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BARYON NUMBER PROBABILITY DISTRIBUTION AT FINITE TEMPERATUR

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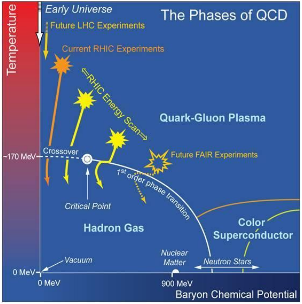
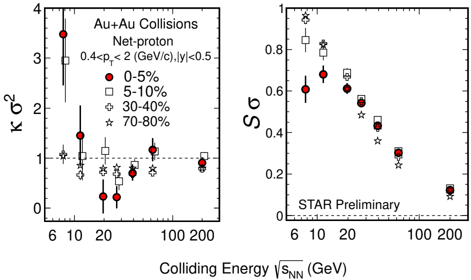
Based on K S,Rui Wen,Wei-jie Fu arXiv:1805.12025

OUTLINE

Introduction





The Hot QCD White Paper(2015)

X.Luo(STAR),PoS CPOD2014, 019(2014)

The baryon number probability distribution

We consider a thermodynamical system, the probability distribution reads

$$P(N_B;T,V,\mu_B) = \frac{Z(T,V,N_B)}{\mathcal{Z}(T,V,\mu_B)} \exp\left(\frac{\mu_B N_B}{T}\right), \tag{2.1}$$

where $Z(T,V,N_B)$ and $\mathcal{Z}(T,V,\mu_B)$ are the canonical and grand canonical partition functions.
$N_B = N_q/3$ with $N_q \in \mathbb{Z}$ is the net quark number.

$$Z(T,V,N_B=N_q/3)=\frac{1}{2\pi}\int_0^{2\pi}d\theta e^{-i\theta N_q}\mathcal{Z}(T,V,e^{i\theta}). \qquad (2.2)$$

The canonical partition function can be deduced from the grand canonical one $\mathcal{Z}(T,V,\mu_q)$ with an imaginary chemical potential, i.e, $\mu_q=i\theta T$.

$$\mathcal{Z}(T,V,\mu_q)=\exp\left(-\frac{V}{T}\Omega(T,V,\mu_q)\right). \qquad (2.3)$$

The low energy effective theory within the FRG approach

The scale-dependent effective action for the low energy effective model

$$\Gamma_k = \int_x \Big\{ Z_{q,k}\bar{q}[\gamma_\mu\partial_\mu - \gamma_0(\mu_q + igA_0)]q + \frac{1}{2}Z_{\phi,k}(\partial_\mu\phi)^2 + h_k\,\bar{q}\left(T^0\sigma + i\gamma_5\boldsymbol{T}\cdot\boldsymbol{\pi}\right)q + V_k(\rho) - c\sigma + V_{\text{glue}}(L,\bar{L})\Big\}, \quad (3.1)$$

$\int_x = \int_0^{1/T} dx_0 \int d^3x$, $\phi = (\sigma, \boldsymbol{\pi})$.

To take into account the deconfinement information, we include the temporal component of the gluon background field A_0 .

$$L(\boldsymbol{x}) = \frac{1}{N_c}\langle tr\mathcal{P}(\boldsymbol{x})\rangle \quad \bar{L}(\boldsymbol{x}) = \frac{1}{N_c}\langle tr\mathcal{P}^{\dagger}(\boldsymbol{x})\rangle \tag{3.2}$$

with

$$\mathcal{P}(\boldsymbol{x}) = \mathcal{P}\exp\left(ig\int_0^{\beta} d\tau A_0(\boldsymbol{x},\tau)\right). \tag{3.3}$$

The Wetterich equation reads

$$\partial_t \Gamma_k = -Tr(G_k^{q\bar{q}} \partial_t R_k^q) + \frac{1}{2} Tr(G_k^{\phi\phi} \partial_t R_k^{\phi}) \tag{3.4}$$

with $t = \ln(k/\Lambda)$ and Λ is the UV cutoff scale. We choose the local potential approximation (LPA), i.e., $Z_{q,k} = Z_{\phi,k} = 1$ and $\partial_t h_k = 0$.

The flow equation for the effective potential

$$\partial_t V_k(\rho) = \frac{k^4}{12\pi^2}\Big[\frac{N_f^2-1}{\sqrt{1+\bar{m}_{\pi,k}^2}}\Big(1+2\,n_B(\bar{m}_{\pi,k}^2;T)\Big) + \frac{1}{\sqrt{1+\bar{m}_{\sigma,k}^2}}\Big(1+2\,n_B(\bar{m}_{\sigma,k}^2;T)\Big) - \frac{4N_cN_f}{\sqrt{1+\bar{m}_{q,k}^2}}\Big(1-n_f(\bar{m}_{q,k}^2;T,\mu_q,L,\bar{L}) - \bar{n}_f(\bar{m}_{q,k}^2;T,\mu_q,L,\bar{L})\Big)\Big] \tag{3.5}$$

Inserting $\mu_q = i\theta T$, $L = |L|e^{i\phi}$, $\bar{L} = |L|e^{-i\phi}$, one obtains the fermionic distribution functions

$$
\begin{aligned}
& n_f(\bar{m}_{q,k}^2; T, \mu_q, L, \bar{L}) \\
=& \Big[1 + 2|L|e^{E_{q,k}/T}e^{-i(\theta+\phi)} + |L|e^{2E_{q,k}/T}e^{i(\phi-2\theta)}\Big] \\
& \Big/\Big[1 + 3|L|e^{E_{q,k}/T}e^{-i(\theta+\phi)} + 3|L|e^{2E_{q,k}/T}e^{i(\phi-2\theta)} \\
& + e^{3E_{q,k}/T}e^{-3i\theta}\Big]
\end{aligned}
\tag{3.6}
$$

$$
\bar{n}_f(\bar{m}_{q,k}^2; T, \mu_q, L, \bar{L}) = n_f^*(\bar{m}_{q,k}^2; T, \mu_q, L, \bar{L}) \tag{3.7}
$$

The thermodynamic potential density is given by

$$\Omega = V_{k=0}(\rho) - c\sigma + V_{\text{glue}}(L, \bar{L}), \tag{3.8}$$

Ω is invariant with the replacement $\theta \to \theta + \frac{2\pi}{3}$, the Roberge-Weiss periodicity.
The modified Polyakov loop

$$L' = |L|e^{i\phi'}, \quad \text{and} \quad \bar{L}' = |L|e^{-i\phi'}, \tag{3.9}$$

with $\phi' = \phi + \theta$.

The periodicity of the thermodynamic potential leads to

$$L'(\theta) = L'(\theta + \frac{2\pi}{3}), \quad \text{and} \quad \bar{L}'(\theta) = \bar{L}'(\theta + \frac{2\pi}{3}), \tag{3.10}$$

$$Z(T, V, N_q) = \frac{1}{2\pi} \int_0^{\frac{2\pi}{3}} d\theta e^{-i\theta N_q} \mathcal{Z}(T, V, \theta)\Big(1 + e^{-i\frac{2\pi}{3}N_q} + e^{-i\frac{4\pi}{3}N_q}\Big) \tag{3.11}$$

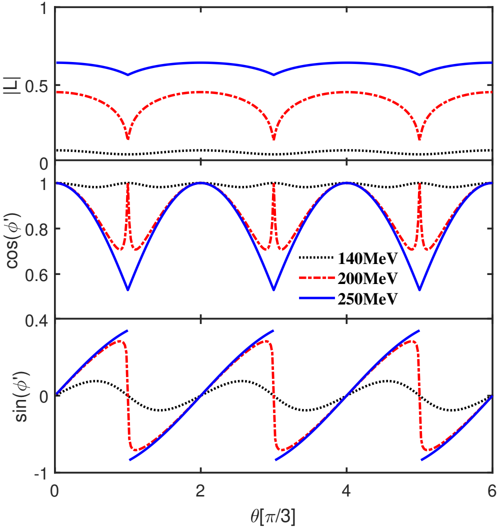
The probability in a state with non-integer baryon numbers is zero, which shows the information of the color confinement.

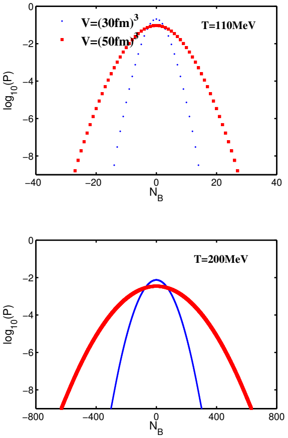
Numerical results

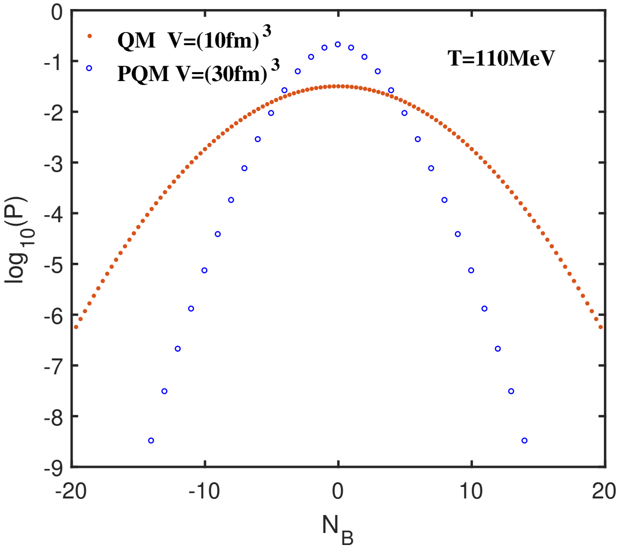
We employ the Taylor expansion around the physical point to solve the flow equation for the effective potential

$$V_k(\rho) = \sum_{n=0}^{N} \frac{\lambda_{n,k}}{n!}(\rho - \kappa_k)^n. \tag{4.1}$$

In our calculations, the maximal order of the Taylor expansion N is chosen to be $N = 5$.


|L|
cos(ϕ')
sin(ϕ')
140MeV
200MeV
250MeV
θ[π/3]
1
0.5
0
1
0.8
0.6
0.4
0
-1
0
2
4
6


V=(30fm)^3
V=(50fm)^3
T=110MeV
T=150MeV
T=200MeV
T=250MeV
log10(P)
N_B


QM V=(10fm)^3
PQM V=(30fm)^3
T=110MeV
$\log_{10}(P)$
N_B
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-20
-10
0
10
20

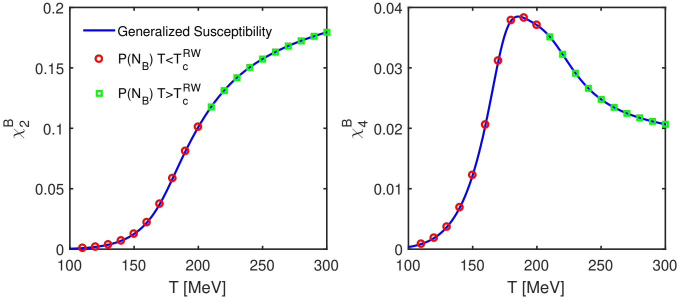
Cumulants of the baryon number distribution

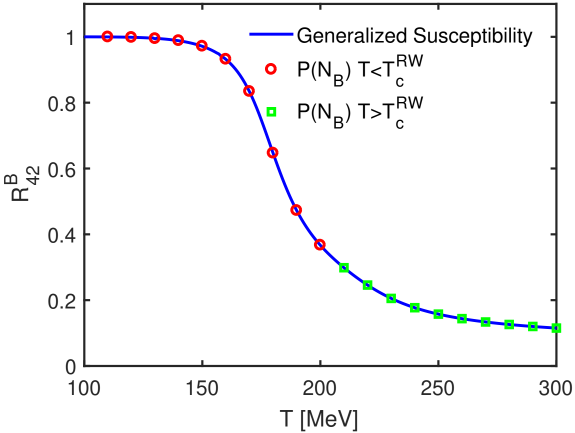
The generalized susceptibilities

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\Big(-\frac{\Omega}{T^4}\Big). \tag{4.2}$$

The baryon number distribution

$$\begin{aligned}\chi_2^B &= \frac{1}{VT^3}\langle(\delta N_B)^2\rangle\,,\\ \chi_4^B &= \frac{1}{VT^3}\Big(\langle(\delta N_B)^4\rangle - 3\langle(\delta N_B)^2\rangle^2\Big)\,,\end{aligned} \tag{4.3}$$


Generalized Susceptibility
P(N_B) T<T_c^RW
P(N_B) T>T_c^RW
χ_2^B
χ_4^B
T [MeV]
T [MeV]


Generalized Susceptibility
P(N_B) T<T_c^RW
P(N_B) T>T_c^RW
R_42^B
0
0.2
0.4
0.6
0.8
1
100
150
200
250
300
T [MeV]

Summary

- The Roberge-Weiss periodicity directly results in that only states of $N_B = N$ are possible, which is an indication of the color confinement from another viewpoint.
- The baryon number probability distribution can be calculated by imaginary chemical potential and the cumlants agree with the generalized susceptibilities method.
- The periodicity is still there in the high temperature regime.

Thank you!