

Production of doubly heavy-flavored hadrons at e^+e^- colliders

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Based on Xu-Chang Zheng and Chao-Hsi Chang et al,

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Outline

1. Introduction
2. Production of doubly heavy-flavored hadrons
3. NLO fragmentation functions for $B_c(B_c^*)$ production
4. Conclusions and outlook

1、Introduction

$\approx 2.4 \text{ MeV}/c^2$ $2/3$ $1/2$ u up	$\approx 1.275 \text{ GeV}/c^2$ $2/3$ $1/2$ c charm	$\approx 172.44 \text{ GeV}/c^2$ $2/3$ $1/2$ t top
$\approx 4.8 \text{ MeV}/c^2$ $-1/3$ $1/2$ d down	$\approx 95 \text{ MeV}/c^2$ $-1/3$ $1/2$ s strange	$\approx 4.18 \text{ GeV}/c^2$ $-1/3$ $1/2$ b bottom

Heavy flavor hadron: A hadron containing c- or b-quark

Heavy quarkonium: $c\bar{c}$, $b\bar{b}$ (J/Ψ , Υ ...)

Explicit double heavy-flavored meson: $c\bar{b}$ (B_c , B_c^* ...)

Double heavy baryon: $QQ'q$

- Production: perturbative, non-perturbative QCD
- Decay: weak interaction

Advantages of the production at e^+e^- colliders:

- The center-of-mass system of the process is known

Angle distributions and forward-backward asymmetry of hadrons have proper meaning in understanding the production.

- There are less backgrounds at an e^+e^- collider

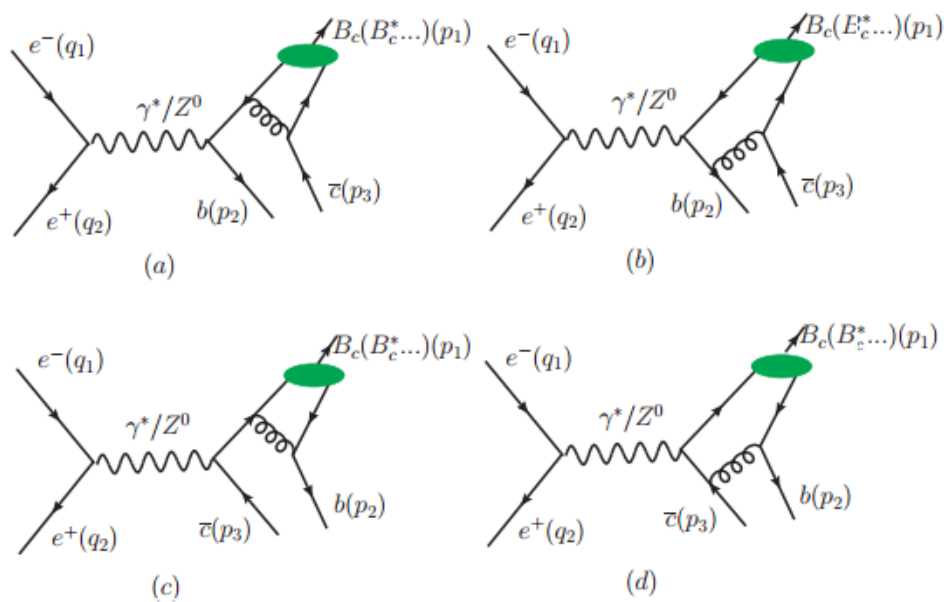
e^+e^- collider is a good platform for precision measurements.

Running at Z-pole:

- Z-resonance effect
- ILC, FCC-ee, CEPC and Super Z-factory

2、 Production of doubly heavy-flavored hadrons

● LO calculation



$$d\sigma(e^+ + e^- \rightarrow Bc + b + \bar{c})$$

$$= \sum_n d\hat{\sigma}(e^+ + e^- \rightarrow c\bar{b}[n] + b + \bar{c}) \langle O^{Bc}(n) \rangle$$

NRQCD factorization

Numerical results

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contribution	total	\bar{b} -frag.	c -frag.	interference
$\sigma(B_c, {}^1S_0)$	2.734	2.613	5.20×10^{-2}	6.90×10^{-2}
$\sigma(B_c^*, {}^3S_1)$	3.823	3.722	4.45×10^{-2}	5.65×10^{-2}
$\sigma(B_c^{**}, {}^1P_1)$	0.271	0.269	3.01×10^{-3}	-1.01×10^{-3}
$\sigma(B_c^{**}, {}^3P_0)$	0.164	0.157	8.13×10^{-3}	-1.13×10^{-3}
$\sigma(B_c^{**}, {}^3P_1)$	0.340	0.331	5.77×10^{-3}	3.23×10^{-3}
$\sigma(B_c^{**}, {}^3P_2)$	0.365	0.366	3.87×10^{-4}	-1.39×10^{-3}

Cross section at Z pole (unit: pb)

Contribution	Total	\bar{b} frag.	c frag.
$\sigma(B_c, {}^1S_0)$	2.851	2.792	5.88×10^{-2}
$\sigma(B_c^*, {}^3S_1)$	3.974	3.923	5.08×10^{-2}
$\sigma(B_c^{**}, {}^1P_1)$	0.296	0.292	3.53×10^{-3}
$\sigma(B_c^{**}, {}^3P_0)$	0.169	0.160	9.08×10^{-3}
$\sigma(B_c^{**}, {}^3P_1)$	0.361	0.354	6.59×10^{-3}
$\sigma(B_c^{**}, {}^3P_2)$	0.395	0.395	4.74×10^{-4}

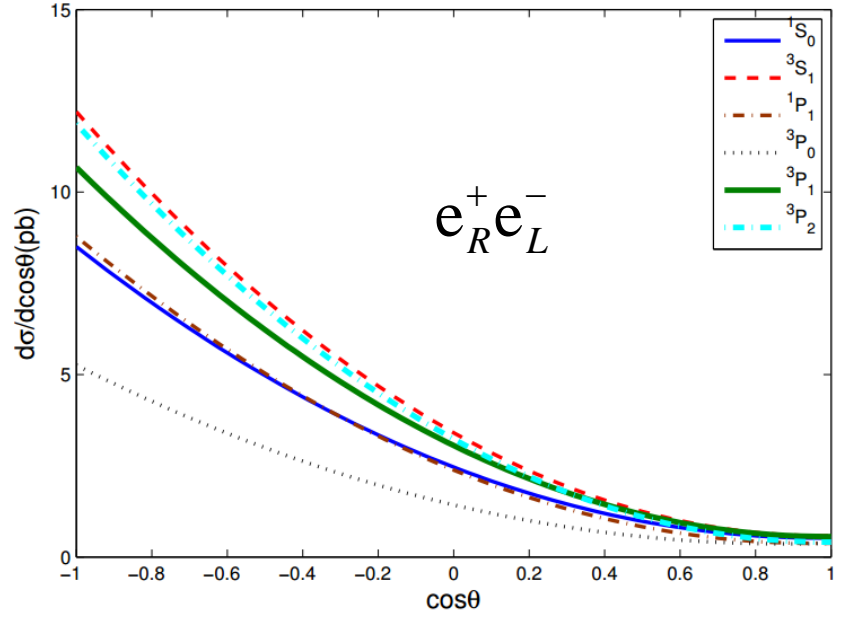
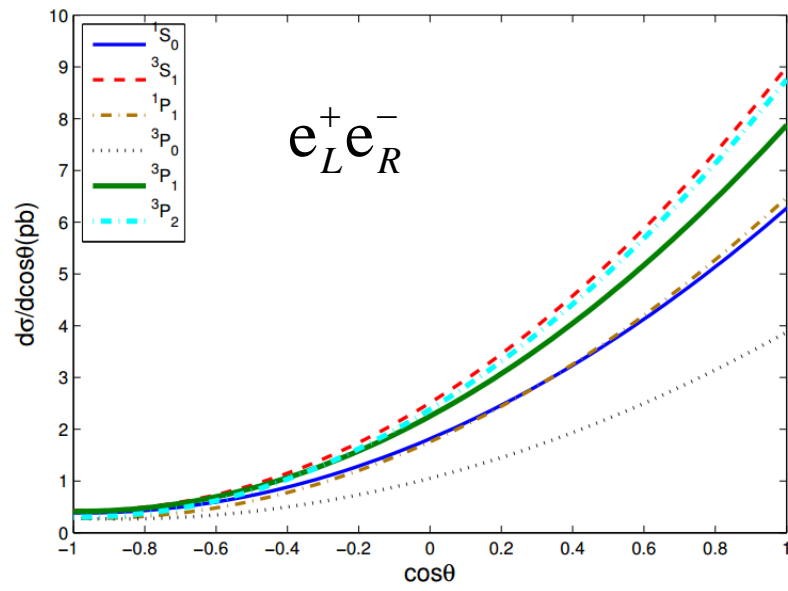
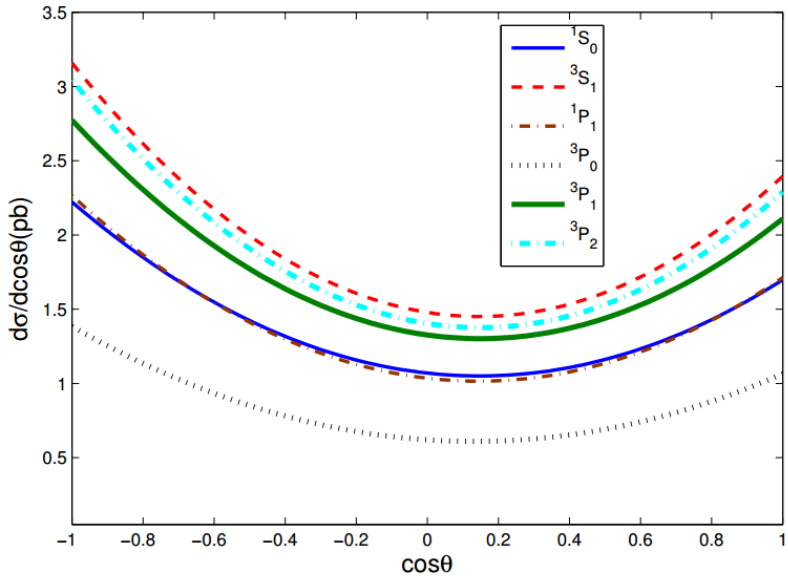
Results under fragmentation approach(unit: pb)

\sqrt{s} (GeV)	180	240
$\sigma(B_c, {}^1S_0)$	1.05	0.47
$\sigma(B_c^*, {}^3S_1)$	1.55	0.72
$\sigma(B_c^{**}, {}^1P_1)$	0.11	0.05
$\sigma(B_c^{**}, {}^3P_0)$	0.07	0.03
$\sigma(B_c^{**}, {}^3P_1)$	0.14	0.07
$\sigma(B_c^{**}, {}^3P_2)$	0.15	0.07

Cross section at other energies (unit: fb)

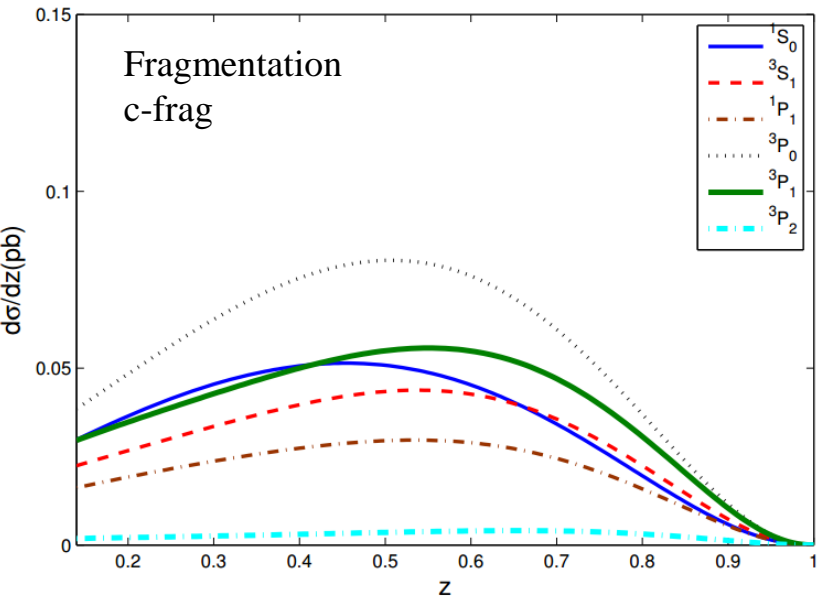
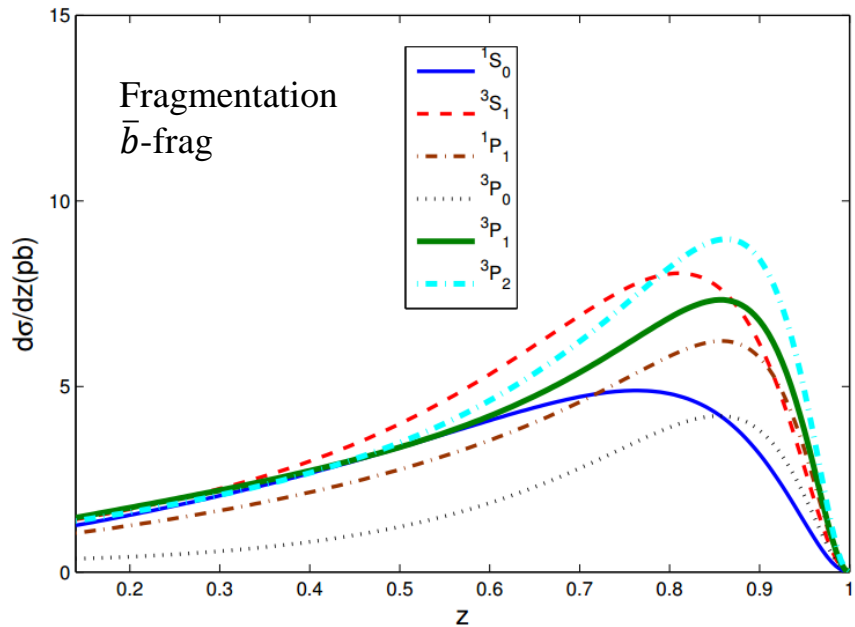
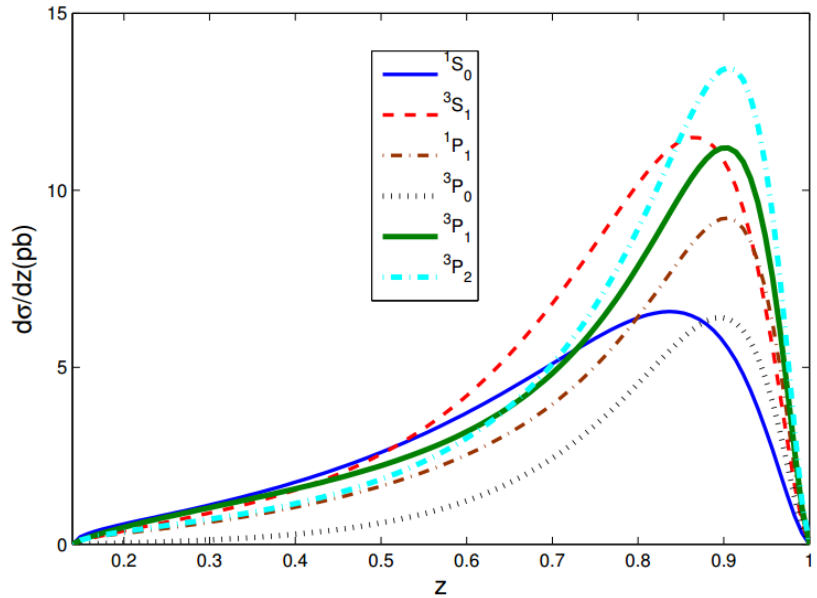
The luminosity of Z factory should be $10^{35-36} \text{cm}^{-2} \text{s}^{-1}$

**Phys. Rev. D 93, 034019, (2016),
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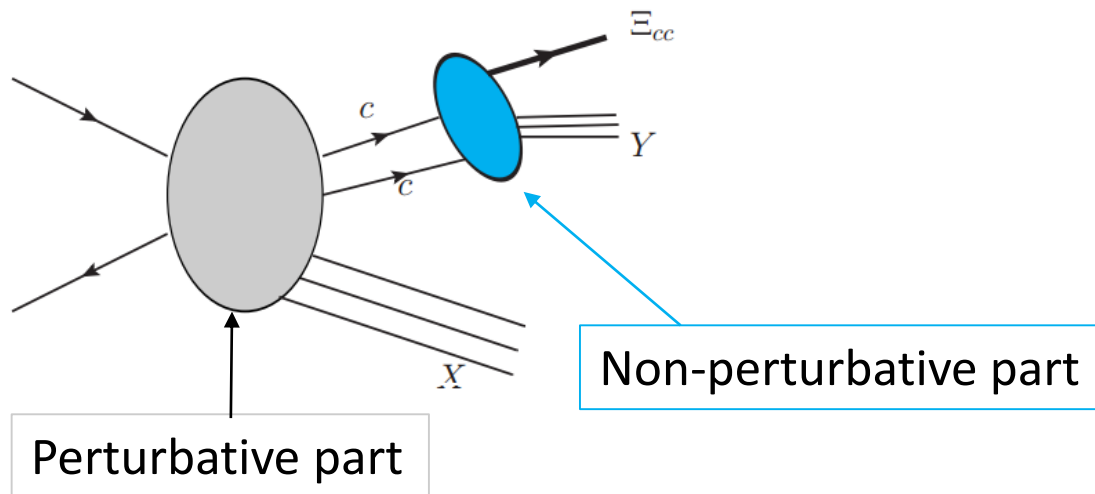


The angle distribution is **forward-backward asymmetric**.

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Production of doubly heavy baryons



1) Production of diquark

2) The diquark fragments into the doubly heavy baryon

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Cross section:

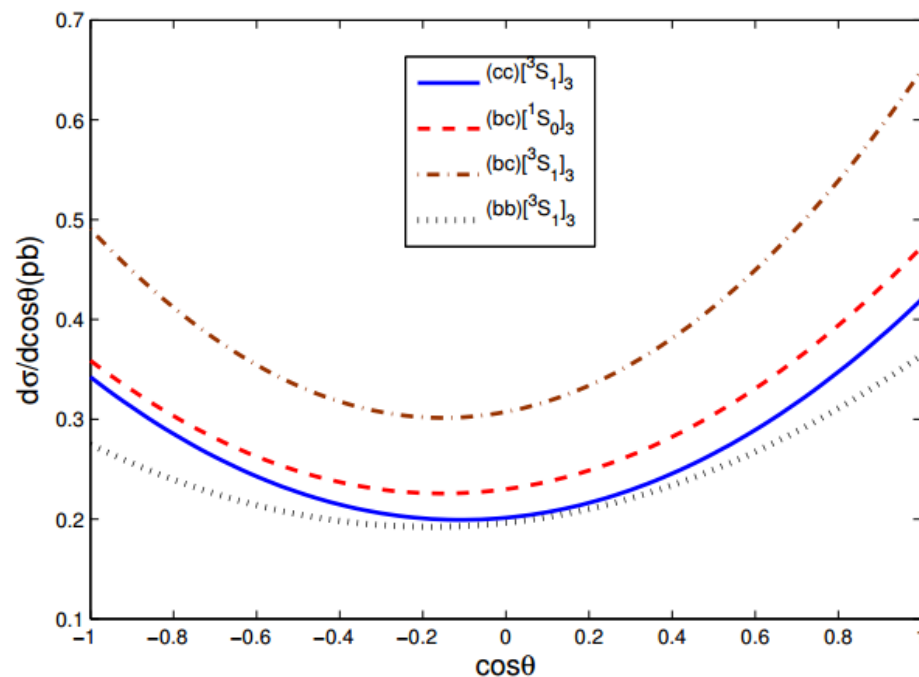
$$\sigma_{|(cc)_{\bar{3}}, {}^3S_1\rangle} = 0.52 \text{ pb},$$

$$\sigma_{|(bc)_{\bar{3}}, {}^1S_0\rangle} = 0.58 \text{ pb},$$

$$\sigma_{|(bc)_{\bar{3}}, {}^3S_1\rangle} = 0.79 \text{ pb},$$

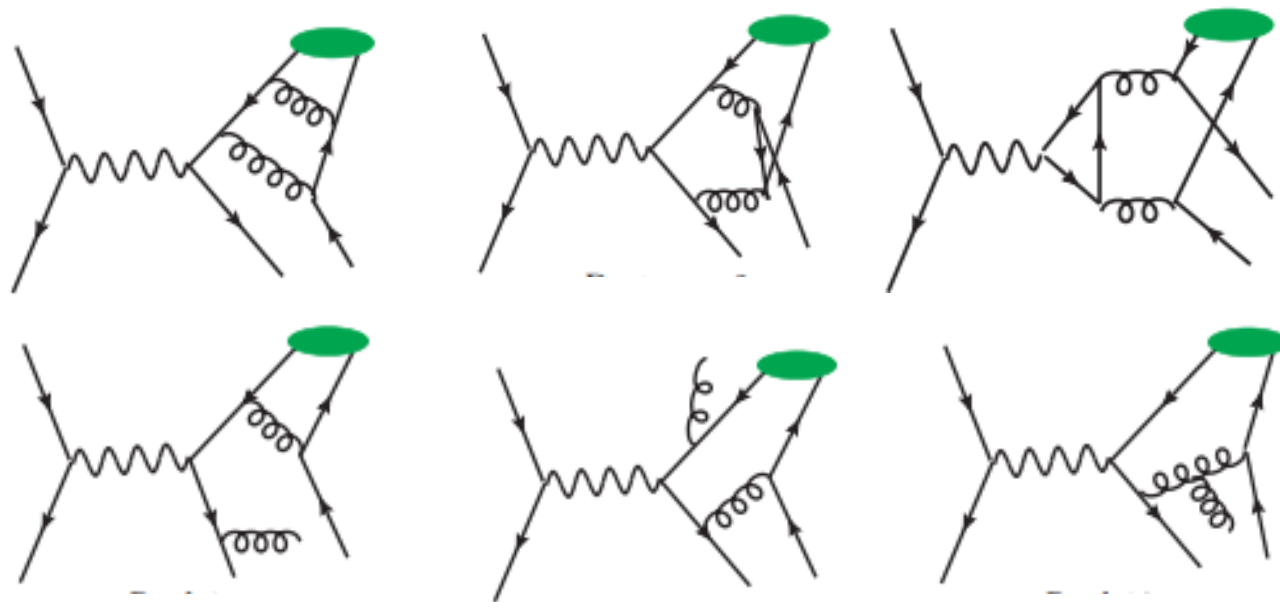
$$\sigma_{|(bb)_{\bar{3}}, {}^3S_1\rangle} = 0.05 \text{ pb}.$$

Differential angle distribution



- NLO calculations for B_c and B_c^*

- To see the changes of the physical observables from the LO calculations to the NLO calculations.
- To see how the dependence on the renormalization scale changes after including the NLO QCD corrections.



Calculation method

- Virtual corrections

 - Passarino-Veltman tensor reduction,

 - Integration-by-parts (IBP) reduction

- Real corrections

 - The two-cutoff phase-space slicing method

Difficulties in the calculations:

- Two mass scales (m_b, m_c)

- Vector and axial-vector couplings

Numerical results

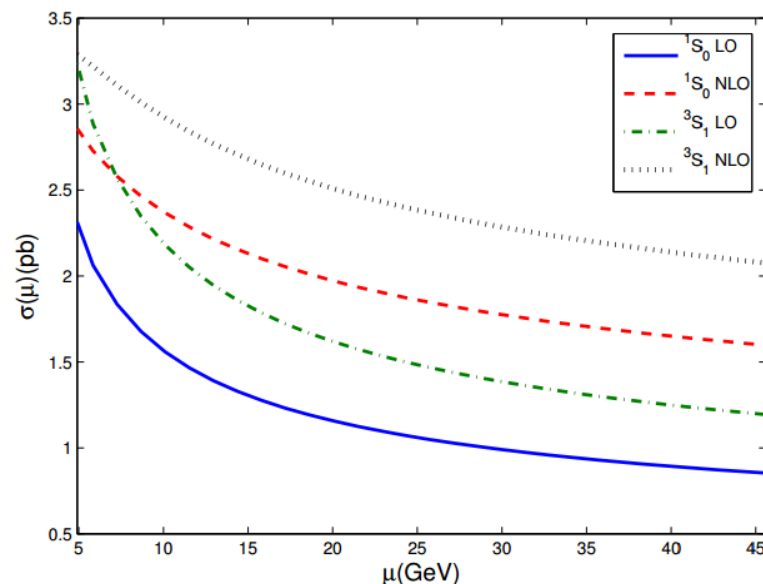
Sci. China-Phys.Mech. Astron. 61, 031012,(2018),
 Xu-Chang Zheng, Chao-Hsi Chang et al.

μ	$\alpha_s(\mu)$	$\sigma_{\text{LO}}(\text{pb})$	$\sigma_{\text{NLO}}(\text{pb})$	$\sigma_{\text{NLO}}/\sigma_{\text{LO}}$
$2m_b$	0.180	1.58	2.38	1.51
$m_Z/2$	0.132	0.85	1.58	1.86

Cross section of Bc

μ	$\sigma_{\text{LO}}(\text{pb})$	$\sigma_{\text{NLO}}(\text{pb})$	$\sigma_{\text{NLO}}/\sigma_{\text{LO}}$
$2m_b$	2.20	2.93	1.33
$m_Z/2$	1.18	2.06	1.74

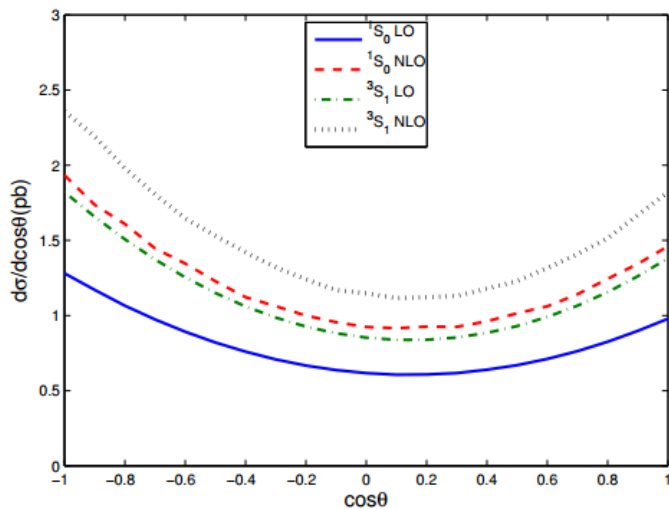
Cross section of Bc*



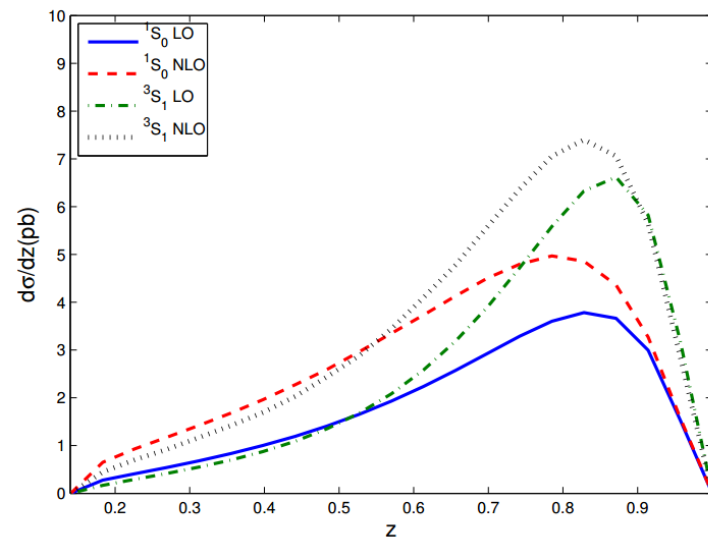
The dependence on μ is weakened significantly due to NLO corrections.

The NLO corrections are significant!

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Differential angle distribution



Differential energy distribution

The K-factor changes very little with different θ ;

The NLO corrections change the energy distribution significantly.

3、 NLO fragmentation functions for $B_c(B_c^*)$ production

Fragmentation function approach

➤ NRQCD factorization

$$\begin{aligned}
 & d\sigma(e^+ + e^- \rightarrow Bc + b + \bar{c}) \\
 &= \sum_n d\sigma(e^+ + e^- \rightarrow (c\bar{b})[n] + b + \bar{c}) \langle O^{Bc}(n) \rangle
 \end{aligned}$$

Energy scales:
 \sqrt{s}, m_Q

Log-terms appear in short-distance coefficients:

$$\alpha_s^m \sum_{n=0}^{\infty} \alpha_s^n \ln^n(s / m_Q^2)$$

Collinear gluon emission

Broke or weak the convergence of the series

$\ln(p_t^2 / m_Q^2)$ appearing in the production at a hadron collider

➤ Perturbative QCD factorization

$$d\sigma(e^+ + e^- \rightarrow B_c(p) + b + \bar{c}) = \sum_i d\hat{\sigma}(e^+ + e^- \rightarrow i + X)(p/z, \mu_F) \otimes D_{i \rightarrow B_c}(z, \mu_F) + O(m_Q^2/s)$$

$\mu_F = O(\sqrt{s})$

Involving $\ln(s/\mu_F^2)$

Fragmentation function

NRQCD factorization:

$$D_{i \rightarrow B_c}(z, \mu_{F0}) = \sum_n d_{i \rightarrow c\bar{b}[n]}(z, \mu_{F0}) \langle O^{B_c}(n) \rangle$$

$\mu_{F0} = O(m_Q)$

Involving $\ln(\mu_{F0}^2/m_Q^2)$

Evolution of fragmentation functions

$$\frac{d}{d \ln \mu_F^2} D_{i \rightarrow B_c}(z, \mu_F) = \sum_j P_{ij}(z/y, \alpha_s(\mu_F)) \otimes D_{i \rightarrow B_c}(y, \mu_F)$$

$$P_{ij}(z, \alpha_s(\mu_F)) = P_{ij}^{(0)}(z) \frac{\alpha_s(\mu_F)}{2\pi} + P_{ij}^{(1)}(z) \left(\frac{\alpha_s(\mu_F)}{2\pi} \right)^2 + O(\alpha_s^3)$$

Collinear log-terms have been resummed through DGLAP evolution.

➤ LO fragmentation function

Extracting from the LO calculation of process $Z^0 \rightarrow B_c + b + \bar{c}$

C-H. Chang, Y-Q. Chen, Phys. Rev. D 46, 3845, (1992);

Calculating from the definition:

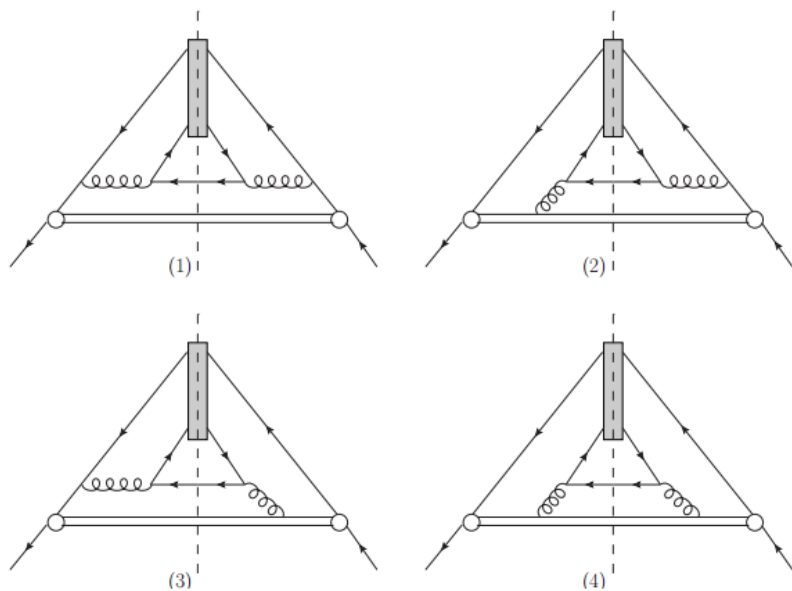
J-P. Ma, Phys. Lett. B332, 398, (1994);

There are no NLO results for $D_{i \rightarrow B_c}(z, \mu_F)$

In order to obtain the theoretical predictions under fragmentation approach up to NLO QCD accuracy, the NLO results for $D_{i \rightarrow B_c}(z, \mu_F)$ are needed.

Fragmentation function calculation

LO cut diagram



Based on the definition of FFs by Collins and Soper.

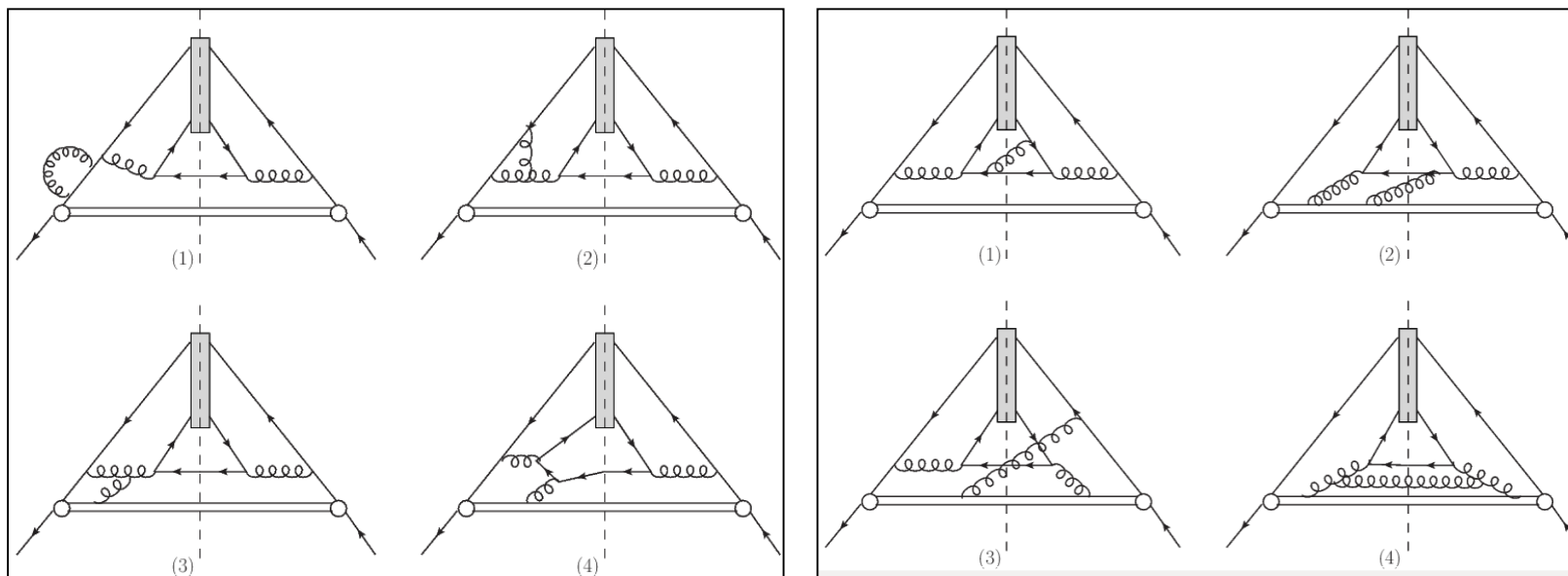
Nucl. Phys. B 194, 445, (1982).

$$D_{\bar{b} \rightarrow B_c}^{\text{LO}}(z) = \frac{2\alpha_s^2 z(1-z)^2 |R_S(0)|^2}{81\pi r_c^2 (1-r_b z)^6 M^3} [6 - 18(1-2r_c)z + (21 - 74r_c + 68r_c^2)z^2 - 2r_b(6 - 19r_c + 18r_c^2)z^3 + 3r_b^2(1 - 2r_c + 2r_c^2)z^4],$$

$$D_{\bar{b} \rightarrow B_c^*}^{\text{LO}}(z) = \frac{2\alpha_s^2 z(1-z)^2 |R_S(0)|^2}{27\pi r_c^2 (1-r_b z)^6 M^3} [2 - 2(3-2r_c)z + 3(3-2r_c+4r_c^2)z^2 - 2r_b(4-r_c+2r_c^2)z^3 + r_b^2(3-2r_c+2r_c^2)z^4].$$

NLO corrections

Typical cut diagrams



54 virtual cut diagrams, 72 real cut diagrams.

➤ Virtual correction

Tensor reduction, IBP reduction

Many integrals containing an eikonal line

$$\int \frac{d^D l}{[(l - p_1)^2 - m_1^2 + i\varepsilon][(l - p_2)^2 - m_2^2 + i\varepsilon][(l - p_3)^2 - m_3^2 + i\varepsilon](l \cdot n + i\varepsilon)}$$

➤ Real correction

UV and IR divergence!

$$D_{\bar{b} \rightarrow B_c}^{real}(z) = \int N_{CS} d\phi_{real} (A_{real} - A_S) + \int N_{CS} d\phi_{real} A_S$$

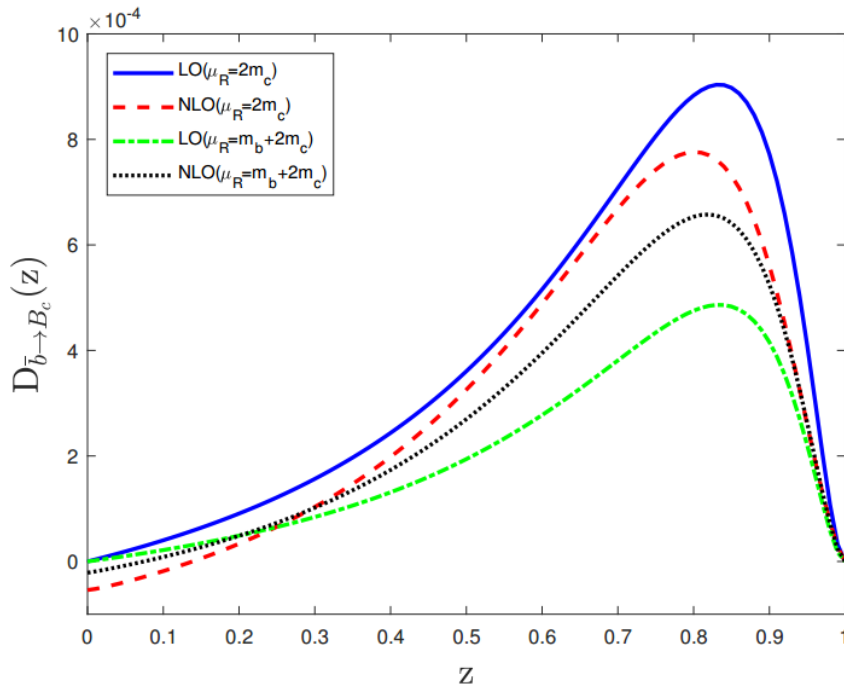
Calculated in
4-dimension

Calculated in
d-dimension

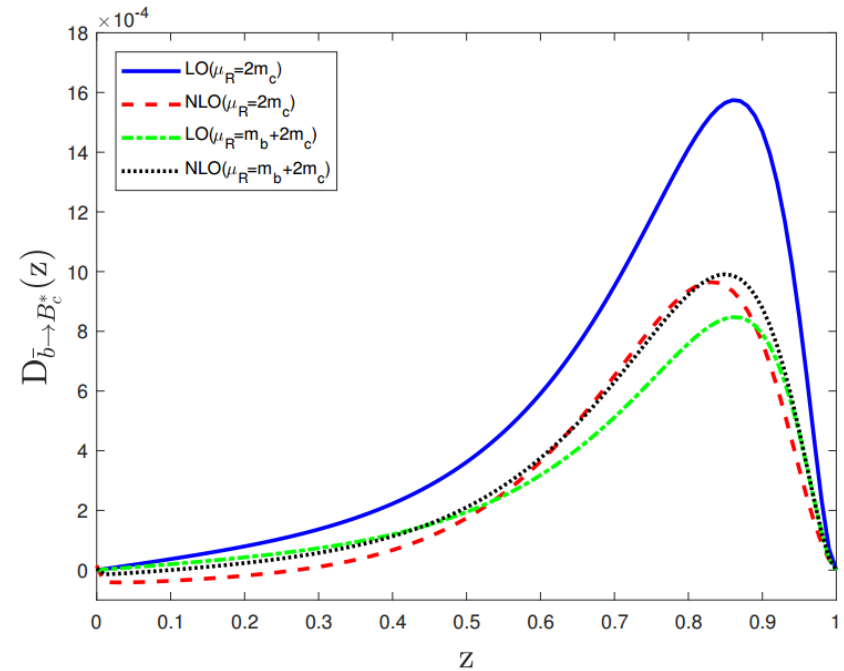
Various types of subtraction terms need to be integrated.

NLO results

Initial fragmentation functions



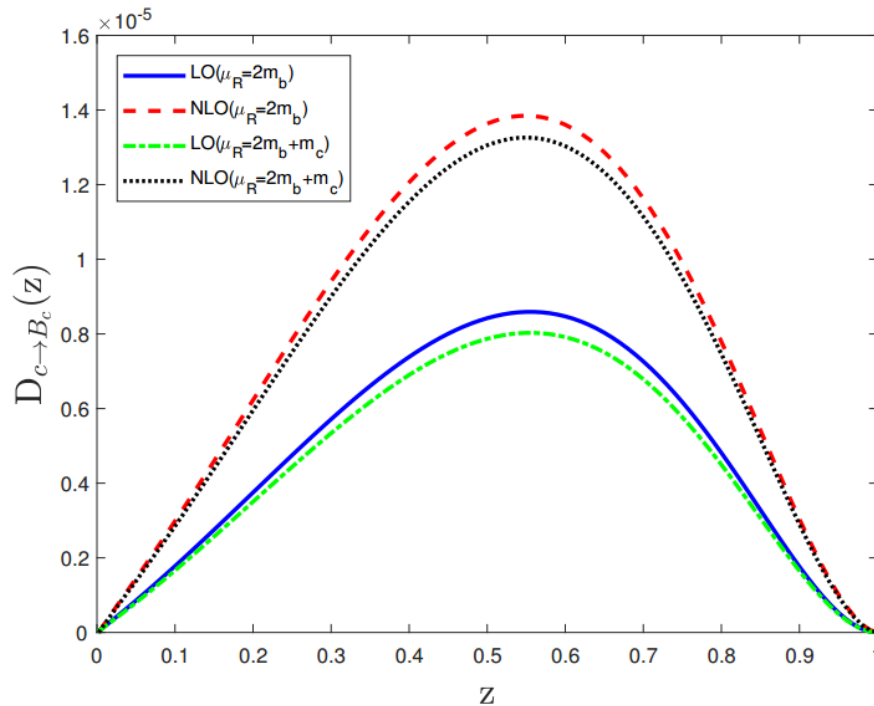
$$D_{\bar{b} \rightarrow B_c}(z, \mu_{F0} = m_b + 2m_c)$$



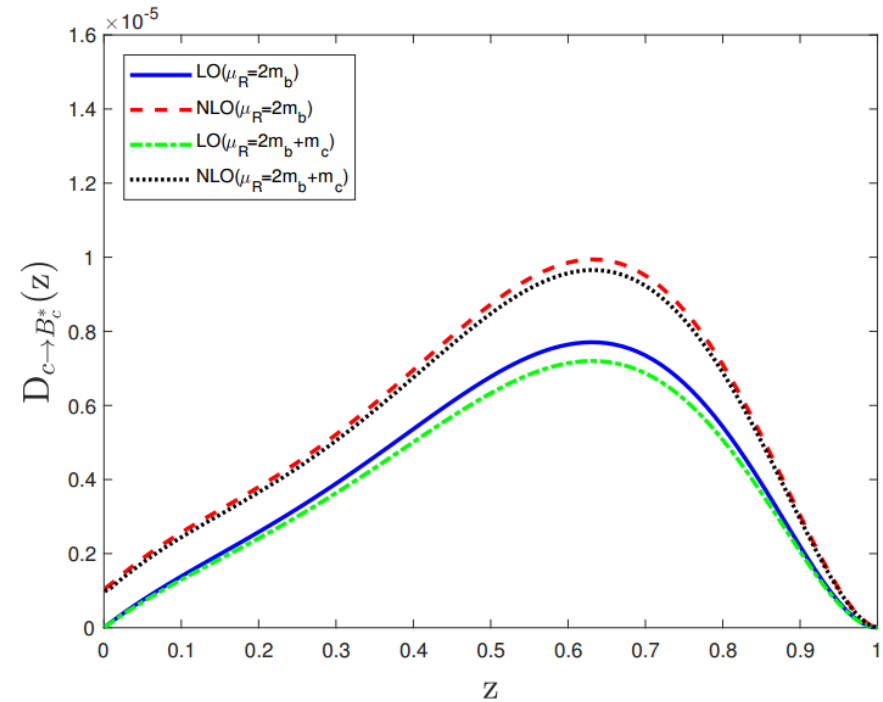
$$D_{\bar{b} \rightarrow B_c^*}(z, \mu_{F0} = m_b + 2m_c)$$

NLO results

Initial fragmentation functions



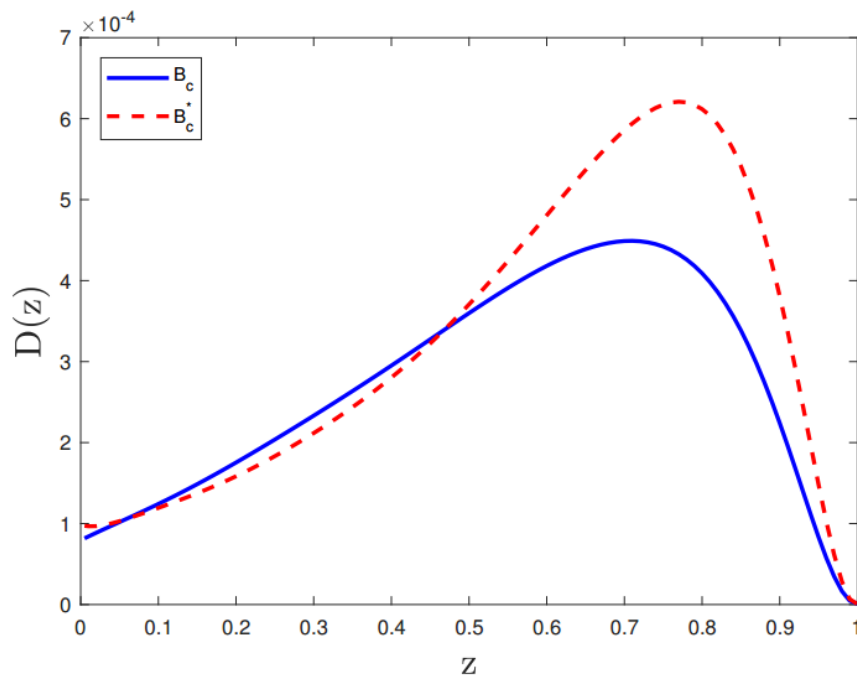
$$D_{c \rightarrow B_c}(z, \mu_{F0} = m_b + 2m_c)$$



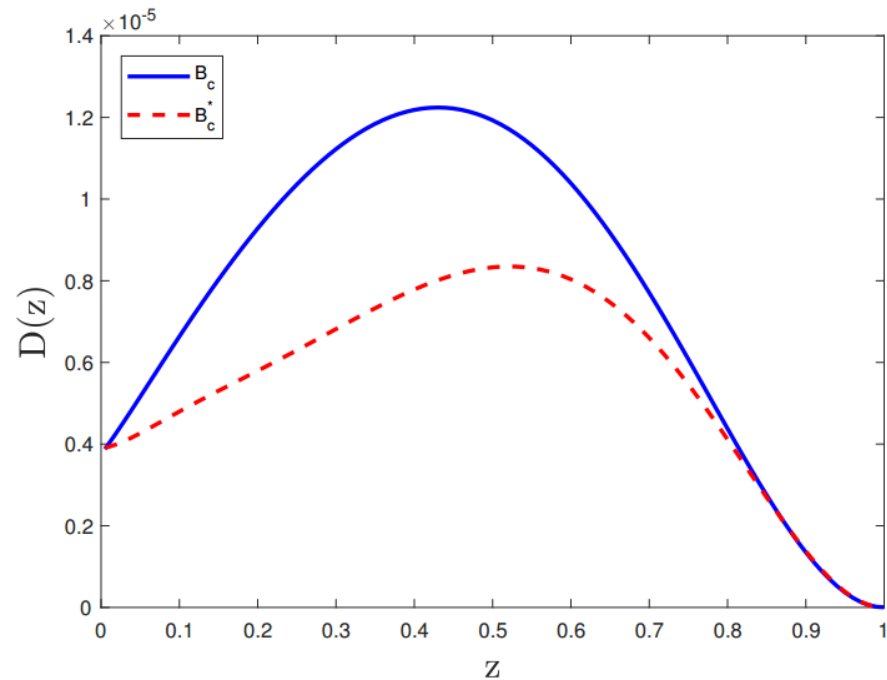
$$D_{c \rightarrow B_c^*}(z, \mu_{F0} = m_b + 2m_c)$$

NLO results

The fragmentation functions at scale of m_Z

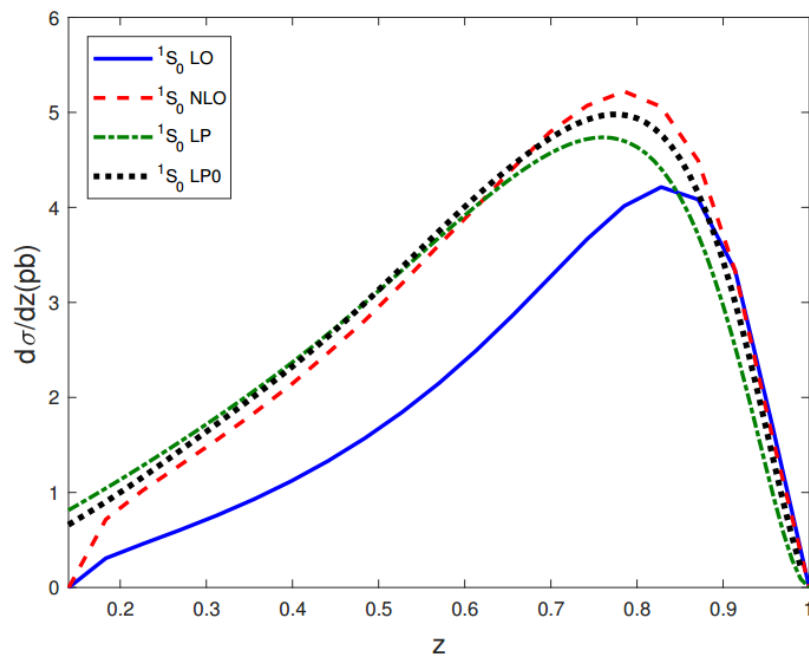


$$D_{\bar{b} \rightarrow B_c(B_c^*)}(z, \mu_F = m_Z)$$

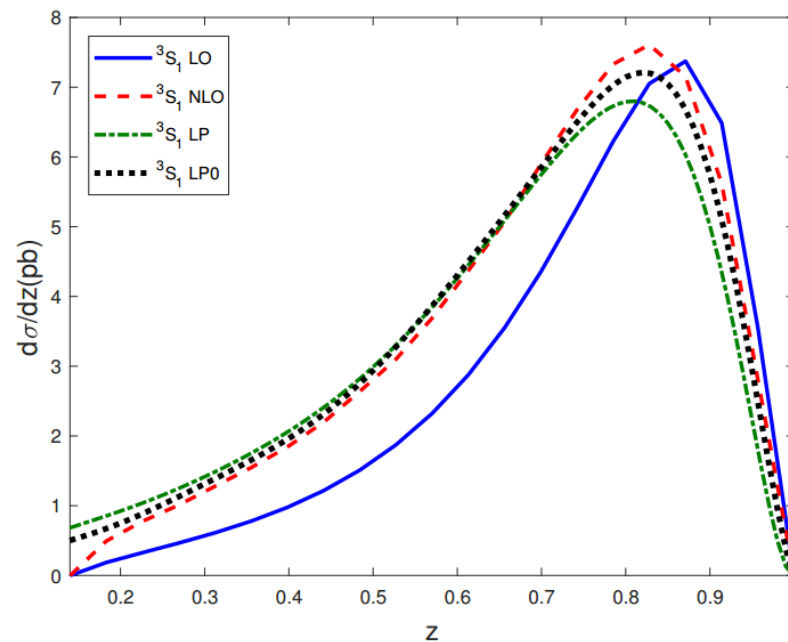


$$D_{c \rightarrow (B_c)B_c^*}(z, \mu_F = m_Z)$$

B_c and B_c^* production at a Z-factory



$d\sigma / dz(B_c)$



$d\sigma / dz(B_c^*)$

LP0: fragmentation approach, no DGLAP evolution

LP: fragmentation approach, evolved with DGLAP equation

Conclusions

- The production of various doubly heavy hadrons at e^+e^- colliders is studied;
- There are many interesting properties in the production, such as the angle distribution is forward-backward asymmetric;
- The NLO corrections to the processes $e^+ + e^- \rightarrow Bc (Bc^*) + b + \bar{c}$ and the FFs for a heavy quark fragmentation into $Bc(Bc^*)$ are significant.

Outlook

- To study the production of the Bc meson at the LHC up to NLO QCD accuracy;
- To calculate the fragmentation functions for the production of other doubly heavy hadrons up to NLO QCD accuracy.

Thank you !