Current correlations and dynamics near the QCD crossover

Rishi Sharma (TIFR)

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Collaborators: Sourendu Gupta

A fermionic theory for long distance physics

- Since the fermionic mode is light near the crossover temperature (T_{co}), a fermionic effective theory may be valid in the crossover region
- Only accurate for length scales much longer than $1/T$, and hence can be used to calculate long distance correlation functions
- Observables like free energy dominated by modes near T. Inaccessible in the theory

EFT approach and matching of Euclidean observables

Symmetry constraints

- Time and space distinguished: $SO(3,1) \to SO(3)$. For example, the kinetic term is

$$\overline{\psi}\partial\!\!\!/_4\psi + d_4\overline{\psi}\partial\!\!\!/_i\psi$$

- Similarly, all vector interaction terms can have different spatial and temporal coefficients
- Dimension 6 terms constrained by chiral symmetry

The Euclidean action

- $$\mathcal{L} = d^{(0)} + \overline{\psi}\partial\!\!\!/_4\psi + d_4\overline{\psi}\partial\!\!\!/_i\psi + d_3 T_0\overline{\psi}\psi + \mathcal{L}_6$$

- $$\begin{aligned}\mathcal{L}_6 = &+\frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma^5 t^a\psi)^2\right] + \frac{d^{62}}{T_0^2}\left[(\overline{\psi}t^a\psi)^2 + (\overline{\psi}i\gamma^5\psi)^2\right] \\ &+ \frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2}(\overline{\psi}i\gamma_i\psi)^2 + \frac{d^{65}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 + \frac{d^{66}}{T_0^2}(\overline{\psi}i\gamma_5\gamma_i\psi)^2 \\ &+ \frac{d^{67}}{T_0^2}\left[(\overline{\psi}\gamma_4 t^a\psi)^2 + (\overline{\psi}\gamma_5\gamma_4 t^a\psi)^2\right] + \frac{d^{68}}{T_0^2}\left[(\overline{\psi}\gamma^i t^a\psi)^2 + (\overline{\psi}\gamma^5\gamma^i t^a\psi)^2\right] \\ &+ \frac{d^{69}}{T_0^2}\left[(\overline{\psi}i\Sigma_{i4}\psi)^2 + (\overline{\psi}i\gamma^5\Sigma_{ij}t^a\psi)^2\right] + \frac{d^{60}}{T_0^2}\left[(\overline{\psi}i\Sigma_{i4}t^a\psi)^2 + (\overline{\psi}\Sigma_{ij}\psi)^2\right] \\ &+ \mathcal{O}(\frac{1}{T_0^5}(\overline{\psi}\psi)^3)\ ,\end{aligned}$$

- In the mean field approximation the long distance axial correlations determined by the constants d_4, $m_q = d_3 T_0$

Fluctuations and π's

- Fluctuations in the mean field can be introduced in the Hubbard-Stratonovich transformation as $\psi_\alpha \bar{\psi}_\beta \to [e^{i\pi^a\tau^a\gamma^5/(2f_{Eu})}]_{\beta\beta'} \langle \psi_{\beta'} \bar{\psi}_{\alpha'} \rangle [e^{i\pi^a\tau^a\gamma^5/(2f_{Eu})}]_{\alpha'\alpha}$
- f_{Eu} is the pion constant at finite T and the subscript $_{Eu}$ symbolizes its value in Euclidean space. Sets the breakdown scale of the π theory
- By integrating out the fermions an effective lagrangian for the π's in Euclidean space can be found

Euclidean π lagrangian

- The long wavelength Euclidean π lagrangian is

$$\mathcal{L}_\pi = \frac{1}{2}(\partial_4 \pi)^2 + \frac{u_{Eu}^2}{2}(\nabla \pi)^2 + \frac{M_{Eu}^2}{2}\pi^2 + \cdot\cdot$$

- They can be computed on the lattice *[Brandt, Francis, Meyer, Robaina (2014)]*
- In *Sourendu Gupta and RS [Phys.Rev. D97 (2018), 036025]*, the parameters in the Euclidean fermionic theory (d_3, d_4) fitted to match the lattice results for (u_{Eu}, M_{Eu})
- f_{Eu} is an independent prediction that agrees with lattice

Real time formalism

Real time π lagrangian

- The simplest ansatz for the Euclidean continuation is that the Minkowski space π action is obtained by the change $x^4 \to ix^0$
- The π lagrangian in Minkowski is

$$-\mathcal{L}_\pi = -\frac{1}{2}(\partial_0 \pi)^2 + \frac{u^2}{2}(\nabla \pi)^2 + \frac{M^2}{2}\pi^2 + \frac{c_5}{2f}\partial_0 \pi^a \nabla^2 \pi^a + \cdots$$

- In this ansatz the Euclidean values of u_{Eu}, f_{Eu}, and M_{Eu} simply go over to Minkowski space
- Motivated by *Son, Stephanov (2002)* we have added a dimension 5 term corresponding to damping

Fermionic analytic continuation

- However the fermion lagrangian is more fundamental
- The π theory is the derived theory
- Therefore more natural to use the Minkowski rotation for the fermion action to perform the analytic continuation and then recalculate the π parameters

Technicality about the continuation of the fermion action

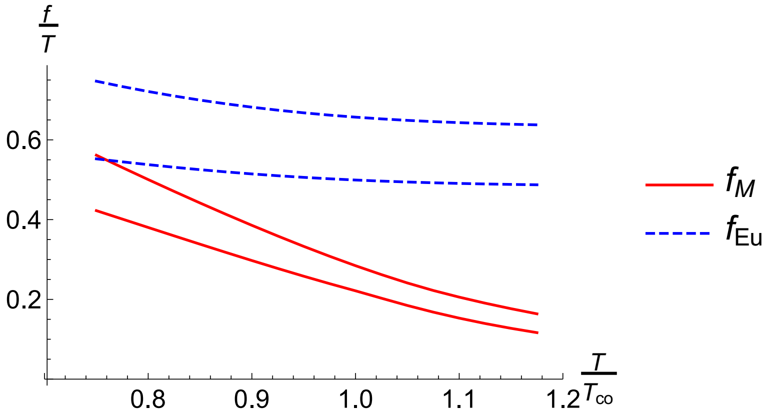
- The Minkowski space fermion action can be obtained from the analytic continuation of the Euclidean fermionic action
- $S_M = \int d^4x[\bar{\psi}(-\partial\!\!\!/ - d_3 T^0)\psi - L_6^M]$ where $\gamma^0 = -i\gamma^4$ and $\partial\!\!\!/ = \gamma^0\partial_0 + d_4\gamma^i\partial_i$
- The diagrams for the π two point functions are unchanged
- However, a subtlety related to order of limits: for the Minkowski results, the injected spatial momentum $\mathbf{q} \to 0$ first and then $q^0 \to 0$. This is because we are now considering dynamical phenomena

Consequences: no damping

- We find that there is no damping: at one loop order $c^5 = 0$
- The values of f, M and u are different in the Minkowski calculation

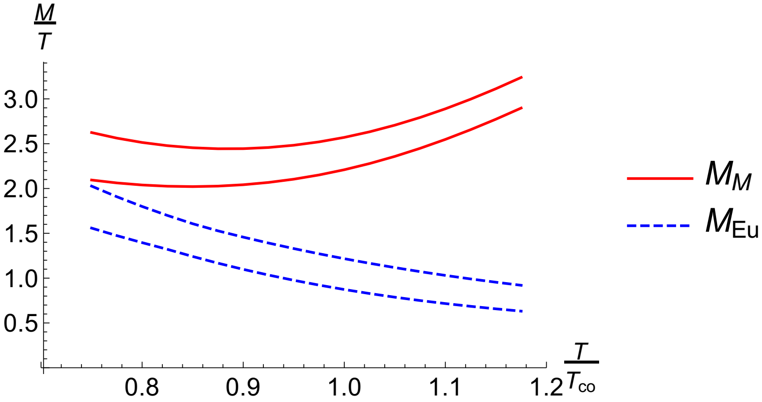
Results

f



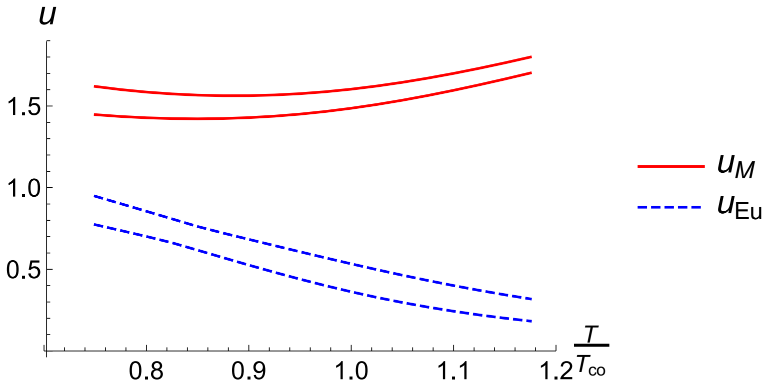
- f determined by the condition that the kinetic term is canonically normalized
- Uncertainty band dominated by the uncertainty in lattice results

Mass of π



- In Minkowski, M can be thought of as the pole mass
- M_M increases near T_{co} because of a decrease in f_M

Speed of π



- The group velocity is < 1 as long as $q < M_K^M / \sqrt{(u^M)^2 - 1}$, well below the cutoff
- u_M increases near T_{co} because of a decrease in f_M

Conclusions

- Parameters of the fermionic EFT in Euclidean space can be matched to the long distance behaviour of static correlators measured on the lattice
- Show good agreement
- Analytic continuation of the fermionic EFT to Minkowski can be used to calculate the π lagrangian in Minkowski
- No damping of the Axial vector to one loop order
- The parameters of the π theory — f, u, M — are different in the Minkowski π theory and the Euclidean π theory

Backup slides

Matching

- Matching u and M_π at $T = 0.84\,T_{co}$
- Error in T associated with $T_{co} = 211(5)\text{MeV}$
- Input from *[Brandt, Francis, Meyer, Robaina (2014)]* (figure below). Heavy π
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$$u = 0.71(4)$$
$$M_\pi / T = 2.05(6)\ .$$

- Fitted values $d_3 = 0.57\left[\pm 6(\text{input})\right]\left[\pm 3(\text{scale})\right]\left[\pm 3(\text{T})\right]$, $d_4 = 1.20\left[\pm 6(\text{input})\right]\left[\pm 4(\text{scale})\right]\left[\pm(4)\text{T}\right]$

T_c

- The peak of the chiral susceptibility in the EFT model occurs at $T_{co} = 1.24\,T_c$
- Taking $T_{co} = 211(5)$, we get $T_c = 170 \pm 6$
- Larger than the value of T_c from the lattice for $2 + 1$ flavors
- However for 2 flavors this agrees with the lattice prediction *[Brandt et. al. (2013)]*

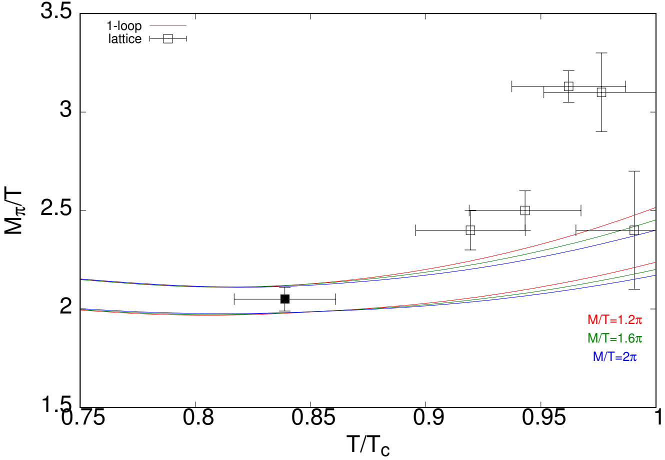
Correlation functions

- A finite temperature generalization of GOR relation is satisfied
- $c^2 T_0^2 = -\frac{\mathcal{N} m_q \langle\bar{\psi}\psi\rangle}{f^2}$
- *[Son, Stephanov (2002)]*
- We can compute f, c^4, M_π in the EFT model and compare to the lattice data
- Interesting behaviour of c_4 at T_c in the chiral limit:

$$c^4 \propto \int \frac{p^2 dp}{1+\exp(p/T)}[\frac{2}{p} - \frac{1}{T(1+\exp(p/T))}] = 0$$

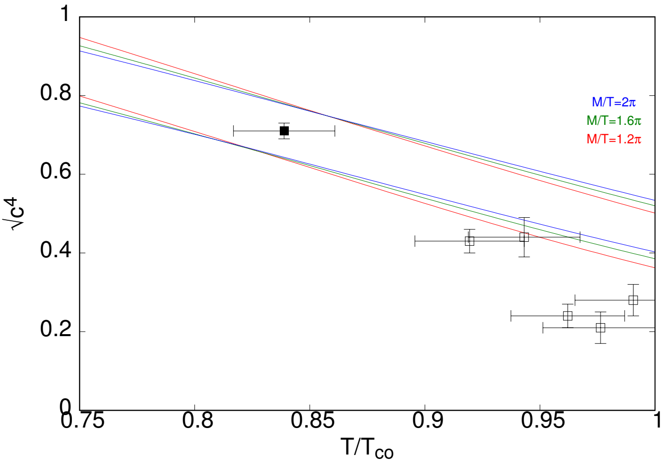
M_π

- Pion Debye screening mass



- Also see *[Ishii et. al. (2013); S Cheng, S Datta et. al. (2011)]*

- Pion velocity



Comparison with the Euclidean normalization

- The Minkowski version

$$(f_M)^2 = \lim_{q^0\to 0}\lim_{\mathbf{q}\to 0} -\frac{\mathcal{N}}{16}\int \frac{d^4k}{(2\pi)^4}\mathrm{tr}[\mathcal{D}_{11}(k)(\gamma^5\gamma^0)\mathcal{D}_{11}(k+q)(\gamma^5\gamma^0)]$$
$$= \frac{\mathcal{N}}{4(d_4)^3}\Big\{\frac{m^2}{(4\pi)^2}[2\log(\frac{(d_4)^2\mu^2}{m^2})] - 2\int \frac{\mathbf{k}^2 d\mathbf{k}}{(2\pi)^2}\frac{1}{e^{E/T}+1}\frac{m^2}{E^3}\Big\}$$

- To be compared with the Euclidean version

$$f_{Eu}^2 = \frac{1}{(d_4)^3 4f^2}\Big\{\frac{m^2}{(4\pi)^2}[2\log(\frac{(d_4)^2\mu^2}{m^2})]$$
$$+\int \frac{\mathbf{k}^2 d\mathbf{k}}{(2\pi)^2}[-\frac{1}{e^{E/T}+1}\frac{m^2}{E^3} + \frac{d_4^2\mathbf{k}^2\exp(E/T)}{(e^{E/T}+1)^2}\frac{1}{E^2T}]\Big\}$$

- $E = \sqrt{m^2 + d_4^2\mathbf{k}^2}$
- The Minkowski version $\sim m^2$ therefore it tends to drop as the condensate decreases near T_{co}
- f_π is defined to set the coefficient of q_0^2 in the Minkowski action to be 1 (canonical normalization of the kinetic term)