Parton Energy Loss in Generalized High-twist Approach

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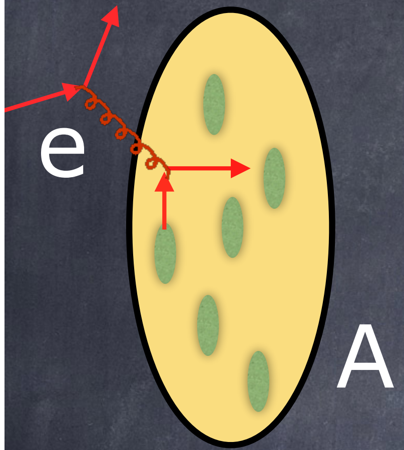
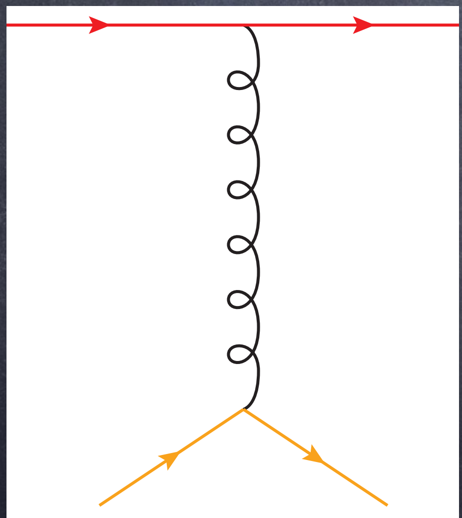
Outline

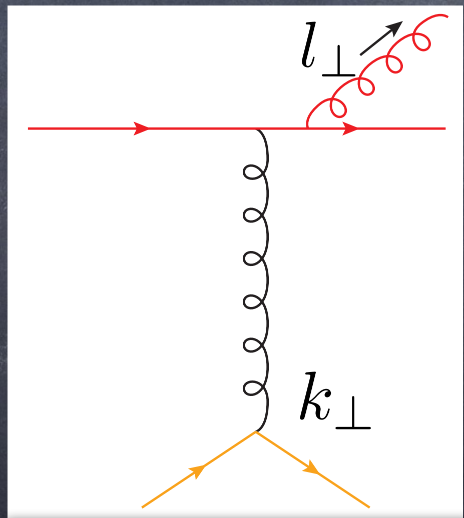
- Introduction
- Generalized High Twist approach
- Results and approximations
- Summary



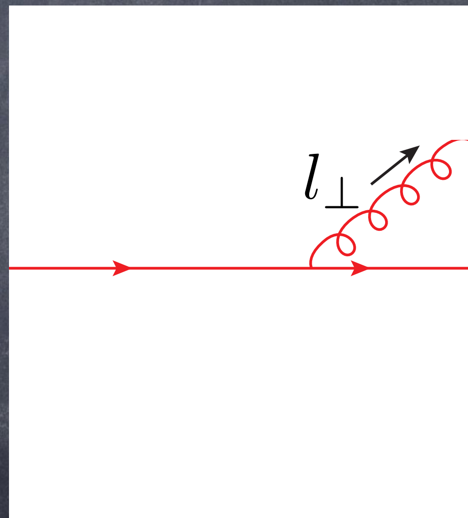
- High energy partons travel through hot QGP or cold nuclei (eA DIS process)
- Energy loss mechanisms reveal medium properties

Parton energy loss mechanisms





medium induced



vacuum

collisional energy loss

radiative energy loss

Radiative energy loss:assumptions

Approaches to radiative energy loss : BDMPS-Z, GLV, AMY, SCET, High Twist

- Scattering Center : Static or Dynamic?

  Static: no energy transfer (BDMPS-Z, GLV)

  Extension of GLV to dynamic S.C. Djordjevic,Heinz PRL 101,022302

  Dynamic: both momentum and energy transfer ✓

- Radiated Gluon : Soft or hard ?

  z → 0 (BDMPS-Z, GLV, SCET)

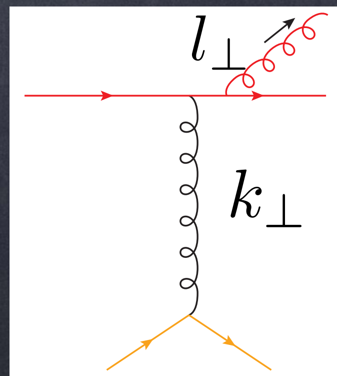
  Discussion on soft appr. of GLV Blagojevic *et al*. arXiv:1804.07593

  z finite ✓

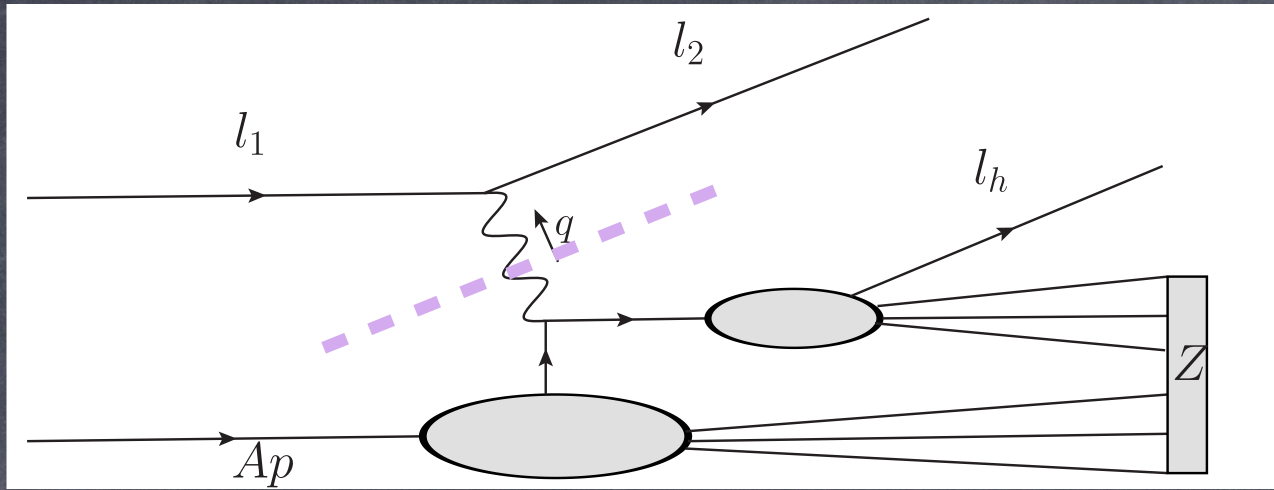
- Transverse momentum transfer : smaller or same order $l_\perp$?

  $k_\perp \ll l_\perp$ (High Twist)

  $k_\perp \sim l_\perp$ ✓



Semi-inclusive Deeply Inelastic Scattering



- lepton-nucleus scattering

$$e(l_1) + A(Ap) \to e(l_2) + h(l_h) + Z$$

identify one hadron in final state

- cross section $d\sigma$ and hadronic tensor $W^{\mu\nu}$

$$d\sigma = \frac{e^4}{2s}\frac{\sum_q e_q^2}{q^4}\int \frac{d^4 l_2}{(2\pi)^4} 2\pi\delta(l_2^2)\frac{1}{2}L_{\mu\nu}W^{\mu\nu}$$

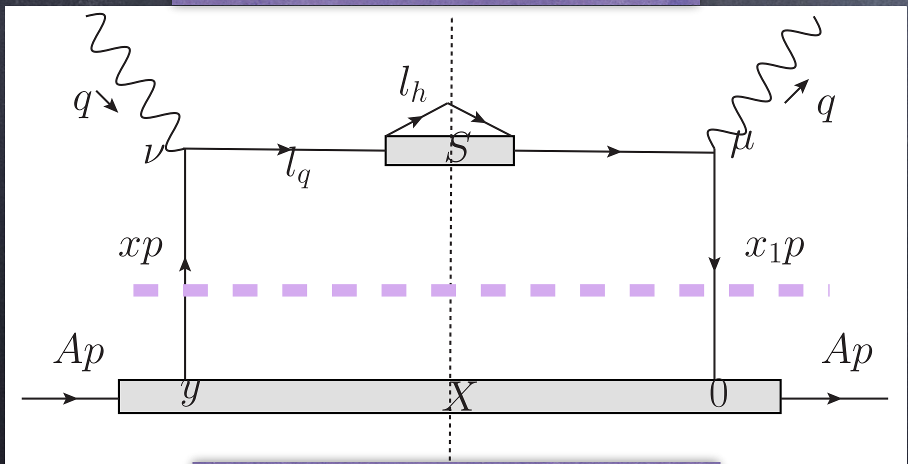
- Factorization

Separate non-perturbative part (pdf, fragmentation function) from perturbative part (hard scattering)

- SIDIS process

Collinear factorization when final hadron $l_{h\perp}$ integrated

hadronic tensor



handbag diagram

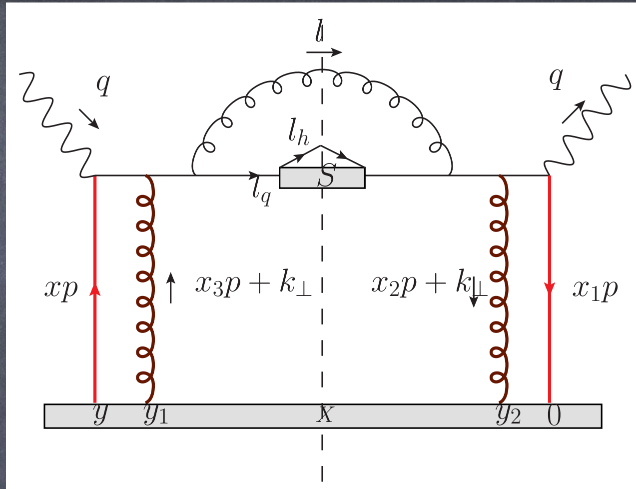
$$\frac{dW^{\mu\nu}_{S(0)}}{dz_h} = \int dx f_q^A(x) H^{\mu\nu}_{(0)} D_{q\to h}(z_h)$$

$f_q^A(x)$ quark distribution function

$D_{q\to h}(z_h)$ quark fragmentation function

Factorization of Medium Induced Radiation

- High Twist approach



Collinear factorization

XF Guo, XN Wang (2000) PRL *85*(17), 3591
XN Wang and *XF Guo (2001)* Nucl. Phys. A 696, 788

$$\frac{dW^{\mu\nu}_{D(1)q}}{dz_h} = \int_{z_h}^{1} \frac{dz}{z} D_{q\to h}(z_h/z) \int \frac{dy^-}{2\pi} dy_1^- dy_2^- \frac{d^2 y_{1\perp}}{(2\pi)^2} d^2 y_{2\perp} d^2 k_\perp e^{-i\vec{k}_\perp\cdot(\vec{y}_{1\perp}-\vec{y}_{2\perp})}$$
$$\frac{1}{2} < A|\bar{\psi}_q(0)\gamma^+ A^+(y_2^-,\vec{y}_{2\perp}) A^+(y_1^-,\vec{y}_{1\perp})\psi_q(y^-)|A > H^{\mu\nu}_D(k_\perp, y^-, y_1^-, y_2^-, p, q, z)$$

Collinear expansion of hard part

$$H^{\mu\nu}_D(k_\perp, y^-, y_1^-, y_2^-, p, q, z) = H^{\mu\nu}_D(k_\perp = 0) + \left.\frac{\partial H^{\mu\nu}_D}{\partial k^\alpha_\perp}\right|_{k_\perp=0} k^\alpha_\perp$$
$$+ \frac{1}{2}\left.\frac{\partial^2 H^{\mu\nu}_D}{\partial k^\alpha_\perp k^\beta_\perp}\right|_{k_\perp=0} k^\alpha_\perp k^\beta_\perp + \cdots$$

$k_\perp \ll l_\perp$approximation

$k_\perp = 0$ contribute to gauge link of initial quark PDF
$k^\alpha_\perp$ contribute zero for unpolarized beam

Factorization of Medium Induced Radiation

- Generalized High Twist approach

relax $k_\perp \ll l_\perp$ without collinear expansion

YY Z, GY Qin, XN Wang (to be published)

factorize quark PDF and gluon PDF directly

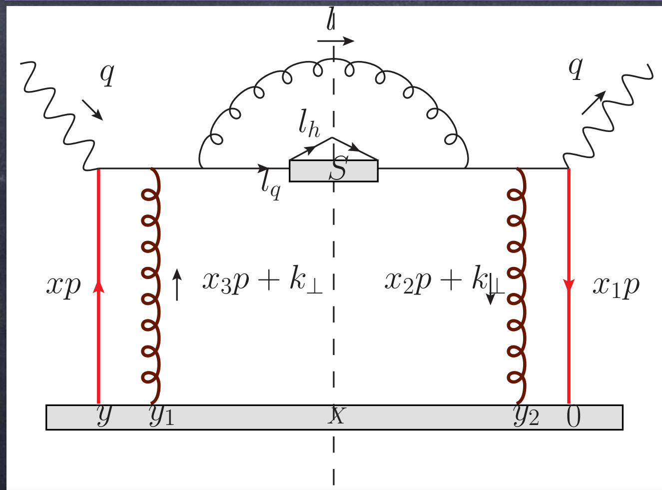
splitting function | nucleon density

$$\frac{dW^{\mu\nu}_{D(1)q}}{dz_h} \sim \frac{\alpha_s}{2\pi} C_F \frac{2\pi\alpha_s}{N_c} \frac{1+z^2}{1-z} H^{\mu\nu}_{(0)}(x) \otimes \rho(y_1^-, \vec{y}_{1\perp}) \otimes \frac{D_{q\to h}(z_h/z)}{z}$$

$$\otimes f_q^A(x) \frac{\pi}{[\vec{l}_\perp - (1-z)\vec{k}_\perp]^2} \frac{\phi(x_L + x_D, \vec{k}_\perp)}{k_\perp^2}$$

TMD gluon pdf | quark pdf

One example diagram

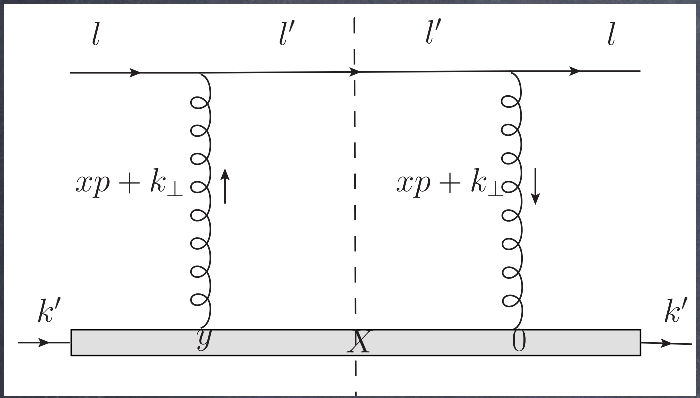


TMD gluon distribution function

- TMD gluon distribution function emerge from TMD transport parameter $\hat{q}(\vec{k}_\perp)$

$$\hat{q} = \rho \int dk_\perp^2 \frac{\langle d\sigma \rangle}{dk_\perp^2} k_\perp^2$$

color source density*average kT broadening per scattering



$$\hat{q} \equiv \int \frac{d^2 k_\perp}{(2\pi)^2} \int dx \delta(x - \frac{k_\perp^2}{2p^+ l^-}) \frac{4\pi\alpha_s C_2(R)}{N_c^2 - 1} \rho(y) \phi(x, \vec{k}_\perp)$$
$$\equiv \int \frac{d^2 k_\perp}{(2\pi)^2} \hat{q}(\vec{k}_\perp)$$

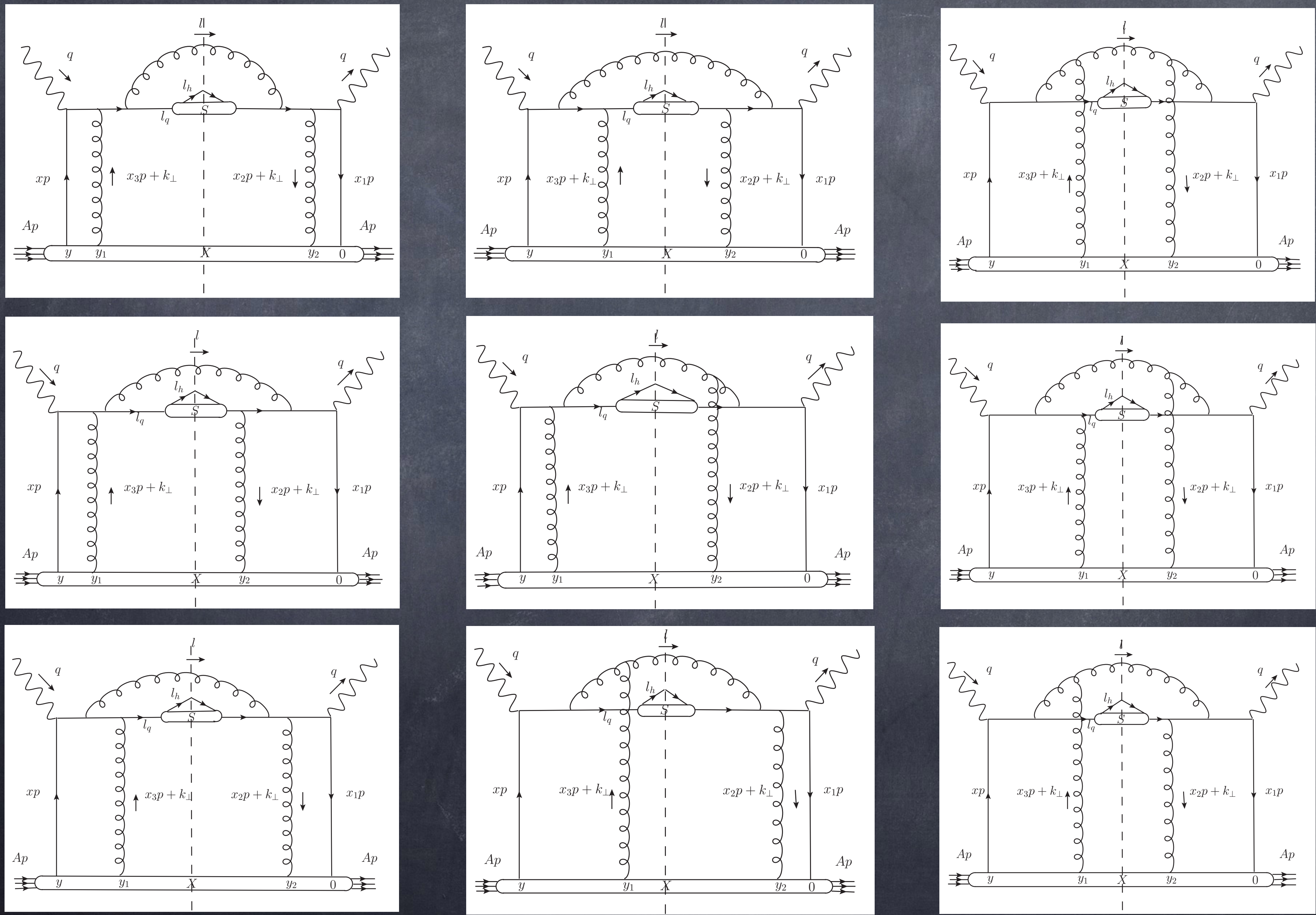
$\phi(x, \vec{k}_\perp)$ emerge naturally

$$\phi(x, \vec{k}_\perp) \equiv \int \frac{dy^-}{2\pi p^+} \int d^2 y_\perp e^{ixp^+ y^- - i\vec{k}_\perp \cdot \vec{y}_\perp} \langle p | F_\alpha{}^{+}(0) F^{+\alpha}(y^-, y_\perp) | p \rangle$$

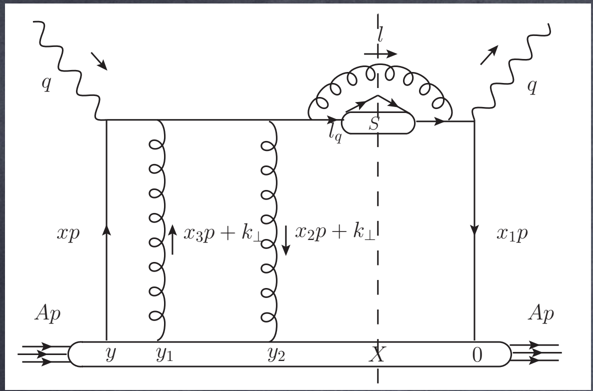
$\hat{q}(\vec{k}_\perp)$ $\phi(x, \vec{k}_\perp)$ depend on the parton energy l^- and medium color source energy p^+ via x

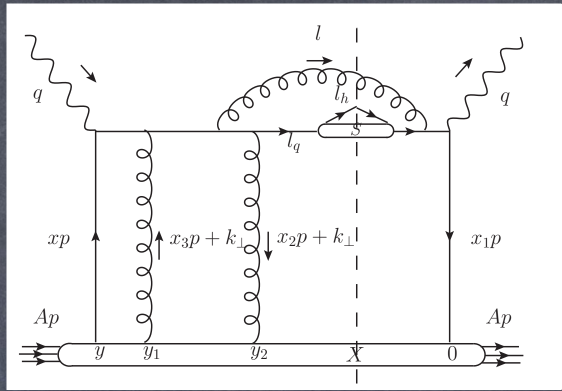
energy transfer via x

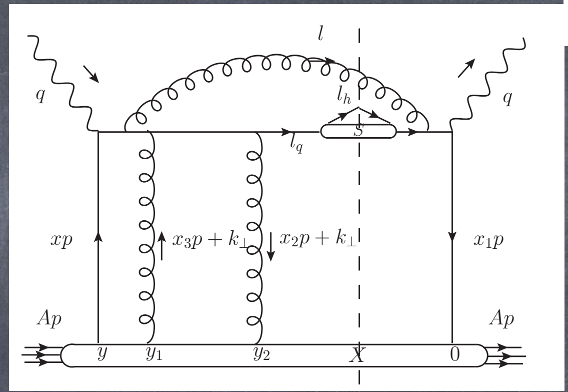
summing up all the diagrams

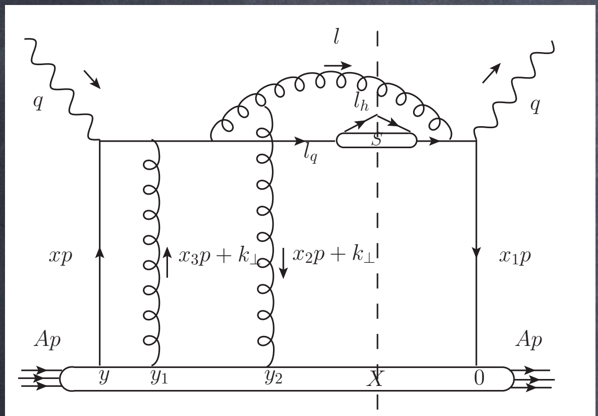


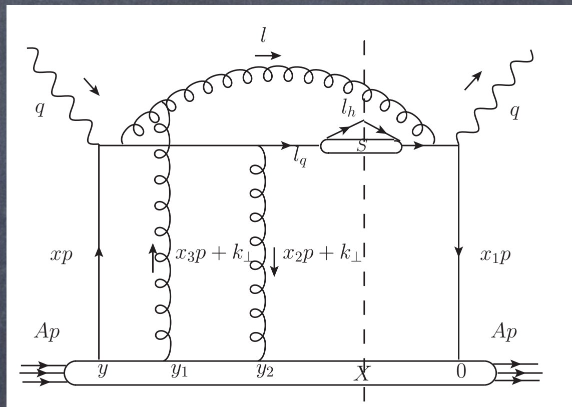
Gluon Spectrum Result

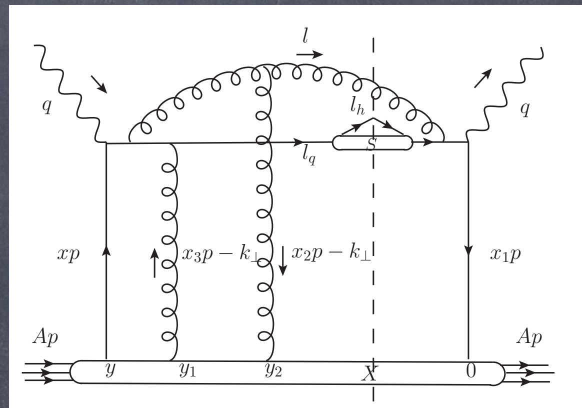


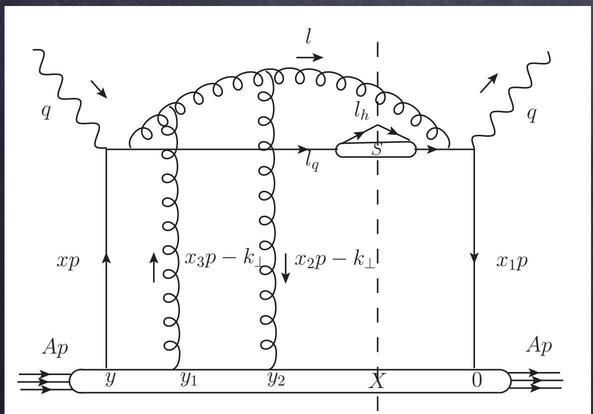












- vacuum + medium induced radiation interference
- left cut diagrams symmetric to right cut diagrams

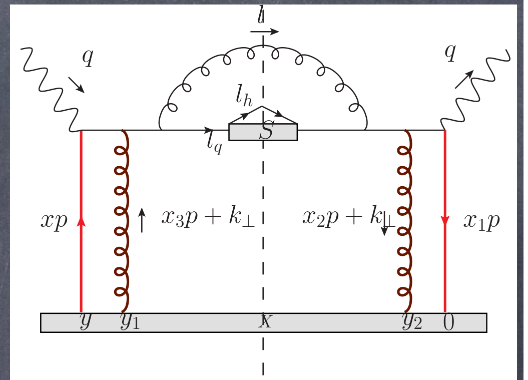
Gluon Spectrum Result: Full Result

We get gluon spectrum from the hadronic tensor

$$\frac{dW^{\mu\nu}_{D(1)g}}{dz_h} = \int dx H^{\mu\nu}_{(0)}(x) \int \frac{dz}{z} D_{g\to h}(z_h/z) \int dl_\perp^2 T_{qg}(x,z,l_\perp^2)$$

Quark-gluon correlation function $T_{qg}(x,z,l_\perp^2)$

nuclear enhancement



$$T_{qg} = \pi \frac{\alpha_s}{2\pi} \frac{1+(1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2k_\perp}{(2\pi)^2} \int dy_1^- \int d^2\vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp})$$
$$\left[H_C^D \theta(y^- - y_1^-)\theta(-y_2^-) + H_L^D \theta(y_1^- - y_2^-)\theta(y^- - y_1^-) + H_R^D \theta(y_2^- - y_1^-)\theta(-y_2^-)\right]$$

Gluon spectrum : probalibility of quark to radiate one gluon with longitudinal momentum zq^- transverse momentum $\vec{l}_\perp$

$$\frac{dN}{dl_\perp^2 dz} \sim \frac{T_{qg}}{f_q^A(x)}$$

Gluon Spectrum Result: Full Result

$$H_C^D = \Bigg\{ \Bigg[\frac{C_A}{(l_\perp - k_\perp)^2} \frac{\phi(-\frac{1-z}{z}x_T, \vec{k}_\perp)}{k_\perp^2} f_q^A(x + x_L + \frac{x_T}{z}) - \frac{C_A}{l_\perp^2} \frac{\phi(\frac{1-z}{z}x_T, -\vec{k}_\perp)}{k_\perp^2} f_q^A(x + x_L) \Bigg]$$

$$+ \Bigg[\Bigg(\frac{C_F}{l_\perp^2} + C_A \frac{\vec{k}_\perp \cdot \vec{l}_\perp}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_F}{[\vec{l}_\perp - z\vec{k}_\perp]^2} + \frac{1}{N} \frac{\vec{l}_\perp \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{l_\perp^2 [\vec{l}_\perp - z\vec{k}_\perp]^2}$$

$$- C_A \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \Bigg) \frac{\phi(x_L + x_T, \vec{k}_\perp)}{k_\perp^2} f_q^A(x) \Bigg]$$

$$\Bigg[\Bigg(-\frac{C_A}{(\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{\vec{l}_\perp \cdot (\vec{l}_\perp - \vec{k}_\perp)}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \Bigg)$$

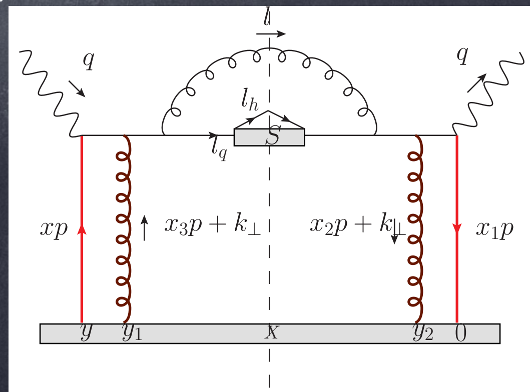
$$\frac{\phi(x_L + x_T, \vec{k}_\perp)}{k_\perp^2} e^{-i(x_L + \frac{x_T}{z})p^+ y_1^-} f_q^A(x + x_L + \frac{x_T}{z}) \Bigg]$$

$$\Bigg[\Bigg(-\frac{C_A}{(\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{\vec{l}_\perp \cdot (\vec{l}_\perp - \vec{k}_\perp)}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} + \frac{C_A}{2} \frac{(\vec{l}_\perp - \vec{k}_\perp) \cdot [\vec{l}_\perp - z\vec{k}_\perp]}{(l_\perp - k_\perp)^2 [\vec{l}_\perp - z\vec{k}_\perp]^2} \Bigg)$$

$$\frac{\phi(-\frac{1-z}{z}x_T, \vec{k}_\perp)}{k_\perp^2} e^{i(x_L + \frac{x_T}{z})p^+ y_1^-} f_q^A(x) \Bigg] \Bigg\}$$

x : energy transfer

$$H_L^D = \cdots \qquad H_R^D = \cdots$$



Gluon Spectrum Result: Contact Terms

There are contact terms, which are negligible

$$\frac{dN_{contact}}{dl_\perp^2 dz} = \frac{\pi}{f_q^A(x)} \frac{\alpha_s}{2\pi} \frac{1+(1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2k_\perp}{(2\pi)^2} \int dy_1^- \int d^2\vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp})$$
$$H_{contact} \left[\theta(y^- - y_1^-)\theta(-y_2^-) - \theta(y_1^- - y_2^-)\theta(y^- - y_1^-) - \theta(y_2^- - y_1^-)\theta(-y_2^-)\right]$$

The θ functions constrain the integration region

$$\int dy_1^- dy_2^- \left[\theta(y^- - y_1^-)\theta(-y_2^-) - \theta(y_1^- - y_2^-)\theta(y^- - y_1^-) - \theta(y_2^- - y_1^-)\theta(-y_2^-)\right] f(y_1^-, y_2^-)$$
$$= \int_0^{y^-} dy_1^- \int_0^{y_1^-} dy_2^- f(y_1^-, y_2^-)$$

gives $0 < y_2^- < y_1^- < y^-$, quark and gluon comes from same nucleon, no nuclear enhancement.

1. Static scattering center approximation:

no energy transfer for medium scattering

$$\phi(x_L + \frac{1-z}{z}x_T, \vec{k}_\perp) \approx \phi(x_T, \vec{k}_\perp) \approx \cdots \approx \phi(0, \vec{k}_\perp)$$

no x dependence

$$f_q^A(x_B + x_L + \frac{x_T}{z}) \approx f(x_B + x_L) \approx f_q^A(x_B)$$

$$Q^2 \gg \frac{l_\perp^2}{z(1-z)}, \frac{k_\perp^2}{z(1-z)} \quad \text{or} \quad x_B \gg x_L, \frac{x_T}{z}$$

gluon spectrum reduces to

$$\frac{d\overline{N}}{dl_\perp^2 dz} = \pi \frac{\alpha_s}{2\pi} \frac{1+(1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2k_\perp}{(2\pi)^2} \int dy_1^- \int d^2\vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp}) \left[\overline{H}_C^D + \frac{1}{2}\overline{H}_L^D + \frac{1}{2}\overline{H}_R^D\right] \frac{\phi(0, \vec{k}_\perp)}{k_\perp^2}$$

$$\overline{H}_C^D = \left\{ \left[\frac{C_A}{(l_\perp - k_\perp)^2} - \frac{C_A}{l_\perp^2} \right] + \cdots \right.$$

$$\overline{H}_R^D = \cdots$$

$$\overline{H}_L^D = \cdots$$

Gluon Spectrum Approximations

2. Soft radiated gluon approximation:

the radiated gluon momentum fraction z ⟶ 0
gluon spectrum reduces to

$$\frac{dN_{\rm soft}}{dl_\perp^2\,dz}=\frac{\pi}{f_q^A(x)}\frac{\alpha_s}{2\pi}\frac{1+(1-z)^2}{z}\frac{2\pi\alpha_s}{N_c}\int\frac{d^2k_\perp}{(2\pi)^2}\int dy_1^-\int d^2\vec{y}_{1\perp}\rho(y_1^-,\vec{y}_{1\perp})\left[(H_C^D)_{\rm soft}\theta(y^--y_1^-)\theta(-y_2^-)\right.$$
$$\left.+(H_L^D)_{\rm soft}\theta(y_1^--y_2^-)\theta(y^--y_1^-)+(H_R^D)_{\rm soft}\theta(y_2^--y_1^-)\theta(-y_2^-)\right]$$

no dependence on z except splitting function, f_q^A and ϕ

$$(H_C^D)_{\rm soft}=\left\{\left[\frac{C_A}{(l_\perp-k_\perp)^2}\frac{\phi(-\frac{1-z}{z}x_T,\vec{k}_\perp)}{k_\perp^2}f_q^A(x+x_L+\frac{x_T}{z})-\frac{C_A}{l_\perp^2}\frac{\phi(\frac{1-z}{z}x_T,-\vec{k}_\perp)}{k_\perp^2}f_q^A(x+x_L)\right]+\cdots\right.$$

$$(H_R^D)_{\rm soft}=\cdots\qquad (H_L^D)_{\rm soft}=\cdots$$

Gluon Spectrum Approximations

3. Static scattering + Soft radiated gluon approximation:

$$\begin{cases} \phi(x_L + \frac{1-z}{z}x_T, \vec{k}_\perp) \approx \phi(x_T, \vec{k}_\perp) \approx \cdots \approx \phi(0, \vec{k}_\perp) \\ f_q^A(x_B + x_L + \frac{x_T}{z}) \approx f(x_B + x_L) \approx f_q^A(x_B) \end{cases}$$ and z → 0 give

notice here $Q^2 \gg \frac{l_\perp^2}{z(1-z)}, \frac{k_\perp^2}{z(1-z)}$, or $x_B \gg x_L, \frac{x_T}{z}$ as z → 0

the gluon spectrum is

$$\frac{d\widetilde{N}}{dl_\perp^2\, dz} = \pi \frac{\alpha_s}{2\pi} \frac{1+(1-z)^2}{z} \frac{2\pi\alpha_s}{N_c} \int \frac{d^2k_\perp}{(2\pi)^2} \int dy_1^- \int d^2\vec{y}_{1\perp} \rho(y_1^-, \vec{y}_{1\perp})$$
$$C_A \frac{2\vec{k}_\perp \cdot \vec{l}_\perp}{l_\perp^2 (\vec{l}_\perp - \vec{k}_\perp)^2} \left(1 - cos[(x_L + \frac{x_T}{z})p^+ y_1^-]\right) \frac{\phi(0, \vec{k}_\perp)}{k_\perp^2}$$

$\phi(x, \vec{k}_\perp)$ non-perturbative

3. Static scattering + Soft radiated gluon approximation:

TMD gluon pdf relation to transport parameter $\hat{q}$

$$\hat{q} = \frac{4\pi\alpha_s C_2(R)}{N_c^2 - 1}\rho(y)\int \frac{d^2k_\perp}{(2\pi)^2}\phi(0,\vec{k}_\perp) \quad \bigstar$$

One method : Static potential model to calculate $\hat{q}$

$$< d\sigma >= \frac{C_2(R)C_2(T)}{d_A}\frac{4\pi\alpha_s^2}{t^2}dt$$

$$\hat{q} = \rho\int dk_\perp^2 \frac{C_2(R)C_2(T)}{d_A}\frac{4\pi\alpha_s^2}{k_\perp^2 + \mu_0^2} \quad \bigstar$$

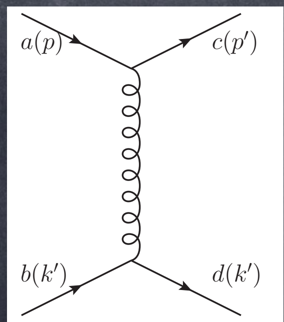
Casimir $C_2(R)$ $C_2(T)$

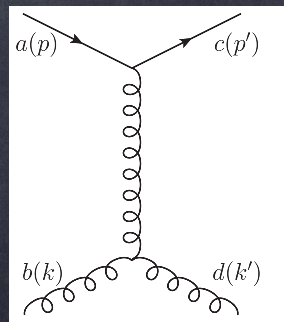
Mandelstam variable t

$$t = k_\perp^2 + \mu_0^2$$

compare two $\hat{q}$

$$\phi(0,\vec{k}_\perp) = C_2(T)\frac{4\alpha_s}{k_\perp^2 + \mu_0^2}$$





Gluon Spectrum Approximations

3. Static scattering + Soft radiated gluon approximation:

substitute $\phi(0,\vec{k}_\perp)$ into gluon spectrum, one get

$$\frac{d\widetilde{N}}{dl_\perp^2\,dz} = 8\pi\alpha_s^3\frac{C_2(T)C_A}{N_c}\frac{1+(1-z)^2}{z}\int\frac{d^2k_\perp}{(2\pi)^2}\int dy_1^-\rho(y_1^-)$$
$$\frac{\vec{k}_\perp\cdot\vec{l}_\perp}{l_\perp^2(\vec{l}_\perp-\vec{k}_\perp)^2}\left(1-cos[(x_L+\frac{x_T}{z})p^+y_1^-]\right)\frac{1}{(k_\perp^2+\mu_0^2)^2}$$

agrees with of GLV formalism at first order in opacity

arguments in Cosine function $\begin{cases} \omega_1 \approx \sqrt{2}(x_L + \frac{1}{z}x_T)p^+ \\ y_{10} = y_1 - y_0 \approx \frac{y_1^-}{\sqrt{2}} \end{cases}$

Summary

- Radiative energy loss in semi-inclusive DIS process

- Generalized High Twist Approach
  - no soft radiated gluon approximation
  - dynamic scattering center (energy transfer)
  - ★ relax $k_\perp \ll l_\perp$ approximation, $k_\perp \sim l_\perp$
  - ★ transverse momentum dependent (TMD) gluon pdf
    TMD transport parameter

- Relation of Generalized High Twist approach result with GLV result

Further work

- gauge link of TMD gluon distribution
- implementation into CoLBT-Hydro Model

Thanks!