



Model independent investigation of R_J/psi and the P-wave observations

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Outlines

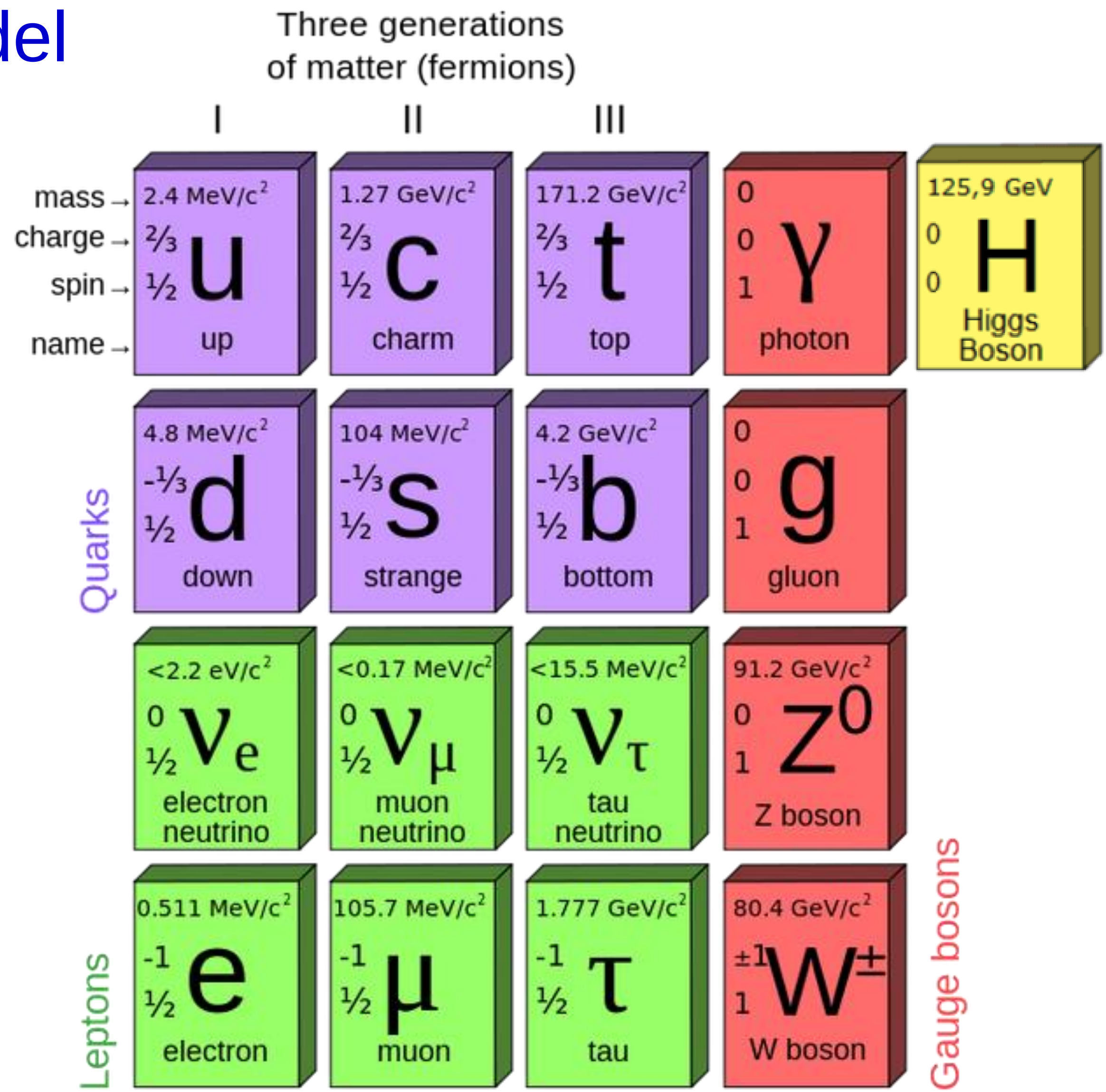
- Background
- b to c/s anomalies
- Heavy quark effective theory (HQET)
- Model independent constraints on the Bc to charmonium form factors

  -R_J/psi

  -R_etac, R_hc, R_chicJ
- Summary

Background

Standard Model



- **Hadron typical scale**

$$\Lambda_{QCD} \approx 200MeV$$

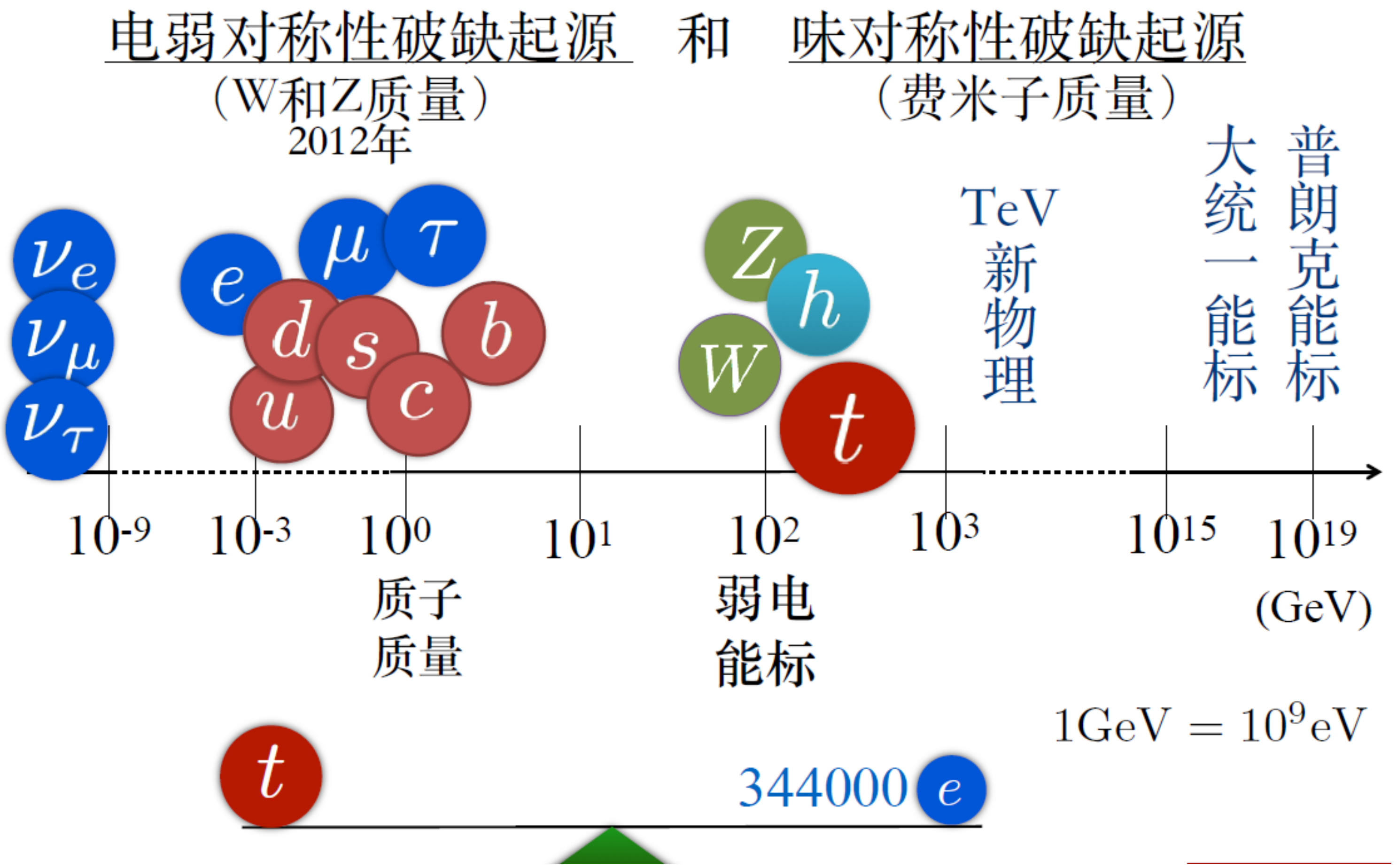
- **Light quark flavor**

up, down, strange

- **Heavy quark flavor**

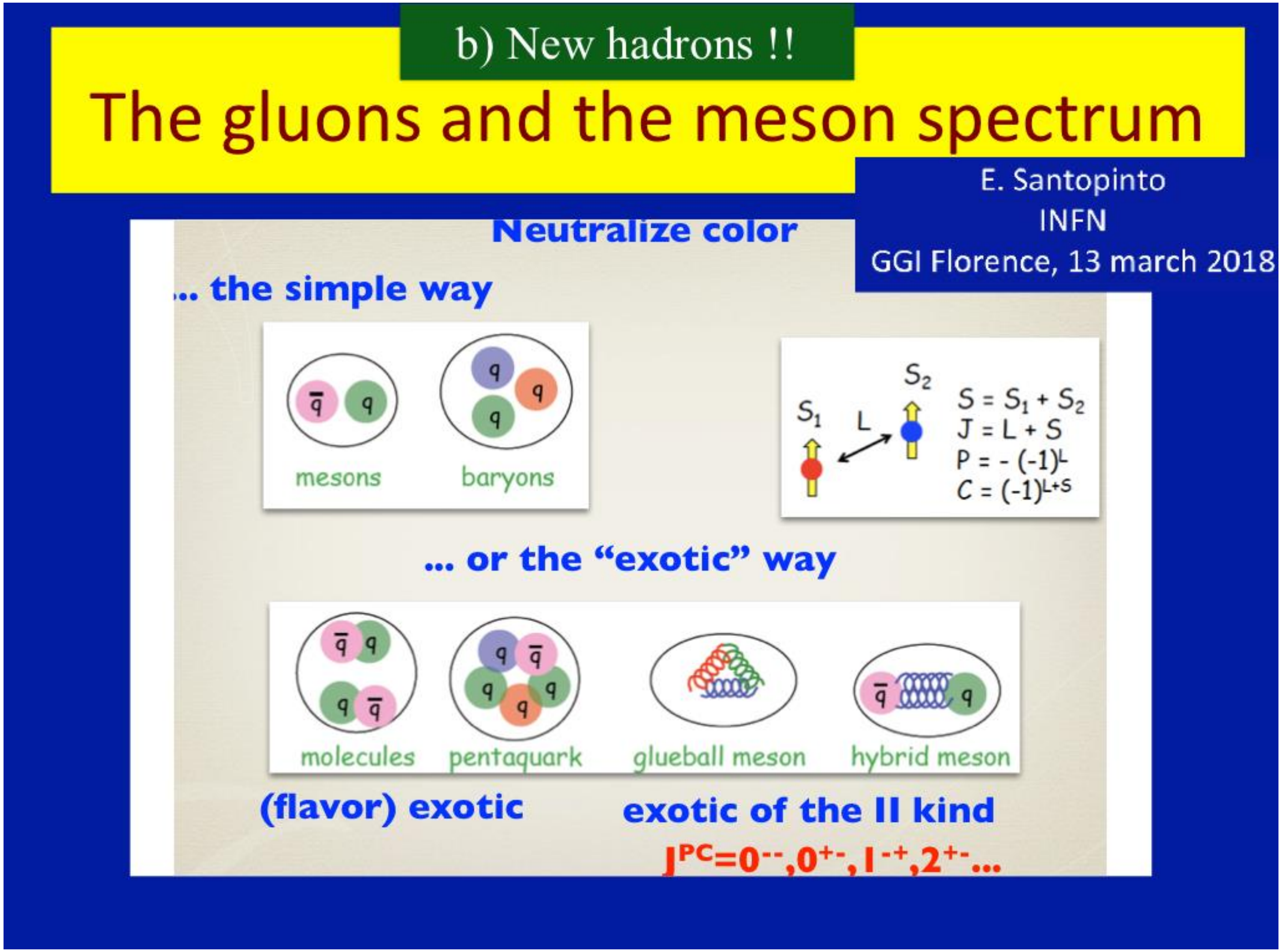
charm, bottom, top

Open question in SM: multi-scales



Qing-Hong Cao, NJNU, 2018

Open question in SM: color confining



Non-perturbative

b to c/s anomalies

- Anomalies in B/Bs/Bc decasy

■2012 R_D R _D*

$$R_{D^{(*)}} = \frac{\Gamma(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau)}{\Gamma(\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell)},$$

	R_D	R_{D^*}
Experimental average	$0.407 \pm 0.039 \pm 0.024$	$0.304 \pm 0.013 \pm 0.007$
SM prediction	0.300 ± 0.010	0.252 ± 0.005

■2014 R_K

2017 R_K*

$$R_K = \frac{\Gamma(B^+ \to K^+\mu^+\mu^-)}{\Gamma(B^+ \to K^+e^+e^-)},$$

SM Predictions: 0.99

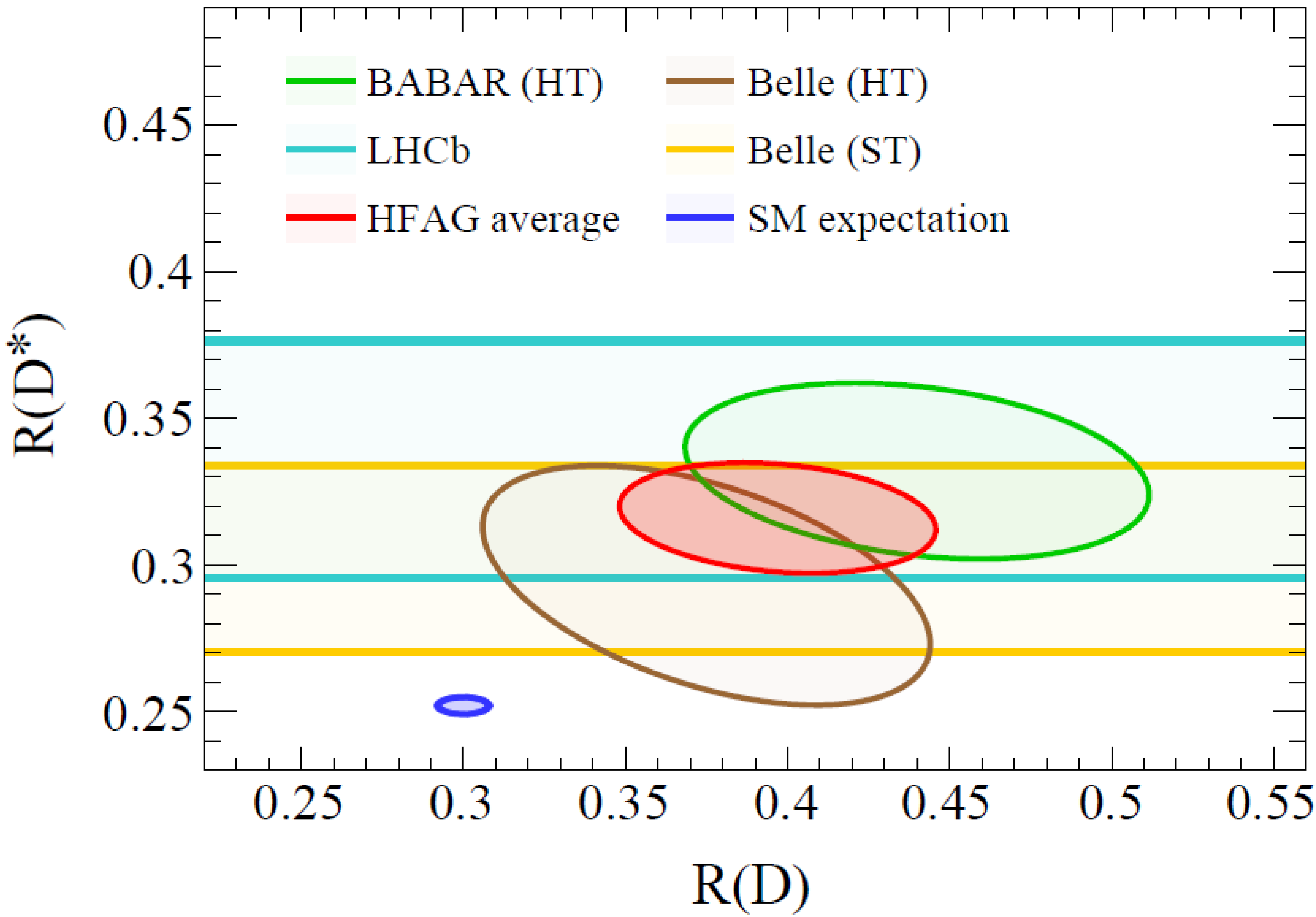
$$\mathscr{R}_K = \frac{\mathscr{B}(B^+ \to K^+\mu^+\mu^-)}{\mathscr{B}(B^+ \to K^+e^+e^-)} = 0.745 \pm^{0.090}_{0.074} \pm 0.036,$$

■2017 R_J/psi

SM Predictions: 0.2-0.4

$$\mathcal{R}(J/\psi) = \frac{\mathcal{B}(B_c^+ \to J/\psi\,\tau^+\nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi\,\mu^+\nu_\mu)} = 0.71 \pm 0.17\,(\mathrm{stat}) \pm 0.18\,(\mathrm{syst}).$$

BABAR, Belle, LHCb



Ciezarek et. Al., 1703.01766

The decay width formulae

Semileptonic decays $B_c \to \eta_c \ell \bar{\nu}_\ell$ have the decay widths:

$$\frac{d\Gamma(B_c \to P l \bar{\nu}_l)}{dq^2} = (\frac{q^2 - m_l^2}{q^2})^2 \frac{\sqrt{\lambda(m_{B_c}^2, m_P^2, q^2)} G_F^2 |V_{\mathrm{CKM}}|^2}{384 m_{B_c}^3 \pi^3} \frac{1}{q^2} \times \left\{ (m_l^2 + 2q^2) \lambda(m_{B_c}^2, m_P^2, q^2) f_+^2(q^2) + 3 m_l^2 (m_{B_c}^2 - m_P^2)^2 f_0^2(q^2) \right\},$$

Decay widths for $B_c \to J/\psi \ell \bar{\nu}_\ell$ are given as:

$$\frac{d\Gamma_L(B_c \to V l \bar{\nu})}{dq^2} = (\frac{q^2 - m_l^2}{q^2})^2 \frac{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)} G_F^2 |V_{\mathrm{CKM}}|^2}{384 m_{B_c}^3 \pi^3} \frac{1}{q^2} \times \left\{ 3 m_l^2 \lambda(m_{B_c}^2, m_V^2, q^2) A_0^2(q^2) + (m_l^2 + 2q^2) | \frac{1}{2m_V} \left[(m_{B_c}^2 - m_V^2 - q^2) \times (m_{B_c} + m_V) A_1(q^2) - \frac{\lambda(m_{B_c}^2, m_V^2, q^2)}{m_{B_c} + m_V} A_2(q^2) \right] |^2 \right\},$$

$$\frac{d\Gamma^{\pm}(B_c \to V l \bar{\nu})}{dq^2} = (\frac{q^2 - m_l^2}{q^2})^2 \frac{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)} G_F^2 |V_{\mathrm{CKM}}|^2}{384 m_{B_c}^3 \pi^3} \times \left\{ (m_l^2 + 2q^2) \lambda(m_{B_c}^2, m_V^2, q^2) \left| \frac{V(q^2)}{m_{B_c} + m_V} \mp \frac{(m_{B_c} + m_V) A_1(q^2)}{\sqrt{\lambda(m_{B_c}^2, m_V^2, q^2)}} \right|^2 \right\}$$

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_\perp}{dq^2}, \quad \frac{d\Gamma_\perp}{dq^2} = \frac{d\Gamma^+}{dq^2} + \frac{d\Gamma^-}{dq^2}.$$

Form factors

$$\langle \eta_c(p)|J_V^\mu|B_c(P)\rangle = f_0^{\eta_c}(q^2)\frac{m_{B_c}^2-m_{\eta_c}^2}{q^2}q^\mu + f_+^{\eta_c}(q^2)(P^\mu+p^\mu-\frac{m_{B_c}^2-m_{\eta_c}^2}{q^2}q^\mu)\,, \quad (1)$$

$$\langle J/\psi(p,\varepsilon^*)|J_V^\mu|B_c(P)\rangle = -\frac{2V^{J/\psi}(q^2)}{m_{B_c}+m_{J/\psi}}\epsilon^{\mu\nu\rho\sigma}\varepsilon_\nu^* p_\rho P_\sigma\,, \quad (2)$$

$$\begin{aligned}\langle J/\psi(p,\varepsilon^*)|J_A^\mu|B_c(P)\rangle &= -i[2m_{J/\psi}A_0^{J/\psi}(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu + (m_{B_c}+m_{J/\psi})A_1^{J/\psi}(q^2)(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu) \\ &\quad -A_2^{J/\psi}(q^2)\frac{\varepsilon^*\cdot q}{m_{B_c}+m_{J/\psi}}(P^\mu+p^\mu-\frac{m_{B_c}^2-m_{J/\psi}^2}{q^2}q^\mu)]\,, \quad (3)\end{aligned}$$

$$\begin{aligned}\langle h_c(p,\varepsilon^*)|J_V^\mu|B_c(P)\rangle &= -i[2m_{h_c}A_0^{h_c}(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu + (m_{B_c}+m_{h_c})A_1^{h_c}(q^2)(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu) \\ &\quad - A_2^{h_c}(q^2)\frac{\varepsilon^*\cdot q}{m_{B_c}+m_{h_c}}(P^\mu+p^\mu-\frac{m_{B_c}^2-m_{h_c}^2}{q^2}q^\mu)]\,, \quad (4)\end{aligned}$$

$$\langle h_c(p,\varepsilon^*)|J_A^\mu|B_c(P)\rangle = \frac{2V^{h_c}(q^2)}{m_{B_c}+m_{h_c}}\epsilon^{\mu\nu\rho\sigma}\varepsilon_\nu^* p_\rho P_\sigma\,, \quad (5)$$

$$\langle \chi_{c0}(p)|J_A^\mu|B_c(P)\rangle = f_0^{\chi_{c0}}(q^2)\frac{m_{B_c}^2-m_{\chi_{c0}}^2}{q^2}q^\mu + f_+^{\chi_{c0}}(q^2)(P^\mu+p^\mu-\frac{m_{B_c}^2-m_{\chi_{c0}}^2}{q^2}q^\mu)\,, \quad (6)$$

$$\begin{aligned}\langle \chi_{c1}(p,\varepsilon^*)|J_V^\mu|B_c(P)\rangle &= -i[2m_{\chi_{c1}}A_0^{\chi_{c1}}(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu + (m_{B_c}+m_{\chi_{c1}})A_1^{\chi_{c1}}(q^2)(\varepsilon^{*\mu}-\frac{\varepsilon^*\cdot q}{q^2}q^\mu) \\ &\quad -A_2^{\chi_{c1}}(q^2)\frac{\varepsilon^*\cdot q}{m_{B_c}+m_{\chi_{c1}}}(P^\mu+p^\mu-\frac{m_{B_c}^2-m_{\chi_{c1}}^2}{q^2}q^\mu)]\,, \quad (7)\end{aligned}$$

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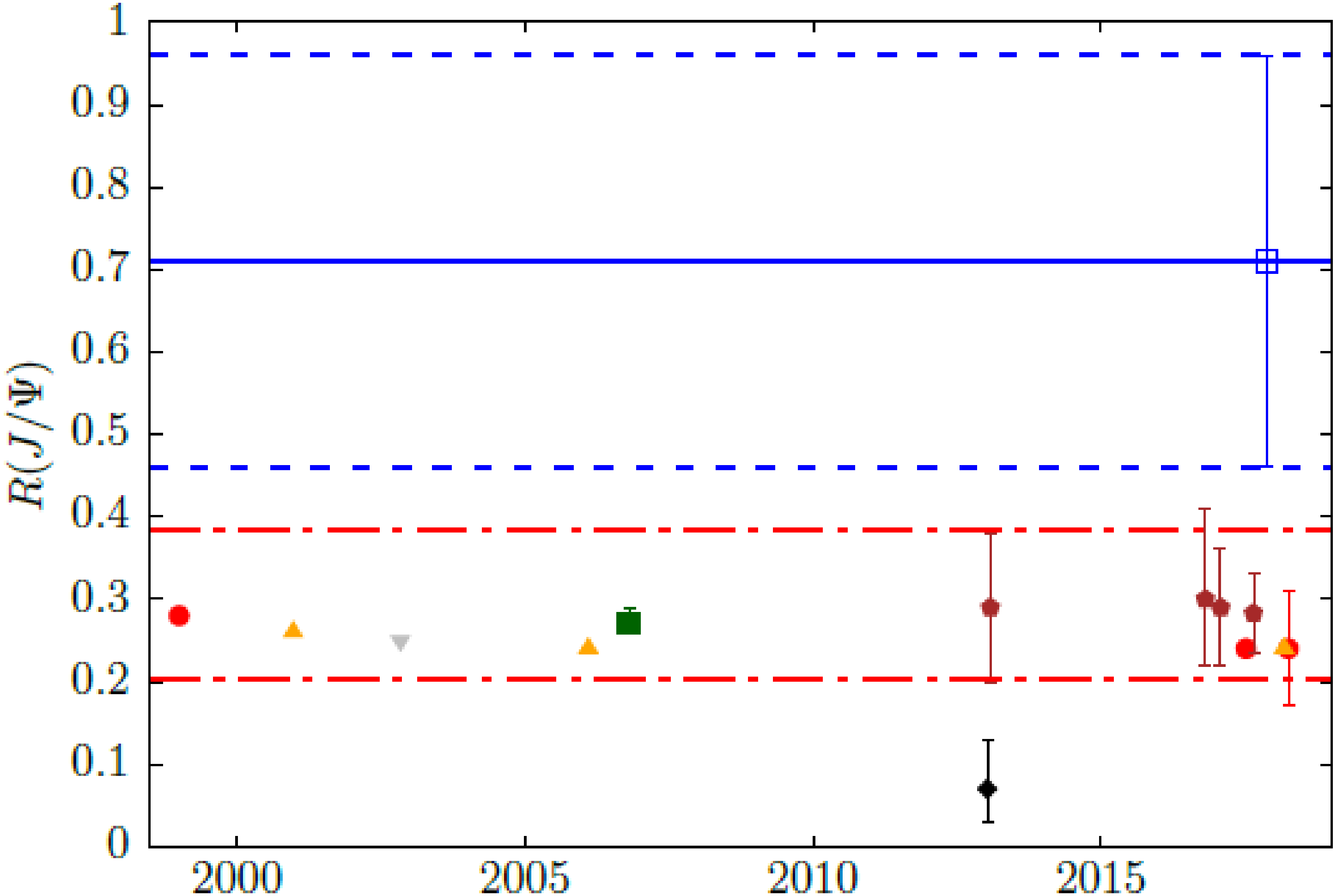


Cohen et. Al., 1807.027306

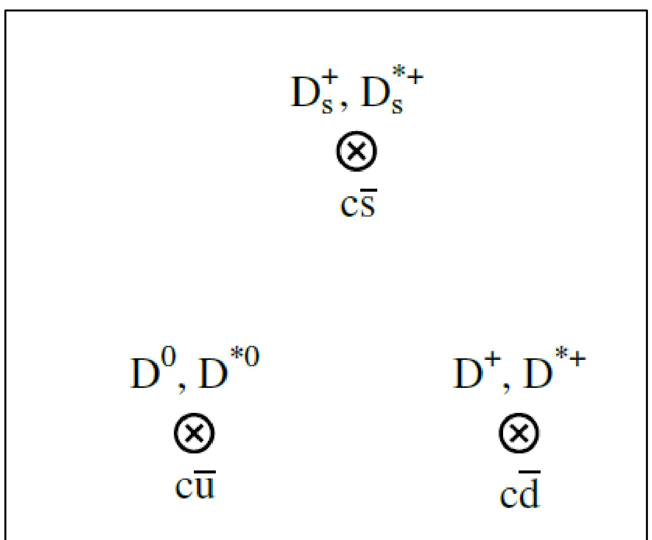
Heavy quark effective theory (HQET)

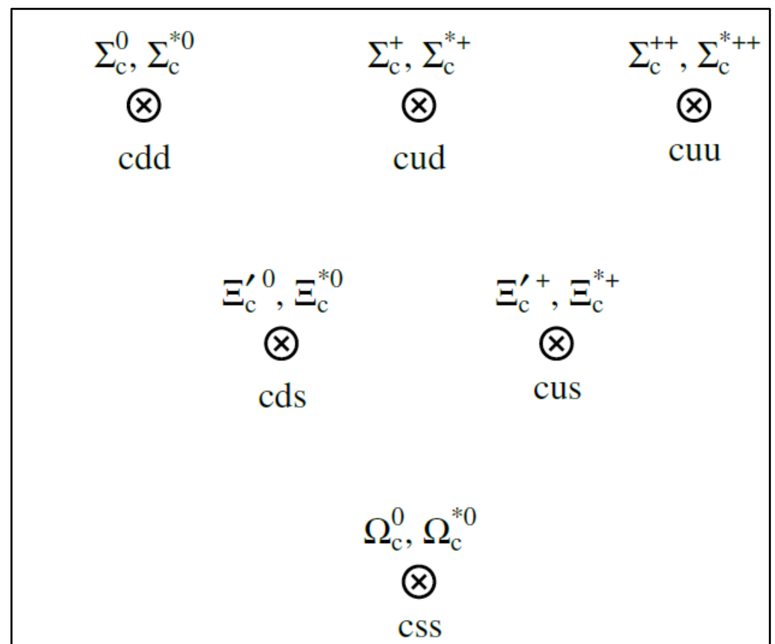
- Q_q heavy-light or Q_{qq} heavy-light-light system

$$m_Q \gg \Lambda_{QCD} \qquad m_q \ll \Lambda_{QCD}.$$

$$\Delta v = \Delta p / m_Q$$

heavy quark flavor symmetry
heavy quark spin symmetry





Manohar-Wise HQP

$$p = m_Q v + k$$

$$Q(x) = e^{-im_Q v\cdot x}[Q_v(x) + \mathfrak{Q}_v(x)],$$

where

$$Q_v(x) = e^{im_Q v\cdot x}\frac{1+\not{v}}{2}Q(x), \qquad \mathfrak{Q}_v(x) = e^{im_Q v\cdot x}\frac{1-\not{v}}{2}Q(x).$$

$$\mathcal{L}_{\text{eff}} = \sum_{i=1}^{N_h} \bar{Q}_v^{(i)}\,(iv\cdot D)\,Q_v^{(i)},$$

$$\left(\frac{1+\not{v}}{2}\right)Q_v = Q_v.$$

$$\mathcal{L}_1 = -\bar{Q}_v\frac{D_\perp^2}{2m_Q}Q_v - g\,\bar{Q}_v\frac{\sigma_{\mu\nu}G^{\mu\nu}}{4m_Q}Q_v.$$

Multiplets in HQET

s wave

$$H_{v'} = \frac{1+\not{v}'}{2}[i\psi^{\beta}\gamma_{\beta} + \eta_c\gamma^5] \, .$$

$$J^P = 0^- \ , \ j = \frac{1}{2} \ ,$$
$$J^P = 1^- \ , \ j = \frac{1}{2} \ ,$$

p wave

$$J^P = 0^+ \ , \ j = \frac{1}{2} \ ,$$
$$J^P = 1^+ \ , \ j = \frac{1}{2} \ ,$$

$$E_{v'} = \frac{1+\not{v}'}{2}[\chi_{c0} + i{h_c}^{\beta}\gamma_{\beta}] \, ,$$

$$F^{\alpha}_{v'} = i\frac{1+\not{v}'}{2}\{\chi^{\alpha\beta}_{c2}\gamma_{\beta} - \sqrt{\frac{3}{2}}\chi^{\beta}_{c1}\gamma^5[g^{\alpha}_{\beta} - \frac{1}{3}\gamma_{\beta}(\gamma^{\alpha} - v'^{\alpha})]\} \, .$$

$$J^P = 1^+ \ , \ j = \frac{3}{2} \ ,$$
$$J^P = 2^+ \ , \ j = \frac{3}{2} \ ,$$

Calculations in HQET

$$\langle H^c(v')|\bar{c}_{v'}\Gamma^\mu b_v|B_c(v)\rangle = \mathrm{Tr}[\xi \bar{H}^c_{v'}\Gamma^\mu \bar{H}^b_v]\,,$$

$$\xi = \xi_0 + \xi_1 \not{v} + \xi_2 \not{v}' + \xi_3 \not{v}\not{v}'$$

$$\langle H^c(v')|\bar{c}_{v'}\Gamma^\mu b_v|B_c(v)\rangle = \mathrm{Tr}[\bar{H}^c_{v'}\Gamma^\mu \bar{H}^b_v]\xi_0(\omega)\,,$$

Isgur Wise function: $\xi_0(\omega)$

$\xi_0(\omega) = \xi_H(\omega)$ for B_c to S-wave charmonium

$\xi_0(\omega) = \xi_E(\omega), \xi_F(\omega)v_\alpha$ for B_c to P-wave charmonium.

Results in HQET

$$\langle \eta_c(v')|\bar{c}_{v'}\gamma^\mu b_v|B_c(v)\rangle = \xi_H(\omega)[v^\mu + v'^\mu]\,, \tag{17}$$

$$\langle J/\psi(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu b_v|B_c(v)\rangle = -\xi_H(\omega)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu v'_\rho v_\sigma\,, \tag{18}$$

$$\langle J/\psi(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu\gamma^5 b_v|B_c(v)\rangle = -i\xi_H(\omega)[(1+\omega)\varepsilon^{*\mu} - \varepsilon^*\cdot v v'^\mu]\,, \tag{19}$$

$$\langle h_c(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu b_v|B_c(v)\rangle = i\xi_E(\omega)[(\omega-1)\varepsilon^{*\mu} - \varepsilon^*\cdot v v'^\mu]\,, \tag{20}$$

$$\langle h_c(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu\gamma^5 b_v|B_c(v)\rangle = \xi_E(\omega)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu v'_\rho v_\sigma\,, \tag{21}$$

$$\langle \chi_{c0}(v')|\bar{c}_{v'}\gamma^\mu\gamma^5 b_v|B_c(v)\rangle = -\xi_E(\omega)[v^\mu - v'^\mu]\,, \tag{22}$$

$$\langle \chi_{c1}(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu b_v|B_c(v)\rangle = \frac{i\xi_F(\omega)}{\sqrt{6}}[(\omega^2-1)\varepsilon^{*\mu} - \varepsilon^*\cdot v(3v^\mu - (\omega-2)v'^\mu)]\,, \tag{23}$$

$$\langle \chi_{c1}(v',\varepsilon^*)\bar{c}_{v'}\gamma^\mu\gamma^5 b_v|B_c(v)\rangle = \frac{(\omega+1)\xi_F(\omega)}{\sqrt{6}}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu v'_\rho v_\sigma, \tag{24}$$

$$\langle \chi_{c2}(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu\gamma^5 b_v|B_c(v)\rangle = -i\xi_F(\omega)v_\alpha[(1+\omega)\varepsilon^{*\alpha\mu} - \varepsilon^{*\alpha\beta}v_\beta v'^\mu]\,, \tag{25}$$

$$\langle \chi_{c2}(v',\varepsilon^*)|\bar{c}_{v'}\gamma^\mu b_v|B_c(v)\rangle = \xi_F(\omega)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_{\alpha\nu}v^\alpha v'_\rho v_\sigma\,. \tag{26}$$

Constraints in HQET

$$f_0^{\eta_c}(\omega) = \frac{(\omega+1)\xi_H(\omega)\sqrt{m_{B_c}}\sqrt{m_{\eta_c}}}{m_{B_c}+m_{\eta_c}},$$

0.94 （w=1)

$$f_+^{\eta_c}(\omega) = \frac{\xi_H(\omega)\left(m_{B_c}+m_{\eta_c}\right)}{2\sqrt{m_{B_c}}\sqrt{m_{\eta_c}}},$$

1.07

$$V^{J/\psi}(\omega) = \frac{\xi_H(\omega)\left(m_{B_c}+m_{J/\psi}\right)}{2\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}},$$

1.06

$$A_0^{J/\psi}(\omega) = \frac{\xi_H(\omega)\left(m_{B_c}+m_{J/\psi}\right)}{2\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}},$$

1.06

$$A_1^{J/\psi}(\omega) = \frac{(\omega+1)\xi_H(\omega)\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}}{m_{B_c}+m_{J/\psi}},$$

0.94

$$A_2^{J/\psi}(\omega) = \frac{\xi_H(\omega)\left(m_{B_c}+m_{J/\psi}\right)}{2\sqrt{m_{J/\psi}}\sqrt{m_{B_c}}}.$$

1.08

Model independent investigation of R_J/psi

- Considering the vacuum expectation of current operators

$$j_V^\mu = \bar{c}\gamma^\mu b\,, \qquad\qquad j_A^\mu = \bar{c}\gamma^\mu\gamma^5 b\,.$$

$$\begin{aligned}\Pi^{\mu\nu}(q^2) &= i\int d^4x\, e^{i\,q\cdot x}\,\langle 0|\,\mathrm{T}\,j^\mu(x)\,j^{\dagger\,\nu}(0)\,|0\rangle \\ &= (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_T(q^2) + q^\mu q^\nu \Pi_L(q^2)\,.\end{aligned}$$

$$\Pi_I(q^2) = P_{\mu\nu,I}(q^2)\,\Pi^{\mu\nu}(q^2)\,, \qquad (I = L, T),$$

$\Pi_I(q^2)$ is an analytic function and it satisfies the dispersion relation

$$\Pi_I(q^2) = \frac{1}{\pi}\int_0^\infty dt\,\frac{\mathrm{Im}\,\Pi_I(t)}{t-q^2}\,.$$

$$\chi_I(n, Q_0^2) = \frac{1}{n!}\,\frac{d^n\Pi_I(q^2)}{dq^{2n}}\bigg|_{q^2=-Q_0^2} = \frac{1}{\pi}\int_0 dt\,\frac{\mathrm{Im}\,\Pi_I(t)}{(t+Q_0^2)^{n+1}}\,.$$

E. de Rafael and J. Taron, Phys. Rev. D **50**, 373 (1994)

Model independent investigation of R_J/psi

- Inserting the hadronic states

$$\mathrm{Im}\,\Pi_I^{BV}(q^2) = \frac{1}{2}\int \frac{d^3p_B}{(2\pi)^3 2E_B}\frac{d^3p_V}{(2\pi)^3 2E_V}(2\pi)^4\delta^4(q-p_B-p_V)\,P_{\mu\nu,I}\,\langle 0|\,j^\mu\,|BV\rangle\,\langle BV|\,j^{\nu\dagger}\,|0\rangle\ ,$$

$$\mathrm{Im}\Pi_I^{BV}(t) \leq \mathrm{Im}\Pi_I(t)\,.$$

- Cross symmetry

$$P_{\mu\nu,T}\,\langle 0|j_V^\mu|BV\rangle\langle BV|j_V^{\nu\dagger}|0\rangle \;=\; \frac{\lambda}{3q^2}\,|\mathcal{B}_V|^2\ ,$$

$$P_{\mu\nu,T}\,\langle 0|j_A^\mu|BV\rangle\langle BV|j_A^{\nu\dagger}|0\rangle \;=\; \frac{\lambda}{3q^2}\sum_{i=1}^{2}|\mathcal{B}_{Ai}|^2\ ,$$

$$P_{\mu\nu,L}\,\langle 0|j_A^\mu|BV\rangle\langle BV|j_A^{\nu\dagger}|0\rangle \;=\; \frac{\lambda}{3q^2}\,|\mathcal{B}_{A0}|^2\ ,$$

$$\mathcal{B}_V(q^2) = \frac{\sqrt{2q^2}}{m_B+m_V}V(q^2)\,,$$

$$\mathcal{B}_{A0}(q^2) = \sqrt{3}A_0(q^2)\,,$$

$$\mathcal{B}_{A1}(q^2) = \frac{(m_B+m_V)^2\,(m_B^2-m_V^2-q^2)\,A_1(q^2)-\lambda\,A_2(q^2)}{2m_V\sqrt{\lambda}\,(m_B+m_V)}\ ,$$

$$\mathcal{B}_{A2}(q^2) = \frac{\sqrt{2\,q^2}\,(m_B+m_V)}{\sqrt{\lambda}}\,A_1(q^2)\,,$$

Model independent investigation of R_J/psi

- Inserting the hadronic states

$$\mathrm{Im}\,\Pi_I^{BV}(q^2) = \frac{1}{2}\int \frac{d^3p_B}{(2\pi)^3 2E_B}\,\frac{d^3p_V}{(2\pi)^3 2E_V}\,(2\pi)^4\delta^4(q-p_B-p_V)\,P_{\mu\nu,I}\,\langle 0|\,j^\mu\,|BV\rangle\,\langle BV|\,j^{\nu\dagger}\,|0\rangle\;,$$

$$\mathrm{Im}\Pi_I^{BV}(t) \leq \mathrm{Im}\Pi_I(t)\,.$$

- Cross symmetry

$$P_{\mu\nu,T}\,\langle 0|j_V^\mu|BV\rangle\langle BV|j_V^{\nu\dagger}|0\rangle \;=\; \frac{\lambda}{3q^2}\,|\mathcal{B}_V|^2\;,$$

$$P_{\mu\nu,T}\,\langle 0|j_A^\mu|BV\rangle\langle BV|j_A^{\nu\dagger}|0\rangle \;=\; \frac{\lambda}{3q^2}\,\sum_{i=1}^{2}|\mathcal{B}_{Ai}|^2\;,$$

$$P_{\mu\nu,L}\,\langle 0|j_A^\mu|BV\rangle\langle BV|j_A^{\nu\dagger}|0\rangle \;=\; \frac{\lambda}{3q^2}\,|\mathcal{B}_{A0}|^2\;,$$

$$\mathcal{B}_V(q^2) = \frac{\sqrt{2q^2}}{m_B+m_V}V(q^2)\,,$$

$$\mathcal{B}_{A0}(q^2) = \sqrt{3}A_0(q^2)\,,$$

$$\mathcal{B}_{A1}(q^2) = \frac{(m_B+m_V)^2\,(m_B^2-m_V^2-q^2)\,A_1(q^2)-\lambda\,A_2(q^2)}{2m_V\sqrt{\lambda}\,(m_B+m_V)}\;,$$

$$\mathcal{B}_{A2}(q^2) = \frac{\sqrt{2\,q^2}\,(m_B+m_V)}{\sqrt{\lambda}}\,A_1(q^2)\,,$$

$$\lambda = \left((m_B-m_V)^2-q^2\right)\left((m_B+m_V)^2-q^2\right),$$

Model independent investigation of R_J/psi

- Integration

$$
\begin{aligned}
\mathrm{Im}\,\Pi^{BV}_{I,i} &= \frac{1}{2}\int \frac{d^3p_B}{(2\pi)^3 2E_B}\frac{d^3p_V}{(2\pi)^3 2E_V}(2\pi)^4\delta^4(q-p_B-p_V)\,\frac{\lambda}{3q^2}\left|A^V_{I,i}\right|^2 \\
&= \frac{1}{48\pi}\frac{\lambda^{3/2}}{q^4}\left|A^V_{I,i}\right|^2 ,
\end{aligned}
$$

$$
\left|A^V_{T,V}\right|^2 = |\mathcal{B}_V|^2 , \qquad \left|A^V_{T,A}\right|^2 = \sum_{i=1}^{2}|\mathcal{B}_{Ai}|^2 , \qquad \left|A^V_{L,A}\right|^2 = |\mathcal{B}_{A0}|^2 .
$$

Similarly，

$$
\begin{aligned}
\mathrm{Im}\,\Pi^{BP}_{I,V} &= \frac{1}{2}\int \frac{d^3p_B}{(2\pi)^3 2E_B}\frac{d^3p_P}{(2\pi)^3 2E_P}(2\pi)^4\delta^4(q-p_B-p_P)\,\frac{\lambda}{3q^2}\left|A^P_{I,V}\right|^2 \\
&= \frac{1}{48\pi}\frac{\lambda^{3/2}}{q^4}\left|A^P_{I,V}\right|^2 ,
\end{aligned}
$$

$$
\left|A^P_{T,V}\right|^2 = \left|\mathcal{B}_{f_+}\right|^2 , \qquad \left|A^P_{L,V}\right|^2 = |\mathcal{B}_{f_0}|^2 .
$$

$$
\begin{aligned}
\mathcal{B}_{f_+}(q^2) &= f_+(q^2) , \\
\mathcal{B}_{f_0}(q^2) &= \frac{\sqrt{3}(m_B^2-m_P^2)}{\sqrt{\lambda}} f_0(q^2) .
\end{aligned}
$$

Model independent investigation of R_J/psi

- operator product expansion

$$i\int dx\, e^{i\,q\cdot x}\,\langle 0|\,\mathrm{T}\,j^{\mu}(x)\,j^{\dagger\,\nu}(0)\,|0\rangle \;=\; \left(\frac{q^{\mu}q^{\nu}}{q^{2}} - g^{\mu\nu}\right)\sum_{n=1}^{\infty} C_{T,n}(q)\;\langle 0|:\mathcal{O}_n(0):|0\rangle + \frac{q^{\mu}q^{\nu}}{q^{2}}\sum_{n=1}^{\infty} C_{L,n}(q)\;\langle 0|:\mathcal{O}_n(0):|0\rangle\;,$$

This expansion is valid for all matrix elements, provided q is much larger than the characteristic momentum in any of the external states.

$$\chi_L^V(n=1)\Big|_{r=0.286} = 4.48\times10^{-3}\left(1+1.34\alpha_s - 6.50\times10^{-4}\left(\frac{4.9\mathrm{GeV}}{m_b}\right)^4\frac{\langle\alpha_s G^2/\pi\rangle}{0.02\mathrm{GeV}^4}\right);$$

$$\chi_L^A(n=1)\Big|_{r=0.286} = 2.09\times10^{-2}\left(1+0.62\alpha_s + 2.87\times10^{-4}\left(\frac{4.9\mathrm{GeV}}{m_b}\right)^4\frac{\langle\alpha_s G^2/\pi\rangle}{0.02\mathrm{GeV}^4}\right);$$

and

$$\chi_T^V(n=2)\Big|_{r=0.286} = \frac{9.94\times10^{-3}}{m_b^2}\left(1+1.38\alpha_s - 8.69\times10^{-6}\left(\frac{4.9\mathrm{GeV}}{m_b}\right)^4\frac{\langle\alpha_s G^2/\pi\rangle}{0.02\mathrm{GeV}^4}\right),$$

$$\chi_T^A(n=2)\Big|_{r=0.286} = \frac{6.10\times10^{-3}}{m_b^2}\left(1+1.32\alpha_s - 8.40\times10^{-4}\left(\frac{4.9\mathrm{GeV}}{m_b}\right)^4\frac{\langle\alpha_s G^2/\pi\rangle}{0.02\mathrm{GeV}^4}\right),$$

C. G. Boyd, B. Grinstein and R. F. Lebed, Phys. Rev. D **56**, 6895 (1997)

Model independent investigation of R_J/psi

■Quark-hadron duality and the inequality

$$\frac{1}{\pi}\int_0^\infty dt\, \frac{\mathrm{Im}\,\Pi_{I,X}^{BH}(t)}{(t-q^2)^{n+1}}\bigg|_{q^2=-Q_0^2} = \frac{1}{\pi}\int_{t_+}^\infty dt\, \frac{\lambda^{3/2}(t)}{48\pi\, t^2 (t+Q_0^2)^{n+1}} \left|A_{I,X}(t)\right|^2 \leq \chi_I^X(n, Q_0^2)\,,$$

where $\chi_X^X \equiv \chi_{I,\mathrm{OPE}}^X$. One can choose $Q_0^2 = 0 \ll t_+$.

■Map to the unit disk

$$z(t) \equiv z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}\,,$$

$0 \leq t < t_+$ can be mapped onto the unit disk $|z(t)| < 1$,

$t \geq t_+$ onto the unit circle $|z(t)| = 1$.

C. G. Boyd, B. Grinstein and R. F. Lebed, Phys. Rev. Lett. **74**, 4603 (1995)

Model independent investigation of R_J/psi

■The inequality

$$\frac{1}{2\pi i}\oint\frac{dz}{z}\,|\phi_I^X(z)\,A_{I,X}(z)|^2\leq 1 \quad\Leftrightarrow\quad \frac{1}{\pi}\int_{t_+}^{\infty}\frac{dt}{t-t_0}\sqrt{\frac{t_+-t_0}{t-t_+}}\,|\phi_I^X(t)\,A_{I,X}(t)|^2\leq 1\,,$$

$$|\phi_I^X(t)|^2=\frac{1}{48\pi\,\chi_I^X(n)}\,\frac{(t-t_+)^2}{(t_+-t_0)^{1/2}}\,\frac{(t-t_-)^{3/2}}{t^{n+2}}\,\frac{t-t_0}{t}\,.$$

■The new parametrization form

$$A_{I,X}(t)=\frac{(\sqrt{-z(t,0)})^m(\sqrt{z(t,t_-)})^l}{B(t)\,\phi_I^X(t)}\sum_{n=0}^{\infty}\alpha_n\,z^n\,,$$

$$\phi_I^X(t)=\sqrt{\frac{1}{48\pi\chi_I^X(n)}}\,\frac{(t-t_+)}{(t_+-t_0)^{1/4}}\left(\frac{z(t,0)}{-t}\right)^{(n+3)/2}\left(\frac{z(t,t_0)}{t_0-t}\right)^{-1/2}\left(\frac{z(t,t_-)}{t_--t}\right)^{-3/4}.$$

The function $B(t)$ is a Blaschke factor with $B(t)=\prod_i z(t,m_{R_i}^2)$, Results:

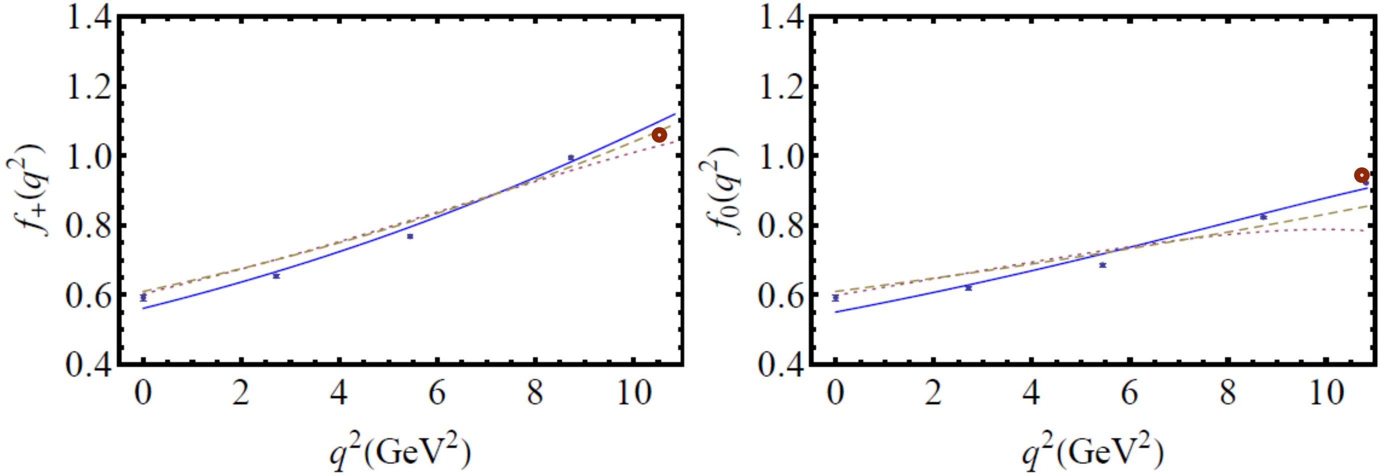


FIG. 1: The form factors of B_c into η_c. The data is from the HPQCD lattice simulations [44]; the blue line is from the z-series based on the lattice data; the dashed line is from the LFQM results [28]; the dotted line is from the the z-series based on the LFQM results.

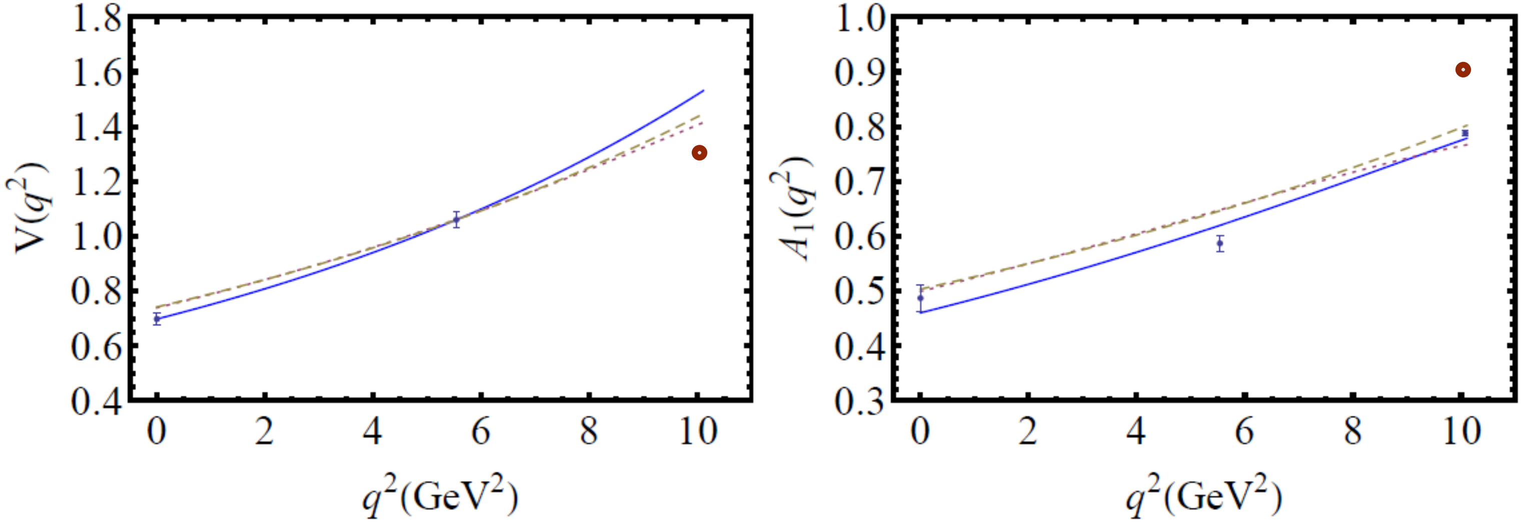


FIG. 2: The form factors of B_c into J/ψ. The data is from the HPQCD lattice simulations [44]; the blue line is from the z-series based on the lattice data; the dashed line is from the LFQM results [28]; the dotted line is from the the z-series based on the LFQM results.

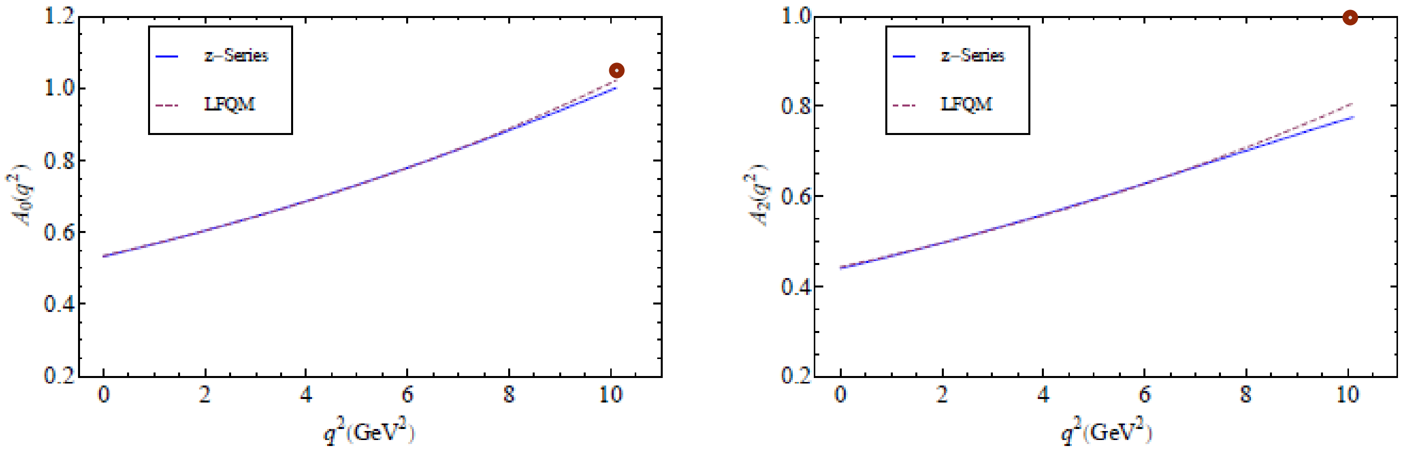


FIG. 3: The form factors of B_c into J/ψ. The dashed line is from the LFQM results [28]; the blue line is from the the z-series based on the LFQM results.

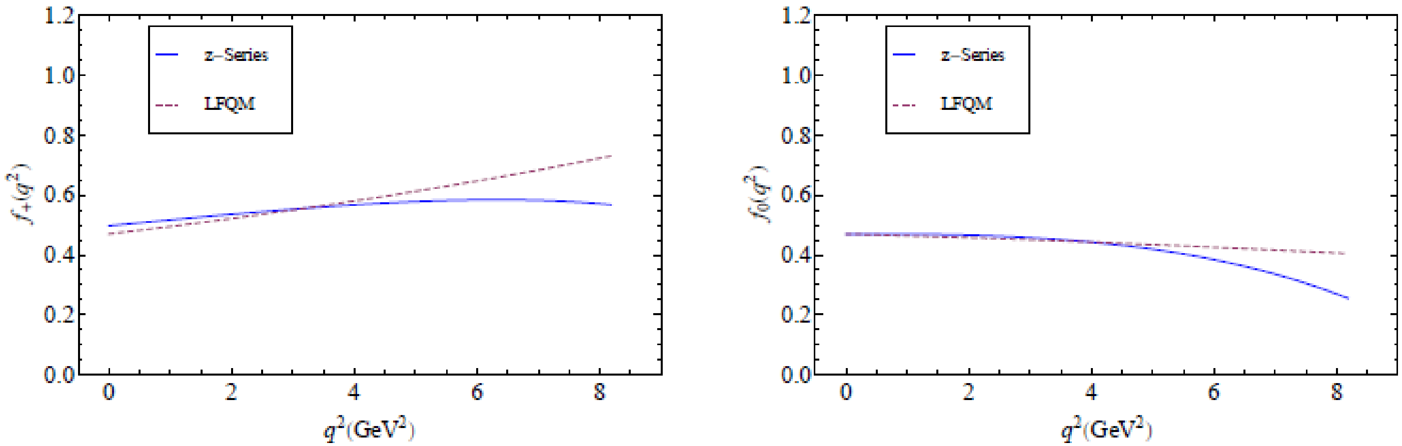


FIG. 4: The form factors of B_c into χ_{c0}. The dashed line is from the LFQM results [29]; the blue line is from the the z-series based on the LFQM results.

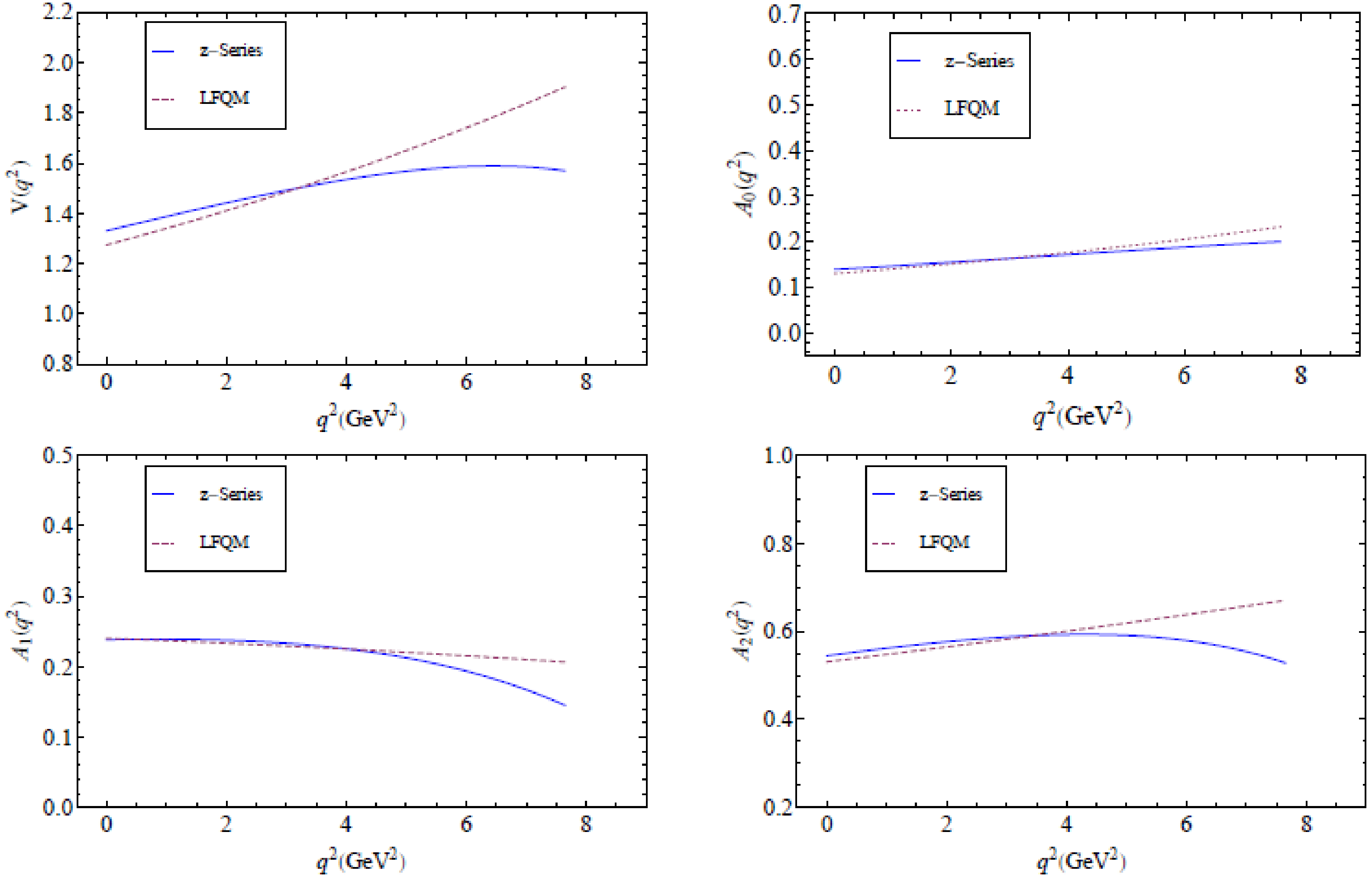


FIG. 5: The form factors of B_c into χ_{c1}. The dashed line is from the LFQM results [29]; the blue line is from the the z-series based on the LFQM results.

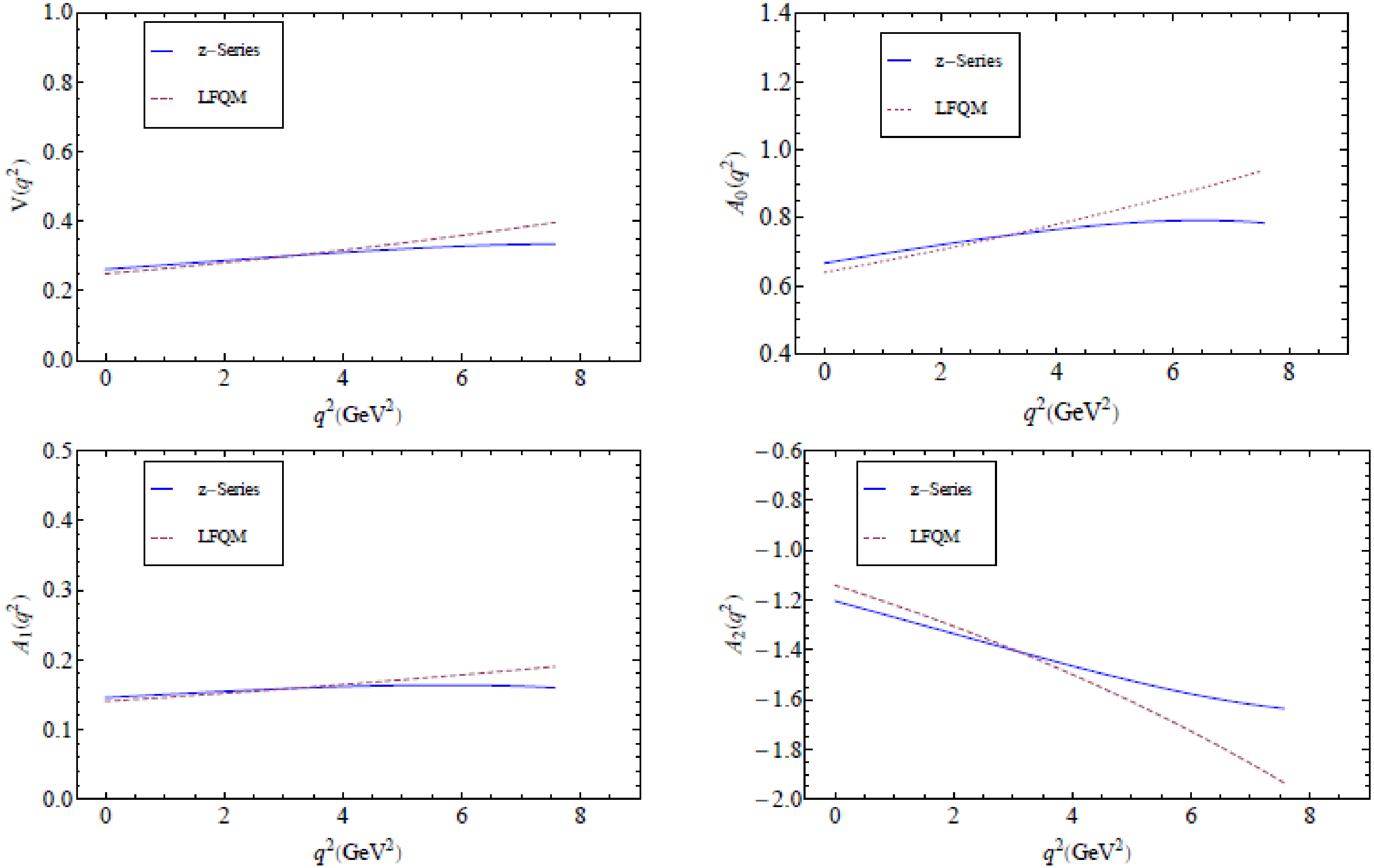


FIG. 6: The form factors of B_c into h_c. The dashed line is from the LFQM results [29]; the blue line is from the the z-series based on the LFQM results.

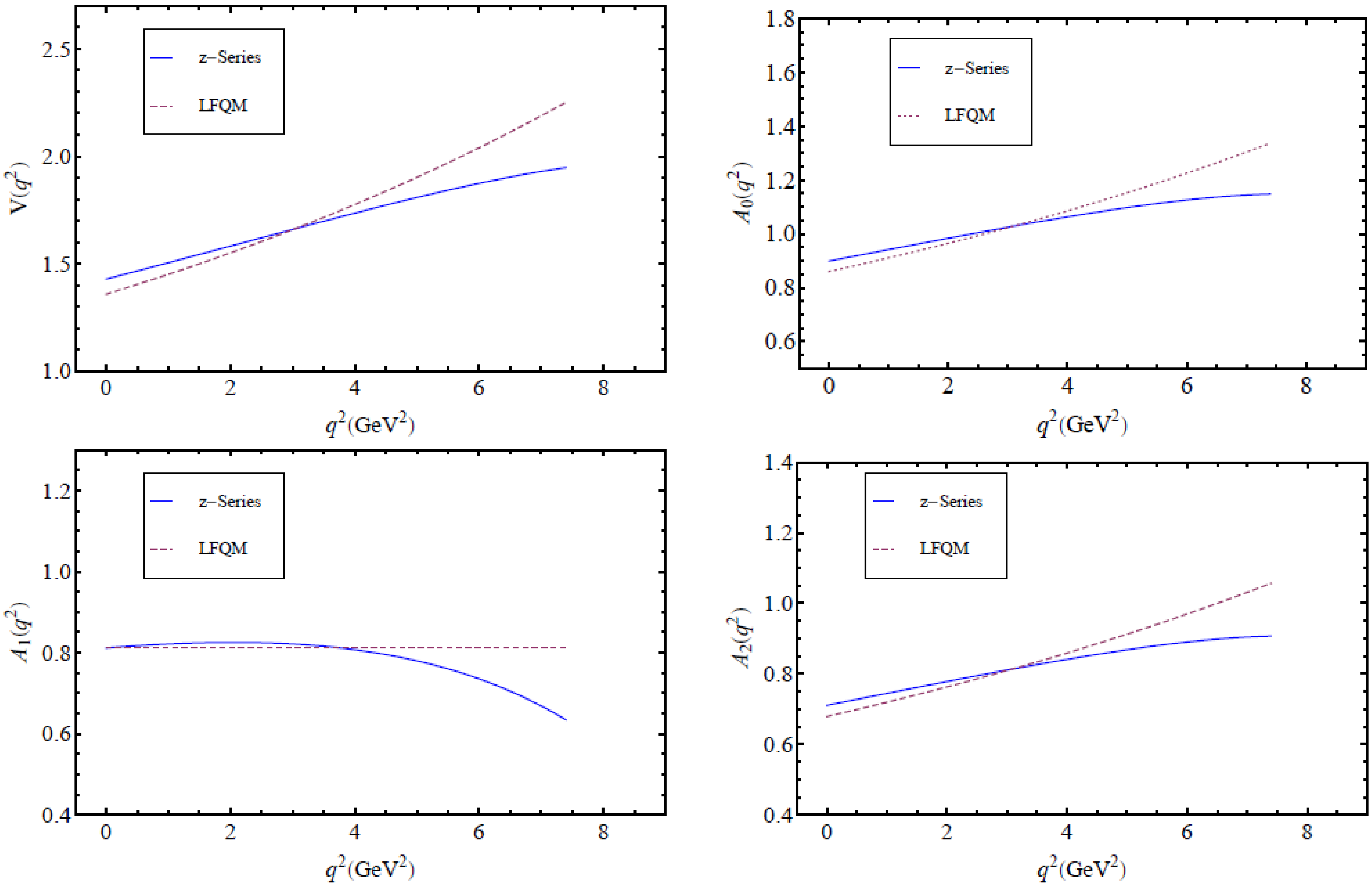


FIG. 7: The form factors of B_c into χ_{c2}. The dashed line is from the LFQM results [29]; the blue line is from the the z-series based on the LFQM results.

Results:

R_H	LHCb data [5]	z-Series approach
$R_{J/\psi}$	$0.71 \pm 0.17(stat) \pm 0.18(syst)$	0.25 ± 0.01
$R^L_{J/\psi}$		0.23 ± 0.01
$R^\perp_{J/\psi}$		0.28 ± 0.01
R_{η_c}		0.31 ± 0.01
$R_{\chi_{c0}}$		0.09 ± 0.01
$R_{\chi_{c1}}$		0.09 ± 0.01
$R^L_{\chi_{c1}}$		0.10 ± 0.02
$R^\perp_{\chi_{c1}}$		0.09 ± 0.01
R_{h_c}		$0.06^{+0.03}_{-0.01}$
$R^L_{h_c}$		$0.06^{+0.02}_{-0.02}$
$R^\perp_{h_c}$		$0.14^{+0.00}_{-0.01}$
$R_{\chi_{c2}}$		$0.04^{+0.00}_{-0.01}$
$R^L_{\chi_{c2}}$		0.03 ± 0.01
$R^\perp_{\chi_{c2}}$		$0.05^{+0.01}_{-0.00}$

$$R_H = \frac{\Gamma(B_c \to H + \tau + \bar{\nu}_\tau)}{\Gamma(B_c \to H + \mu + \bar{\nu}_\mu)},$$

$$R_H^L = \frac{\Gamma_L(B_c \to H + \tau + \bar{\nu}_\tau)}{\Gamma_L(B_c \to H + \mu + \bar{\nu}_\mu)},$$

$$R_H^\perp = \frac{\Gamma_\perp(B_c \to H + \tau + \bar{\nu}_\tau)}{\Gamma_\perp(B_c \to H + \mu + \bar{\nu}_\mu)}.$$

The new physics effects?

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb}[(1 + C_{V_1})O_{V_1} + C_{V_2}O_{V_2} + C_{S_1}O_{S_1} + C_{S_2}O_{S_2} + C_T O_T],$$

$$O_{S_1} = (\bar{c}_L b_R)(\bar{\tau}_R \nu_L), \quad O_{S_2} = (\bar{c}_R b_L)(\bar{\tau}_R \nu_L),$$
$$O_{V_1} = (\bar{c}_L \gamma^\mu b_L)(\bar{\tau}_L \gamma_\mu \nu_L), \quad O_{V_2} = (\bar{c}_R \gamma^\mu b_R)(\bar{\tau}_L \gamma_\mu \nu_L),$$
$$O_T = (\bar{c}_R \sigma^{\mu\nu} b_L)(\bar{\tau}_R \sigma_{\mu\nu} \nu_L),$$

A. K. Alok, D. Kumar, J. Kumar, S. Kumbhakar and S. U. Sankar, arXiv:1710.04127

Z. R. Huang, Y. Li, C. D. Lü, M. A. Paracha and C. Wang, arXiv:1808.03565

TABLE IV: Fitted values of the Wilson coefficients in different NP scenarios.

NP scenario	value	χ^2/dof	Correlation
C_{V_1}	$(1 + Re[C_{V_1}])^2 + (Im[C_{V_1}])^2 = 1.27(6)$	7.42/8	–
C_{V_2}	$0.057(50) \pm 0.573(73)i$	6.19/8	0.750
C_{S_1}	0.405(91)	15.5/8	–
C_{S_2}	$-1.05(30) \pm 1.09(12)i$	5.98/8	0.589
C_T	$0.24(11) \pm 0.13(8)i$	8.39/8	−0.993

Inputting the fitted parameters

Z. R. Huang, Y. Li, C. D. Lü, M. A. Paracha and C. Wang, arXiv:1808.03565

$$[1+\mathrm{Re}(V_1)]^2+\mathrm{Im}(C_{V_1})^2=1.27\pm0.06,\quad C_{V_2}=(0.057+0.573i)\pm(0.050+0.072i).$$

R_H	LHCb data [5]	z-Series approach	S1	S2
$R_{J/\psi}$	$0.71\pm0.17(stat)\pm0.18(syst)$	0.25 ± 0.01	0.31 ± 0.02	0.31 ± 0.01
$R^L_{J/\psi}$		0.23 ± 0.01	0.29 ± 0.01	0.28 ± 0.01
$R^\perp_{J/\psi}$		0.28 ± 0.01	0.36 ± 0.02	0.35 ± 0.01
R_{η_c}		0.31 ± 0.01	0.39 ± 0.02	0.41 ± 0.02
$R_{\chi_{c0}}$		0.09 ± 0.01	0.11 ± 0.01	0.12 ± 0.01
$R_{\chi_{c1}}$		0.09 ± 0.01	0.11 ± 0.01	0.12 ± 0.01
$R^L_{\chi_{c1}}$		0.10 ± 0.02	0.11 ± 0.02	0.13 ± 0.01
$R^\perp_{\chi_{c1}}$		0.09 ± 0.01	0.11 ± 0.01	0.12 ± 0.01
R_{h_c}		$0.06^{+0.03}_{-0.01}$	$0.08^{+0.03}_{-0.01}$	$0.08^{+0.03}_{-0.01}$
$R^L_{h_c}$		$0.06^{+0.02}_{-0.02}$	$0.08^{+0.02}_{-0.02}$	$0.07^{+0.02}_{-0.02}$
$R^\perp_{h_c}$		$0.14^{+0.00}_{-0.01}$	$0.18^{+0.01}_{-0.01}$	$0.17^{+0.01}_{-0.01}$
$R_{\chi_{c2}}$		$0.04^{+0.00}_{-0.01}$	$0.05^{+0.01}_{-0.01}$	$0.05^{+0.01}_{-0.01}$
$R^L_{\chi_{c2}}$		0.03 ± 0.01	0.04 ± 0.01	0.04 ± 0.01
$R^\perp_{\chi_{c2}}$		$0.05^{+0.01}_{-0.00}$	$0.06^{+0.01}_{-0.01}$	$0.07^{+0.01}_{-0.01}$

Summary

- The theoretical uncertainties are reduced. The unitary constraint on the form factors is model independent.

- Issue of b to c/s anomalies is still an open question, which is hard to understand within the current data.

- The helicity dependent observations and other independent channels can provide more information to understand these anomalies.

Thank You

for your attention !