

The 7th Asian Triangle Heavy-Ion Conference (ATHIC 2018)

BARYON NUMBER FLUCTUATIONS IN THE 2+1 FLAVOR LOW ENERGY EFFECTIVE THEORY

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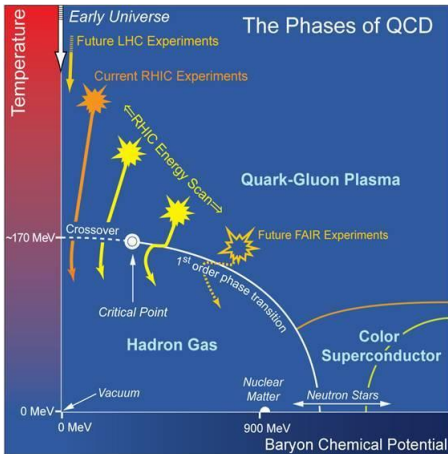
November 3, 2018

*based on : R. W. ,Chuang Huang,Wei-jie Fu ,arXiv:1809.04233

OUTLINE

- 1 Introduction
- 2 Effective action and flow equation
- 3 Result
- 4 Summary and outlook

Introduction



Effective action

The PQM scale-dependant effective action

$$\Gamma_k = \int_x [\bar{q}(\gamma_\mu \partial_\mu - \gamma_0(\mu + igA_0))]q + h\bar{q}\Sigma_5 q + \text{tr}(\partial_\mu \Sigma \cdot \partial_\mu \Sigma^\dagger) + \tilde{U}_k(\Sigma) + V_{glue}(L, \bar{L})] \quad (2.1)$$

$$\int_x \equiv \int_0^{1/T} dx_0 \int d^3x_i$$

local potential approximation (LPA)

$$\partial_t Z_{\phi/q} = 0, \partial_t h_{l/s} = 0$$

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Σ means the meson field

$$\Sigma = T_a(\sigma_a + i\pi_a), a = 0, 1, \dots, 8,$$

$$\Sigma_5 = T_a(\sigma_a + i\gamma_5\pi_a)$$

$T_a = \lambda_a/2$, λ_a are the Gell-Mann matrices.

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$$\tilde{U}_k(\Sigma) = U_k(\rho_1, \tilde{\rho}_2) - c_k \xi - j_L \sigma_L - j_S \sigma_S$$

$U_k(\rho_1, \tilde{\rho}_2)$ chirally invariant term

$SU_V(3) \times SU_A(3) \times U_V(1) \times U_A(1)$

$\xi = \det(\Sigma) + \det(\Sigma^\dagger)$ Kobayashi-Maskawa-'t Hooft(KMT) term

$j_L \sigma_L, j_S \sigma_S$ the linear sigma terms

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The Polyakov loops

$$L(x) = \frac{1}{N_c} \langle \text{Tr} \mathcal{P}(x) \rangle, \quad \bar{L} = \frac{1}{N_c} \langle \text{Tr} \mathcal{P}^\dagger(x) \rangle$$

with $\mathcal{P}(x) = \mathcal{P} \exp(ig \int_0^\beta d\tau A_0(x, \tau))$

Effective action

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$$\frac{V_{glue}}{T^4} = -\frac{1}{2}a(t)\bar{L}L + \frac{c(t)}{2}(L^3 + \bar{L}^3) + d(t)(\bar{L}L)^2 + b(t)\ln(1 - 6\bar{L}L + 4(L^3 + \bar{L}^3) - 3(\bar{L}L))$$

reduced temperature $t = (T - T_c)/T_c$, $t_{YM} = \alpha t_{glue}$

[arXiv:1307.5958] Pok Man Lo et al.

Effective action

The PQM scale-dependant effective action

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The Wetterich equation

$$\partial_t \Gamma_k = -\text{Tr}(G_k^{q\bar{q}} \partial_t R_k^q) + \frac{1}{2} \text{Tr}(G_k^{\phi\phi} \partial_t R_k^\phi)$$

$$t = \ln(k/\Lambda)$$

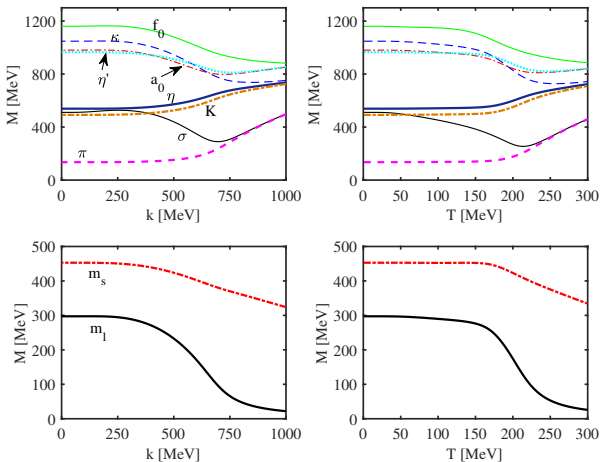
The flow equation

$$\begin{aligned}
\partial_t U_k(\rho_1, \tilde{\rho}_2) = & \frac{k^4}{4\pi^2} \{ 3l_0^{(B)}(m_{\pi,k}; T) + 4l_0^{(B)}(m_{K,k}; T) \\
& + l_0^{(B)}(m_{\eta,k}; T) + l_0^{(B)}(m_{\eta',k}; T) + 3l_0^{(B)}(m_{a0,k}; T) \\
& + 4l_0^{(B)}(m_{\kappa,k}; T) + l_0^{(B)}(m_{f0,k}; T) + 3l_0^{(B)}(m_{\sigma,k}; T) \\
& - 4N_c [2l_0^{(F)}(m_{l,k}; T, \mu) + l_0^{(F)}(m_{s,k}; T, \mu)] \}
\end{aligned} \tag{2.2}$$

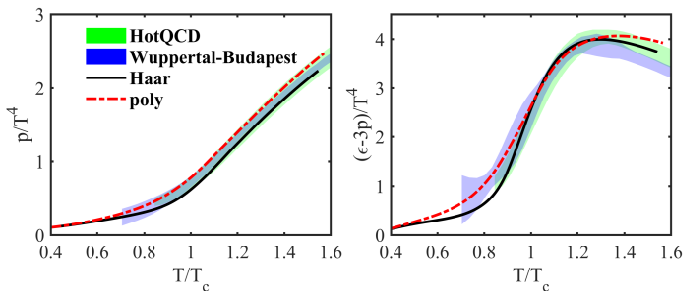
here

$$\begin{aligned}
l_0^{(B)}(m) &= \frac{k}{3E_{\phi,k}} (1 + 2n_B(E_{\phi,k})) \\
l_0^{(F)}(m) &= \frac{k}{3E_{q,k}} (1 - n_F(E_{q,k} - \mu) - n_F(E_{q,k} + \mu))
\end{aligned} \tag{2.3}$$

The meson and quark masses changed with scale k and temperature T



The pressure p and the trace anomaly $\Delta = \epsilon - 3p$
 ($\alpha = 0.52, T_c^{glue} = 270MeV$)



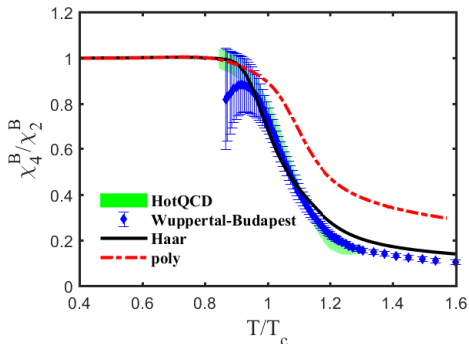
$T_\chi = 194MeV$

hotQCD [arXiv:1407.6387] $T_\chi = 154MeV$

Wuppertal-Budapest [arXiv:1309.5258] $T_\chi = 156MeV$

The cumulants $\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n} \frac{P}{T^4}, n \in \mathbb{Z}$

The kurtosis ratio $R_{42} = \chi_4^B / \chi_2^B$

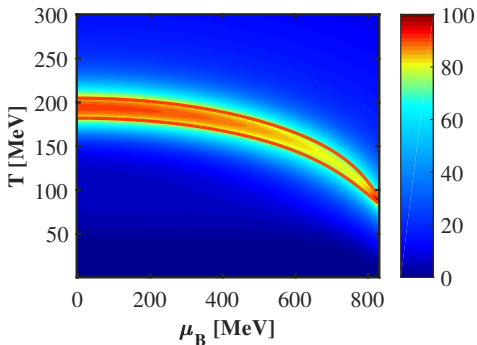


hotQCD [arXiv:1708.04897]

Wuppertal-Budapest[arXiv:1305.5161]

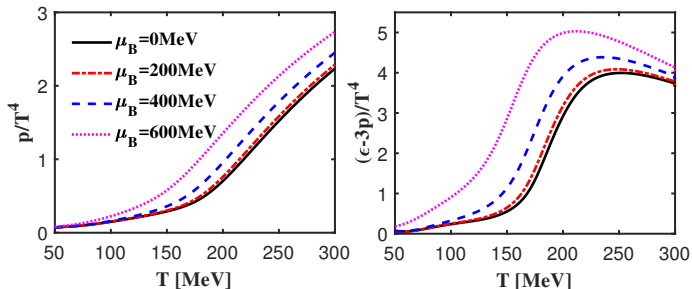
Chiral transition diagram

(identified by $\left| \frac{\partial \rho_1}{\partial T} \right|$) 90%

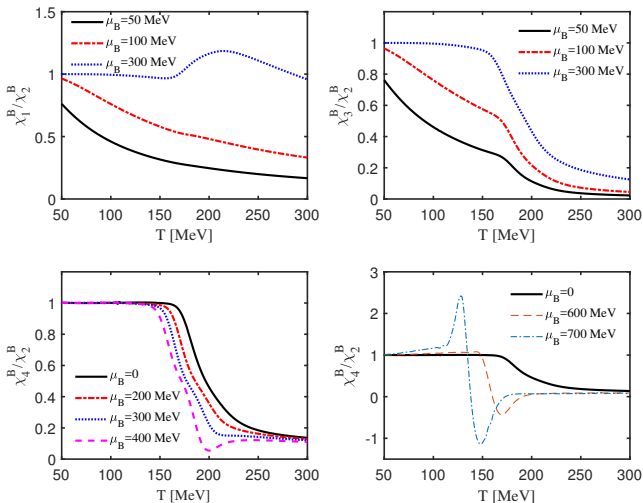


$(T_C = 84 \text{ MeV}, \mu_{BC} = 840 \text{ MeV})$

The pressure and the trace anomaly



Ratios of baryon number fluctuations



Summary and outlook

- The pressure, trace anomaly and the kurtosis agree with the lattice results.
- The EoS and baryon number fluctuations up to the four order at finite baryon chemical potential are also calculated.
- In the future, lots of things can be done, for instance, going beyond the LPA truncation, considering the electric charge and strangeness chemical potentials, extending the low energy effective theory to the 2+1 flavor QCD.

Thank you

Summary and outlook

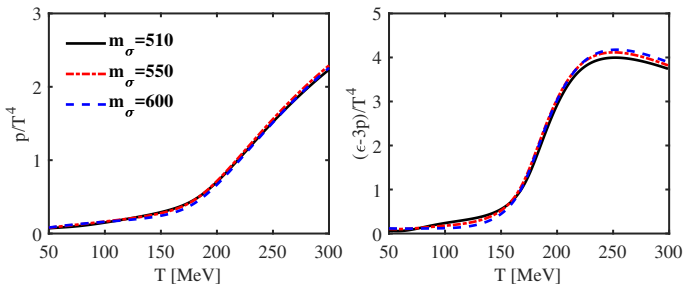
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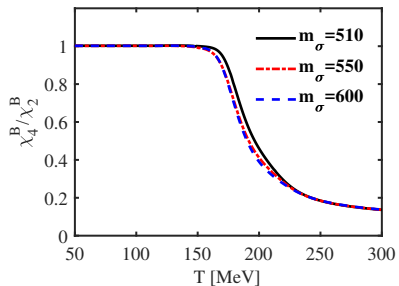
Thank you

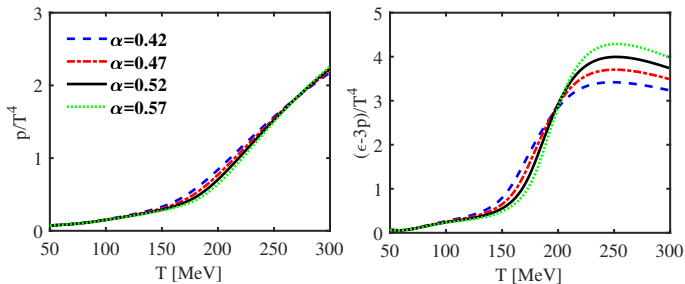
Backup

The pressure and the trace anomaly

m_σ

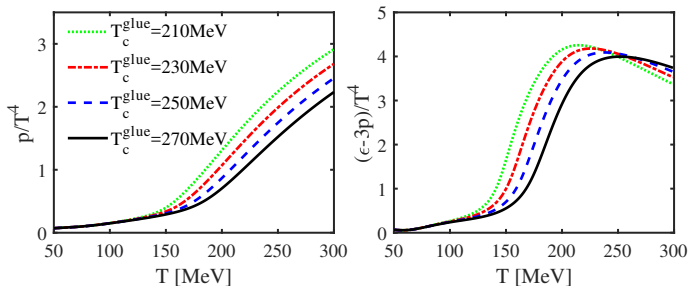


The kurtosis ratio m_σ 

The pressure and the trace anomaly α 

The pressure and the trace anomaly

$$T_c^{glue}$$





$$\begin{aligned}
 U_{k=\Lambda} = & - \int_{\Lambda}^{\infty} dk \frac{k^5 N_c}{3\pi^2} \left(\frac{2}{E_l} (n_F(x_l, T, L, \bar{L}) + n_F(\bar{x}_l, T, \bar{L}, L)) \right. \\
 & \left. + \frac{1}{E_s} (n_F(x_s, T, L, \bar{L}) + n_F(\bar{x}_s, T, \bar{L}, L)) \right)
 \end{aligned}
 \tag{4.1}$$

$$m_{l/s, k > \Lambda} = m_{l/s, k = \Lambda},$$



$$\frac{\partial U_{total}}{\partial \sigma_l} = \frac{\partial U_{total}}{\partial \sigma_s} = \frac{\partial U_{total}}{\partial L} = \frac{\partial U_{total}}{\partial \bar{L}} = 0
 \tag{4.2}$$

Result

Initial conditions

- the UV cutoff $\Lambda = 1\text{GeV}$

$$U(\rho, \tilde{\rho}_2)_\Lambda = \lambda_{10}\rho_1 + \frac{\lambda_{20}}{2}\rho_1^2 + \lambda_{01}\rho_2$$

- $h_l = h_s = 6.5$, $c_k = 4808\text{MeV}$

$$j_l = (121\text{MeV})^3, j_s = (336\text{MeV})^3$$

$$\lambda_{10} = (654\text{MeV})^2, \lambda_{20} = 31, \lambda_{01} = 11.9$$

- $f_\pi = 92\text{MeV}$, $f_K = 115\text{MeV}$

$$m_\pi = 135\text{MeV}, m_K = 492\text{MeV},$$

$$m_\sigma = 510\text{MeV}, m_\eta^2 + m_\eta'^2 = 1.22 \times 10^6\text{MeV}^2$$

$$m_l = 299\text{MeV}, m_s = 454\text{MeV}$$