^4}{5} - \frac{67a^3 b}{18} + \frac{68a^3}{135} + \frac{301a^3}{36}$$
$$- \frac{4a^2 b^2}{9} + \frac{83a^2 b^2}{18} + \frac{61a^2 b}{36} - \frac{13a^2}{245} - \frac{47a^2}{12} + \frac{13ab^2}{18} + \frac{b^3}{36a} - \frac{188ab^2}{36} + \frac{b^2}{72a}$$
$$+ \frac{37ab}{36} + \frac{47a}{72} - \frac{13b^2}{36} + \frac{83b^2}{36} - \frac{41b}{72}\Big),$$

$$c_6 = d_2^2 d_3 d_4 d_5\Big(\frac{80a^7 b}{9} + \frac{14a^7}{45} - \frac{70a^7}{9} + \frac{176a^6 b^2}{3} - \frac{292a^6 b}{3} - \frac{80a^6}{45} + \frac{146a^6}{3} + 48a^5 b^3$$
$$- \frac{628a^5 b^2}{3} + 228a^5 b + \frac{77a^5}{45} - 87a^5 - \frac{16a^4 b^3}{9} - \frac{748a^4 b^2}{9} + \frac{2218a^4 b^2}{9} - \frac{1963a^4 b}{9} - \frac{49a^4}{45}$$
$$+ \frac{1261a^4}{18} + \frac{154a^3 b^3}{3} - \frac{410a^3 b^2}{3} + \frac{881a^3 b}{9} + \frac{119a^3}{725} - \frac{88a^3}{2} + \frac{157a^2 b^2}{6} - \frac{367a^2 b}{18}$$
$$- \frac{7a^2}{385} + \frac{29a^2}{5} + \frac{77ab}{72} - \frac{a}{4}\Big),$$

$$c_7 = \frac{a}{25}, c_8 = -\frac{a^2}{25} - \frac{b}{2},$$

$$c_9 = -\frac{5a^2}{288 b^2} - \frac{b^2}{32a^2} - \frac{5a^2}{135} + \frac{13b}{288a^2} + \frac{5a}{36 b^2} + \frac{b^2}{24a} - \frac{47a}{1440} - \frac{19b}{144a} + \frac{7a}{8} + \frac{11}{96a} + \frac{b}{18} + \frac{35}{2880} - \frac{89}{144},$$

$$c_{10} = \frac{5a^2}{135} + \frac{5a}{48} - \frac{a}{36} - \frac{1}{55} + \frac{13}{144},$$

$$c_{11} = d_5 d_6^{-1} d_7^{-1}\Big(-\frac{a^3}{144 b^2} + \frac{a^2}{48 b^2} - \frac{5a^2}{48} + \frac{a^2}{5} - \frac{ab}{18} + \frac{a}{36} - \frac{a}{18} + \frac{1}{144}\Big). \quad (15)$$

For $\tilde{A}_q$, $\tilde{c}_i^{00} = 0$, $\tilde{c}_i^{++} = -\tilde{c}_i^{--} = c_i$,

$$c_5 = d_5^{-2}\frac{2}{45}, c_6 = d_2 d_5^{-1} y_2^2 \frac{4}{9}. \quad (20)$$

For A_g^{00}, $c_i^{00} = -\sqrt{y_2}\, c_i$,

$$c_0 = d_1^2 d_3 d_4\Big(\frac{7\pi^2 a^2}{540 b^2} + \frac{28a^2 b}{9} + \frac{8}{27}\pi^2 a^2 b - \frac{28\pi^2 a^2}{545} + \frac{7\pi^2 a^2}{27} - \frac{16a^2}{9}$$
$$- \frac{7\pi^2 a}{1080 b^2} - \frac{16ab}{9} + \frac{17}{27}\pi^2 ab + \frac{31\pi^2 a}{540} - \frac{43\pi^2 a}{27} + \frac{8a}{9} - \frac{37\pi^2}{216a} + \frac{29\pi^2}{54}\Big) - d_1 \frac{5}{24}\ln\frac{\mu^2}{\mu_F^2},$$

$$c_1 = -\frac{7}{9ab} - \frac{2}{45}, c_2 = y_1^{-1}\Big(-\frac{2a}{45} + \frac{7}{18ab} - \frac{7}{45}\Big),$$

$$c_3 = d_1^2 d_5^2\Big(\frac{2a^4}{45} + \frac{8a^3}{45} - \frac{a^3}{3} + \frac{17a^2 b}{9} - \frac{7a^2}{45} - \frac{14a^2}{9} - \frac{23ab}{9} + \frac{23a}{9}\Big),$$

$$c_4 = d_1^3 y_2\Big(-\frac{2a^2}{45} - \frac{7a}{45} - \frac{2a}{9} + \frac{1}{45} - \frac{1}{3}\Big),$$

$$c_5 = d_3 d_5\Big(\frac{32a^5}{45} - \frac{28a^4}{45} - \frac{49a^4}{9} - 2a^3 b + \frac{a^3}{35} + \frac{148a^3}{18} + \frac{19a^2 b^2}{9} + \frac{a^2 b}{3}$$
$$+ \frac{a^2}{45} - \frac{41a^2}{9} - \frac{8ab^2}{6} + \frac{7b^2}{18a} + \frac{13ab}{9} + \frac{17a}{18} - \frac{7b^2}{9} - \frac{11b}{18}\Big),$$

$$c_6 = d_2 d_3 d_4 d_5\Big(-\frac{16}{9}a^5 b^2 + 12a^5 b + \frac{28a^5}{45} - \frac{98a^5}{9} - \frac{80a^4 b^3}{3}$$
$$+ \frac{488a^4 b^2}{9} - \frac{428a^4 b}{9} - \frac{14a^4}{35} + \frac{194a^4}{9} + \frac{384a^3 b^3}{9} - \frac{914a^3 b^2}{9} + \frac{514a^3 b}{9} + \frac{7a^3}{35} - \frac{281a^3}{18} + \frac{388a^2 b^2}{9}$$
$$- \frac{847a^2 b}{18} - \frac{7a^2}{185} + \frac{43a^2}{9} + \frac{32ab}{9} - \frac{a}{2}\Big),$$

$$c_7 = \frac{a}{25}, c_8 = -\frac{a^2}{25} - \frac{b}{2}, c_9 = -\frac{a^2}{25} - \frac{7b}{36a^2} + \frac{a}{35} + \frac{8b}{9a} + \frac{b}{9} - \frac{5}{12a} - \frac{7b}{18} - \frac{2}{45} + \frac{8}{9},$$

$$c_{10} = \frac{a^2}{25} - \frac{a}{35} + \frac{b}{2} + \frac{1}{125} - \frac{2}{9}, c_{11} = d_5 d_6^{-1} d_7^{-1}\Big(\frac{a^2}{18} - \frac{ab}{6} - \frac{a}{185} + \frac{2a}{9} - \frac{1}{18}\Big). \quad (16)$$

For A_q^{++} and A_q^{--}, $c_i^{++} = c_i^{--} = c_i$,

$$c_0 = d_1^2 \frac{a^2}{45}, c_3 = d_1^2 d_5 y_2 \frac{8}{9}, c_4 = d_1^3 y_2 \frac{2}{35},$$
$$c_5 = -\frac{2a^2}{45} + \frac{2a}{45} - \frac{8a}{9} + \frac{4b}{9} + \frac{4}{9}, c_6 = d_2\Big(\frac{8a^3}{9} - 8a^2 b + \frac{8a^2}{3} + \frac{32ab}{9} - \frac{32a}{9} + \frac{4}{9}\Big),$$
$$c_7 = -\frac{4}{45}, c_8 = c_9 = \frac{4a}{45} - \frac{4}{9}, c_{10} = \frac{1}{35} - \frac{4a}{45}, c_{11} = -\frac{2a^2}{35} + \frac{8a^2}{3} - \frac{8ab}{9} - \frac{4a}{9}. \quad (17)$$

For A_q^{00}, $c_i^{00} = -\sqrt{y_2}\, c_i$,

$$c_3 = d_1^2 d_5 (1 - y_2)\frac{8}{9}, c_5 = \frac{8a^2}{45} - \frac{4a}{45} - \frac{4}{9}, c_6 = \frac{16ab}{9} - \frac{8a}{3} + \frac{8}{9},$$
$$c_7 = -\frac{4}{45}, c_8 = c_9 = \frac{4a}{45} - \frac{4}{9}, c_{10} = -\frac{4a}{45} - \frac{4}{9}, c_{11} = \frac{8a^2}{9} - \frac{8ab}{3} + \frac{16a}{9}. \quad (18)$$

For $\tilde{A}_g$, $\tilde{c}_i^{00} = 0$, $\tilde{c}_i^{++} = -\tilde{c}_i^{--} = c_i$,

$$c_1 = -\frac{1}{36 y_1}, c_2 = \frac{a}{36 y_1}, c_4 = -d_1 y_2 \frac{a}{135},$$
$$c_5 = d_3 d_5^{-1}\Big(\frac{19a^2}{18} - \frac{ab}{6} - \frac{7a}{12} + \frac{8b}{35}\Big), c_6 = d_3 d_4 d_5^{-2}\Big(-\frac{2a^2 b}{3} + \frac{5a^2}{9} + \frac{14ab}{9} - \frac{29a}{18} + \frac{13}{36}\Big),$$
$$c_9 = d_1\Big(\frac{1}{24} - \frac{5a}{72b}\Big), c_{11} = d_1 d_5^{-2} d_6^{-1} d_7^{-1} \frac{1}{72ab}. \quad (19)$$

Electroproduction

At high energy limit:

$$\mathcal{M} \sim \alpha_s F^g(\xi,\xi,\mu_F)$$
$$+\frac{\alpha_s^2}{\pi}\ln\frac{\bar{m}^2}{\mu_F^2}\ln\frac{1}{\xi}\left[3F^g(\xi,\xi,\mu_F)+\frac{4}{3}F^{q(+)}(\xi,\xi,\mu_F)\right],$$

where

$$\bar{m} = \sqrt{(m^2+\frac{Q^2}{4})}$$

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Electroproduction

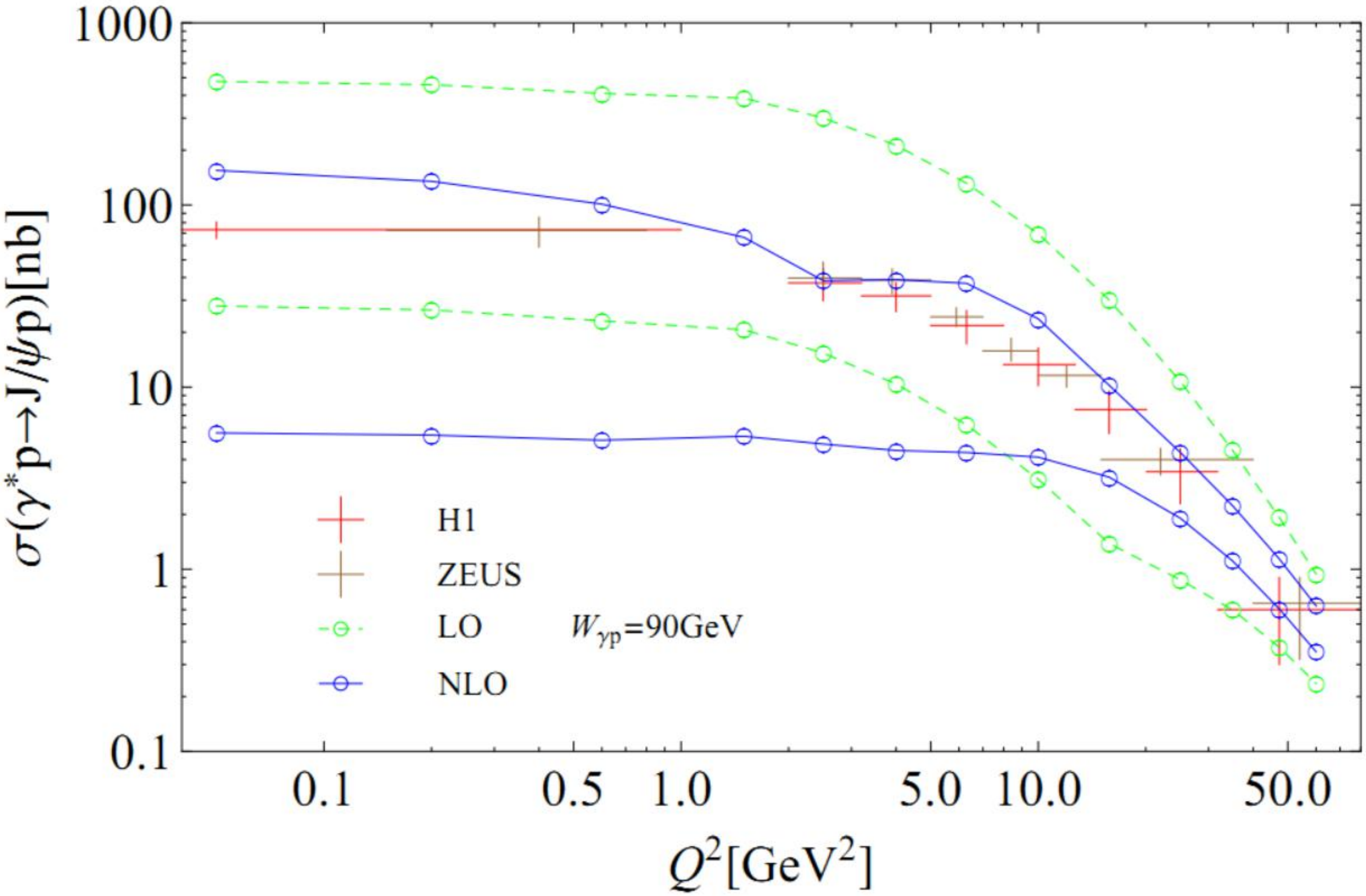
Parameter list:

- $\Lambda^3_{\rm QCD} = 332{\rm MeV}$, $\Lambda^4_{\rm QCD} = 292{\rm MeV}$, $\Lambda^5_{\rm QCD} = 210{\rm MeV}$;
- $|R_{J/\psi}(0)|^2 = 0.903{\rm GeV}^3$, $|R_\Upsilon(0)|^2 = 7.76{\rm GeV}^3$;
- $1.4 \leq m_c \leq 1.6$, $4.8 \leq m_b \leq 5.0$;
- $max\{1, \frac{1}{2}\sqrt{m^2 + Q^2/4}\} \leq \mu_F \leq 2\sqrt{m^2 + Q^2/4}$;
- $\mu_R = \mu_F$.

The choice of μ_R eliminates the contribution from β_0 term ($\beta_0 \ln\frac{\mu_F^2}{\mu_R^2}$), which related to the BLM(PMC) method.

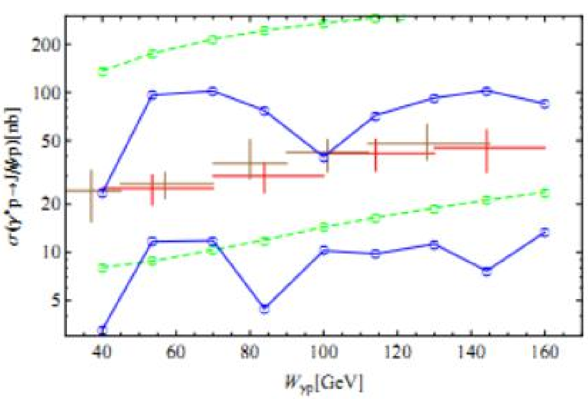
Electroproduction

HERA上J/ψ 产生:

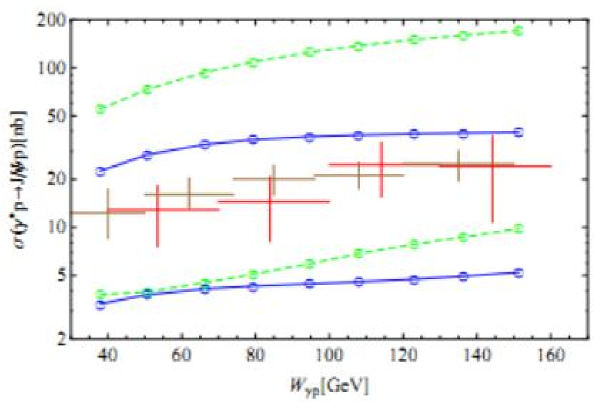


Electroproduction

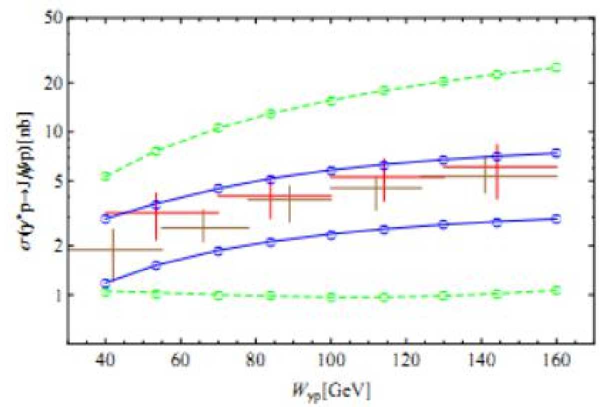
HERA上J/ψ 产生:



(a)$Q^2 = 3.2\text{GeV}^2$



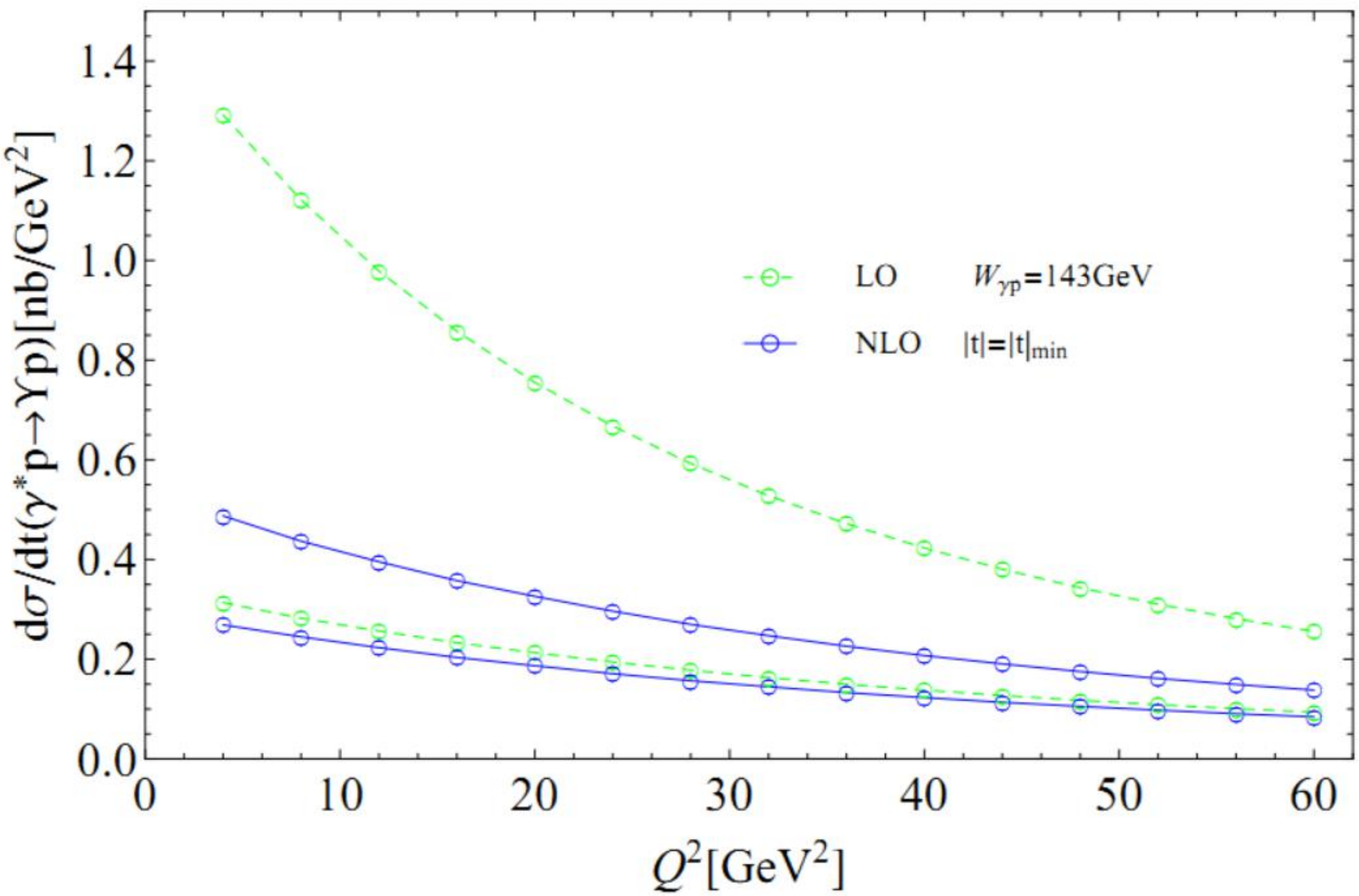
(b)$Q^2 = 7.0\text{GeV}^2$



(c)$Q^2 = 22.4\text{GeV}^2$

Electroproduction

HERA上ϒ产生:



Conclusion

- The quarkonium exclusive production process can potentially serve to constrain the parton distributions in low x_b region.
- For J/ψ electroproduction, the NLO result agrees with data nicely, with theoretical uncertainty largely suppressed at large Q^2 region.
- Precise phenomenological study require more reliable approach for the evaluation of GPDs.

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