# The QED Radiative Corrections to Chiral Magnetic Effect

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# Outline

- Charged massless quarks in a magnetic field

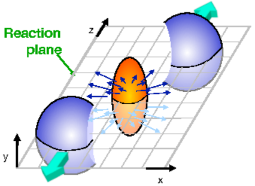
| Chirality | Right-handed | | Left-handed | |
|---|---|---|---|---|
| Charge | $+$ | $-$ | $+$ | $-$ |
| Spin polarization | $\uparrow$ | $\downarrow$ | $\uparrow$ | $\downarrow$ |
| Momentum | $\uparrow$ | $\downarrow$ | $\downarrow$ | $\uparrow$ |
| Electric current | $\Uparrow$ | $\Uparrow$ | $\Downarrow$ | $\Downarrow$ |

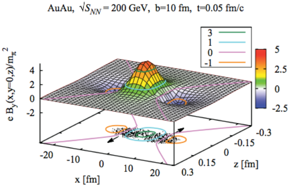
- with a net axial charge $Q_5$(chiral imbalance)

$$\mathbf{J} = \mathbf{J}_R + \mathbf{J}_L \neq 0$$

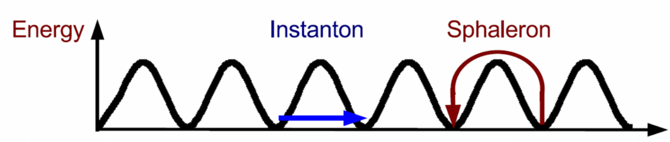
# CME in non-central HIC

- an extremely large magnetic field created in non-central HIC



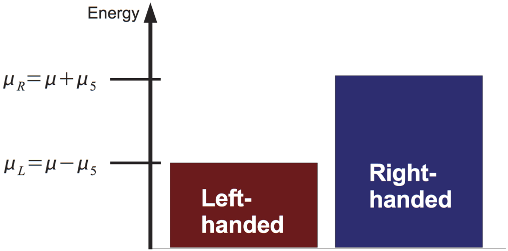

- transition between vacuum with different topologies in QCD




axial anomaly

$$\Delta Q_5 = \frac{N_f g^2}{32\pi^2} \int d^4x F^l_{\mu\nu} \tilde{F}^l_{\mu\nu} \equiv n_w$$

- Introduce chiral imbalance by hand




Chiral imbalance $\Leftrightarrow$ Nonzero chiral chemical potential $\mu_5$

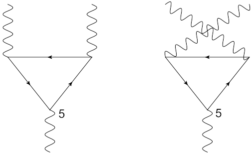
$$H \to H - \mu_5 \int d^3x \bar{\psi}\gamma^0\gamma^5\psi$$

## The relation of CME current to chiral anomaly

The CME current

$$J_i(p) = \eta\mu_5 K_{ij}(p) A_j(p) + \mathcal{O}(\mu_5^3)$$

- In terms of the AVV three point function $\Lambda_{\mu\nu\rho}(Q_1, Q_2)$




$$Q_1 = (\mathbf{q}, i(\omega + \frac{k_0}{2})),$$

$$Q_2 = (-\mathbf{q}, i(-\omega + \frac{k_0}{2}))$$

- the coefficient

$$K_{ij}(q) = \Lambda_{ij4}(q, -q) = -i \lim_{k_0 \to 0} \frac{1}{k_0} (Q_1 + Q_2)_\rho \Lambda_{ij\rho}(Q_1, Q_2)$$

- the chiral anomaly

$$(Q_1 + Q_2)_\rho \Lambda_{\mu\nu\rho}(Q_1, Q_2) = -i \frac{e^2}{2\pi^2} \epsilon_{\mu\nu\alpha\beta} Q_{1\alpha} Q_{2\beta}$$

It follows that

$$K_{ij}(q) = i\frac{e^2}{2\pi^2}\epsilon_{ikj}q_k$$

Then the CME current

$$\mathbf{J} = \frac{e^2}{2\pi^2}\mu_5\mathbf{B} \tag{1}$$

There are, however, two shortcomings in the above establishment

1. distinction between chiral anomaly at the operator level and its matrix element

   *only the former one is free from radiative corrections.*

2. the constant $\mu_5$ limit in eq.(1) becomes subtle at finite temperature

$$\lim_{k_0\to 0}\lim_{\mathbf{k}\to 0} \neq \lim_{\mathbf{k}\to 0}\lim_{k_0\to 0} \tag{2}$$

*note that in the limiting process $\lim_{\mathbf{k}\to 0}\lim_{k_0\to 0}$, the relation of CME current to chiral anomaly becomes unclear.*

# Chiral anomaly at operator level and its matrix element

- The operator equation of the anomaly

$$\partial_\mu j^5_\mu = 2imj^5 + i\frac{\alpha_0}{4\pi}\epsilon_{\rho\sigma\lambda\nu}F_{\rho\sigma}F_{\lambda\nu}$$

  the coefficient of the anomalous term is exactly $\alpha_0/4\pi$ and *does not involve an unknown power series in the coupling constant coming from higher orders in perturbation theory.* *Adler and Bardeen (1969')*

- The matrix element between the vacuum and a state with two photons of momenta $Q_1, Q_2$

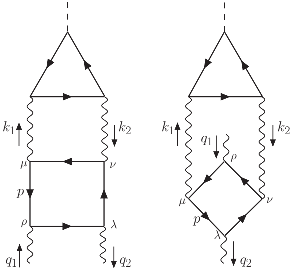
$$(Q_1+Q_2)_\mu\Lambda_{\mu\rho\lambda}(Q_1,Q_2) = -i\left[2mG\left(\frac{Q_1^2}{m^2},\frac{Q_2^2}{m^2},\frac{Q_1\cdot Q_2}{m^2}\right)+H\left(\frac{Q_1^2}{m^2},\frac{Q_2^2}{m^2},\frac{Q_1\cdot Q_2}{m^2}\right)\right] \times \epsilon_{\rho\lambda\alpha\beta}Q_{1\alpha}Q_{2\beta}$$

  in low energy limit

$$2mG(0,0,0)+H(0,0,0)=0, \quad H(0,0,0)=\frac{2\alpha}{\pi}$$

- For massless fermions, the low energy kinematic point cannot be attained, the matrix elements receive radiative corrections.

# Radiative corrections from photon-photon rescattering




Photon rescattering contribution to the AVV function

*Ansel'm and Ioganson (1989')*

- The anomalous Ward identity

$$(Q_1 + Q_2)_\rho \Lambda_{\mu\nu\rho}(Q_1, Q_2) = -i\frac{e^2}{2\pi^2}\epsilon_{\mu\nu\alpha\beta}Q_{1\alpha}Q_{2\beta} \times \left(1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{k^2}\right)$$

- The kernel of CME current becomes

$$K_{ij}(q) = i\frac{e^2}{2\pi^2}\mu_5\epsilon_{ikj}q_k\left(1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{k^2}\right)$$

- Likewisely, the same diagrams with two internal photons replaced by two gluons may also contribute to CME.

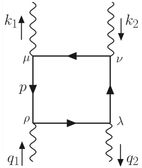
# Radiative corrections at finite temperature

At finite temperature, the results correspond to the two limits

$$\lim_{Q_0\to 0}\lim_{\mathbf{Q}\to 0}, \qquad \lim_{\mathbf{Q}\to 0}\lim_{Q_0\to 0}$$

may be different.

- The order $\lim_{Q_0\to 0}\lim_{\mathbf{Q}\to 0}$
  1. The AVV triangle in real-time formulation(e.g., CTP) is diagonal, i.e., only 111 and 222 components survive.
  2. For static external momenta, the real-time formulation reduces to Mastubara one.

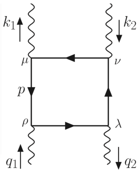



The photon box is free from IR singularity, thus

$$\Gamma_{mnij}(0,0;k) = \frac{\partial}{\partial q_l}\Gamma_{mnij}(q,q;k)\Big|_{q=0} = 0$$

the three-loop diagrams would be at order of $\mathcal{O}(\mathbf{q}^2)$ and thus do not contribute to CME.

- The order $\lim_{\mathbf{Q}\to 0}\lim_{Q_0\to 0}$
  1. The AVV triangle may not be diagonal W.R.T the CTP indices.
  2. For static external momenta, the real-time formulation reduces to Mastubara one.




$$\Gamma_{mnij}(0,0;k) = \frac{\partial}{\partial q_l}\Gamma_{mnij}(q,q;k)\Big|_{q=0} = 0$$

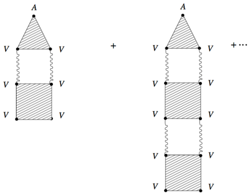
$$\Gamma_{mnij}(q,q;0) = \frac{\partial}{\partial k_l}\Gamma_{mnij}(q,q;k)\Big|_{k=0} = 0$$

Thus, the amplitude of the box

$$\Gamma \simeq \mathcal{O}(\mathbf{q}^2\mathbf{k}^2)$$

Then Coleman-Hill theorem( for 3D QED) can apply and the chain diagrams can be ruled out.

*Coleman and Hill (1985')*



# Summary and Conclusions

Summary of our results of CME up to three-loop massless QED:

- the kernel of CME current

$$K_{ij}(\mathbf{q}) = i\frac{e^2}{2\pi^2}F_s\left(\frac{|\mathbf{q}|}{T}\right)\epsilon_{ikj}q_j$$

1. In low temperature limit($T << |\mathbf{q}|$): $F_s(|\mathbf{q}|/T) \to 1 - \frac{3e^4}{64\pi^4}\ln\frac{\Lambda^2}{q^2}$
2. At finite temperature(T>$|\mathbf{q}|$): for $\lim_{Q_0\to 0}\lim_{\mathbf{Q}\to 0}$, $F_s(|\mathbf{q}|/T) \to 1$
   for $\lim_{\mathbf{Q}\to 0}\lim_{Q_0\to 0}$, $F_s(|\mathbf{q}|/T) \to 0$

- Out results apply to the case with static external magnetic field, i.e., $|\mathbf{q}| >> q_0$. In non-central HIC, according to simulations, the magnetic field created therein has a life time smaller than its spatial size rendering $|\mathbf{q}| << q_0$.

  *Kharzeev and Warringa (2009'); Hou, Liu and Ren (2011')*

- The higher order contributions in QCD is much more complicated due to the gluon's self-coupling and the potential IR singularity is more severe than QED.