第十六届重味物理和CP破坏研讨会

郑州，10.26 - 10.29.2018

Chiral study of open charm meson resonances

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Outline:

1. Introduction

2. Chiral EFT in finite box

3. Fits to the lattice energy levels

   and physical predictions

4. Summary

Introduction

First glance from PDG

CHARMED MESONS
($C = \pm1$)
$D^+ = c\bar{d}$, $D^0 = c\bar{u}$, $\bar{D}^0 = \bar{c}u$, $D^- = \bar{c}d$, similarly for D^*'s

INSPIRE search

$D_0^*(2400)^\pm$ $\quad I(J^P) = 1/2(0^+)$

J, ^{P} need confirmation.

$D_0^*(2400)^\pm$ MASS	2351 ± 7 MeV
$D_0^*(2400)^\pm$ WIDTH	230 ± 17 MeV (S = 1.1)

CHARMED, STRANGE MESONS
($C = S = \pm1$)
$D_s^+ = c\bar{s}$, $D_s^- = \bar{c}s$, similarly for D_s^*'s

INSPIRE search

$D_{s0}^*(2317)^\pm$ $\quad I(J^P) = 0(0^+)$

AUBERT 2006P and CHOI 2015A do not observe neutral and doubly charged partners of the $D_{s0}^*(2317)^+$.

$D_{s0}^*(2317)^\pm$ MASS	2317.7 ± 0.6 MeV (S = 1.1)
$m_{D_{s0}^*(2317)^{\pm}} - m_{D_{s\pm}}$	349.4 ± 0.6 MeV (S = 1.1)
$D_{s0}^*(2317)^\pm$ WIDTH	< 3.8 MeV CL=95.0%

$D^*_{s0}(2317)$: a hot topic at *B*-Factory

1. **The BaBar detector**
(2145) BaBar Collaboration (Bernard Aubert (Annecy, LAPP) *et al.*). Apr 2001. 119 pp.
Published in **Nucl.Instrum.Meth. A479 (2002) 1-116**
SLAC-PUB-8569, BABAR-PUB-01-08
DOI: 10.1016/S0168-9002(01)02012-5
e-Print: **hep-ex/0105044** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; BaBar Publications Database; BaBar Password Protected Publications Data
Detailed record - Cited by 2145 records 1000+

2. **Observation of CP violation in the B^0 meson system**
(867) BaBar Collaboration (Bernard Aubert (Annecy, LAPTH) *et al.*). Jul 2001. 8 pp.
Published in **Phys.Rev.Lett. 87 (2001) 091801**
SLAC-PUB-8904, BABAR-PUB-01-18
DOI: 10.1103/PhysRevLett.87.091801
e-Print: **hep-ex/0107013** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; OSTI.gov Server; BaBar Publications Database; BaBar Password Protecte
Document Server
Detailed record - Cited by 867 records 500+

3. **Observation of a narrow meson decaying to $D_s^+\pi^0$ at a mass of 2.32-GeV/c^2**
(823) BaBar Collaboration (B. Aubert (Annecy, LAPP) *et al.*). Apr 2003.
Published in **Phys.Rev.Lett. 90 (2003) 242001**
SLAC-PUB-9711, BABAR-PUB-03-011
DOI: 10.1103/PhysRevLett.90.242001
e-Print: **hep-ex/0304021** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac |
ADS Abstract Service; OSTI.gov Server; BaBar Publications
Document Server
Detailed record - Cited by 823 records 500+

Top 3 @BaBar

5. **Measurements of the meson - photon transition form-factors of light pseudosca**
(610) CLEO Collaboration (J. Gronberg (UC, Santa Barbara) *et al.*). Jul 1997. 30 pp.
Published in **Phys.Rev. D57 (1998) 33-54**
SLAC-PUB-9838, CLNS-97-1477, CLEO-97-7
DOI: 10.1103/PhysRevD.57.33
e-Print: **hep-ex/9707031** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
KEK scanned document; ADS Abstract Service; Cornell U., LNS Server; SLAC Document Server; SLA
Data: INSPIRE | HepData
Detailed record - Cited by 610 records 500+

6. **Observation of a narrow resonance of mass 2.46-GeV/c**2 decaying to D*+(s) pi**
(575) CLEO Collaboration (D. Besson (Kansas U.) *et al.*). May 2003. 16 pp.
Published in **Phys.Rev. D68 (2003) 032002**, Erratum: **Phys.Rev. D75 (2007) 119908**
CLNS-03-1826, CLEO-03-09, CLNS03-1826
DOI: 10.1103/PhysRevD.68.032002, 10.1103/PhysRevD.75.119908
e-Print: **hep-ex/0305100** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 575 records 500+

Top 6 @CLEO

7. **Observation of B Meson Semileptonic Decays to Noncharmed Final States**
(419) CLEO Collaboration (R. Fulton *et al.*). Nov 1989. 14 pp.
Published in **Phys.Rev.Lett. 64 (1990) 16-20**
CLNS-89/951, CLEO-89-14
DOI: 10.1103/PhysRevLett.64.16
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
OSTI.gov Server
Detailed record - Cited by 419 records 250+

Published in Phys.Rev.Lett. 97 (2006) 251802
BELLE-PREPRINT-2006-11
DOI: 10.1103/PhysRevLett.97.251802
e-Print: **hep-ex/0604018** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 347 records 250+

16. **Study of B- ---> D**0 pi- (D**0 ---> D(*)+ pi-) decays**
(338) Belle Collaboration (Kazuo Abe (KEK, Tsukuba) *et al.*). Jul 2003. 20 pp.
Published in **Phys.Rev. D69 (2004) 112002**
DOI: 10.1103/PhysRevD.69.112002
e-Print: **hep-ex/0307021** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 338 records 250+

Top 20 @Belle

17. **Observation of the D(sJ)(2317) and D(sJ)(2457) in B decays**
(337) Belle Collaboration (P. Krokovny (Novosibirsk, IYF) *et al.*). Aug 2003. 6 pp.
Published in **Phys.Rev.Lett. 91 (2003) 262002**
DOI: 10.1103/PhysRevLett.91.262002
e-Print: **hep-ex/0308019** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 337 records 250+

18. **Observation of double c anti-c production in e+ e- annihilation at**
(337) Belle Collaboration (Kazuo Abe (KEK, Tsukuba) *et al.*). May 2002. 7 pp.
Published in **Phys.Rev.Lett. 89 (2002) 142001**
BELLE-PREPRINT-2002-13, KEK-PREPRINT-2002-27
DOI: 10.1103/PhysRevLett.89.142001
e-Print: **hep-ex/0205104** | **PDF**

Continuing efforts from LHCb, Belle-II, PANDA,…

and also Lattice QCD

Theoretical interpretations

recent reviews: H.X.Chen, et al., Rept.Prog.Phys.2017; F.K.Guo, et al., Rev.Mod.Phys.2018

- $c\bar{s}$ state: Y.B.Dai, et al., PRD2003 ; S.Narison, PLB2005; E. van Beveren, et al., PRL 2003; Z.G.Wang, PRD2006 ……

- Hadronic molecular state: T.Barnes, et al., PRD'03; F.K.Guo, et al., PLB'06; Altenbuchinger, et al., PRD'14; Du, et al., 1712.07957; Albaladejo, et al. EPJC'18 …

- Four-quark state: H.Y.Cheng, et al., PRD 2003; K. Terasaki, PRD2003; L.Maiani et.al., PRD2005; M.Bracco, et al., PLB2005; ……

- Mixing of molecular and four-quark states: T. Browder, et al., PLB2004;……

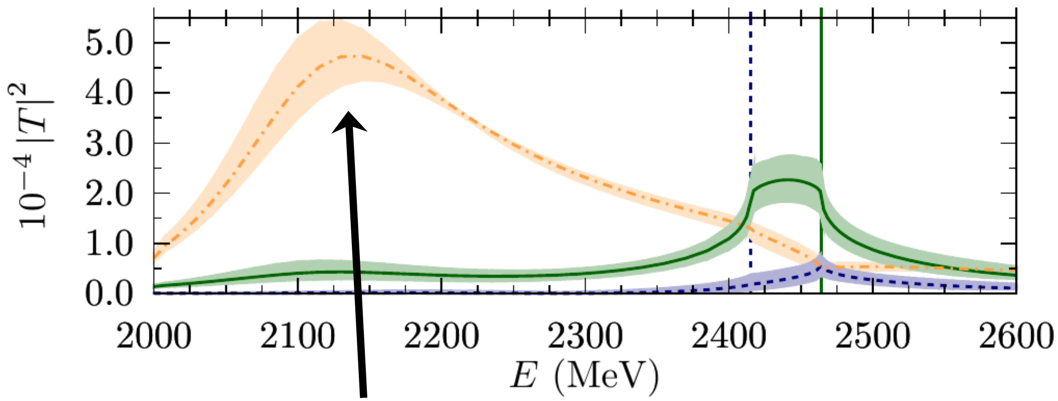
- ……

$D^*_0(2400)$: two-pole structure ?

[ZHG,Meissner,Yao, PRD'15]

(S, I)	RS	$\sqrt{s_{pole}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV]
$(0, \frac{1}{2})$	II	$2114^{+3}_{-3} - i\,111^{+8}_{-7}$	$9.66^{+0.15}_{-0.13}(D\pi)$
	III	$2473^{+29}_{-22} - i\,140^{+8}_{-7}$	$5.36^{+0.40}_{-0.28}(D\pi)$

[Albaladejo,Fernandez-Soler,F.K.Guo,Nieves, PLB'17]



Absent in PDG!

The above results are predicted from other channels with different quantum numbers.

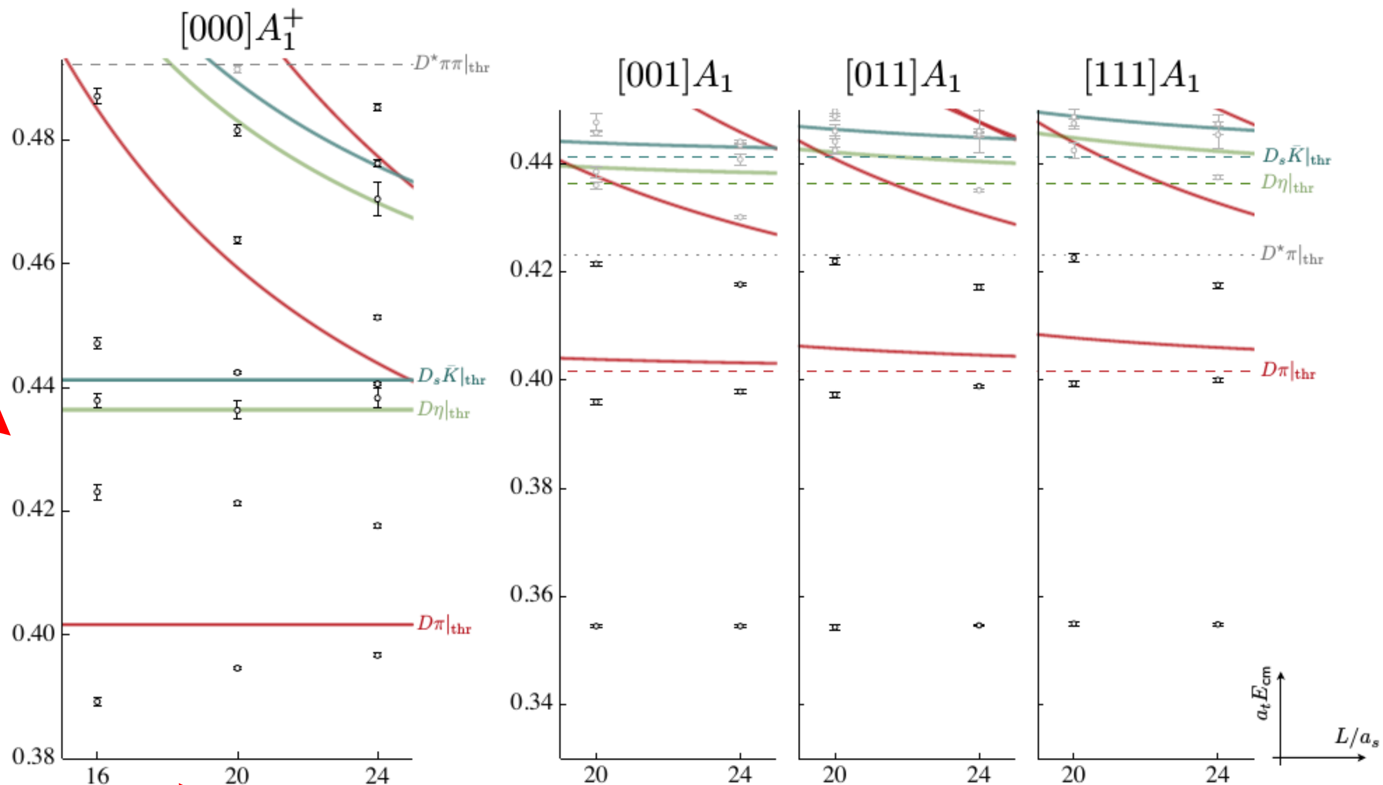
Direct Exp measurements, like phase shifts & inelasticities, are not available at present!

Any concrete confirmation about the resonance structures in D-pi, D-eta, Ds-Kbar coupled channel scattering?

Yes, Lattice QCD can help !

[Moir, et al., JHEP'16]

Eigenenergies in finite box



Length of finite box

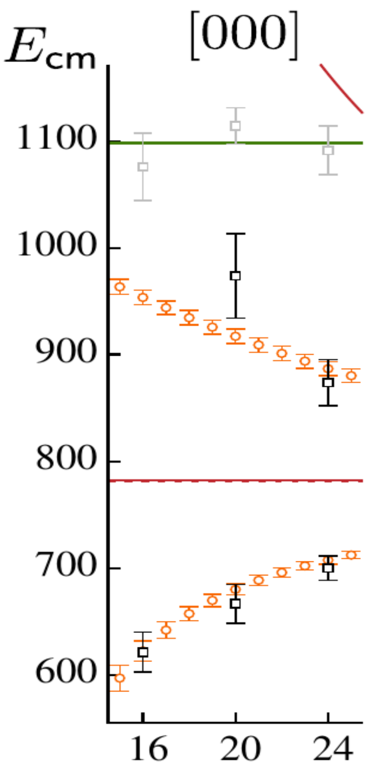
Luscher's Approach:

connect the discrete spectra in finite box to the scattering amplitudes in the infinite volume

[Luscher, NPB '91]

Elastic scattering case:

$$\tan\phi(q) = -\frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$



Phase shifts

Luscher's Zeta function
(function of *L*, parameter free)

- **For the elastic case, one has the one-to-one correspondance between the phase shifts and energy levels.**
- **The one-to-one correspondance will be lost in the inelastic case.**

[He,Feng,Liu,JHEP'05] [Wilson,Briceno,Dudek,Edwards,Thomas, PRD '15]
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn, PRD'14] [Fu,PRD'12]
[Gockeler,Horsley,Lage,Meissner,Rakow,Rusetksy,Schierholz,Zanotti,PRD'12] ......

A widely used approach in the inelastic scattering case:

Luscher function + K matrix

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (\mathbf{1} + i\mathcal{M})] = 0$$

Kinematical factor

K matrix: polynomial + pole terms

Luscher's finite-volume functions (complex objects)

- **Free parameters in K matrix are determined by the finite-volume spectra. Then one can determine amplitudes in infinite volume.**
- **K matrix does not automatically respect the QCD symmetries, such as the chiral symmetry. It could be problematic for chiral extrapolation.**

Our approach:

Step 1: Put chiral perturbation theory (ChPT) in finite volume.

Step 2: The free parameters in ChPT, which are indepdent of quark masses and volumes, are fitted to the finite-volume energy levels obtained at (un)physical quark masses.

Step 3: Perform the chiral extrapolation and give the predictions in infinite volume with physical quark masses, including phase shifts, inelasticities, resonance poles, etc.

I will show some results for the D-pi, D-eta and Ds-Kbar scattering.

Theoretical framework

LO ChPT with heavy-light mesons

$$\mathcal{L}_{D\phi}^{(1)} = \mathcal{D}_\mu D \mathcal{D}^\mu D^\dagger - \overline{M}_D^2 D D^\dagger$$

NLO ChPT with heavy-light mesons [F.K.Guo,et al.,PLB'08]

$$\begin{aligned}\mathcal{L}_{D\phi}^{(2)} &= D\left(-h_0\langle\chi_+\rangle - h_1\chi_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu\right) D^\dagger \\ &+\mathcal{D}_\mu D\left(h_4\langle u_\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\}\right)\mathcal{D}_\nu D^\dagger\ ,\end{aligned}$$

with $u_\mu = i\left(u^\dagger \partial_\mu u - u\,\partial_\mu u^\dagger\right)$ $u = \exp\left(\frac{i\phi}{\sqrt{2}F_0}\right)$ $\chi_\pm = u^\dagger \chi u^\dagger \pm u\chi u$

$$\phi = \begin{pmatrix} \frac{\sqrt{3}\pi^0 + \eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\sqrt{3}\pi^0 + \eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta_8}{\sqrt{6}} \end{pmatrix}$$

$D_{(s)}$ and light pseudoscalar meson scattering amplitudes

(S,I)	Channels	C_{LO}	C_0	C_1	C_{24}	C_{35}
$(-1,0)$	$D\bar{K}\to D\bar{K}$	-1	m_K^2	m_K^2	1	-1
$(-1,1)$	$D\bar{K}\to D\bar{K}$	1	m_K^2	$-m_K^2$	1	1
$(2,\frac{1}{2})$	$D_sK\to D_sK$	1	m_K^2	$-m_K^2$	1	1
$(0,\frac{3}{2})$	$D\pi\to D\pi$	1	m_π^2	$-m_\pi^2$	1	1
$(1,1)$	$D_s\pi\to D_s\pi$	0	m_π^2	0	1	0
	$DK\to DK$	0	m_K^2	0	1	0
	$DK\to D_s\pi$	1	0	$-(m_K^2+m_\pi^2)/2$	0	1
$(1,0)$	$DK\to DK$	-2	m_K^2	$-2m_K^2$	1	2
	$DK\to D_s\eta$	$-\sqrt{3}$	0	$\frac{-5m_K^2+3m_\pi^2}{2\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$
	$D_s\eta\to D_s\eta$	0	$\frac{4m_K^2-m_\pi^2}{3}$	$\frac{4(m_\pi^2-2m_K^2)}{3}$	1	$\frac{4}{3}$
$(0,\frac{1}{2})$	$D\pi\to D\pi$	-2	m_π^2	$-m_\pi^2$	1	1
	$D\eta\to D\eta$	0	$\frac{4m_K^2-m_\pi^2}{3}$	$\frac{-m_\pi^2}{3}$	1	$\frac{1}{3}$
	$D_s\bar{K}\to D_s\bar{K}$	-1	m_K^2	$-m_K^2$	1	1
	$D\eta\to D\pi$	0	0	$-m_\pi^2$	0	1
	$D_s\bar{K}\to D\pi$	$-\frac{\sqrt{6}}{2}$	0	$\frac{-\sqrt{6}(m_K^2+m_\pi^2)}{4}$	0	$\frac{\sqrt{6}}{2}$
	$D_s\bar{K}\to D\eta$	$-\frac{\sqrt{6}}{2}$	0	$\frac{5m_K^2-3m_\pi^2}{2\sqrt{6}}$	0	$\frac{-1}{\sqrt{6}}$

Unitarization: Algebraic approximation of N/D (a variant version of K-matrix)

[Oller,Oset, PRD '99]

$$T_J = \frac{N(s)}{1 - N(s)\,G(s)}$$

- **The s-channel unitariy is exact. The crossed-channel dyanmics is included in a perturbative manner.**
- **Unitarity condition:** $\mathrm{Im}G(s) = -\rho(s)$

$$G(s) = \boxed{a^{SL}(s_0)} - \frac{s - s_0}{\pi} \int_{4m^2}^{\infty} \frac{\rho(s')}{(s' - s)(s' - s_0)} ds'$$

- ***N*(*s*): given by the partial wave chiral amplitudes**

$$\mathcal{V}^{(S,I)}_{J,D_1\phi_1\to D_2\phi_2}(s) = \frac{1}{2}\int_{-1}^{+1} \mathrm{d}\cos\varphi P_J(\cos\varphi) V^{(S,I)}_{D_1\phi_1\to D_2\phi_2}(s, t(s,\cos\varphi))$$

Finite-volume effects

Two types of finite volume dependence of scattering amplitudes:

- **Exponentially suppressed type ∝ *exp*(-m$_p$L) : *s*, *t*, *u* channels**
- **Power suppressed type ∝ *1/L*^{3} : only *s* channel**

We ignore the exponentially suppressed terms, indicating that finite-volume effects only enter through s channel.

$$T_J = \frac{N(s)}{1 - N(s)\, G(s)}$$

I.e. We only consider the finite-volume corrections for $G(s)$.

$$G(s) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(q^2-m_1^2+i\epsilon)[(P-q)^2-m_2^2+i\epsilon]}\,, \qquad s\equiv P^2$$

Sharp momentum cutoff to regularize G(s)

$$G(s)^{\rm cutoff} = \int^{|\vec q|<q_{\rm max}} \frac{d^3\vec q}{(2\pi)^3} I(|\vec q|)\,, \qquad I(|\vec q|) = \frac{w_1+w_2}{2w_1w_2\,[E^2-(w_1+w_2)^2]}\,, \quad w_i = \sqrt{|\vec q|^2+m_i^2}\,, \quad s=E^2$$

G(s) in a finite box of length L with periodic boundary condition

$$\widetilde{G} = \frac{1}{L^3}\sum_{\vec n}^{|\vec q|<q_{\rm max}} I(|\vec q|)\,, \qquad \vec q = \frac{2\pi}{L}\vec n, \quad \vec n\in\mathbb{Z}^3$$

Finite-volume correction ΔG [Doring, Meissner, Oset, Rusetsky, EPJA11]

$$\Delta G = \widetilde{G} - G^{\rm cutoff}$$

$$= \left\{\frac{1}{L^3}\sum_{\boldsymbol q}^{|\boldsymbol q|<q_{\rm max}} - \int^{|\boldsymbol q|<q_{\rm max}}\frac{d^3\boldsymbol q}{(2\pi)^3}\right\}\frac{1}{2\omega_1(\boldsymbol q)\,\omega_2(\boldsymbol q)}\,\frac{\omega_1(\boldsymbol q)+\omega_2(\boldsymbol q)}{E^2-(\omega_1(\boldsymbol q)+\omega_2(\boldsymbol q))^2}$$

Finite-volume effects in the moving frames

Lorentz invariance is lost in finite box. One needs to work out the explicit form of the loops when boosting from one frame to another.

transforming $\vec{q}_{i=1,2}$ to $\vec{q}^{\,*}_{i=1,2}$ → **CM quantities**

$$\vec{q}^{\,*}_i = \vec{q}_i + \left[\left(\frac{P^0}{E} - 1\right)\frac{\vec{q}_i \cdot \vec{P}}{|\vec{P}|^2} - \frac{q_i^0}{E}\right]\vec{P}$$

moving frame with total four-momentum $P^\mu = (P^0, \vec{P})$ $\quad s = E^2 = (P^0)^2 - |\vec{P}|^2$

Impose on-shell condition

$$q_i^{*\,0} = \sqrt{|\vec{q}^{\,*}_i|^2 + m_i^2}$$

$$q_i^0 = \frac{q_i^{*\,0} E + \vec{q}_i \cdot \vec{P}}{P^0} \longrightarrow q_i^0 = \sqrt{|\vec{q}_i|^2 + m_i^2}$$

G function in the moving frame

$$\int^{|\vec{q}_1|^* < q_{\max}} \frac{d^3 \vec{q}^{\,*}_1}{(2\pi)^3} I(|\vec{q}^{\,*}_1|) \Longrightarrow \widetilde{G}^{\rm MV} = \frac{E}{P^0 L^3} \sum_{\vec{q}_1}^{|\vec{q}^{\,*}_1| < q_{\max}} I(|\vec{q}^{\,*}_1(\vec{q}_1)|)$$

$$\vec{q}_1 = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3, \qquad \vec{P} = \frac{2\pi}{L}\vec{N}, \quad \vec{N} \in \mathbb{Z}^3$$

Finite-volume correction $\Delta G^{\rm MV}$:

$$\Delta G^{\rm MV} = \widetilde{G}^{\rm MV} - G^{\rm cutoff}$$

[Doring, Meissner, Oset, Rusetsky, EPJA12]

Mixing of different partial waves in finite volume

The mixing is absent in the infinite volume:
$$\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta Y_{\ell m}(\theta,\phi) Y^*_{\ell' m'}(\theta,\phi) = \delta_{\ell\ell'}\delta_{mm'}$$

The mixing appears in finite-volume case, due to the absence of the general orthogonal conditions.

Finite-volume correction to G function:
$$\Delta G^{\rm MV}_{\ell m} = \widetilde{G}^{\rm MV}_{\ell m} - G^{\rm cutoff}$$

$$\widetilde{G}^{\rm MV}_{\ell m} = \sqrt{\frac{4\pi}{2\ell+1}}\frac{1}{L^3}\frac{E}{P^0}\sum_{\vec{n}}^{|\vec{q}^*|<q_{\max}}\left(\frac{|\vec{q}^*|}{|\vec{q}^{\rm on*}|}\right)^{\ell} Y_{\ell m}(\hat{q}^*)\, I(|\vec{q}^*|)$$

Final expression for the G function:
$$\widetilde{G}_{\ell m} = G^{\rm Infinite\ volume} + \Delta G^{\rm MV}_{\ell m}$$

To determine the energy levels in different frames with only S and P waves:

A_1^+ (0,0,0) : $\det[I + N_0(s)\cdot \widetilde{G}_{00}] = 0$

T_1^- (0,0,0): $\det[I + N_1(s)\cdot (\widetilde{G}_{00} + 2\widetilde{G}_{20})] = 0$

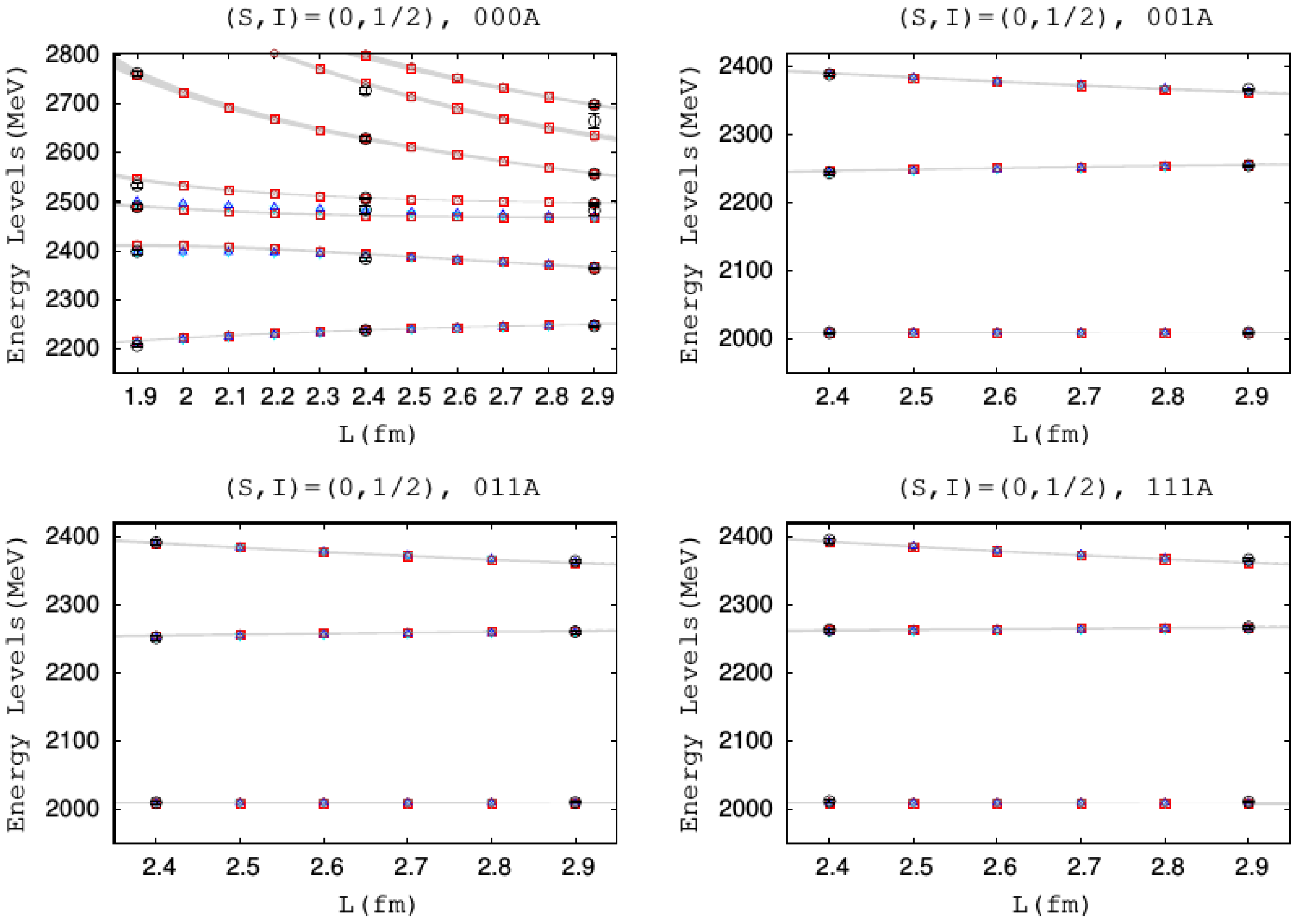
A_1 (0,0,1): $\det[I + N_{0,1}\cdot \mathcal{M}^{A_1}_{0,1}] = 0\,,\quad N_{0,1} = \begin{pmatrix} N_0 & 0 \\ 0 & N_1 \end{pmatrix}\,,\quad \mathcal{M}^{A_1}_{0,1} = \begin{pmatrix} \widetilde{G}_{00} & i\sqrt{3}\widetilde{G}_{10} \\ -i\sqrt{3}\widetilde{G}_{10} & \widetilde{G}_{00}+2\widetilde{G}_{20} \end{pmatrix}$

……

[ZHG,Liu,Meissner,Oller,Rusetsky, to appear]

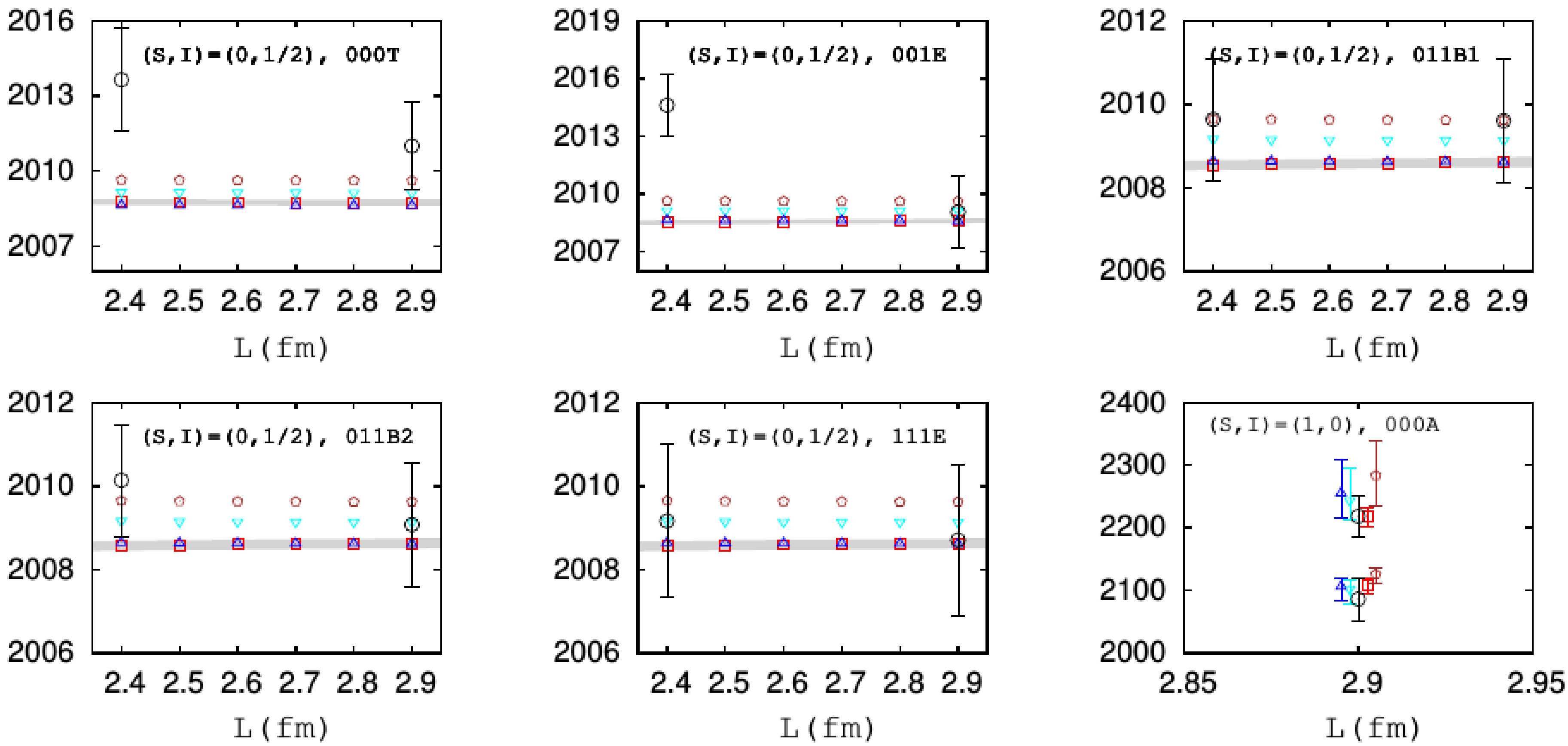
Reprduction of the lattice energy levels

[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]



[ZHG,Liu,Meissner,Oller,Rusetsky, to appear]

Reprduction of the lattice energy levels

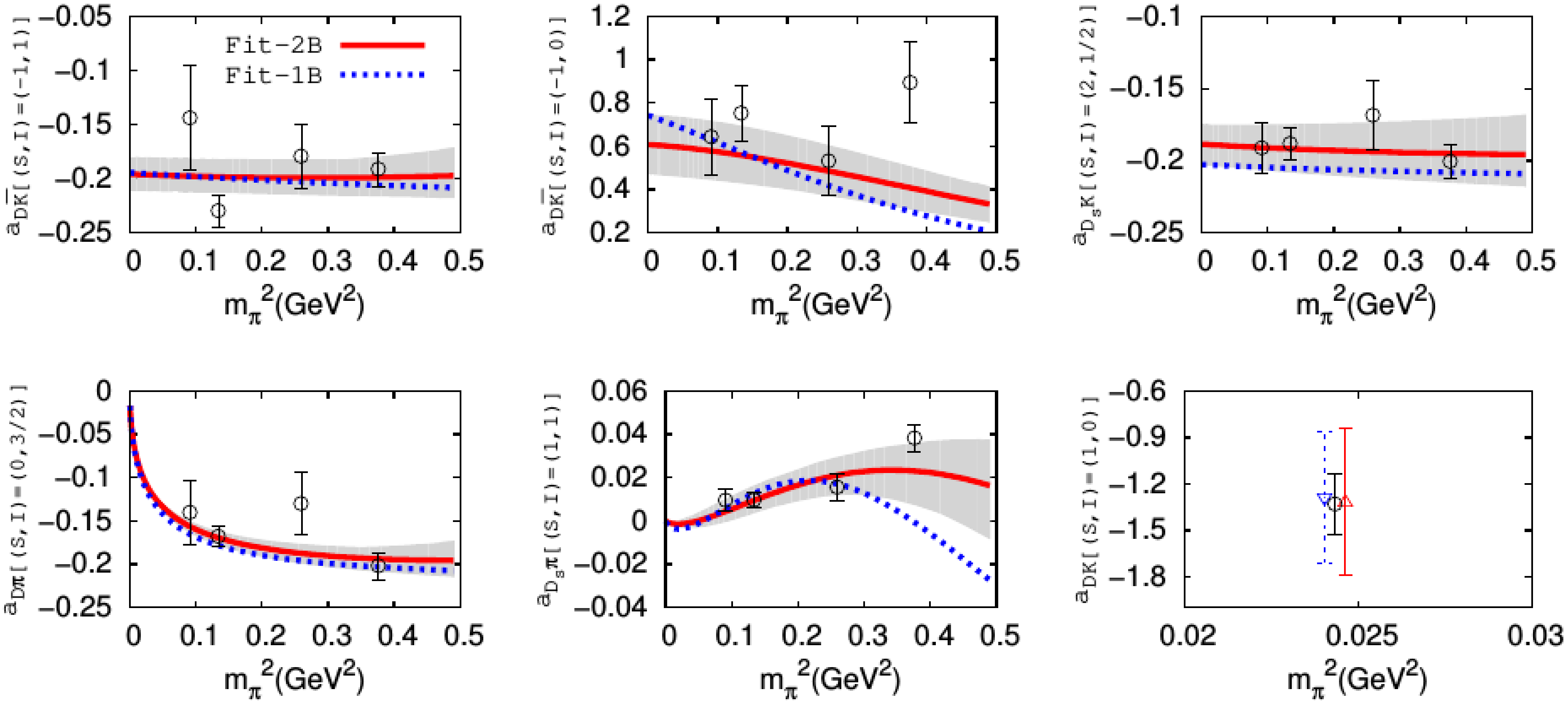


[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]

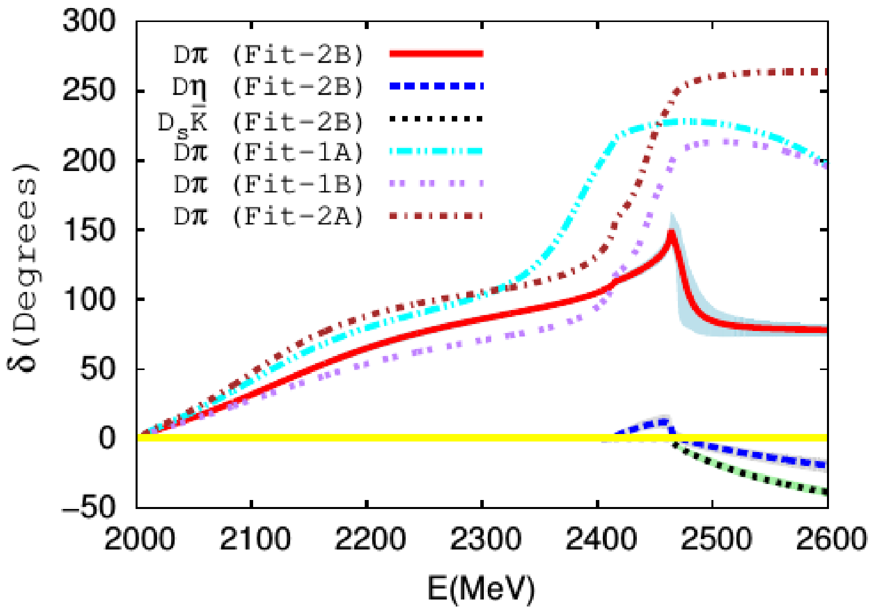
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]

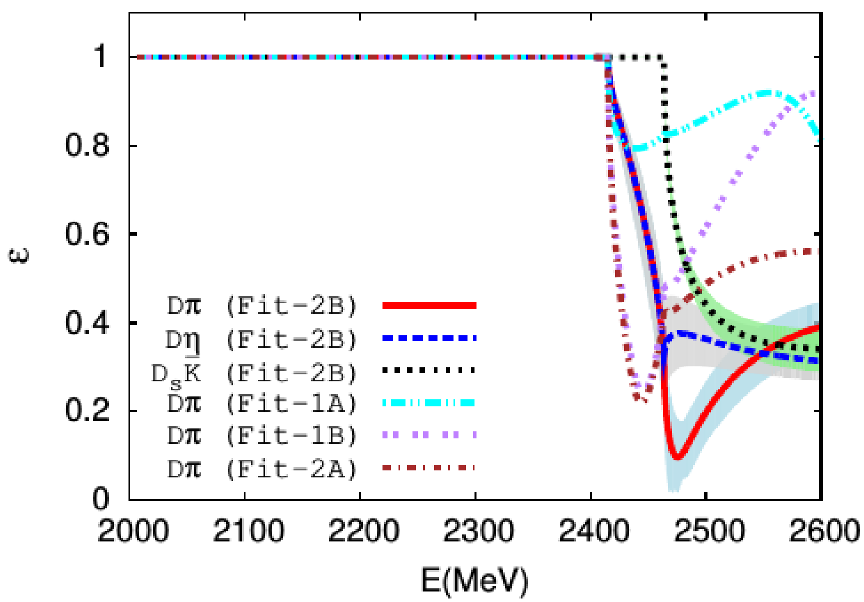
Reproduction of scattering lengths

[L.Liu,Orginos,F.K.Guo,Meissner, PRD'13]
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]



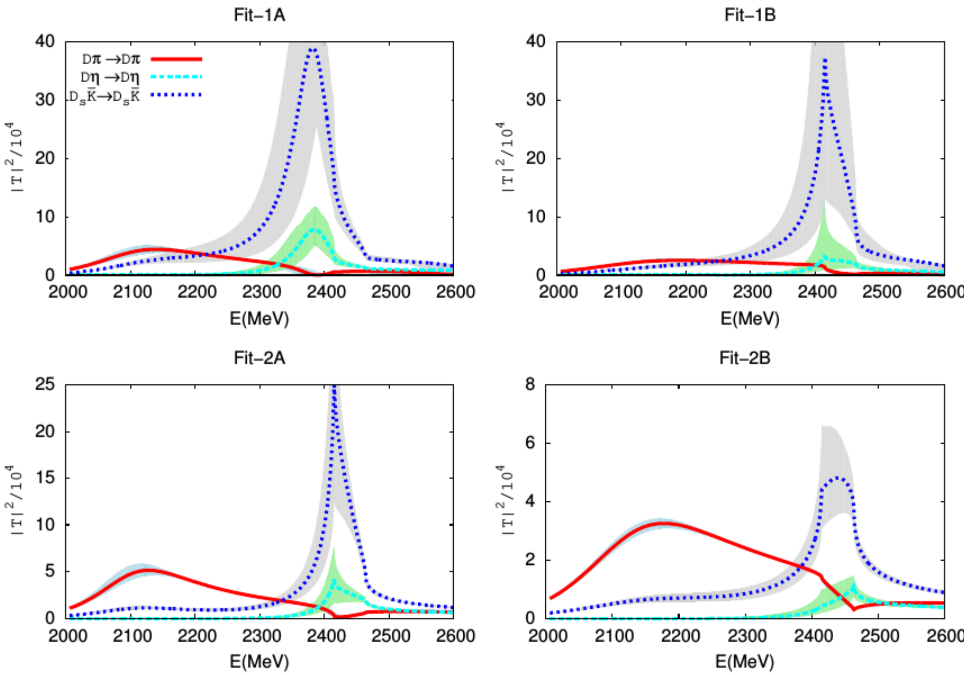
Prediction of the physical phase shifts and inelasticities

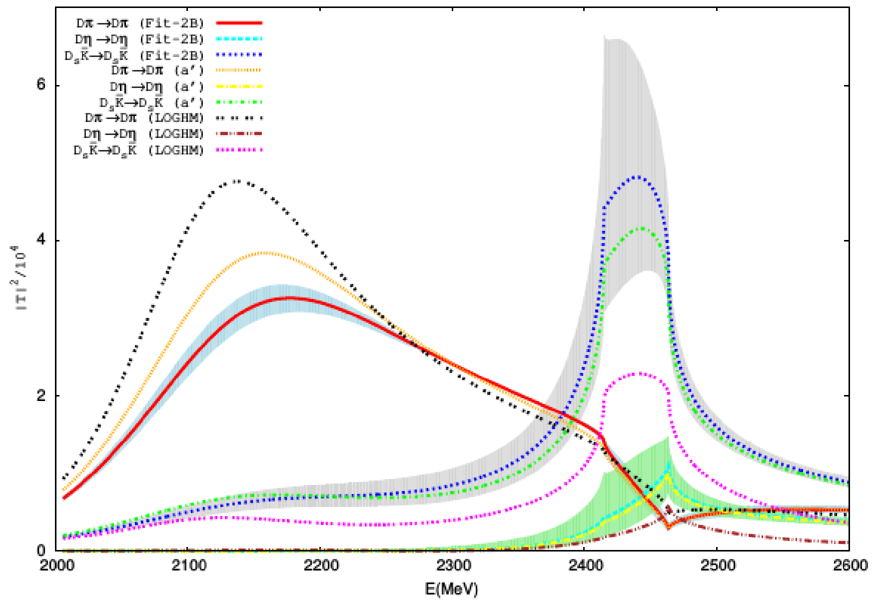




[ZHG,Liu,Meissner,Oller,Rusetsky, to appear]

Prediction of the physical scattering amplitudes





[ZHG,Liu,Meissner,Oller,Rusetsky, to appear]

Summary

- The chiral approach illustrated in this talk provides an efficient way to study the finite-volume energy levels.

- It can build a bridge to connect the lattice eigenenergies in finite box obtained at unphysical masses with the physical observables, such as phase shifts, inelasticities, at physical meson masses.

- We have successfully applied this approach to the pi-eta, K-Kbar and pi-eta' coupled-channel scattering.

- Similar study in the D-pi, D-eta and Ds-Kbar is in progress.

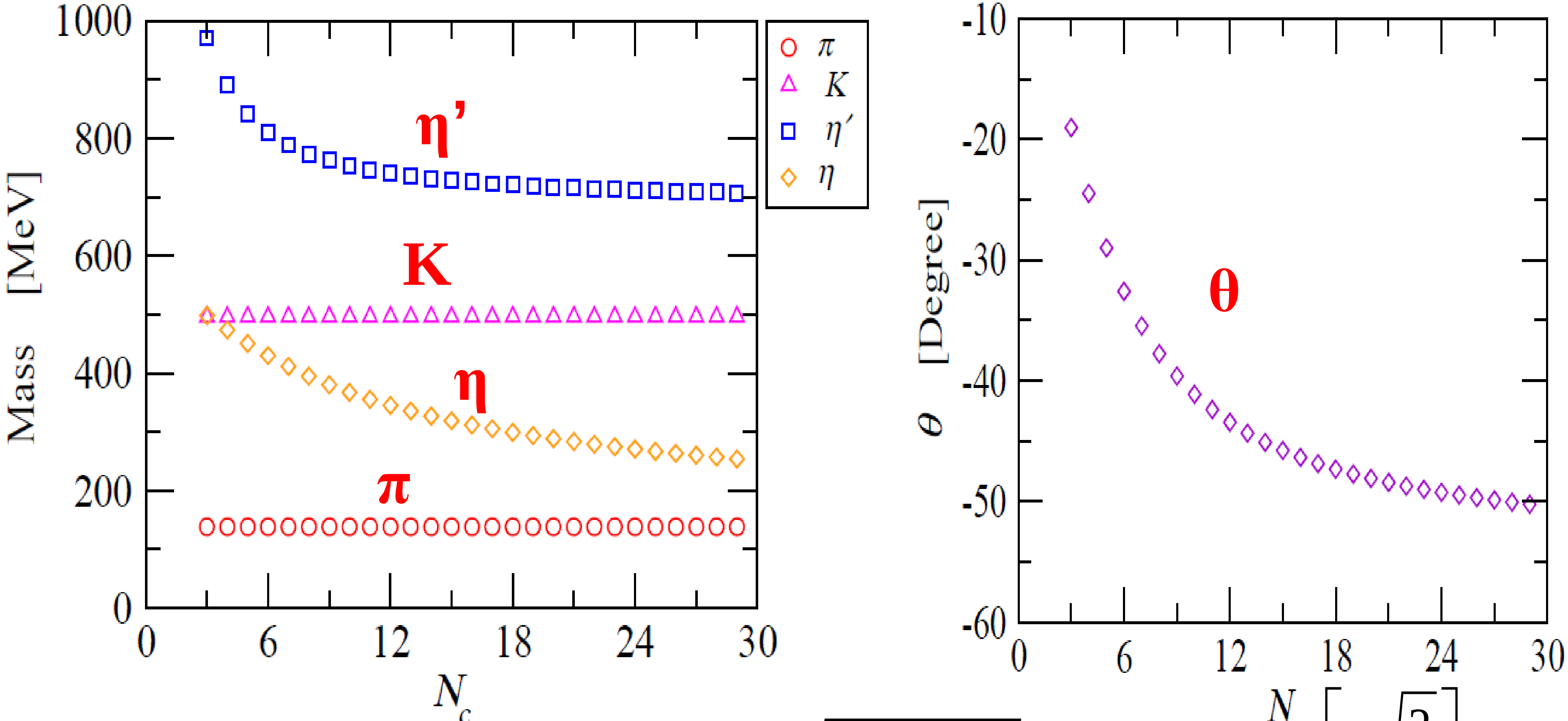
谢谢大家!

Chiral Lagrangian for pseudo-Goldstone with $U_A(1)$ anomaly

$$\mathcal{L}_\chi = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle\chi_+\rangle + \boxed{\frac{F^2}{3}M_0^2\ln^2 \det u}$$

Leads to a massive η_0

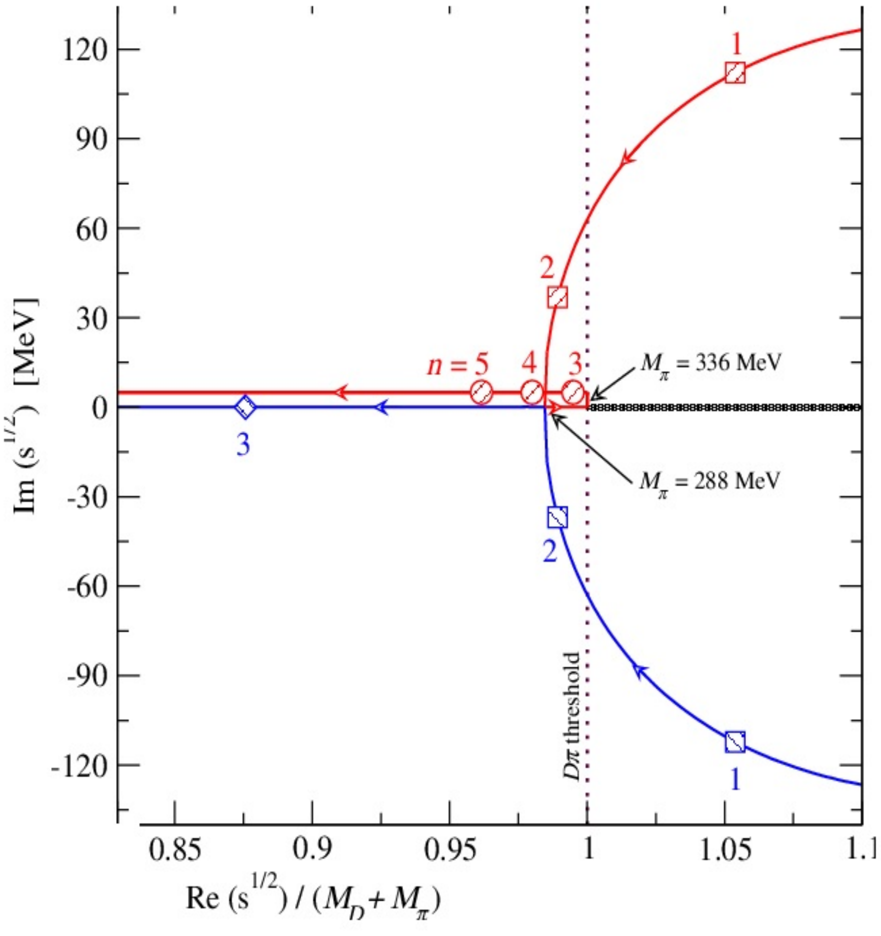
$M_0^2 \to 1/Nc, \quad m_\pi^2, m_K^2 \to Const.$

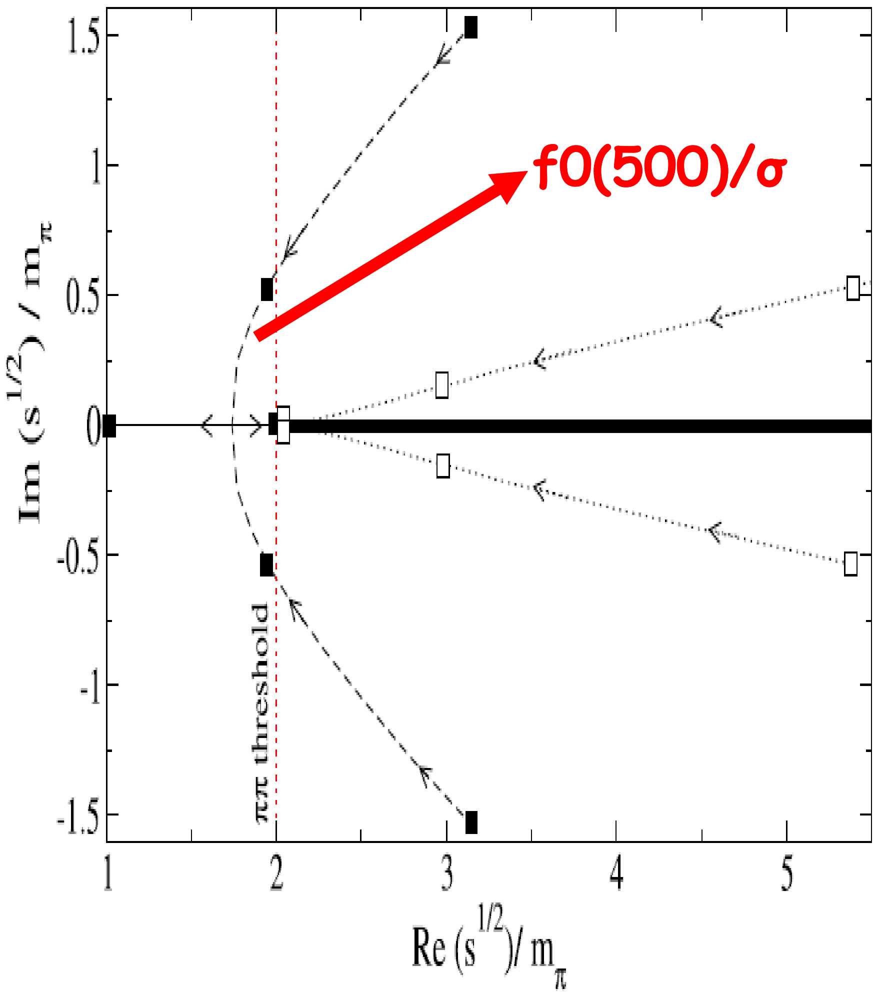


$$\mathbf{Nc} \to \infty: \quad m_\eta \to m_\pi, \quad m_{\eta'} \to \sqrt{2m_K^2 - m_\pi^2}, \quad ,\theta \to \mathrm{Arcsin}\left[-\sqrt{\frac{2}{3}}\right]$$

Pole trajectories with varying m_π

[ZHG,Meissner, Yao, PRD'15]





(S,I)=(0,1/2) channel

Hanhart, et al., PLB14

Nc trajectories for $D^*_{s0}(2317)$ [ZHG, Meissner, Yao, PRD'15]