# Recent Progress in pQCD and Application to LHC Physics

李重生

北京大学物理学院理论物理研究所

全国重味物理和CP破坏讨论会，河南工业大学

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# introduction

## Progress in perturbative QCD calculations

### 1. Building blocks of QCD corrections up to NNLO

- At next-to-next-to leading order (NNLO), one can distinguish three basic contributions to the scattering amplitude: The two-loop virtual part AVV, the one-loop amplitude with one extra parton that can become unresolved (soft or collinear) ARV, and the tree-level amplitude ARR, where two extra partons can become unresolved.

## 2. NNLO double real radiation

- The isolation of the infrared singularities from ARR was for a long time a bottleneck which hampered progress in the construction of fully differential Monte Carlo programs for NNLO predictions.

- This situation however changed drastically in the last few years, and partly can be traced back to the fact that insights about the universal infrared behaviour of QCD, partly gained from resummation or Soft-Collinear Effective Theory.

- The method of qT -subtraction or N-jettiness has been employed to obtain NNLO results for LHC processes involving colourless final states or at most one jet .

- The proceeses H+jet , Z+jet and di-jets at NNLO, as well as single jet inclusive and di-jet production in DIS have been calculated based on antenna subtraction .

# 3. Loop integrals and two-loop amplitudes

- At two loops, many remarkable achievements can be reported, and an impressive number of differential NNLO results for 2 ->2 processes became available recently.

- The integrals entering top quark pair production at NNLO  have been calculated numerically , analytic results are partially available . The NNLO corrections to top quark decay have been calculated.

- The analytic calculation of two-loop four-point integrals with both massive propagators and massive/off-shell external legs is currently one of the most vibrant topics in the field of precision calculations.

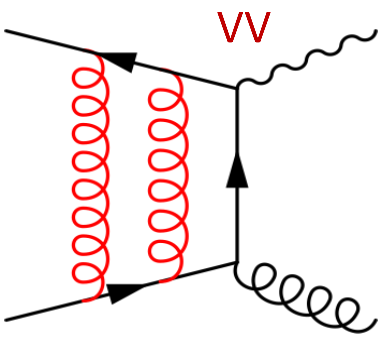
- Note that the NLO corrections to these processes involve two loops, as the leading order already proceeds via a loop. The analytic calculation of the two-loop integrals entering Higgs+jet and di-Higgs involves a new level of complexity due to the fact that the results contain new function classes, involving elliptic integrals. Nonetheless, results for the planar case have been achieved .

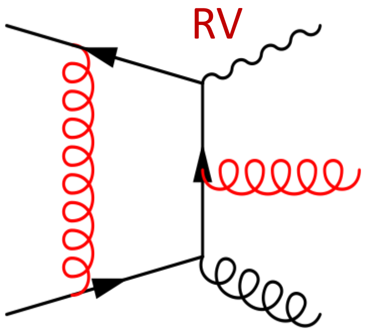
# 4. Multi-loop front

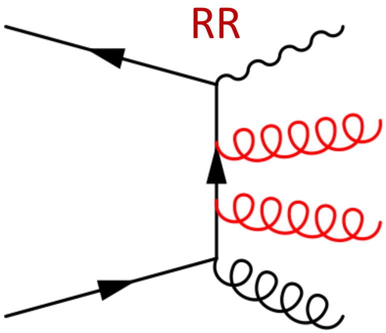
- At the multi-loop front, among the most remarkable recent achievements are the five-loop QCD beta function and four-loop contributions to the cusp anomalous dimension and N3LO splitting functions , three-loop corrections to the heavy flavour Wilson coefficients in DIS with two different masses , and the N3LO calculations for Higgs boson production in gluon fusion and in vector boson fusion .

# Ingredients for NNLO calculation

- Typical NNLO calculation consists of double virtual (VV), real-virtual (RV), and double real (RR) corrections.







- **VV: explicit IR poles from loops**
  - **Challenge: construction of amplitudes integrand and the resulting integrals**
- **RV and RR: implicit IR poles in phase space integrals**
  - **Challenge: regularizing IR divergent phase space integral and extracting IR poles**

# Two-loop virtual amplitudes

- Advances in methodology:
  - Integration by Parts relation, and its automation
  - Differential equation for Feynman integral
  - Better understanding of (generalized) polylogarithms
- These advances lead to the calculation of all the 2→2 scattering amplitudes with phenomenological relevance
    - Di-jet production: Anastasiou, Glover, Oleari, Tejeda-Yeomans; Bern, De Freitas, Dixon
  - Photon+jet production: Anastasiou, Glover, Tejeda-Yeomans
  - W+jet production: Gehrmann, Tancredi
  - Z+jet production: Gehrmann, Tancredi, Weihs
  - H+jet production: Gehrmann, Jaquier, Glover, Koukoutsakis
  - diboson production: Gehrmann, von Manteuffel, Tancredi; Caola, Henn, Melnikov, Smirnov, Smirnov
  - top-quark pair production: Czakon, Mitov; Bonciani, Ferroglia, Gehrmann, von Manteuffel, Studerus
  - ……
- Foundation for the flourish of NNLO calculations in recent years

# Methods for IR regularization

- It is only after the mature of IR regularization scheme that leads to phenomenology relevant NNLO predictions
- The flourish of IR regularization methods under the name of subtraction:
    - Sector decomposition: Anastasiou, Melnikov, Petriello; Binoth, Heinrich
    - Antenna subtraction: Gehrmann, Gehrmann-De Ridder, Glover
    - Sector-improved residue subtraction: Czakon
    - CoLoRFulNNLO subtraction: Del Duca, Somogyi, Trocsanyi
    - Projection-to-Born: Cacciari, Dreyer, Karlberg, Salam, Zanderighi
- Or phase space slicing
    - qT subtraction: Catani, Grazzini
    - Inclusive jet mass: JGao, CSLi, HXZ
    - N-jettiness subtraction: Boughezal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh

# Towards N$^3$LO QCD calculation

In principle, N-jettiness subtraction is still available at N$^3$LO!

N-jettiness: $$\tau_N = \frac{2}{Q^2}\sum_k \min\{q_a\cdot p_k, q_b\cdot p_k, q_1\cdot p_k, ..., q_N\cdot p_k\}$$

Phase space cut: $$\sigma(X) = \underbrace{\int_0^{\tau_N^{\rm cut}} d\tau_N \frac{d\sigma(X)}{d\tau_N}}_{\text{unresolved, IR safe}} + \underbrace{\int_{\tau_N^{\rm cut}} d\tau_N \frac{d\sigma(X)}{d\tau_N}}_{\text{resolved, IR safe}}$$

In the limit $\tau_N^{\rm cut} \ll 1$, all the radiations should be soft or collinear to a beam or jet axis, and the cross section can be factorized in SCET

$$\frac{d\sigma}{d\tau_N} = \int dt_a B_a(t_a, x_a, \mu)\int dt_b B_b(t_b, x_b, \mu)\prod_{k=1}^{N}\int dp_k^2\, J(p_k^2, \mu)$$
$$\times\; H(\{p_a, p_b, p_k\}, \mu) S(\mathcal{T}_s, \{\hat{p}_a, \hat{p}_b, \hat{p}_k\}, \mu)\delta\left(\tau_N - \frac{t_a + t_b + Q\mathcal{T}_s + \sum p_k^2}{Q^2}\right) + \mathcal{O}(\tau_N)$$

At N$^3$LO, all the hard, soft and beam functions need to be calculated at three-loop level.

**A first step: three-loop quark jet function!** **Brueser, Liu, Stahlhofen, 18**

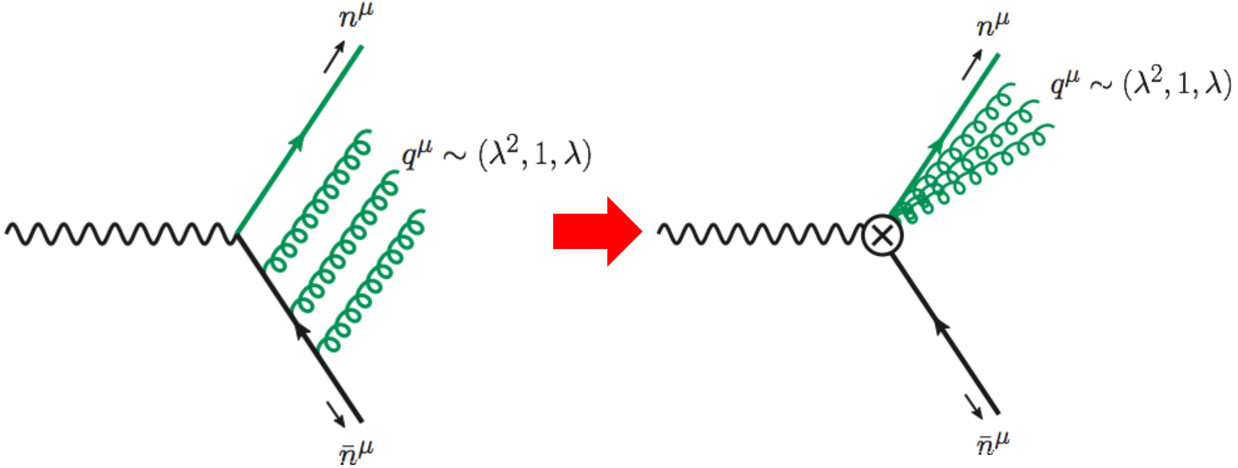
@ N$^3$LO, resolved part = X+1 jet @ NNLO

Brueser, 刘泽龙, Stahlhofen, PRL，2018

# Three-loop Quark Jet Function

Jet function is a universal ingredient in SCET factorization for many collider and decay process

$$q^\mu=(n\cdot q)\frac{\bar{n}^\mu}{2}+(\bar{n}\cdot q)\frac{n^\mu}{2}+q_\perp^\mu=(q^+,q^-,q_\perp)$$



The jet function can be extracted from hard collinear propagators:

$$\frac{\not{n}}{2}\bar{n}\cdot p\,\mathcal{J}(p^2,\mu)=\int d^4x\,e^{-ip\cdot x}\langle 0|\mathbf{T}\{\frac{\not{n}\not{\bar{n}}}{4}\boxed{W^\dagger(0)}\psi(0)\bar{\psi}(x)\boxed{W(x)}\frac{\not{\bar{n}}\not{n}}{4}\}|0\rangle$$

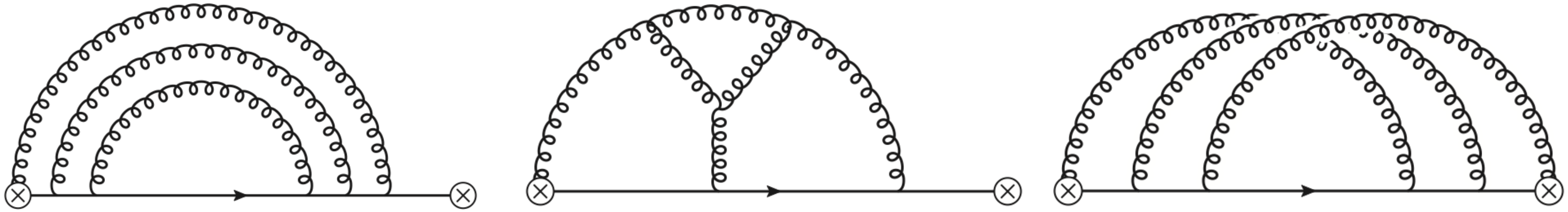
$$J(p^2,\mu)=\frac{1}{\pi}\mathrm{Im}\left[i\mathcal{J}(p^2,\mu)\right]$$

$$\boxed{W(x)=\mathbf{P}\exp\left(ig\int_{-\infty}^{0}ds\,\bar{n}\cdot A(x+s\bar{n})\right)}$$

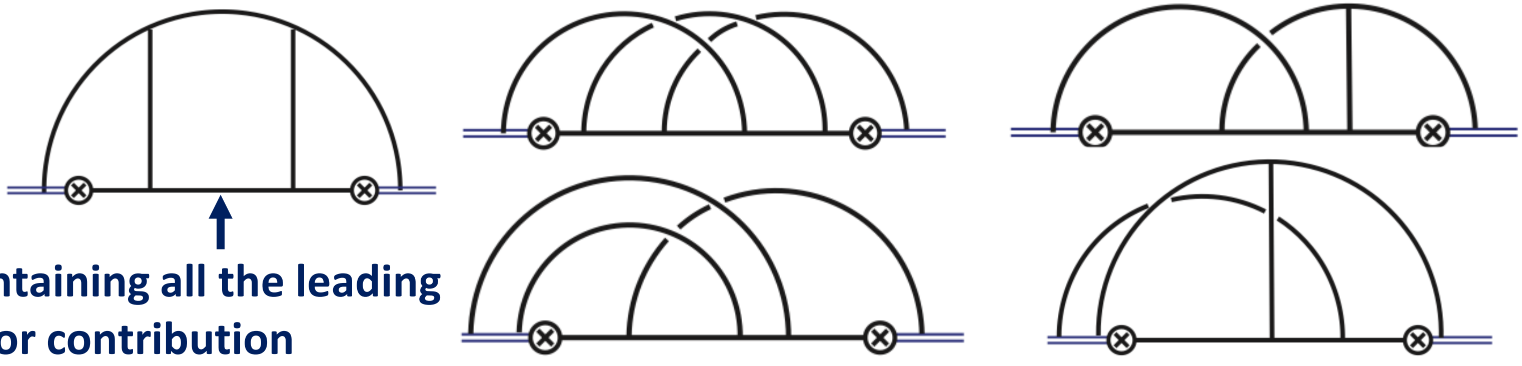
**The collinear Wilson Line encodes the collinear radiations.**

# Methods

1. Generate 3-loop Feynman diagrams and amplitudes
    - By QGRAF and Private codes
    - More than 600 diagrams and 8000 scalar Feynman integrals



2. Reduction of Feynman integrals
    - All the scalar integrals can be mapped to 5 topologies



**Containing all the leading color contribution**

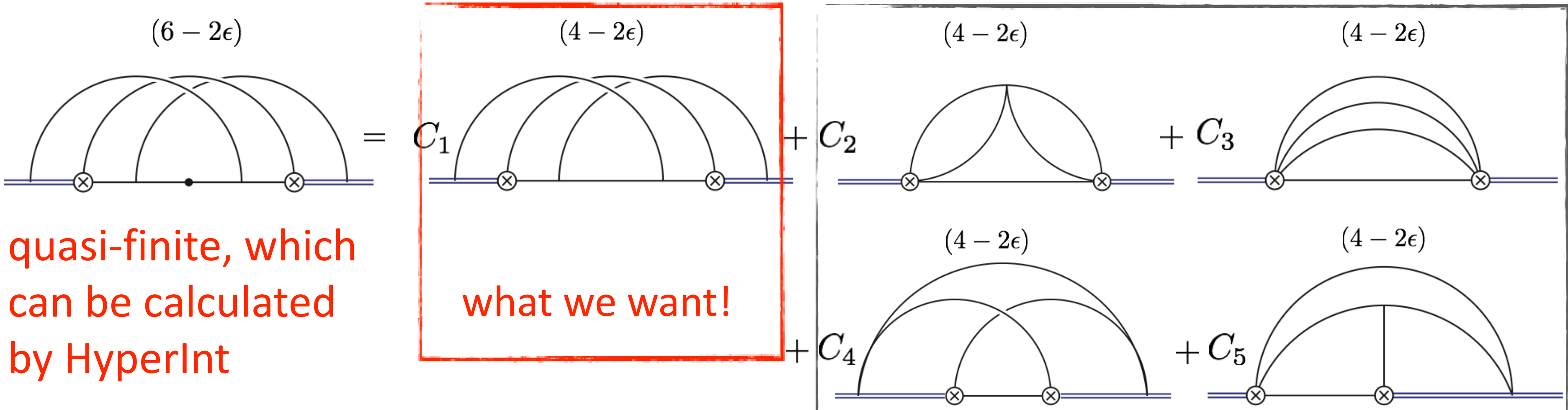
- Integration by part (IBP) reduction with FIRE5

3. Calculate 34 master integrals (MIs) with dimensional recurrence relation

# Calculation of the Master Integrals

- Mellin-Barnes + PSLQ algorithm
  - not easy to get analytical result for MIs with > 7 propagators
- Dimensional Recurrence Relation (DRR)
  - relate to a quasi-finite integrals in d+2 by dimensional recurrence

    A quasi-finite integral means either convergent or its divergence can be factored out in a simple way.



quasi-finite, which can be calculated by HyperInt

what we want!

with less propagators, easier to be obtained

**Manteuffel, Panzer, Schabinger, ’14 ’15**

# Analytical Result for 3-loop quark jet function

**Brueser, 刘泽龙, Stahlhofen, PRL，2018**

The result in momentum space can be expressed as

$$\boxed{J_i^{(m)}(s,\mu) = J_{i,-1}^{(m)}\,\delta(s) + \sum_{n=0}^{2m-1} J_{i,n}^{(m)}\,\frac{1}{\mu^2}\mathcal{L}_n\Big(\frac{s}{\mu^2}\Big)}$$

$$\begin{aligned}
J_{q,-1}^{(3)} = {}& C_F^3\Big(274\zeta_3 + \frac{22\pi^2\zeta_3}{3} - \frac{400\zeta_3^2}{3} - 88\zeta_5 + \frac{1173}{8} - \frac{3505\pi^2}{72} + \frac{622\pi^4}{45} - \frac{9871\pi^6}{8505}\Big)\\
&+C_A C_F^2\Big(-\frac{28241\zeta_3}{27} + \frac{2200\pi^2\zeta_3}{27} + \frac{424\zeta_3^2}{3} + \frac{560\zeta_5}{9} + \frac{206197}{324} - \frac{17585\pi^2}{72} + \frac{18703\pi^4}{1215} + \frac{1547\pi^6}{4860}\Big)\\
&+C_A^2 C_F\Big(-\frac{187951\zeta_3}{243} + \frac{394\pi^2\zeta_3}{9} + \frac{1528\zeta_3^2}{9} - \frac{380\zeta_5}{9} + \frac{50602039}{52488} - \frac{464665\pi^2}{4374} + \frac{1009\pi^4}{1620} + \frac{221\pi^6}{5103}\Big)\\
&+C_A C_F n_f T_F\Big(\frac{14828\zeta_3}{81} - \frac{64\pi^2\zeta_3}{9} + \frac{32\zeta_5}{3} - \frac{2942843}{6561} + \frac{136648\pi^2}{2187} - \frac{418\pi^4}{405}\Big)\\
&+C_F^2 n_f T_F\Big(\frac{22432\zeta_3}{81} - \frac{272\pi^2\zeta_3}{27} + \frac{160\zeta_5}{3} - \frac{261587}{486} + \frac{4853\pi^2}{54} - \frac{5876\pi^4}{1215}\Big)\\
&+C_F n_f^2 T_F^2\Big(\frac{1504\zeta_3}{243} + \frac{249806}{6561} - \frac{1864\pi^2}{243} + \frac{8\pi^4}{45}\Big)
\end{aligned}$$

The three-loop quark jet function is also necessary for the resummation of various jet shapes at e+e- collider beyond N$^3$LL accuracy. This is important to precise determination of QCD $\alpha_s$.

# NNLO massive N-jettiness soft function

Hai Tao Li, Jian Wang, Phys.Lett.B.784(2018)397

- So far, N-jettiness slicing method is successful for the NNLO differential calculation of jet production at hadron colliders.
- How about massive colored particle production, ttbar or tW?

$$\frac{d\sigma}{d\mathcal{O}} = \boxed{\left.\frac{d\sigma}{d\mathcal{O}}\right|_{\mathcal{T}_N<\Delta}} + \left.\frac{d\sigma}{d\mathcal{O}}\right|_{\mathcal{T}_N>\Delta}$$

$$\mathcal{T}_N = \sum_k \min_i \{ n_i \cdot q_k \} \,,$$

$$\Downarrow$$

$$\frac{d\sigma}{dY\,d\tau} = \int d\Phi_2 \frac{d\hat{\sigma}_0}{d\Phi_2} \int dt_a dt_b d\tau_s H(\beta_t, \cos\theta_t, \mu) B_1(t_a, x_a, \mu) B_2(t_b, x_b, \mu)$$

$$\times \boxed{S(\tau_s, \beta_t, \cos\theta_t, \mu)} \delta\left(\tau - \tau_s - \frac{t_a + t_b}{\sqrt{\hat{s}}}\right) \left(1 + \mathcal{O}\left(\frac{\tau}{\sqrt{\hat{s}}}\right)\right)$$

$$\Downarrow$$

$$\sum_{X_s} \left\langle 0 \middle| \bar{\mathbf{T}} Y_n^\dagger Y_{\bar{n}} Y_v \middle| X_s \right\rangle \delta\left(\tau - \sum_k \min\left(n \cdot \hat{P}_k, \bar{n} \cdot \hat{P}_k\right)\right) \left\langle X_s \middle| \mathbf{T} Y_n Y_{\bar{n}}^\dagger Y_v^\dagger \middle| 0 \right\rangle$$

$$v^+ = \frac{1 - \beta_t \cos\theta_t}{\sqrt{1 - \beta_t^2}}, \quad v^- = \frac{1 + \beta_t \cos\theta_t}{\sqrt{1 - \beta_t^2}}, \quad |v_\perp| = \frac{\beta_t \sin\theta_t}{\sqrt{1 - \beta_t^2}},$$

# NNLO massive N-jettiness soft function

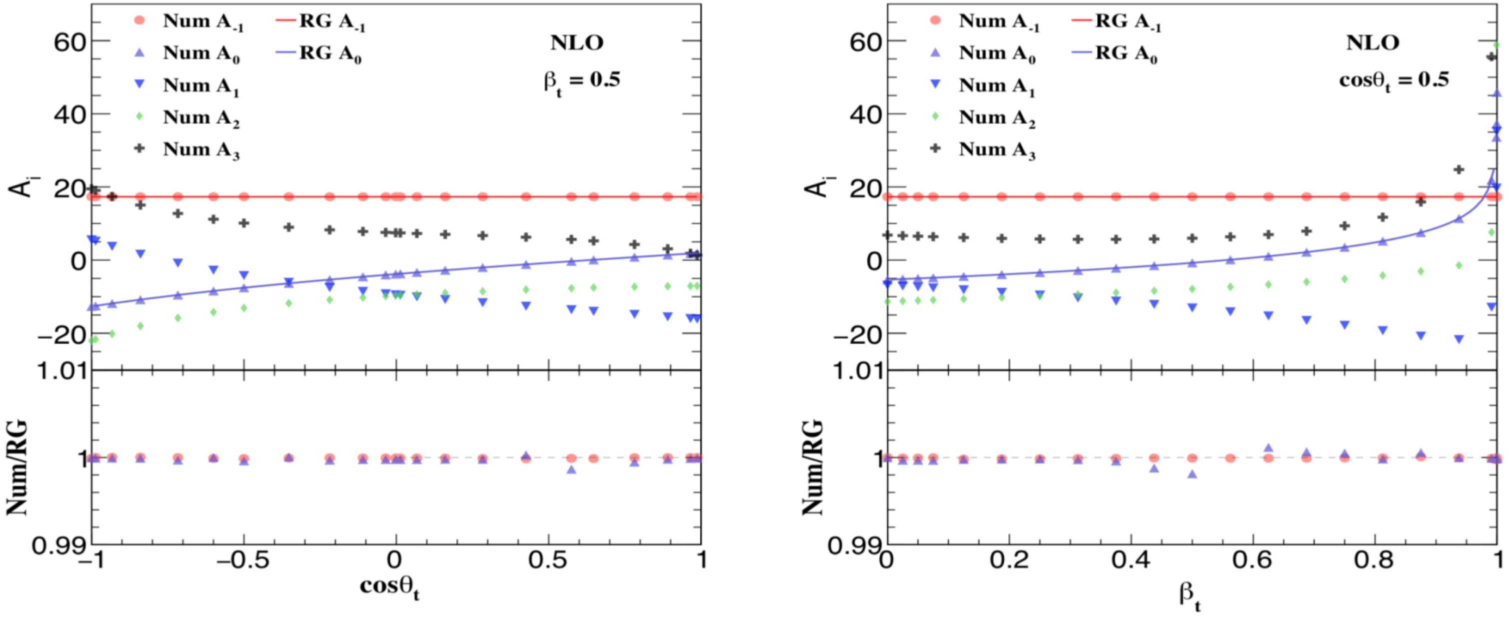
Hai Tao Li, Jian Wang, Phys.Lett.B.784(2018)397

$$S(\tau, \beta_t, \cos\theta_t, \mu) = \delta(\tau) + \frac{1}{\tau}\sum_{n=1}^{\infty}\left(\frac{Z_{\alpha_s}\alpha_s}{4\pi}\right)^n \left(\frac{\tau}{\mu}\right)^{-2n\epsilon} s^{(n)}(\beta_t, \cos\theta_t)$$

LO

n=1,2 corresponds NLO and NNLO

$$s^{(1)} = \frac{A_{-1}}{\epsilon} + A_0 + A_1\epsilon + A_2\epsilon^2 + A_3\epsilon^3 + \mathcal{O}(\epsilon^4)$$



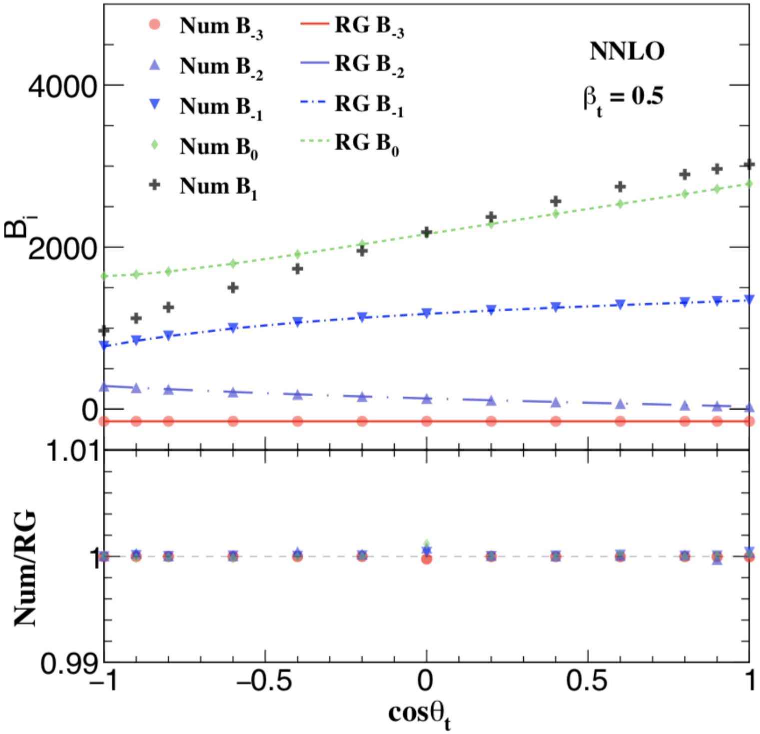
Our numerical results for $A_{-1}$ and $A_0$ agree well with the renormalization group predictions.

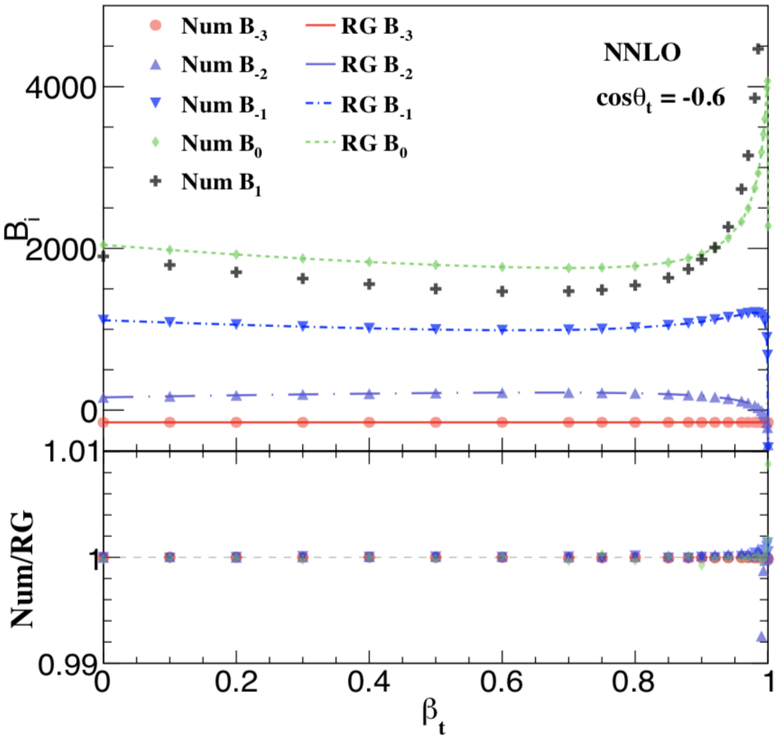
$A_2$ and $A_3$ are needed for NNLO calculation and are obtained in our work.

# NNLO massive N-jettiness soft function

Hai Tao Li, Jian Wang, Phys.Lett.B.784(2018)397

$$s^{(2)} = \frac{B_{-3}}{\epsilon^3} + \frac{B_{-2}}{\epsilon^2} + \frac{B_{-1}}{\epsilon^1} + B_0 + B_1\epsilon + \mathcal{O}(\epsilon^2)$$





These numerical results for $B_{-3}$ $B_{-2}$ $B_{-1}$ and $B_0$ agree well with the renormalization group prediction

$B_1$ is needed for NNLO calculation and is obtained in our work.

# Resummation of double logarithms in loop-induced processes

Matthias Neubert, Jian Wang, 2018

The Sudakov double logarithms in a cross section have the following expansion form:

$$\sigma = \sigma_0(1+\alpha_s L^2+\alpha_s^2 L^4+\cdots) \sim \exp(\alpha_s L^2)$$

This is resummation

This kind of resummation has been used in the threshold and transverse momentum resummation in the production of Higgs/top/…

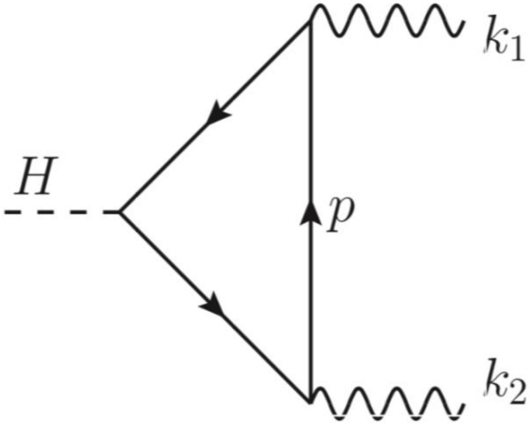
All these resummations have a common feature:

Leading order: $\sigma_0$ contains no large logarithms

What if $\sigma_0$ contains large logarithms?

# Resummation of double logarithms in loop-induced processes

Matthias Neubert, Jian Wang, 2018



A new resummation method needed!

If the quark in the loop has a small mass, compared to Higgs boson, then there is a series of large logarithms,

$$\alpha_s^n \ln^{2n}(m_h^2/m_b^2)$$

A method to resum the large double logarithms in loop induced processes with an effective field theory approach has been proposed . This study can help to understand the all order structure of this kind of logarithms.

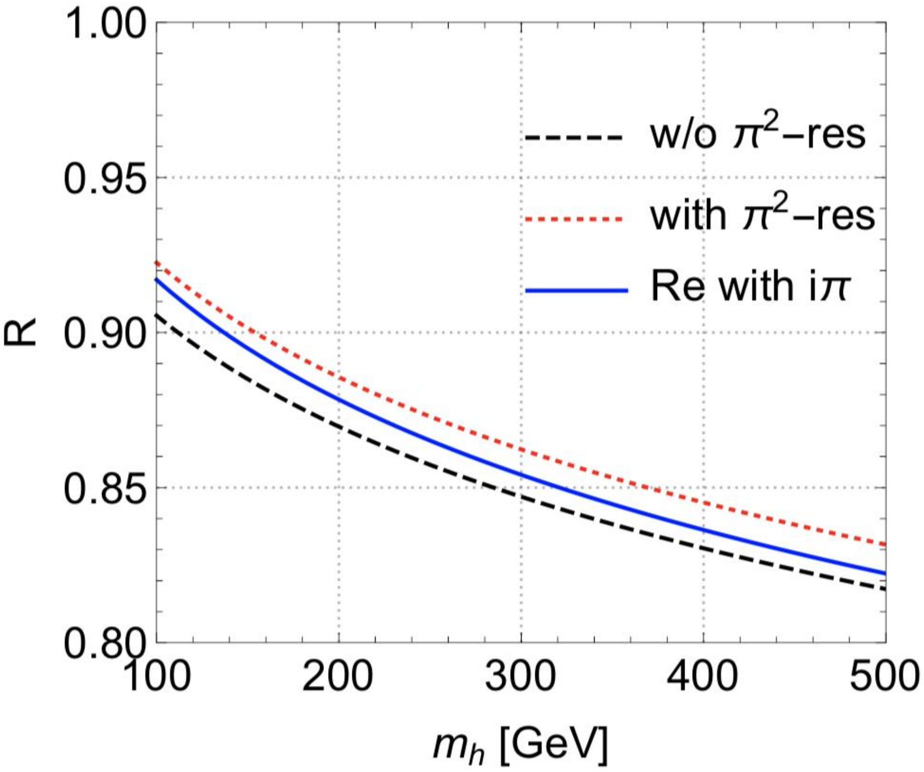
$$\underbrace{\frac{y_b e_q^2}{8\pi^2} N_c m_b \epsilon_\perp(k_1) \cdot \epsilon_\perp(k_2) \frac{1}{2} L^2}_{\text{LO}} \times \underbrace{{}_2F_2(1,1;\frac{3}{2},2;-\frac{\alpha_s C_F}{8\pi}L^2)}_{\text{Resummation}}$$

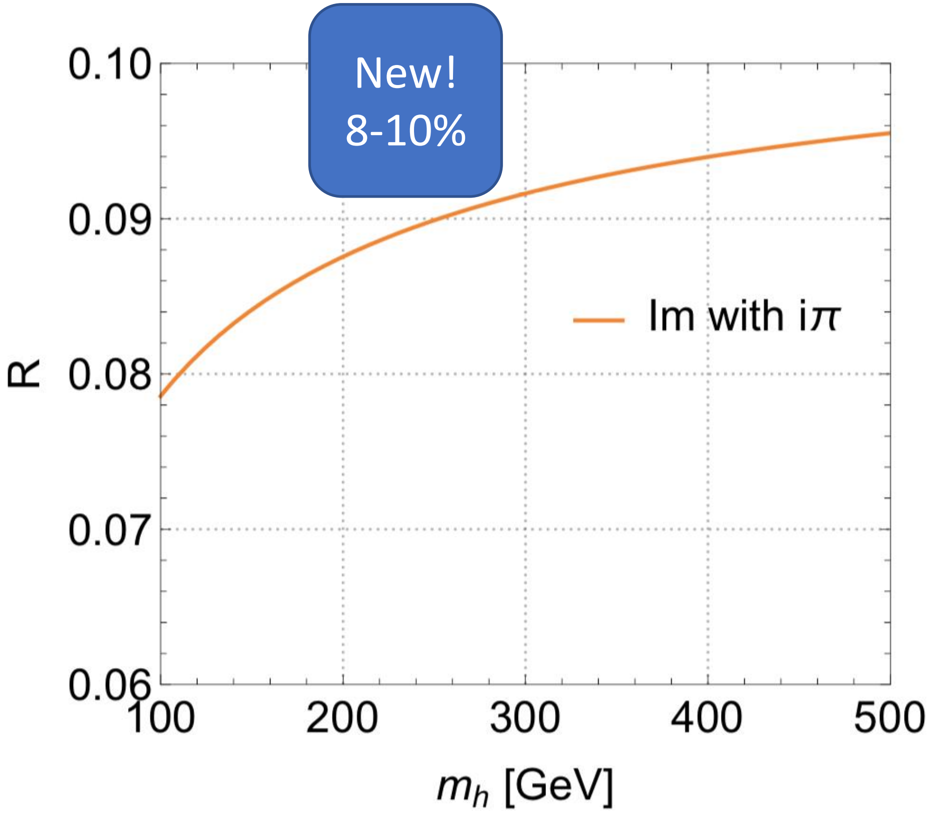
$$L = \ln(-m_h^2/m_b^2 - i0)$$

# Resummation of double logarithms in loop-induced processes

Matthias Neubert, Jian Wang, 2018

The ratios of the resummed results over the leading order result.





New!
8-10%

The imaginary part appears because we have resummed the double logarithms in the form ln^2(-mb^2/mh^2-i0).

# Anomalous dimensions of subleading power operators

M.Beneke, M.Garny, R.Szafron, J. Wang, arXiv:1712.04416, 1712.07462, 1808.04742

The cross section can be expanded in a series of a small variable τ,

$$\sigma(\tau) = \underset{LP}{C\delta(\tau)} + \sum_n \alpha_s^n \left[ \underset{LP}{\frac{\ln^{2n-1}\tau}{\tau}} + \underbrace{\ln^{2n-1}\tau}_{NLP} + \cdots \right]$$

Here τ can be the N-jettiness variable, the threshold variable (1 − M2/s).

- Phenomenology: Useful for NNLO differential calculations in N-jettiness slicing methods, Moult et al, 2016; Boughezal et al, 2016
- Theory: NLP factorization and resummation, Bonocore et al,2015, ‘16, ‘18
- Low subleading soft theorem 1958 and its relation to an asymptotic symmetry at null infinity, Strominger 2013

# Anomalous dimensions of subleading power operators

M.Beneke, M.Garny, R.Szafron, J. Wang, arXiv:1712.04416, 1712.07462, 1808.04742

Large log ↔ IR pole

The infrared divergences of the N-parton scattering amplitude are governed by the soft-collinear anomalous dimension, Becher, Neubert 2009

$$\mathbf{\Gamma} = -\gamma_{\rm cusp}(\alpha_s) \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \ln\left(\frac{-s_{ij}}{\mu^2}\right) + \sum_i \gamma_i(\alpha_s)$$

*Subleading power results?*

We should first identify the subleading power operators.
In soft-collinear effective theory, the collinear-gauge-invariant collinear building blocks

$$\psi_i(t_i n_{i+}) \in \begin{cases} \chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i & \text{collinear quark} \\ \mathcal{A}^\mu_{\perp i}(t_i n_{i+}) \equiv W_i^\dagger [iD^\mu_{\perp i} W_i] & \text{collinear gluon} \end{cases}$$

This gives the Leading Power operators $J_i^{A0}(t_i) = \psi_i(t_i n_{i+})$

# Anomalous dimensions of subleading power operators

M.Beneke, M.Garny, R.Szafron, J. Wang, arXiv:1712.04416, 1712.07462, 1808.04742

Subleading power operators:

$$
\begin{aligned}
J_i^{A1}(t_i) &= i\partial^{\nu}_{\perp i} J_i^{A0} & \mathcal{O}(\lambda)\,, \\
J_i^{A2}(t_i) &= i\partial^{\nu}_{\perp i}\, i\partial^{\rho}_{\perp i} J_i^{A0} & \mathcal{O}(\lambda^2)\,.
\end{aligned}
$$

$$
J_i^{B1}(t_{i_1}, t_{i_2}) = \psi_{i_1}(t_{i_1} n_{i+})\psi_{i_2}(t_{i_2} n_{i+}) \in \begin{cases} \mathcal{A}^{\mu}_{\perp i}(t_{i_1} n_{i+})\chi_i(t_{i_2} n_{i+}) \\ \chi_i(t_{i_1} n_{i+})\chi_i(t_{i_2} n_{i+}) \\ \mathcal{A}^{\mu}_{\perp i}(t_{i_1} n_{i+})\mathcal{A}^{\nu}_{\perp i}(t_{i_2} n_{i+}) \\ \chi_i(t_{i_1} n_{i+})\bar{\chi}_i(t_{i_2} n_{i+})\,. \end{cases}
$$

$$
J_i^{B2}(t_{i_1}, t_{i_2}) \in \begin{cases} \psi_{i_1}(t_{i_1} n_{i+}) i\partial^{\mu}_{\perp i}\psi_{i_2}(t_{i_2} n_{i+}) \\ i\partial^{\mu}_{\perp i}\left[\psi_{i_1}(t_{i_1} n_{i+})\psi_{i_2}(t_{i_2} n_{i+})\right] \end{cases}
$$

$$
J_i^{C2}(t_{i_1}, t_{i_2}, t_{i_3}) = \psi_{i_1}(t_{i_1} n_{i+})\psi_{i_2}(t_{i_2} n_{i+})\psi_{i_3}(t_{i_3} n_{i+})
$$

And non-local operators

$$
J_i^{T1}(t_i) = i \int d^4x\, T\left\{ J_i^{A0}(t_i), \mathcal{L}_i^{(1)}(x) \right\}
$$

$$
J^{T2}_{\chi\chi,\xi}(t_{i_1}, t_{i_2}) = i \int d^4x\, T\left\{ J^{B1}_{\chi\chi}(t_{i_1}, t_{i_2}), \mathcal{L}_{\xi}^{(1)}(x) \right\}
$$

We can use these operators to reproduce Low’s subleading soft theorem.

# Anomalous dimensions of subleading power operators

M.Beneke, M.Garny, R.Szafron, J. Wang, arXiv:1712.04416, 1712.07462, 1808.04742

Main formula at subleading power:

$x_i$ : momentum fraction in a collinear direction

$$\Gamma_{PQ}(x,y) =$$

$$\delta_{PQ}\delta(x-y)\left[-\gamma_{\text{cusp}}(\alpha_s)\sum_{i<j}\sum_{k,l}\mathbf{T}_{i_k}\cdot\mathbf{T}_{j_l}\ln\left(\frac{-s_{ij}x_{i_k}x_{j_l}}{\mu^2}\right)+\sum_i\sum_k\gamma_{i_k}(\alpha_s)\right]$$

$$+2\sum_i\delta^{[i]}(x-y)\gamma^i_{PQ}(x,y)+2\sum_{i<j}\delta(x-y)\gamma^{ij}_{PQ}(y)\,,$$

collinear

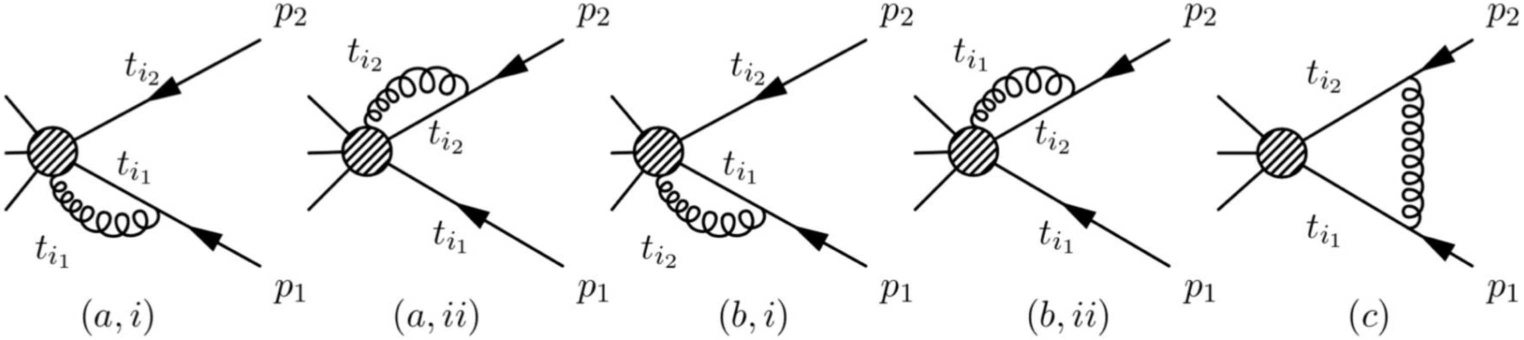
soft

$$\gamma_{\text{cusp}}(\alpha_s)=\frac{\alpha_s}{\pi} \qquad \text{and} \qquad \gamma_{i_k}(\alpha_s)=\begin{cases}-\dfrac{3\alpha_s C_F}{4\pi} & \text{(q)}\\ 0 & \text{(g)}\end{cases}$$

# Anomalous dimensions of subleading power operators

M.Beneke, M.Garny, R.Szafron, J. Wang, arXiv:1712.04416, 1712.07462, 1808.04742

Collinear anomalous dimensions, B1 to B1 example, $\gamma^{i}_{\chi_\alpha\chi_\beta,\chi_\gamma\chi_\delta}(x,y)$



$$\frac{\alpha_s \mathbf{T}_{i_1}\cdot\mathbf{T}_{i_2}}{2\pi}\left\{\delta_{\alpha\gamma}\delta_{\beta\delta}\left(\theta(x-y)\left[\frac{1}{x-y}\right]_+ +\theta(y-x)\left[\frac{1}{y-x}\right]_+ \right.\right.$$

$$\left. -\theta(x-y)\frac{1-\frac{\bar{x}}{2}}{\bar{y}}-\theta(y-x)\frac{1-\frac{x}{2}}{y}\right)$$

$$\left. -\frac{1}{4}\left(\sigma_\perp^{\nu\mu}\right)_{\alpha\gamma}\left(\sigma_{\perp\nu\mu}\right)_{\beta\delta}\left(\theta(x-y)\frac{\bar{x}}{\bar{y}}+\theta(y-x)\frac{x}{y}\right)\right\}.$$

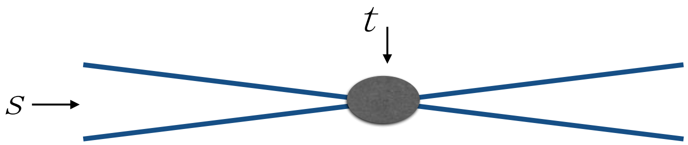
More complete results are included in our papers.

# Theoretical progress in SCET

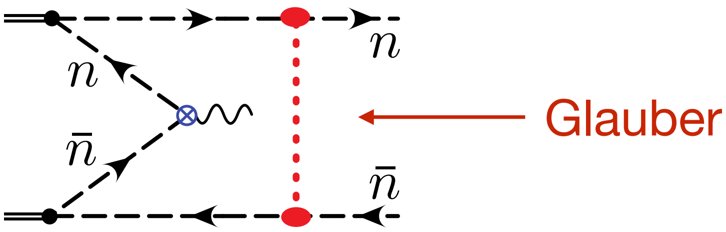
- Forward scattering, Glauber gluons, factorization violation at hadron colliders. (Rothstein & Stewart ‘16)
- Resummation for non-global observables. (Becher, Neubert, Rothen, Shao ‘16 & Becher, Peckjack, Shao ’16 & Becher, Rahn, Shao ’17)

# Forward scattering

- Glauber gluons are important to describe forward process.
- Glauber gluons are off-shell, and Glauber modes are not well defined without rapidity regulator.
- Rothstein and Stewart developed a systematical SCET framework including Glauber mode.(Rothstein & Stewart '16)

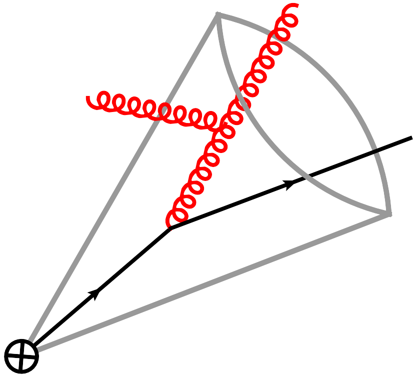


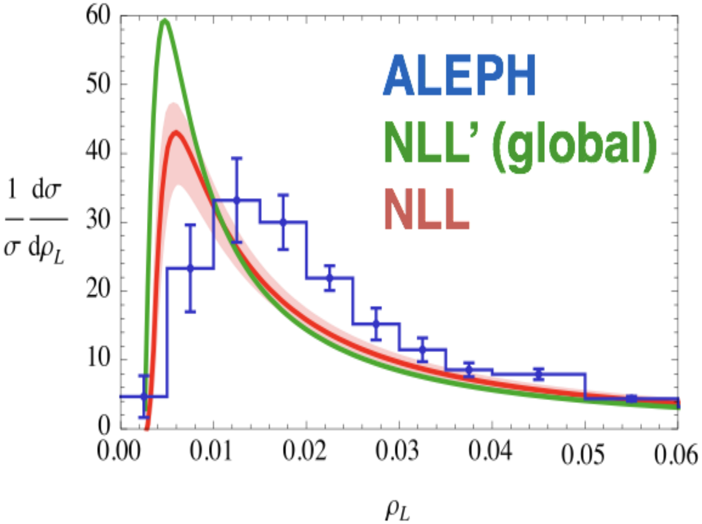
$\alpha_s^n \ln^n (s/-t)$



# Resummation for Non-global observables

- First observed by Dasgupta and Salam (Dasgupta & Salam et.al. '01). Non-global logarithms (NGLs) can not be described in the usual resummation formula.
- From phenomenology side, non-global effects are very important. E.g. jet mass distribution (see right figure)
- From theory side, NGLs resummation help to understand pQCD.
- A new effective field theory which fully factorizes non-global observables and resum NGLs to all order. (Becher et.al. '16,'17)





# Application of SCET

- N-Jettiness subtraction for NNLO calculations. (Gao,CSL & Zhu ‘13; Boughezal, et.al. ‘15; Gaunt et.al. ‘15)
- Three-loop soft anomalous dimension for transverse momentum resummation. (Li, Zhu, ‘16)
- Two-loop leading power corrections for jettiness (Moult et.at. ’ 16,’17)
- Resummation for boosted top quark pair production at LHC (Peckjak et.at. ’ 16,’17)
- Jet radius resummation for inclusive and exclusive process (Chien et.at ’16; Idible et.al. ‘16; Dai et.al. ‘16)
- Analytical jet substructure resummation. (Frye et.at. ’ 16)

# Automated resummation within SCET

- Automated computation for two-loop soft functions (Bell, et.al. ‘15)
- NNLL jet veto cross section within Madgraph5 framework (Becher, et.al. ’15)
- GENEVA parton shower framework, combining higher-order resummation results (Alioli & Baur ‘15)

# Summary

- QCD NNLO 计算已经成为LHC 许多观察量的标配。
- QCD NNNLO 计算也正在推进之中。
- 虽然 Antenna 减除在NNLO和 NNNLO 计算中是一种比较成功的方法，但N-Jettiness 也正在得到很成功的应用。
- 应用SCET 不但是因子化的利器，同样也是分析红外发散结构及提供减除方法的有效途径。

**Thank you for your attention!**