# The direct coupling of light quarks to heavy diquarks

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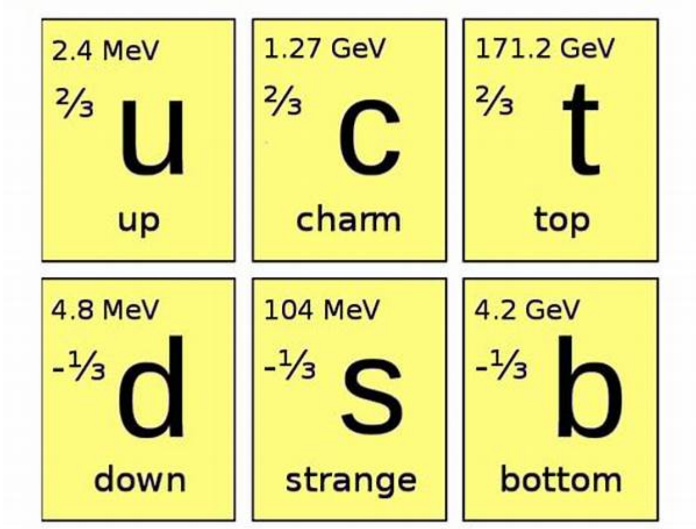
# Heavy quarks

- 6 quarks



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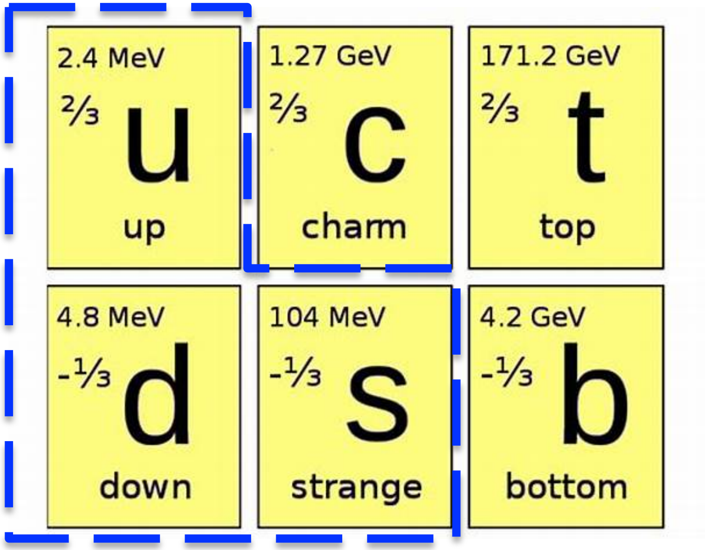


Light quarks $m_q \ll \Lambda_{\rm QCD}$

# Heavy quarks

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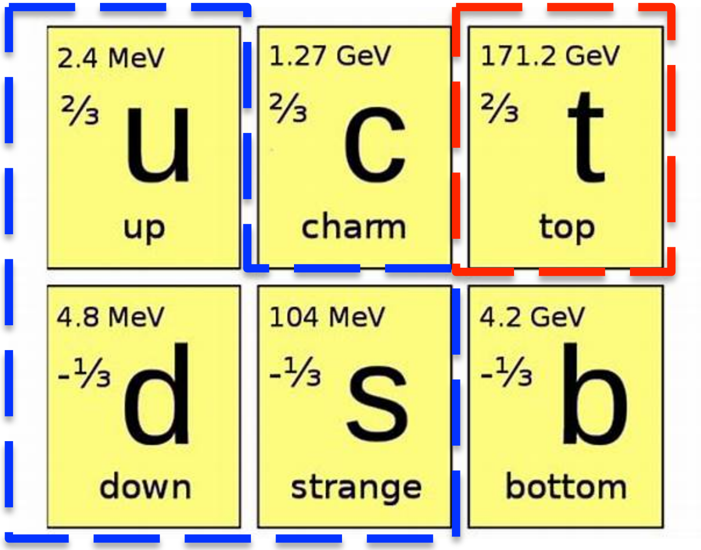


Short lived

$\Gamma_t \gg \Lambda_{\rm QCD}$

Light quarks $m_q \ll \Lambda_{\rm QCD}$

# Heavy quarks

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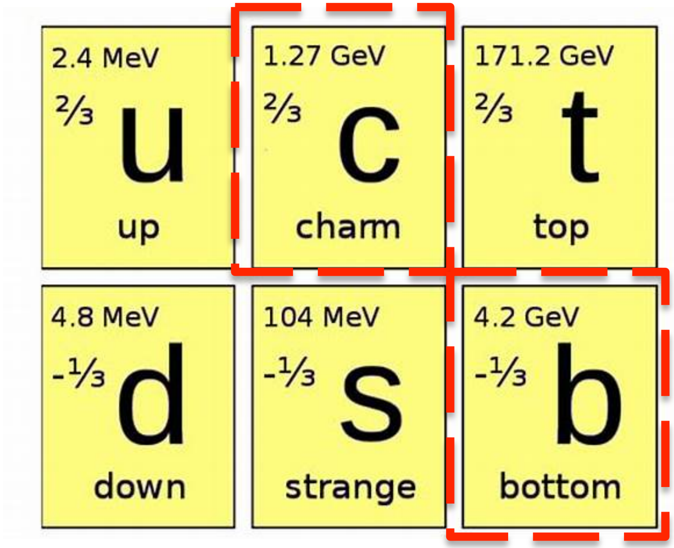


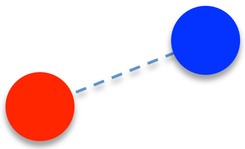

$$m_Q \gg \Lambda_{\mathrm{QCD}}$$

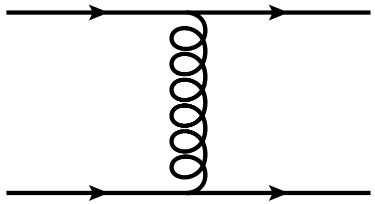
$$\Gamma_Q \ll \Lambda_{\mathrm{QCD}}$$

1. Might be able to be treated perturbatively
2. Can form non-trivial structures

# Heavy quark binding

- Effective Coulomb potential





$$q \sim \alpha_s \times m_Q$$

$$\alpha_S \approx 0.3 \sim 0.5$$

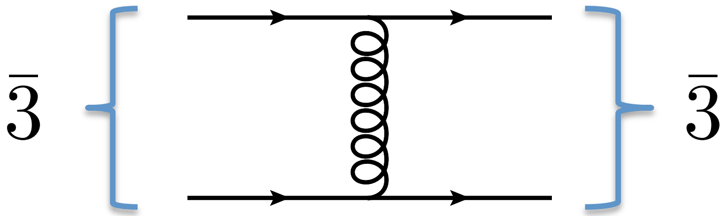
For *b* quark: $q \sim \alpha_S m_b \gg \Lambda_{\mathrm{QCD}}$

For *c* quark: $q \sim \alpha_S m_c \gtrsim \Lambda_{\mathrm{QCD}}$

- It is consistent to treat the strong interaction perturbatively.

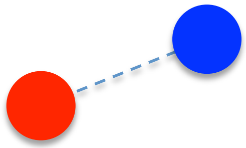
# Heavy quark binding

- Diquark $3 \times 3 = \bar{3} + 6$




$$V_{\rm eff}(r) = \frac{\alpha_S}{r} \times \frac{1}{2}\epsilon_{abc}\epsilon_{ab'c'}T^A_{bb'}T^A_{cc'}$$
$$= -\frac{2}{3}\frac{\alpha_S}{r}$$

Attractive force

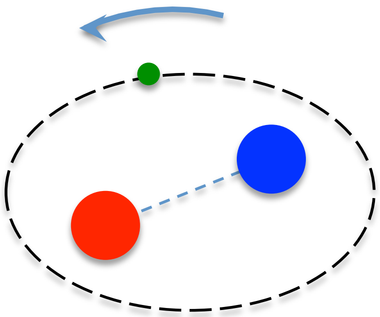


$$d_D \approx \left(\frac{2}{3}\alpha_S \mu_Q\right)^{-1} \gg \Lambda_{\rm QCD}^{-1}$$

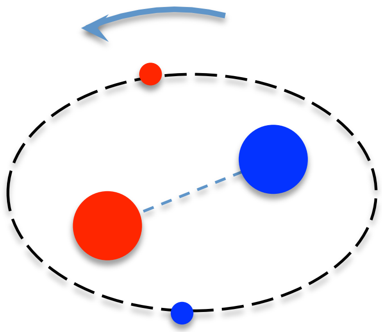
# QQq and QQqbarqbar

- Diquark picture

$$d_D \approx \left(\frac{2}{3}\alpha_S \mu_D\right)^{-1} \ll \Lambda_{\mathrm{QCD}}^{-1} \sim d_{QQq}, d_{QQ\bar{q}\bar{q}}$$



$QQq$



$QQ\bar{q}\bar{q}$

# Why Tetra-quark is long lived?

- $$(QQ\bar{q}\bar{q}) \not\Rightarrow (QQq) + (\bar{q}\bar{q}\bar{q})$$

  ↓

  It cannot happen since it cost about 600 MeV to get an additional quark pair.

- $$(QQ\bar{q}\bar{q}) \not\Rightarrow (Q\bar{q}) + (Q\bar{q})$$

  ↓

  It cannot happen since no potential in the final state.

# Di-quark effective theory

- Diquark field
  - Scalar
  - $\bar{3}$

$$\mathcal{L} = \left|(\partial_\mu - ig\bar{T}^A G^A_\mu)S\right|^2 - m_S^2|S|^2$$

- Non-relativistic

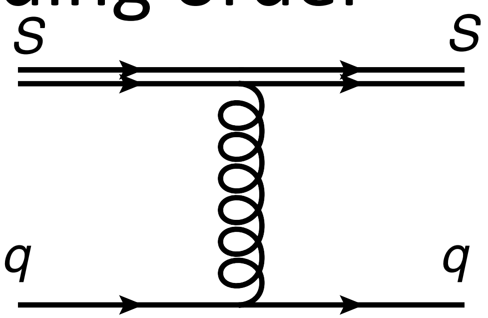
$$S \to (2m_S)^{-1/2} e^{-imv_\mu x^\mu} S$$

$$\mathcal{L} \to S^\dagger i v^\mu D_\mu S - \frac{1}{4} G^{A\mu\nu} G^A_{\mu\nu} + \sum_q \bar{q}(i\gamma^\mu D_\mu - m_q)q$$

No dependence on $m_S$, heavy diquark symmetry.

# Diquark light quark interactions

- Leading order




Independence on $m_S$

- Corrections breaks heavy diquark symmetry

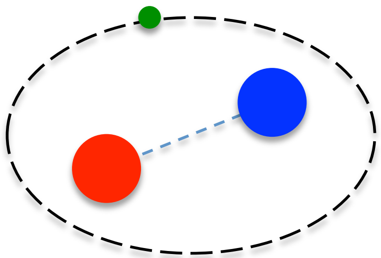
$$[S] = \frac{3}{2}\ , \quad [q] = \frac{3}{2}$$

$$S^\dagger S\ \bar{q}\gamma_\mu v^\mu q \qquad\qquad C \sim \frac{1}{m_S^2}?$$

Similar to heavy quark effective theory.

# Diquark light quark interactions

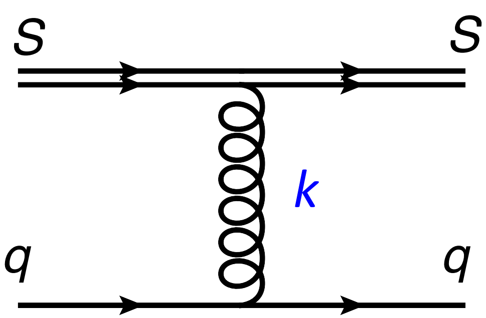
- For diquark there is another subtlety



$QQq$

The light quark moves inside the chromo-electric field emitted by the diquark.

- Calculate the electric field
- Calculate the motion of the light particles

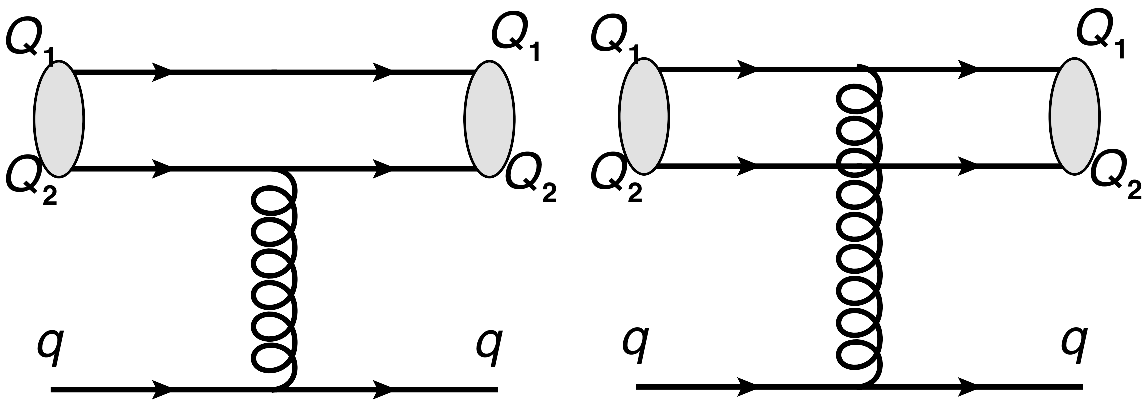



$$k \sim \Lambda_{\text{QCD}} \ll \alpha_S m_Q$$

$$e^{i\mathbf{k}\cdot\mathbf{x}} \approx 1 + i\mathbf{k}\cdot\mathbf{x} - \frac{1}{2}\mathbf{kk}\cdot\mathbf{xx} + \cdots$$

We can do a multi-pole expansion just like in classical electrodynamics.

# Matching




Q1 and Q2 are heavy quarks within the bound state.

$$\mathcal{A} \simeq \frac{g^2}{2k^2}\left(2m_{Q_1} + 2m_{Q_2}\right)\bar{u}(k_f)T^A_{\beta'\beta}\gamma^0 u(k_i)\left(-T^A\right)_{\alpha\alpha'} \times$$

$$\int \frac{d^3p}{(2\pi)^3}\left(\tilde{\phi}^*\left(\mathbf{p} - \frac{m_{Q_2}}{m_{Q_1}+m_{Q_2}}\mathbf{k}\right) + \tilde{\phi}^*\left(\mathbf{p} + \frac{m_{Q_1}}{m_{Q_1}+m_{Q_2}}\mathbf{k}\right)\right)\tilde{\phi}(\mathbf{p})$$

↓

Heavy quark wave function inside diquark in Fourier space

# Matching

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$$\phi(\mathbf{p}+\mathbf{k}) = \phi(\mathbf{p}) + \nabla_p\phi\cdot\mathbf{k} + \frac{1}{2}\nabla_p\nabla_p\phi\cdot\mathbf{kk}+\cdots$$

Zeroth order: Effective Coulomb interaction

First order: vanishes since the ground state is S-wave

Second order: contact interaction $\bar{q}T^A\gamma^0 q S^\dagger \bar{T}^A S$

# Matching

$$\int \frac{d^3p}{(2\pi)^3}\tilde{\phi}^*(\mathbf{p})\nabla_p^2\tilde{\phi}(\mathbf{p}) \sim \int d^3x\ r^2\phi^*(\mathbf{r})\phi(\mathbf{r}) = \langle r^2\rangle$$

$$\langle r^2\rangle \sim \frac{1}{\alpha_S^2 m_S^2}$$

# Matching

$$\int \frac{d^3p}{(2\pi)^3}\tilde{\phi}^*(\mathbf{p})\nabla_p^2\tilde{\phi}(\mathbf{p}) \sim \int d^3x\ r^2\phi^*(\mathbf{r})\phi(\mathbf{r}) = \langle r^2\rangle$$

$$\langle r^2\rangle \sim \frac{1}{\alpha_S^2 m_S^2}$$

$$\Delta\mathcal{L} = C\left(S_v^\dagger \bar{T}^A S_v\right)\sum_q \left(\bar{q}T^A\gamma^\mu v_\mu q\right)$$

$$C = \frac{\pi\alpha_s\langle r^2\rangle}{3}\left(\frac{m_{Q_1}^2 + m_{Q_2}^2}{(m_{Q_1}+m_{Q_2})^2}\right) = \frac{9\pi}{4\alpha_s}\left(\frac{m_{Q_1}^2 + m_{Q_2}^2}{m_{Q_1}^2 m_{Q_2}^2}\right)$$

↓

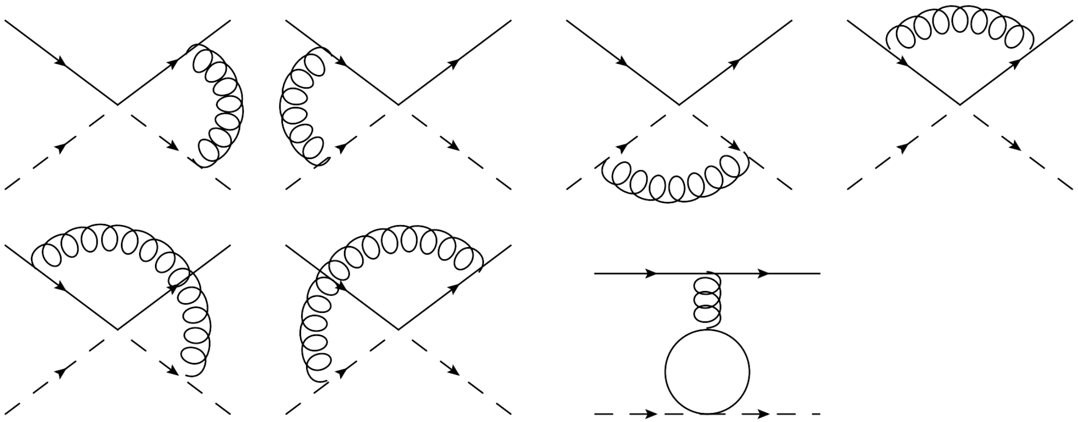
Parametrically larger than from heavy quark expansion

# Running

- Two independent operators

$$O_1 = \left(S^\dagger_{v\alpha} S_{v\alpha}\right) \sum_q \left(\bar{q}_\beta \gamma^\mu v_\mu q_\beta\right), \quad O_2 = \left(S^\dagger_{v\alpha} S_{v\beta}\right) \sum_q \left(\bar{q}_\alpha \gamma^\mu v_\mu q_\beta\right)$$

$$C_1 = C/6, \quad C_2 = -C/2.$$



+ wave function renormalization diagrams

# Running

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$$C_1 = C/6, \quad C_2 = -C/2.$$

$$\gamma(g) = \frac{g^2}{16\pi^2} \begin{pmatrix} 0 & 0 \\ 3 - \frac{4}{9} n_q & -9 + \frac{4}{3} n_q \end{pmatrix}$$

# Applications to spectrum calculation

- Effective Lagrangian

$$\Delta L = \left(\frac{3\pi}{8\alpha_s(m_Q v)}\right)\left(\frac{m_{Q_2}^2 + m_{Q_1}^2}{m_{Q_2}^2 m_{Q_1}^2}\right)\left[\frac{\alpha_s(m_Q v)}{\alpha_s(\mu)}\right]^{\left(\frac{-\frac{9}{2}+\frac{2}{3}n_q}{11-\frac{2}{3}n_q}\right)} O_-(\mu)$$

$$O_- = \left(S^\dagger_{v\alpha} S_{v\alpha}\right) \sum_q \left(\bar{q}_\beta \gamma^\mu v_\mu q_\beta\right) - 3\left(S^\dagger_{v\alpha} S_{v\beta}\right) \sum_q \left(\bar{q}_\alpha \gamma^\mu v_\mu q_\beta\right)$$

$$\Delta E = -\int d^3x \langle Sq|\Delta L(x)|Sq\rangle$$

# Applications to spectrum calculation

- Strong interaction is hard, let's use a model
  - Assuming factorization

$$\langle Sq|S^\dagger S\ \bar{q}\gamma_\mu v^\mu q|Sq\rangle \approx \langle S|S^\dagger S|S\rangle\langle q|\bar{q}\gamma_\mu v^\mu q|q\rangle$$

# Applications to spectrum calculation

- Strong interaction is hard, let's use a model
  - Assuming factorization

$$\left\langle \int d^3x O_-(\mathbf{x}) \right\rangle_{Sq} = -8 \int d^3x n_S(\mathbf{x}) n_q(\mathbf{x}) = -8|\phi_q(\mathbf{0})|^2$$

Can be got from heavy meson decay

Using the same model:

$$\phi_q(\mathbf{0}) = \frac{f_B\sqrt{m_B}}{2\sqrt{3}} \left[\frac{\alpha_S(m_B)}{\alpha_S(\mu)}\right]^{\frac{6}{33-2n_q}}$$

$$\Delta m_{\Xi_{bbq}} \simeq \frac{\pi}{2\alpha_s(m_b v)} \frac{f_B^2}{m_B} \sim 30 \text{ MeV}$$

# Applications to spectrum calculation

- A cruder estimation for $S\bar{q}\bar{q}$
  - We assume the diquark and light quark factorize
  - We neglect the interaction between the light quarks

$$\Delta m_{T_{bb\bar{q}\bar{q}}} \approx 30 \text{ MeV}$$

# Excited states

- If $Q_1 \neq Q_2$, there is a 3-bar vector diquark.
- Transition EDM from scalar diquark to vector diquark

$$\Delta L = \frac{1}{2\sqrt{3}} \left( \frac{m_{Q_2} - m_{Q_1}}{m_{Q_2} + m_{Q_1}} \right) S_j^\dagger \bar{T}^A S g E^{Aj}_{\text{color}} \langle r \rangle_{\text{trans}} + \text{h.c.}$$

$$\Delta m_{\Xi_{Q_1 Q_2 q}} \Big|_{\text{Excited}} \sim \Lambda^4_{\text{QCD}} / (\alpha_S (m_Q v_{\text{rel}})^4 m_Q^3)$$

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$$\Delta m_{\Xi_{Q_1 Q_2 q}}\Big|_{\rm Excited} \sim \Lambda^4_{\rm QCD} / (\alpha_S (m_Q v_{\rm rel})^4 m_Q^3)$$

$$\Delta m_{\Xi_{Q_1 Q_2 q}}\Big|_{\rm Ground} \sim \Lambda^3_{\rm QCD} / (\alpha_S (m_Q v_{\rm rel}) m_Q^2)$$

$$\Delta m_{\Xi_{Q_1 Q_2 q}}\Big|_{\rm Ground} > \Delta m_{\Xi_{Q_1 Q_2 q}}\Big|_{\rm Excited} \quad \text{only if } \alpha_{\rm S}^2 {\rm m_Q} \gg \Lambda_{\rm QCD}$$

# Summary

- In the region $m_Q > m_Q v_{\rm rel} > m_Q v_{\rm rel}^2 \gg \Lambda_{QCD}$ we calculated the leading direct interaction between heavy diquarks and light quarks.

- To get a better and realistic understanding we need to consider the excited states of 3-bar bound states.

- We may also need to include the contribution from the 6 scattering states (repulsive).