

QCD phase diagram from Lattice QCD

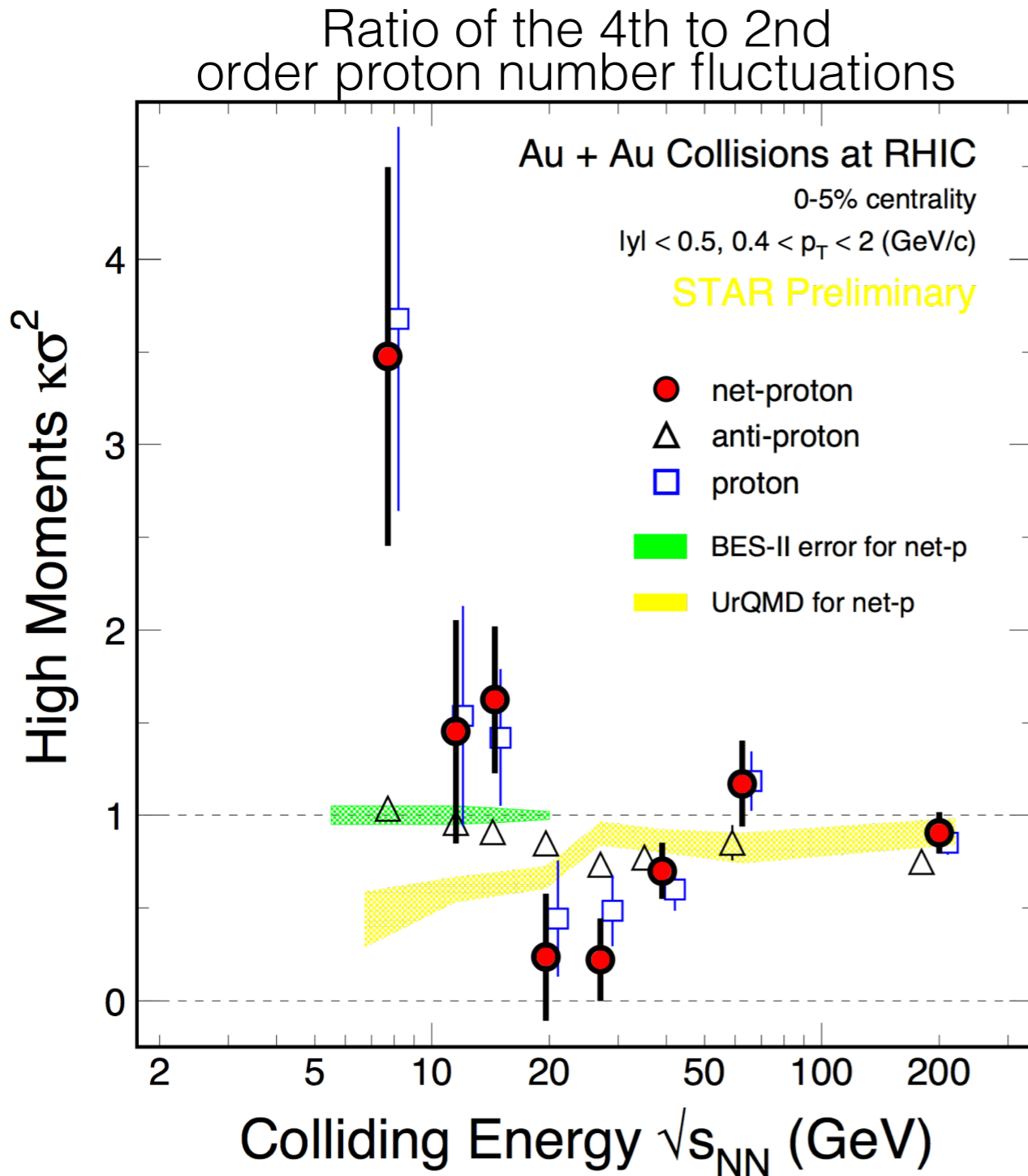
Heng-Tong Ding (丁亨通)

Central China Normal University

7th Asian Triangle Heavy-Ion Conference

3-6 Nov., 2018@USTC, Hefei

Search for the QCD critical point with fluctuations of conserved charges in HIC



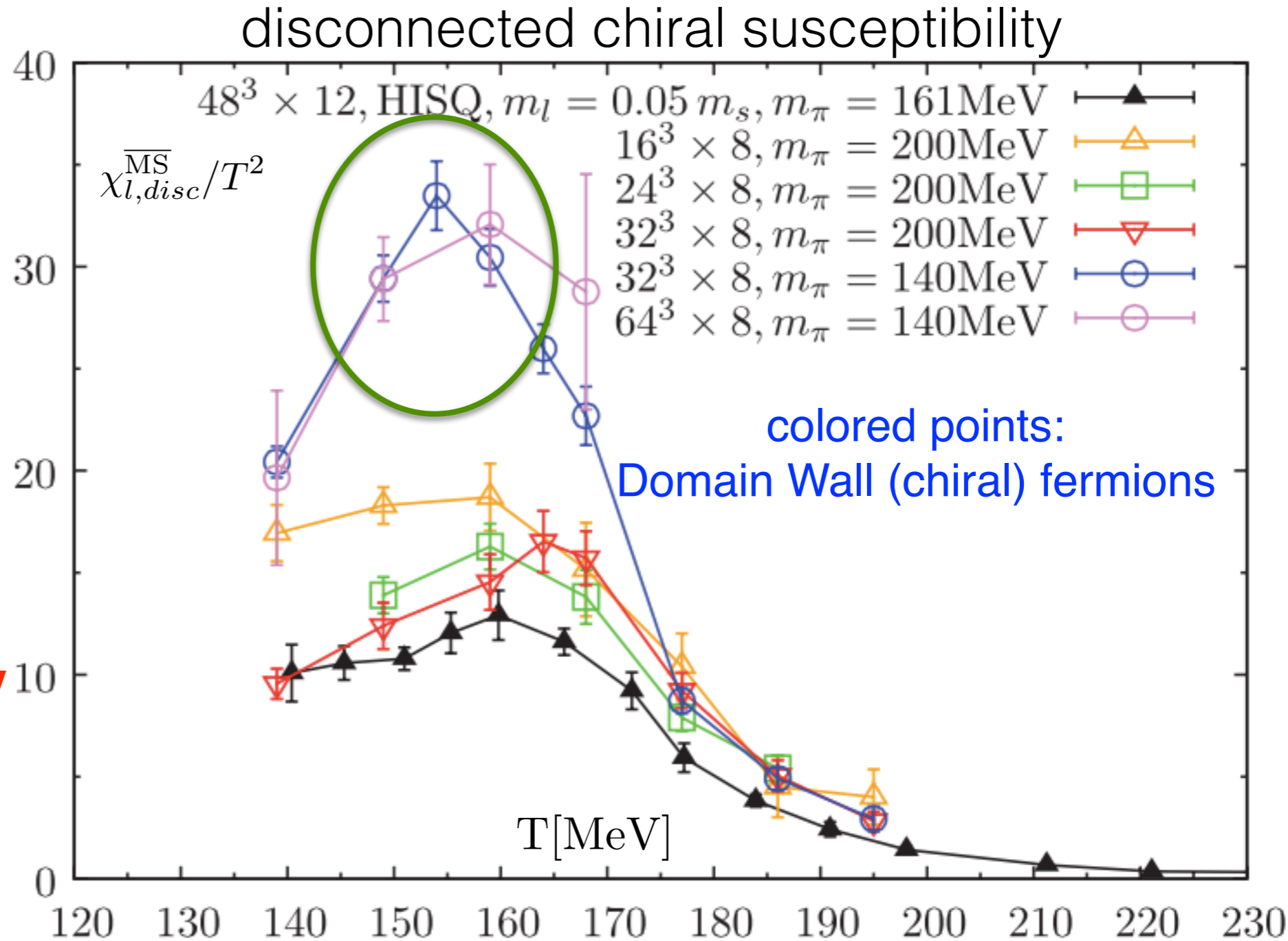
Relation to
QCD criticality?

QCD transition with $m_\pi = 140$ MeV
 at $\mu_B = 0 / \sqrt{s_{NN}} \approx 200$ GeV

**“cross over”
 type of transition**

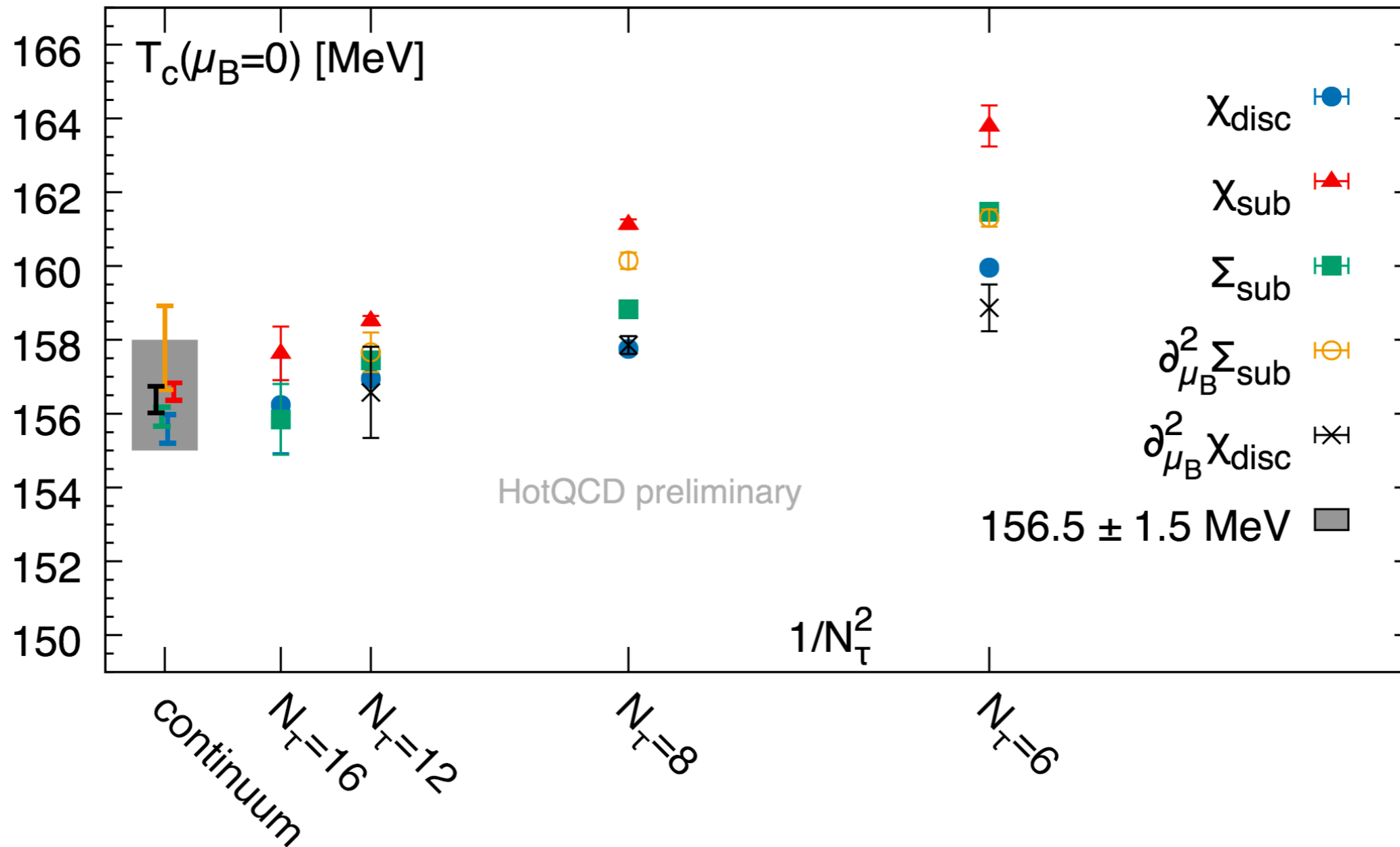
No true
 “PHASE transition”

$T_{pc} = 155(1)(8)$ MeV



T. Bhattacharya, ...HTD, ...et al. [HotQCD collaboration],
 Phys. Rev. Lett., 113(2014) 082001 (Editor's suggestion)

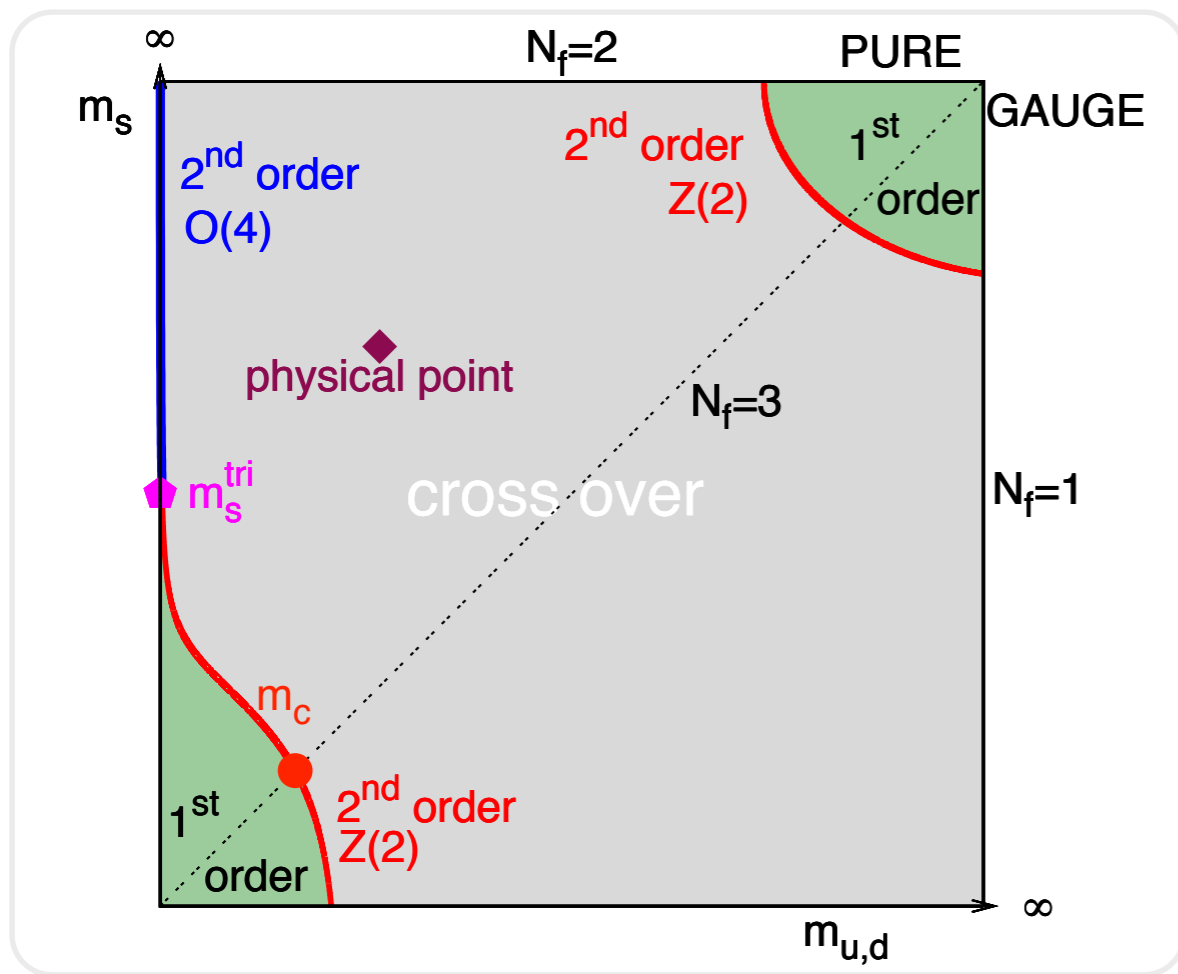
QCD transition with $m_\pi = 140$ MeV at $\mu_B = 0$



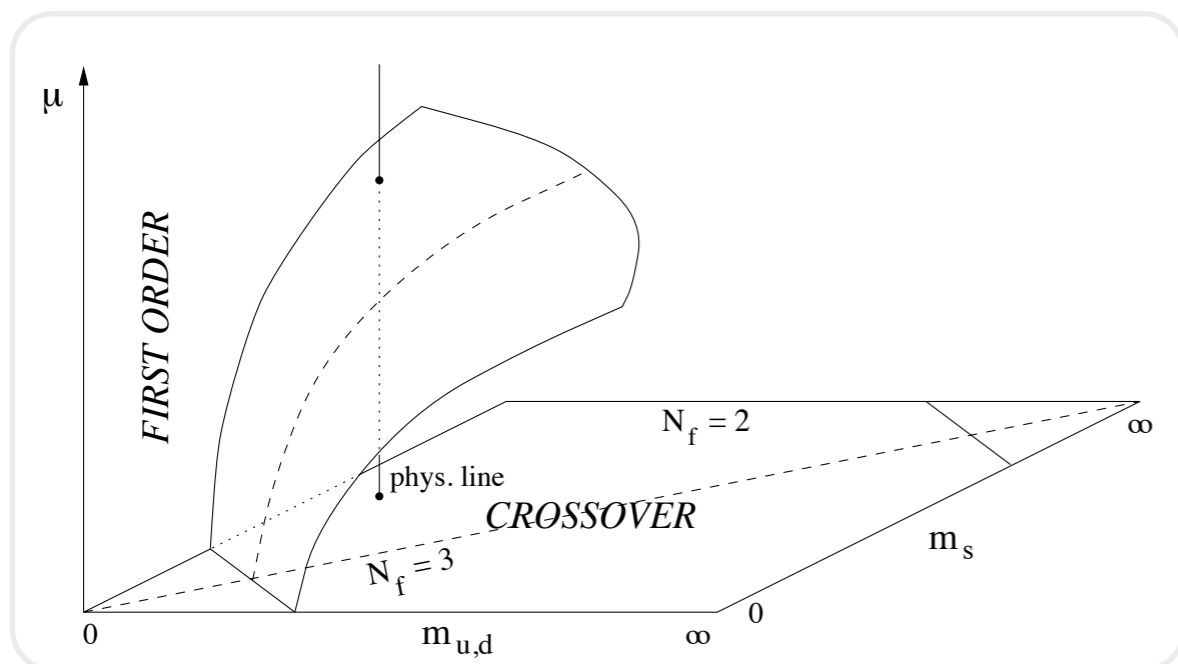
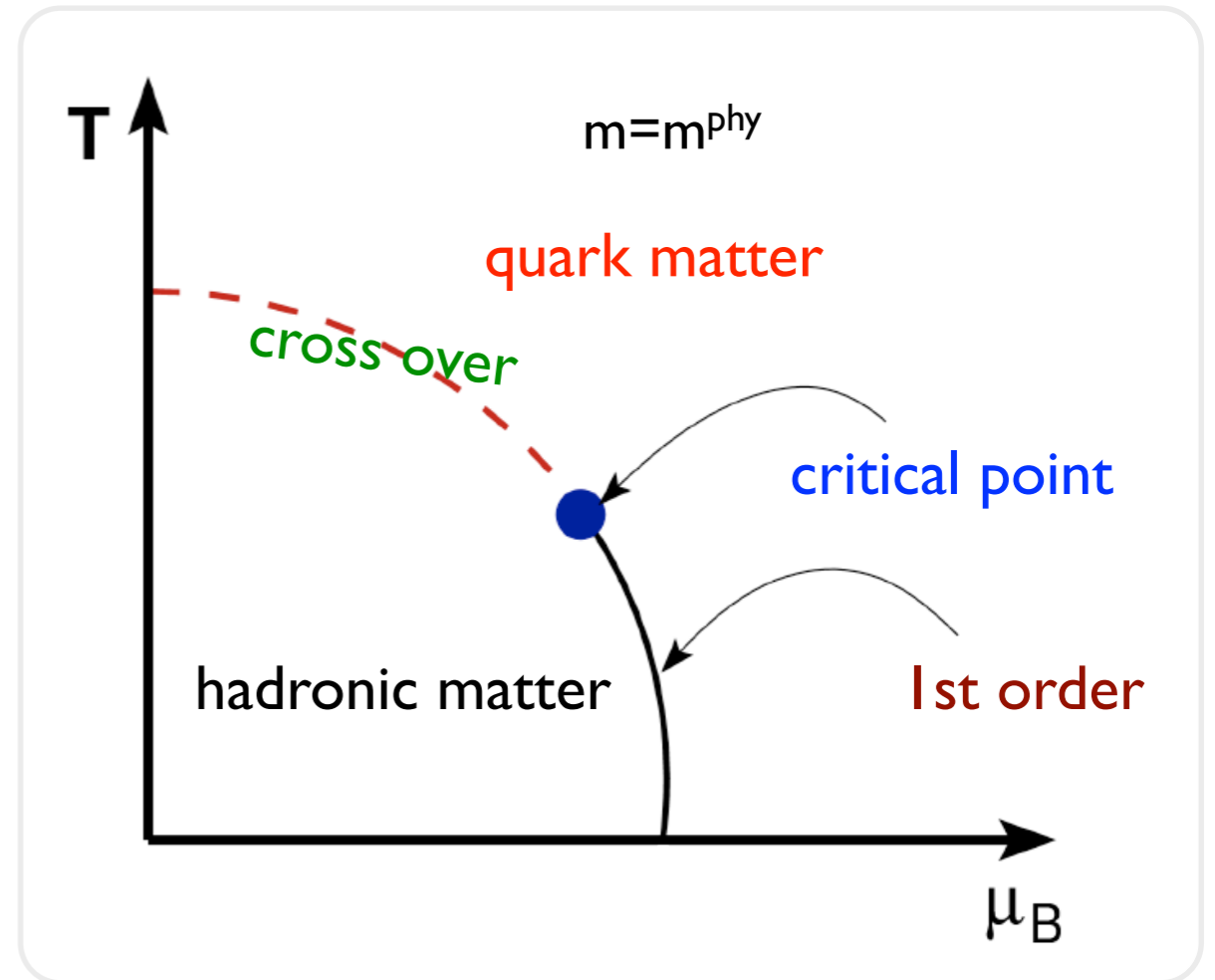
Higher precision in the continuum limit:

$$T_{pc} = 156.5(1.5) \text{ MeV}$$

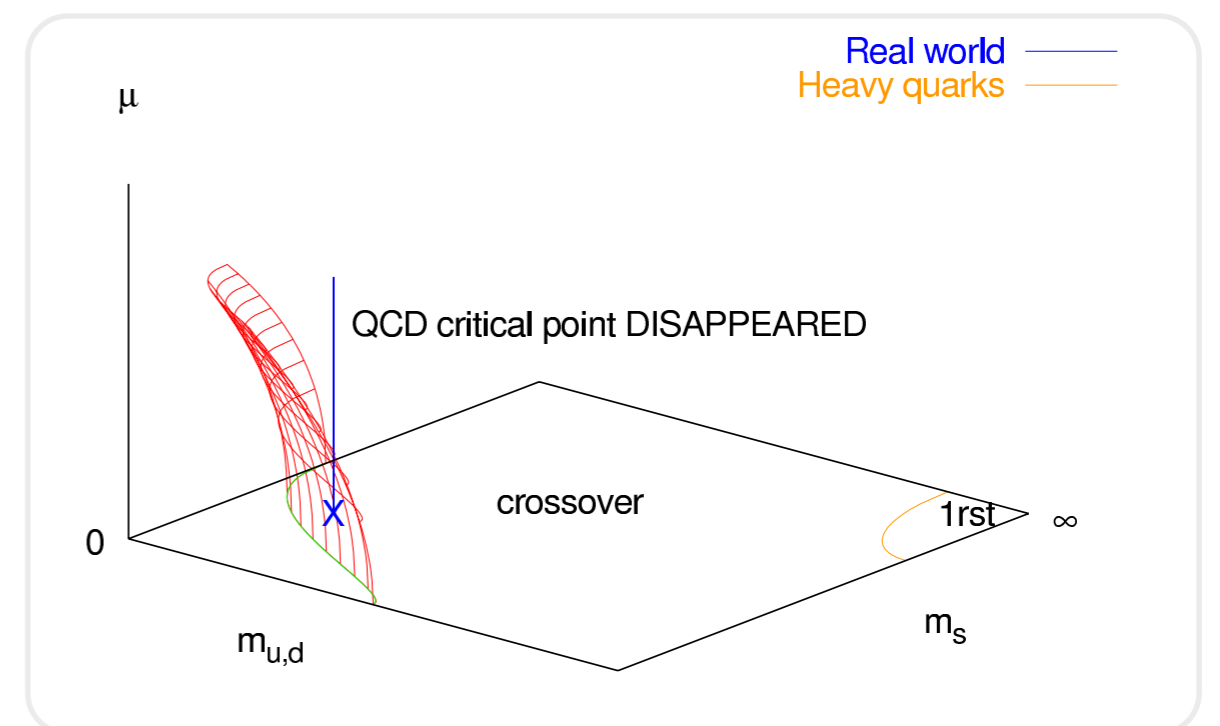
QCD transitions at the physical point



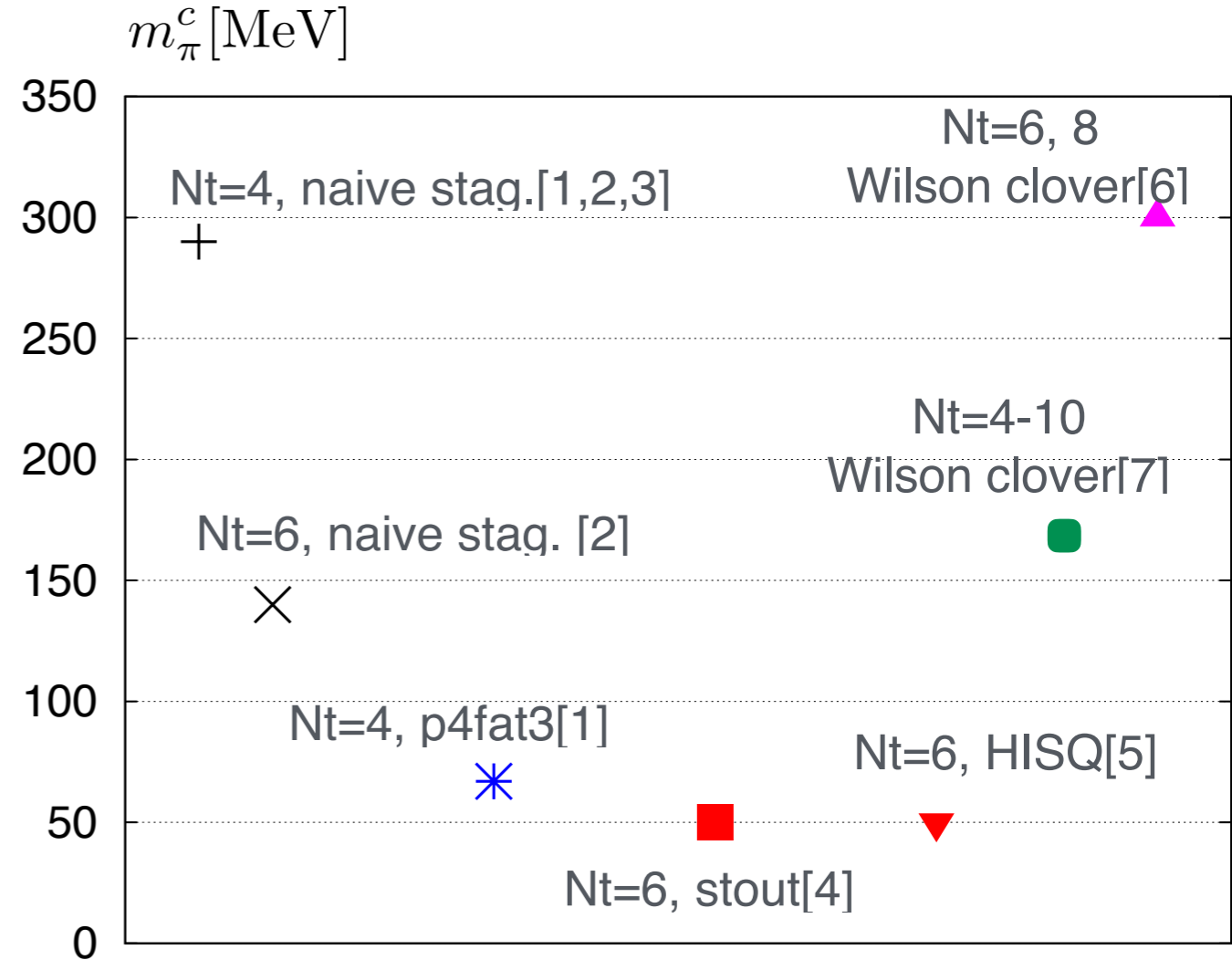
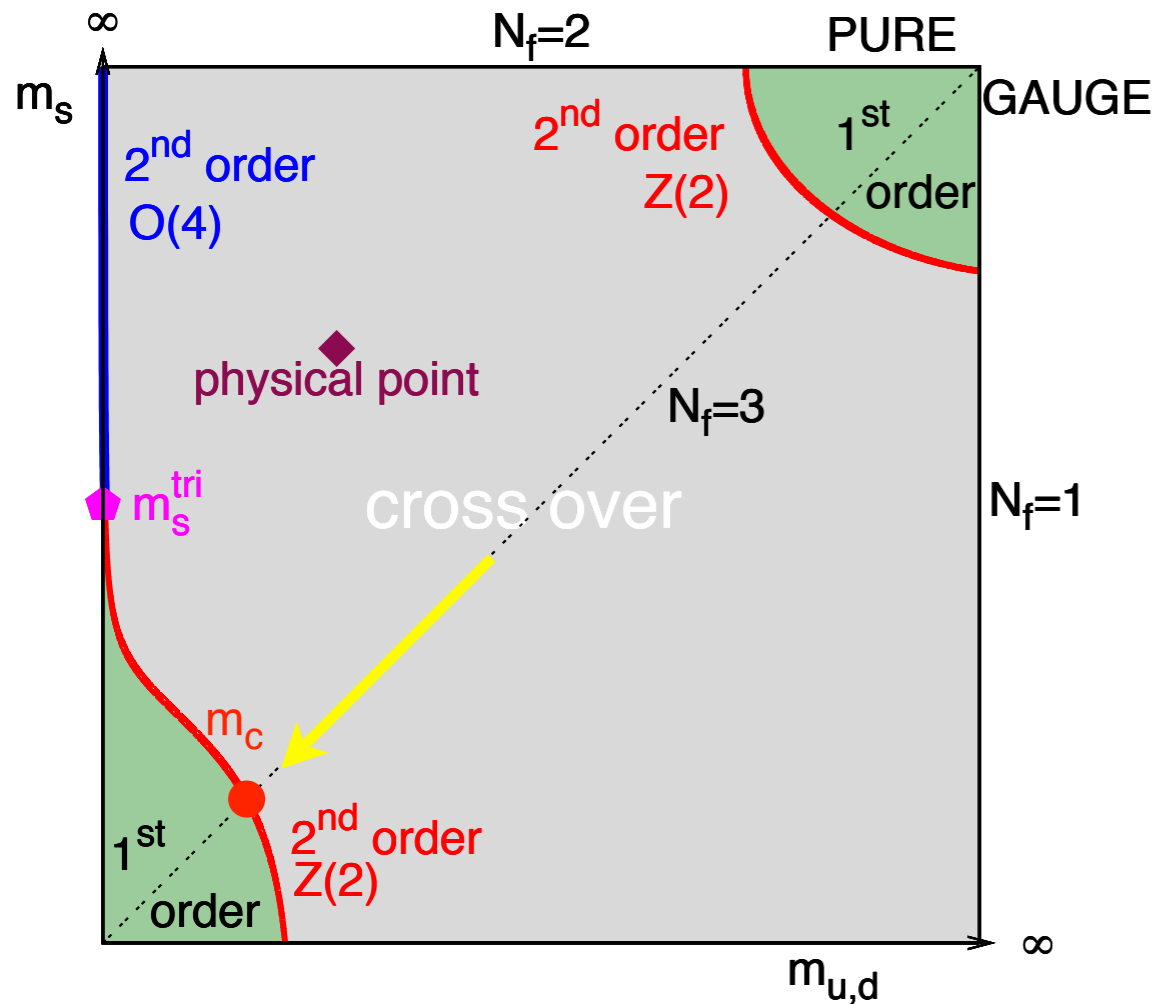
HTD, F. Karsch, S. Mukherjee, arXiv:1504.05274



Karsch et al., '03, X.-Y. Jin et al., '15



1st order chiral phase transition region



[1] F. Karsch et al., Nucl.Phys.Proc.Suppl. 129 (2004) 614

[3] D. Smith & C. Schmidt, Lattice 2011, arXiv:1109.6729

[5] Bazavov, HTD et al., Phys.Rev. D95 (2017) no.7, 074505

[7] Jin et al., PRD96 (2017) no.3, 034523

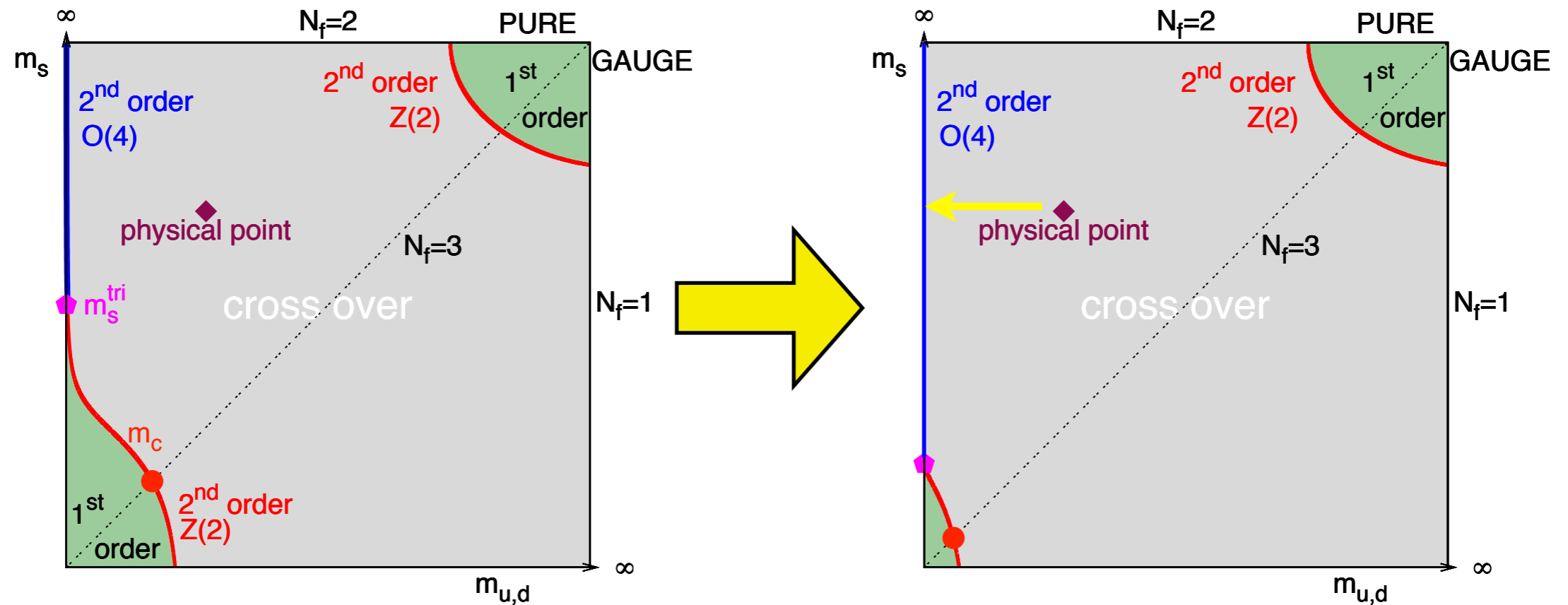
[2] P. de Forcrand et al, PoS LATTICE2007 (2007) 178

[4] G. Endrodi et al., PoS LAT2007 (2007) 228

[6] Y. Nakamura, Lattice 15', PRD92 (2015) no.11, 114511

The region shrinks from LQCD results with improved actions and finer lattices

Chiral phase transition region in $N_f=3$ QCD at $\mu_B=0$



1st order chiral phase transition seem to be not much relevant for thermodynamics at the physical point

How about the 2nd order $O(4)$ transition line?



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计算决定未来!

N: Nuclear **S**: Science **C³**: Color 3 -> QCD

“道生一，一生二，二生三，三生万物。” — 《道德经》老子 600 BC

”Tao gives birth to One, One gives birth to Two, Two gives birth to Three, Three gives birth to everything.“- Lao Tzu



18 computing nodes
(144 V100 GPUs)
Peak performance:
1 PFlops/s
Storage:
1 PB





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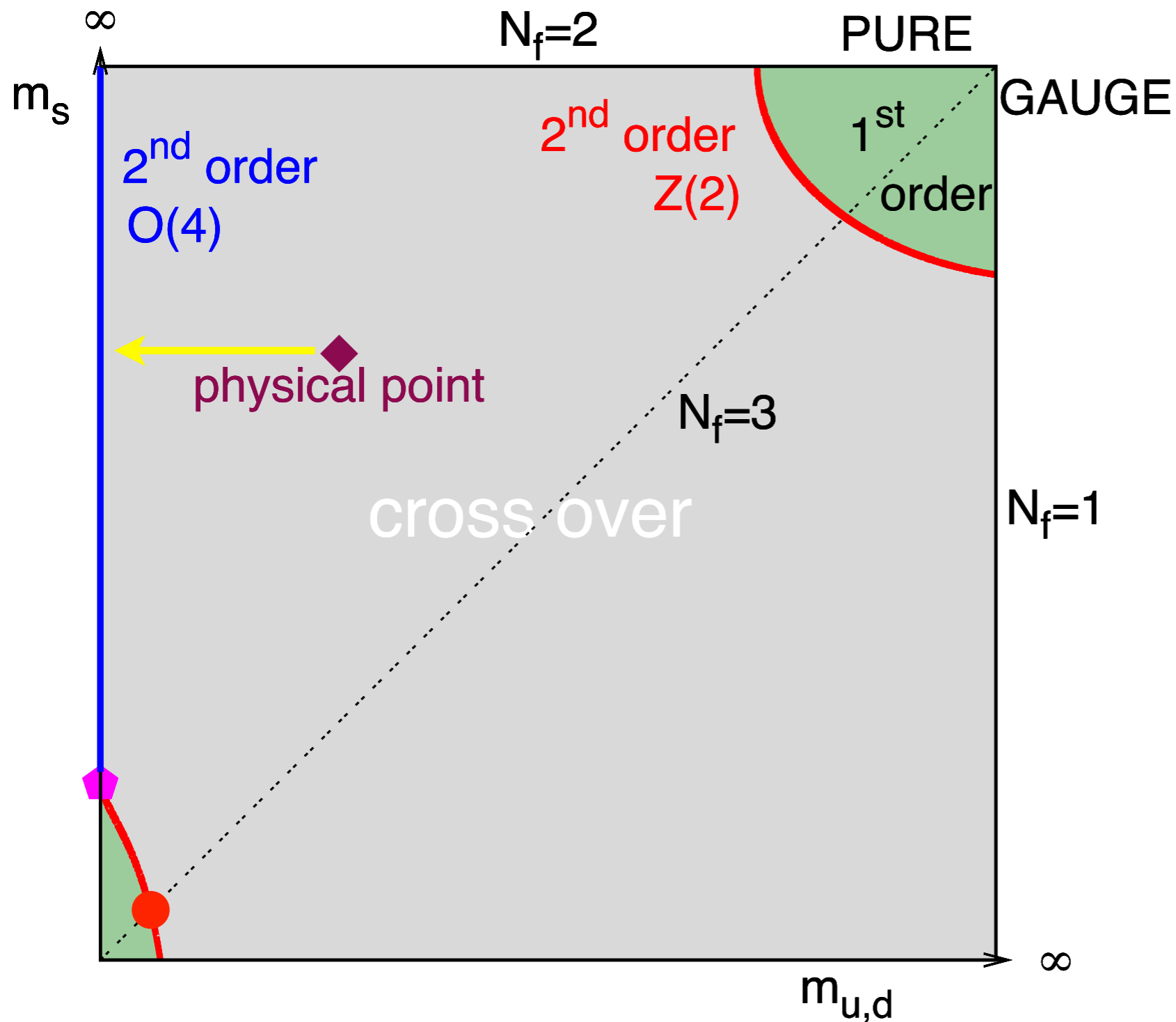


18 computing nodes
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1 PFlops/s
Storage:
1 PB

On 1-single GPU:
64³x16, 72³x12
8 GPUs in each node
8 nodes in each rack



Towards chiral limit of (2+1)-flavor QCD

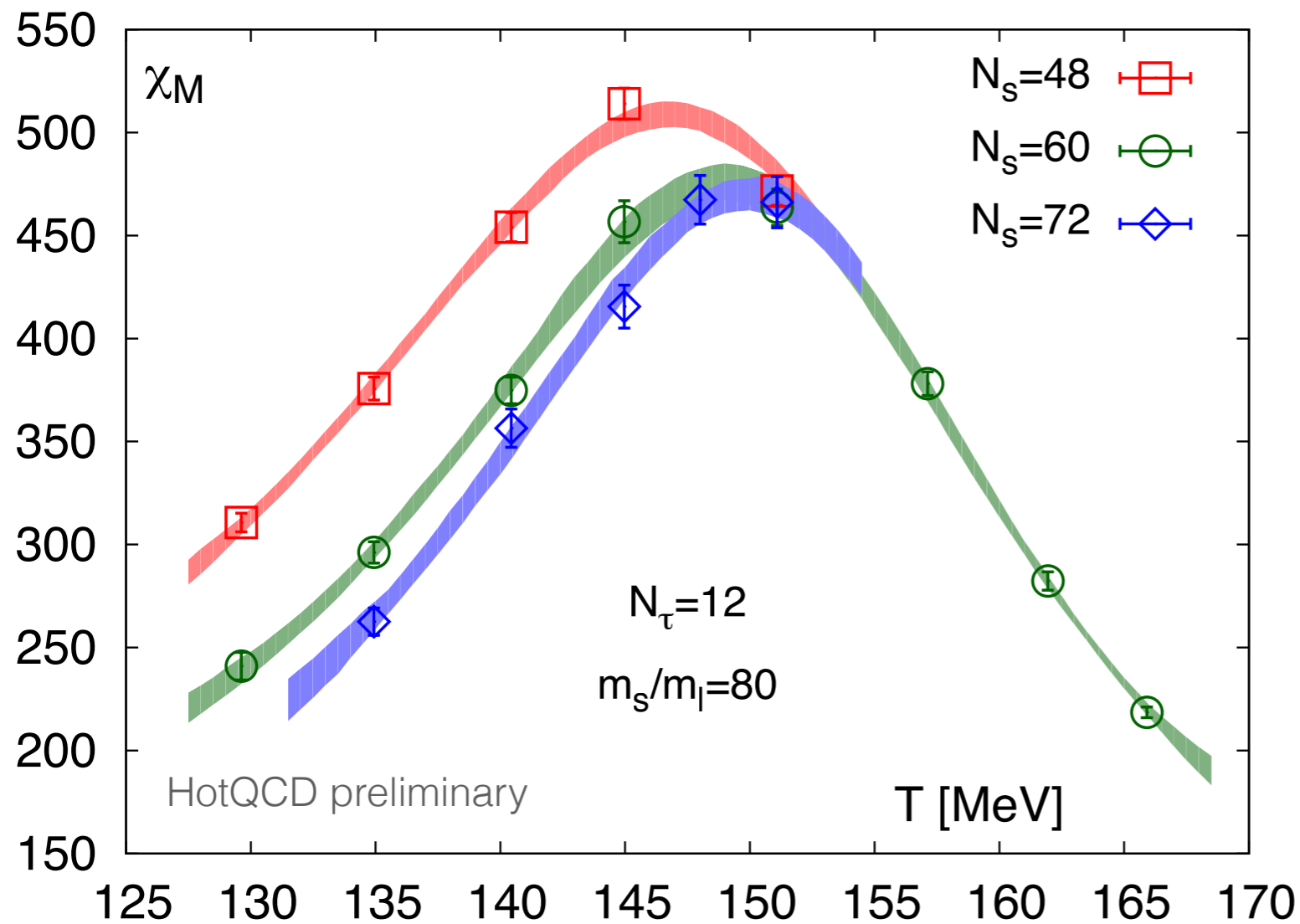


$$m_u = m_d \rightarrow 0$$

$$m_s = m_s^{\text{phy}}$$

Volume dependence of chiral susceptibility

$m_\pi=80$ MeV



No direct evidence of 1st order phase transition down to $m_\pi = 80$ MeV

hot from the machine

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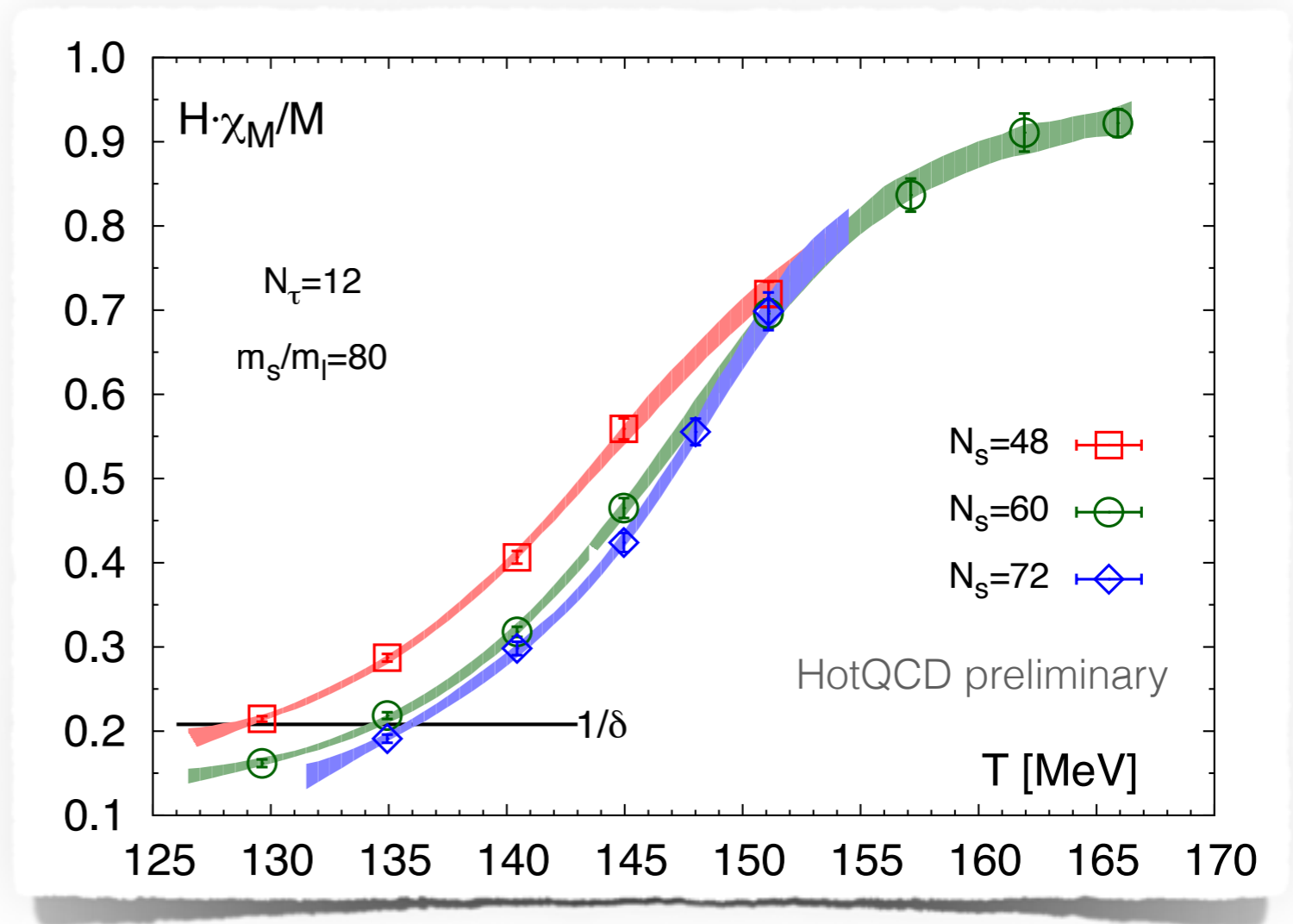
Chiral phase transition temperature T_c

$$H \chi_M / M \rightarrow 1/\delta @ T_c^0$$

H: m_l/m_s

M: chiral condensate

χ_M : chiral susceptibility



hot from the machine



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A novel approach to estimate T_c^0

• Pseudo-critical temperature at H

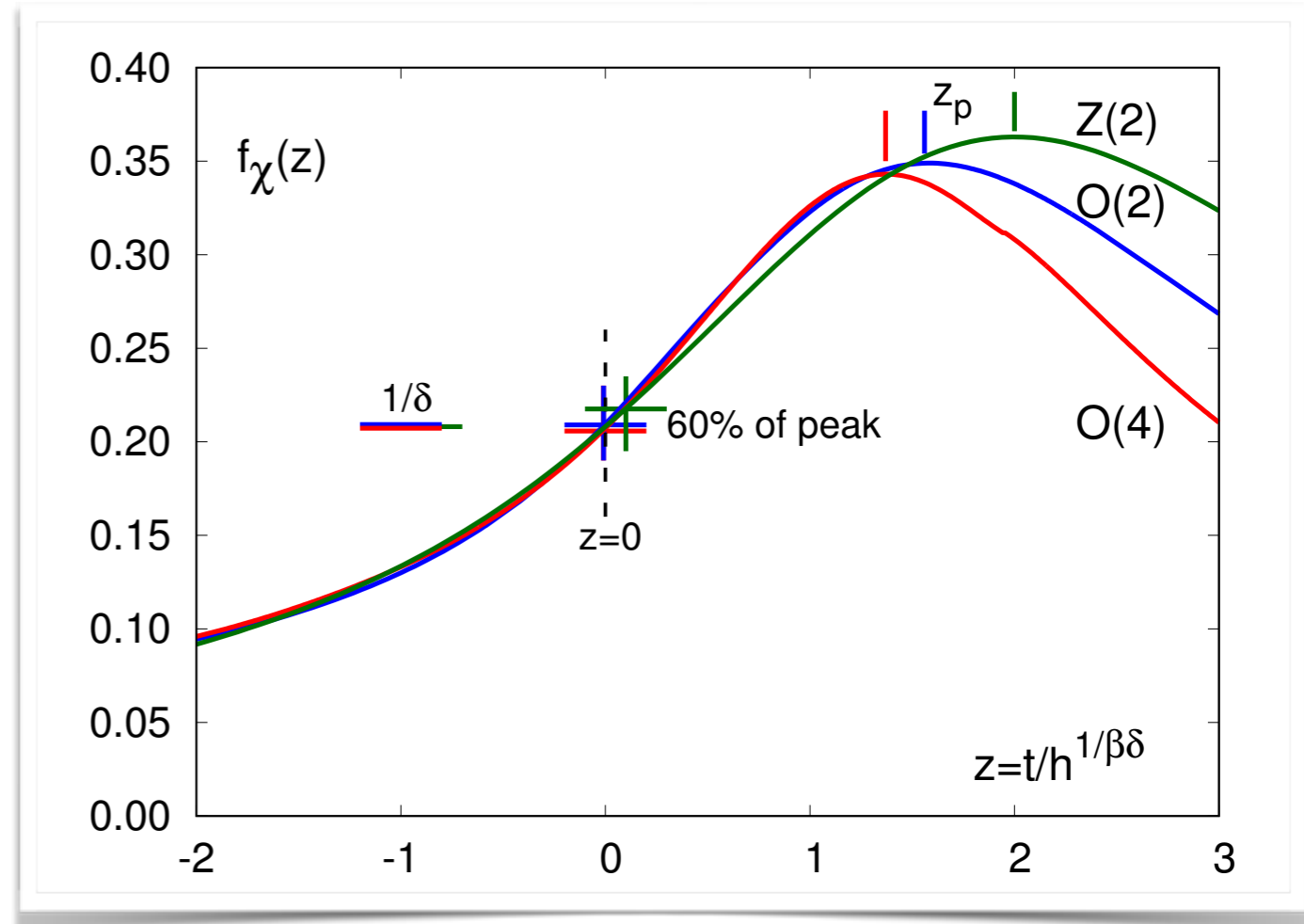
$$T_{pc}(H) = T_c^0 \left(1 + \frac{z_p}{z_0} H^{\frac{1}{\beta\delta}} \right)$$

• Estimate of the chiral transition T_c^0

$$T_{60\%}^-(H) = T_c^0 \left(1 + \frac{z_{60\%}^-}{z_0} H^{\frac{1}{\beta\delta}} \right)$$

☑ small quark mass dependence

☑ small variations among universality classes

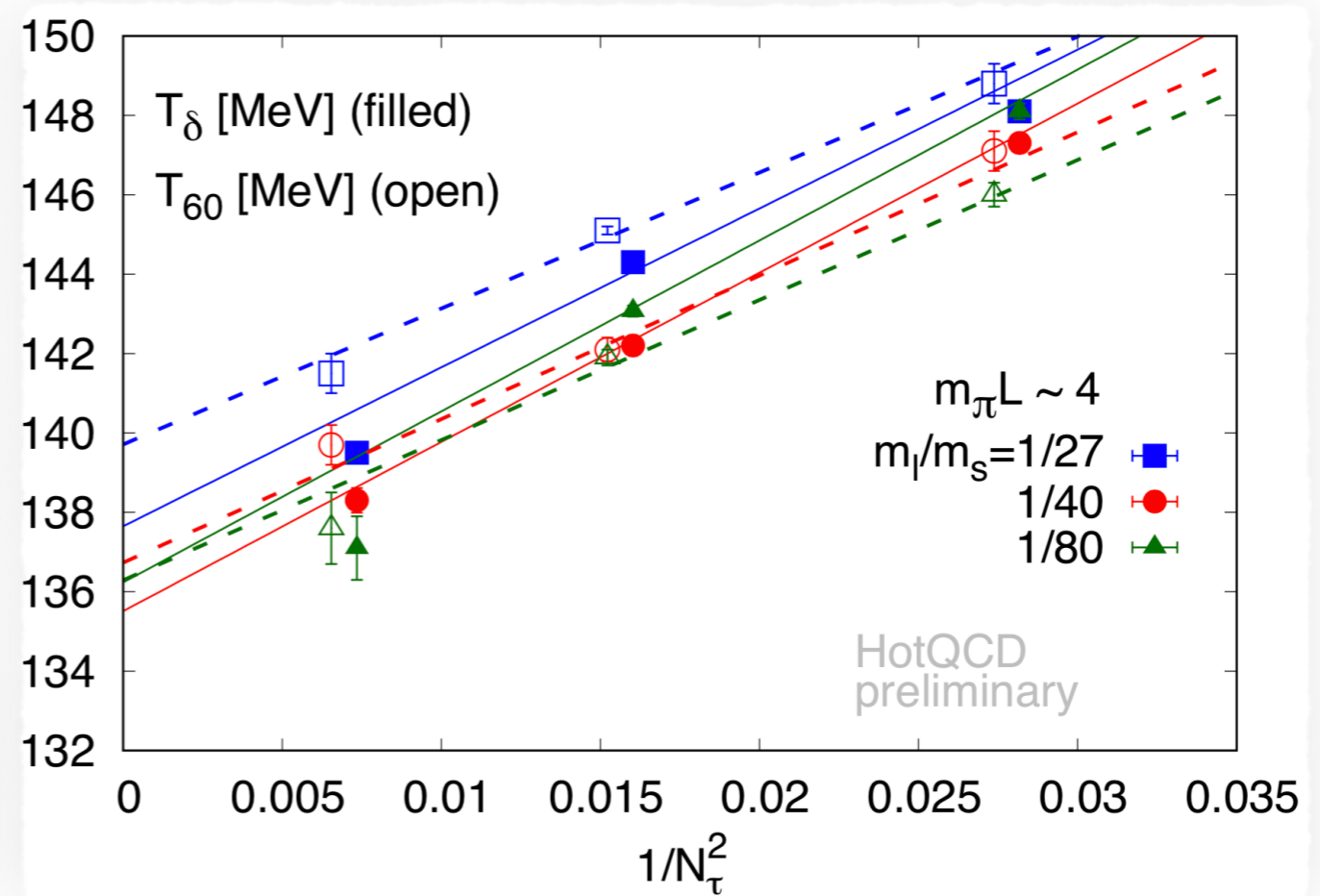
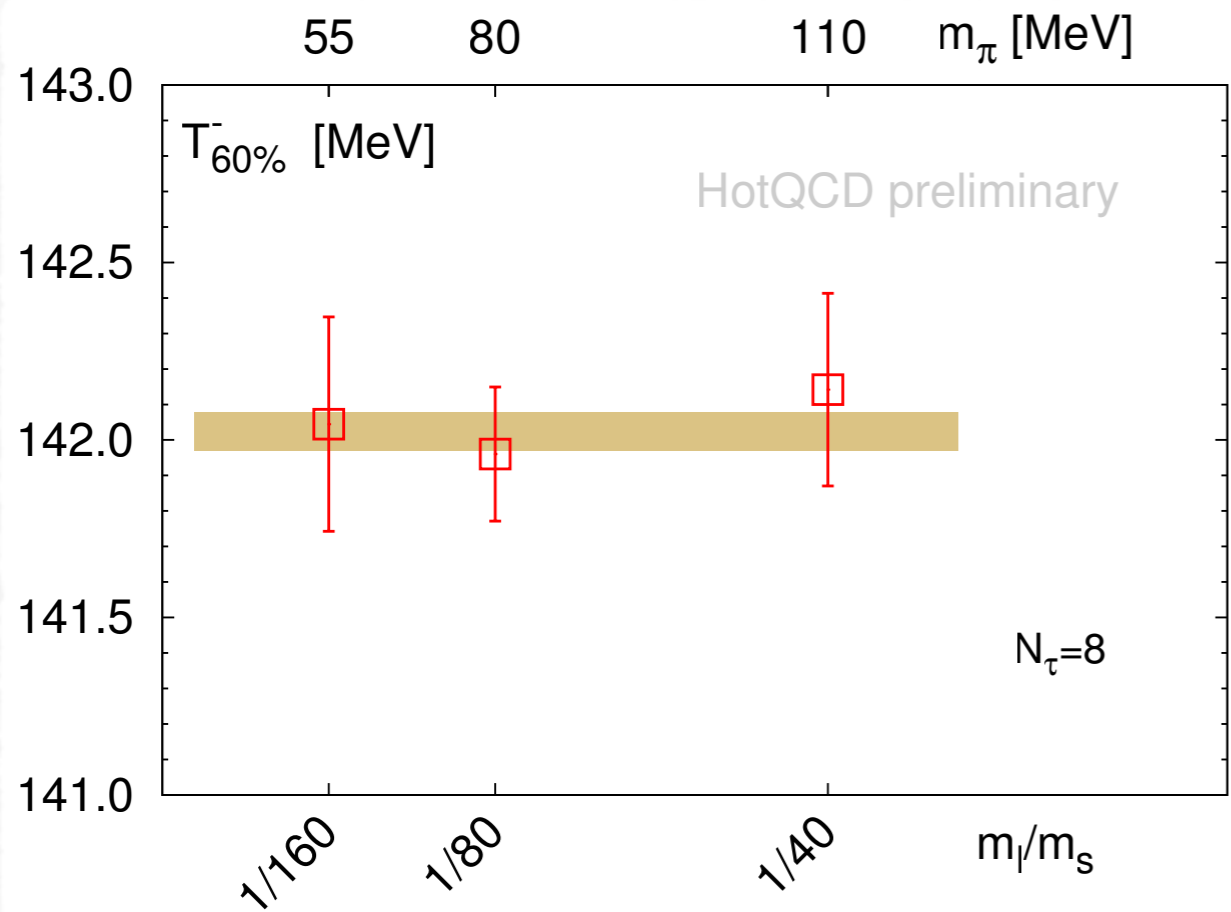


z_p : peak location of the susceptibility

$z_{60\%}^-$: location of 60% of peak height from left

	δ	z_p	$z_{60\%}^-$
Z(2)	4.805	2.00(5)	0.10(1)
O(2)	4.780	1.58(4)	-0.005(9)
O(4)	4.824	1.37(3)	-0.013(7)

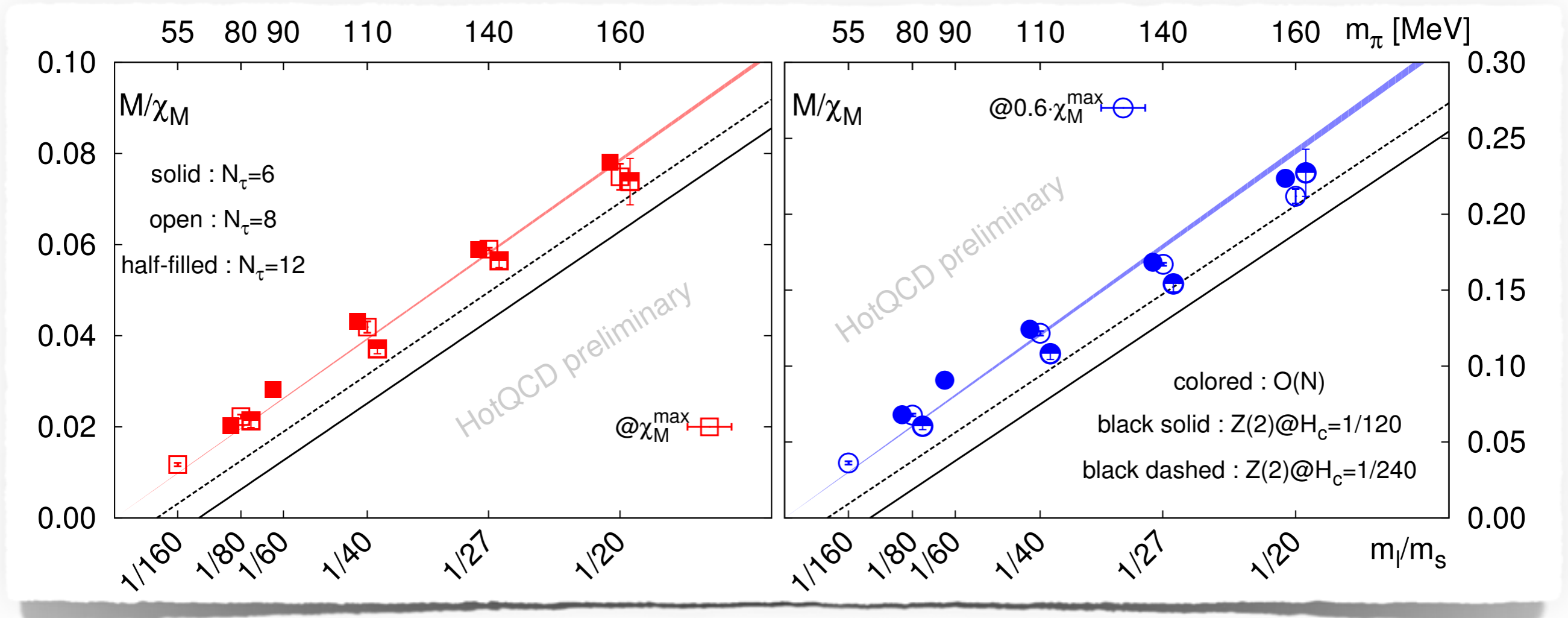
Estimate of chiral phase transition T_c



S.-T. Li, Lattice 2018, A. Lahiri, QM 2018

$T_c \approx 135 \text{ MeV}$

Consistency of QCD chiral phase transition with $O(N)$ universality class



S.-T. Li, Lattice 2018, A. Lahiri, QM 2018

$$M/\chi_M = \frac{m_l - m_l^{\text{critical}}}{m_s^{\text{phys}}} \frac{f_M}{f_{\chi_M}}$$

Taylor expansion coefficients of QCD pressure at $\mu_B \neq 0$

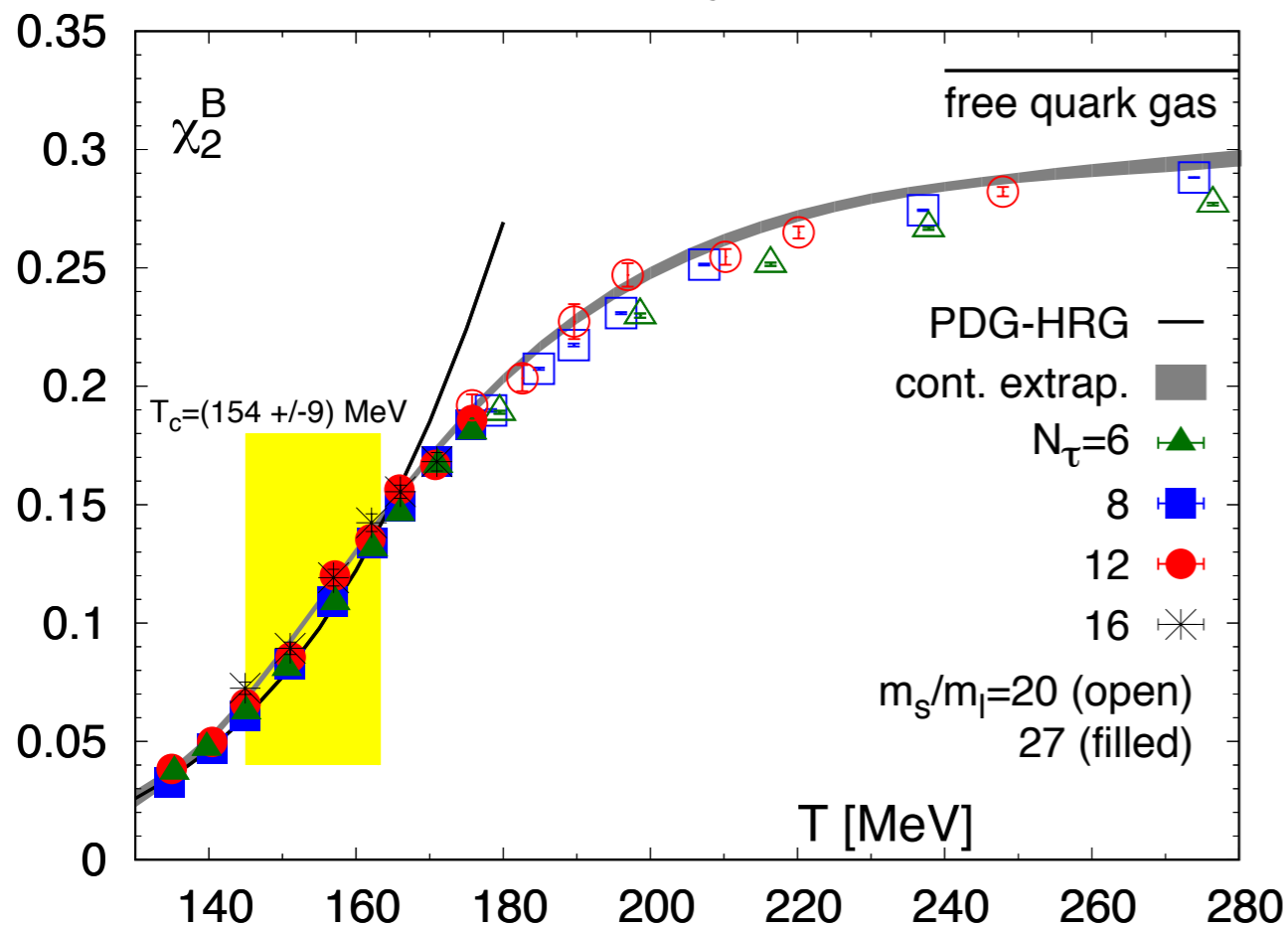
Allton et al., Phys.Rev. D66 (2002) 074507
 Gavai & Gupta, Phys.Rev. D68 (2003) 034506

$\mu_Q = \mu_S = 0$:

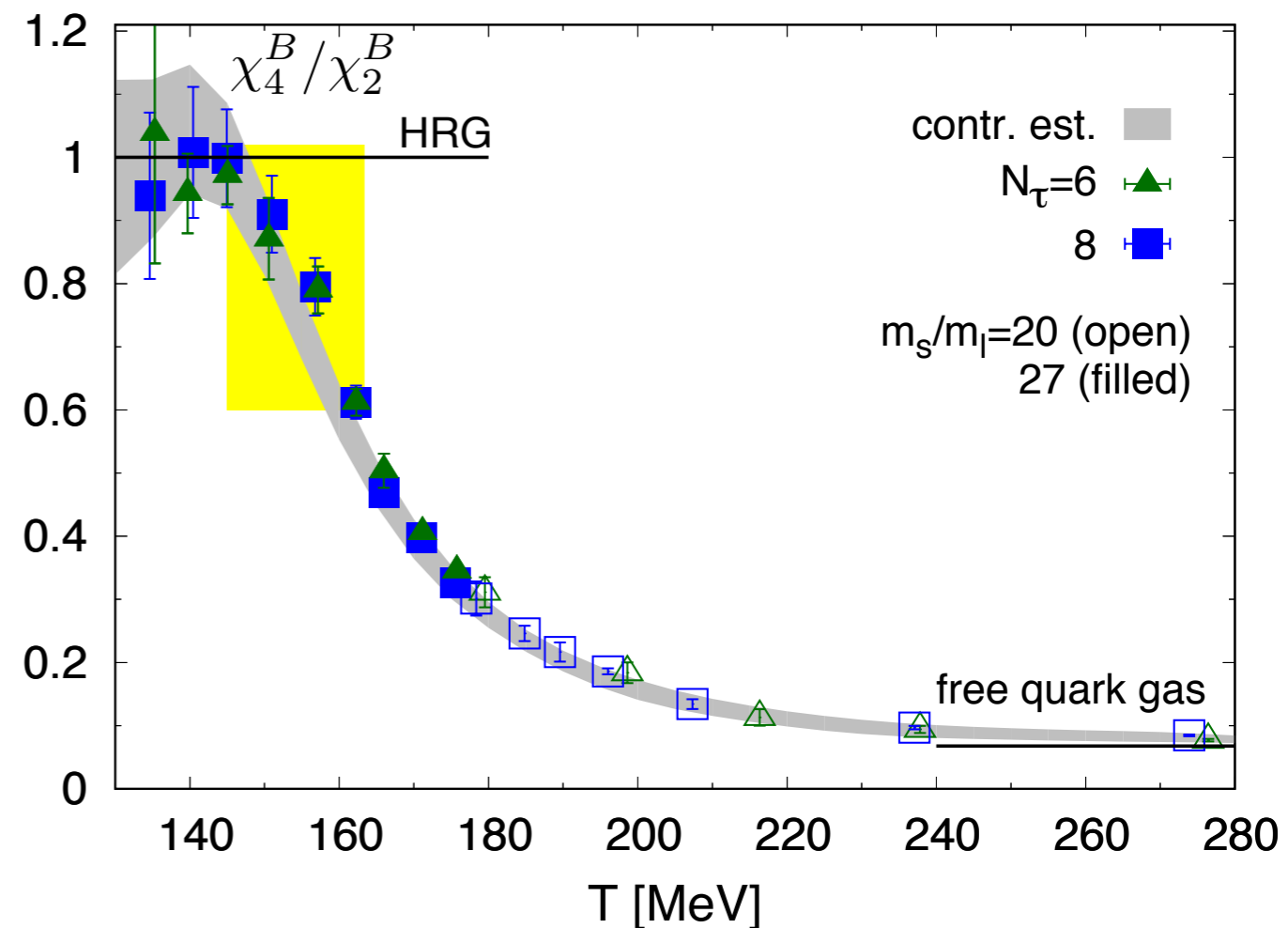
$$\Delta(P/T^4) = \frac{P(T, \mu_B) - P(T, 0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

$$= \frac{1}{2} \chi_2^B(T) \hat{\mu}_B^2 \left(1 + \frac{1}{12} \frac{\chi_4^B(T)}{\chi_2^B(T)} \hat{\mu}_B^2 + \frac{1}{360} \frac{\chi_6^B(T)}{\chi_2^B(T)} \hat{\mu}_B^4 + \dots \right)$$

LO expansion coefficient
 variance of net-baryon number distri.



NLO expansion coefficient
 kurtosis * variance



Bielefeld-BNL-CCNU, Phys.Rev. D95 (2017) no.5, 054504

Taylor expansion coefficients of QCD pressure at $\mu_B \neq 0$

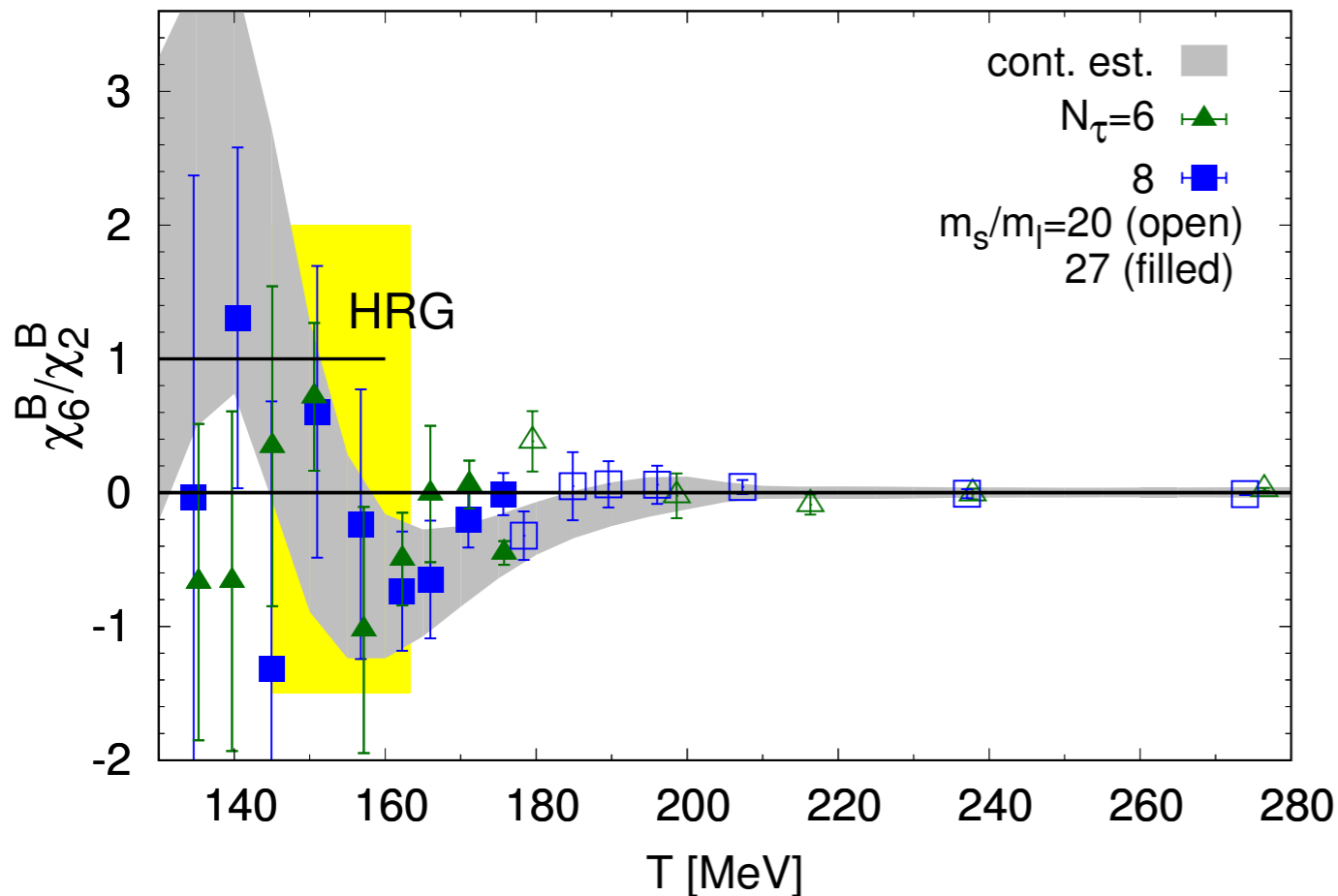
Allton et al., Phys.Rev. D66 (2002) 074507
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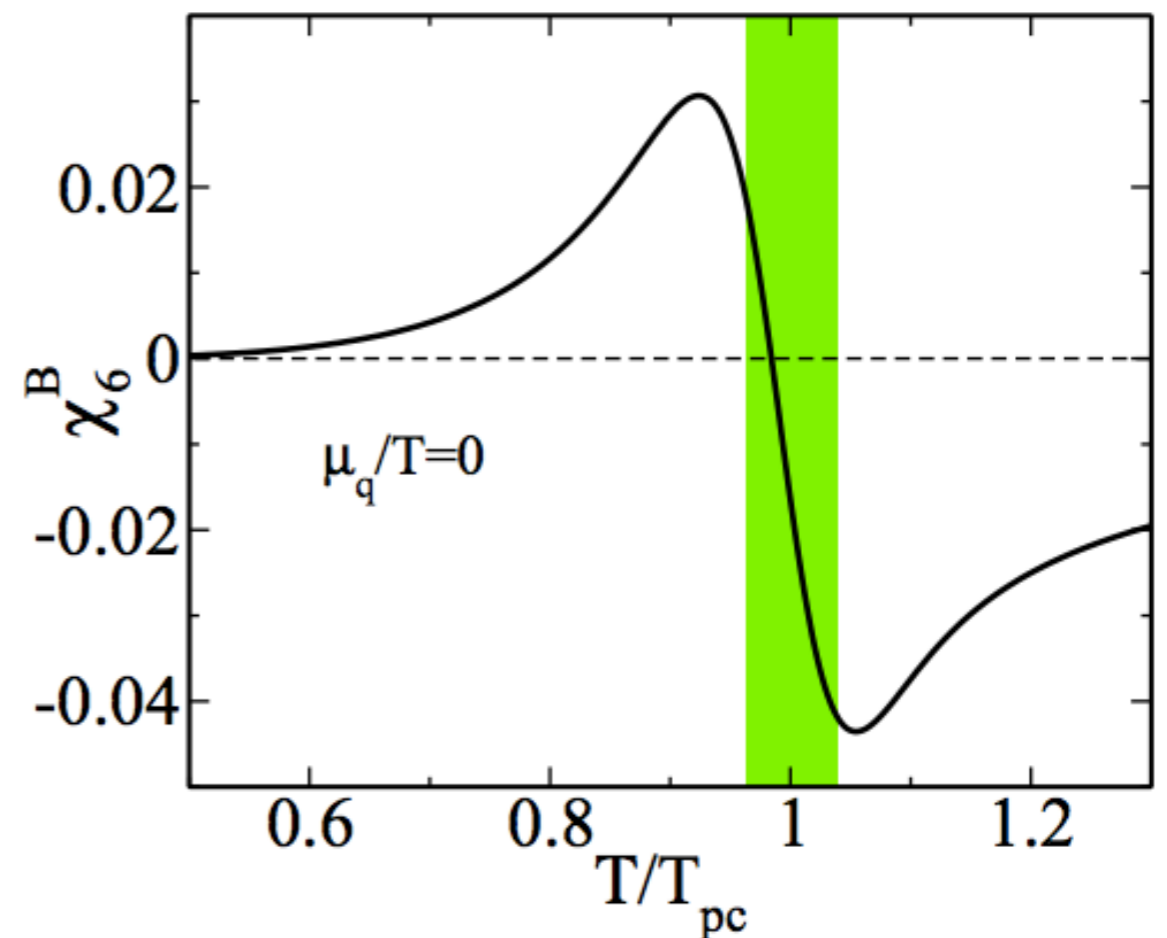
$$= \frac{1}{2} \chi_2^B(T) \hat{\mu}_B^2 \left(1 + \frac{1}{12} \frac{\chi_4^B(T)}{\chi_2^B(T)} \hat{\mu}_B^2 + \frac{1}{360} \frac{\chi_6^B(T)}{\chi_2^B(T)} \hat{\mu}_B^4 + \dots \right)$$

NNLO expansion coefficient



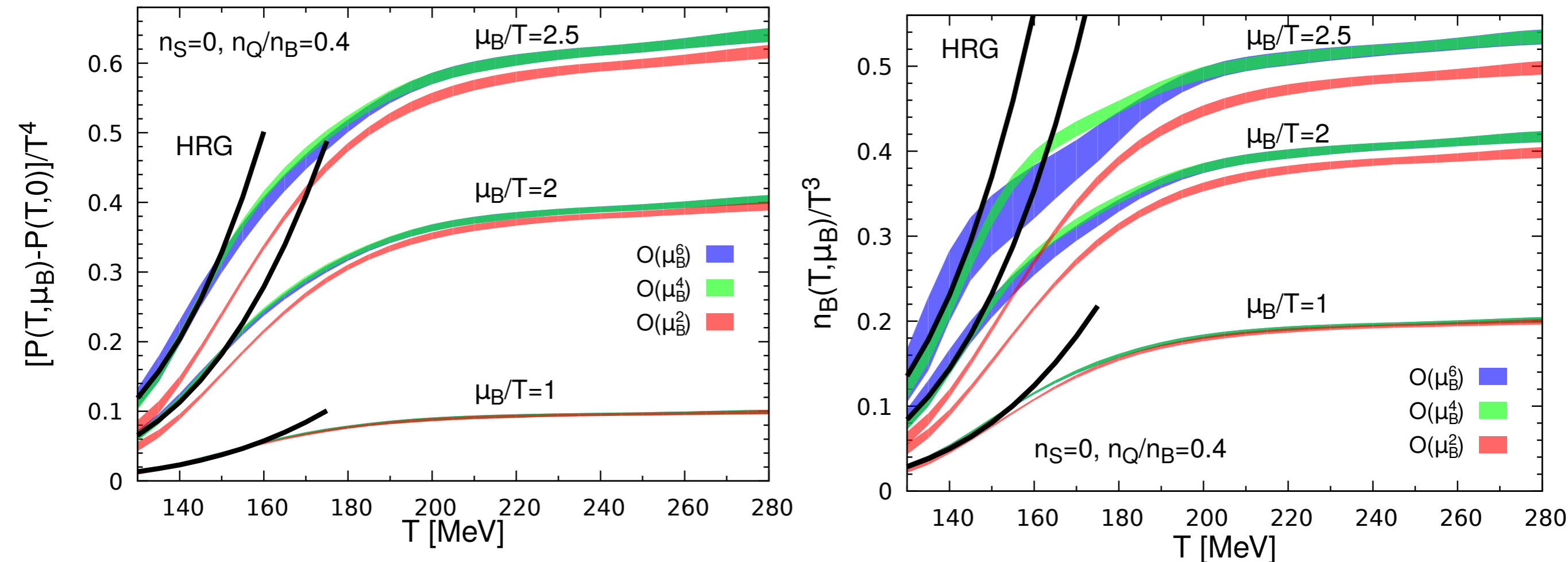
Bielefeld-BNL-CCNU, Phys.Rev. D95 (2017) no.5, 054504

PQM with O(4) symmetry



B. Friman et al., EPJC71 (2011) 1694

Pressure and baryon number density in the strangeness neutral case



Bielefeld-BNL-CCNU, Phys.Rev. D95 (2017) no.5, 054504

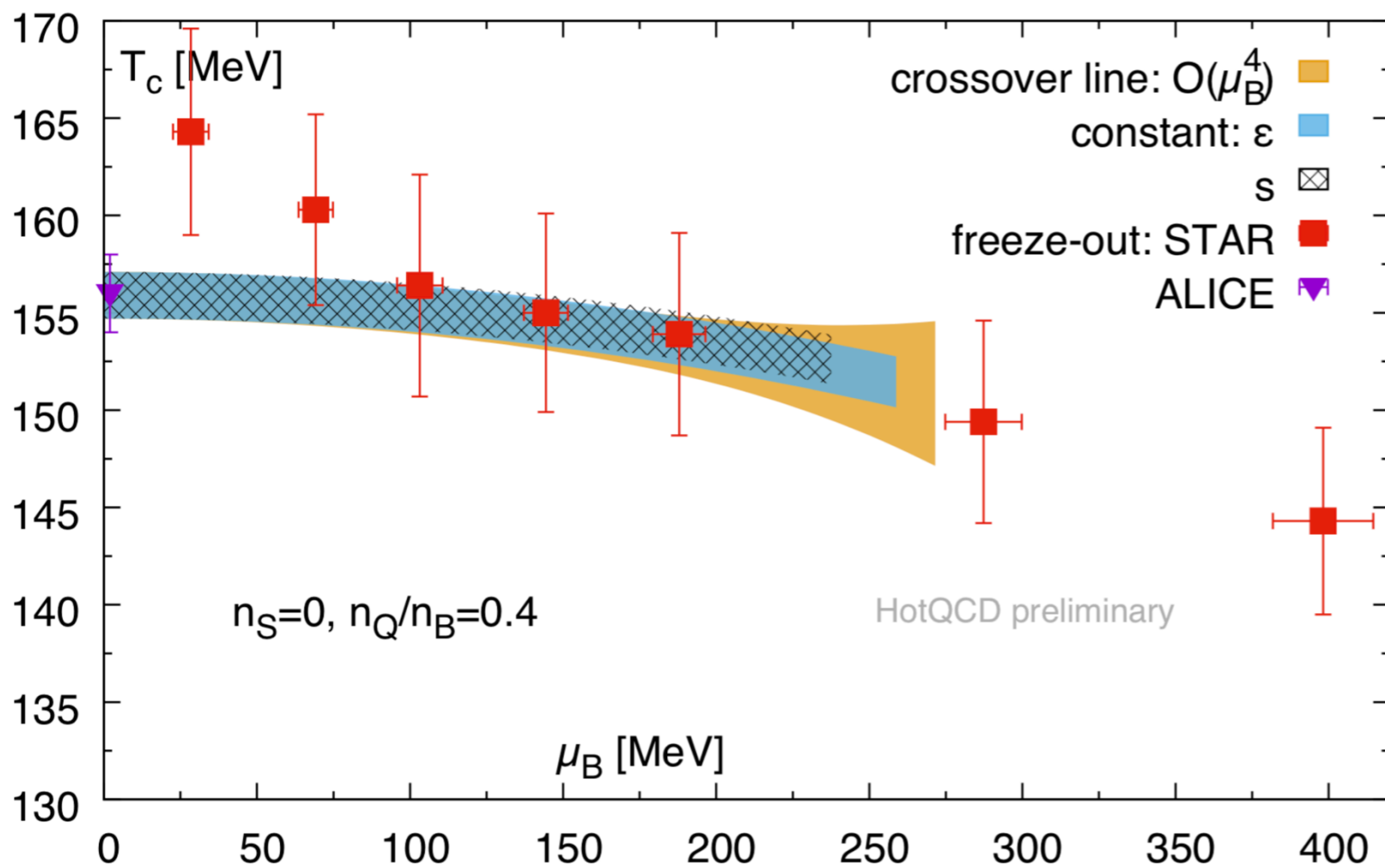
The EoS is well under control at $\mu_B/T \lesssim 2$ or $\sqrt{s_{NN}} \gtrsim 12$ GeV

Consistent results obtained using analytic continuations
from the imaginary μ

Wuppertal-Budapest-Houston:
EPJ Web Conf. 137(2017) 07008

Crossover, line of constant physics & freeze-out

$$T(\mu_B) = T(0) \left(1 - \kappa_2 \left(\frac{\mu_B}{T} \right)^2 - \kappa_4 \left(\frac{\mu_B}{T} \right)^4 + \mathcal{O} \left(\frac{\mu_B}{T} \right)^6 \right)$$



curvature of crossover line

HotQCD preliminary

$$\kappa_2 = 0.0123 \pm 0.003$$

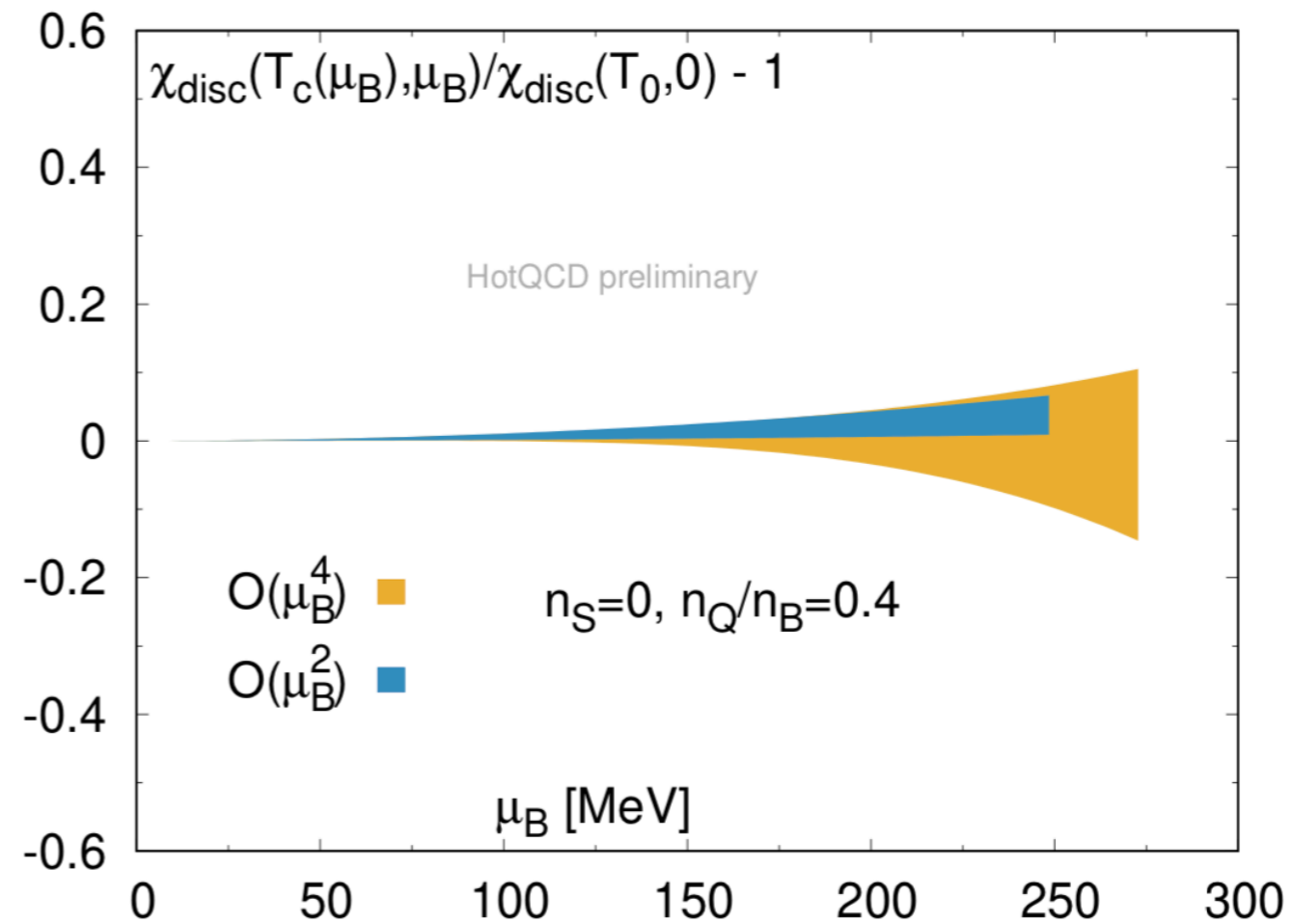
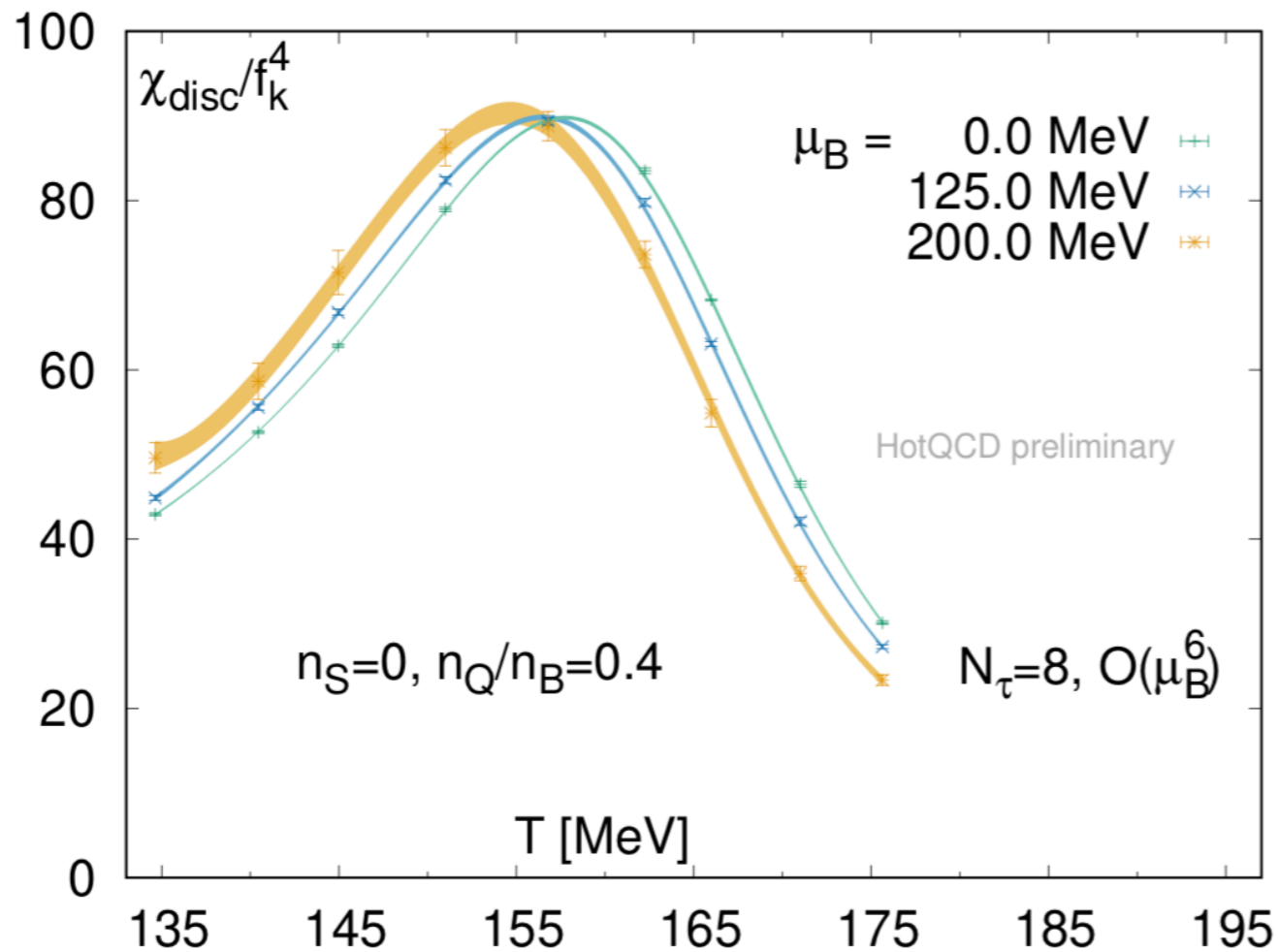
$$\kappa_4 = 0.000131 \pm 0.0041$$

curvature at constant b:

$$0.006 \leq \kappa_2^b \leq 0.012, \quad b = P, \epsilon, s$$

Bielefeld-BNL-CCNU, PRD95 (2017) no.5, 054504

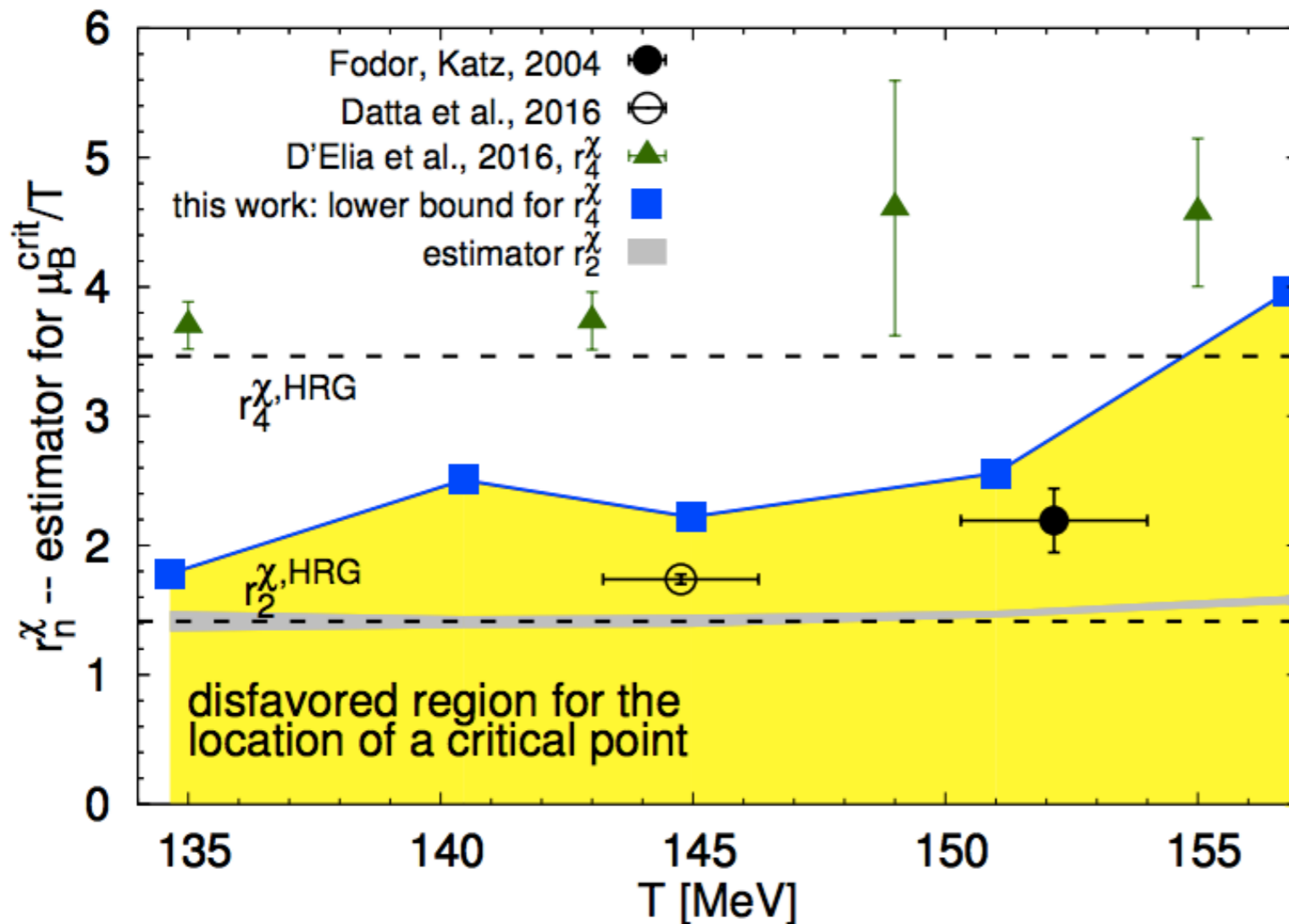
Order Parameter Susceptibility at $\mu_B \neq 0$



No indication of a stronger phase transition at larger μ_B

Estimates of the radius of convergence

$$\text{radius of convergence} = \lim_{n \rightarrow \infty} r_{2n}^\chi = \lim_{n \rightarrow \infty} \left| \frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B} \right|^{1/2}$$



HISQ + Taylor Exp. (this work):
 Nf=2+1, Nt=8
 Bielefeld-BNL-CCNU,
 PRD 95 (2017) no.5, 054504

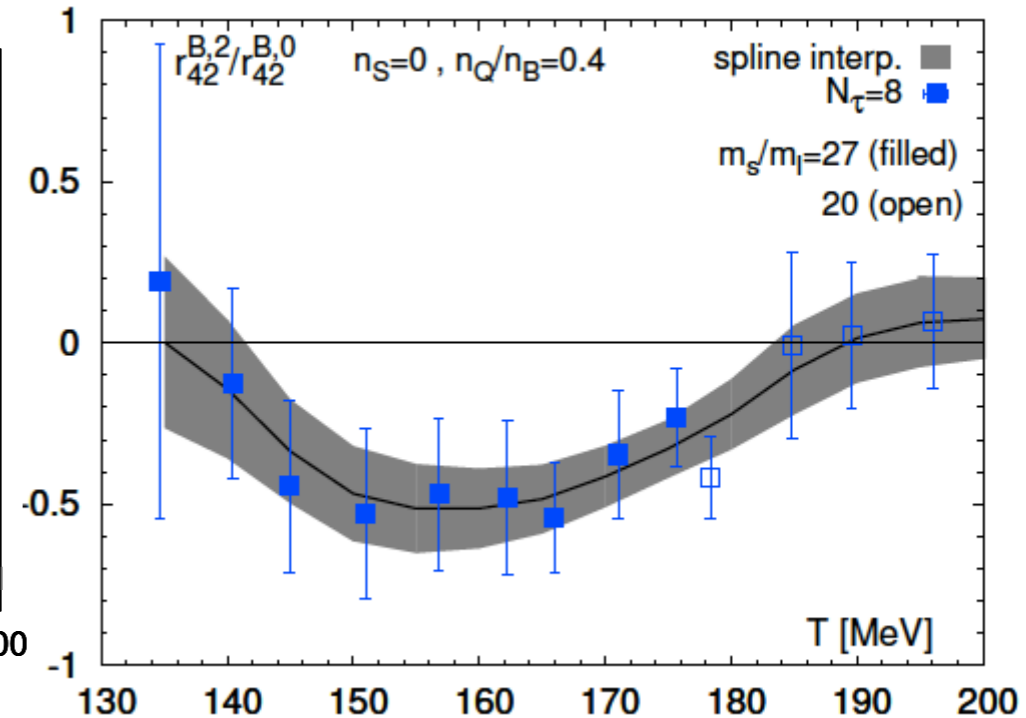
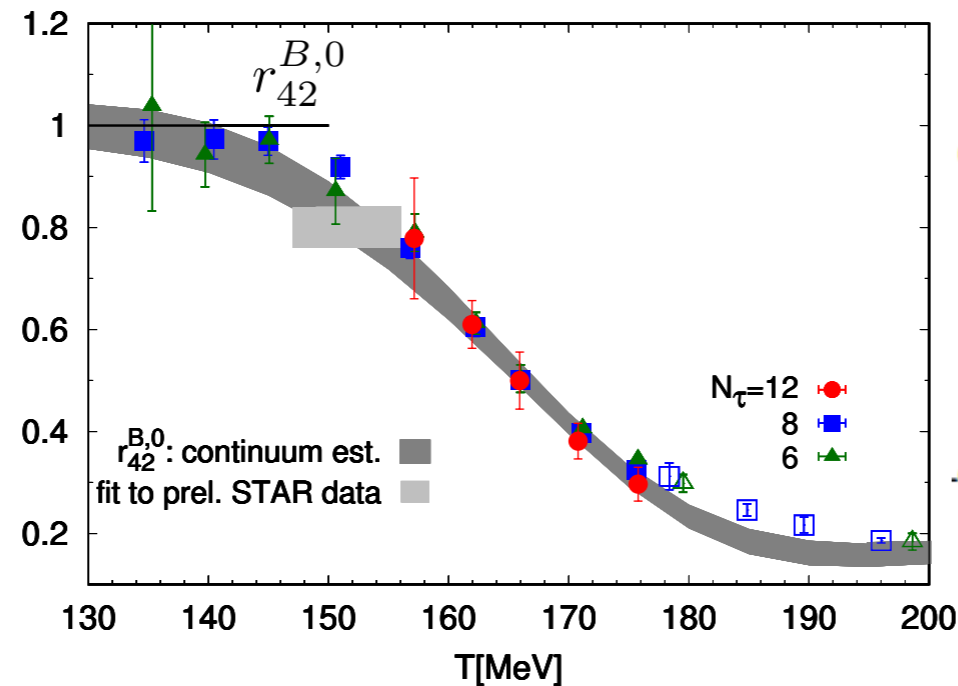
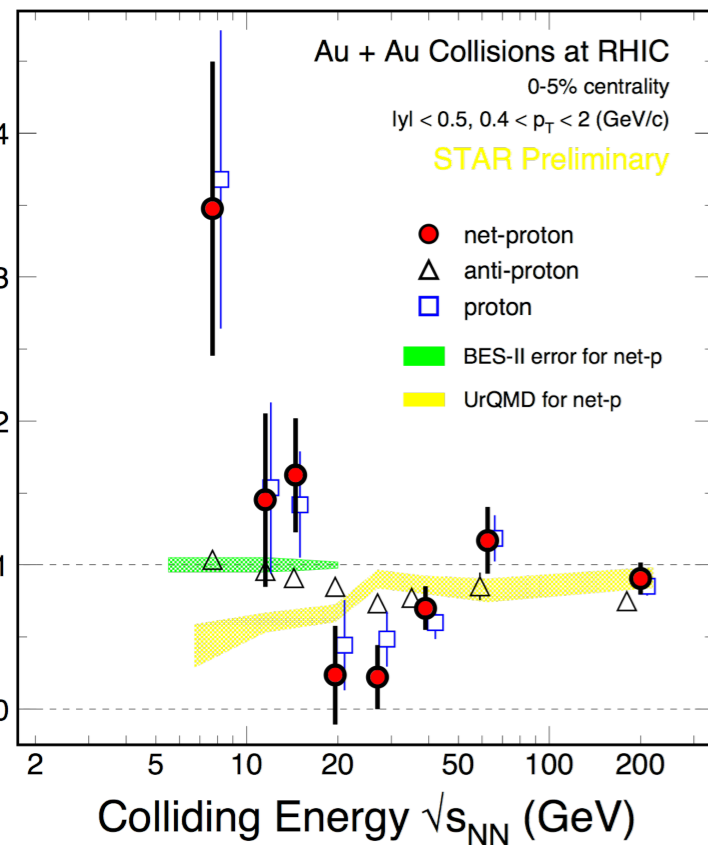
stout + Img. mu:
 Nf=2+1, Nt=8
 D'Elia et al., PRD 95 (2017) 094503

unimproved staggered + Taylor Exp.:
 Nf=2, Nt=4,6,8
 Datta et al., PRD 95 (2017) 054512

unimproved staggered + Reweighting:
 Nf=2+1, Nt=4
 Fodor and Katz, JHEP 0404 (2004) 050

A QCD critical point is disfavored at $\mu_B/T \lesssim 2$ at $T \gtrsim 135$ MeV

Cumulant ratios of proton (baryon) number fluctuations: HIC data v.s. Lattice results



Bazavov et al., [HotQCD], Phys.Rev. D96 (2017) no.7, 074510

**strangeness neutral
case:**

$$(\kappa\sigma^2)_B = \frac{\chi_{4,\mu}^{B,SN}}{\chi_{2,\mu}^{B,SN}} = r_{42}^{B,0} \left[1 + \frac{r_{42}^{B,2}}{r_{42}^{B,0}} \left(\frac{\mu_B}{T} \right)^2 + \dots \right]$$

$\sqrt{s_{NN}} \gtrsim 20$ GeV:

$\kappa\sigma^2$ is consistent with QCD in equilibrium

Conclusions

- ☑ The chiral T_c of $N_f=2+1$ QCD is ~ 135 MeV, and the $O(N)$ universality class of the chiral phase transition is preferred
- ☑ EoS from Taylor expansions of QCD partition functions are now reliable in the region $\mu_B/T \approx 2$ or $\sqrt{s_{NN}} \gtrsim 12$ GeV
- ☑ Properties of cumulants measured in BES-I for $\sqrt{s_{NN}} \gtrsim 20$ GeV clearly differs from HRG thermodynamics but are consistent to QCD thermodynamics close to the transition region
- ☑ A QCD critical point is disfavored at $\mu_B/T \approx 2$ at $T \gtrsim 135$ MeV

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Wuhan, China

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QM in HIC community

~ 400 participants annually

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to join us!

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- Theoretical Developments
- Vacuum Structure and Confinement
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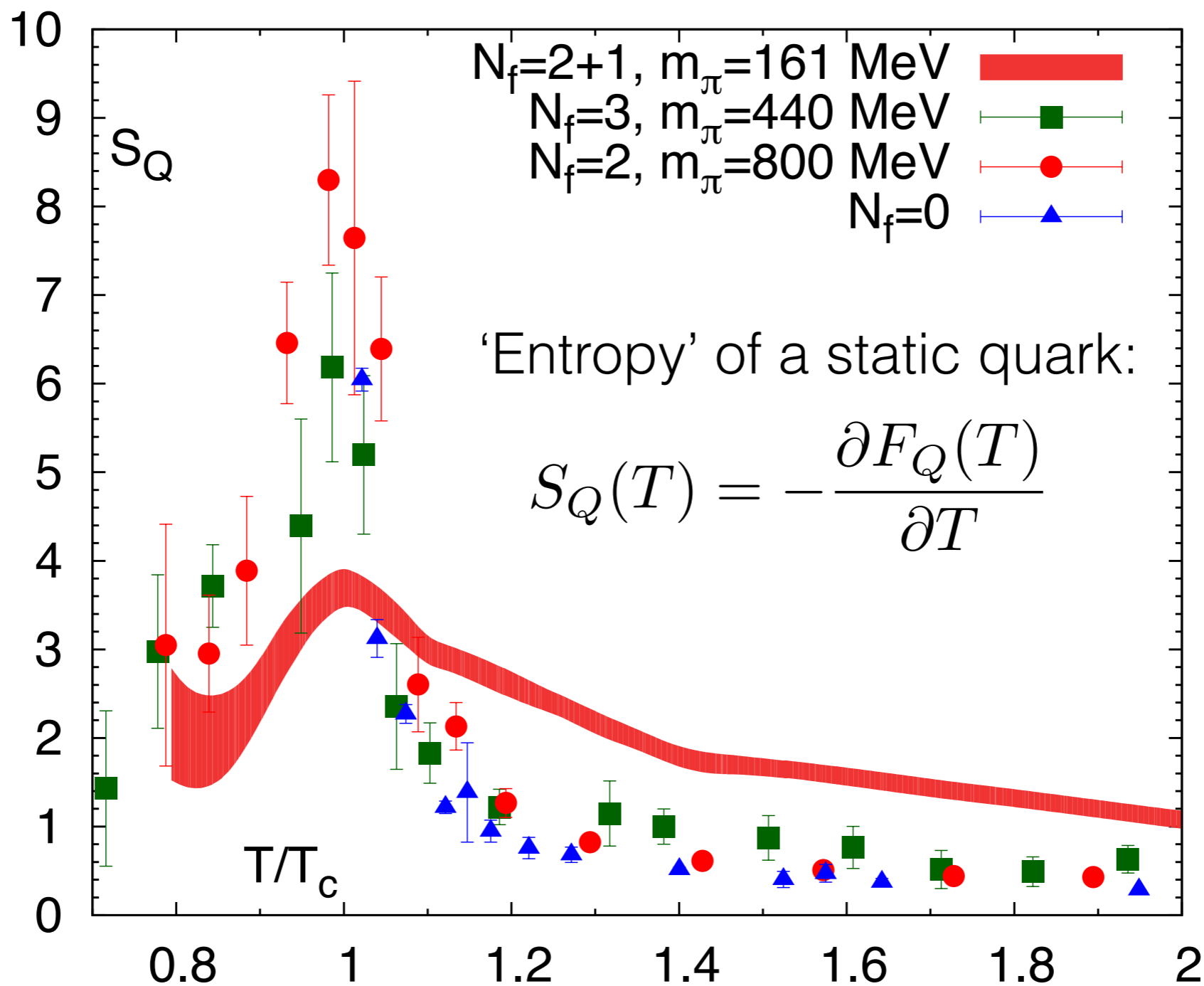


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谢谢!

Thanks for your attention!

Possible connections between deconfinement & chiral aspects of the cross over



A. Bazavov, N. Brambilla, HTD et al., PRD93 (2016) no.11, 114502

chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$$f(m, T) = h^{1+1/\delta} f_s(z), \quad z = t/h^{1/\beta\delta}$$

h : external field, t : reduced temperature, β, δ : universal critical exponents

$f_s(z)$: universal scaling function, O(N) etc.

$$h = \frac{|m_l|}{h_0 m_s}$$

$$t = \frac{T - T_c}{T_c}$$

Magnetic Equation of State (MEoS):

$$M = -\partial f_s(t, h) / \partial H = h^{1/\delta} f_G(z) \quad \chi_M = \partial M / \partial H = \frac{h^{1/\delta}}{H} \left(f_G(z) - \frac{z}{\beta} \frac{df_G(z)}{dz} \right)$$

