

Optimization of data taking

Data taking scheme

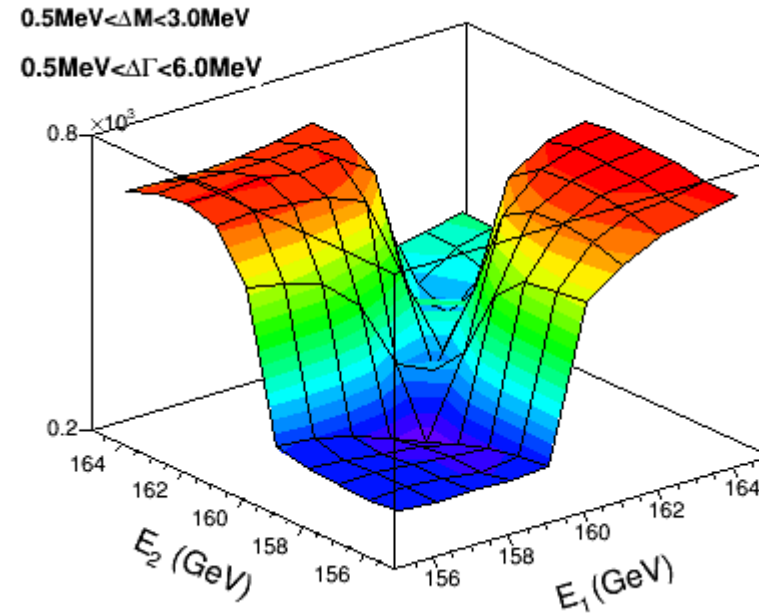
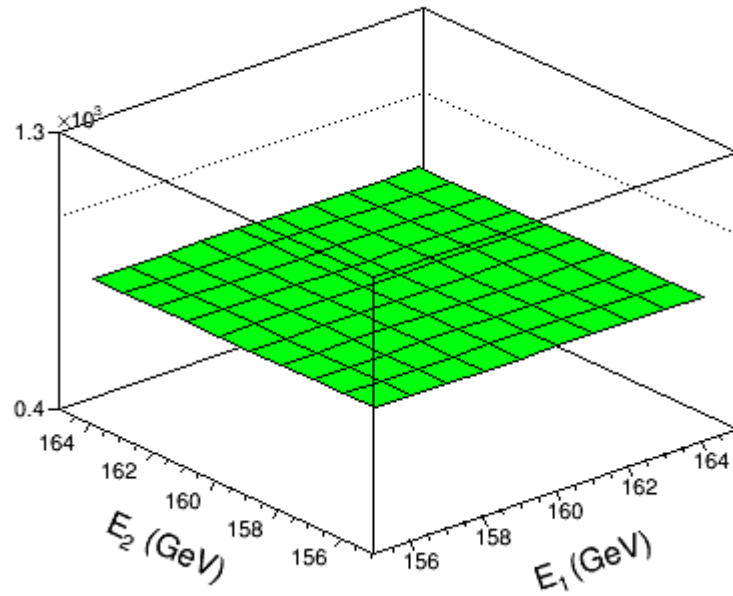
1. Taking data at one point (just for m_W) ✓
2. Taking data at two points (both m_W and Γ_W) ✓
3. Taking data at three points (m_W , Γ_W and the correlated syst. uncertainties).

With $L = 3.2 \text{ ab}^{-1}$, $\epsilon_P = 0.72$

Taking data at three point

1. Fit parameters: m_W, Γ_W, h (associated with σ_{sys}^{corr})
2. Scan parameters: E_1, E_2, E_3, F_1, F_2 (L normalization factors)
3. Scan procedure:
 - A. $E_1, E_2, E_3 \in (154, 165)\text{GeV}, F_1, F_2 \in (0, 1), \Delta E_i = 1, \Delta F_i = 0.1$ (σ_{stat})
 - B. $E_1 \in (154, 160), E_2, E_3 \in (160, 164), F_1 \in (0, 0.5), F_2 \in (0, 1), \Delta F_2 = 0.2$ ($\sigma_{stat} + \sigma_{sys}^{corr}$)
 - C. Get the $\Delta m_W, \Delta \Gamma_W$ with optimization result from $b(\sigma_{stat} + \sigma_{sys}^{corr} + \Delta E + \Delta E_{BS})$

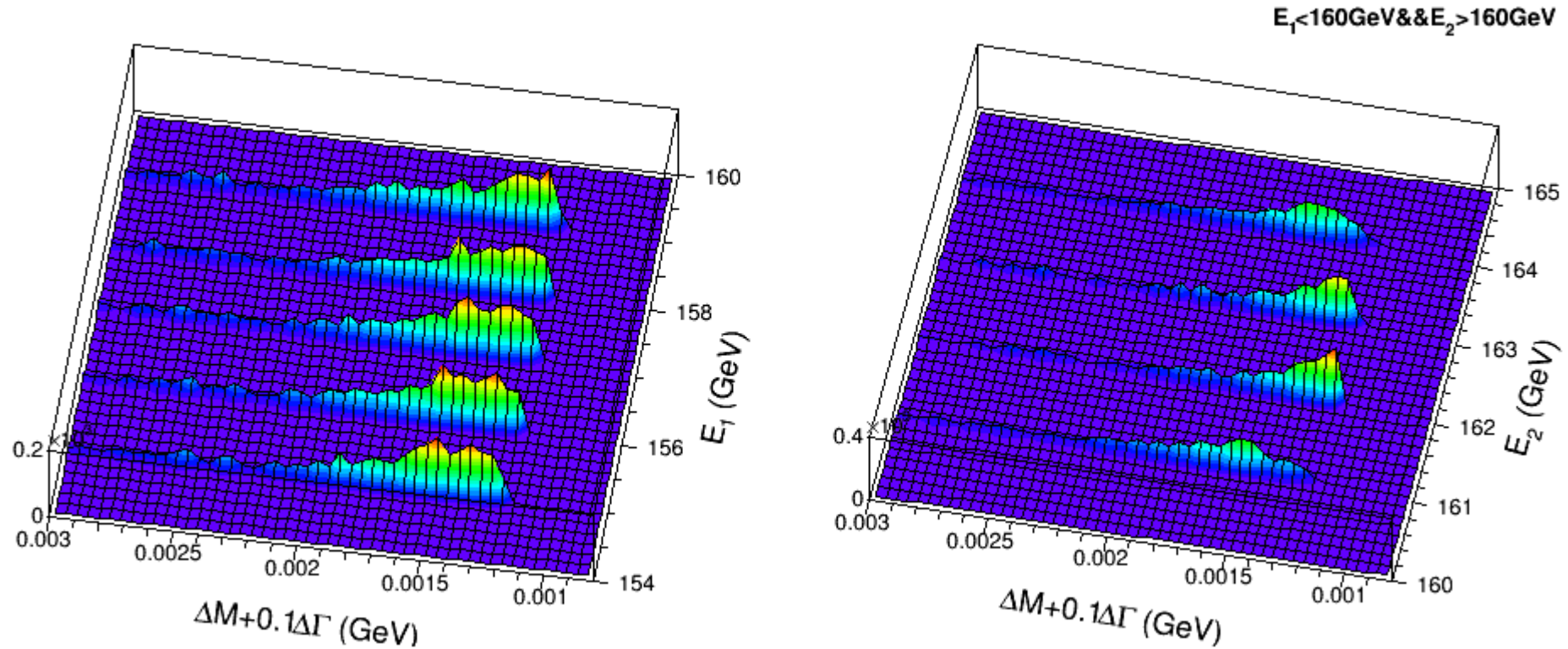
Step A: E_1, E_2



The z axis is the cumulation of the fit result. The edge of the distributions will affect the optimization results.

$E_1 < 160, E_2 > 160$ GeV is used in further optimization

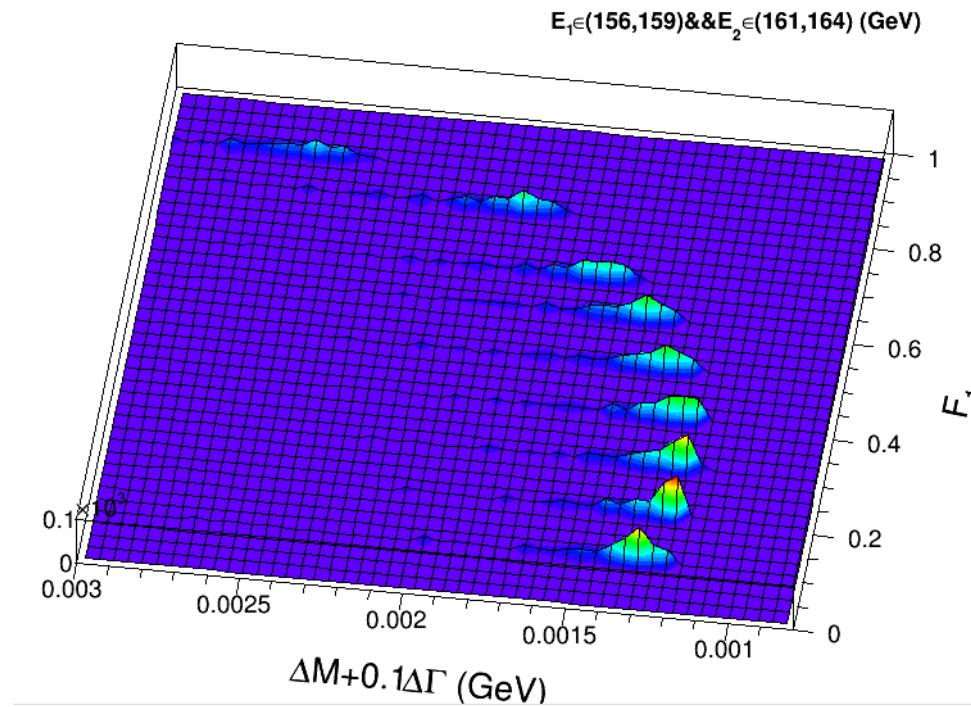
Step A: E_1, E_2



The optimal regions of E_1, E_2 are similar as two data points:

$$E_1 \sim (157, 158) \text{ GeV}, \quad E_2 \sim (162, 163) \text{ GeV}$$

Step A: F_1



The optimal region of F_1 is similar as two data points: $F_1 \sim 0.3$

Step B

1. Use the rough results from step A, the requirements below are used:

$$E_1 \in (155, 160)$$

$$E_2 \in (160, 164)$$

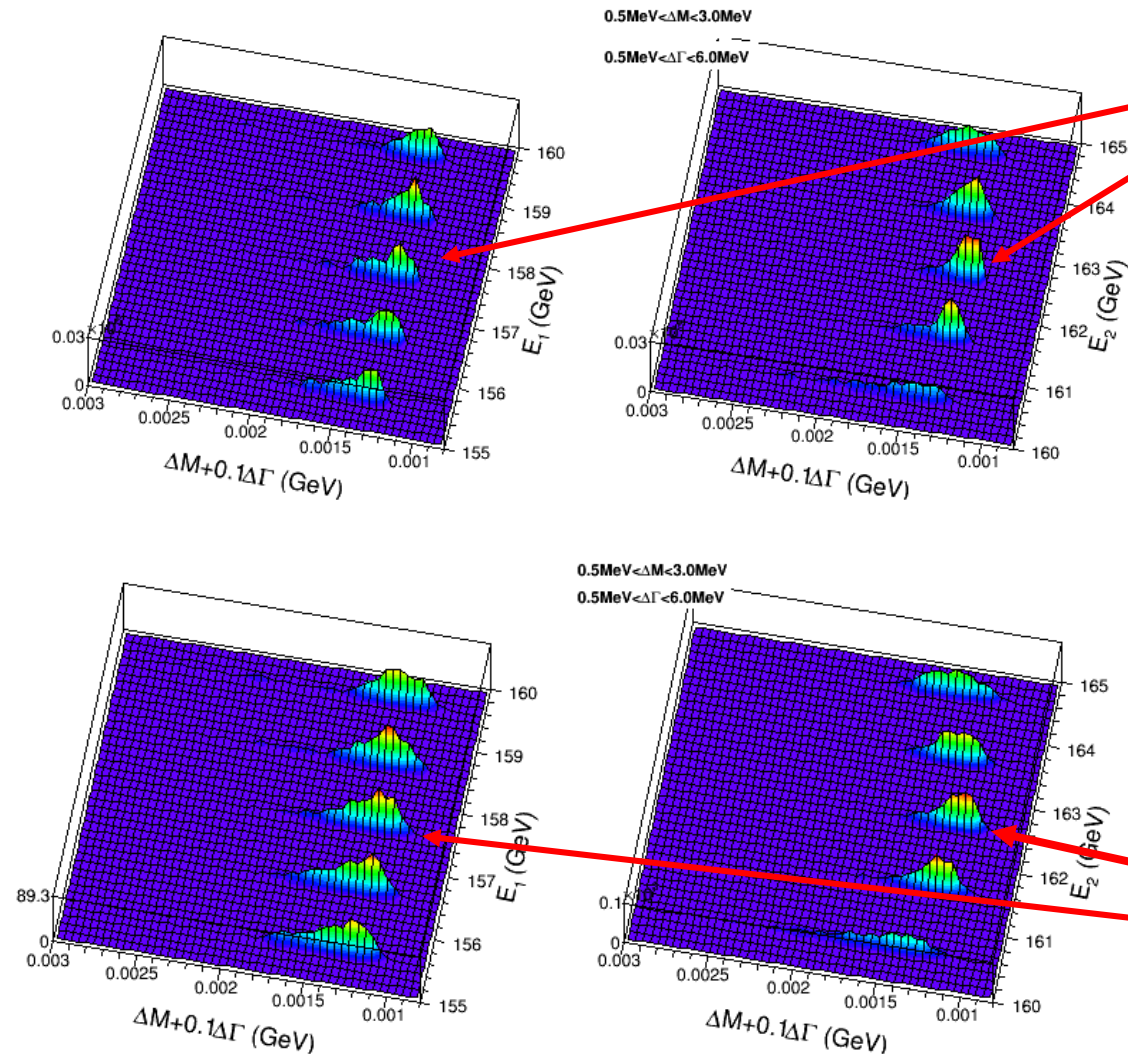
$$E_3 \in (160, 164)$$

$$F_1 = 0.3, F_2 \in (0, 1)$$

the $\sigma_{sys}^{corr} = 2 \times 10^{-4}$ is considered in the fit.

2. For each specific fit, 200 samplings are used, $\sigma_{WW} \sim G(\sigma_{WW}^0, \sigma_{sys}^{corr})$
3. So we can get the direct fit results ($N_1 = N_{scan} \cdot 200$), and the results by fitting the distributions of m_W, Γ_W of each fit result ($N_2 = N_{scan}$).

Step B: E_1, E_2



Direct fit results

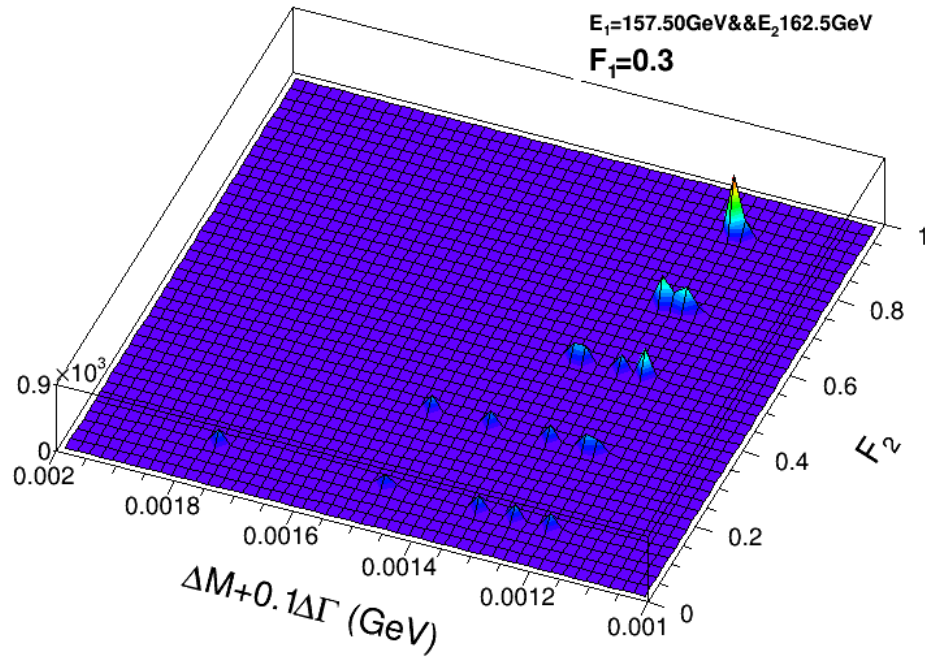
The optimal regions of E_1, E_2 from these two results are consistent and the results are similar as two data points:

$$E_1 \sim 157.5 \text{ GeV}, \quad E_2 \sim 162.5 \text{ GeV}$$

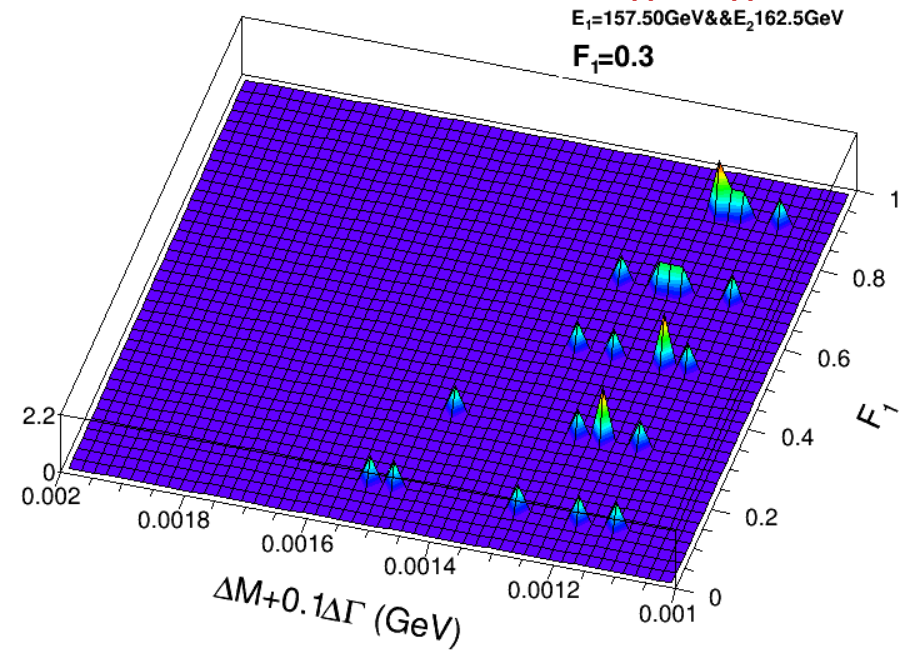
Fit the m_W, Γ_W of each fit results

Step B: F_2

Direct fit results



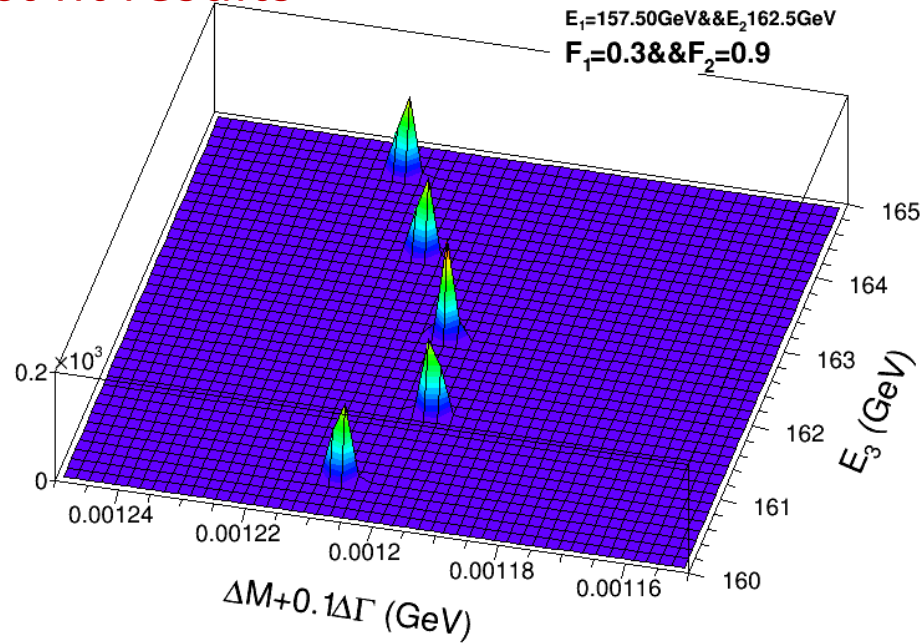
Fit the m_W, Γ_W of each fit results



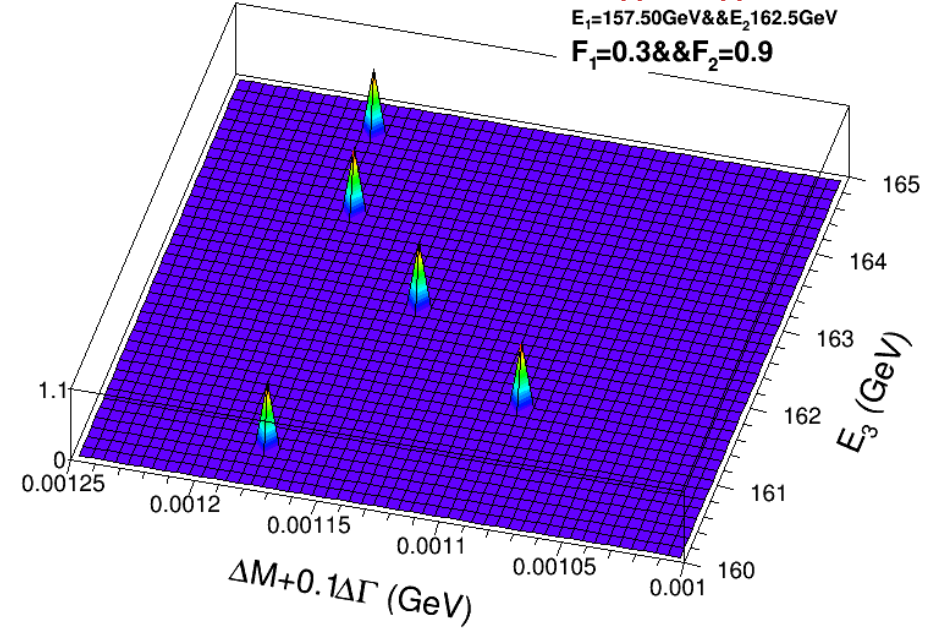
The $F_2 = 0.9$ is used in further study

Step B: E_3

Direct fit results



Fit the m_W, Γ_W of each fit results



The minimal result favors $E_3 \sim 161.5$ GeV

Step C

1. Use the rough results from step A, the requirements below are used:

$$E_1 = 157.5, E_2 = 162.5, E_3 = 161.5, F_1 = 0.3, F_2 = 0.9$$

$$\sigma_{sys}^{corr} = 2 \times 10^{-4}, \Delta E = 0.5 \text{ MeV}, E_{BS} = 1.6 \times 10^{-3}, \Delta E_{BS} = 0.01$$

2. $\sigma_{WW} \sim G(\sigma_{WW}^0, \sigma_{sys}^{corr})$, $E \sim G(E_p^0, \Delta E) + G(E_m^0, \Delta E)$, the description about E_{BS} can be found in backup.

3. By 500 samplings, we fit the distributions of m_W, Γ_W , and the corresponding uncertainties are : $\Delta m_W \sim 1 \text{ MeV}$, $\Delta \Gamma_W \sim 2.8 \text{ MeV}$

Summary and next to do

1. With the configurations :

$$L = 3.2 \text{ ab}^{-1}, \epsilon P = 0.72, \sigma_{sys}^{corr} = 2 \times 10^{-4}$$

$$\Delta E = 0.5 \text{ MeV}, E_{BS} = 1.6 \times 10^{-3}, \Delta E_{BS} = 0.01$$

If we taking data at:

a. One points: $\Delta m_W \sim 0.9 \text{ MeV}$ at 162.5 GeV

b. Two points: $\Delta m_W \sim 1.0 \text{ MeV}$, $\Delta \Gamma_W \sim 2.9 \text{ MeV}$ ($E_1 = 157.5$, $E_2 = 162.5 \text{ GeV}$, $F_1 = 0.3$)

c. Three points: $\Delta m_W \sim 1.0 \text{ MeV}$, $\Delta \Gamma_W \sim 2.8 \text{ MeV}$ ($E_1 = 157.5$, $E_2 = 161.5$, $E_3 = 162.5 \text{ GeV}$, $F_1 = 0.3$, $F_2 = 0.1$)

2. The more precise scan will be performed with the preliminary results.

Backup

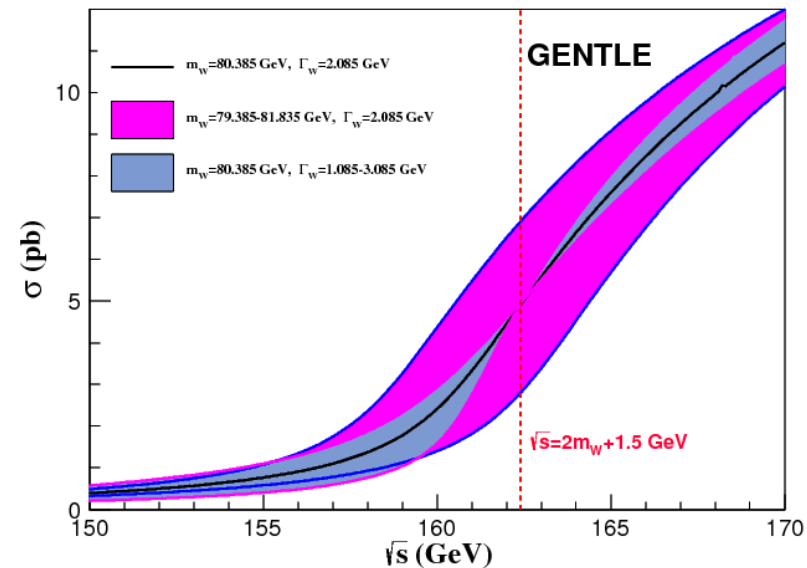
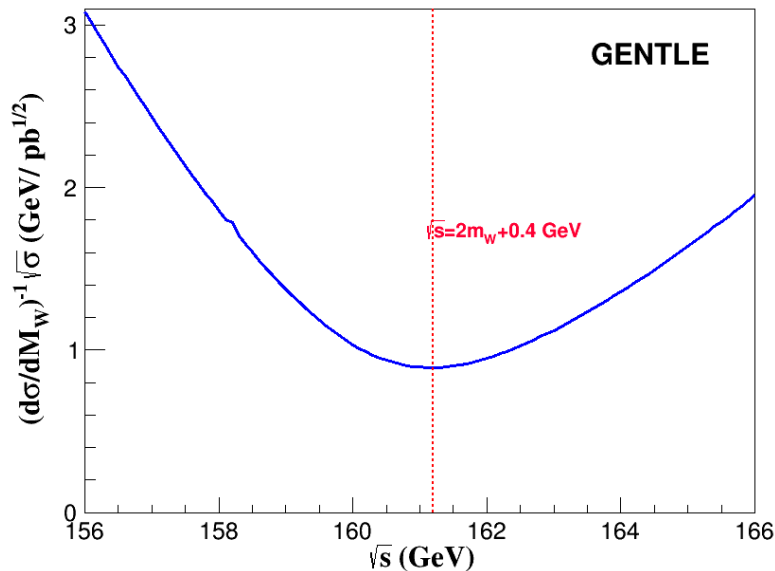
Taking data at one point (just for m_W)

There are two special energy points for just measuring m_W :

1. The one where most statistical sensitivity to m_W :

$$\Delta m_W(\text{stat.}) = \left(\frac{d\sigma_{WW}}{dm_W} \right)^{-1} \frac{\sqrt{\sigma_{WW}}}{\sqrt{L\epsilon P}} \approx 0.59 \text{ MeV at } E=161.2 \text{ GeV (with } \Delta\Gamma_W \text{ effect)}$$

2. The one where $\frac{\partial\sigma_{WW}}{\partial\Gamma_W} = 0$ at $E \approx 162.5$ GeV (Δm_W 0.68 MeV, but no $\Delta\Gamma_W$ effect)



Systematic uncertainty for data taking at one point

$$N_{tot} = L \cdot \sigma_{WW}(E) \cdot \frac{\epsilon}{P}$$

$$\Delta m_W(\sigma_{WW}) = \frac{\partial m_W}{\partial \sigma_{WW}} \Delta \sigma_{WW}$$

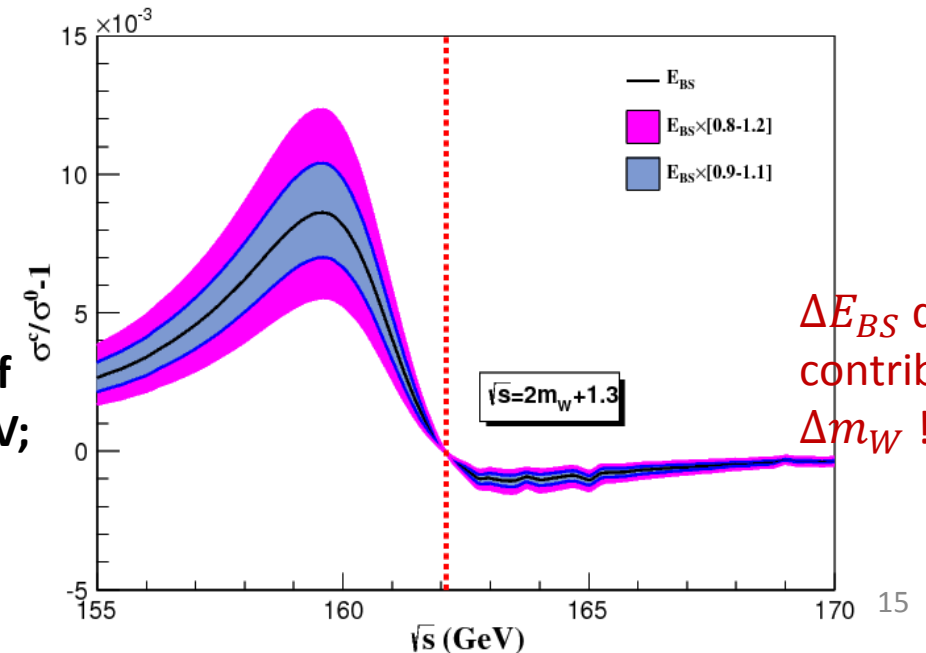
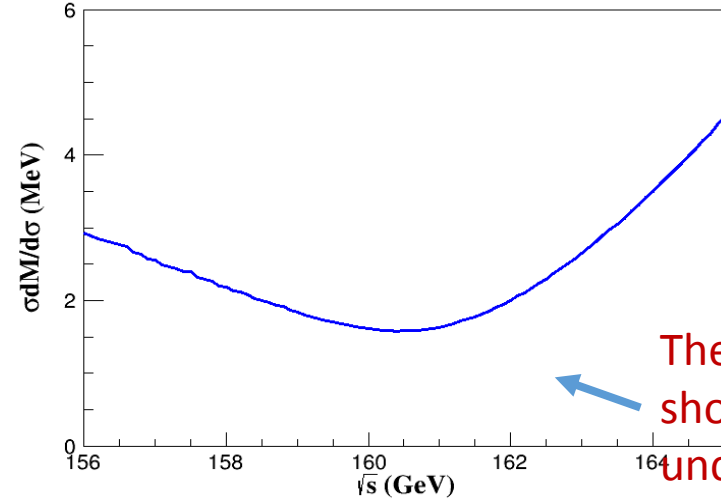
$$\Delta m_W(\Gamma_W) = \frac{\partial m_W}{\partial \sigma_{WW}} \frac{\partial \sigma_{WW}}{\partial \Gamma_W} \Delta \Gamma_W \dots\dots$$

$$\sigma^{sys}(corr.) = \sqrt{\Delta L^2 + \Delta \sigma_{WW}^2 + \Delta \epsilon^2 + \Delta P^2}$$

With ΔL ($\Delta \sigma_{WW}$, $\Delta \epsilon$, ΔP) $< 10^{-4}$, $\sigma^{sys}(corr.) < 2 \times 10^{-4}$:

| | $E = 161.2 \text{ GeV}$ | $E = 162.5 \text{ GeV}$ |
|----------------------------|-------------------------|-------------------------|
| $\sigma^{sys}(corr.)$ | 0.35 | 0.44 |
| ΔE (0.5 MeV) | 0.36 | 0.37 |
| $\Delta E_{BS}(1\%)$ | 0.12 | - |
| $\Delta \Gamma_W$ (42 MeV) | 8 | - |

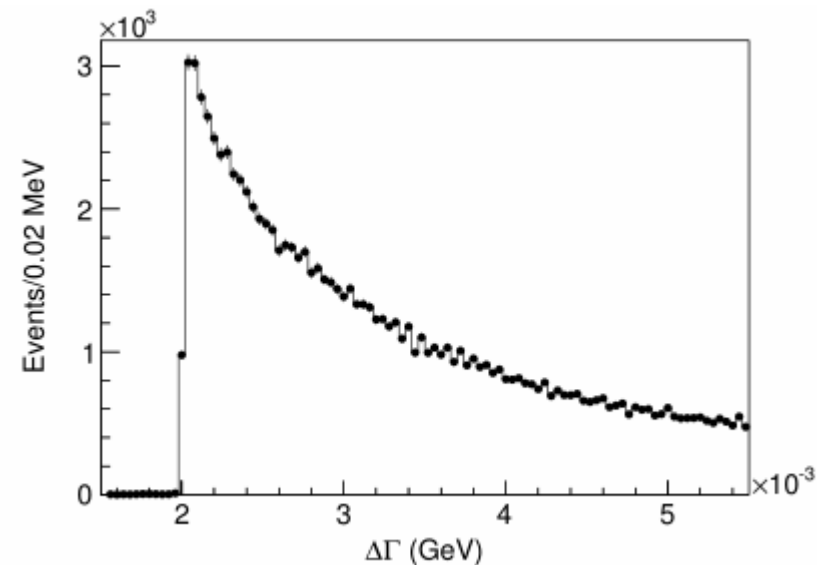
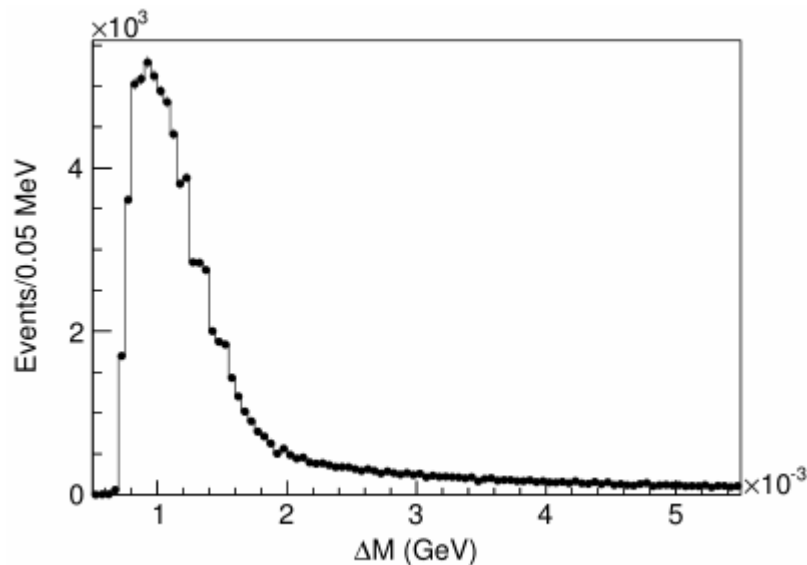
$\Delta m_W(tot) \sim 0.9 \text{ MeV}$, if taking data at 162.5 GeV;



Taking data at two energy points

To measure both Δm_W and $\Delta \Gamma_W$, we scan the energies and the luminosity fraction of the two data points:

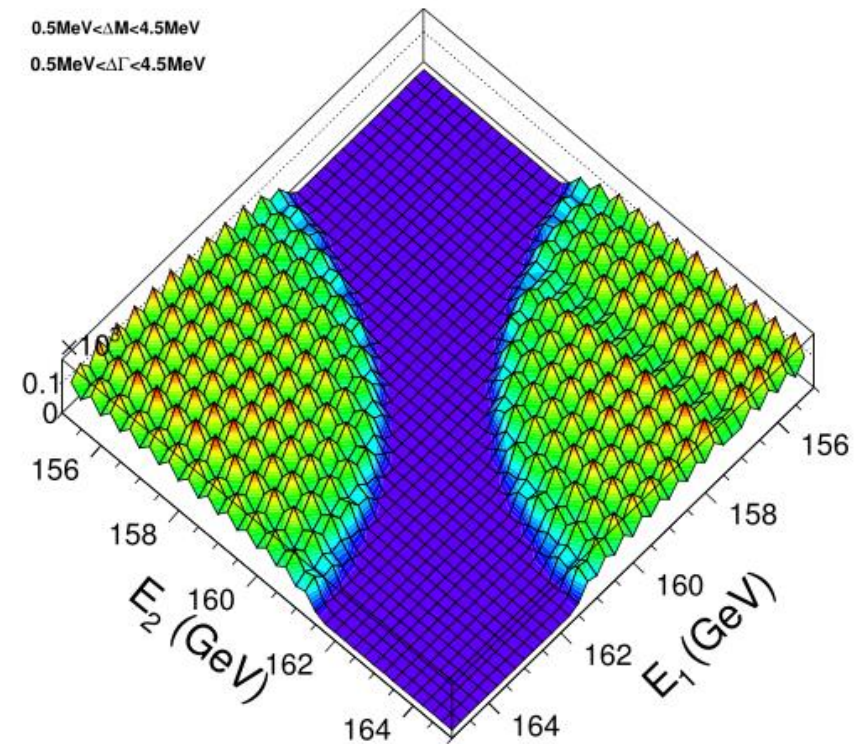
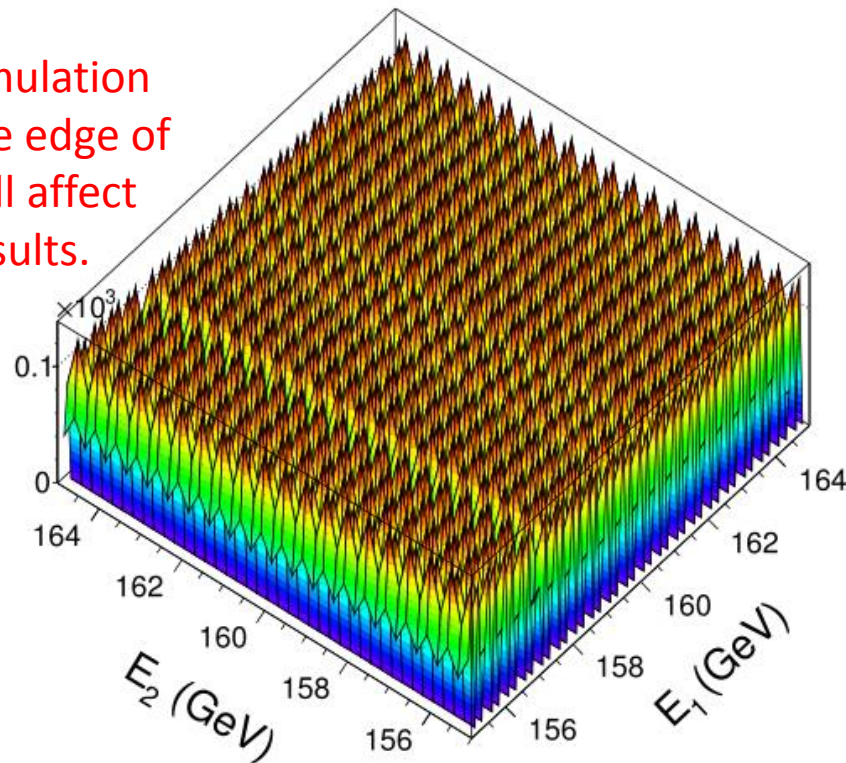
1. $E_1, E_2 \in [155, 165] \text{ GeV}, \Delta E = 0.1 \text{ GeV}$
2. $F \left(\frac{L_1}{L_2} \right) \in (0, 1), \Delta F = 0.05$



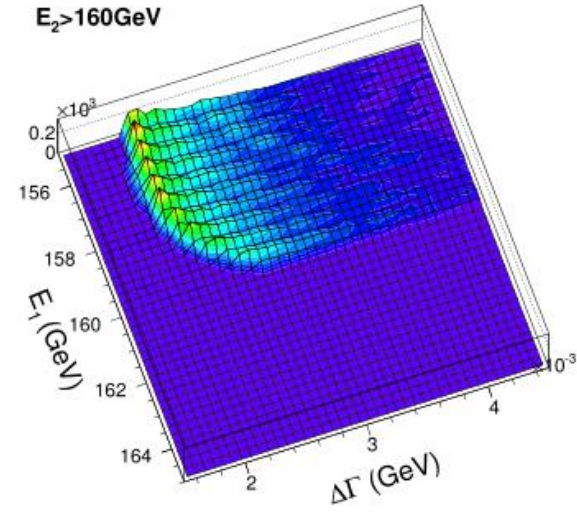
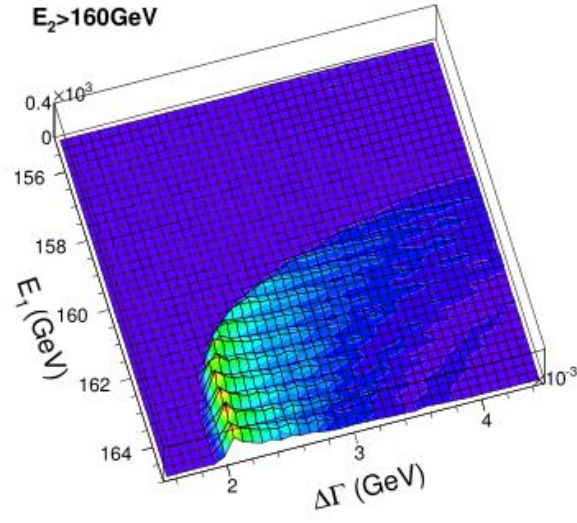
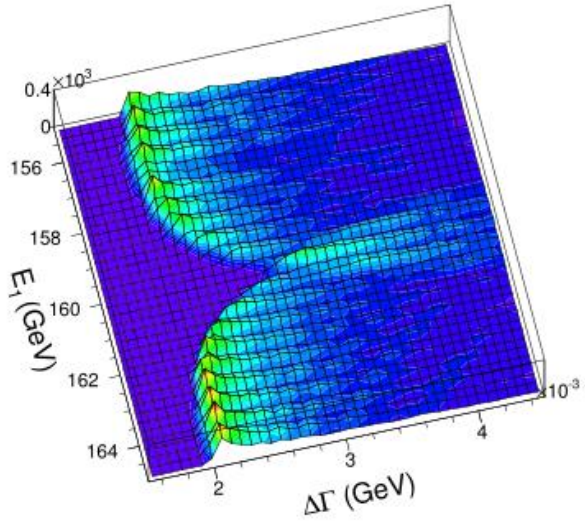
$$E_1, E_2$$

For further study, the two requirements are preformed: $\Delta m_W(\Delta\Gamma_W) \in (0.5, 4.5)\text{MeV}$, the scatter plot of E_1, E_2 is divided into two parts corresponding.

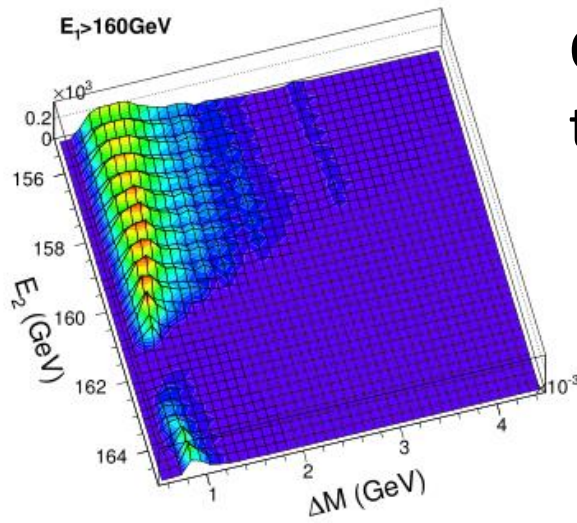
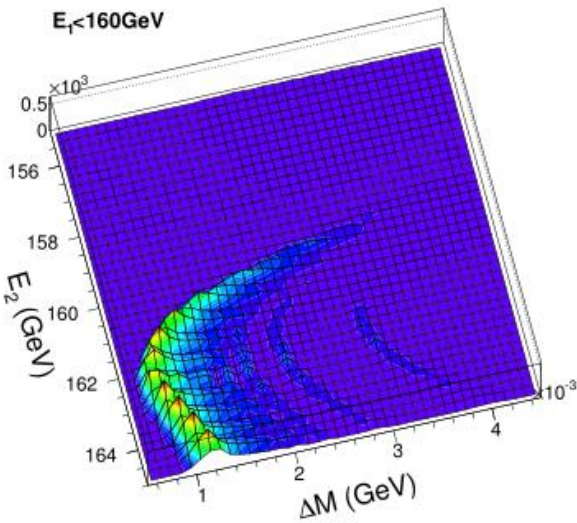
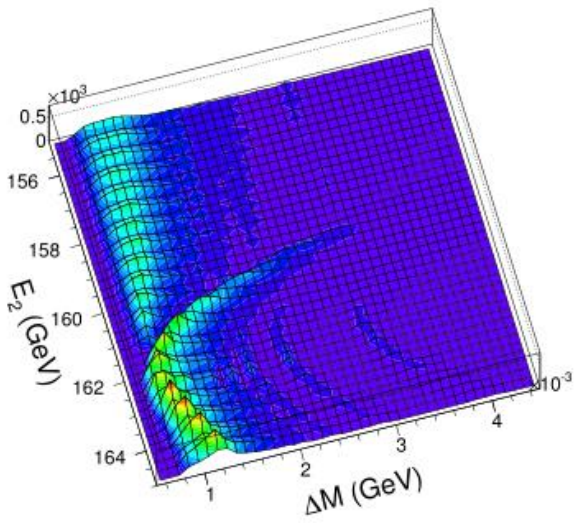
The z axis is the cumulation of the fit result. The edge of the distributions will affect the optimization results.



$\Delta m_W, \Delta \Gamma_W$ vs E_1, E_2



Both the energies of the two data points will affect $\Delta \Gamma_W$



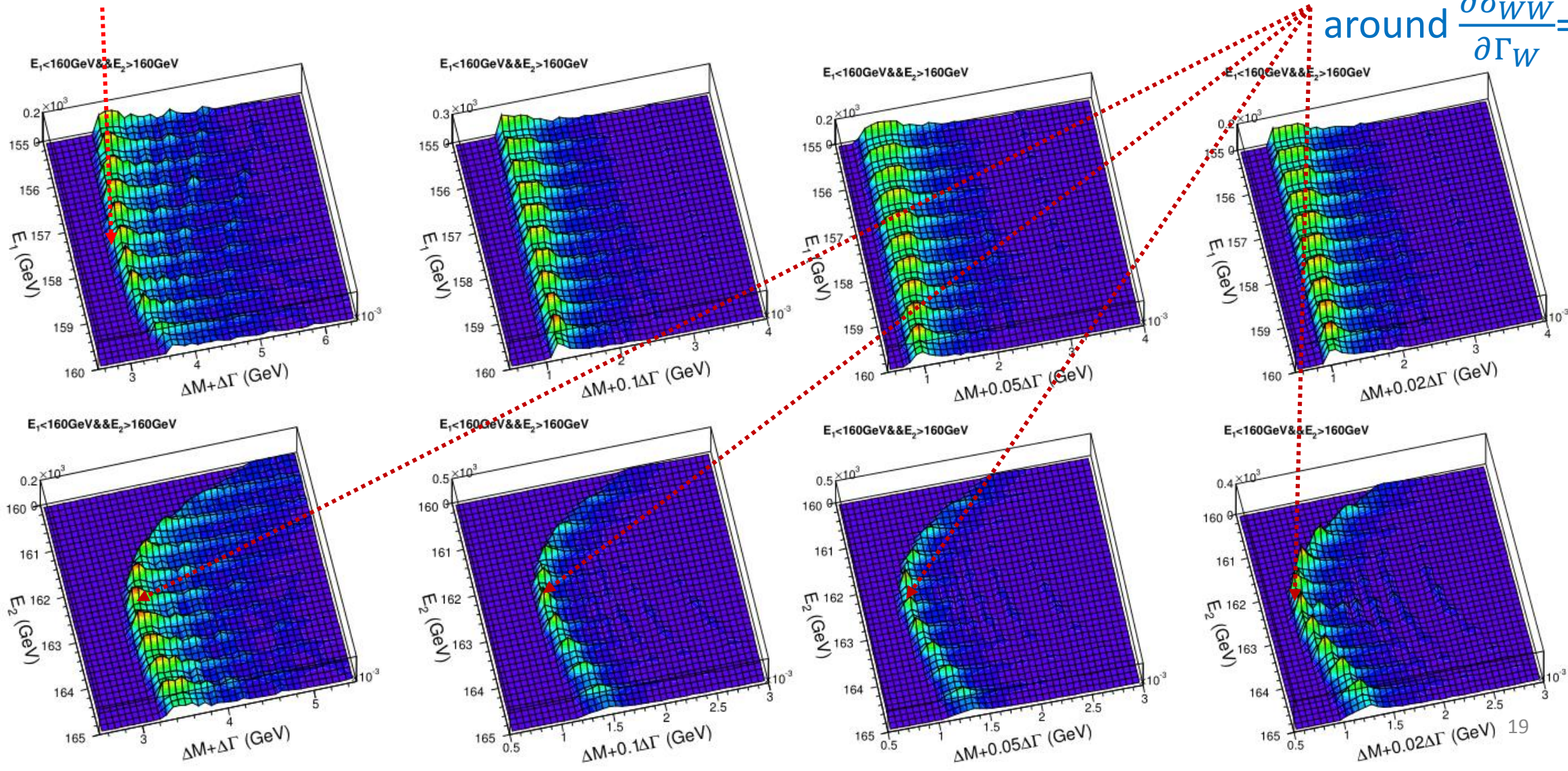
Only the one above threshold affect Δm_W

$E_1 \sim 157.5$ GeV

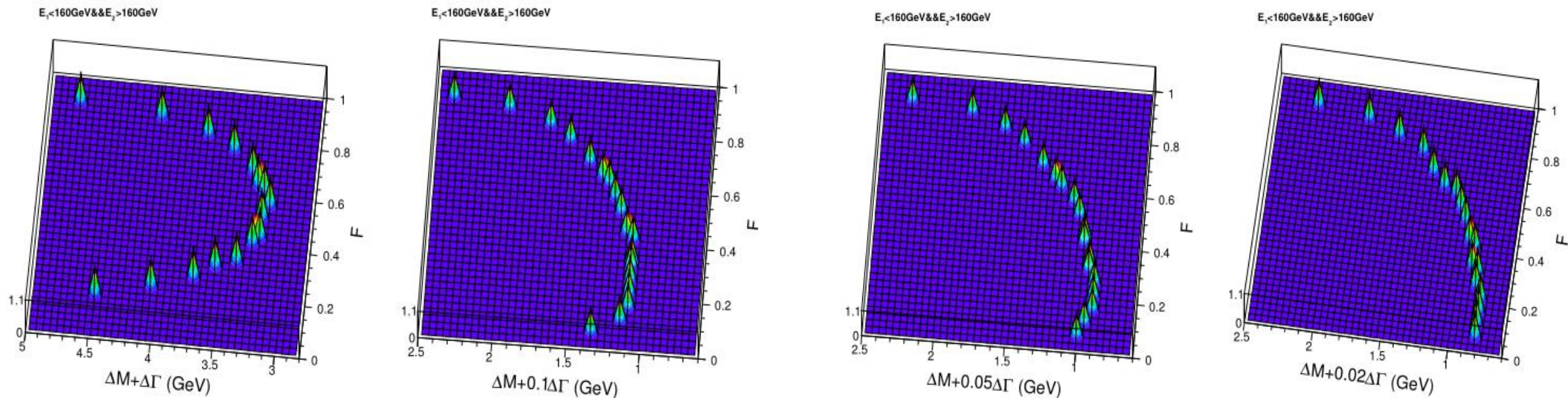
$(\Delta m_W + A \cdot \Delta \Gamma_W)$ vs E_1

$E_2 \sim 162.5$ GeV,

around $\frac{\partial \sigma_{WW}}{\partial \Gamma_W} = 0$



$$(\Delta m_W + A \cdot \Delta \Gamma_W) \text{ vs } F$$



Systematic uncertainty for data taking at two point

With : $E_1=157.5\text{GeV}$, $E_2=162.5\text{ GeV}$, $\sigma^{sys}(\text{corr.}) = 2 \times 10^{-4}(\text{relative})$
 $\Delta E_{BS}=1.6 \times 10^{-3}(\text{relative})$, $\Delta E=0.5\text{ MeV}$

Just the quadratic sum
without the ΔE_{BS}

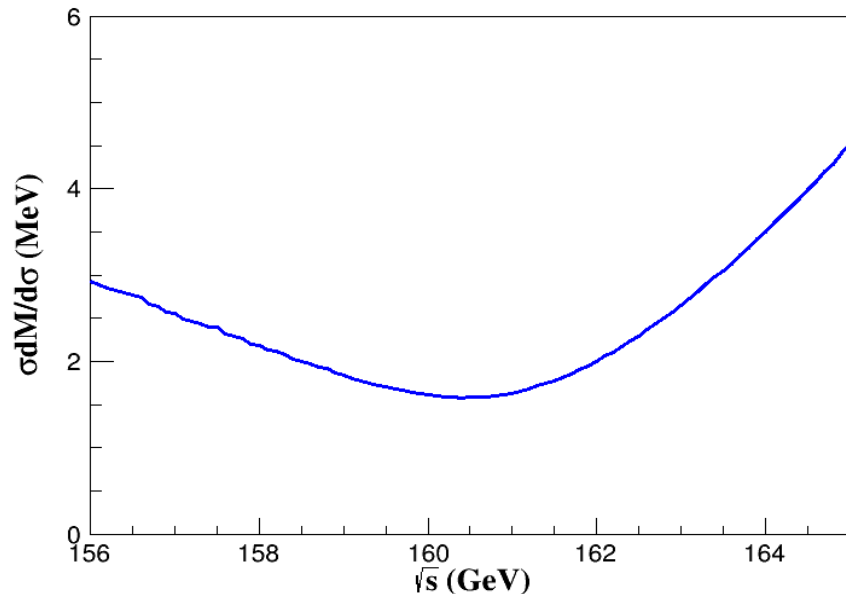
| F | Δm_W (MeV) | | | | | | $\Delta \Gamma_W$ (MeV) | | | | | |
|------|--------------------|------------------------|------------|-----------------|----------------------|-------|-------------------------|------------------------|------------|-----------------|----------------------|-------|
| | Stat. | Sys. | | | | Total | Stat. | Sys. | | | | Total |
| | | $\sigma(\text{corr.})$ | ΔE | ΔE_{BS} | σ_{tot}^{sys} | | | $\sigma(\text{corr.})$ | ΔE | ΔE_{BS} | σ_{tot}^{sys} | |
| 0.1 | 0.71 | 0.47 | 0.35 | - | | 0.92 | 4.6 | 0.31 | 0.52 | 0.43 | 0.74 | 4.7 |
| 0.15 | 0.73 | 0.47 | 0.37 | - | | 0.94 | 3.7 | 0.28 | 0.52 | 0.55 | 0.8 | 3.8 |
| 0.2 | 0.76 | 0.45 | 0.37 | - | | 0.96 | 3.3 | 0.26 | 0.52 | 0.60 | 0.84 | 3.4 |
| 0.25 | 0.78 | 0.46 | 0.37 | - | | 0.98 | 3.0 | 0.23 | 0.51 | 0.76 | 0.94 | 3.1 |
| 0.3 | 0.81 | 0.48 | 0.38 | - | | 1.02 | 2.7 | 0.22 | 0.54 | 0.88 | 1.06 | 2.9 |

$$\sigma^{sys}(\text{corr.}) (\sqrt{\Delta L^2 + \Delta\sigma_{WW}^2 + \Delta\epsilon^2 + \Delta P^2})$$

Considering the $\sigma^{sys}(\text{corr.})$, the σ_{WW} becomes: $\sigma_{WW} \sim G(\sigma_{WW}^0, \sigma^{sys}(\text{corr.}))$

We simulate data with σ_{WW} , and use σ_{WW}^0 in fit.

$\sigma^{sys}(\text{corr.}) = 2 \times 10^{-4}$ (relative). By 500 samplings, the results are shown below (the uncertainty of each value is $1.5 - 2.0 \times 10^{-5}$)



| F | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 |
|-------------------------|------|------|------|------|------|
| Δm_W (MeV) | 0.47 | 0.47 | 0.45 | 0.46 | 0.48 |
| $\Delta \Gamma_W$ (MeV) | 0.31 | 0.28 | 0.26 | 0.23 | 0.22 |

$$\Delta E$$

With the ΔE , the total energy becomes:

$$E = G(E_p, \Delta E) + G(E_m + \Delta E)$$

With $\Delta E=0.5$ MeV and 500 samplings:

| F | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 |
|-------------------------------|------|------|------|------|------|
| $\Delta m_W(\text{MeV})$ | 0.35 | 0.37 | 0.37 | 0.37 | 0.38 |
| $\Delta \Gamma_W(\text{MeV})$ | 0.52 | 0.52 | 0.52 | 0.51 | 0.54 |

Uncertainty of each value is $0.6 - 1 \times 10^{-5}$

$$\Delta E_{BS}$$

With the ΔE_{BS} , the σ_{WW} becomes:

$$\begin{aligned}\sigma_{WW}(E) &= \int_0^{\infty} \sigma(E') \times G(E, E') dE' \\ &= \int_{E-6\sqrt{2}E_{BS}}^{E+6\sqrt{2}E_{BS}} \sigma(E') \times \frac{1}{\sqrt{2\pi}\sqrt{2}E_{BS}} e^{\frac{-(E-E')^2}{2(\sqrt{2\pi}E_{BS})^2}} dE'\end{aligned}$$

For simulation $E_{BS} = E_{BS}^0 + \Delta E_{BS}$, and E_{BS}^0 for fit.