HFCPV-2018

Study of semileptonic decays $B_c \to (\eta_c, J/\psi) l\nu$ in PQCD factorization approach

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Outline

- *Motivation*
- *Framework*
- *Contents*
- *Results*
- *Summary*

Motivation

$B_{(u,d,s,c)}$ meson semileptonic decay

- Determination of CKM

  (For example: $|V_{cb}|$ in $B \to D l v$)

- Examination of SM

  (HQET, NRQCD, LFQM etc.)

- Hints of new physics

  (2HDM, MSSM, Leptoquark mode etc.)

$R(D^{(*)})$ Anomaly

$$R(D) = \frac{Br(B \to D\tau\nu)}{Br(B \to Dl\nu)}$$

l=e,μ

$$R(D^*) = \frac{Br(B \to D^*\tau\nu)}{Br(B \to D^*l\nu)}$$

$R(D^{(*)})$ Anomaly

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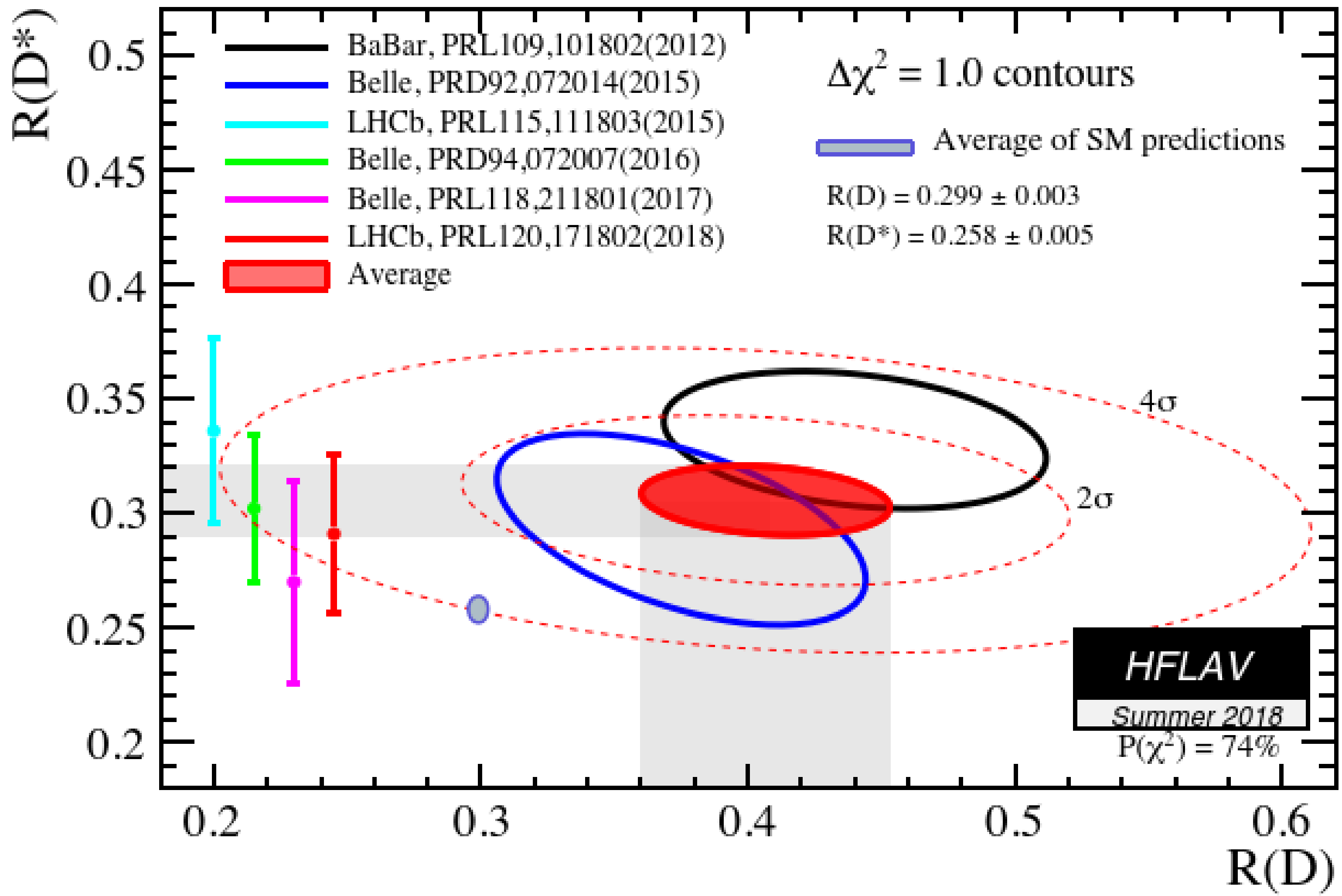
1. The theoretical uncertainties of each branching ratio are large.

2. It mainly comes from the Form Factors and V_{cb}.

3. The theoretical uncertainties in $R(D^{(*)})$ are significantly reduced.

$R(D^{(*)})$ Anomaly


BaBar, PRL109,101802(2012)
Belle, PRD92,072014(2015)
LHCb, PRL115,111803(2015)
Belle, PRD94,072007(2016)
Belle, PRL118,211801(2017)
LHCb, PRL120,171802(2018)
Average
Δχ² = 1.0 contours
Average of SM predictions
R(D) = 0.299 ± 0.003
R(D*) = 0.258 ± 0.005
4σ
2σ
HFLAV
Summer 2018
P(χ²) = 74%
R(D*)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.2
0.3
0.4
0.5
0.6
R(D)

R(J/ψ) Anomaly

$$R(J/\psi) = 0.71 \pm 0.17(stat) \pm 0.18(syst)$$

LHCb

$$R(J/\psi) \equiv \frac{Br(Bc \to J/\psi\tau\nu)}{Br(Bc \to J/\psi\mu\nu)} \in [0.25, 0.28]$$

SM

PhysRevLett. 120 (2018) 121801

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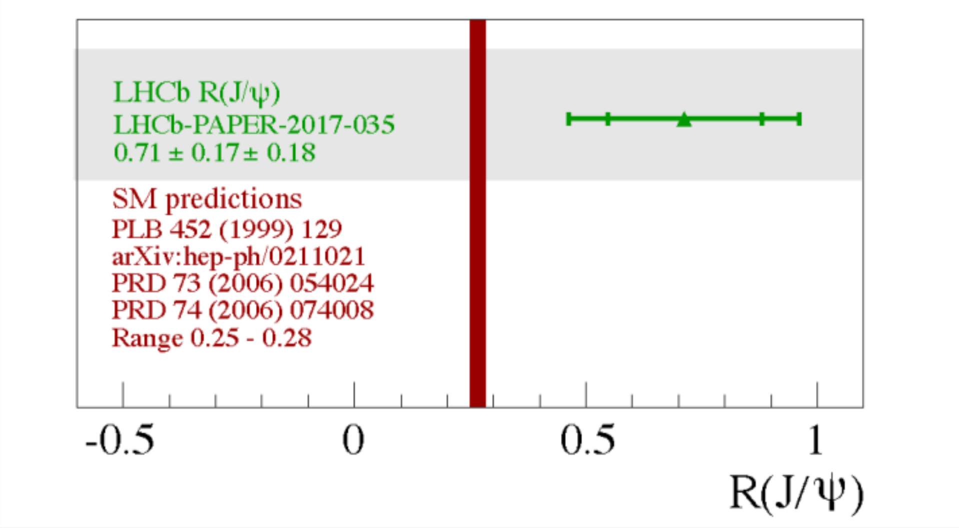

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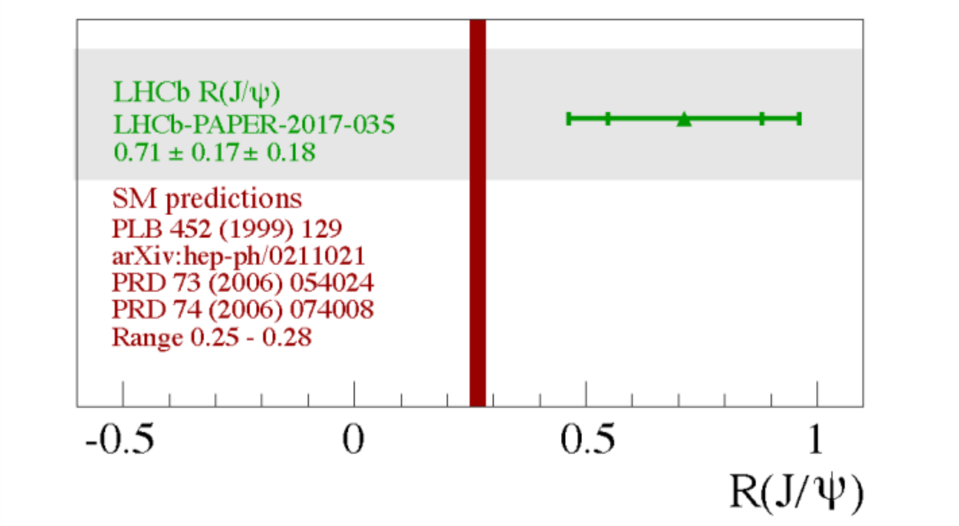
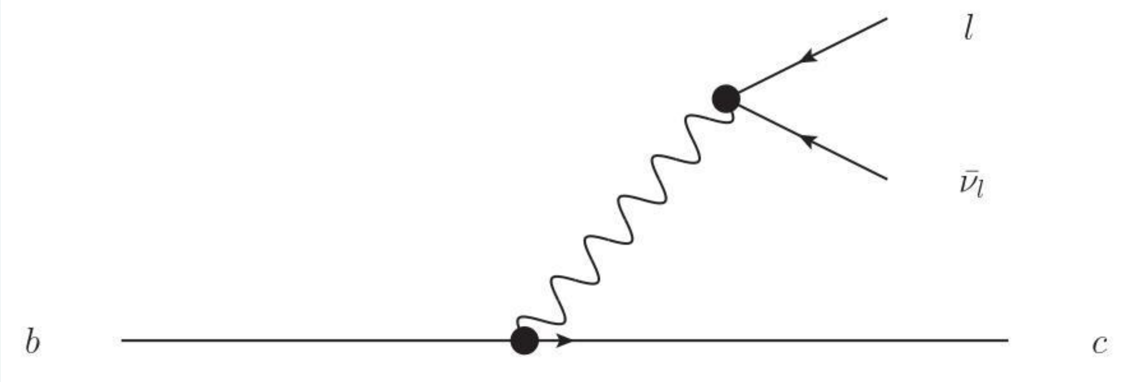


Calculate the R(J/ψ) in PQCD factorization approach

Framework

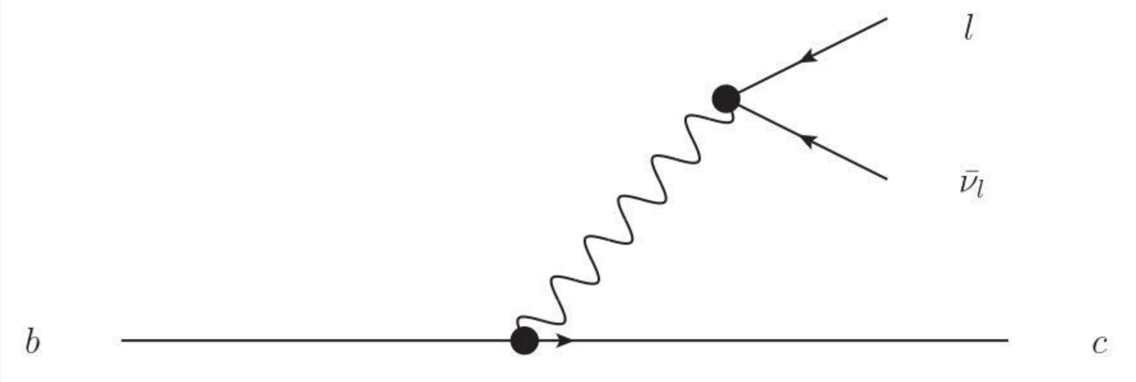
$B_c \to (\eta_c, J/\psi) l\nu \qquad l = e, \mu, \tau$



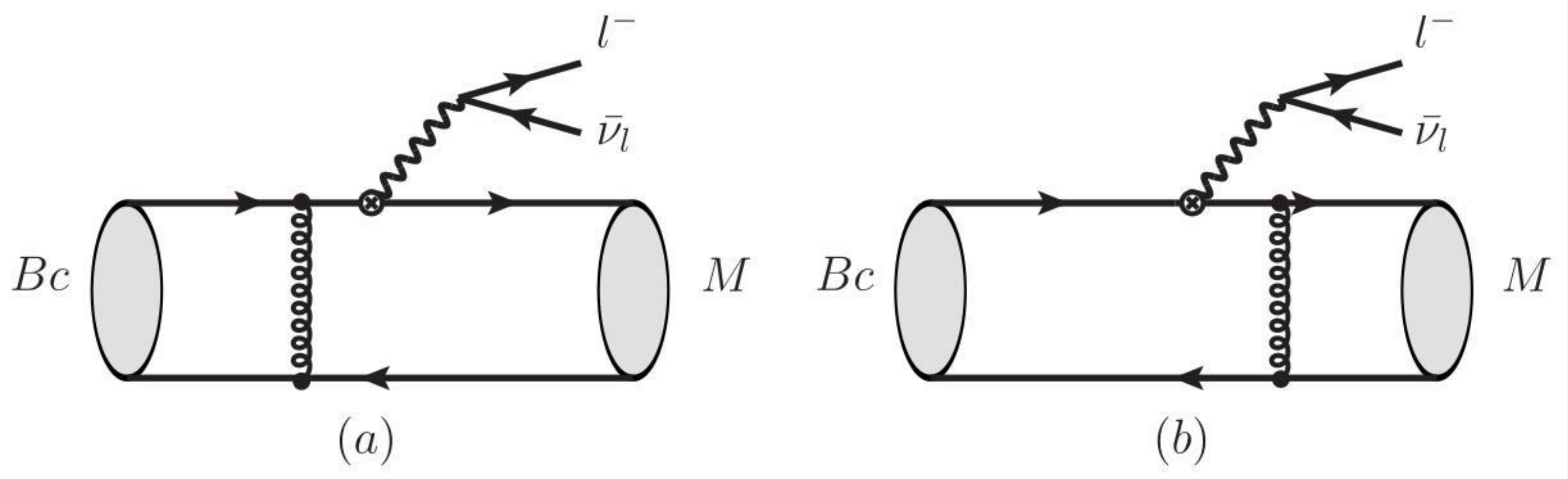
Quark level

Framework

$$B_c \to (\eta_c, J/\psi) l\nu \qquad l = e, \mu, \tau$$



Quark level



LO diagrams

Framework

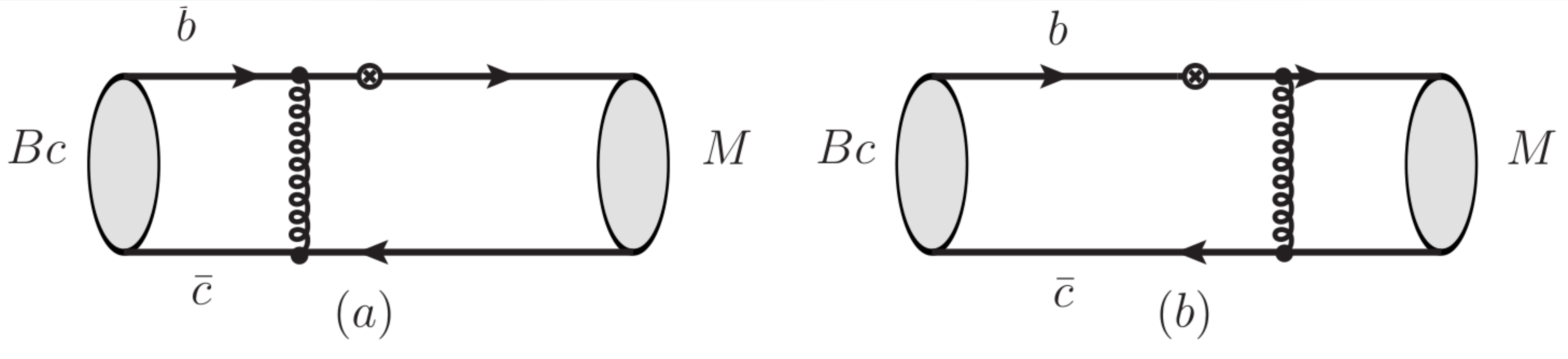
$$\mathcal{H}_{eff}(b \to c l \bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{cb}\, \bar{c}\gamma_\mu(1-\gamma_5)b \cdot \bar{l}\gamma^\mu(1-\gamma_5)\nu_l$$

Effective Hamiltonian

Framework

$$\mathcal{H}_{eff}(b \to c l \bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{cb}\, \bar{c}\gamma_\mu(1-\gamma_5)b \cdot \bar{l}\gamma^\mu(1-\gamma_5)\nu_l$$

Effective Hamiltonian



Effective Diagrams

Form Factors

Definition of Form Factors:

Form Factors

Definition of Form Factors:

For $B_c \to \eta_c$ transition

$$\langle M(p_2)|\bar{q}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = \left[(p_1+p_2)_\mu - \frac{m_{B_c}^2 - m^2}{q^2}q_\mu\right]F_+(q^2) + \frac{m_{B_c}^2 - m^2}{q^2}q_\mu F_0(q^2),$$

↓ Pseudo-scalar meson

Form Factors

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For $B_c \to \eta_c$ transition

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Pseudo-scalar meson

For $B_c \to J/\psi$ transition

$$\langle M(p_2)|\bar{q}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = \epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu *} p_1^\alpha p_2^\beta \frac{2V(q^2)}{m_{B_c}+m},$$

$$\langle M(p_2)|\bar{q}(0)\gamma_\mu\gamma_5 b(0)|B_c(p_1)\rangle = i\left[\epsilon_\mu^* - \frac{\epsilon^*\cdot q}{q^2} q_\mu\right](m_{B_c}+m) A_1(q^2) - i\left[(p_1+p_2)_\mu - \frac{m_{B_c}^2 - m^2}{q^2} q_\mu\right](\epsilon^*\cdot q)\frac{A_2(q^2)}{m_{B_c}+m} + i\frac{2m(\epsilon^*\cdot q)}{q^2} q_\mu A_0(q^2),$$

Vector meson

Branching Ratio

Branching Ratio

For $B_c \to \eta_c\, l\, \nu$

$$\frac{d\Gamma(b \to c l \bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_B^3} \left(1 - \frac{m_l^2}{q^2}\right)^2 \frac{\lambda^{1/2}(q^2)}{2q^2} \cdot \left\{3m_l^2 \left(m_B^2 - m_D^2\right)^2 |F_0(q^2)|^2 + \left(m_l^2 + 2q^2\right) \lambda(q^2) |F_+(q^2)|^2\right\},$$

$$\lambda(q^2) = (m_B^2 + m_{D*}^2 - q^2)^2 - 4m_B^2 m_{D*}^2$$

Branching Ratio

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$$\lambda(q^2) = (m_B^2 + m_{D^*}^2 - q^2)^2 - 4 m_B^2 m_{D^*}^2$$

For $B_c \to J/\psi \, l \, \nu$

Branching Ratio

Longitude part:

$$\frac{d\Gamma_{\mathrm{L}}}{dq^2} = \frac{G_{\mathrm{F}}^2|V_{\mathrm{ub}}|^2}{192\pi^3 m_{\mathrm{B_c}}^3}\left(1-\frac{m_{\mathrm{l}}^2}{q^2}\right)^2\frac{\lambda^{1/2}(q^2)}{2q^2}\cdot\left\{3m_{\mathrm{l}}^2\lambda(q^2)A_0^2(q^2)\right.$$

$$\left.+\frac{m_{\mathrm{l}}^2+2q^2}{4m_{\mathrm{M}}^2}\cdot\left[(m_{\mathrm{B_c}}^2-m_{\mathrm{M}}^2-q^2)(m_{\mathrm{B_c}}+m_{\mathrm{M}})A_1(q^2)-\frac{\lambda(q^2)}{m_{\mathrm{B_c}}+m_{\mathrm{M}}}A_2(q^2)\right]^2\right\},$$

Branching Ratio

Longitude part:

$$\frac{d\Gamma_{\rm L}}{dq^2}=\frac{G_{\rm F}^2|V_{\rm ub}|^2}{192\pi^3 m_{\rm B_c}^3}\left(1-\frac{m_{\rm l}^2}{q^2}\right)^2\frac{\lambda^{1/2}(q^2)}{2q^2}\cdot\Bigg\{3m_{\rm l}^2\lambda(q^2)A_0^2(q^2)$$

$$+\frac{m_{\rm l}^2+2q^2}{4m_{\rm M}^2}\cdot\left[(m_{\rm B_c}^2-m_{\rm M}^2-q^2)(m_{\rm B_c}+m_{\rm M})A_1(q^2)-\frac{\lambda(q^2)}{m_{\rm B_c}+m_{\rm M}}A_2(q^2)\right]^2\Bigg\},$$

Transverse part:

$$\frac{d\Gamma_{\pm}}{dq^2}=\frac{G_F^2|V_{\rm ub}|^2}{192\pi^3 m_{\rm B_c}^3}\left(1-\frac{m_{\rm l}^2}{q^2}\right)^2\frac{\lambda^{3/2}(q^2)}{2}$$

$$\cdot\left\{(m_{\rm l}^2+2q^2)\left[\frac{V(q^2)}{m_{\rm B_c}+m_{\rm M}}\mp\frac{(m_{\rm B_c}+m_{\rm M})A_1(q^2)}{\sqrt{\lambda(q^2)}}\right]^2\right\},$$

Branching Ratio

Longitude part:

$$\frac{d\Gamma_{\rm L}}{dq^2}=\frac{G_{\rm F}^2|V_{\rm ub}|^2}{192\pi^3 m_{\rm B_c}^3}\left(1-\frac{m_{\rm l}^2}{q^2}\right)^2\frac{\lambda^{1/2}(q^2)}{2q^2}\cdot\left\{3m_{\rm l}^2\lambda(q^2)A_0^2(q^2)\right.$$
$$\left.+\frac{m_{\rm l}^2+2q^2}{4m_{\rm M}^2}\cdot\left[(m_{\rm B_c}^2-m_{\rm M}^2-q^2)(m_{\rm B_c}+m_{\rm M})A_1(q^2)-\frac{\lambda(q^2)}{m_{\rm B_c}+m_{\rm M}}A_2(q^2)\right]^2\right\},$$

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$$\cdot\left\{(m_{\rm l}^2+2q^2)\left[\frac{V(q^2)}{m_{\rm B_c}+m_{\rm M}}\mp\frac{(m_{\rm B_c}+m_{\rm M})A_1(q^2)}{\sqrt{\lambda(q^2)}}\right]^2\right\},$$

Total differential decay widths:

$$\frac{d\Gamma_T}{dq^2}=\frac{d\Gamma^+}{dq^2}+\frac{d\Gamma^-}{dq^2},\qquad \frac{d\Gamma}{dq^2}=\frac{d\Gamma_L}{dq^2}+\frac{d\Gamma_T}{dq^2}.$$

Contents

Transition matrix elements:

$$=> \qquad < M \,|\, J_{(A/V)} \,|\, Bc >$$

In PQCD factorization approach

$$A \sim \int dx_1\, dx_2 b_1 db_1 b_2 db_2$$

$$\times Tr[\Phi_{Bc}(x_1, b_1)\Phi_M(x_2, b_2)H(x_i, b_i, t)e^{-s(t)}]$$

Contents

Bc meson wave function

$$\Phi_{B_c}(x,b) = \frac{i}{\sqrt{2N_c}}(\not{p}_1 + m_{B_c})\gamma_5\phi_{B_c}(x,b).$$

Bc meson distribution amplitudes

$$\phi_{B_c}(x,b) = \frac{f_{B_c}}{2\sqrt{2N_c}} N_{B_c} x(1-x) exp\left[-\frac{(1-x)m_c^2 + x m_b^2}{8\beta_{B_c}^2 x(1-x)}\right] exp\left[-2\beta_{B_c}^2 x(1-x)b^2\right]$$

normalization constant N_{Bc} *is fixed by the relation:*

$$\int_0^1 \phi_{B_c}(x, b=0)dx \equiv \int_0^1 \phi_{B_c}(x)dx = \frac{f_{B_c}}{2\sqrt{2N_c}}$$

In order to analyze the uncertainties of theoretical predictions induced by the inputs, we set $\beta_{Bc} = 1.0 \pm 0.2$

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Contents

η_c meson wave function

$$\Phi_{\eta_c}(x) = \frac{i}{\sqrt{2N_c}}\gamma_5[\not{p}\phi^v(x) + m_{\eta_c}\phi^s(x)].$$

η_c meson distribution amplitudes

$$\phi^v(x) = 9.58\frac{f_{\eta_c}}{2\sqrt{2N_c}}x(1-x)\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7},$$

$$\phi^s(x) = 1.97\frac{f_{\eta_c}}{2\sqrt{2N_c}}\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7}.$$

Contents

J/ψ meson wave function

$$\Phi^L_{J/\Psi}(x) = \frac{1}{\sqrt{2N_c}}\left\{m_{J/\Psi}\not\epsilon_L\phi^L(x) + \not\epsilon_L\not p\phi^t(x)\right\},$$

$$\Phi^T_{J/\Psi}(x) = \frac{1}{\sqrt{2N_c}}\left\{m_{J/\Psi}\not\epsilon_T\phi^V(x) + \not\epsilon_T\not p\phi^T(x)\right\}.$$

J/ψ meson distribution amplitudes

$$\phi^L(x) = \phi^T(x) = 9.58\frac{f_{J/\Psi}}{2\sqrt{2N_c}}x(1-x)\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7},$$

$$\phi^t(x) = 10.94\frac{f_{J/\Psi}}{2\sqrt{2N_c}}(1-2x)^2\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7},$$

$$\phi^V(x) = 1.67\frac{f_{J/\Psi}}{2\sqrt{2N_c}}[1+(2x-1)^2]\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7}.$$

Contents

For example : $B_c \to \eta_c$

$$\langle M(p_2)|\bar{q}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = \left[(p_1+p_2)_\mu - \frac{m_{B_c}^2 - m^2}{q^2}q_\mu\right]F_+(q^2) + \frac{m_{B_c}^2 - m^2}{q^2}q_\mu F_0(q^2),$$

Pseudo-scalar meson

Contents

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↓ Pseudo-scalar meson

Define the auxiliary form factors:

$$F_+(q^2) = \frac{1}{2}\left[f_1(q^2) + f_2(q^2)\right],$$

$$F_0(q^2) = \frac{1}{2} f_1(q^2)\left[1 + \frac{q^2}{m_{\mathrm{B_c}}^2 - m^2}\right] + \frac{1}{2} f_2(q^2)\left[1 - \frac{q^2}{m_{\mathrm{B_c}}^2 - m^2}\right],$$

Contents

For example : $B_c \to \eta_c$

$$\langle M(p_2)|\bar{q}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = \left[(p_1+p_2)_\mu - \frac{m_{B_c}^2 - m^2}{q^2} q_\mu\right] F_+(q^2) + \frac{m_{B_c}^2 - m^2}{q^2} q_\mu F_0(q^2),$$

Pseudo-scalar meson

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Then:

$$\langle M(p_2)|\bar{u}(0)\gamma^\mu b(0)|B_c(p_1)\rangle = p_1^\mu f_1(q^2) + p_2^\mu f_2(q^2).$$

Contents

$$f_1(q^2) = 8\pi m_{B_c}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \; \phi_{B_c}(x_1, b_1)$$

$$\times \Big\{ \big[(-2r^2 x_2)\phi^v(x_2) + 2r(2 - r_b)\phi^s(x_2)\big]$$

$$\times\, h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp\left[-S_{ab}(t_1)\right]$$

$$+ \left[(-2r^2 + \frac{r x_1 \eta^+ \eta^+}{\sqrt{\eta^2 - 1}})\phi^v(x_2) + (4 r r_c - \frac{2 x_1 r \eta^+}{\sqrt{\eta^2 - 1}})\phi^s(x_2)\right]$$

$$\times\, h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp\left[-S_{ab}(t_2)\right] \Big\},$$

$$f_2(q^2) = 8\pi m_{B_c}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \; \phi_{B_c}(x_1, b_1)$$

$$\times \Big\{ \big[(4r_b - 2 + 4x_2 r \eta)\phi^v(x_2) + (-4 r x_2)\phi^s(x_2)\big]$$

$$\times\, h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp\left[-S_{ab}(t_1)\right]$$

$$+ \left[(-2r_c - \frac{x_1 \eta^+}{\sqrt{\eta^2 - 1}})\phi^v(x_2) + (4r + \frac{2x_1}{\sqrt{\eta^2 - 1}})\phi^s(x_2)\right]$$

$$\times\, h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp\left[-S_{ab}(t_2)\right] \Big\},$$

Contents

1. PQCD predictions for the considered form factors are much more reliable at low q^2 region

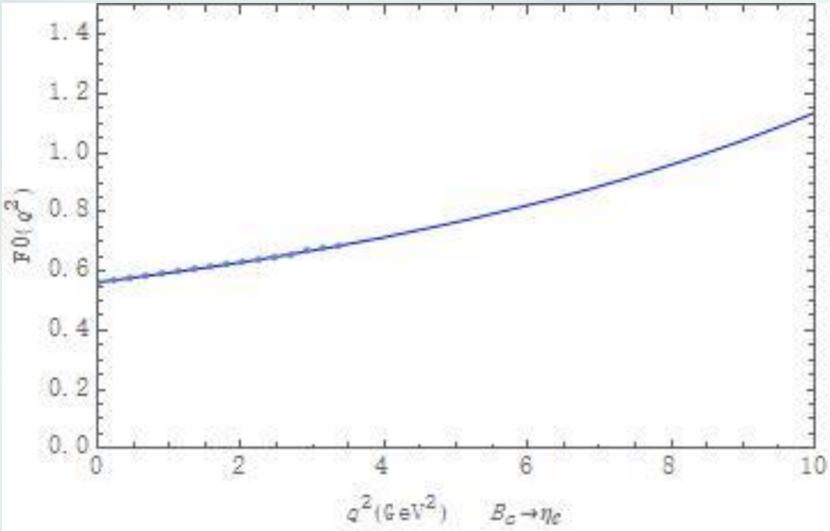
2. For the form factors in the larger q^2 region, one has to make an extrapolation

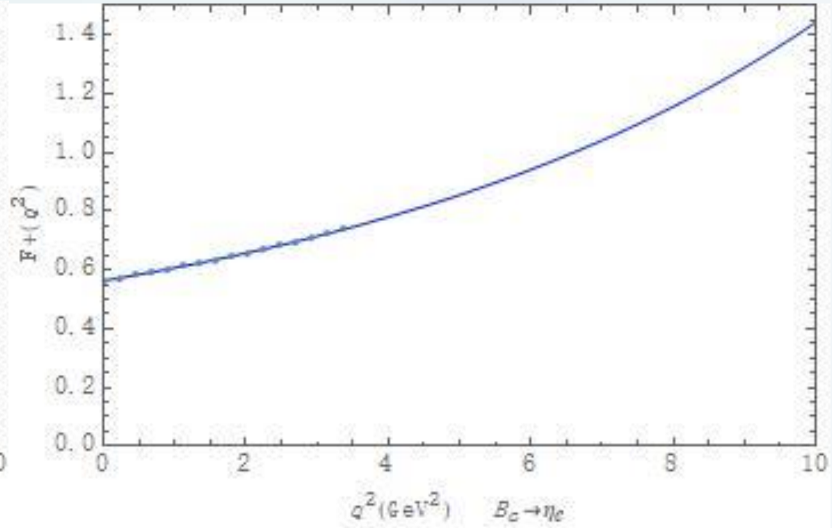
3. In this work we make the extrapolation by using the formula:

$$F(q^2) = F(0) \cdot \exp\left[a \cdot q^2 + b \cdot (q^2)^2\right].$$

Contents

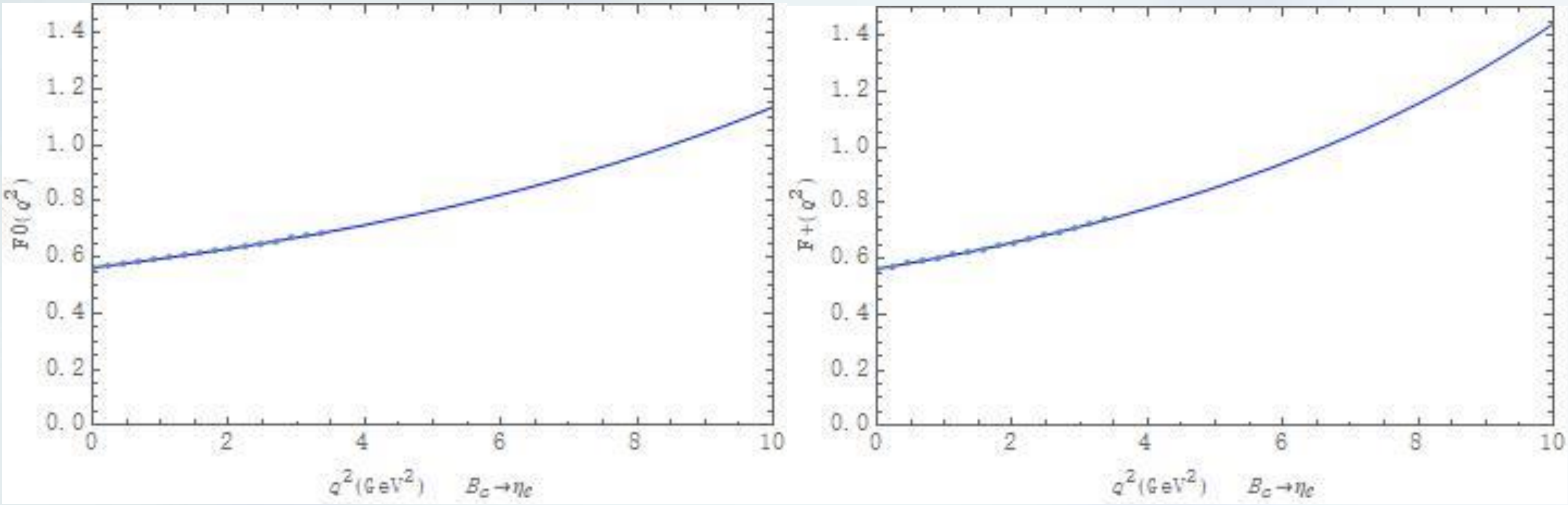
$$B_c \to \eta_c\, l\, \nu \qquad (l = e, \mu, \tau)$$





Contents

$$B_c \to \eta_c \, l \, \nu \qquad (l = e, \mu, \tau)$$

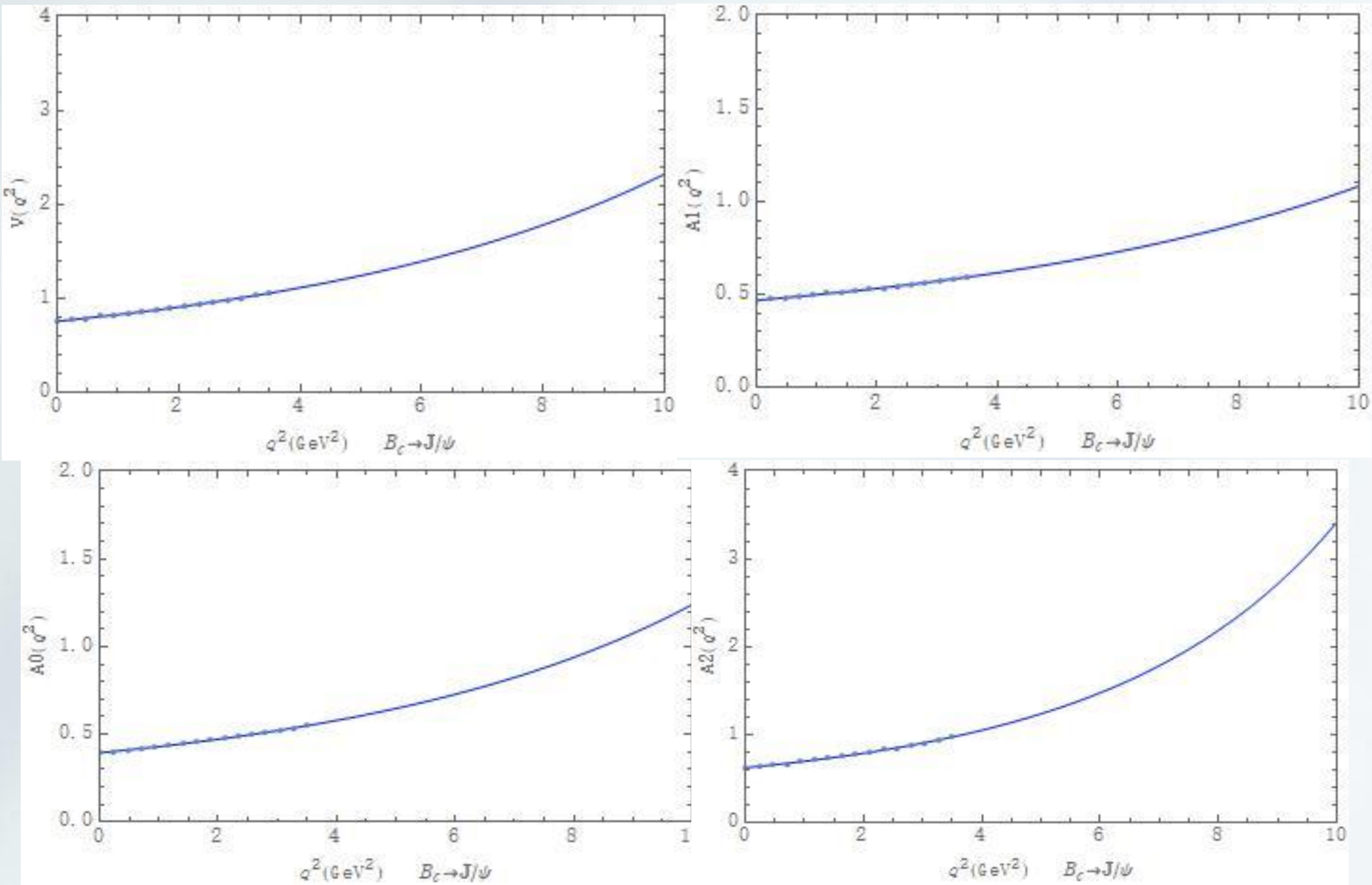


q^2 at 0~4 GeV^2 region : PQCD prediction

q^2 at 4~10 GeV^2 region : Extrapolation

Contents

$$B_c \to J/\psi\, l\, \nu \qquad (l = e, \mu, \tau)$$



Result

Numerical results of form factors:

	this work	LFQM	BSW	NRCQM
$F_0(0)$	0.56	0.61	0.58	0.49
$F_+(0)$	0.56	0.61	0.58	0.49
$V(0)$	0.75	0.74	0.91	0.61
$A_0(0)$	0.40	0.53	0.58	0.45
$A_1(0)$	0.47	0.5	0.63	0.49
$A_2(0)$	0.62	0.44	0.74	0.56

Result

Numerical results of Br:

	this work	LFQM	SMEFT	PQCD
$B_c \to \eta_c \mu \nu$	0.78	0.67	0.48	0.44
$B_c \to \eta_c \tau \nu$	0.29	0.19	0.15	0.14
$B_c \to J/\psi \mu \nu$	1.66	1.49	1.14	1.03
$B_c \to J/\psi \tau \nu$	0.50	0.37	0.33	0.29

Result

The theoretical uncertainties of Brs are large

$$\mathcal{B}(B_c \to \eta_c \tau \bar{\nu}_\tau) = \left(2.93^{+2.09}_{-1.19}(\beta_{\mathrm{B_c}}) \pm 0.22(V_{\mathrm{cb}}) \pm 0.08(m_{\mathrm{c}})\right) \times 10^{-3},$$
$$\mathcal{B}(B_c \to \eta_c l \bar{\nu}_{\mathrm{l}}) = \left(7.87^{+5.55}_{-3.00}(\beta_{\mathrm{B_c}}) \pm 0.53(V_{\mathrm{cb}}) \pm 0.27(m_{\mathrm{c}})\right) \times 10^{-3},$$
$$\mathcal{B}(B_c \to J/\psi \tau \bar{\nu}_\tau) = \left(4.98^{+3.77}_{-2.01}(\beta_{\mathrm{B_c}}) \pm 0.37(V_{\mathrm{cb}}) \pm 0.19(m_{\mathrm{c}})\right) \times 10^{-3},$$
$$\mathcal{B}(B_c \to J/\psi l \bar{\nu}_{\mathrm{l}}) = \left(1.66^{+1.16}_{-0.64}(\beta_{\mathrm{B_c}}) \pm 0.12(V_{\mathrm{cb}}) \pm 0.06(m_{\mathrm{c}})\right) \times 10^{-2}.$$

Main errors come from the parameter β_{Bc}

Result

However, the theoretical uncertainties can be significantly reduced in the prediction for the ratio of branching ratios .

$$R(\eta_c) = 0.373^{+0.001}_{-0.016}$$

$$R(J/\psi) = 0.300^{+0.010}_{-0.008}$$

The error of the PQCD predictions for all R(X)-ratios are around ~10% only

Summary

1. *PQCD predictions for the branching ratios* $B_c \to (\eta_c, J/\psi)\, l\, \nu$ *agree well with other SM predictions.*

2. *Although the theoretical uncertainties of Brs are large, The error of the* $R(\eta_c)$ *and* $R(J/\psi)$ *are around* $\sim 10\%$ *only.*

3. *There is still a discrepancy since the experiment result is* $R(J/\psi)=0.71\pm0.17(stat)\pm0.18(syst)$.

Thank you