第五届XYZ粒子学术研讨会
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Charmed hadrons from chiral EFT

郭志辉

河北师范大学

Outline:

1. Introduction

2. Scalar charmed meson resonances:

   $D^*_{s0}(2317)$、$D^*_0(2400)$

3. Possible exotic doubly charmed baryons

4. Summary

Introduction

First glance from PDG

CHARMED MESONS
($C = \pm1$)
$D^+ = c\bar{d}$, $D^0 = c\bar{u}$, $\bar{D}^0 = \bar{c}u$, $D^- = \bar{c}d$, similarly for D^*'s

INSPIRE search

$D_0^*(2400)^\pm$ $\quad I(J^P) = 1/2(0^+)$

J, P need confirmation.

$D_0^*(2400)^\pm$ MASS	2351 ± 7 MeV
$D_0^*(2400)^\pm$ WIDTH	230 ± 17 MeV (S = 1.1)

CHARMED, STRANGE MESONS
($C = S = \pm1$)
$D_s^+ = c\bar{s}$, $D_s^- = \bar{c}s$, similarly for D_s^*'s

INSPIRE search

$D_{s0}^*(2317)^\pm$ $\quad I(J^P) = 0(0^+)$

AUBERT 2006P and CHOI 2015A do not observe neutral and doubly charged partners of the $D_{s0}^*(2317)^+$.

$D_{s0}^*(2317)^\pm$ MASS	2317.7 ± 0.6 MeV (S = 1.1)
$m_{D_{s0}^*(2317)^{\pm}} - m_{D_{s\pm}}$	349.4 ± 0.6 MeV (S = 1.1)
$D_{s0}^*(2317)^\pm$ WIDTH	< 3.8 MeV CL=95.0%

$D_{s0}^{*}(2317)$： a hot topic at *B*-Factory

1. **The BaBar detector**
(2145) BaBar Collaboration (Bernard Aubert (Annecy, LAPP) *et al.*). Apr 2001. 119 pp.
Published in **Nucl.Instrum.Meth. A479 (2002) 1-116**
SLAC-PUB-8569, BABAR-PUB-01-08
DOI: 10.1016/S0168-9002(01)02012-5
e-Print: **hep-ex/0105044** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; BaBar Publications Database; BaBar Password Protected Publications Data
Detailed record - Cited by 2145 records 1000+

2. **Observation of CP violation in the B^0 meson system**
(867) BaBar Collaboration (Bernard Aubert (Annecy, LAPTH) *et al.*). Jul 2001. 8 pp.
Published in **Phys.Rev.Lett. 87 (2001) 091801**
SLAC-PUB-8904, BABAR-PUB-01-18
DOI: 10.1103/PhysRevLett.87.091801
e-Print: **hep-ex/0107013** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service; OSTI.gov Server; BaBar Publications Database; BaBar Password Protected
Document Server
Detailed record - Cited by 867 records 500+

3. **Observation of a narrow meson decaying to $D_s^+\pi^0$ at a mass of 2.32-GeV/c^2**
(823) BaBar Collaboration (B. Aubert (Annecy, LAPP) *et al.*). Apr 2003.
Published in **Phys.Rev.Lett. 90 (2003) 242001**
SLAC-PUB-9711, BABAR-PUB-03-011
DOI: 10.1103/PhysRevLett.90.242001
e-Print: **hep-ex/0304021** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac |
ADS Abstract Service; OSTI.gov Server; BaBar Publications
Document Server
Detailed record - Cited by 823 records 500+

Top 3 @BaBar

5. **Measurements of the meson - photon transition form-factors of light pseudosca**
(610) CLEO Collaboration (J. Gronberg (UC, Santa Barbara) *et al.*). Jul 1997. 30 pp.
Published in **Phys.Rev. D57 (1998) 33-54**
SLAC-PUB-9838, CLNS-97-1477, CLEO-97-7
DOI: 10.1103/PhysRevD.57.33
e-Print: **hep-ex/9707031** | PDF
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
KEK scanned document; ADS Abstract Service; Cornell U., LNS Server; SLAC Document Server; SLA
Data: INSPIRE | HepData
Detailed record - Cited by 610 records 500+

6. **Observation of a narrow resonance of mass 2.46-GeV/c**2 decaying to D*+(s) pi**
(575) CLEO Collaboration (D. Besson (Kansas U.) *et al.*). May 2003. 16 pp.
Published in **Phys.Rev. D68 (2003) 032002**, Erratum: **Phys.Rev. D75 (2007) 119908**
CLNS-03-1826, CLEO-03-09, CLNS03-1826
DOI: 10.1103/PhysRevD.68.032002, 10.1103/PhysRevD.75.119908
e-Print: **hep-ex/0305100** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 575 records 500+

Top 6 @CLEO

7. **Observation of B Meson Semileptonic Decays to Noncharmed Final States**
(419) CLEO Collaboration (R. Fulton *et al.*). Nov 1989. 14 pp.
Published in **Phys.Rev.Lett. 64 (1990) 16-20**
CLNS-89/951, CLEO-89-14
DOI: 10.1103/PhysRevLett.64.16
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
OSTI.gov Server
Detailed record - Cited by 419 records 250+

BELLE-PREPRINT-2006-11
DOI: 10.1103/PhysRevLett.97.251802
e-Print: **hep-ex/0604018** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 347 records 250+

16. **Study of B- ---> D**0 pi- (D**0 ---> D(*)+ pi-) decays**
(338) Belle Collaboration (Kazuo Abe (KEK, Tsukuba) *et al.*). Jul 2003. 20 pp.
Published in **Phys.Rev. D69 (2004) 112002**
DOI: 10.1103/PhysRevD.69.112002
e-Print: **hep-ex/0307021** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 338 records 250+

Top 20 @Belle

17. **Observation of the D(sJ)(2317) and D(sJ)(2457) in B decays**
(337) Belle Collaboration (P. Krokovny (Novosibirsk, IYF) *et al.*). Aug 2003. 6 pp.
Published in **Phys.Rev.Lett. 91 (2003) 262002**
DOI: 10.1103/PhysRevLett.91.262002
e-Print: **hep-ex/0308019** | **PDF**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 337 records 250+

18. **Observation of double c anti-c production in e+ e- annihilation at**
(337) Belle Collaboration (Kazuo Abe (KEK, Tsukuba) *et al.*). May 2002. 7 pp.
Published in **Phys.Rev.Lett. 89 (2002) 142001**
BELLE-PREPRINT-2002-13, KEK-PREPRINT-2002-27
DOI: 10.1103/PhysRevLett.89.142001
e-Print: **hep-ex/0205104** | **PDF**

Continuing efforts from LHCb, Belle-II, PANDA,…

and also Lattice QCD

Theoretical interpretations

recent reviews: H.X.Chen, et al., Rept.Prog.Phys.2017; F.K.Guo, et al., Rev.Mod.Phys.2018

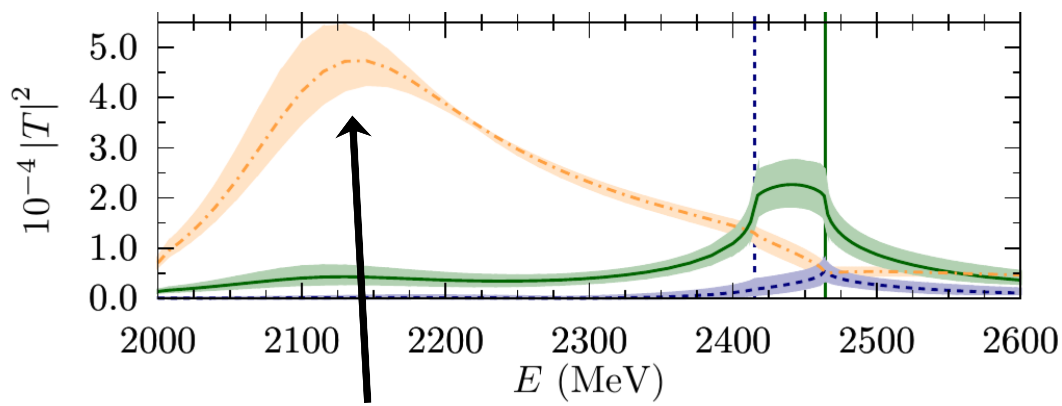
- $c\bar{s}$ state: Y.B.Dai, et al., PRD2003 ; S.Narison, PLB2005; E. van Beveren, et al., PRL 2003; Z.G.Wang, PRD2006 ……
- Hadronic molecular state: T.Barnes, et al., PRD'03; F.K.Guo, et al., PLB'06; Altenbuchinger, et al., PRD'14; Du, et al., 1712.07957; Albaladejo, et al. EPJC'18 …
- Four-quark state: H.Y.Cheng, et al., PRD 2003; K. Terasaki, PRD2003; L.Maiani et.al., PRD2005; M.Bracco, et al., PLB2005; ……
- Mixing of molecular and four-quark states: T. Browder, et al., PLB2004;……
- ……

$D^*_0(2400)$: two-pole structure ?

[ZHG,Meissner,Yao, PRD'15]

(S,I)	RS	$\sqrt{s_{pole}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV]
$(0,\frac{1}{2})$	II	$2114^{+3}_{-3} - i\,111^{+8}_{-7}$	$9.66^{+0.15}_{-0.13}(D\pi)$
	III	$2473^{+29}_{-22} - i\,140^{+8}_{-7}$	$5.36^{+0.40}_{-0.28}(D\pi)$

[Albaladejo,Fernandez-Soler,F.K.Guo,Nieves, PLB'17]



Absent in PDG!

The above results are predicted from other channels with different quantum numbers.

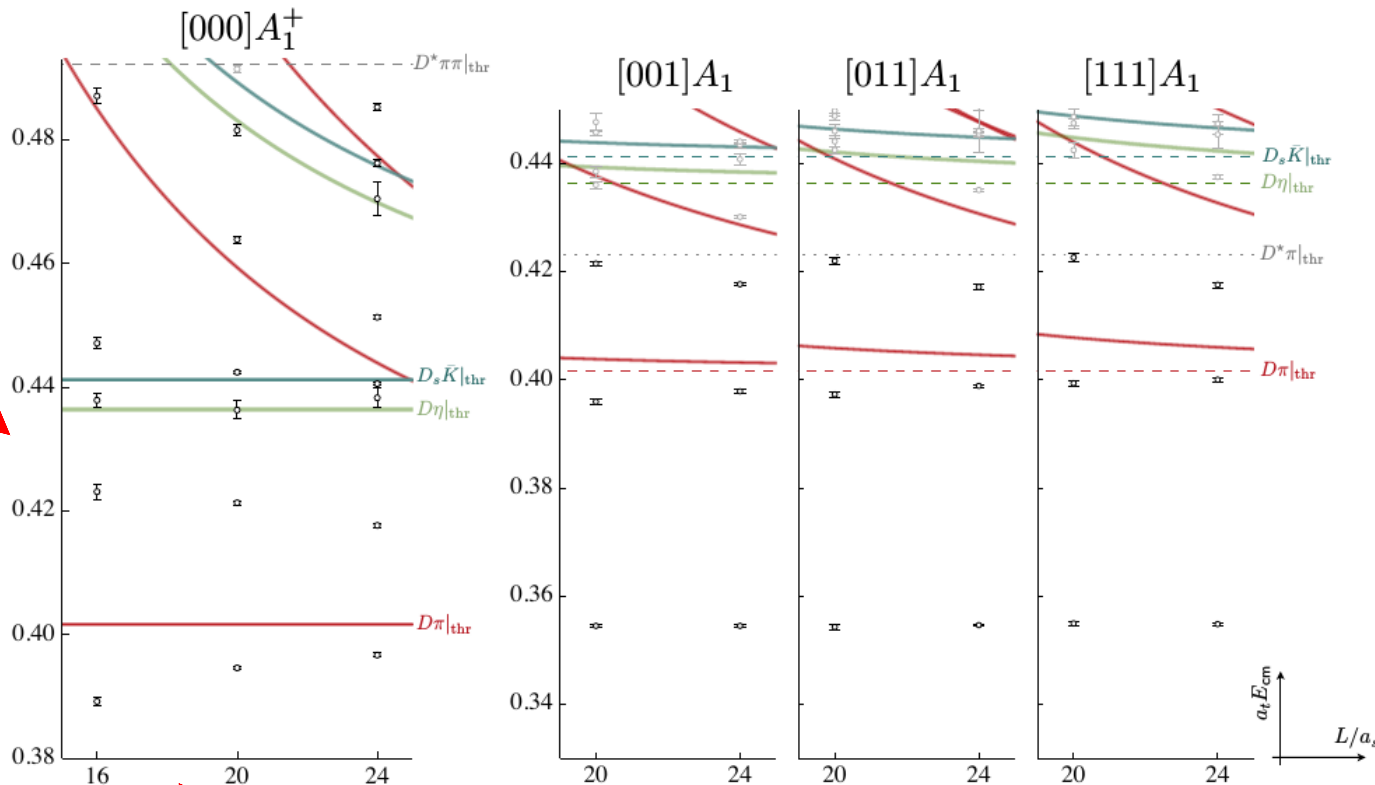
Direct Exp measurements, like phase shifts & inelasticities, are not available at present!

Any concrete confirmation about the resonance structures in D-pi, D-eta, Ds-Kbar coupled channel scattering?

Yes, Lattice QCD can help !

[Moir, et al., JHEP'16]

Eigenenergies in finite box



Length of finite box

Our improvements:

- Scalar Charmed states from $D_{(s)} + \pi(K,\eta,\eta')$ scattering

& their Nc and m_π dependences

- We also try to use the lattice energy levels to constrain the physical scattering amplitudes

Scalar charmed meson resonances:

Ds*0(2317), D*0(2400)

& their Nc and mpi dependences

Theoretical framework

LO ChPT with heavy-light mesons

$$\mathcal{L}_{D\phi}^{(1)} = \mathcal{D}_\mu D \mathcal{D}^\mu D^\dagger - \overline{M}_D^2 D D^\dagger$$

NLO ChPT with heavy-light mesons [F.K.Guo,et al.,PLB'08]

$$\begin{aligned}\mathcal{L}_{D\phi}^{(2)} &= D\left(-h_0\langle\chi_+\rangle - h_1\chi_+ + h_2\langle u_\mu u^\mu\rangle - h_3 u_\mu u^\mu\right) D^\dagger \\ &\quad + \mathcal{D}_\mu D\left(h_4\langle u_\mu u^\nu\rangle - h_5\{u^\mu, u^\nu\}\right)\mathcal{D}_\nu D^\dagger\ ,\end{aligned}$$

with $u_\mu = i\left(u^\dagger\partial_\mu u - u\,\partial_\mu u^\dagger\right)$ $\quad u = \exp\left(\dfrac{i\phi}{\sqrt{2}F_0}\right)$ $\quad \chi_\pm = u^\dagger\chi u^\dagger \pm u\chi u$

It is essential to generalize to U(3) case to study the Nc behaviors:

$$\phi = \begin{pmatrix} \frac{\sqrt{3}\pi^0+\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\sqrt{3}\pi^0+\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta_8}{\sqrt{6}} \end{pmatrix} \longrightarrow \begin{pmatrix} \frac{\sqrt{3}\pi^0+\eta_8+\sqrt{2}\eta_0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\sqrt{3}\pi^0+\eta_8+\sqrt{2}\eta_0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta_8+\sqrt{2}\eta_0}{\sqrt{6}} \end{pmatrix}$$

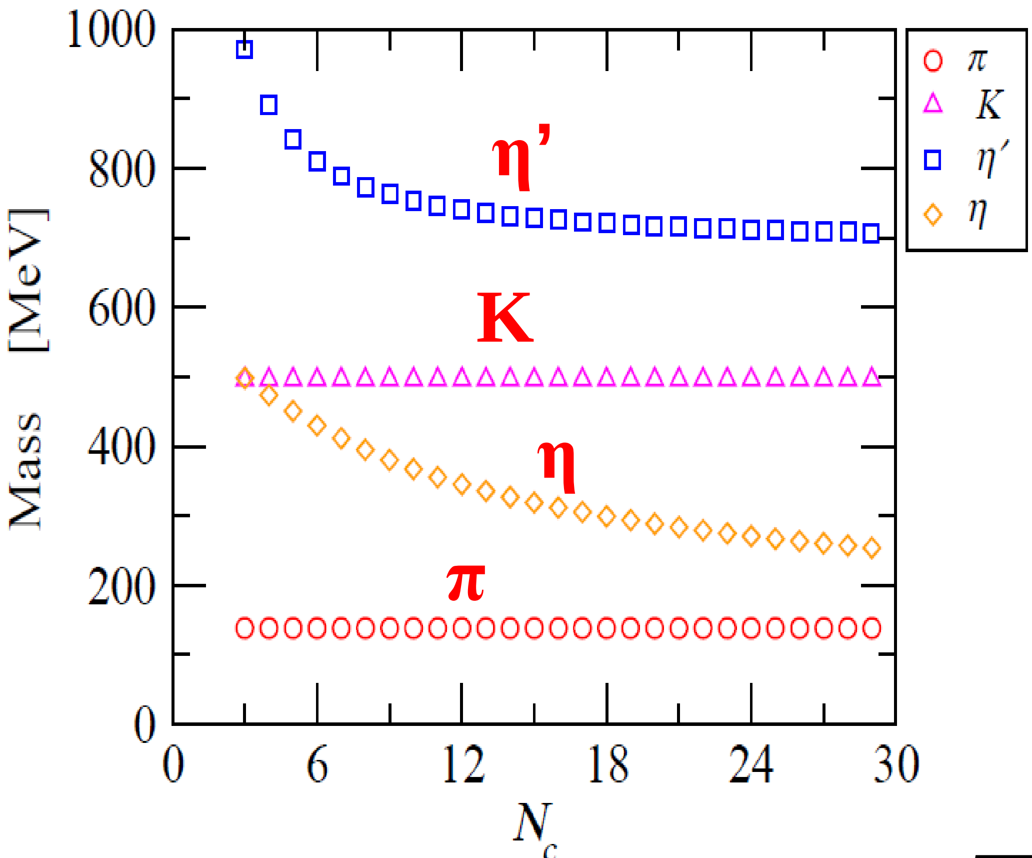
Possible new operators with singlet η_0 can appear, but they are much less relevant in the present analysis.

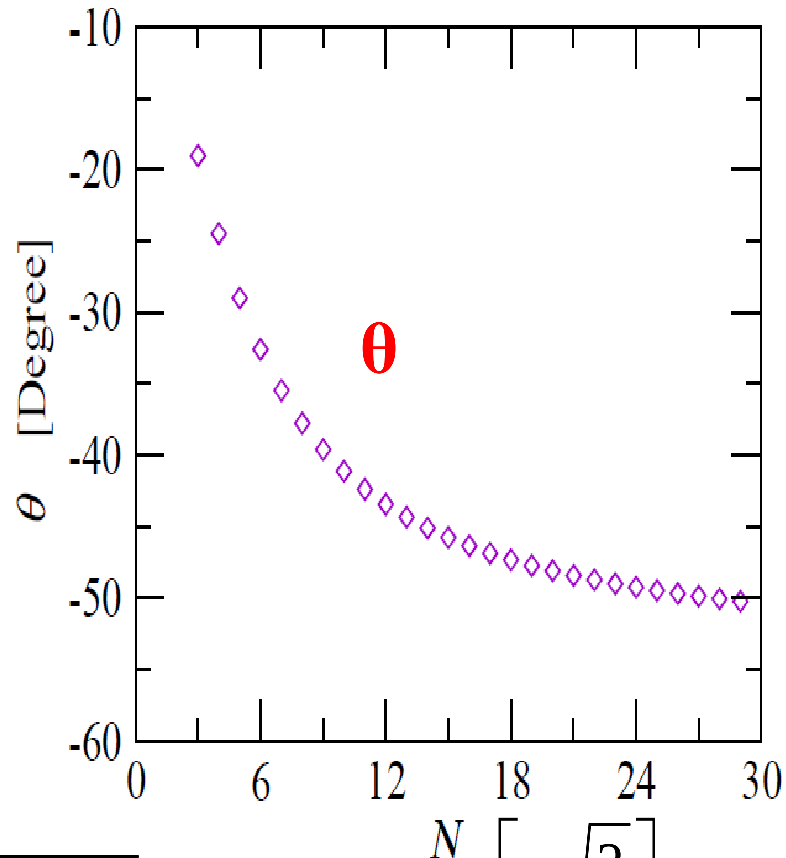
Chiral Lagrangian for pseudo-Goldstone with $U_A(1)$ anomaly

$$\mathcal{L}_\chi = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle + \boxed{\frac{F^2}{3} M_0^2 \ln^2 \det u}$$

Leads to a massive η_0

$$M_0^2 \to 1/Nc, \quad m_\pi^2, m_K^2 \to Const.$$





$$\mathbf{Nc \to \infty:} \quad m_\eta \to m_\pi, \quad m_{\eta'} \to \sqrt{2m_K^2 - m_\pi^2}, \quad ,\theta \to \mathrm{Arcsin}\left[-\sqrt{\frac{2}{3}}\right]$$

$D_{(s)}$ and light pseudoscalar meson scattering amplitudes

$$V^{(S,I)}_{D_1\phi_1\to D_2\phi_2}(s,t,u)=\frac{1}{F^2}\left[\frac{C_{\rm LO}}{4}(s-u)-4C_0h_0+2C_1h_1\ -2C_{24}H_{24}(s,t,u)+2C_{35}H_{35}(s,t,u)\right]$$

(S,I)	Channels	$C_{\rm LO}$	C_0	C_1	C_{24}	C_{35}
$(-1,0)$	$D\bar{K}\to D\bar{K}$	-1	M_K^2	M_K^2	1	-1
$(-1,1)$	$D\bar{K}\to D\bar{K}$	1	M_K^2	$-M_K^2$	1	1
$(2,\frac{1}{2})$	$D_sK\to D_sK$	1	M_K^2	$-M_K^2$	1	1
$(0,\frac{3}{2})$	$D\pi\to D\pi$	1	M_π^2	$-M_\pi^2$	1	1
$(1,1)$	$D_s\pi\to D_s\pi$	0	M_π^2	0	1	0
	$DK\to DK$	0	M_K^2	0	1	0
	$DK\to D_s\pi$	1	0	$-(M_K^2+M_\pi^2)/2$	0	1
$(1,0)$	$DK\to DK$	-2	M_K^2	$-2M_K^2$	1	2
	$DK\to D_s\eta$	$-\sqrt{3}c_\theta$	0	$C_1^{1,0\ DK\to D_s\eta}$	0	$C_{35}^{1,0\ DK\to D_s\eta}$
	$D_s\eta\to D_s\eta$	0	$C_0^{1,0\ D_s\eta\to D_s\eta}$	$C_1^{1,0\ D_s\eta\to D_s\eta}$	1	$C_{35}^{1,0\ D_s\eta\to D_s\eta}$
	$DK\to D_s\eta'$	$-\sqrt{3}s_\theta$	0	$C_1^{1,0\ DK\eta\to D_s\eta'}$	0	$C_{35}^{1,0\ DK\eta\to D_s\eta'}$
	$D_s\eta\to D_s\eta'$	0	$C_0^{1,0\ D_s\eta\to D_s\eta'}$	$C_1^{1,0\ D_s\eta\to D_s\eta'}$	0	$C_{35}^{1,0\ D_s\eta\to D_s\eta'}$
	$D_s\eta'\to D_s\eta'$	0	$C_0^{1,0\ D_s\eta'\to D_s\eta'}$	$C_1^{1,0\ D_s\eta'\to D_s\eta'}$	1	$C_{35}^{1,0\ D_s\eta'\to D_s\eta'}$
$(0,\frac{1}{2})$	$D\pi\to D\pi$	-2	M_π^2	$-M_\pi^2$	1	1
	$D\eta\to D\eta$	0	$C_0^{0,\frac{1}{2}\ D\eta\to D\eta}$	$C_1^{0,\frac{1}{2}\ D\eta\to D\eta}$	1	$C_{35}^{0,\frac{1}{2}\ D\eta\to D\eta}$
	$D_s\bar{K}\to D_s\bar{K}$	-1	M_K^2	$-M_K^2$	1	1
	$D\eta\to D\pi$	0	0	$M_\pi^2(\sqrt{2}s_\theta-c_\theta)$	0	$c_\theta-\sqrt{2}s_\theta$
	$D_s\bar{K}\to D\pi$	$-\frac{\sqrt{6}}{2}$	0	$-\sqrt{6}(M_K^2+M_\pi^2)/4$	0	$\frac{\sqrt{6}}{2}$
	$D_s\bar{K}\to D\eta$	$-\frac{\sqrt{6}}{2}c_\theta$	0	$C_1^{0,\frac{1}{2}\ D_sK\to D\eta}$	0	$C_{35}^{0,\frac{1}{2}\ D_sK\to D\eta}$
	$D\eta'\to D\pi$	0	0	$-M_\pi^2(\sqrt{2}c_\theta+s_\theta)$	0	$s_\theta+\sqrt{2}c_\theta$
	$D\eta\to D\eta'$	0	$C_0^{0,\frac{1}{2}\ D\eta\to D\eta'}$	$C_1^{0,\frac{1}{2}\ D\eta\to D\eta'}$	0	$C_{35}^{0,\frac{1}{2}\ D\eta\to D\eta'}$
	$D_s\bar{K}\to D\eta'$	$-\frac{\sqrt{6}}{2}s_\theta$	0	$C_1^{0,\frac{1}{2}\ D_s\bar{K}\to D\eta'}$	0	$C_{35}^{0,\frac{1}{2}\ D_s\bar{K}\to D\eta'}$
	$D\eta'\to D\eta'$	0	$C_0^{0,\frac{1}{2}\ D\eta'\to D\eta'}$	$C_1^{0,\frac{1}{2}\ D\eta'\to D\eta'}$	1	$C_{35}^{0,\frac{1}{2}\ D\eta'\to D\eta'}$

Unitarization: Algebraic approximation of N/D (a variant version of K-matrix)

[Oller, Oset, PRD '99]

$$T_J = \frac{N(s)}{1 - N(s)\,G(s)}$$

- **The s-channel unitariy is exact. The crossed-channel dyanmics is included in a perturbative manner.**
- **Unitarity condition:** $\mathrm{Im}G(s) = -\rho(s)$

$$G(s) = \boxed{a^{SL}(s_0)} - \frac{s - s_0}{\pi} \int_{4m^2}^{\infty} \frac{\rho(s')}{(s'-s)(s'-s_0)} ds'$$

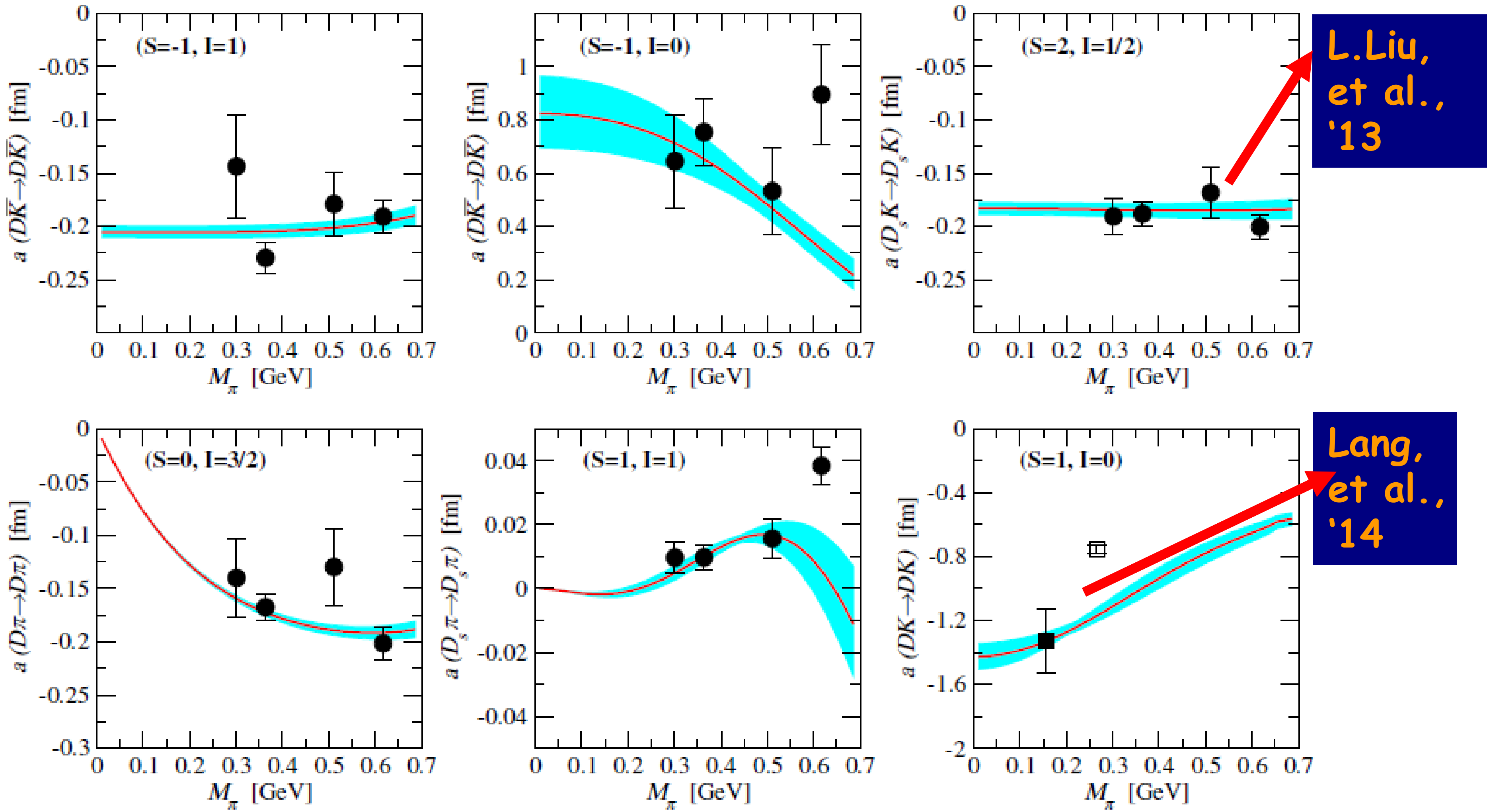
- ***N*(*s*): given by the partial wave chiral amplitudes**

$$\mathcal{V}^{(S,I)}_{J,D_1\phi_1\to D_2\phi_2}(s) = \frac{1}{2}\int_{-1}^{+1} \mathrm{d}\cos\varphi P_J(\cos\varphi) V^{(S,I)}_{D_1\phi_1\to D_2\phi_2}(s, t(s,\cos\varphi))$$

$$D\left(-h_0\langle\chi_+\rangle - h_1\chi_+\right)D^\dagger$$

h_0 h_1 determined by masses of D and Ds

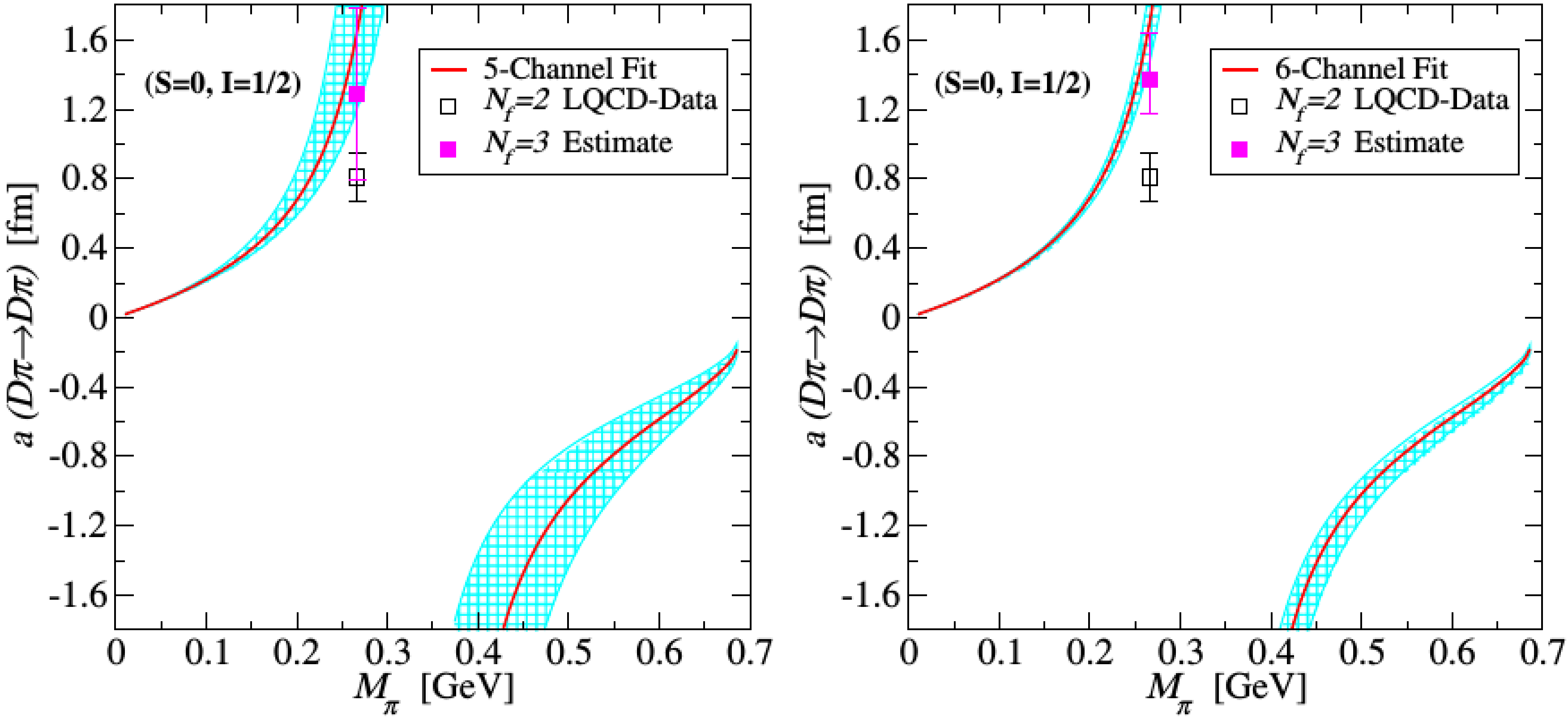
Six-channel fit (6c): 4 LECs and 1 common a_{SL}



Radical Assmuption

All the channels, regardless of the quantum numbers and the dynamical states, share a common subtraction constant in the unitarized amplitudes !

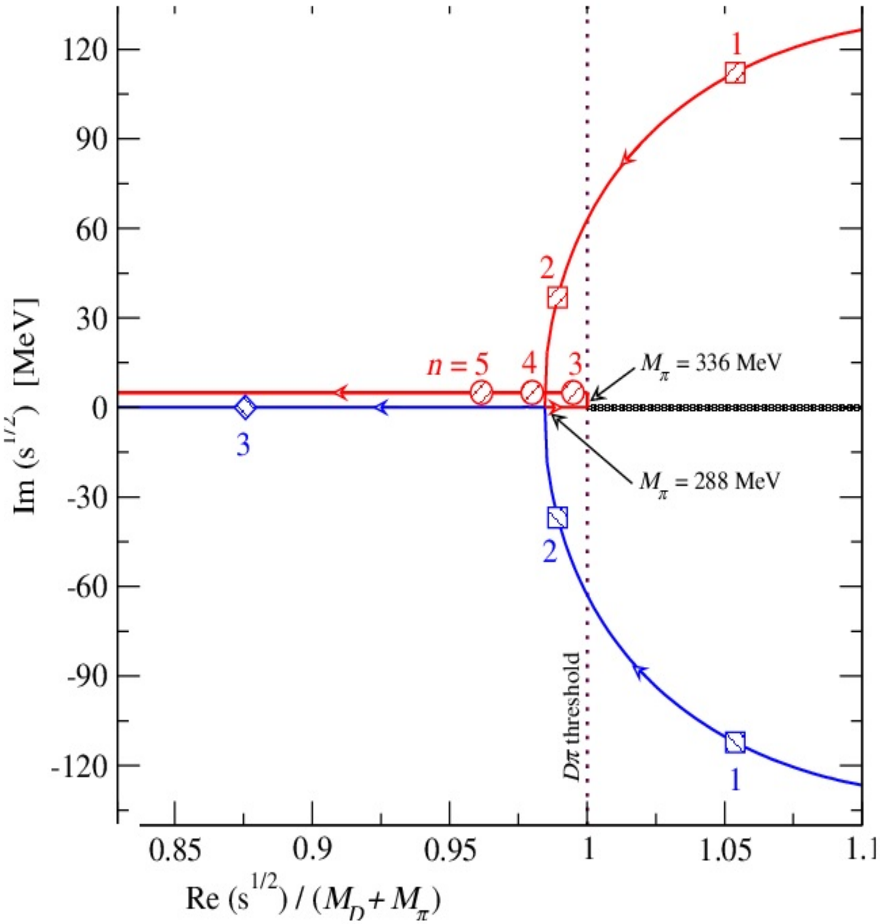
Prediciton of scattering lengths for (S,I)=(0,1/2) channel

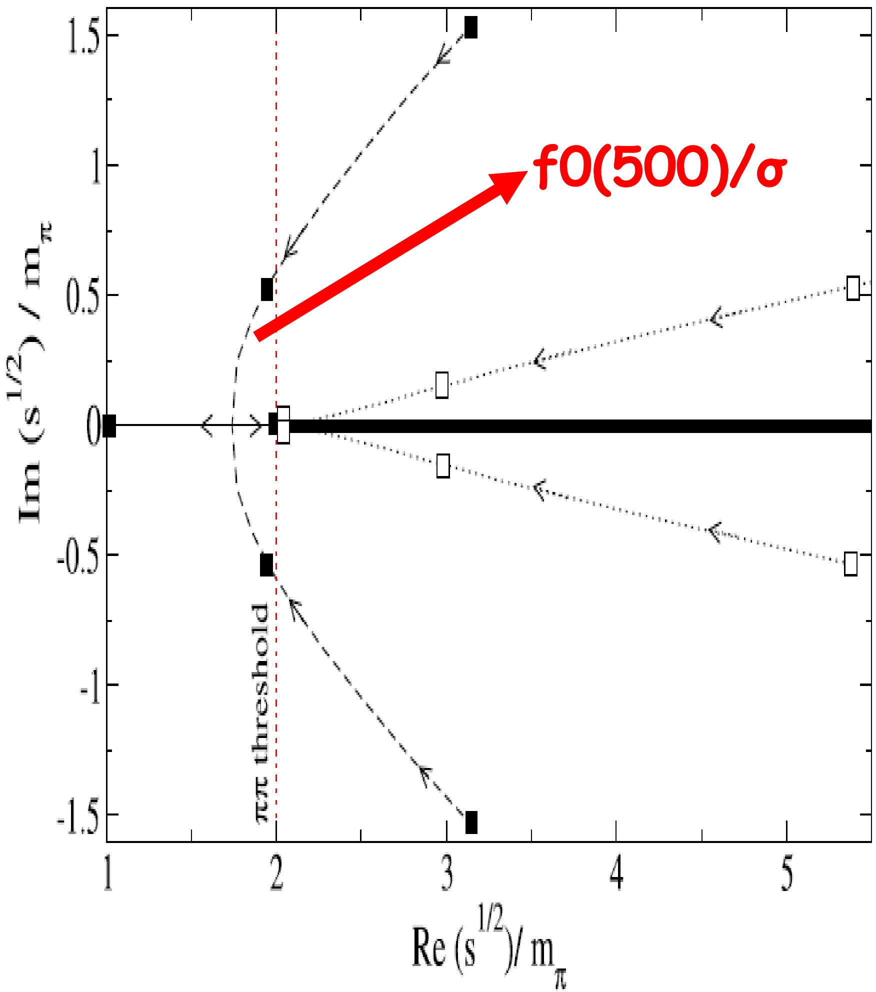


It clearly indicates that "exotics" things happen around m_π=0.3~0.4GeV

Pole trajectories with varying m_π

[ZHG,Meissner, Yao, PRD'15]





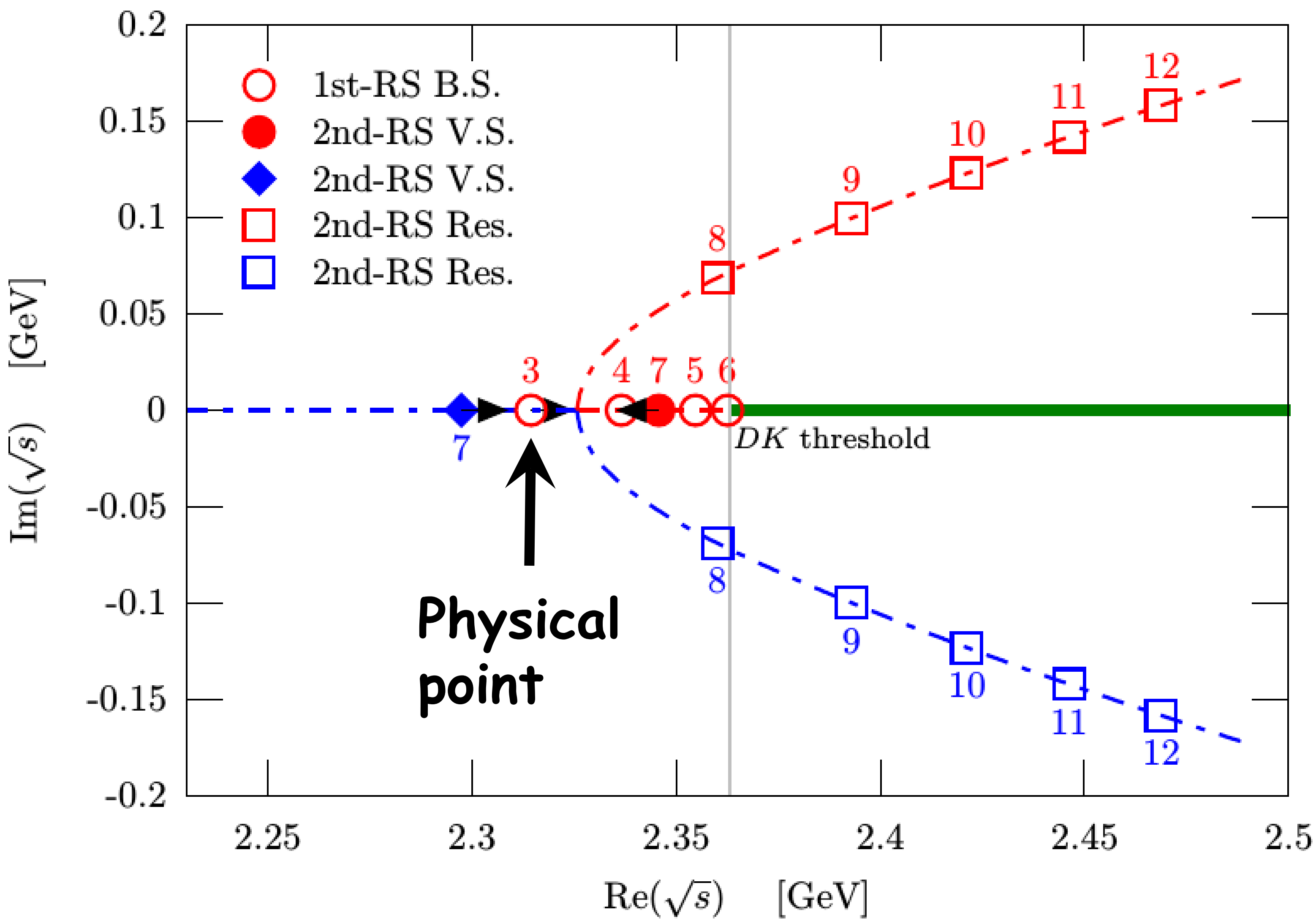
(S,I)=(0,1/2) channel

Hanhart, et al., PLB14

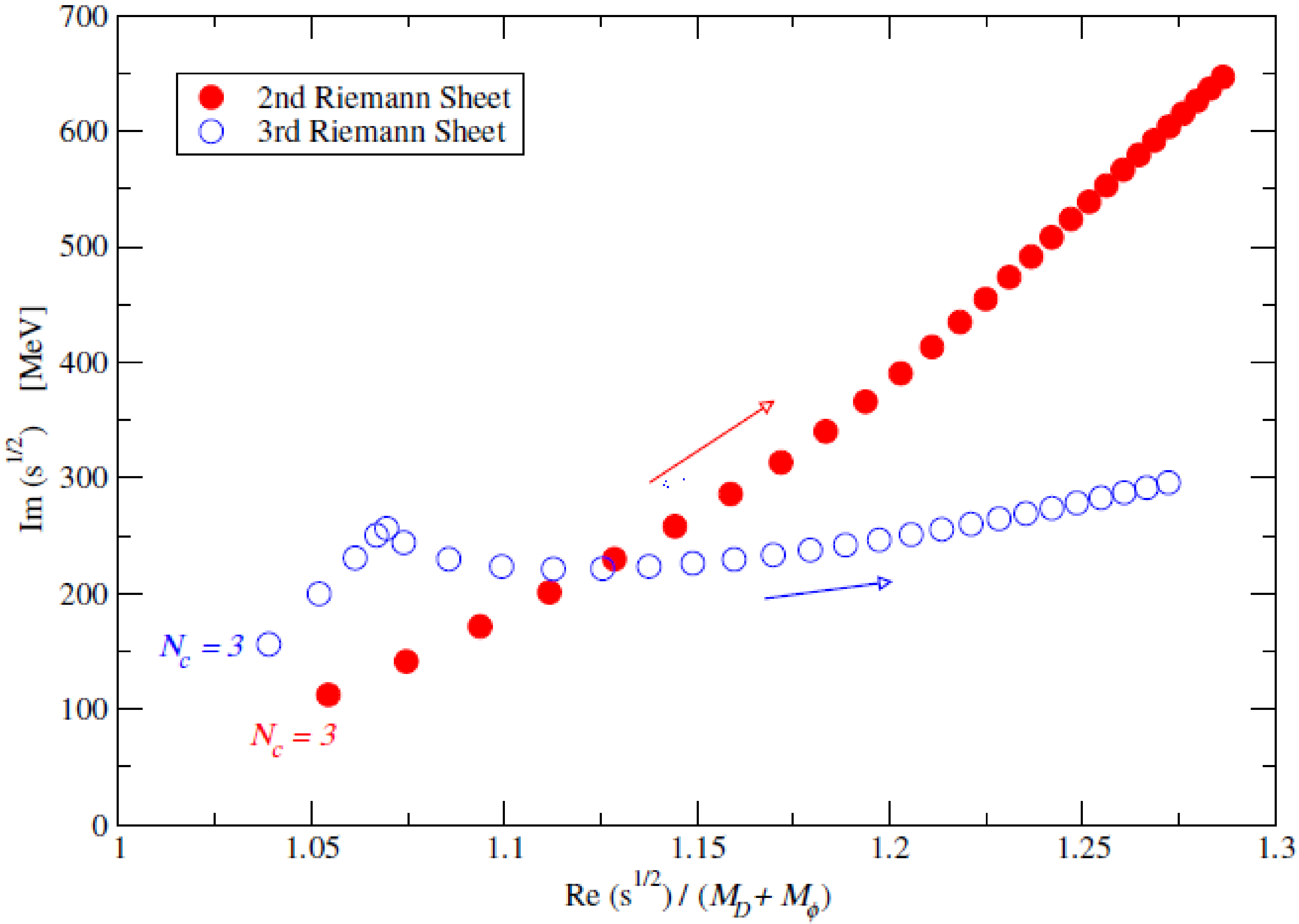
(S, I)	RS	$\sqrt{s_{pole}}$ [MeV]	$\lvert\text{Residue}\rvert^{1/2}$ [GeV]	Ratios	
$(-1,0)$	II	2333^{+15}_{-36}	$7.45^{+3.56}_{-1.38}(D\bar{K})$		
$(1,1)$	II	$2466^{+32}_{-27} - i\,271^{+4}_{-5}$	$6.95^{+0.60}_{-0.37}(D_s\pi)$	$1.72^{+0.12}_{-0.15}(DK/D_s\pi)$	
	III	$2225^{+12}_{-9} - i\,178^{+19}_{-17}$	$7.35^{+0.19}_{-0.13}(D_s\pi)$	$0.80^{+0.04}_{-0.04}(DK/D_s\pi)$	
(1,0)	I	2321^{+6}_{-3}	$9.30^{+0.04}_{-0.12}(DK)$	$0.77^{+0.02}_{-0.02}(D_s\eta/DK)$	$0.43^{+0.15}_{-0.13}(D_s\eta'/DK)$
$(0,\frac{1}{2})$	II	$2114^{+3}_{-3} - i\,111^{+8}_{-7}$	$9.66^{+0.15}_{-0.13}(D\pi)$	$0.31^{+0.03}_{-0.03}(D\eta/D\pi)$ $0.49^{+0.08}_{-0.08}(D\eta'/D\pi)$	$0.46^{+0.02}_{-0.02}(D_s\bar{K}/D\pi)$
	III	$2473^{+29}_{-22} - i\,140^{+8}_{-7}$	$5.36^{+0.40}_{-0.28}(D\pi)$	$1.09^{+0.06}_{-0.05}(D\eta/D\pi)$ $1.12^{+0.18}_{-0.16}(D\eta'/D\pi)$	$2.12^{+0.06}_{-0.08}(D_s\bar{K}/D\pi)$

Pole positions and their residues with physical masses

Nc trajectories for $D^*_{s0}(2317)$ [ZHG,Meissner,Yao, PRD'15]



Nc trajectories for D^*_0 pole in (S,I)=(0,1/2) channel



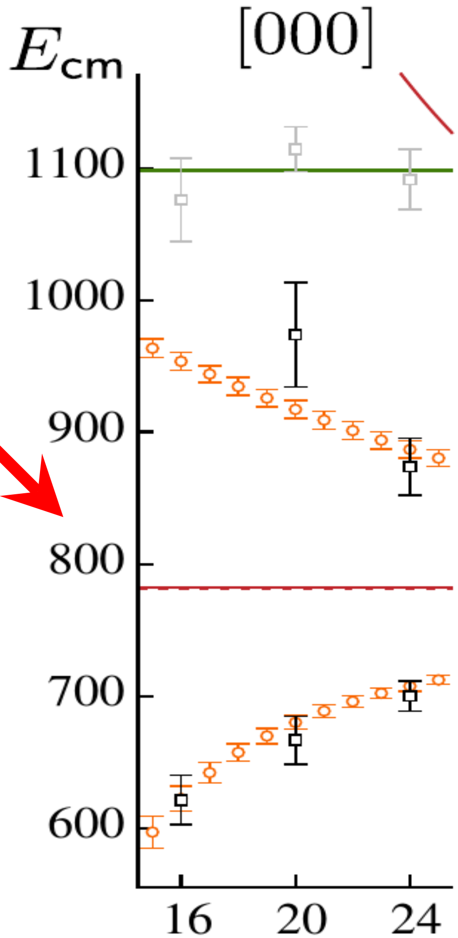
Precise determination of the scattering amplitudes using the lattice energy levels:

coupled D-pi, D-eta and Ds-Kbar channels

Lattice simulation data:

finite-volume spectra in meson-meson scattering

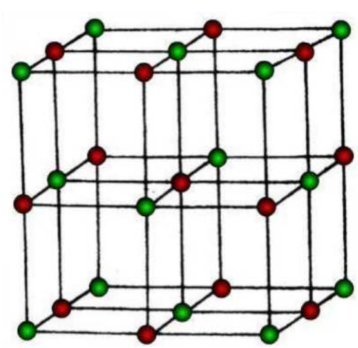
Eigenenergies in finite box



An example:
Isoscalar S-wave pi-pi scattering

[Briceno, Dudek, Edwards, Wilson, PRL'17]

Length of finite box



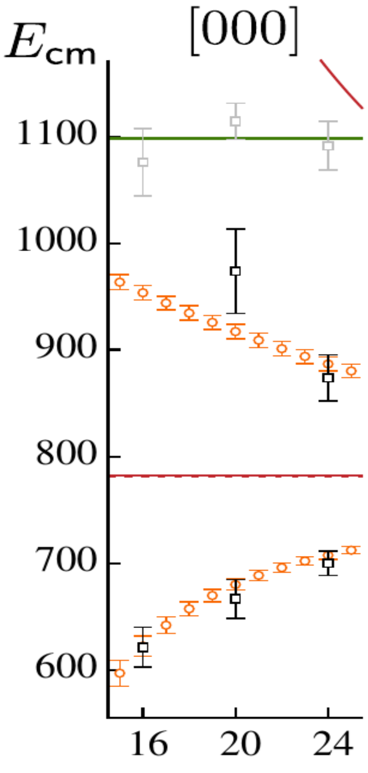
Luscher's Approach:

connect the discrete spectra in finite box to the scattering amplitudes in the infinite volume

[Luscher, NPB '91]

Elastic scattering case:

$$\tan\phi(q) = -\frac{\pi^{3/2}q}{\mathcal{Z}_{00}(1;q^2)}$$



Phase shifts

Luscher's Zeta function
(function of L, parameter free)

- **For the elastic case, one has the one-to-one correspondance between the phase shifts and energy levels.**
- **The one-to-one correspondance will be lost in the inelastic case.**

[He,Feng,Liu,JHEP'05] [Wilson,Briceno,Dudek,Edwards,Thomas, PRD '15]
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn, PRD'14] [Fu,PRD'12]
[Gockeler,Horsley,Lage,Meissner,Rakow,Rusetksy,Schierholz,Zanotti,PRD'12] ......

A widely used approach in the inelastic scattering case:

Luscher function + K matrix

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (\mathbf{1} + i\mathcal{M})] = 0$$

Kinematical factor

K matrix: polynomial + pole terms

Luscher's finite-volume functions (complex objects)

- **Free parameters in K matrix are determined by the finite-volume spectra. Then one can determine amplitudes in infinite volume.**
- **K matrix does not automatically respect the QCD symmetries, such as the chiral symmetry. It could be problematic for chiral extrapolation.**

Our approach:

Step 1: Put chiral perturbation theory (ChPT) in finite volume.

Step 2: The free parameters in ChPT, which are indepdent of quark masses and volumes, are fitted to the finite-volume energy levels obtained at (un)physical quark masses.

Step 3: Perform the chiral extrapolation and give the predictions in infinite volume with physical quark masses, including phase shifts, inelasticities, resonance poles, etc.

I will show some results for the D-pi, D-eta and Ds-Kbar scattering.

Finite-volume effects

Two types of finite volume dependence of scattering amplitudes:

- **Exponentially suppressed type $\propto$ *exp*(-m$_p$L) :** ***s*, *t*, *u* channels**
- **Power suppressed type $\propto$ $1/L^3$:** **only *s* channel**

We ignore the exponentially suppressed terms, indicating that finite-volume effects only enter through s channel.

$$T_J = \frac{N(s)}{1 - N(s)\,G(s)}$$

I.e. We only consider the finite-volume corrections for $G(s)$.

$$G(s)=i\int\frac{\mathrm{d}^4q}{(2\pi)^4}\frac{1}{(q^2-m_1^2+i\epsilon)[(P-q)^2-m_2^2+i\epsilon]}\,,\qquad s\equiv P^2$$

Sharp momentum cutoff to regularize G(s)

$$G(s)^{\rm cutoff}=\int^{|\vec{q}|<q_{\rm max}}\frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}I(|\vec{q}|)\,,\qquad \begin{aligned} I(|\vec{q}|) &= \frac{w_1+w_2}{2w_1w_2\,[E^2-(w_1+w_2)^2]}\,,\\ w_i &= \sqrt{|\vec{q}|^2+m_i^2}\,,\quad s=E^2\end{aligned}$$

G(s) in a finite box of length L with periodic boundary condition

$$\widetilde{G}=\frac{1}{L^3}\sum_{\vec{n}}^{|\vec{q}|<q_{\rm max}}I(|\vec{q}|)\,,\qquad \vec{q}=\frac{2\pi}{L}\vec{n},\qquad \vec{n}\in\mathbb{Z}^3$$

Finite-volume correction ΔG [Doring, Meissner, Oset, Rusetsky, EPJA11]

$$\Delta G = \widetilde{G}-G^{\rm cutoff}$$

$$=\left\{\frac{1}{L^3}\sum_{\boldsymbol{q}}^{|\boldsymbol{q}|<q_{\rm max}}-\int^{|\boldsymbol{q}|<q_{\rm max}}\frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}\right\}\frac{1}{2\omega_1(\boldsymbol{q})\,\omega_2(\boldsymbol{q})}\,\frac{\omega_1(\boldsymbol{q})+\omega_2(\boldsymbol{q})}{E^2-(\omega_1(\boldsymbol{q})+\omega_2(\boldsymbol{q}))^2}$$

Finite-volume effects in the moving frames

Lorentz invariance is lost in finite box. One needs to work out the explicit form of the loops when boosting from one frame to another.

transforming $\vec{q}_{i=1,2}$ to $\boxed{\vec{q}^{\,*}_{i=1,2}}$ → **CM quantities**

$$\vec{q}^{\,*}_i = \vec{q}_i + \left[\left(\frac{P^0}{E} - 1\right)\frac{\vec{q}_i \cdot \vec{P}}{|\vec{P}|^2} - \frac{q_i^0}{E}\right]\vec{P}$$

moving frame with total four-momentum $P^\mu = (P^0, \vec{P})$ $\quad s = E^2 = (P^0)^2 - |\vec{P}|^2$

Impose on-shell condition $\quad q_i^{*\,0} = \sqrt{|\vec{q}^{\,*}_i|^2 + m_i^2}$

$$q_i^0 = \frac{q_i^{*\,0} E + \vec{q}_i \cdot \vec{P}}{P^0} \longrightarrow q_i^0 = \sqrt{|\vec{q}_i|^2 + m_i^2}$$

G function in the moving frame

$$\int^{|\vec{q}_1|^* < q_{\max}} \frac{d^3\vec{q}^{\,*}_1}{(2\pi)^3} I(|\vec{q}^{\,*}_1|) \Longrightarrow \widetilde{G}^{\rm MV} = \frac{E}{P^0 L^3} \sum_{\vec{q}_1}^{|\vec{q}^{\,*}_1| < q_{\max}} I(|\vec{q}^{\,*}_1(\vec{q}_1)|) \qquad \vec{q}_1 = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3, \qquad \vec{P} = \frac{2\pi}{L}\vec{N}, \quad \vec{N} \in \mathbb{Z}^3$$

Finite-volume correction $\Delta G^{\rm MV}$: $\quad \Delta G^{\rm MV} = \widetilde{G}^{\rm MV} - G^{\rm cutoff}$

[Doring, Meissner, Oset, Rusetsky, EPJA12]

Mixing of different partial waves in finite volume

The mixing is absent in the infinite volume: $$\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta Y_{\ell m}(\theta,\phi) Y^*_{\ell' m'}(\theta,\phi) = \delta_{\ell\ell'}\delta_{mm'}$$

The mixing appears in finite-volume case, due to the absence of the general orthogonal conditions.

Finite-volume correction to G function: $$\Delta G^{\rm MV}_{\ell m} = \widetilde{G}^{\rm MV}_{\ell m} - G^{\rm cutoff}$$

$$\widetilde{G}^{\rm MV}_{\ell m} = \sqrt{\frac{4\pi}{2\ell+1}} \frac{1}{L^3} \frac{E}{P^0} \sum_{\vec{n}}^{|\vec{q}^*|<q_{\rm max}} \left(\frac{|\vec{q}^*|}{|\vec{q}^{\rm on*}|}\right)^{\ell} Y_{\ell m}(\hat{q}^*)\, I(|\vec{q}^*|)$$

Final expression for the G function: $$\widetilde{G}_{\ell m} = G^{\rm Infinite\ volume} + \Delta G^{\rm MV}_{\ell m}$$

To determine the energy levels in different frames with only S and P waves:

A_1^+ (0,0,0) : $$\det[I + N_0(s)\cdot \widetilde{G}_{00}] = 0$$

T_1^- (0,0,0): $$\det[I + N_1(s)\cdot (\widetilde{G}_{00} + 2\widetilde{G}_{20})] = 0$$

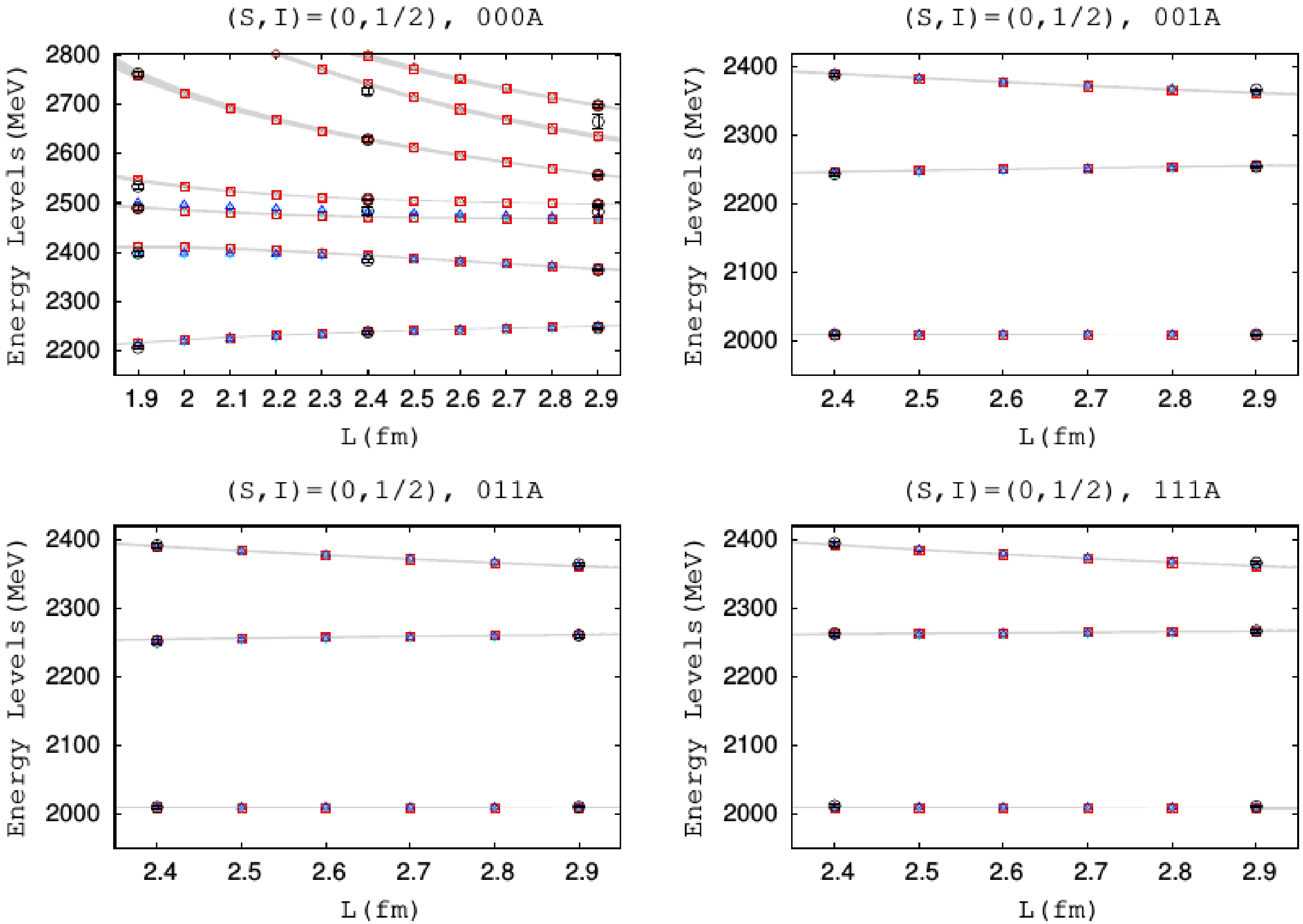
A_1 (0,0,1): $$\det[I + N_{0,1}\cdot \mathcal{M}^{A_1}_{0,1}] = 0\,, \quad N_{0,1} = \begin{pmatrix} N_0 & 0 \\ 0 & N_1 \end{pmatrix}, \quad \mathcal{M}^{A_1}_{0,1} = \begin{pmatrix} \widetilde{G}_{00} & i\sqrt{3}\widetilde{G}_{10} \\ -i\sqrt{3}\widetilde{G}_{10} & \widetilde{G}_{00} + 2\widetilde{G}_{20} \end{pmatrix}$$

……

[ZHG,Liu,Meissner,Oller,Rusetsky, to appear]

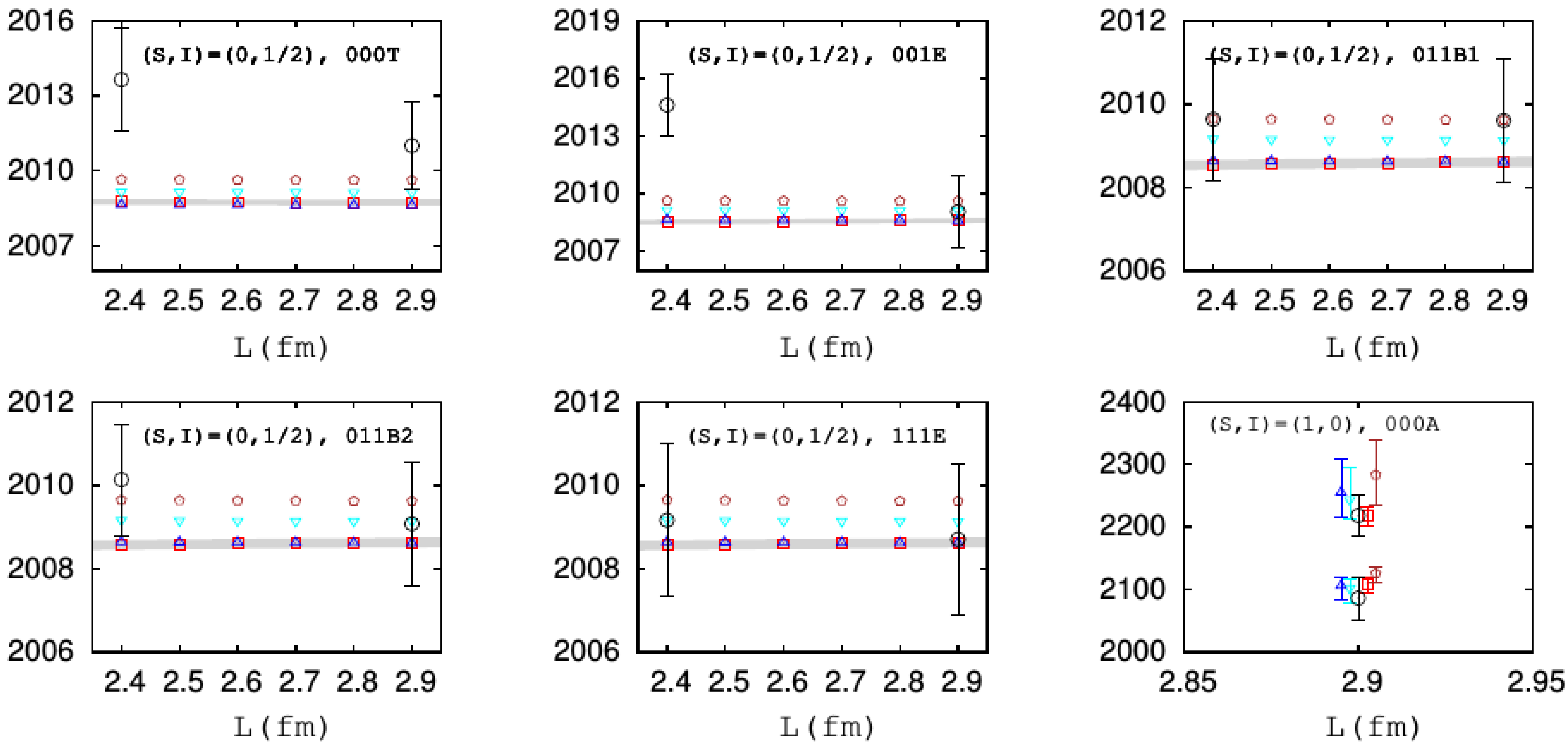
Reprduction of the lattice energy levels

[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]



[ZHG,Liu,Meissner,Oller,Rusetsky, to appear]

Reprduction of the lattice energy levels

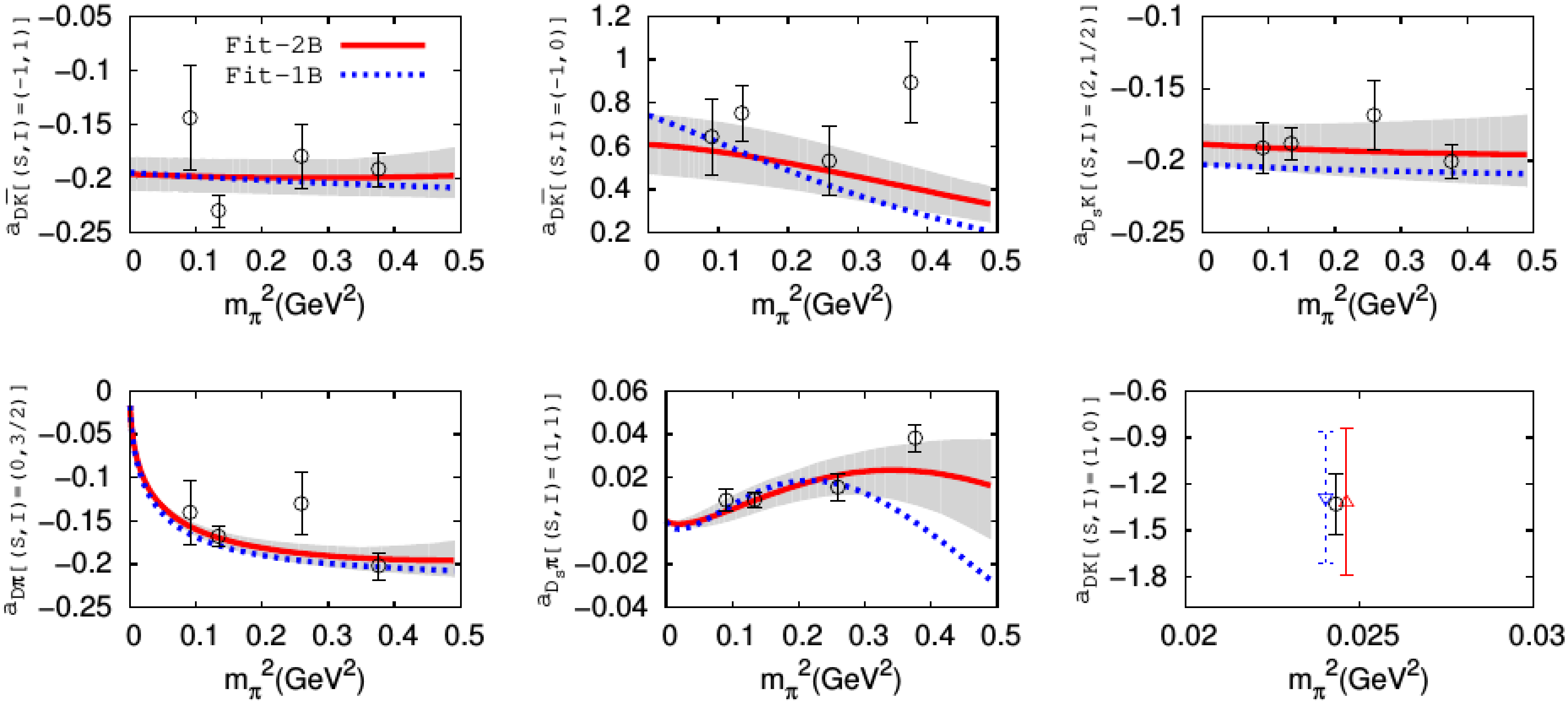


[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]

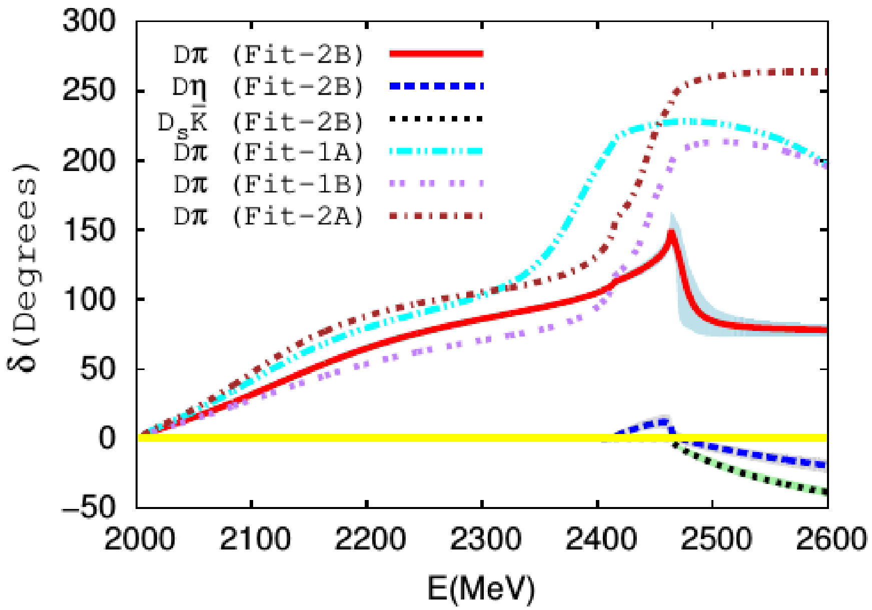
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]

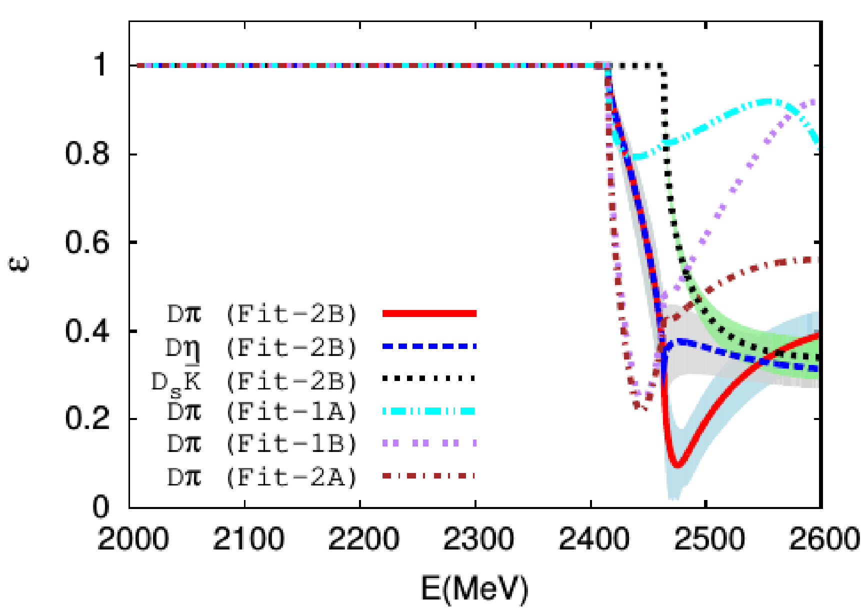
Reproduction of scattering lengths

[L.Liu,Orginos,F.K.Guo,Meissner, PRD'13]
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]



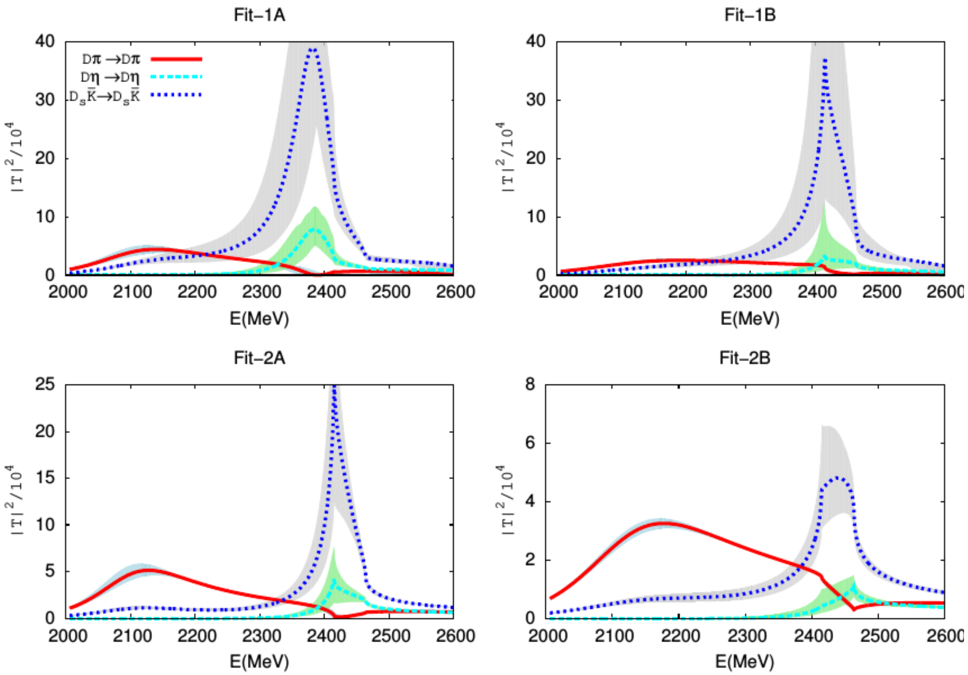
Prediction of the physical phase shifts and inelasticities

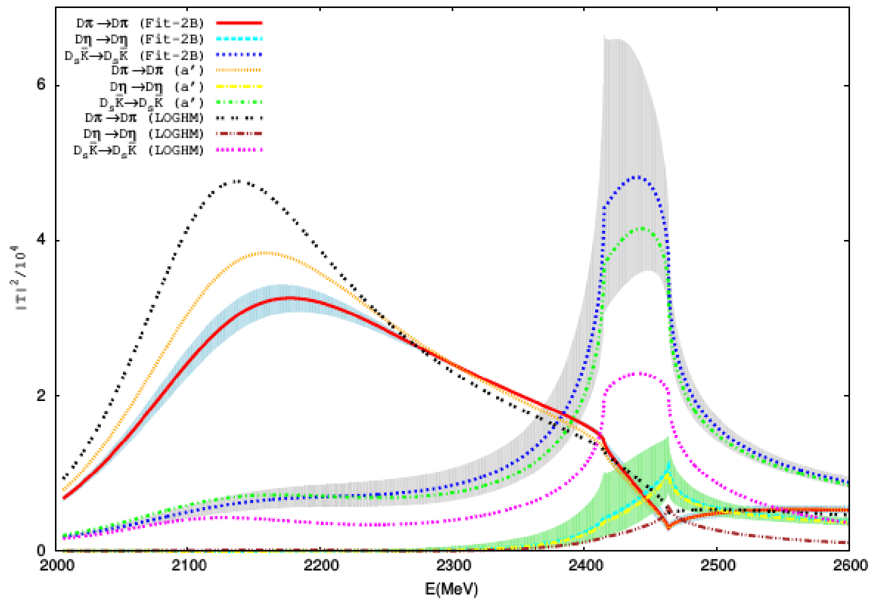




[ZHG,Liu,Meissner,Oller,Rusetsky, to appear]

Prediction of the physical scattering amplitudes





[ZHG,Liu,Meissner,Oller,Rusetsky, to appear]

Exploratory study of exotic doubly charmed baryons

Status of ground-state doubly heavy flavored baryons

Only doubly charmed baryons are observed now.

Long standing experimental puzzle:

- **SELEX:** $\Xi_{cc}^{+} \rightarrow \Lambda_c^{+} K^{-} \pi^{+}$, $p\, D^{+} K^{-}$

  [SELEX Coll., PRL'02, PLB'05]

  mass 3519 ÷ 2 MeV

- **However the results are NOT confirmed by FOCUS, Belle, BaBar !**

  [Ratti, NPPS'03, BaBar Coll., PRD'06, Belle Coll.,PRL'06]

- **New measurements from LHCb:**

$\Xi_{cc}^{++} \rightarrow \Lambda_c^{+} K^{-} \pi^{+} \pi^{+}$, **mass 3621.40 ÷ 0.78 MeV**

[LHCb Coll., PRL'17]

Our exploratory work aims at pushing forward the study of the EXCITED doubly charmed baryons.

- **Methodology:** Scattering of ground-state doubly charmed baryons (Ξ_{cc}^{++}, Ξ_{cc}^{+}, Ω_{cc}^{+}) and light pseudoscalar mesons (π, K, η)
- **Theoretical framework:**

($\Xi_{cc}^{++} = ccu$, $\Xi_{cc}^{+} = ccd$, $\Omega_{cc}^{+} = ccs$) : charm quarks behave like static color source. The dynamics is governed by the light flavors.

Then chiral EFT is a perfect tool to study such processes.

- **Benefits:** Our results could provide useful guides for future experimental measurements and lattice QCD simulations !

Construction of the leading order chiral Lagrangian

- **Ground-state doubly charmed baryons form a chiral triplet**

$$\psi_{cc} = \begin{pmatrix} \Xi_{cc}^{++} \\ \Xi_{cc}^{+} \\ \Omega_{cc}^{+} \end{pmatrix}$$

- **Light pseudoscalar mesons identified as pseudo Nambu-Goldstone bosons**

$$U = u^2 \quad = \quad e^{i\sqrt{2}\Phi/F}$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & \frac{-2}{\sqrt{6}}\eta \end{pmatrix}$$

The leading order Lagrangian

$$D_\mu = \partial_\mu + \frac{1}{2}\left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right], \; u_\mu = i u^+ D_\mu U u^+$$

$$\mathcal{L} = \bar{\psi}_{cc}\left(i\not{D} - m_0 + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu\right)\psi_{cc}$$

(wt) (s) (u)

$$T^{Ai\to Bj} = T_{wt}^{Ai\to Bj} + T_s^{Ai\to Bj} + T_u^{Ai\to Bj},$$

$$T^{Ai\to Bj} = T_{wt}^{Ai\to Bj} + T_s^{Ai\to Bj} + T_u^{Ai\to Bj}\,,$$

$$T_{wt}^{Ai\to Bj} = -\mathcal{F}_{wt}\frac{1}{F^2}\,\bar{u}_B(\not{q} + \not{q}')u_A\,,$$

$$T_s^{Ai\to Bj} = -\mathcal{F}_s\frac{g_A^2}{F^2}\,\bar{u}_B\not{q}'\gamma_5\,\frac{1}{\not{p}_C - m_C}\,\not{q}\gamma_5 u_A\,,$$

$$T_u^{Ai\to Bj} = -\mathcal{F}_u\frac{g_A^2}{F^2}\,\bar{u}_B\not{q}\gamma_5\,\frac{1}{\not{p}_C - m_C}\,\not{q}'\gamma_5 u_A\,,$$

(S,I)	Processes	$\mathcal{F}_{wt}$	$\mathcal{F}_s\ (C)$	$\mathcal{F}_u\ (C)$	(S,I)	Processes	$\mathcal{F}_{wt}$	$\mathcal{F}_s\ (C)$	$\mathcal{F}_u\ (C)$
$(-2,\frac{1}{2})$	$\Omega_{cc}\bar{K}\to\Omega_{cc}\bar{K}$	$\frac{1}{4}$	0	$\frac{1}{2}\ (\Xi_{cc})$	$(1,0)$	$\Xi_{cc}K\to\Xi_{cc}K$	$-\frac{1}{4}$	0	$-\frac{1}{2}\ (\Omega_{cc})$
$(0,\frac{3}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	$\frac{1}{4}$	0	$\frac{1}{2}\ (\Xi_{cc})$	$(1,1)$	$\Xi_{cc}K\to\Xi_{cc}K$	$\frac{1}{4}$	0	$\frac{1}{2}\ (\Omega_{cc})$
$(-1,1)$	$\Omega_{cc}\pi\to\Omega_{cc}\pi$	0	0	0	$(0,\frac{1}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	$-\frac{1}{2}$	$\frac{3}{4}\ (\Xi_{cc})$	$-\frac{1}{4}\ (\Xi_{cc})$
	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	0	0	0		$\Xi_{cc}\eta\to\Xi_{cc}\eta$	0	$\frac{1}{12}\ (\Xi_{cc})$	$\frac{1}{12}\ (\Xi_{cc})$
	$\Omega_{cc}\pi\to\Xi_{cc}\bar{K}$	$\frac{1}{4}$	0	$\frac{1}{2}\ (\Xi_{cc})$		$\Omega_{cc}K\to\Omega_{cc}K$	$-\frac{1}{4}$	$\frac{1}{2}\ (\Xi_{cc})$	0
$(-1,0)$	$\Xi_{cc}\bar{K}\to\Xi_{cc}\bar{K}$	$-\frac{1}{2}$	$1\ (\Omega_{cc})$	0		$\Xi_{cc}\pi\to\Xi_{cc}\eta$	0	$\frac{1}{4}\ (\Xi_{cc})$	$\frac{1}{4}\ (\Xi_{cc})$
	$\Omega_{cc}\eta\to\Omega_{cc}\eta$	0	$\frac{1}{3}\ (\Omega_{cc})$	$\frac{1}{3}\ (\Omega_{cc})$		$\Xi_{cc}\pi\to\Omega_{cc}K$	$-\frac{\sqrt{3}}{4\sqrt{2}}$	$\frac{\sqrt{3}}{2\sqrt{2}}\ (\Xi_{cc})$	0
	$\Xi_{cc}\bar{K}\to\Omega_{cc}\eta$	$\frac{\sqrt{3}}{4}$	$-\frac{1}{\sqrt{3}}\ (\Omega_{cc})$	$\frac{1}{2\sqrt{3}}\ (\Xi_{cc})$		$\Xi_{cc}\eta\to\Omega_{cc}K$	$-\frac{\sqrt{3}}{4\sqrt{2}}$	$\frac{1}{2\sqrt{6}}\ (\Xi_{cc})$	$-\frac{1}{\sqrt{6}}\ (\Omega_{cc})$

[ZHG, PRD2017]

Partial-wave projection and the unitarization

- **S-wave partial-wave projection formula for baryon-meson scattering**

$$\mathcal{T}(W) = \frac{1}{8\pi}\sum_{\sigma=1,2}\int d\Omega\, T(W,\Omega;\sigma,\sigma)$$

- **Unitarization: to take into account the possible strong interactions of the two-body scattering**

$$\mathrm{Im}\mathbb{T} = -\rho(s) \qquad \rho(s)_i = q(s)_i/8\pi\sqrt{s}$$ **($q(s)$: CM three-momentum)**

$$\boxed{\mathbb{T}(s) = \left[1 - \mathcal{T}(s)\cdot G(s)\right]^{-1}\cdot\mathcal{T}(s)}$$

✓ **G(s) takes care of the right-hand/unitarity cuts**

$$G(s) = \frac{1}{i}\int\frac{\mathrm{d}^4q}{(2\pi)^4}\frac{1}{(q^2-m_1^2+i\epsilon)[(P-q)^2-m_2^2+i\epsilon]}$$

$$G^\Lambda(s) = -\int^{|\vec{q}|<\Lambda}\frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\frac{w_1+w_2}{2w_1w_2\left[s-(w_1+w_2)^2\right]}, \qquad \left(w_i = \sqrt{|\vec{q}|^2+m_i^2}\right)$$

✓ $\mathcal{T}(s)$ **contains the remaining contributions except the right-hand cuts**

Inputs

$$m_{\Xi_{cc}^{++}} = m_{\Xi_{cc}^{+}} = 3621.4 \text{ MeV}$$ [LHCb, PRL'17 & Isospin symmetry]

$$m_{\Omega_{cc}^{+}} = 3700 \text{ MeV}$$ [Chen, et al., PRD'17]

$$F = F_\pi = 92.1 \text{ MeV}$$ [PDG, CPC'16]

$$|g_A| = 0.2, \quad g_A = 0$$ [Sun, Vicente Vacas, PRD'16]

(The final results are barely affected by the g_A term.)

- The most sensitive parameter: cutoff scale Λ

Our conservative estimation: Λ = 1.0 ÷ 0.2 GeV

Scattering lengths

$$a^{S,I}_{\psi_{cc}\phi\to\psi_{cc}\phi} = \frac{1}{8\pi(m_{\psi_{cc}} + m_\phi)}\mathbb{T}^{S,I}_{\psi_{cc}\phi\to\psi_{cc}\phi}(s_{\text{thr}})$$

$(S,\ I)$	Processes	Scattering lengths (fm)
$(-2, 1/2)$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	$-0.19^{+0.02}_{-0.02}$
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	$-1.4,\ -3.6,\ 5.2$
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	$-0.19^{+0.02}_{-0.02}$
$(0, 3/2)$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	$-0.095^{+0.003}_{-0.004}$
$(-1, 1)$	$\Omega_{cc}\pi \to \Omega_{cc}\pi$	$0.03^{+0.01}_{-0.01}$
	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$	$-0.22^{+0.14}_{-0.14} + i0.45^{+0.00}_{-0.09}$
$(-1, 0)$	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$	$-0.49^{+0.10}_{-0.19}$
	$\Omega_{cc}\eta \to \Omega_{cc}\eta$	$-0.26^{+0.03}_{-0.03} + i\,0.02^{+0.02}_{-0.01}$
$(0, 1/2)$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	$0.55^{+0.16}_{-0.10}$
	$\Xi_{cc}\eta \to \Xi_{cc}\eta$	$-0.72^{+0.21}_{-0.17} + i\,0.30^{+1.10}_{-0.18}$
	$\Omega_{cc}K \to \Omega_{cc}K$	$-0.55^{+0.11}_{-0.16} + i\,0.13^{+0.19}_{-0.07}$

Resonance poles in the complex energy plane

[ZHG, PRD'17]

(S,I)	RS	Mass (MeV)	Width/2 (MeV)	$\lvert\text{Residue}\rvert_{11}^{1/2}$ (GeV)	$\lvert\text{Residue}\rvert_{22}^{1/2}$ (GeV)	$\lvert\text{Residue}\rvert_{33}^{1/2}$ (GeV)
$(1,0)$				$\Xi_{cc}K$		
	I	$(--, 4112, 4096)$	0	$(--, 10.0, 14.9)$		
	II	$(4114, 4115, 4113)$	0	$(6.5, 3.9, 4.1)$		
$(-1,1)$				$\Omega_{cc}\pi$	$\Xi_{cc}\bar{K}$	
	II	$(4191, 4134, 4090)$	$(89, 83, 74)$	$(15.7, 14.5, 13.2)$	$(21.0, 18.3, 16.4)$	
$(-1,0)$				$\Xi_{cc}\bar{K}$	$\Omega_{cc}\eta$	
	I	$(4018, 3957, 3907)$	0	$(22.4, 21.7, 19.8)$	$(13.4, 12.5, 11.1)$	
	II	$(4105, 4095, 4083)$	0	$(5.7, 6.6, 7.4)$	$(3.0, 3.2, 3.3)$	
$(0,\frac{1}{2})$				$\Xi_{cc}\pi$	$\Xi_{cc}\eta$	$\Omega_{cc}K$
	II	$(3830, 3816, 3800)$	$(76, 50, 33)$	$(15.7, 14.6, 13.4)$	$(1.0, 1.2, 1.2)$	$(8.3, 7.6, 6.9)$
	II	$(4170, 4146, 4116)$	$(8, 18, 22)$	$(4.4, 5.7, 6.4)$	$(9.9, 12.1, 13.2)$	$(12.0, 13.6, 14.1)$

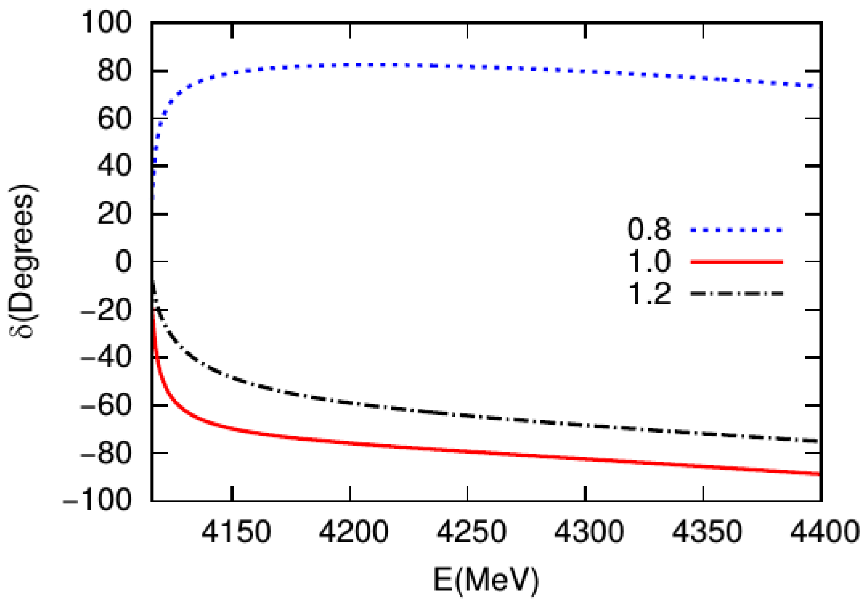
Resembles $\Lambda(1405)$ & $D^*_{s0}(2317)$

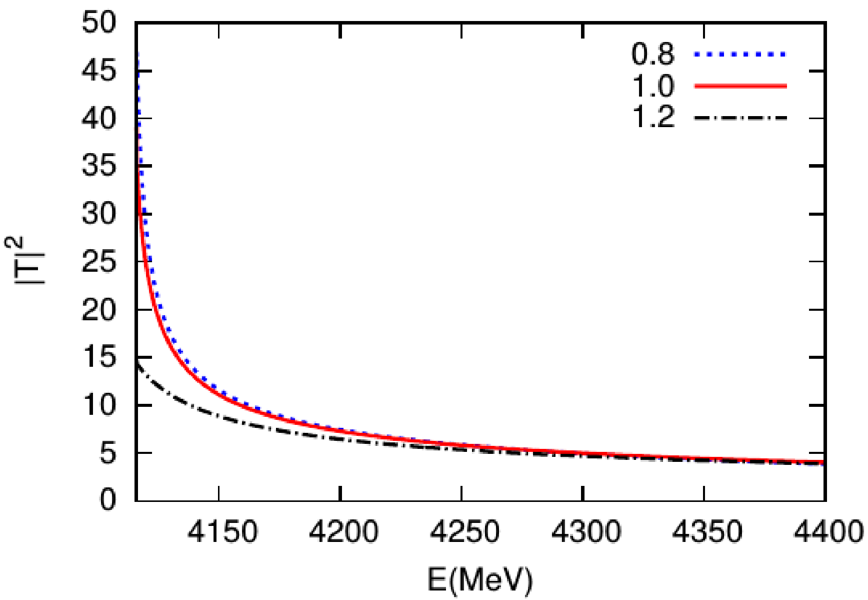
See also a pioneer study of the bound state pole in the Ξ_{cc}Kbar scattering by F.K.Guo & Ulf.-G.Meissner, PRD'11; and a following detailed study by M.J.Yuan, et al., 1805.10972 and J.M.Dias et al., 1805.03286

Phase shifts, inelasticities, line shapes

$\Xi_{cc}K \to \Xi_{cc}K$ scattering with $(S, I) = (1, 0)$

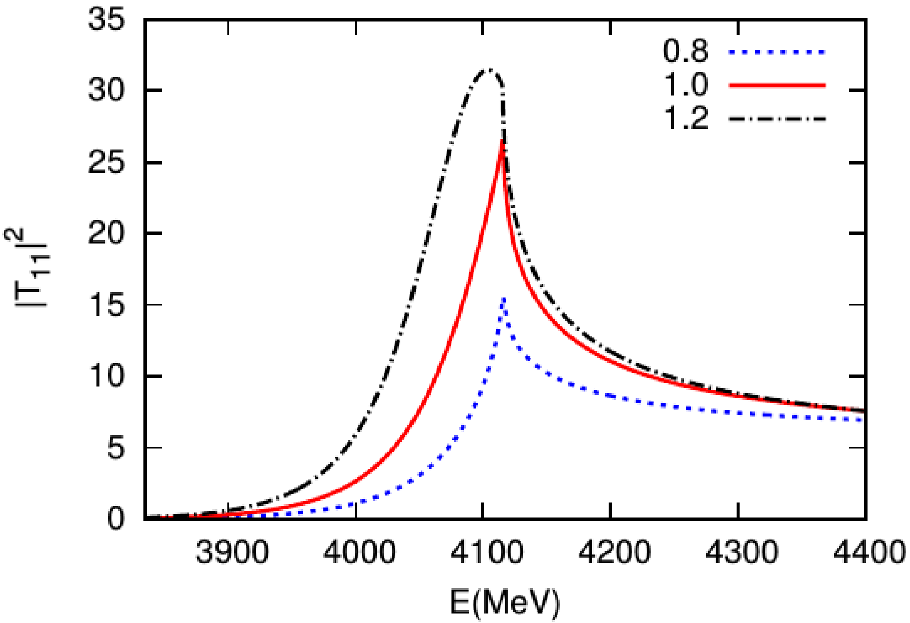
(S, I)	RS	Mass(MeV)	Width/2(MeV)	$\lvert\text{Residue}\rvert_{11}^{1/2}$(GeV)
$(1, 0)$	I	$(--,\ 4112,\ 4096)$	0	$(--,\ 10.0,\ 14.9)$
	II	$(4114,\ 4115,\ 4113)$	0	$(6.5,\ 3.9,\ 4.1)$

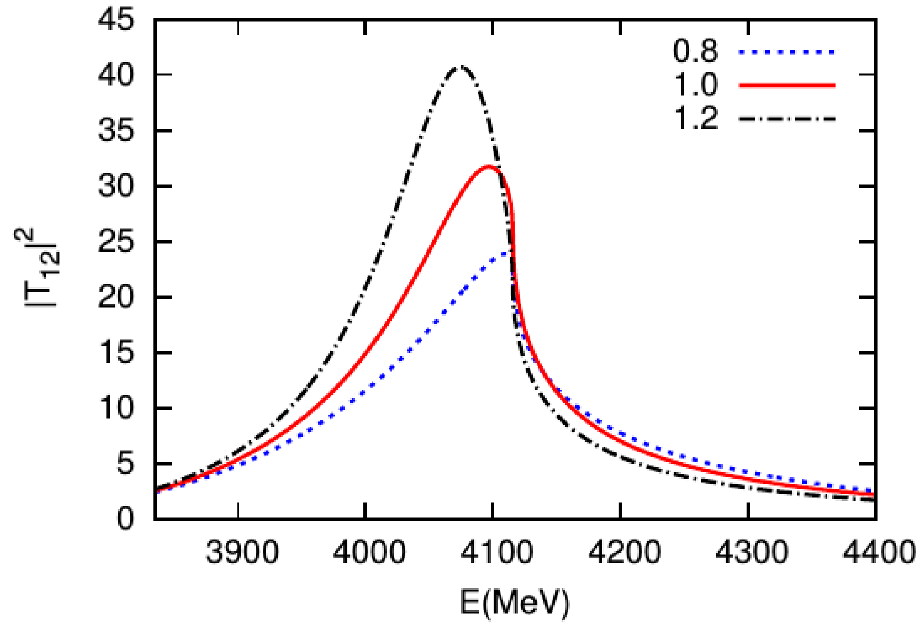




$\Omega_{cc}\pi \to \Omega_{cc}\pi$ and $\Omega_{cc}\pi \to \Xi_{cc}\bar{K}$ scattering with $(S, I) = (-1, 1)$

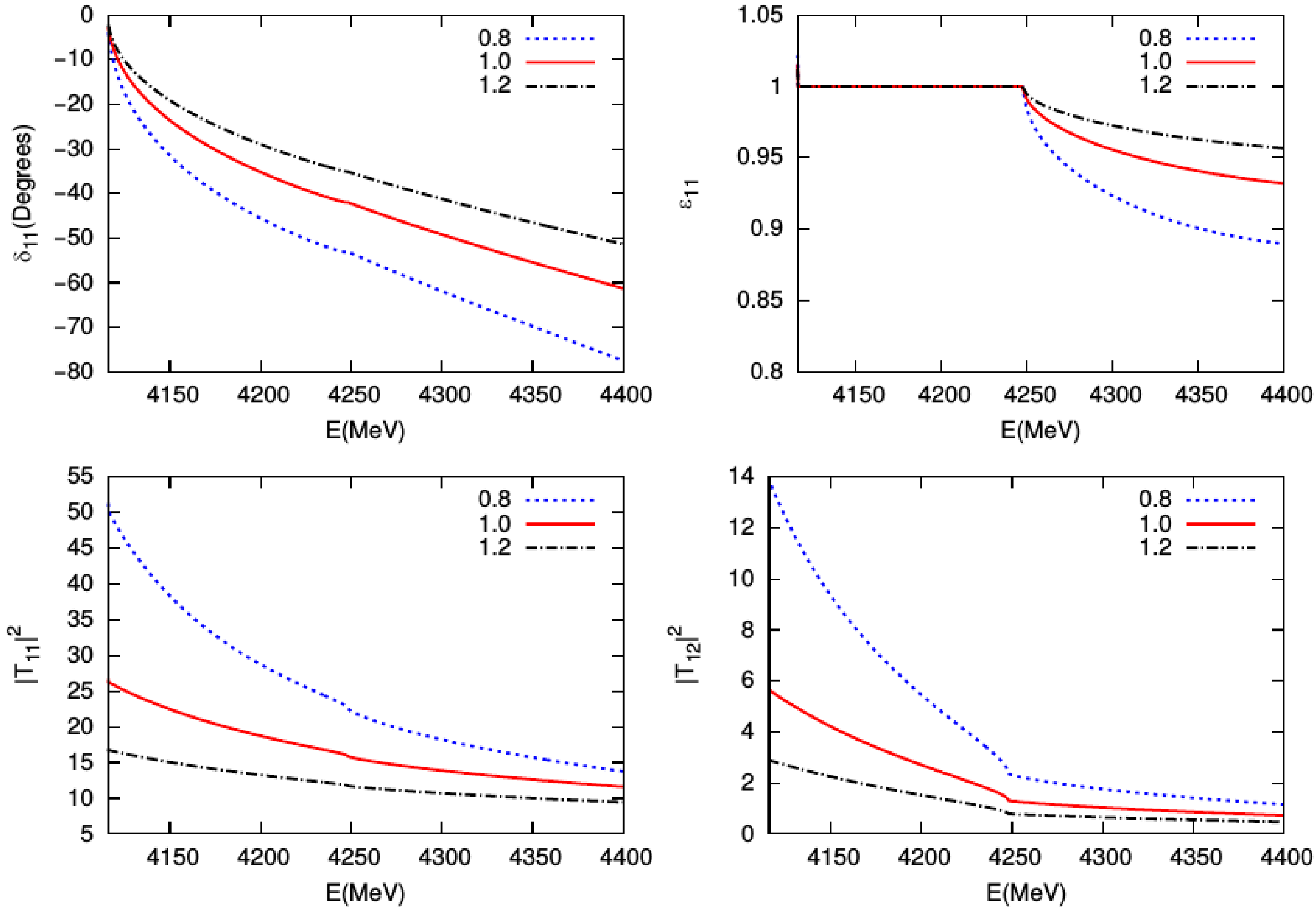
(S, I)	RS	Mass(MeV)	Width/2(MeV)	$\lvert\text{Residue}\rvert_{11}^{1/2}$(GeV)	$\lvert\text{Residue}\rvert_{22}^{1/2}$(GeV)
$(-1, 1)$	II	(4191, 4134, 4090)	(89, 83, 74)	(15.7, 14.5, 13.2)	(21.0, 18.3, 16.4)





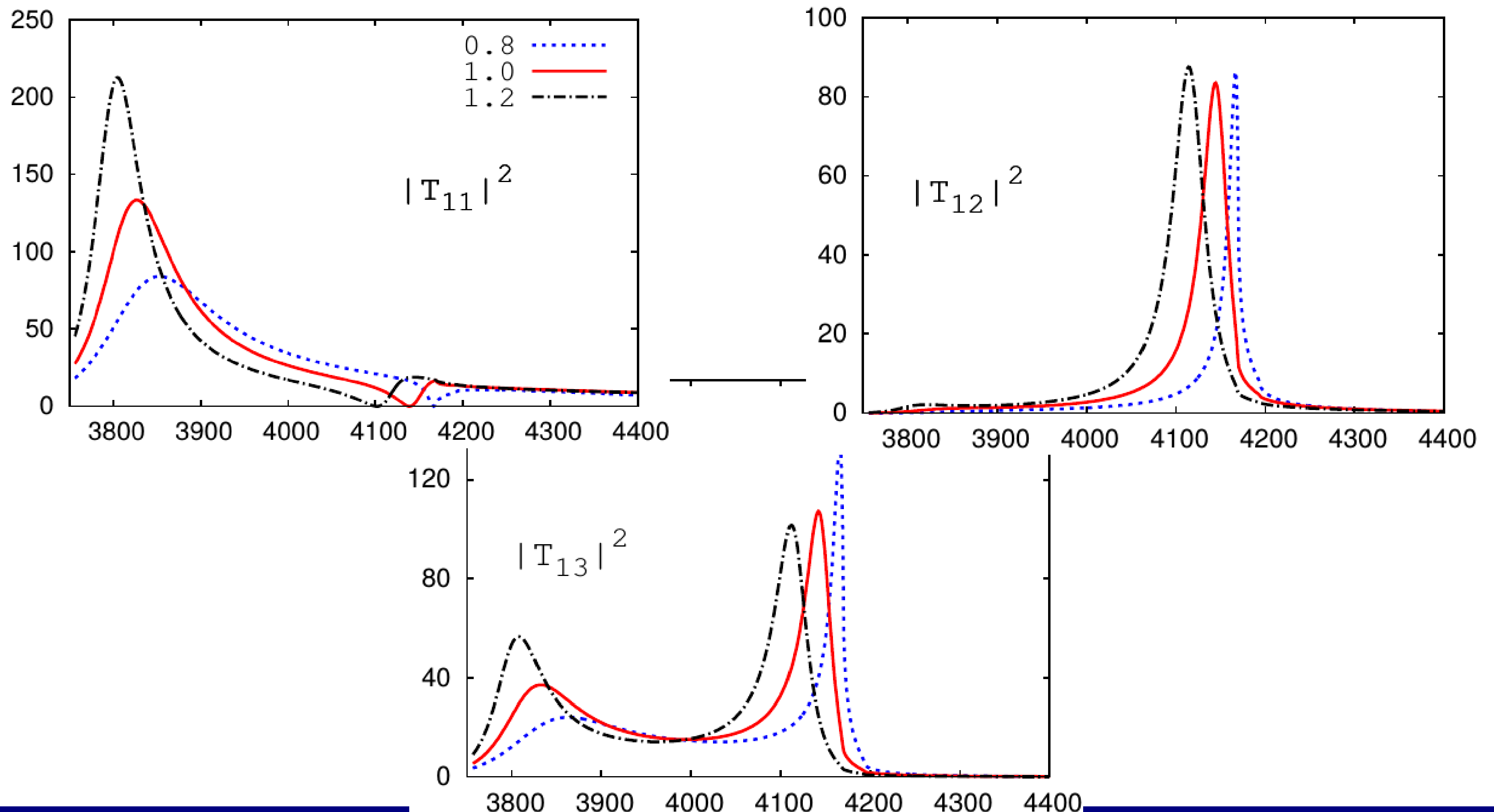
$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$ and $\Xi_{cc}\bar{K} \to \Omega_{cc}\eta$ scattering with $(S,I) = (-1,0)$

(S,I)	RS	Mass(MeV)	Width/2(MeV)	$\lvert\text{Residue}\rvert_{11}^{1/2}$(GeV)	$\lvert\text{Residue}\rvert_{22}^{1/2}$(GeV)
$(-1,0)$	I	(4018, 3957, 3907)	0	(22.4, 21.7, 19.8)	(13.4, 12.5, 11.1)
	II	(4105, 4095, 4083)	0	(5.7, 6.6, 7.4)	(3.0, 3.2, 3.3)



$\Xi_{cc}\pi$, $\Xi_{cc}\eta$ and $\Omega_{cc}K$ coupled-channel scattering with $(S, I) = (0, 1/2)$

(S, I)	RS	Mass(MeV)	Width/2(MeV)	$\lvert\text{Residue}\rvert_{11}^{1/2}$(GeV)	$\lvert\text{Residue}\rvert_{22}^{1/2}$(GeV)	$\lvert\text{Residue}\rvert_{33}^{1/2}$(GeV)
$(0, \frac{1}{2})$	II	(3830, 3816, 3800)	(76, 50, 33)	(15.7, 14.6, 13.4)	(1.0, 1.2, 1.2)	(8.3, 7.6, 6.9)
	II	(4170, 4146, 4116)	(8, 18, 22)	(4.4, 5.7, 6.4)	(9.9, 12.1, 13.2)	(12.0, 13.6, 14.1)



Summary

- Chiral EFT provides a useful tool to study the charmed hadron spectra.

- Pole trajectories with varying m_π for the (S,I)=(0,1/2) channel are found to be quite similar as those from f0(500).

- Pole trajectories with varying Nc for $D^*_{s0}(2317)$ and the (S,I)=(0,1/2) channel are given.

- To further constrain the scatteing amplitudes using lattice finite-volume energy levels and scattering lengths appears soon !

- **Two clearly exotic doubly charmed baryons are predicted, due to their exotic quantum numbers of (S,I)=(1,0) and (-1,1).**

- **Two resonance poles in the $\Xi_{cc}\pi, \Xi_{cc}\eta, \Omega_{cc}K$ coupled-channel scattering with (S,I)=(0,1/2) are predicted.**

- **Similar to the $\Lambda(1405)$ in $\bar{K}N$ and $D^{*}_{s0}(2317)$ in DK scattering, one bound/virtual state pole is found below the $\Xi_{cc}\bar{K}$ threshold with (S,I)=(-1,0).**

- **These findings can provide useful guides for future experimental and lattice studies.**

谢谢各位！