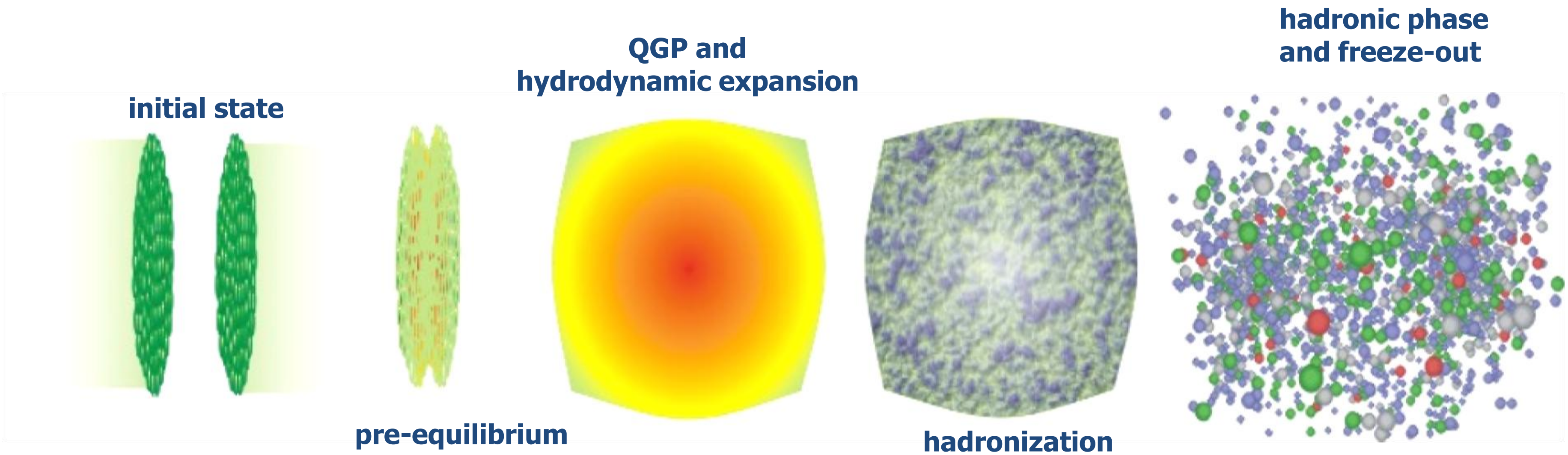
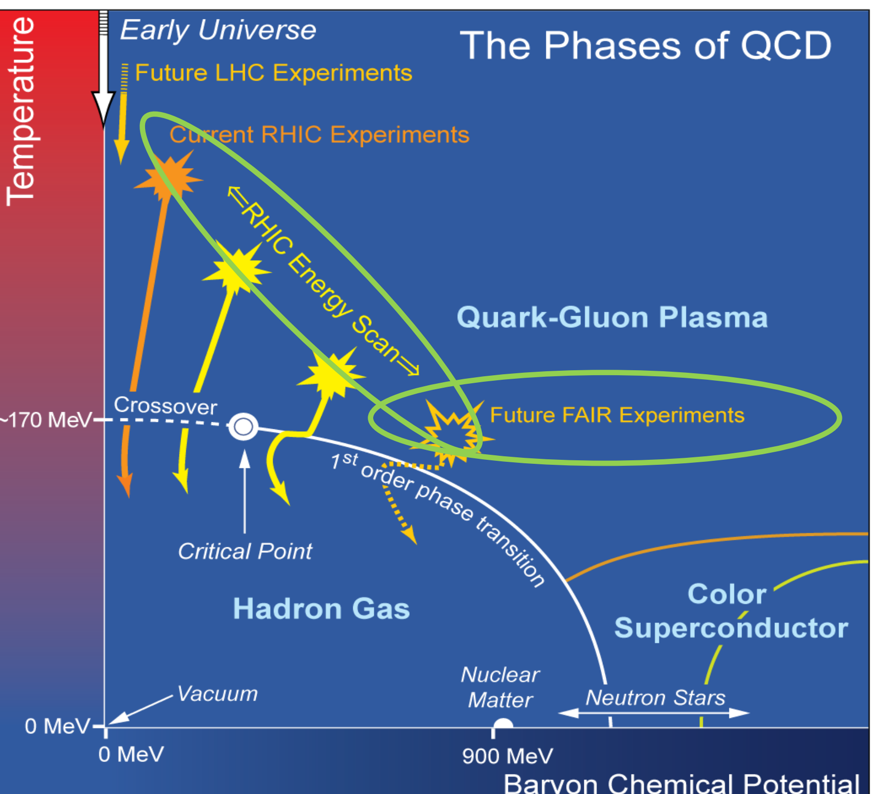
Study QCD phase diagram from an extended AMPT model

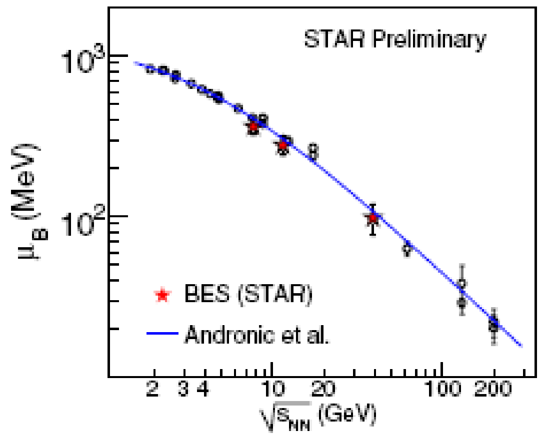
Jun Xu[1,2] (徐骏)

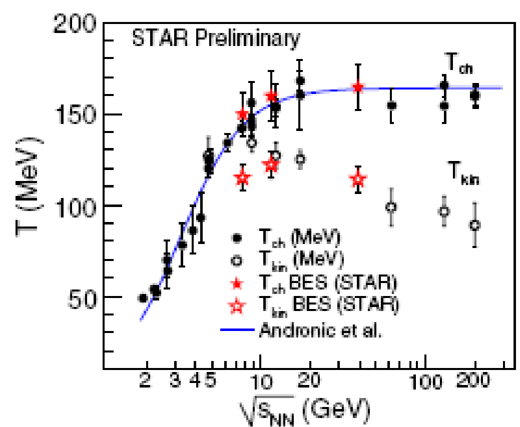
1.Shanghai Advanced Research Institute, Chinese Academy of Sciences

2.Shanghai Institute of Applied Physics, Chinese Academy of Sciences







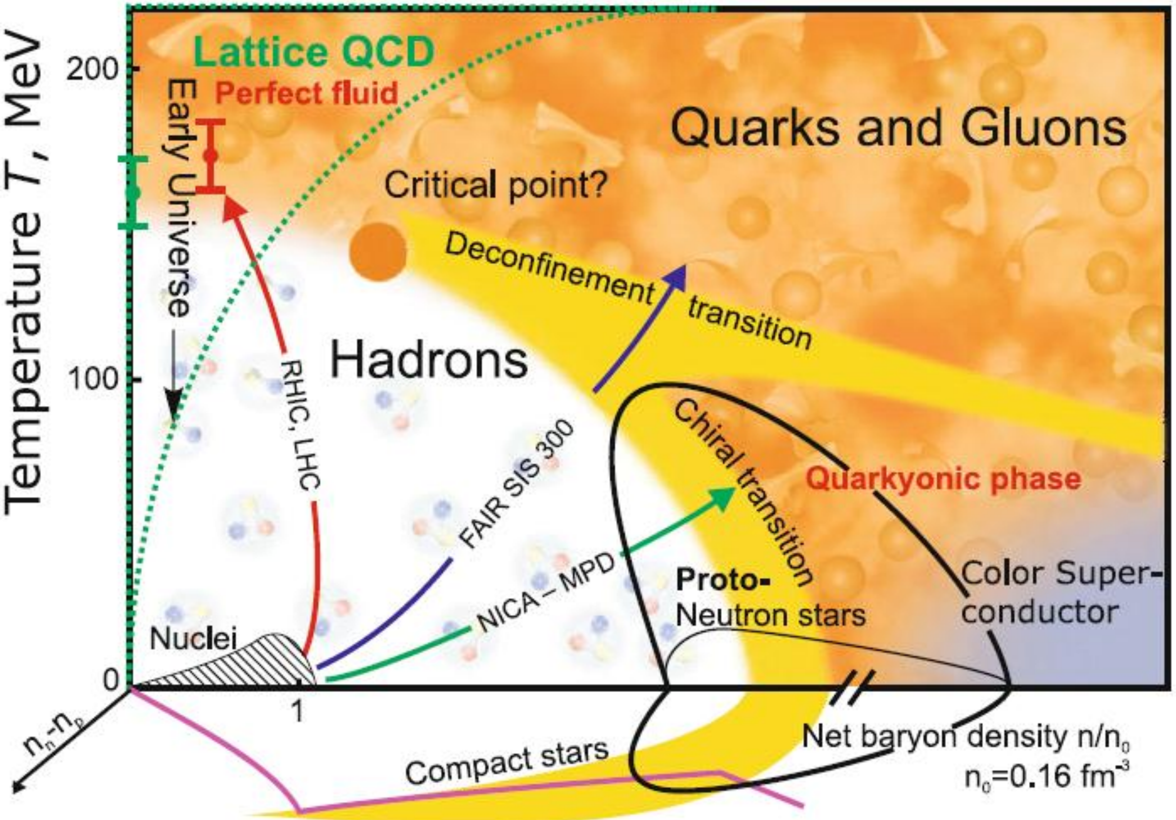


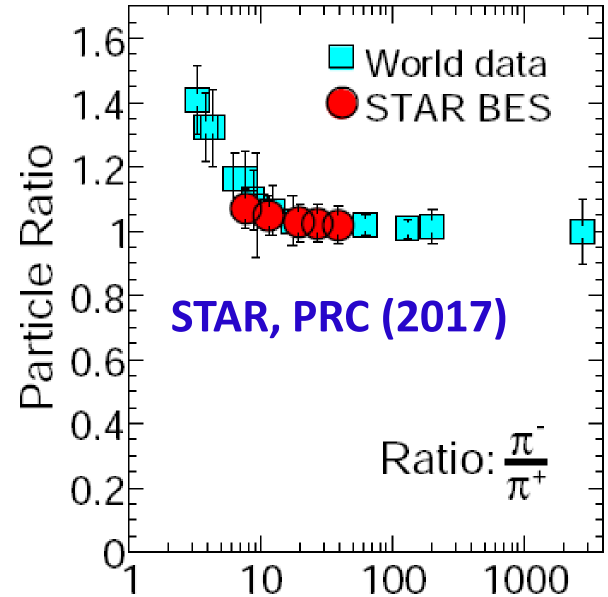
Search for signals of critical point at finite μ_B!

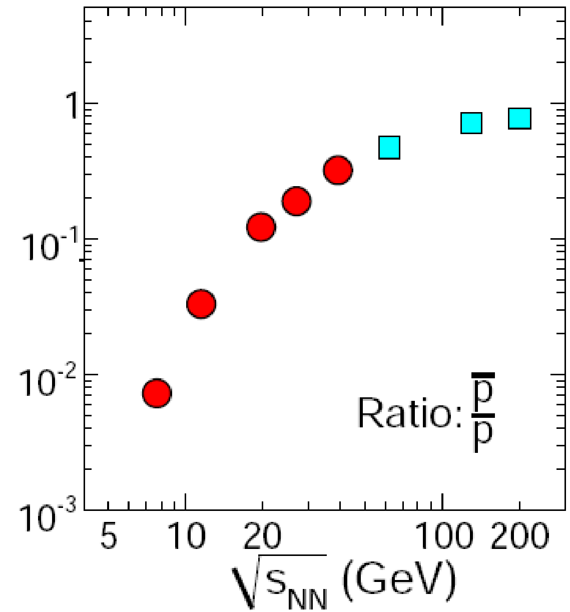
RHIC-BES:
$\sqrt{s} \sim$ **7.7-39 GeV**

FAIR-CBM:
$\sqrt{s}$ **< 9 GeV**

QCD phase structure at finite baryon and isospin chemical potentials







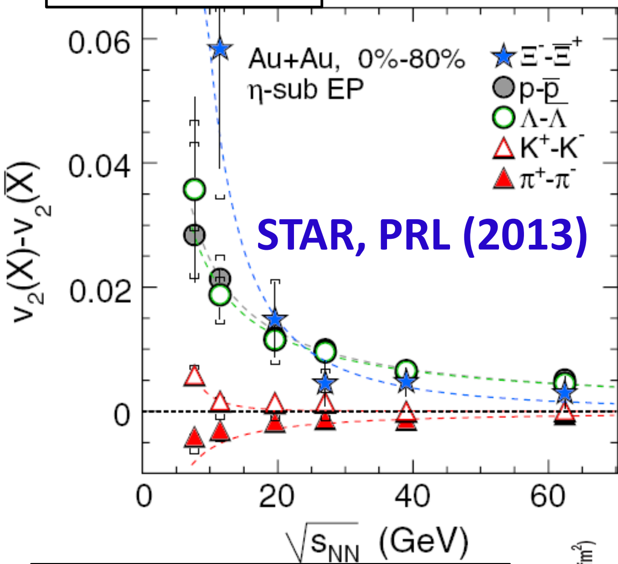
$$\mu_I = -0.0308\mu_B + 2.77 \cdot 10^{-8}\mu_B^3$$

$$\mu_I = -0.293 - 0.0264\mu_B$$

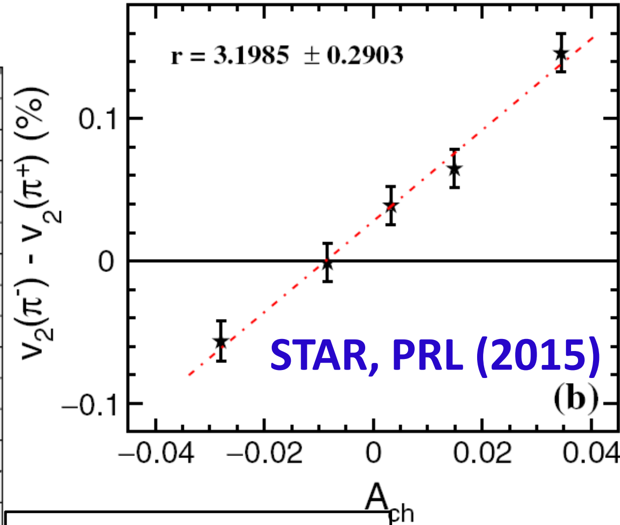
Y. Hatta, A. Monnai, and B.W. Xiao, NPA (2016)

Highlights of recent RHIC physics

v_2 splitting

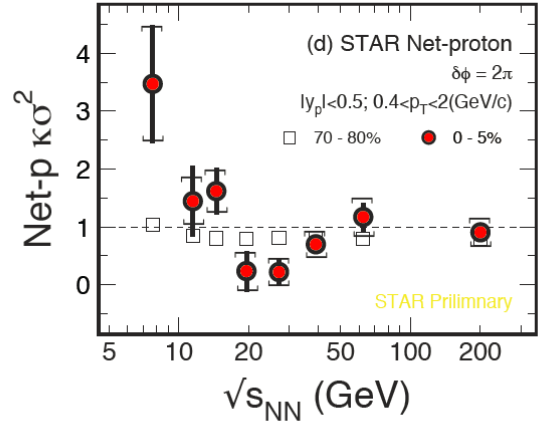


STAR, PRL (2013)

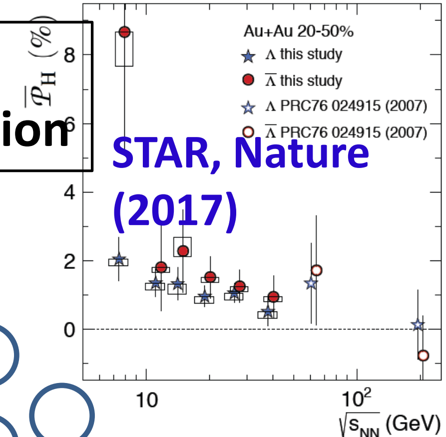


STAR, PRL (2015)

Net proton fluctuation

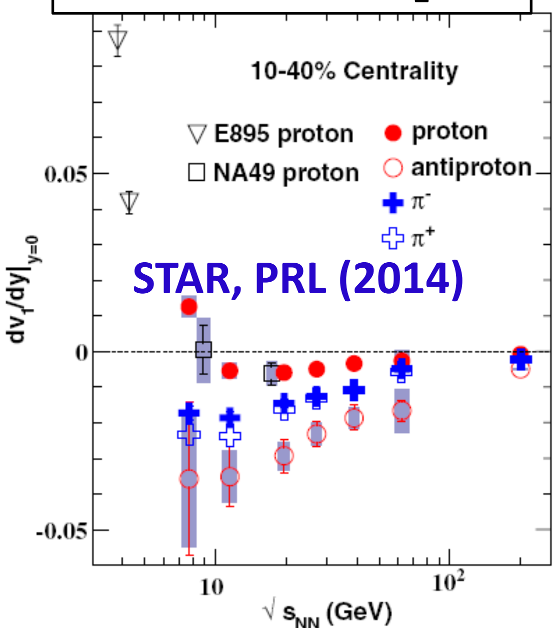


Λ and Λ̄ polarization



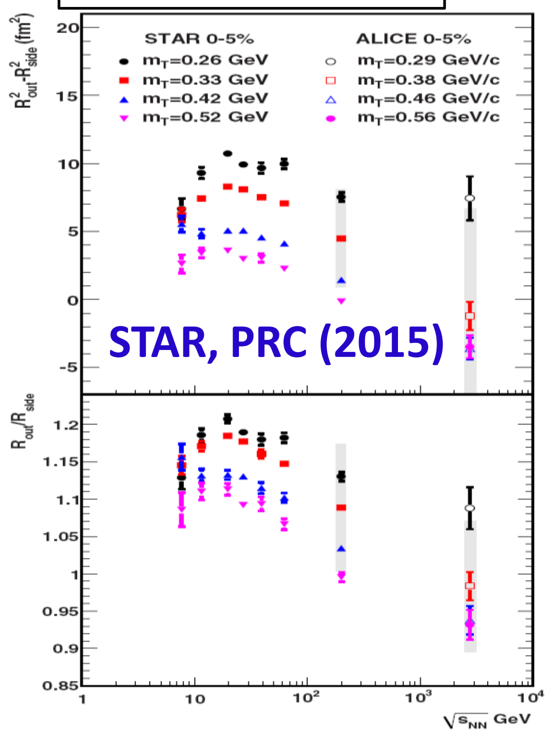
STAR, Nature (2017)

Minimum of dv_1/dy

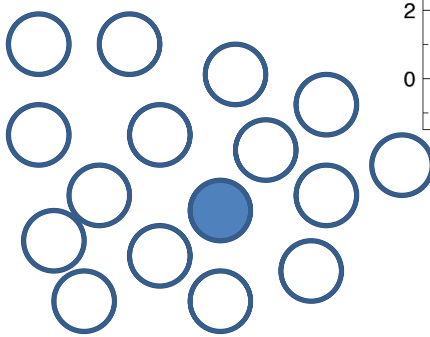


STAR, PRL (2014)

HBT correlation



STAR, PRC (2015)



$$U_i \sim \sum_{i \neq j} V_{ij} + \ldots$$

Effects of mean-field potentials on these observables?

Transport model simulations of intermediate-energy heavy-ion collisions

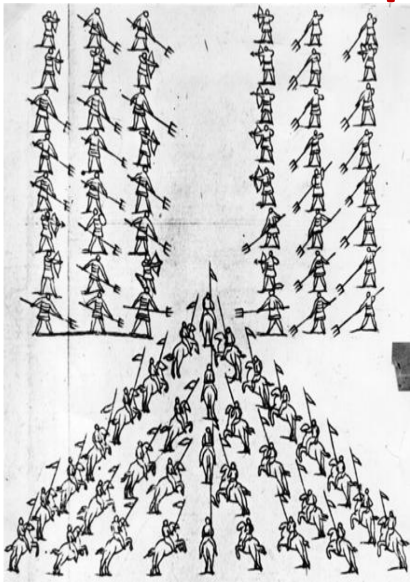
Heavy-ion experiments ⟷ Mean-field potential

Transport simulations

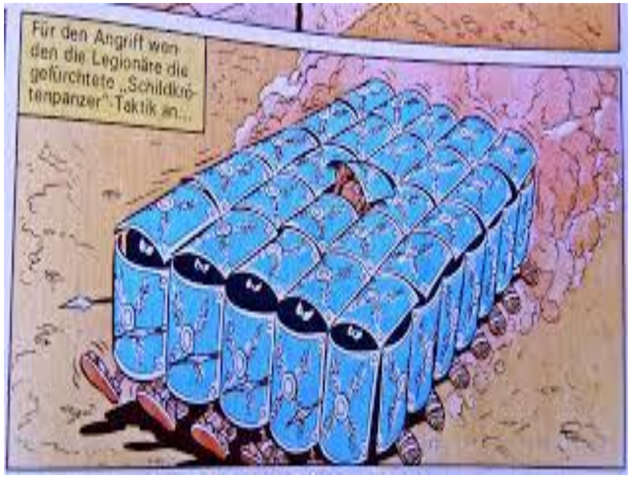
Mean-field potential ⟷ Nuclear EOS, E_{sym}

Main approaches:

Boltzmann transport and Quantum Molecular Dynamics



Initialization

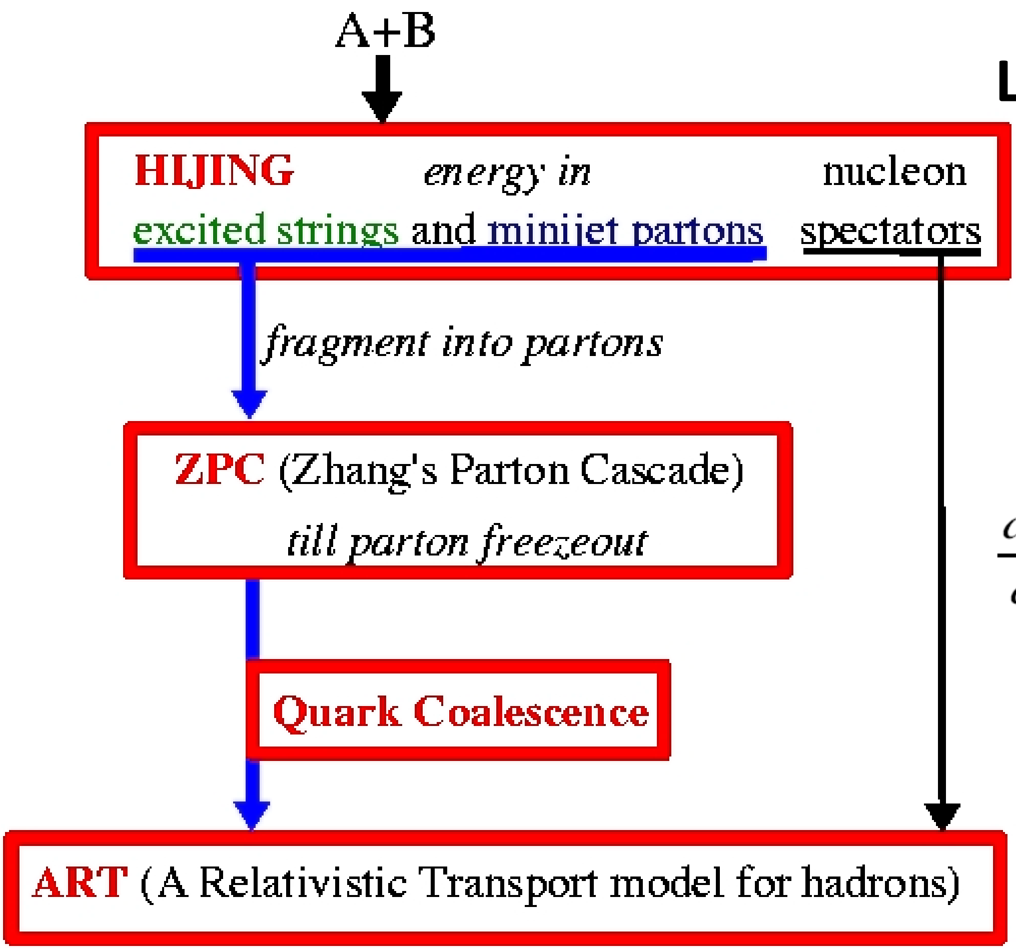


Mean-field potential

NN scatterings

A multiphase transport (AMPT) model with string melting

Structure of AMPT model with string melting



Lund string fragmentation function

$$f(z) \approx z^{-1}(1-z)^{a}\exp\left[-\frac{b(m^2+p_t^2)}{z}\right]$$

z : light-cone momentum fraction

Parton scattering cross section

$$\frac{d\sigma}{dt} \approx \frac{9\pi\alpha^2}{2s^2}\left(1+\frac{\mu^2}{s}\right)\left(\frac{1}{t-\mu^2}\right)^2, \quad \sigma \approx \frac{9\pi\alpha^2}{2\mu^2}$$

α: strong coupling constant

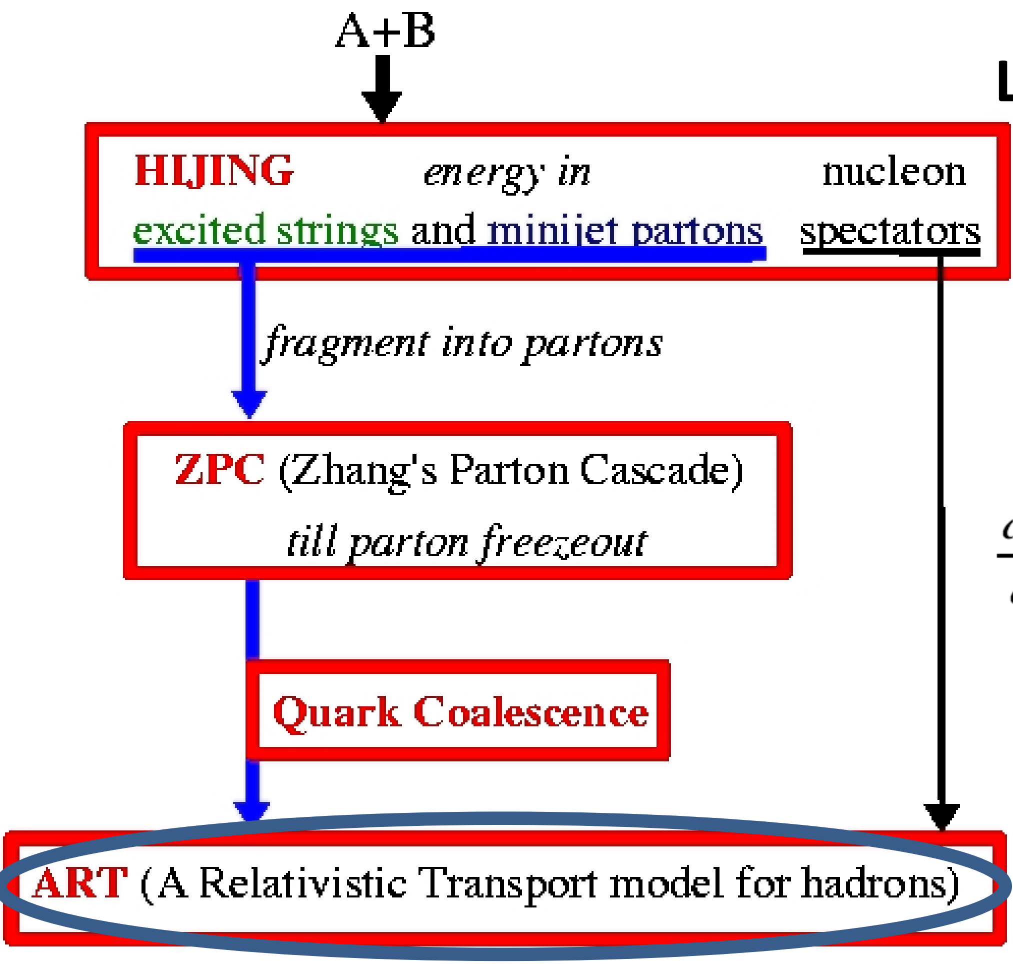
μ: screening mass

a, b: particle multiplicity

α, μ: partonic interaction

A multiphase transport (AMPT) model with string melting

Structure of AMPT model with string melting



Turn on hadronic mean-field potentials

Lund string fragmentation function

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Parton scattering cross section

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α: strong coupling constant

μ: screening mass

a, b: particle multiplicity

α, μ: partonic interaction

hadronic potentials for particles and antiparticles

Nucleon and antinucleon potential

$$\mathcal{L} = \overline{\psi}[i\gamma_\mu\partial^\mu - m - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu]\psi + \frac{1}{2}(\partial^\mu\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}b\sigma^3 - \frac{1}{4}c\sigma^4 - \frac{1}{4}(\partial_\mu\omega^\nu - \partial_\nu\omega^\mu)^2 + \frac{1}{2}m_\omega^2\omega^{\mu 2},$$

G.Q. Li, C.M. Ko, X.S. Fang, and Y.M. Zheng, PRC (1994)

$$\Sigma_s = g_\sigma\langle\sigma\rangle, \qquad \Sigma_{v\mu} = g_\omega\langle\omega_\mu\rangle$$

$$U_{N,\bar{N}} = \Sigma_s(\rho_B, \rho_{\bar{B}}) \pm \Sigma_v^0(\rho_B, \rho_{\bar{B}})$$

$$U_{\Lambda,\overline{\Lambda}} \sim \frac{2}{3}U_{N,\overline{N}}, U_{\Xi,\overline{\Xi}} \sim \frac{1}{3}U_{N,\overline{N}}$$

Vector potential changes sign for antiparticles!

(e[+]e[-] exchange γ)

Kaon and antikaon potential

$$\omega_{K,\bar{K}} = \sqrt{m_K^2 + p^2 - a_K\rho_s + (b_K\rho_B^{\rm net})^2} \pm b_K\rho_B^{\rm net}$$

$$U_{K(\overline{K})} = \omega_{K(\overline{K})} - \omega_0 \qquad \omega_0 = \sqrt{m_K^2 + p^2}$$

G.Q. Li, C.H. Lee, and G.E. Brown, PRL (1997); NPA (1997)

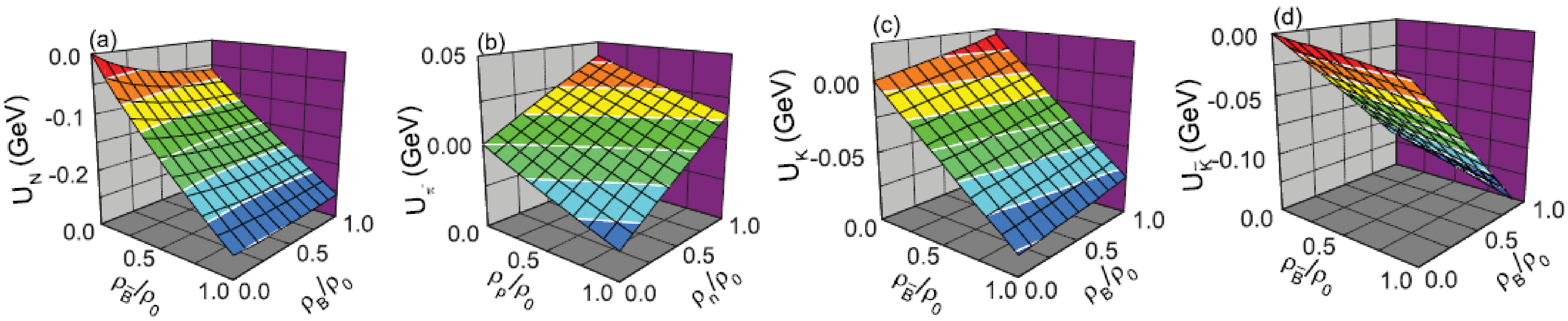
Pion s-wave potential

$$\begin{aligned}
\Pi_s^-(\rho_p, \rho_n) &= \rho_n[T_{\pi N}^- - T_{\pi N}^+] - \rho_p[T_{\pi N}^- + T_{\pi N}^+] \\
&\quad + \Pi_{\rm rel}^-(\rho_p, \rho_n) + \Pi_{\rm cor}^-(\rho_p, \rho_n) \\
\Pi_s^+(\rho_p, \rho_n) &= \Pi_s^-(\rho_n, \rho_p) \\
\Pi_s^0(\rho_p, \rho_n) &= -(\rho_p + \rho_n)T_{\pi N}^+ + \Pi_{\rm cor}^0(\rho_p, \rho_n).
\end{aligned}$$

$$U_{\pi^{\pm 0}} = \Pi_s^{\pm 0}/(2m_\pi)$$

N. Kaiser and W. Weise, PLB (2001)

hadronic potentials for particles and antiparticles



In baryon-rich and neutron-rich matter:

Incorporated via test-particle method

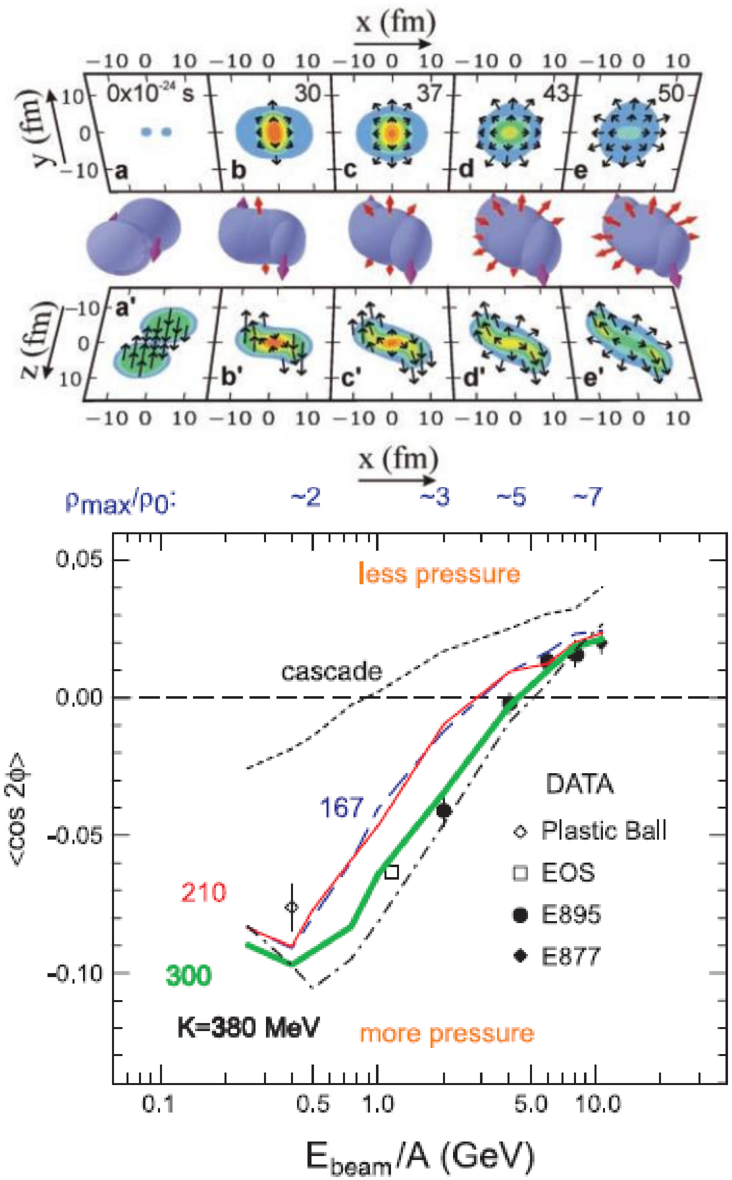
- **Baryon potential: weakly attractive**
- **Antibaryon potential: deeply attractive**

Sub threshold particle production

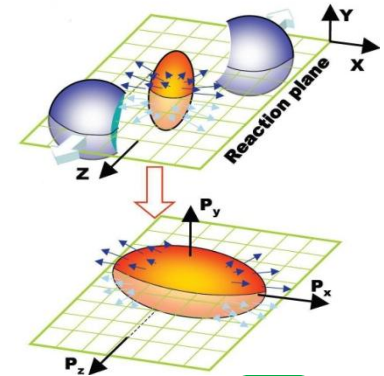
- **K^+ potential: weakly repulsive**
- **K^- potential: deeply attractive**
- **π^+ potential: weakly attractive**
- **π^- potential: weakly repulsive**

Chiral perturbation theory

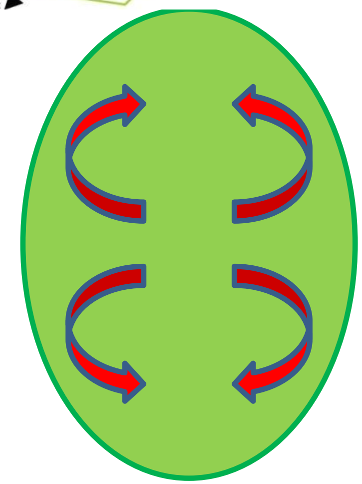
Effects of mean-field potentials on elliptic flow



$$v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

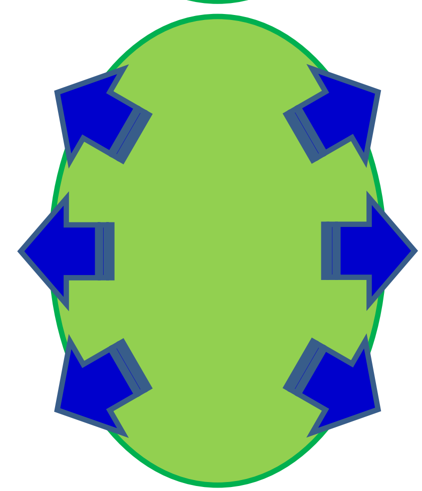


Particles with attractive potentials are more likely to be trapped in the system



v_2 decrease

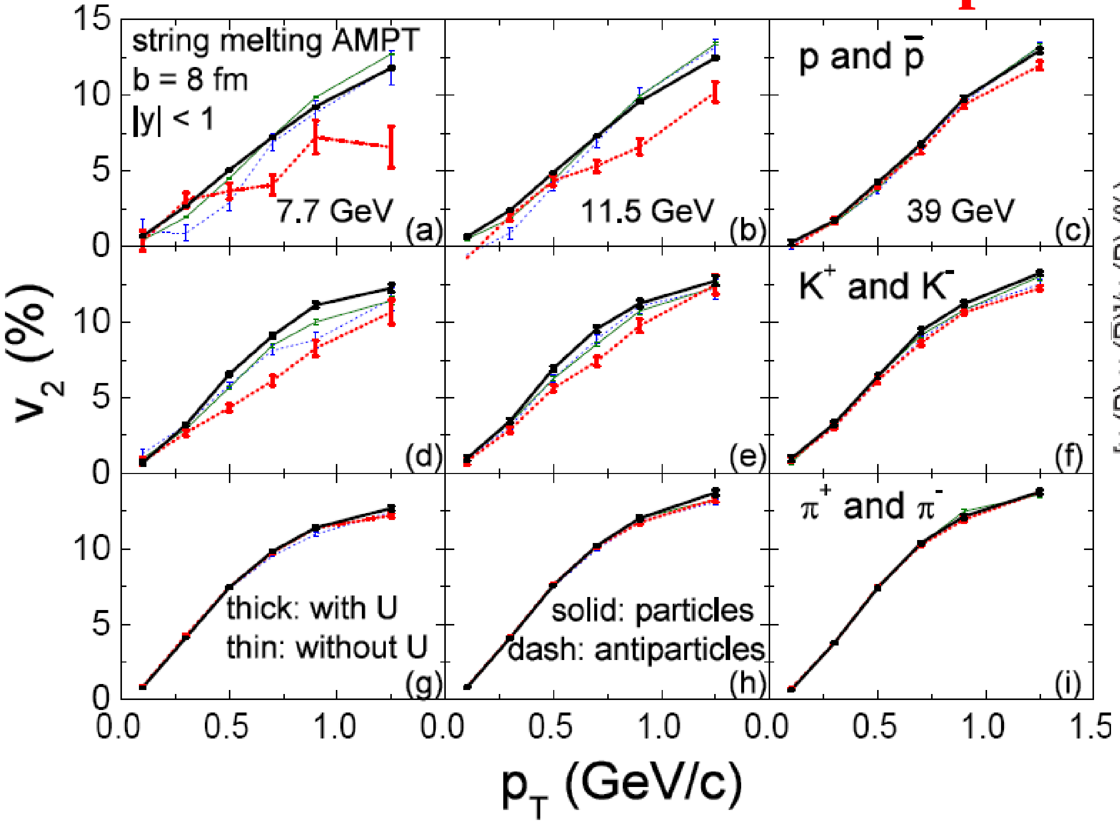
Particles with repulsive potentials are more likely to leave the system

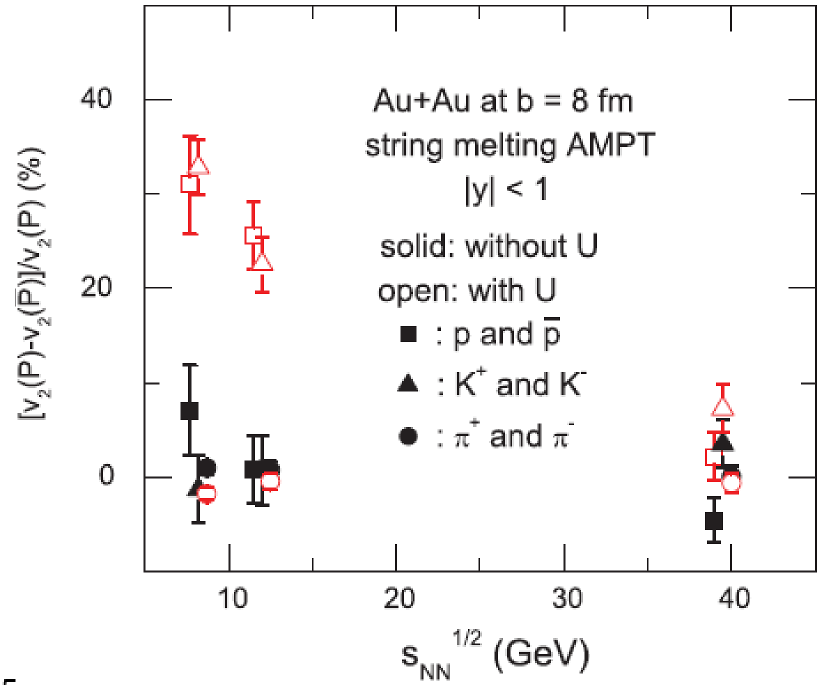


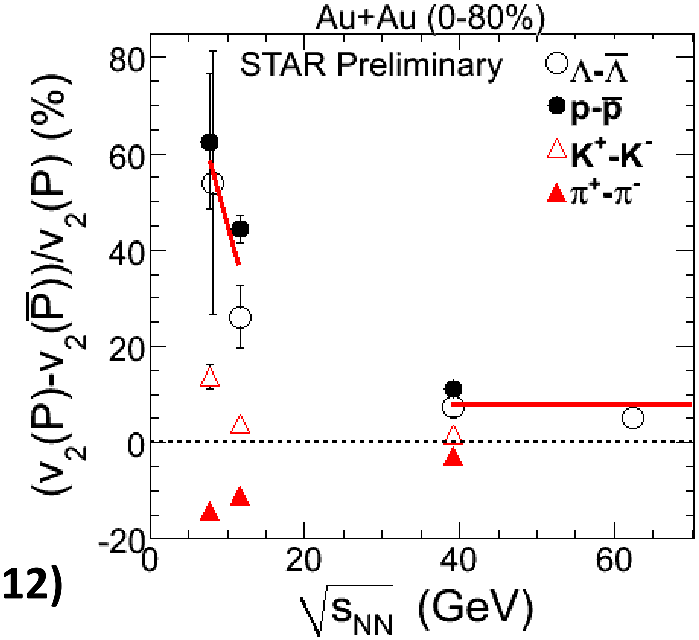
v_2 increase

P. Danielewicz, R. Lacey, and W. G. Lynch, Science (2002).

Effects of hadronic mean-field potentials on elliptic flow splitting







Qualitatively consistent

proton and antiproton: underestimate

K^+ and K^-: overestimate

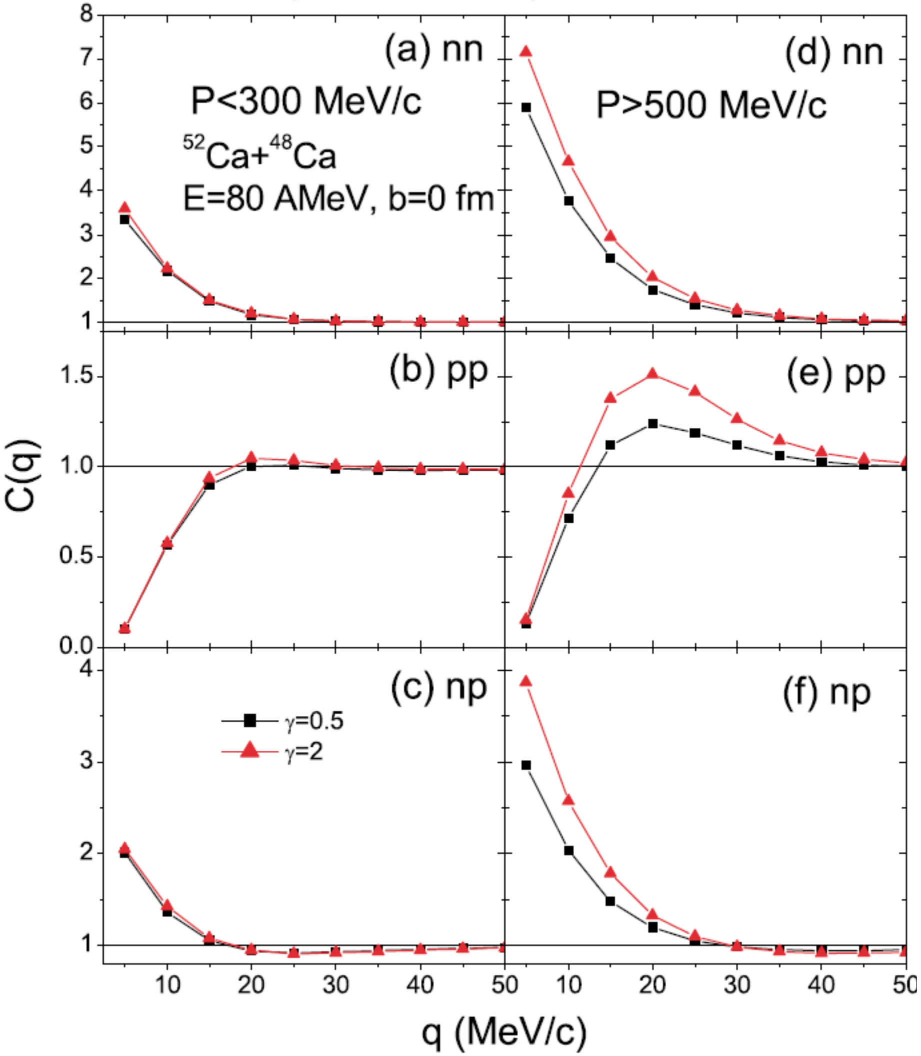
π^+ and π^-: underestimate

JX, L. W. Chen, C. M. Ko, and Z. W. Lin, PRC 85, 041901(R) (2012)

Effects of mean-field potentials on HBT correlation

a probe of neutron-proton U difference based on IBUU

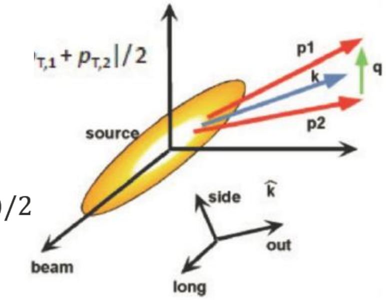
$$E_{\rm sym}(\rho) = E_{\rm sym}(\rho_0) \cdot u^{\gamma}$$



Two-particle correlation function:

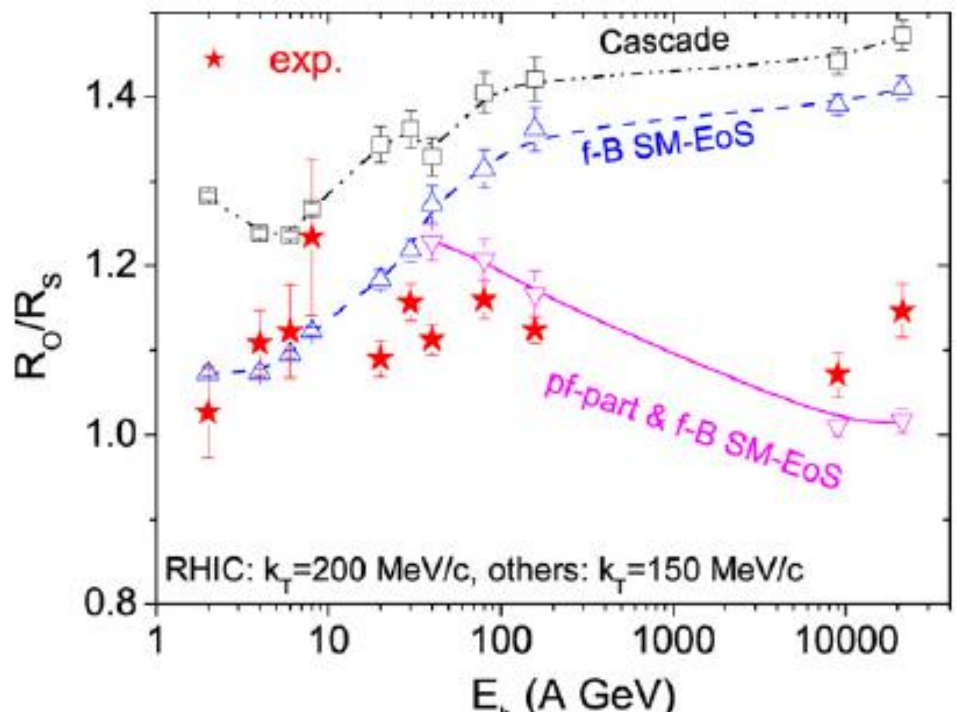
$$C(\mathbf{k}^*) = \frac{\int S(\mathbf{r}^*,\mathbf{k}^*) \left|\Psi^{S(+)}_{-\mathbf{k}^*}(\mathbf{r}^*)\right|^2 d^4\mathbf{r}^*}{\int S(\mathbf{r}^*,\mathbf{k}^*)\, d^4\mathbf{r}^*}$$

where $\mathbf{r}^* = \mathbf{x}_1 - \mathbf{x}_2$ and $\mathbf{k}^* = q_{\rm inv}/2 = (\mathbf{p}_1 - \mathbf{p}_2)/2$



$$C(\vec{q}) = (1-\lambda) + \lambda K_{\rm coul}(q_{\rm inv}) \times \left(1 + e^{-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2 - 2 q_o q_s R_{os}^2 - 2 q_o q_l R_{ol}^2}\right)$$

affect the HBT radii based on UrQMD

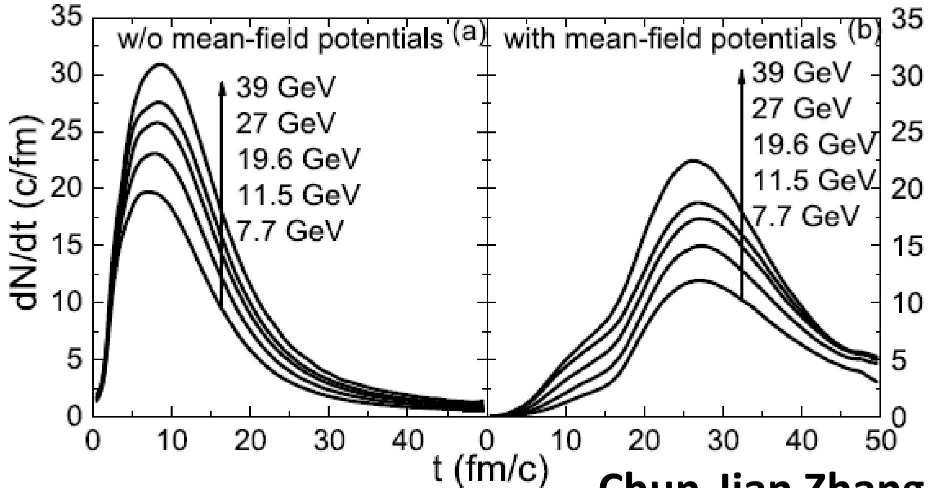


Q.F. Li, M. Bleicher, and H. Stocker, PLB (2008)

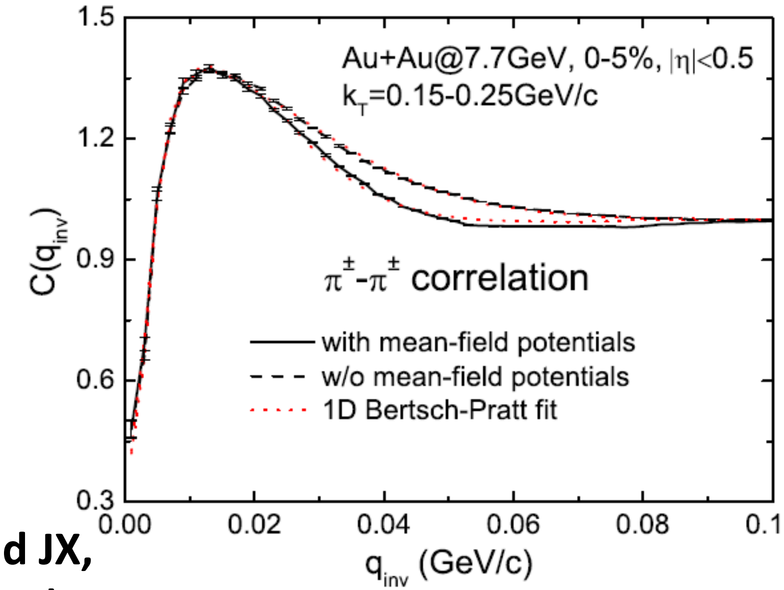
L.W. Chen, V. Greco, C.M. Ko, and B.A. Li, PRL (2003)

Effects of hadronic mean-field potentials on HBT correlation

later emission and
broader emission time distribution

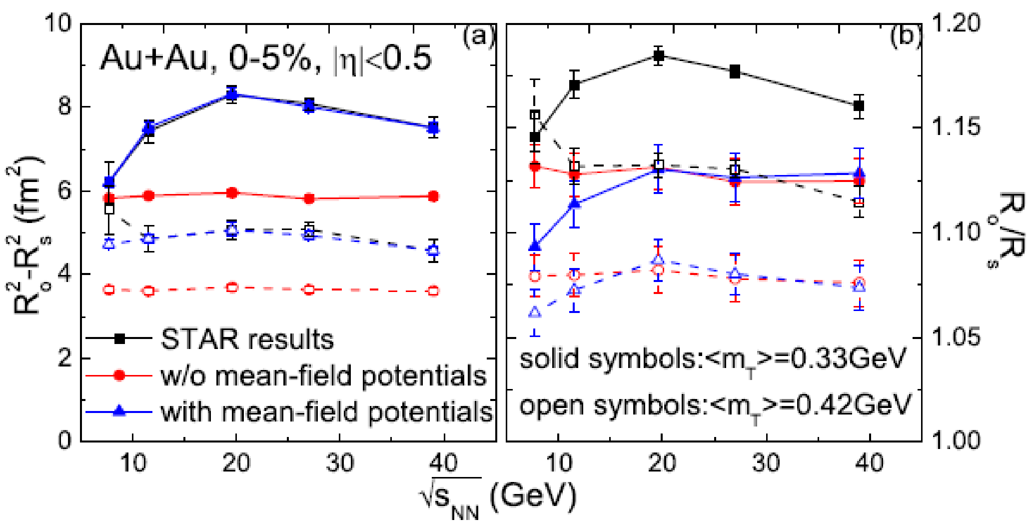


larger HBT radius

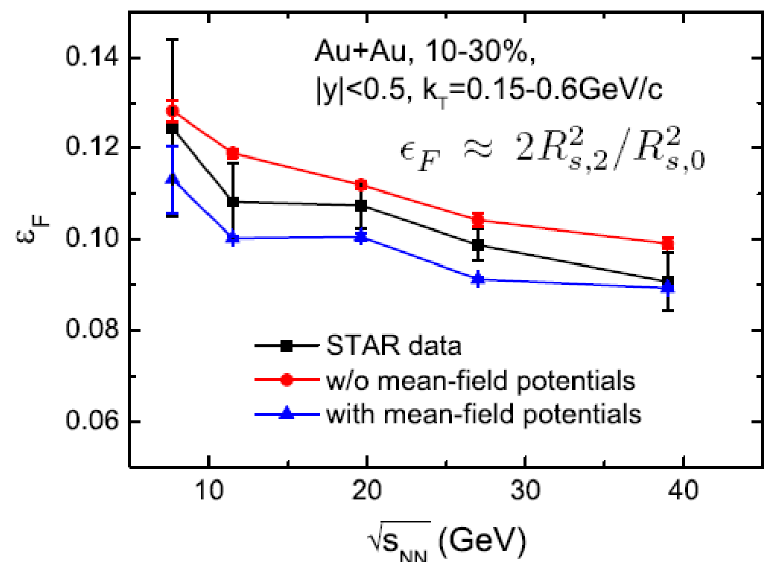


Chun-Jian Zhang and JX,
PRC 96, 044907 (2017)

Affect R_{out} and R_{side}

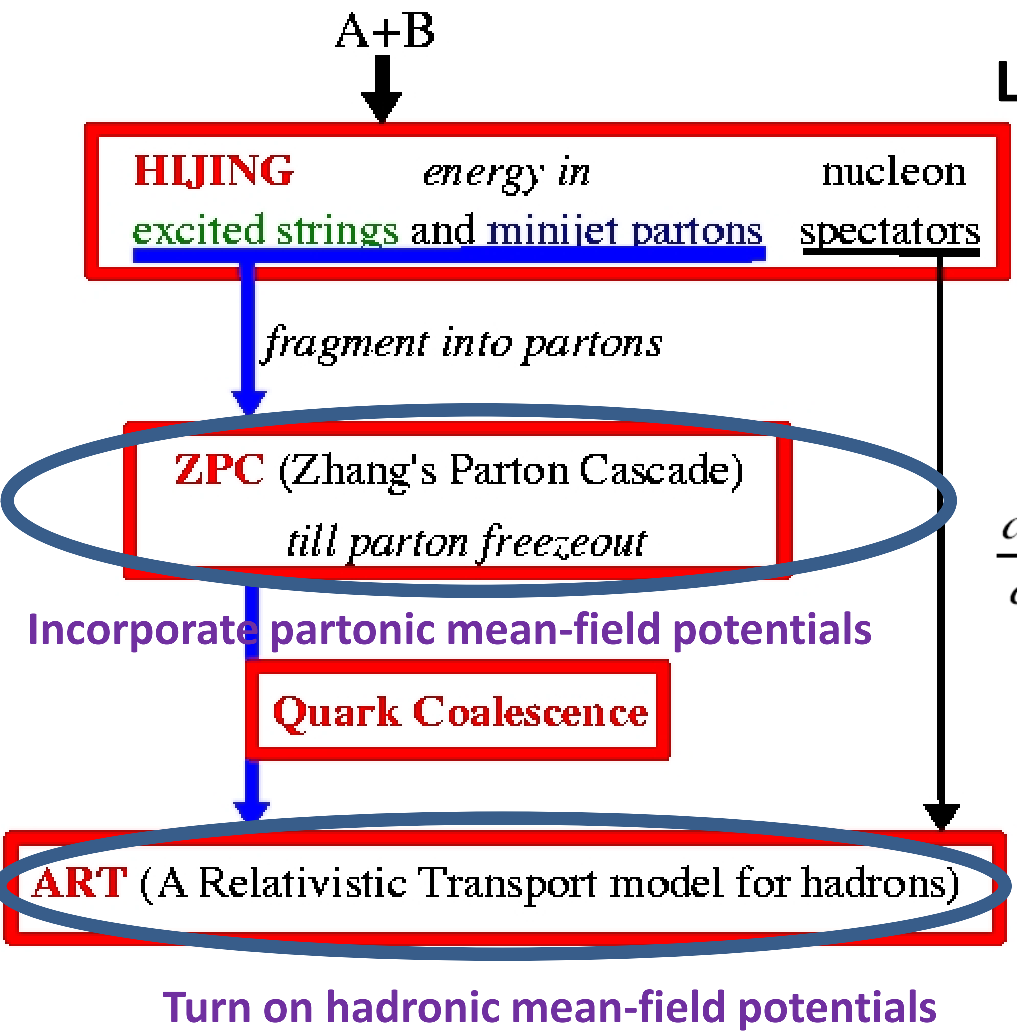


Lead to a smaller freeze-out eccentricity



A multiphase transport (AMPT) model with string melting

Structure of AMPT model with string melting



Lund string fragmentation function

$$f(z) \approx z^{-1}(1-z)^{a}\exp\left[-\frac{b(m^2+p_t^2)}{z}\right]$$

z : light-cone momentum fraction

Parton scattering cross section

$$\frac{d\sigma}{dt} \approx \frac{9\pi\alpha^2}{2s^2}\left(1+\frac{\mu^2}{s}\right)\left(\frac{1}{t-\mu^2}\right)^2, \quad \sigma \approx \frac{9\pi\alpha^2}{2\mu^2}$$

α: strong coupling constant

μ: screening mass

a, b: particle multiplicity

α, μ: partonic interaction

3-flavor Nambu-Jona-Lasinio transport model

Lagrangian:

scalar — pseudoscalar — vector — pseudovector

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - M)\psi + \frac{G}{2}\sum_{a=0}^{8}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] - \sum_{a=0}^{8}\Big[\frac{G_V}{2}(\bar{\psi}\gamma_\mu\lambda^a\psi)^2 + \frac{G_A}{2}(\bar{\psi}\gamma_\mu\gamma_5\lambda^a\psi)^2\Big]$$

Kobayashi-Maskawa-t'Hooft interaction

$$- K[\det_f(\bar{\psi}(1+\gamma_5)\psi) + \det_f(\bar{\psi}(1-\gamma_5)\psi)]$$

Yoichiro *Nambu*

2008年
Nobel Prize
For Physics

Parameters taken from
M. Lutz, S. Klimt, and W. Weise, NPA (1992)
to reproduce meson properties

$$\rho^\mu = \langle \bar{\psi}\gamma^\mu\psi \rangle$$

Boltzmann equation:

$$\frac{\partial}{\partial t}f + \vec{v}\cdot\nabla_x f - \nabla_x H \cdot \nabla_p f = \mathcal{C}$$

$$\langle \bar{\psi}\gamma^\mu\psi \rangle = 2N_c \sum_{i=u,d,s} \int \frac{d^3\mathbf{k}}{(2\pi)^3 E_i} k^\mu [f_i(k) - \bar{f}_i(k)]$$

$$\rho^0 \equiv \langle \bar{\psi}\gamma^0\psi \rangle$$

Net quark density

Single-quark Hamiltonian:

$$H = \sqrt{M^{*2} + p^{*2}} \pm g_V \rho^0$$

$$M_u = m_u - 2G\langle \bar{u}u \rangle + 2K\langle \bar{d}d \rangle\langle \bar{s}s \rangle$$
$$M_d = m_d - 2G\langle \bar{d}d \rangle + 2K\langle \bar{s}s \rangle\langle \bar{u}u \rangle$$
$$M_s = m_s - 2G\langle \bar{s}s \rangle + 2K\langle \bar{u}u \rangle\langle \bar{d}d \rangle$$

$$\mathbf{p}^* = \mathbf{p} \mp g_V \boldsymbol{\rho} \qquad g_V \equiv (2/3)G_V$$

Equations of motion:

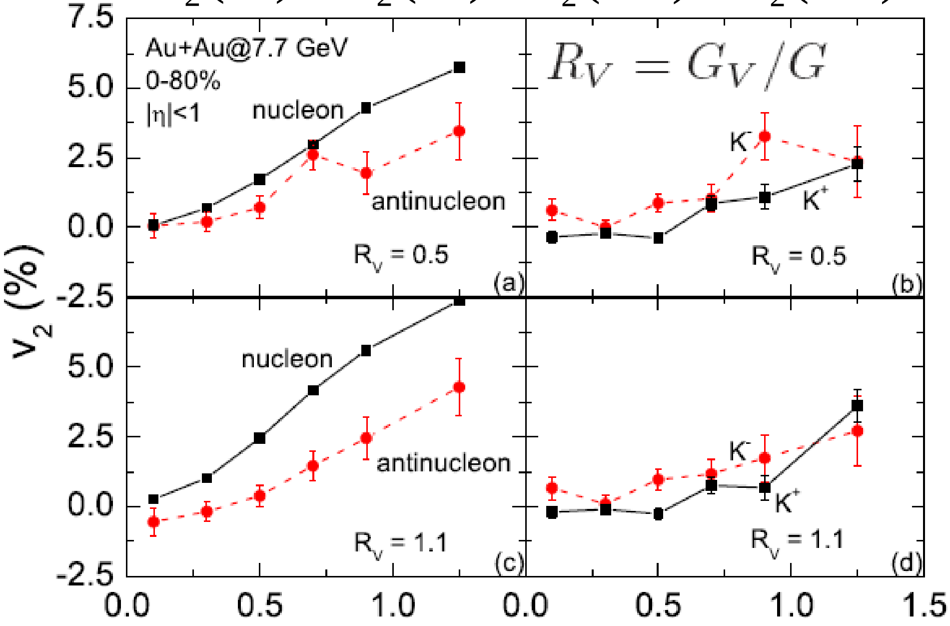
$$\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i} = \frac{p_i^*}{E^*}$$

$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i} = -\frac{M^*}{E^*}\frac{\partial M^*}{\partial x_i} \pm g_V\left(v_j\frac{\partial \rho_j}{\partial x_i} - \frac{\partial \rho_0}{\partial x_i}\right)$$

Solve with
test particle method

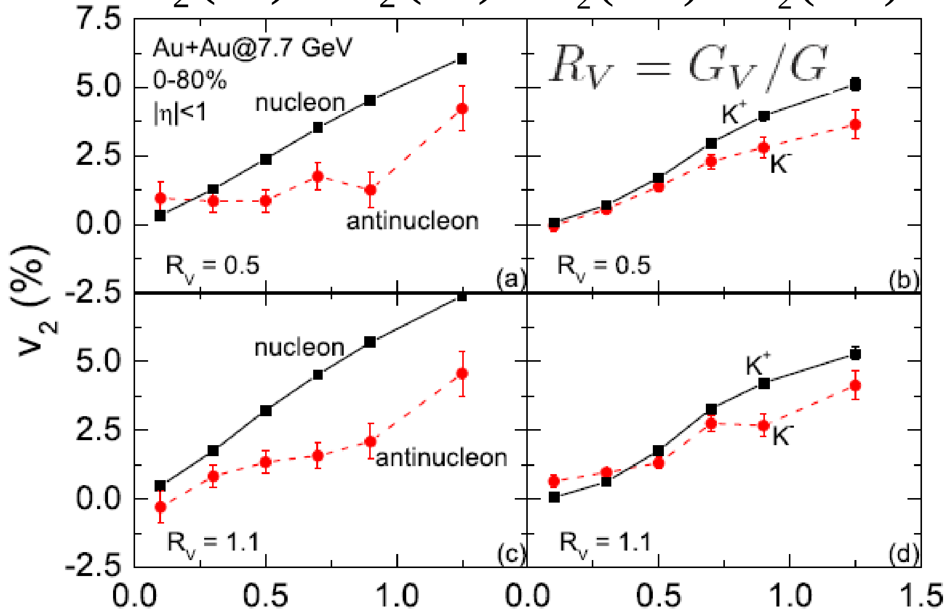
v_2 right after hadronization

$v_2(N) > v_2(\bar{N})$ $v_2(K^+) < v_2(K^-)$

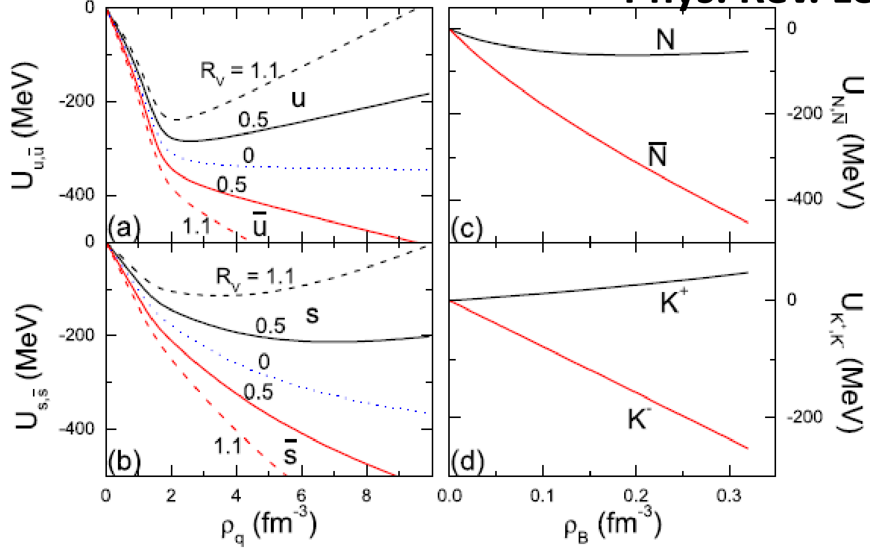


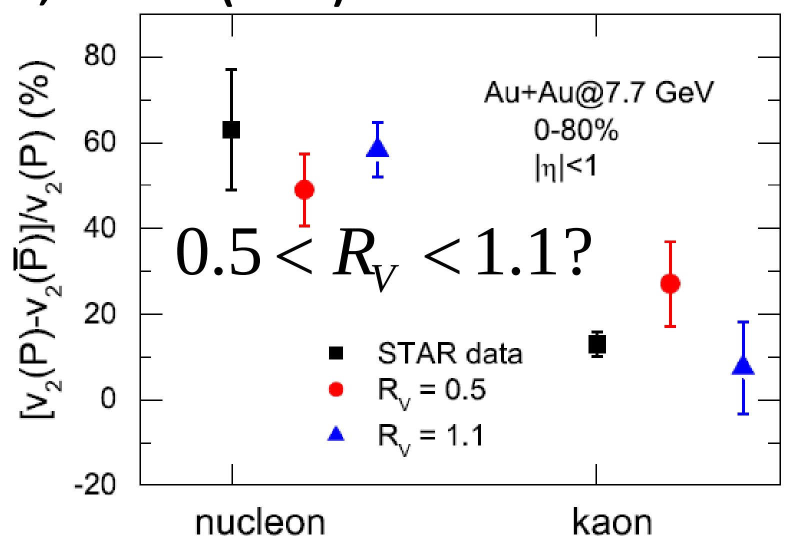
Final v_2

$v_2(N) > v_2(\bar{N})$ $v_2(K^+) > v_2(K^-)$

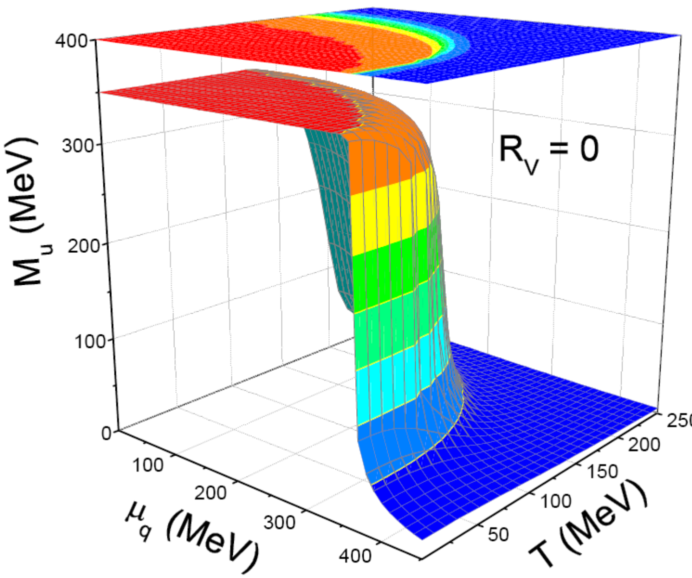


JX, T. Song, C. M. Ko, and F. Li,
Phys. Rev. Lett. 110, 012301 (2014)

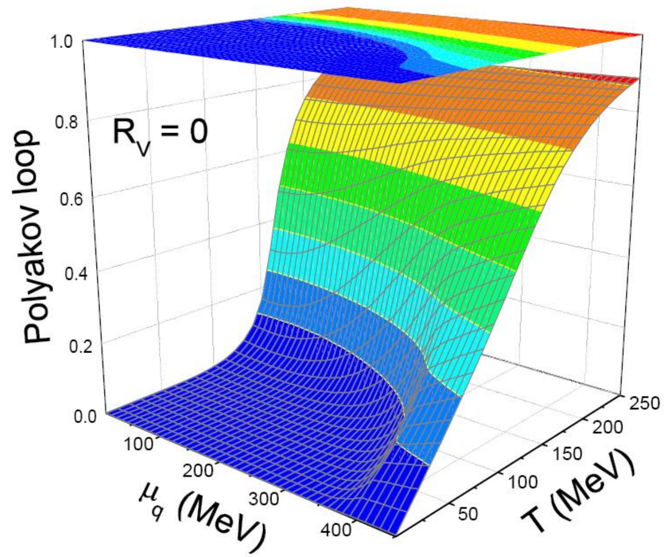




Quark mass/condensate (chiral transition)

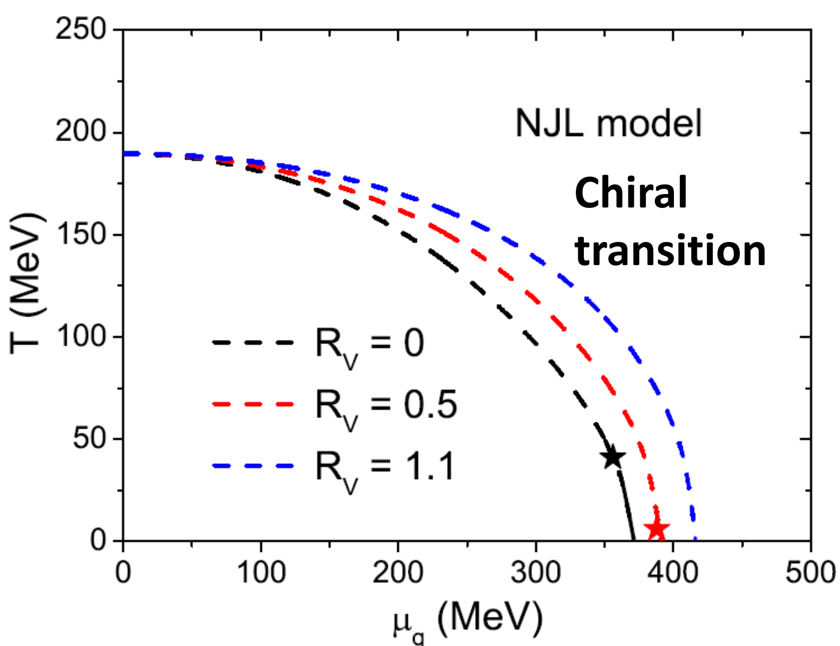


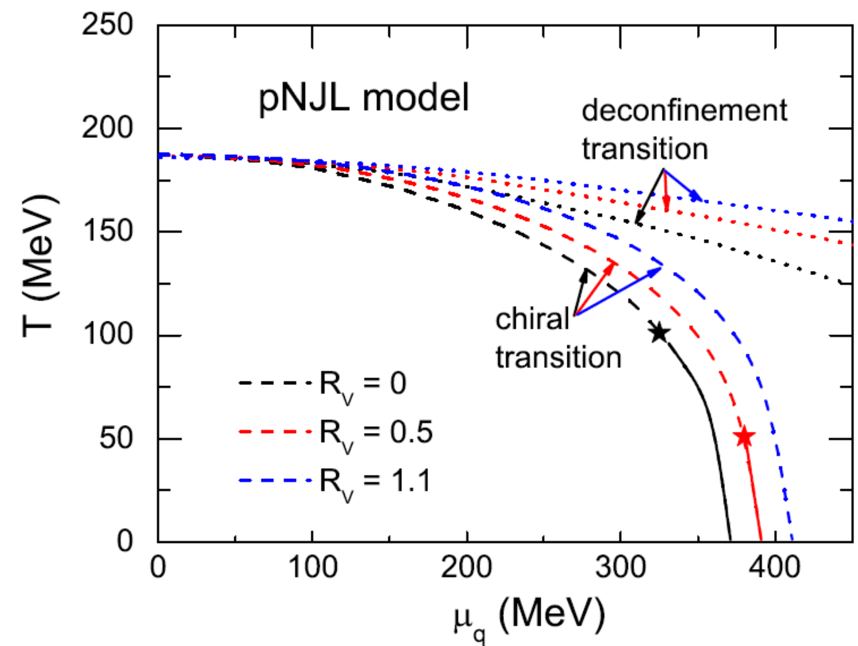
Polyakov loop (deconfinement transition)



Polyakov potential:

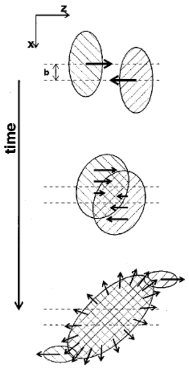
$$\mathcal{U}(\Phi, \bar{\Phi}, T) = -b \cdot T\{54e^{-a/T}\Phi\bar{\Phi} + \ln[1 - 6\Phi\bar{\Phi} - 3(\Phi\bar{\Phi})^2 + 4(\Phi^3 + \bar{\Phi}^3)]\}$$

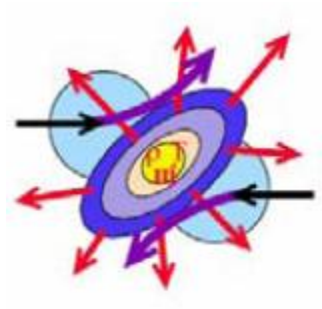


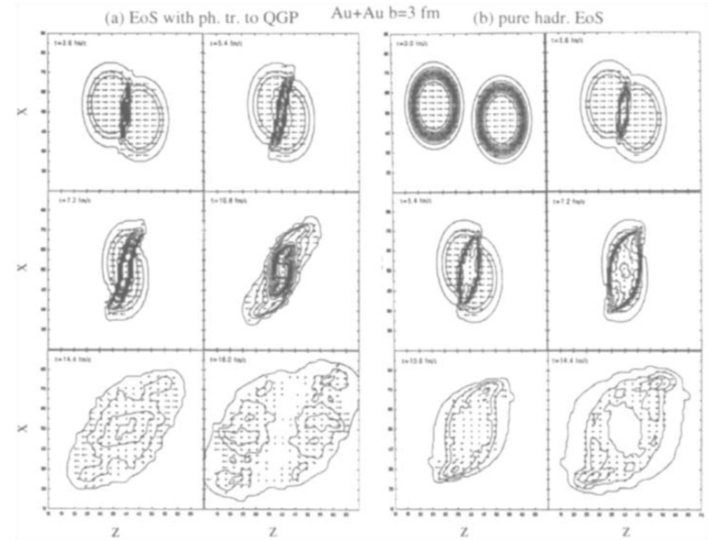


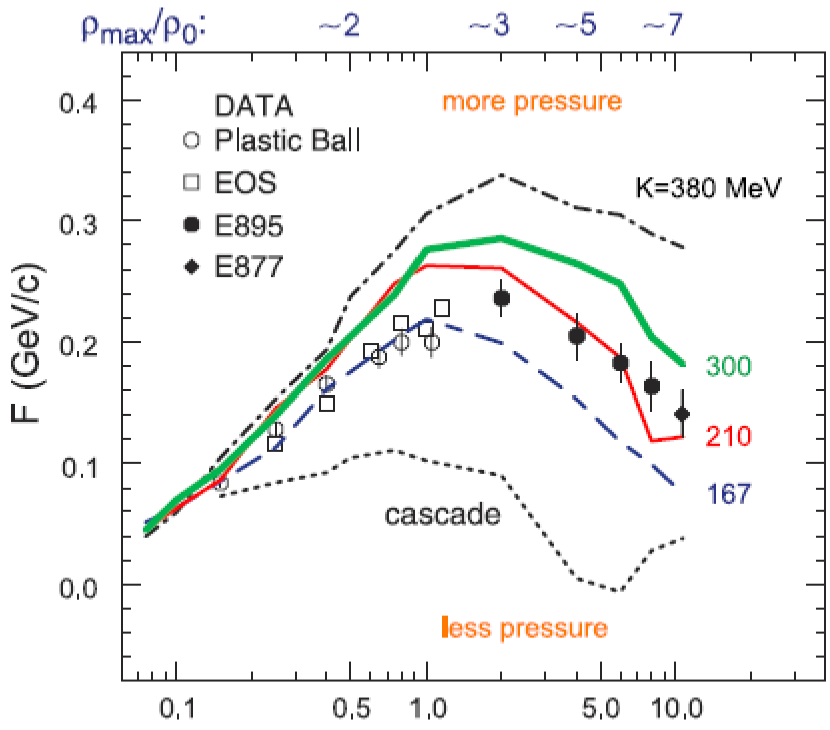
EOS effects on the directed flow

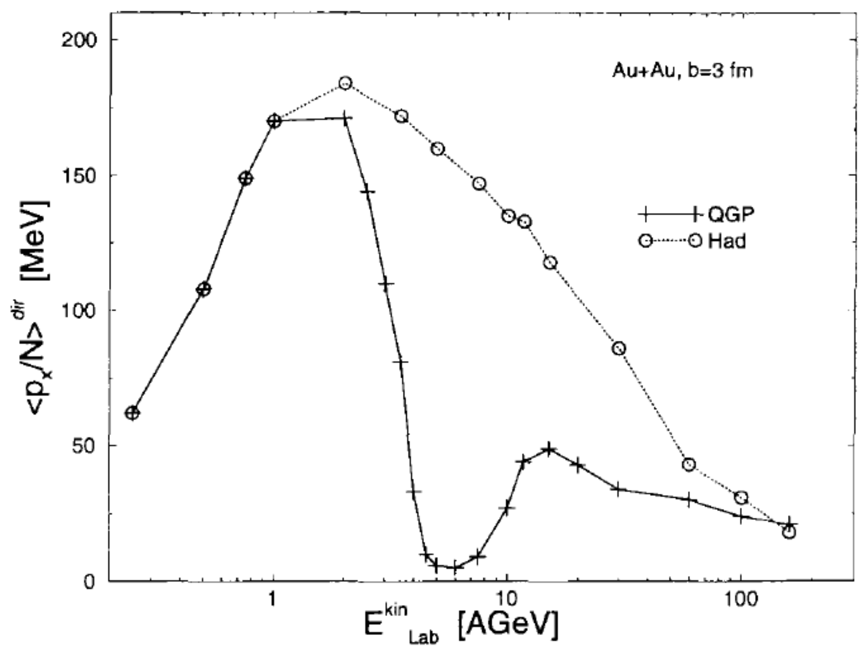
$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$$





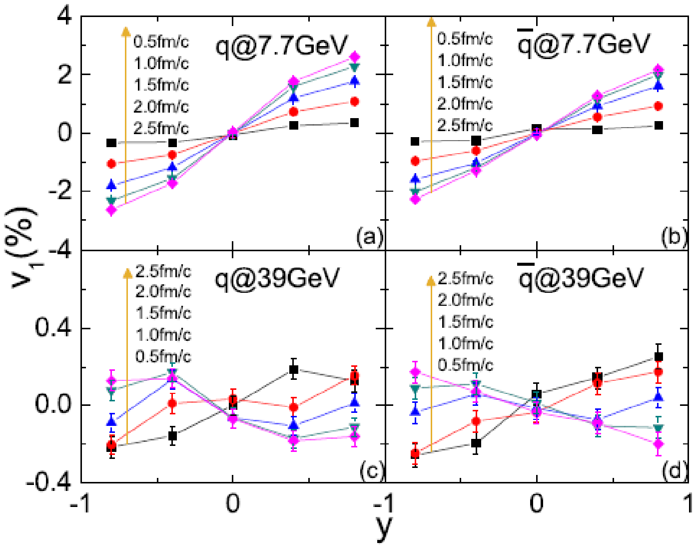


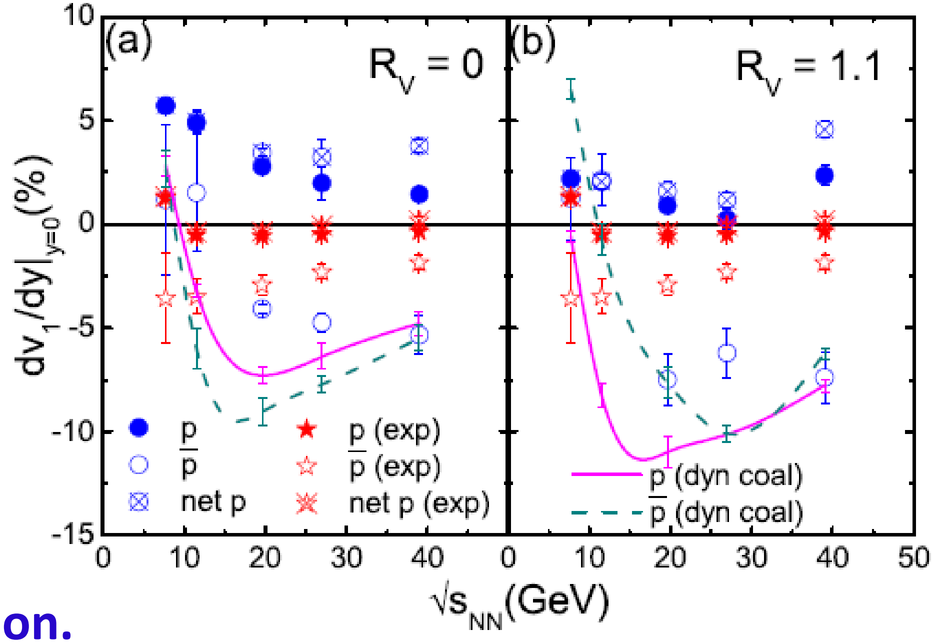




D.H. Rischke et al., APH N.S. Heavy Ion Physics (1995)

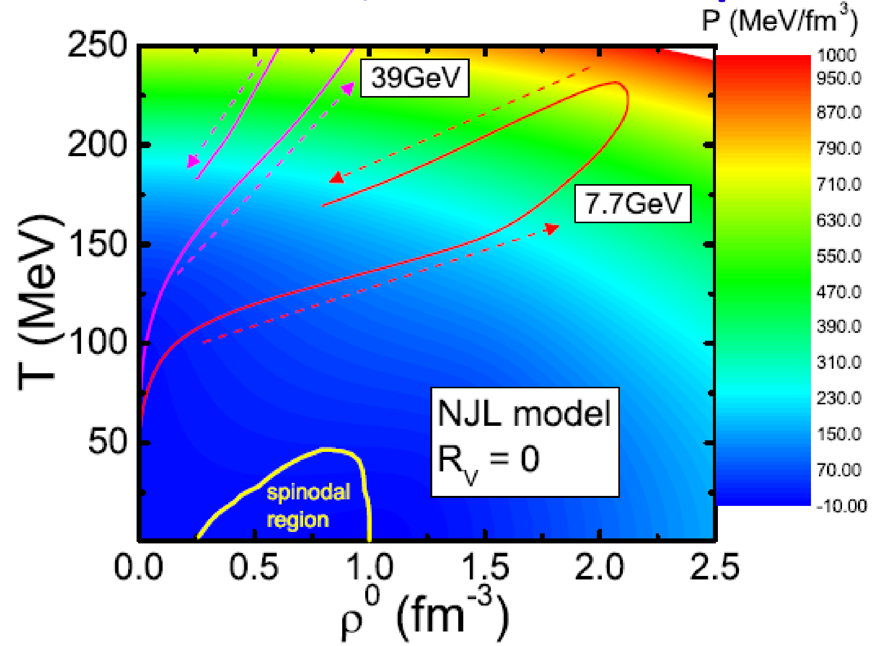
P. Danielewicz, R. Lacey, and W.G. Lynch, Science (2002)

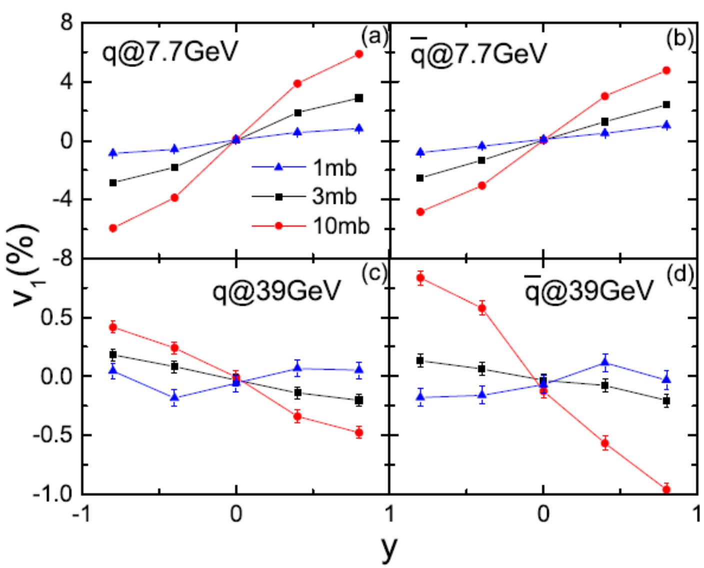




The negative final parton v_1 doesn't need the system to pass through a spinodal region.

The coalescence, which dominates the hadron flow, needs to be improved.





The final parton v_1 is dominated by the partonic scatterings.

C.Q. Guo, H. Liu, and JX, PRC (2018)

Isovector couplings in NJL model

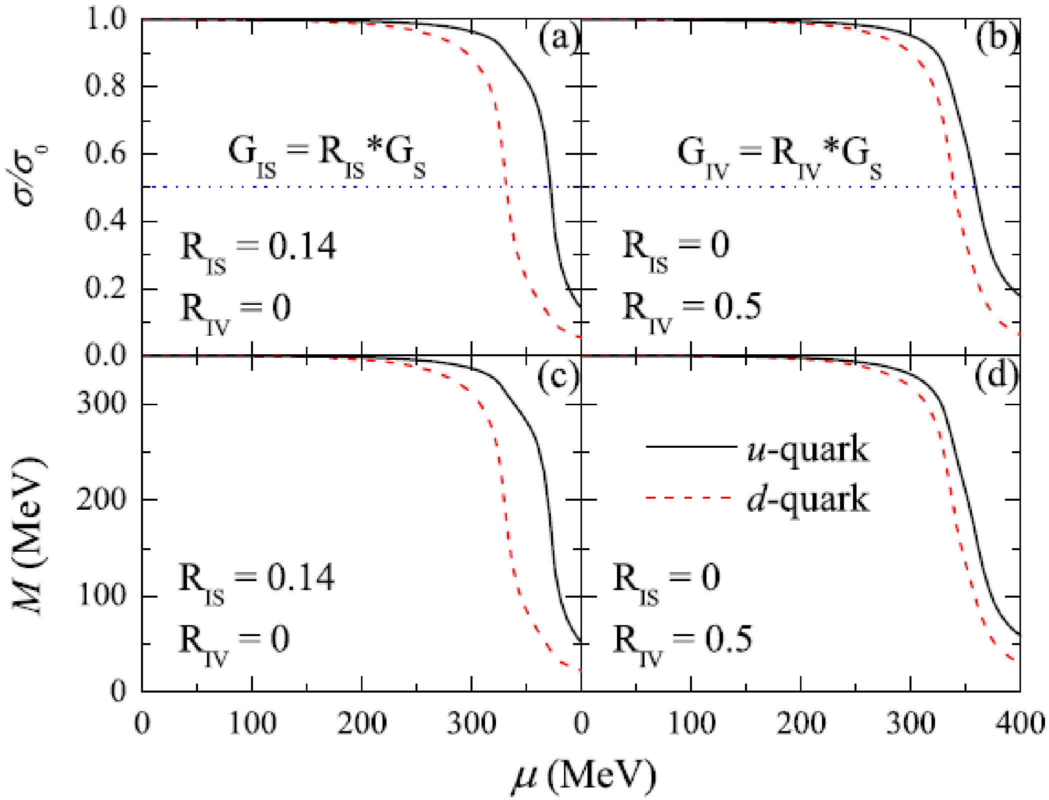
Scalar-isovector coupling $G_{IS}\sum_{a=1}^{3}[(\bar{q}\lambda_a q)^2+(\bar{q}i\gamma_5\lambda_a q)^2]$

Vector-isovector coupling $G_{IV}\sum_{a=1}^{3}[(\bar{q}\gamma_\mu\lambda_a q)^2+(\bar{q}\gamma_5\gamma_\mu\lambda_a q)^2]$

a=1~3
Pauli Matrics
In isospin space

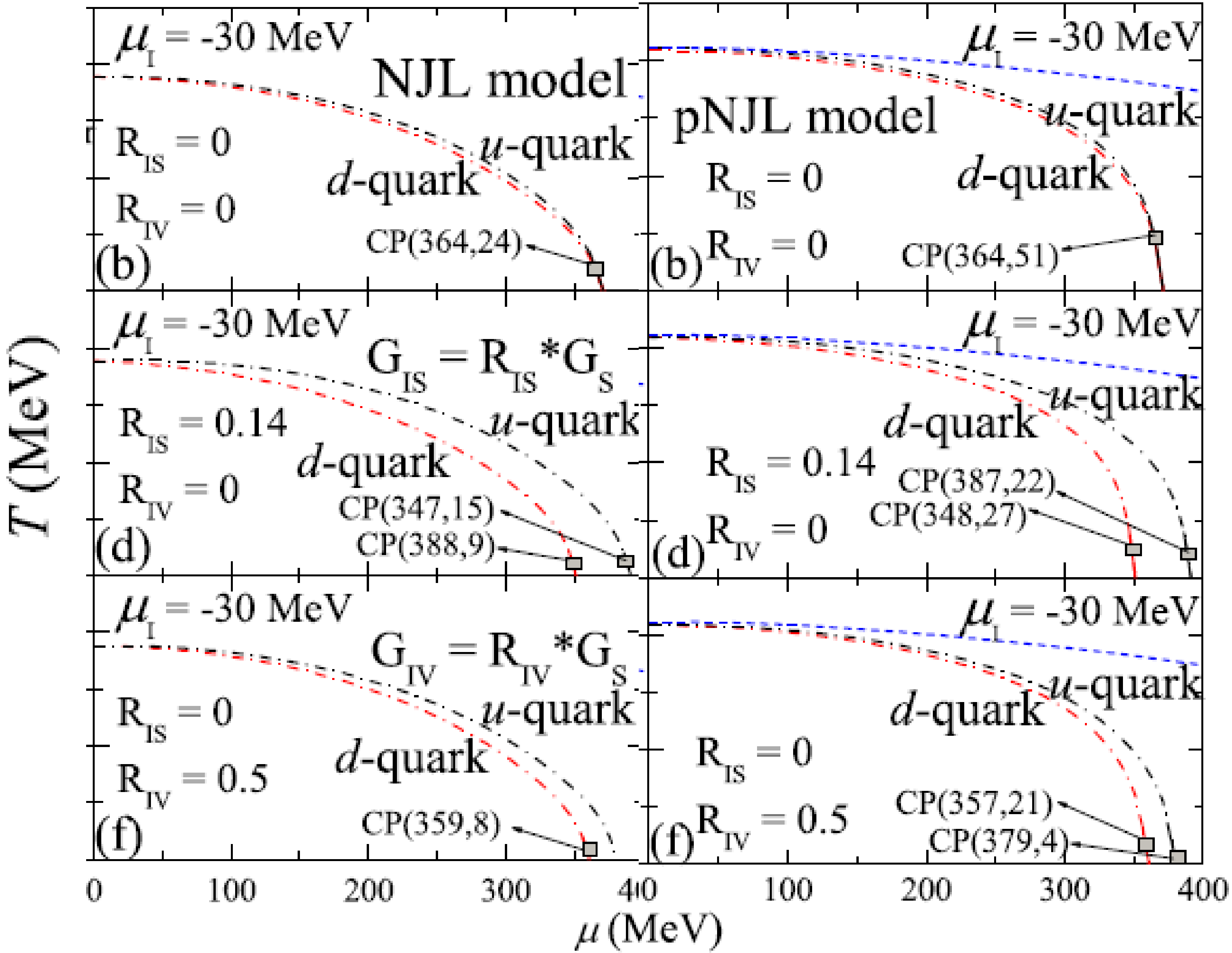
Dynamical mass $M_i = m_i - 2G_S\sigma_i + 2K\sigma_j\sigma_k - 2G_{IS}\tau_{3i}(\sigma_u-\sigma_d)$

Mass splitting
for u and d quarks,
especially near
the phase boundary



H. Liu, JX, L.W. Chen, and K.J. Sun, PRD (2016)

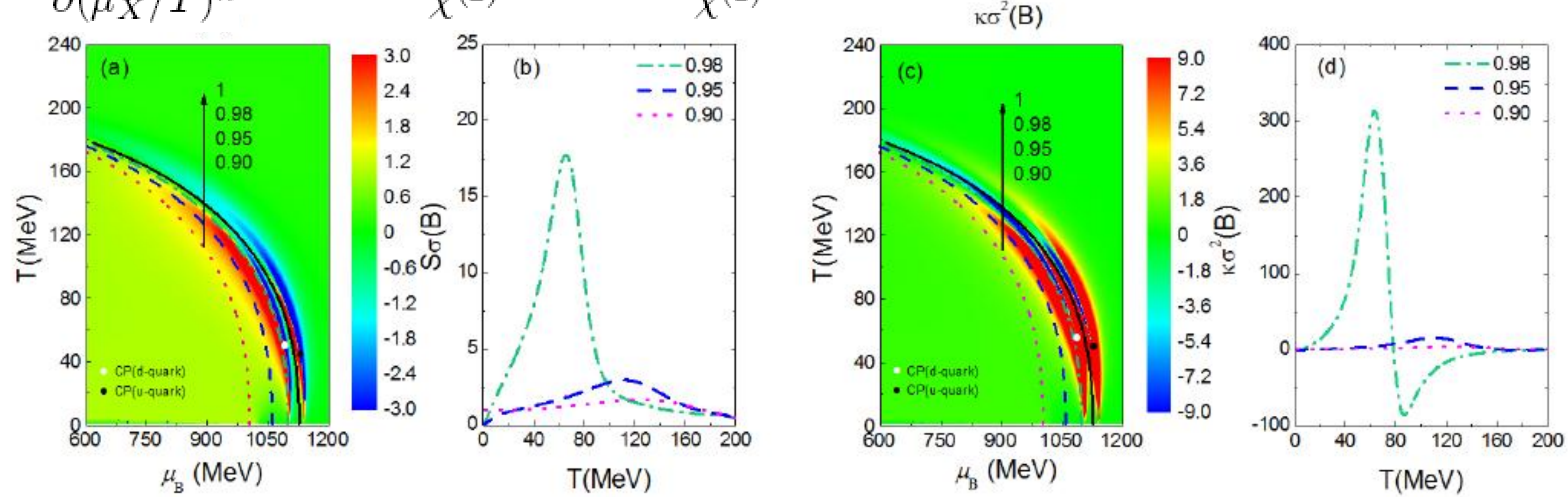
Phase diagram from (p)NJL model



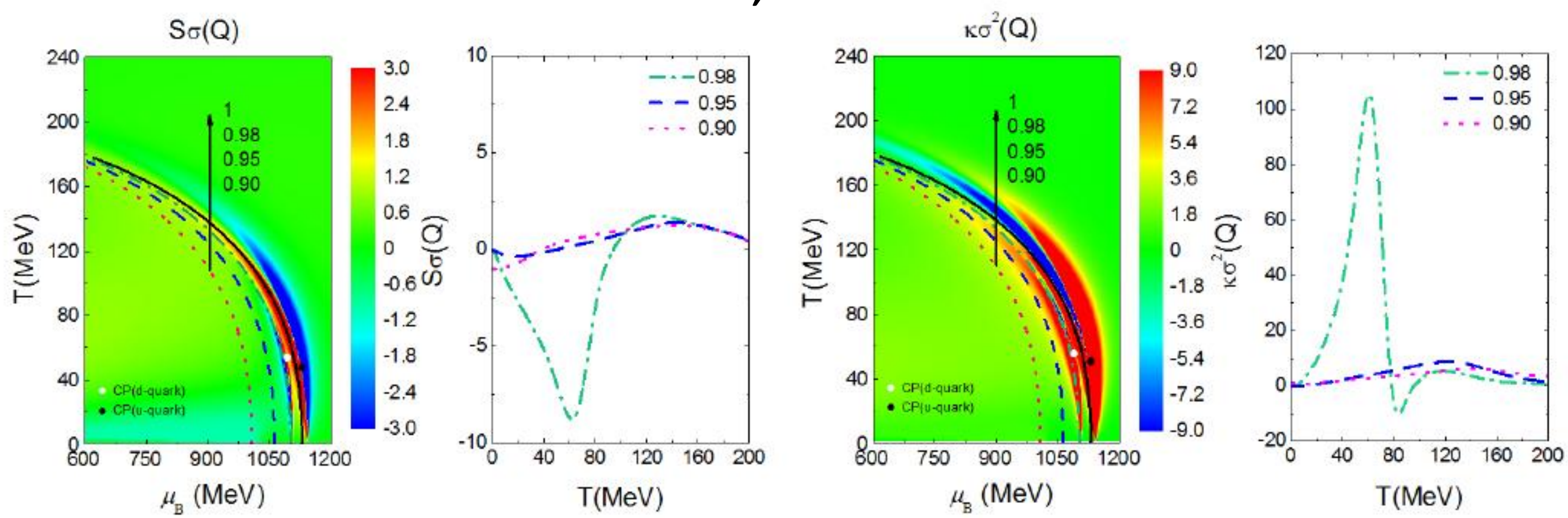
H. Liu, JX, L.W. Chen, and K.J. Sun, PRD (2016)

Susceptibility from splittings of u-d chiral phase transition

$$\chi_X^{(n)} = \frac{\partial^n(-\Omega/T)}{\partial(\mu_X/T)^n} \qquad S\sigma = \frac{\chi^{(3)}}{\chi^{(2)}}, \quad \kappa\sigma^2 = \frac{\chi^{(4)}}{\chi^{(2)}} \qquad \mu_I = -0.293 - 0.0264\mu_B \text{ (MeV)}$$



H. Liu and JX, arXiv: 1709.05178



Chiral magnetic wave effect on pion v_2 splitting

Dirac equation for massless particles

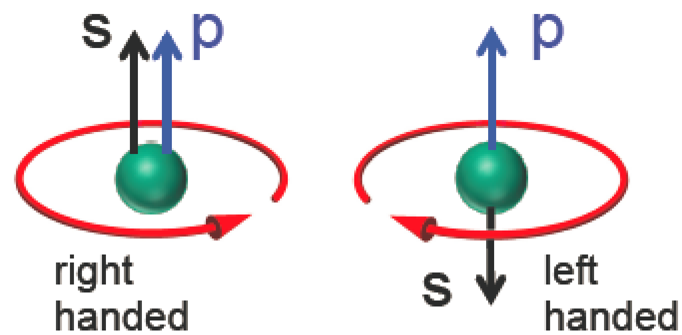
$$\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi = \bar{\psi}(\gamma^{0}\partial_{t} - \gamma^{k}\partial_{k})\psi = 0$$

$$\bar{\psi}\left[\begin{pmatrix} 0 & h \\ h & 0 \end{pmatrix} + \begin{pmatrix} 0 & \vec{\sigma}\cdot\vec{p} \\ -\vec{\sigma}\cdot\vec{p} & 0 \end{pmatrix}\right]\psi = 0$$

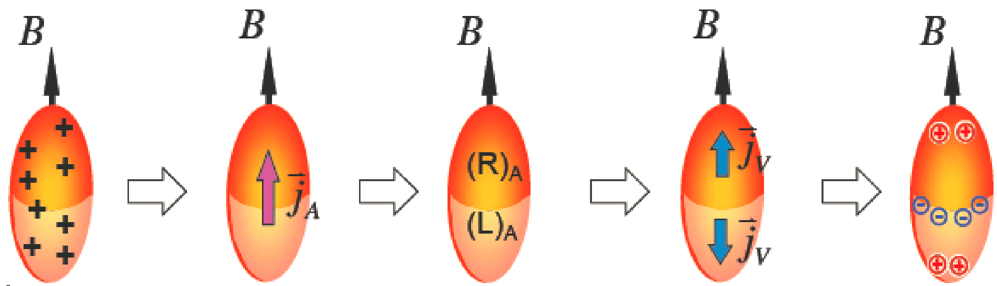
Weyl SOC

$h \sim +\vec{\sigma}\cdot\vec{p}$ for $\psi_L = \frac{1}{2}(1-\gamma^5)\psi$

$h \sim -\vec{\sigma}\cdot\vec{p}$ for $\psi_R = \frac{1}{2}(1+\gamma^5)\psi$

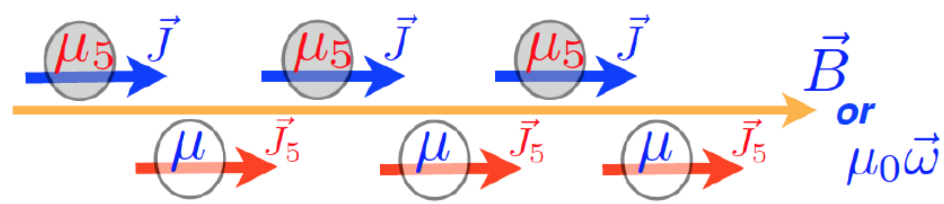


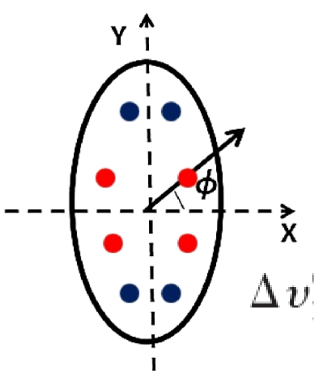
$$j_A = \frac{N_c e}{2\pi^2}\mu_V B \qquad j_V = \frac{N_c e}{2\pi^2}\mu_A B$$



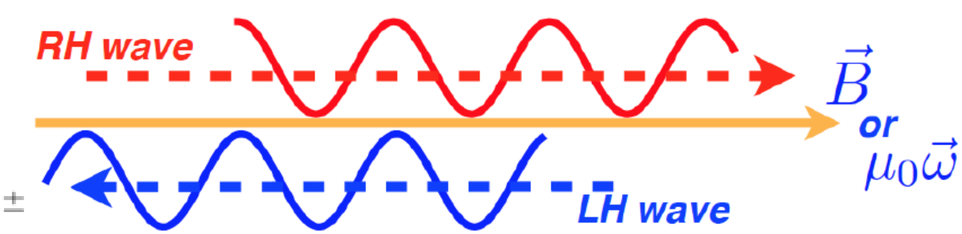
$$\mu_V \sim N_+ - N_-$$

$$\mu_A \sim N_L - N_R$$





D.E. Kharzeev and H.U. Yee, PRD (2011); Y. Burnier, D.E. Kharzeev, J.F. Liao, and H.U. Yee, PRL (2011)



$$\Delta v_2^{\mathrm{CMW}} \equiv v_2(\pi^-) - v_2(\pi^+) \simeq rA_{\pm}$$

Results from chiral dynamics

Chiral kinetic equations of motion

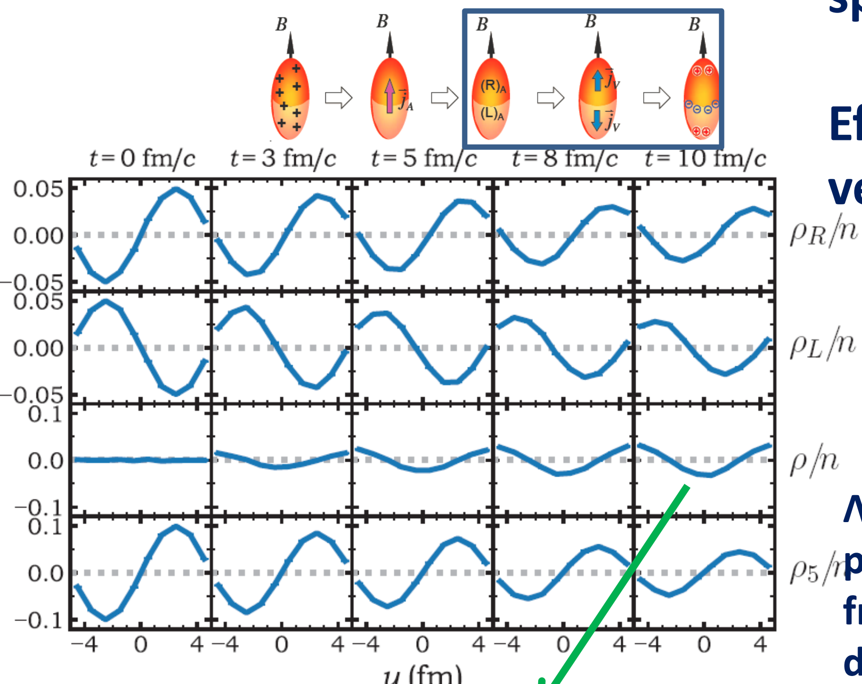
$$\sqrt{G}\frac{d\vec{r}}{dt} = \hat{k} + c\frac{\hbar}{2k^2}\vec{B} + c\frac{\hbar}{2k^3}\vec{E}\times\vec{k}$$

$$\sqrt{G}\frac{d\vec{k}}{dt} = \vec{k}\times\vec{B} + c\frac{\hbar\vec{k}}{2k^3}(\vec{E}\cdot\vec{B}) + \vec{E}$$

$$\sqrt{G} = 1 + c\frac{\vec{B}\cdot\vec{k}}{2k^3}$$

M.A. Stephenov and Y. Yin, PRL (2012)
J.W. Chen, S. Pu, Q. Wang, and X.N. Wang, PRL (2013)
D.T. Son and N. Tamamoto, PRD (2013)

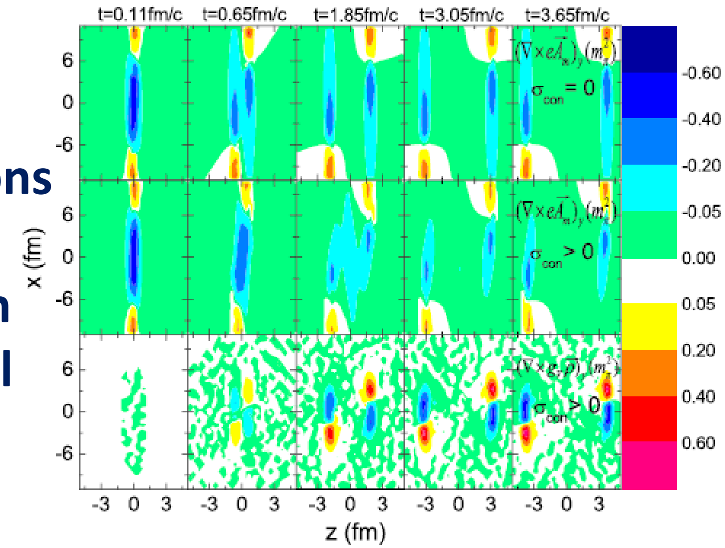
Simulating chiral magnetic wave in a box



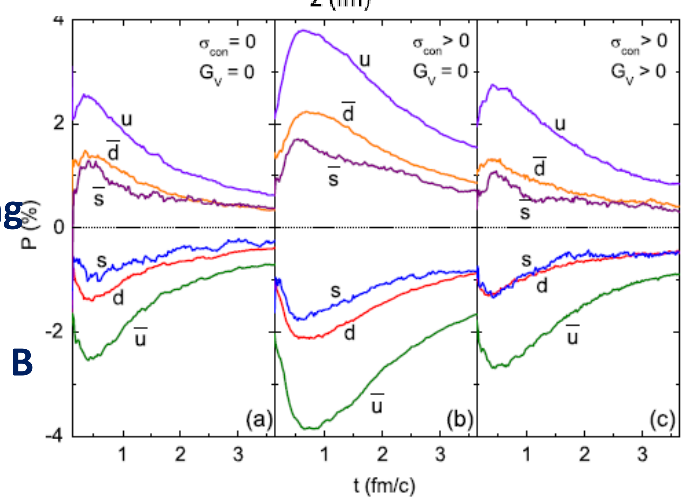
Electric quadruple moment

W.H. Zhao and JX, PRC (2018)

Real B from spectator protons
vs
Effective B from vector potential



Λ and $\bar{\Lambda}$ polarization splitting from s and $\bar{s}$ depends on both real B and effective B



CMW in HIC from real B and effective B in progress

Z.Z. Han and JX, PLB (2018)

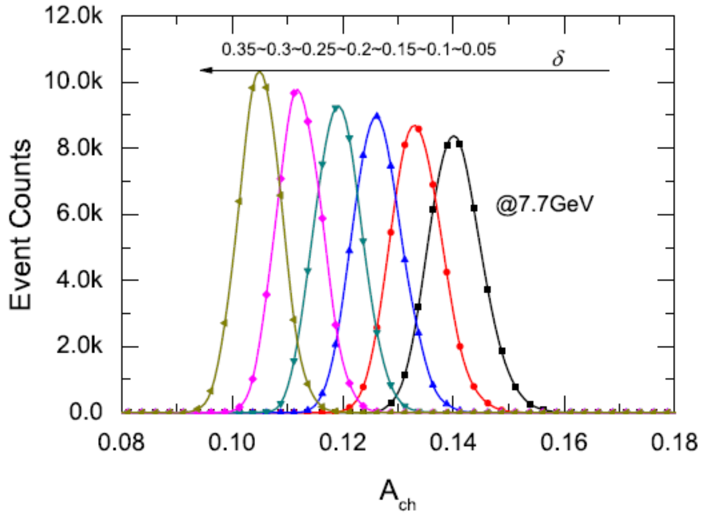
pion v_2 splitting from mean-field potentials

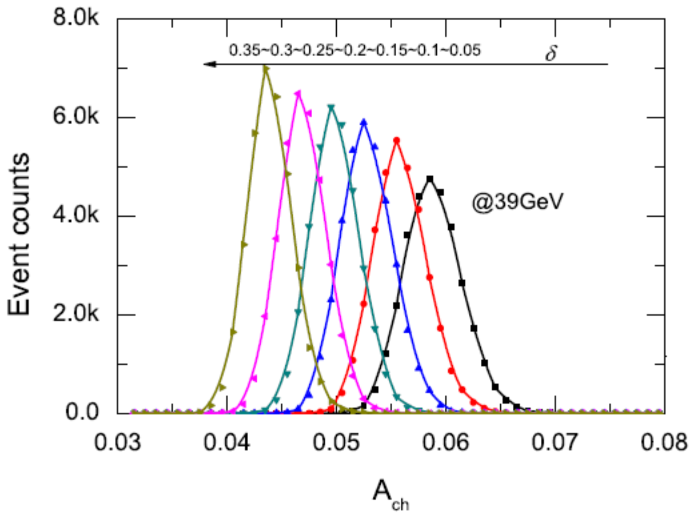
$$H_i = \sqrt{p_i^{*2} + M_i^2} \pm G_V \rho_i^0 \pm g_V \rho^0 + G_{IV} \tau_{3i} \left(\rho_u^0 - \rho_d^0\right)$$

$$\delta = 3\frac{\rho_d^0 - \rho_u^0}{\rho_d^0 + \rho_u^0} \qquad \vec{p}_i^{\,*} = \vec{p} \mp G_V \vec{\rho}_i \mp g_V \vec{\rho} \mp G_{IV} \tau_{3i} \left(\vec{\rho}_u - \vec{\rho}_d\right)$$

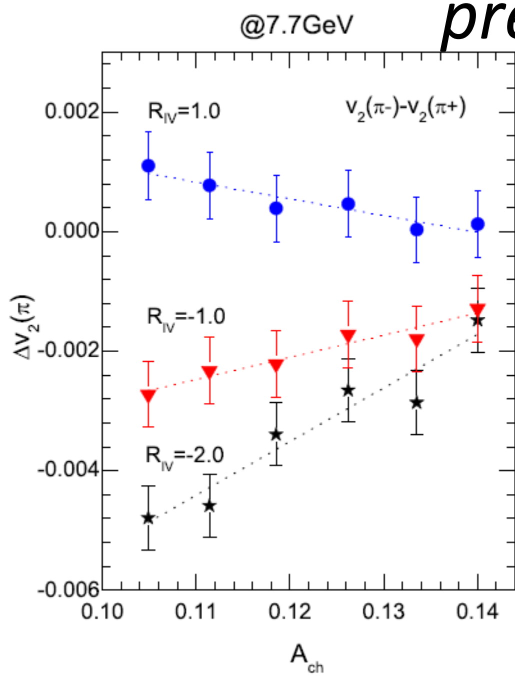
$$A_{ch} = \frac{N_+ - N_-}{N_+ + N_-}$$

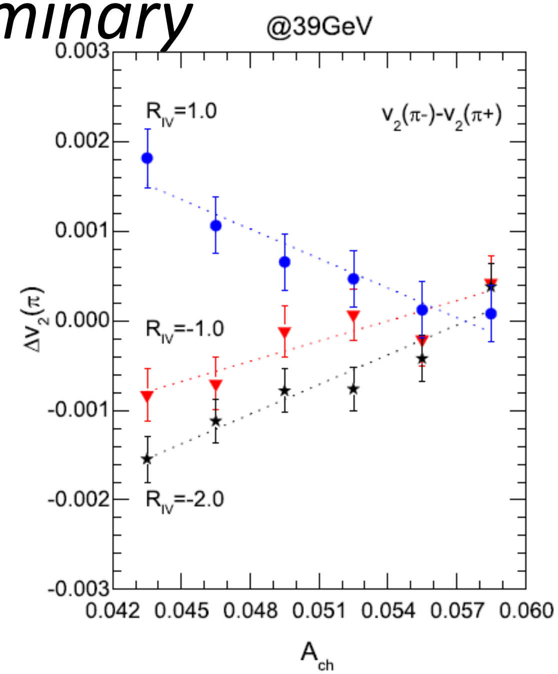
$$R_{IV} = G_{IV}/G_S$$





preliminary

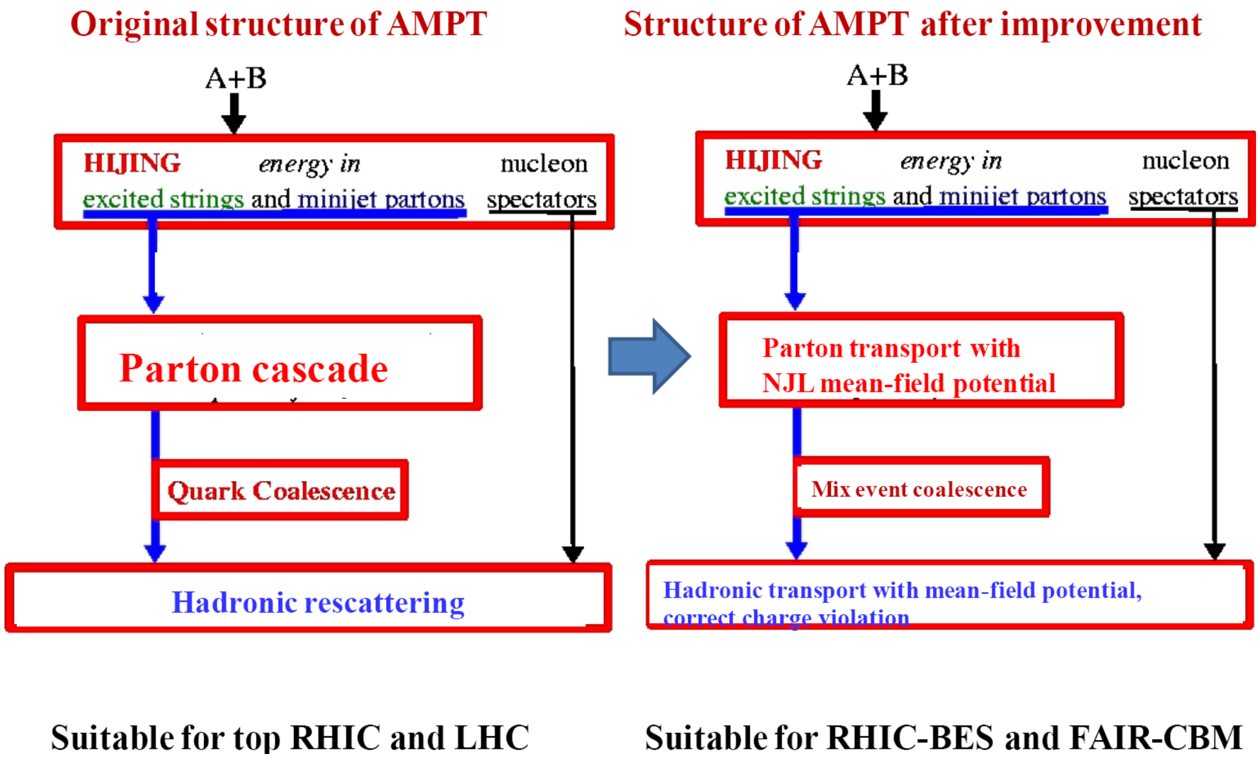




Slope of charged pions v_2 splitting is sensitive to the vector-isovector coupling.

H. Liu, C.J. Zhang, JX, and C.M. Ko, in preparation

Concluding remarks



RHIC-BES,
FAIR-CBM, etc
Flow, HBT,
Fluctuation, etc

an extended AMPT model with NJL Lagrangian

Mean-field potential

QM EOS, QCD phase diagram

Acknowledge

Collaborators：

Che Ming Ko (TAMU)
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Kai-Jia Sun (SJTU)

Students in SINAP：

Chun-Jian Zhang
He Liu
Zhang-Zhu Han
Chong-Qiang Guo
Wen-Hao Zhou

Thank you!

xujun@sinap.ac.cn

Backup

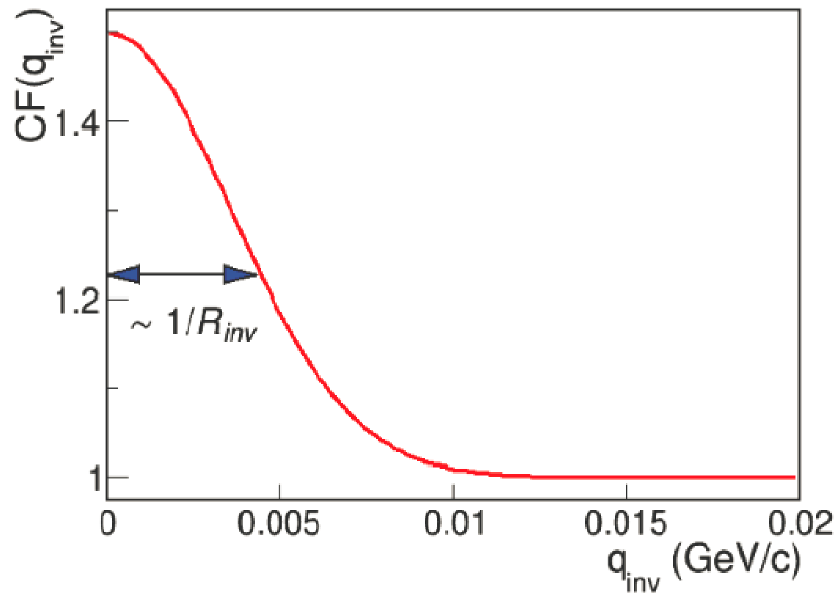
Hanbury-Brown and Twiss (HBT) Correlation

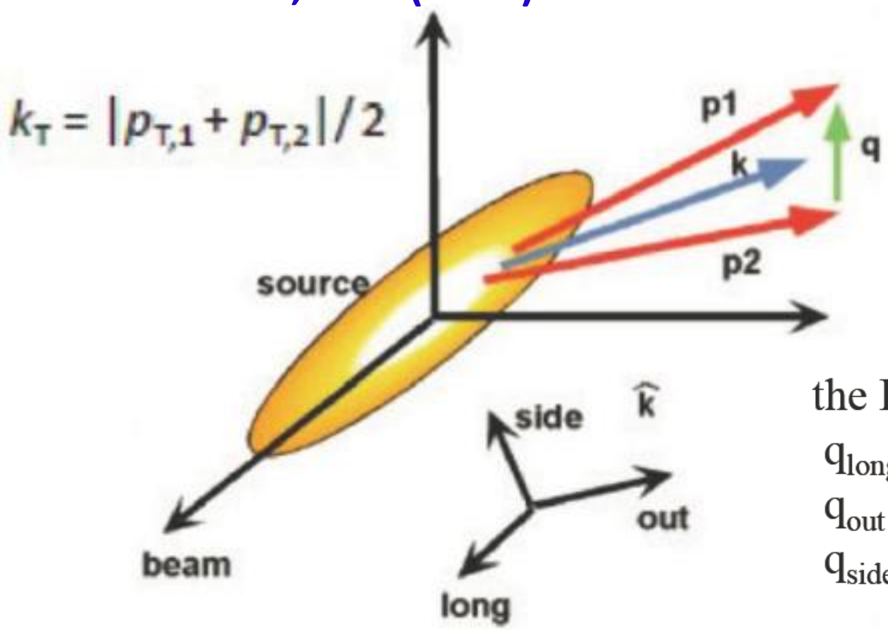
Two-particle correlation function:

$$C(\mathbf{k}^*)=\frac{\int S(\mathbf{r}^*,\mathbf{k}^*)\left|\Psi^{S(+)}_{-\mathbf{k}^*}(\mathbf{r}^*)\right|^2 d^4\mathbf{r}^*}{\int S(\mathbf{r}^*,\mathbf{k}^*)\, d^4\mathbf{r}^*}$$

where $\mathbf{r}^*=\mathbf{x}_1-\mathbf{x}_2$ and $\mathbf{k}^* = q_{inv}/2 = (\mathbf{p}_1-\mathbf{p}_2)/2$

R.H. Brown and R.Q. Twiss, Nature (1956)
S. Pratt, PRD (1986)





One Dimension :

$$C(q_{inv})=(1-\lambda)+\lambda K_{coul}(q_{inv})\left(1+e^{-q_{inv}^2R_{inv}^2}\right)$$

Three Dimension:

$$C(\vec{q})=(1-\lambda)+\lambda K_{coul}(q_{inv})\times\left(1+e^{-q_o^2R_o^2-q_s^2R_s^2-q_l^2R_l^2-2q_oq_sR_{os}^2-2q_oq_lR_{ol}^2}\right)$$

the Bertsch-Pratt, out-side-long system:

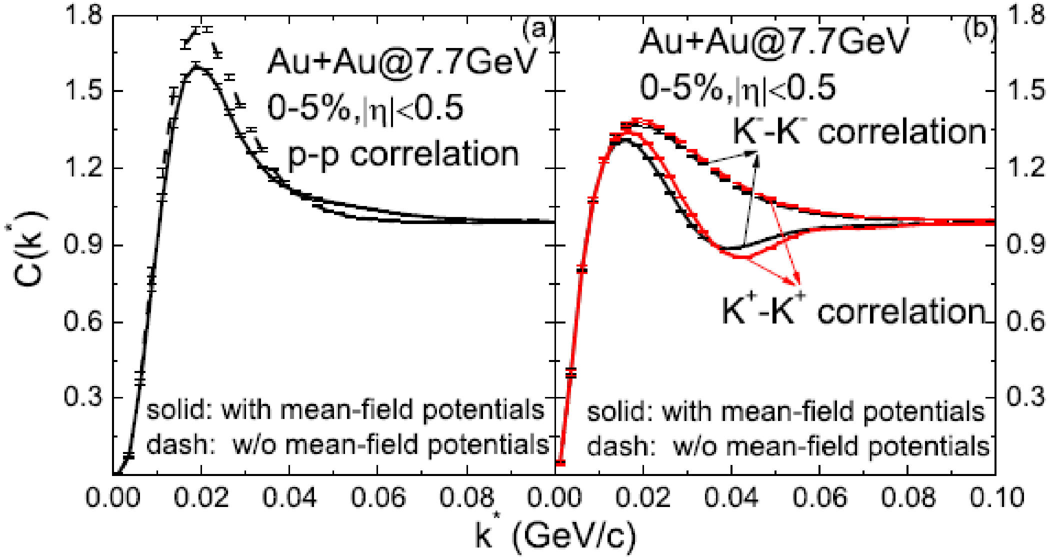
q_{long} - along the beam direction

q_{out} – along the transverse momentum of the pair

q_{side} – perpendicular to longitudinal and outwards directions

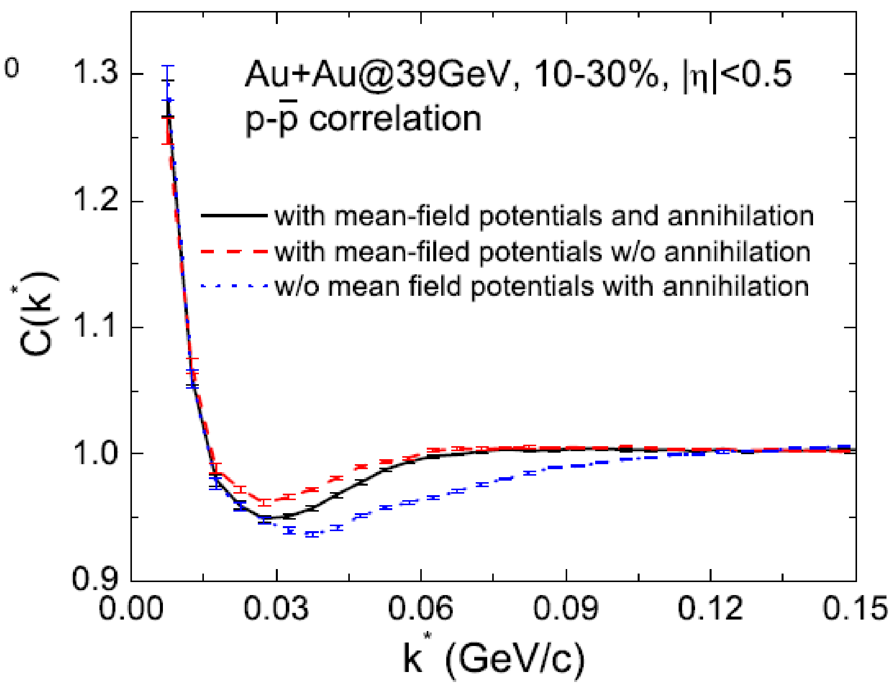
Effects of hadronic mean-field potentials on HBT correlation

Affect correlation for indentified particles



Affect global system evolution
+
Affect emission of individual particles

Interplay with p-pbar annihilation

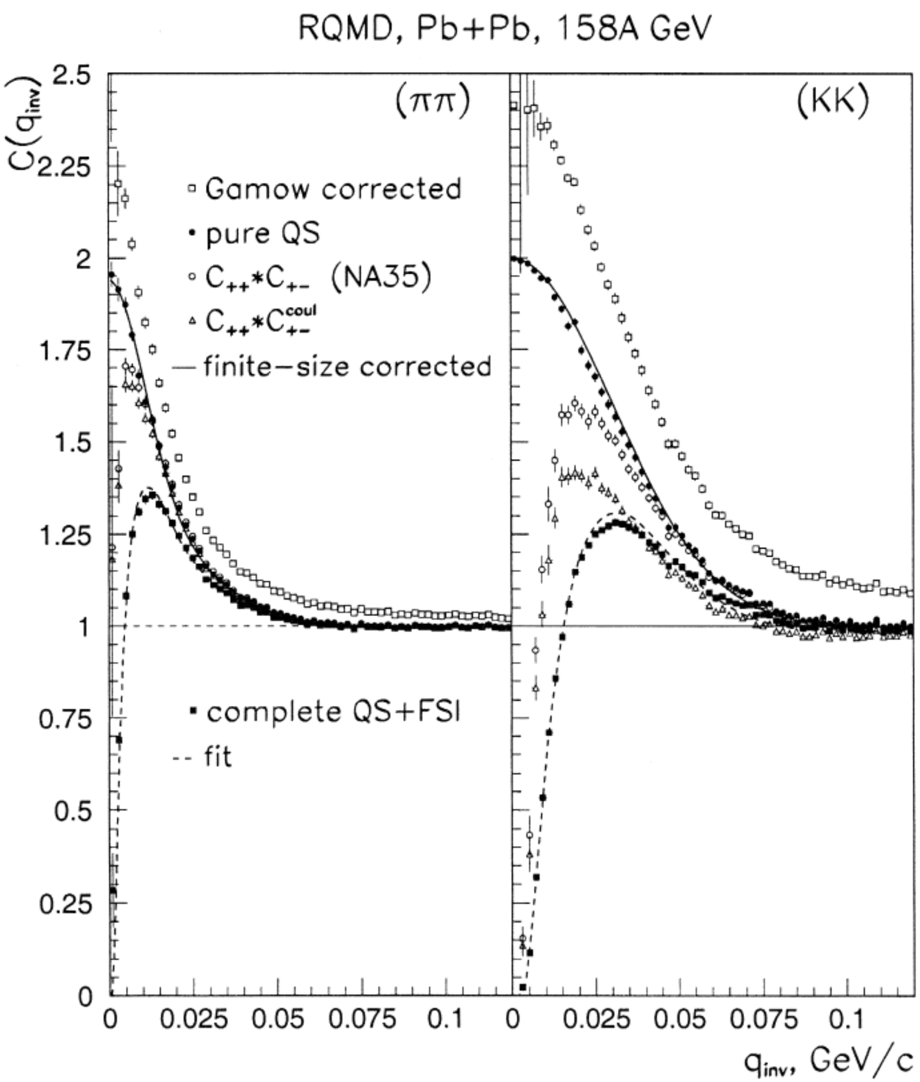


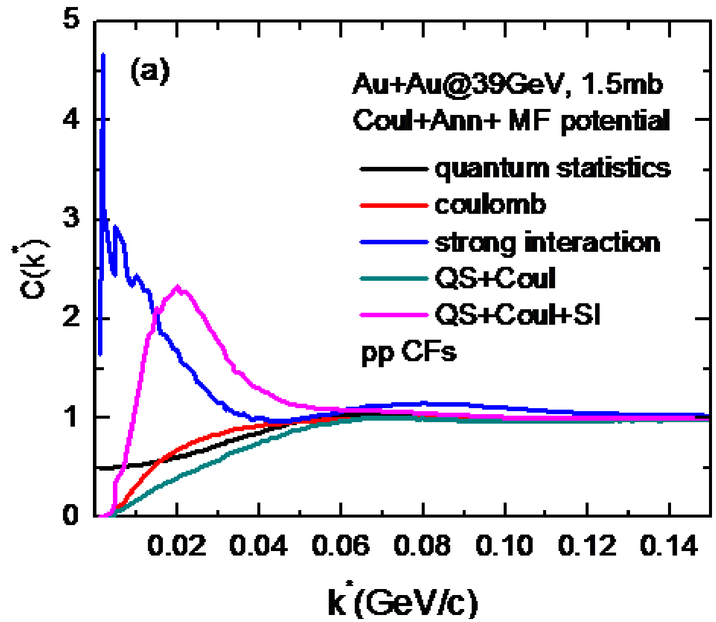
Attractive potential and later emission
=> weaken anti-correlation

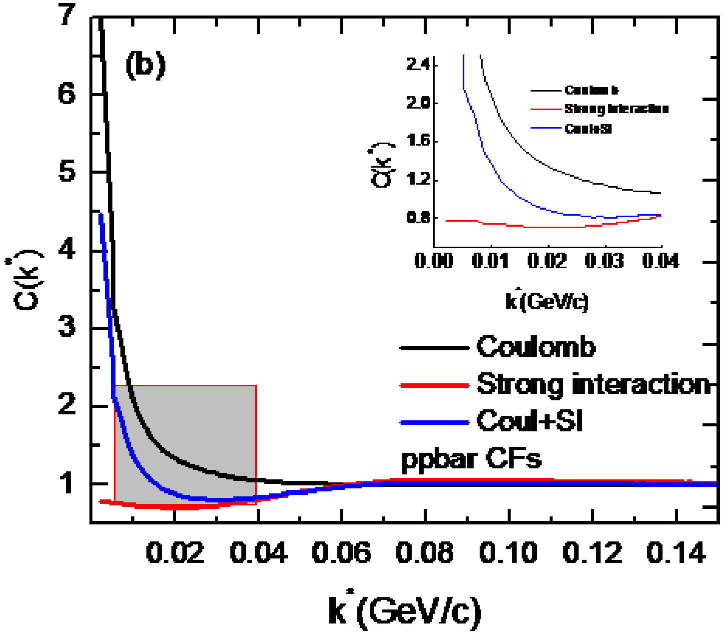
Annihilation
=> enhance anti-correlation

Chun-Jian Zhang and JX,
PRC 96, 044907 (2017)

Various contributions to HBT correlations

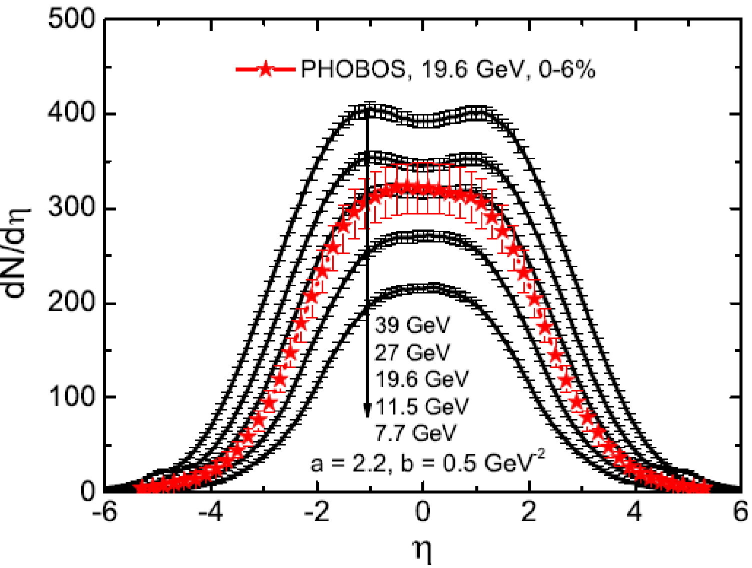


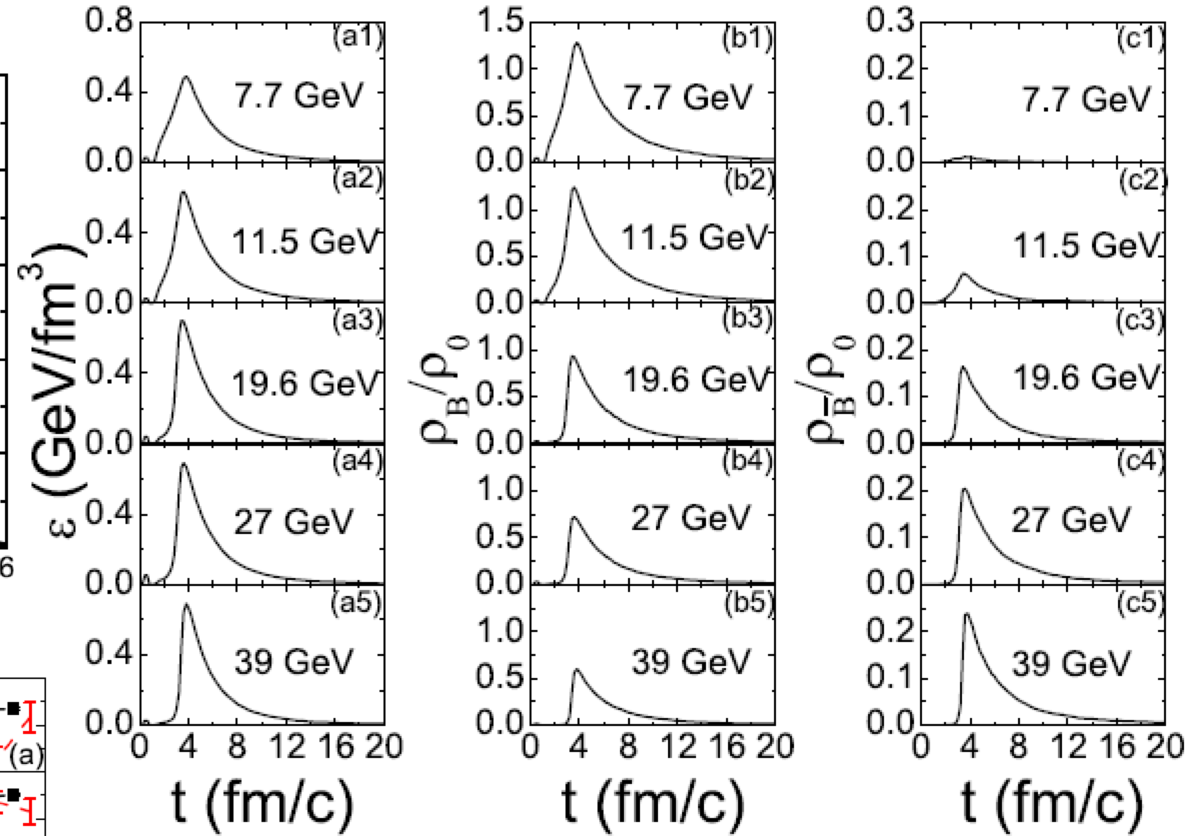


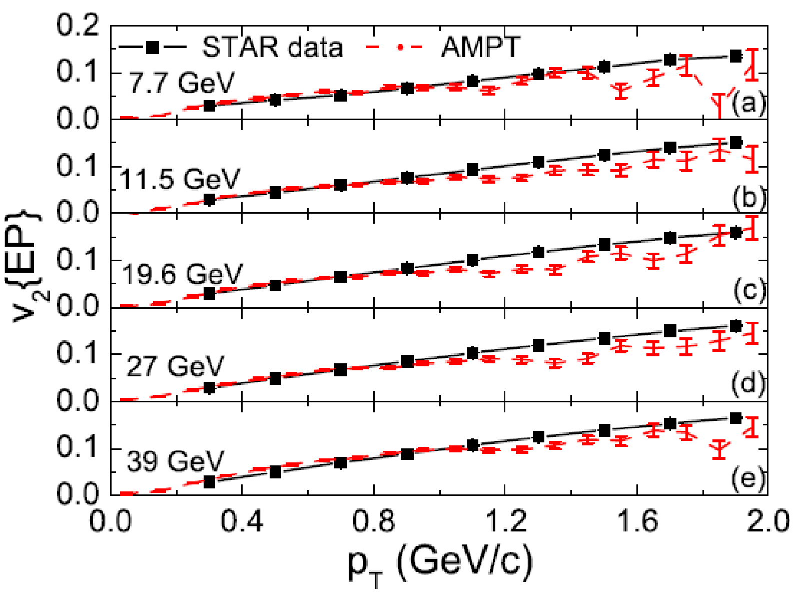


Yu.M. Sinyukov, R. Lednicky, S.V. Akkelin, J. Pluta, and B. Erazmus, PLB (1998)

Fit AMPT parameters at RHIC-BES energies







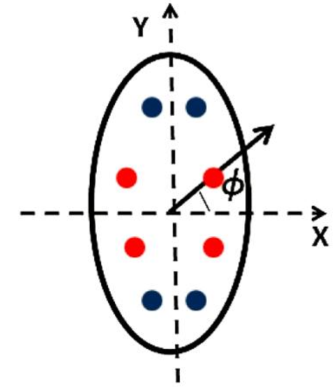
Lund string fragmentation parameters, Parton scattering cross section, parton life time

⇓

Pseudorapidity distribution, elliptic flow, energy density at chemical freeze-out

Explanations for v_2 splitting

$v_2(\pi^+) < v_2(\pi^-)$



- Chiral magnetic wave

  => electric quadrupole moment

  Y. Burnier, D. E. Kharzeev, J. F. Liao, and H. U. Yee, PRL (2011)

- Different v_2 of transported and produced partons

  J. C. Dunlop, M. A. Lisa, and P. Sorensen, PRC (2011)

- Different rapidity distributions of quarks and antiquarks

  V. Greco, M. Mitrovski, and G. Torrieri, PRC (2012)

- Conservation of baryon charge, strangeness, and isospin

  J. Steinheimer, V. Koch, and M. Bleicher, PRC (2012)

- Different mean-field potentials for particles and their antiparticles

  JX, L. W. Chen, C. M Ko, and Z. W. Lin, PRC (2012);
  T. Song, S. Plumari, V. Greco, C. M. Ko, and F. Li, arXiv:1211.5511 [nucl-th];
  JX, T. Song, C. M. Ko, and F. Li, PRL (2014)

- Different radial flows of protons and antiprotons

  X. Sun, H. Masui, A.M. Poskanzer, and A. Schmach, PRC (2015)

- Hydrodyanmics at finite baryon chemical potential

  Y. Hatta, A. Monnai, and B.W. Xiao, arXiv: 1505.04226 [nucl-th]; 1507.04909 [nucl-th]

Quark condensate:

$$\langle \bar{q}_i q_i \rangle = -2M_i N_c \int \frac{d^3\mathbf{k}}{(2\pi)^3 E_i} \left[1 - f_i(k) - \bar{f}_i(k)\right],$$

Quark 4-dimensional density:

$$\langle \bar{\psi} \gamma^\mu \psi \rangle = 2N_c \sum_{i=u,d,s} \int \frac{d^3\mathbf{k}}{(2\pi)^3 E_i} \, k^\mu \left[f_i(k) - \bar{f}_i(k)\right],$$

$$E_i(p) = \sqrt{p^2 + M_i^2}$$

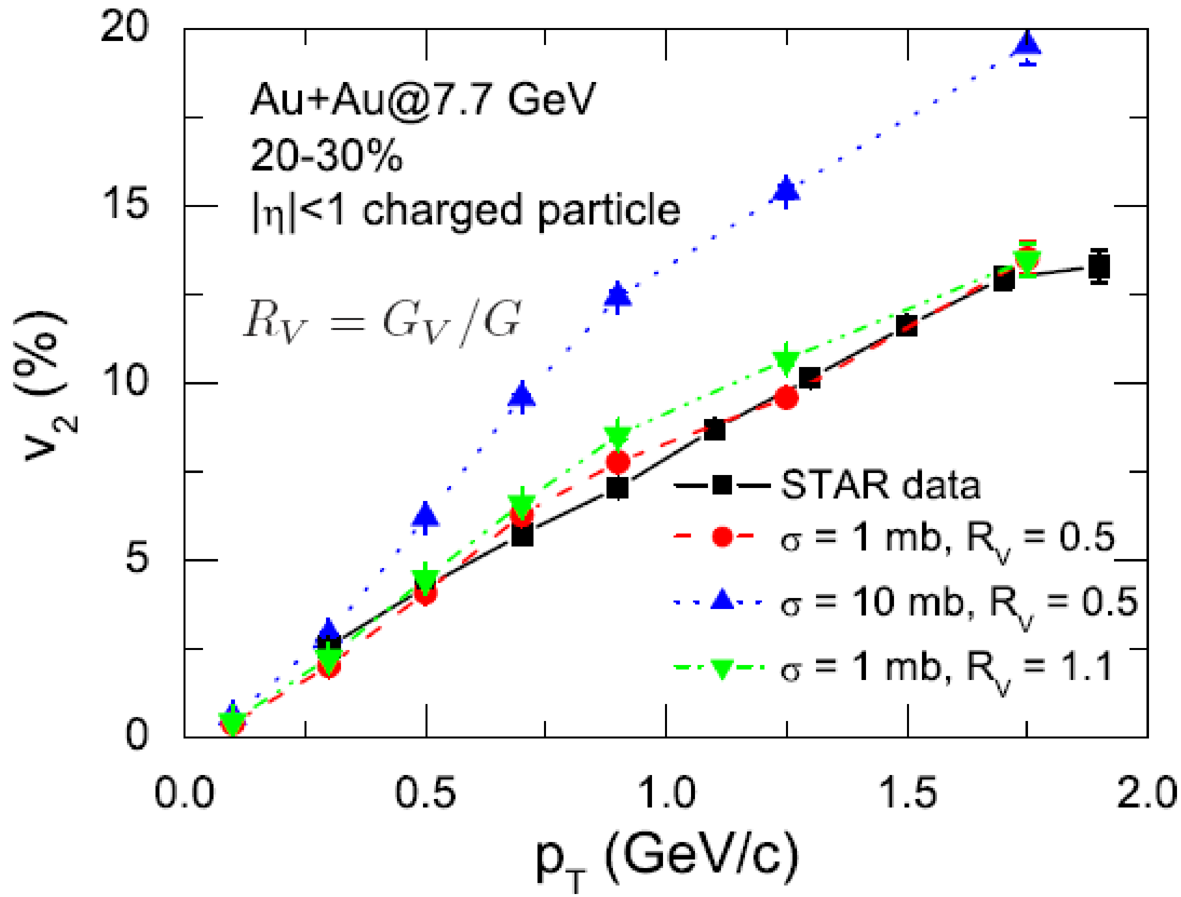
$$\begin{aligned} M_u &= m_u - 2G\langle \bar{u}u \rangle + 2K\langle \bar{d}d \rangle \langle \bar{s}s \rangle, \\ M_d &= m_d - 2G\langle \bar{d}d \rangle + 2K\langle \bar{s}s \rangle \langle \bar{u}u \rangle, \\ M_s &= m_s - 2G\langle \bar{s}s \rangle + 2K\langle \bar{u}u \rangle \langle \bar{d}d \rangle, \end{aligned}$$

iteration needed

test particle method:

$$f(\vec{x}, \vec{k}) = \frac{1}{N_{test}} \sum_i g(\vec{x} - \vec{x}_i) g'(\vec{k} - \vec{k}_i)$$

Fit the parton scattering cross section with charged-particle v_2



Hadronization happens when chiral symmetry is broken, i.e., $M^*>M_{vac}/2$

(global hadronization,
can be improved by local hadronization)

Fierz transformation: $R_V = 0.5$
Vector meson-mass spectrum: $R_V = 1.1$

Total v_2 is less sensitive to R_V

EOS of quark phase from NJL

$$\Omega_{\rm NJL} = -2N_c \sum_{i=u,d,s} \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} [E_i + T\ln(1+e^{-\beta(E_i-\tilde{\mu}_i)})$$

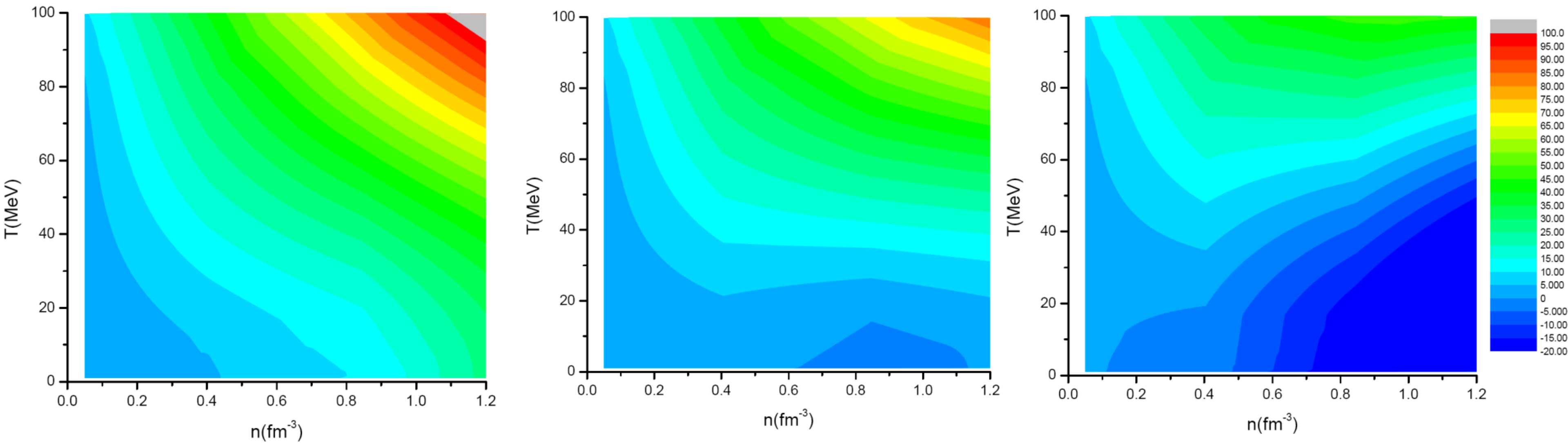
$$+ T\ln(1+e^{-\beta(E_i+\tilde{\mu}_i)})] + G_S(\sigma_u^2+\sigma_d^2+\sigma_s^2)$$

$$-4K\sigma_u\sigma_d\sigma_s - \frac{1}{3}G_V(\rho_u+\rho_d+\rho_s)^2$$

$$P = -\Omega_{NJL}$$

Pressure in the temperature-density (T-n) plane

$G_V = 1.1G$ **$G_V = 0$** **$G_V = -1.1G$?**



Phase diagram from NJL model

Lagrangian:

$$L=\bar{q}(i\gamma^{\mu}\partial_{\mu}-M)q+\frac{G}{2}\sum_{a=0}^{8}\left[\left(\bar{q}\lambda^{a}q\right)^{2}+\left(\bar{q}i\gamma_{5}\lambda^{a}q\right)^{2}\right]-\frac{G_{V}}{2}\sum_{a=0}^{8}\left[\left(\bar{q}\gamma_{\mu}\lambda^{a}q\right)^{2}+\left(\bar{q}\gamma_{\mu}\gamma_{5}\lambda^{a}q\right)^{2}\right]$$

$$-K\left[\det{}_{f}\left(\bar{q}(1+\gamma_{5})q\right)+\det{}_{f}\left(\bar{q}(1-\gamma^{5})q\right)\right]$$

After mean-field approximation:

$$L_{MF}=\sum_{q=u,d,s}\bar{q}(\gamma^{\mu}i\partial'_{\mu}-M'_{q})q+L_{den}$$

$$M_{u}=m_{u}-2G\phi_{u}+2K\phi_{d}\phi_{s}$$

$$M_{d}=m_{d}-2G\phi_{d}+2K\phi_{u}\phi_{s}$$

$$M_{s}=m_{s}-2G\phi_{s}+2K\phi_{u}\phi_{d}$$

$$i\partial'_{\mu}=i\partial_{\mu}-\frac{2}{3}G_{V}\rho$$

Quark condensate $\phi_{q}=\langle\bar{q}q\rangle$

Quark density $\rho_{q}^{\mu}=\langle\bar{q}\gamma^{\mu}q\rangle$

Net quark density $\rho=\rho_{u}^{0}+\rho_{d}^{0}+\rho_{s}^{0}$

$$L_{den}=-G\left(\phi_{u}^{2}+\phi_{d}^{2}+\phi_{s}^{2}\right)+4K\phi_{u}\phi_{d}\phi_{s}-\frac{1}{3}G_{V}\rho^{2}$$

Free Fermions:

$$L = \bar{\psi}(\gamma^{\mu} i\partial_{\mu} - m)\psi \qquad H = \pi \frac{\partial \psi}{\partial t} - L \qquad \pi = \frac{\partial L}{\partial(\partial\psi/\partial t)}$$

Partition function:

$$Z = Tr[e^{-\beta(H - \mu\bar{\psi}\gamma^0\psi)}]$$

converse baryon charge

$$\ln Z = 2V \int \frac{d^3p}{(2\pi)^3}\left[\beta\omega + \ln(1+e^{-\beta(\omega-\mu)}) + \ln(1+e^{-\beta(\omega+\mu)})\right] \qquad \Omega = -\frac{T}{V}\ln Z$$

$$\omega = \sqrt{p^2 + m^2}$$

NJL system

$$L_{MF} = \sum_{q=u,d,s} \bar{q}(\gamma^{\mu} i\partial'_{\mu} - M_q)q + L_{den} \qquad i\partial'_{\mu} = i\partial_{\mu} - \frac{2}{3}G_V\rho$$

$$H - \mu\bar{q}\gamma^0 q = \tilde{H} - \tilde{\mu}\bar{q}\gamma^0 q$$

reduced chemical potential

$$\tilde{\mu} = \mu - \frac{2}{3}G_V\rho$$

Thermal potential

$$\Omega = \Omega_{cond} + \Omega_{quark} + \Omega_{zero}$$

$$\Omega_{cond} = -L_{den}$$

$$\frac{\partial \Omega}{\partial \phi_q} = 0$$

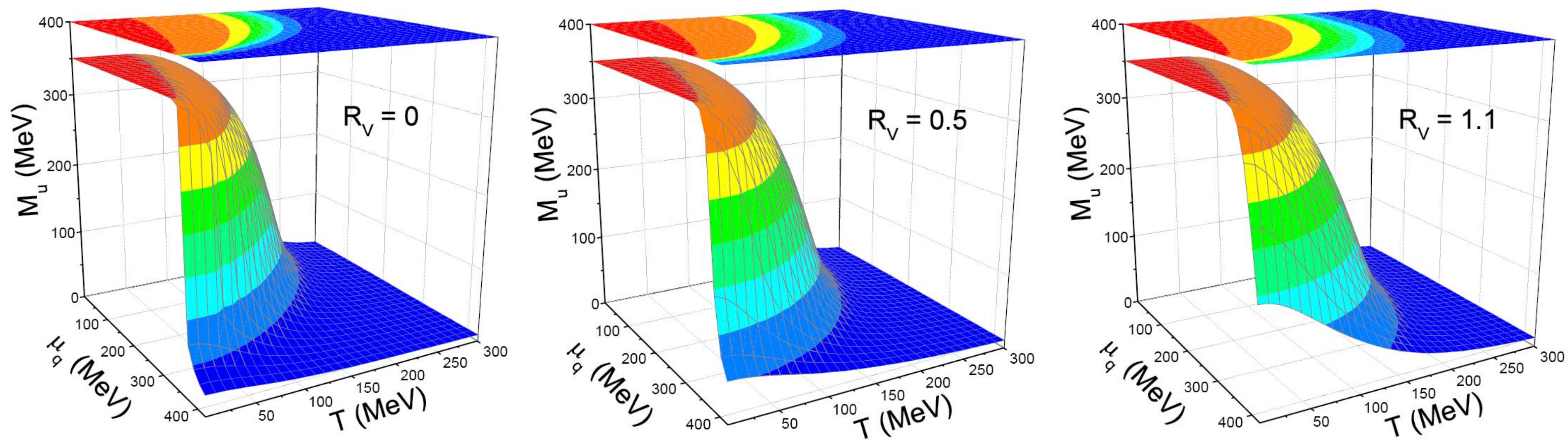
$$\Omega_{quark} = -2TN_c \sum_{q=u,d,s} \int \frac{d^3p}{(2\pi)^3}\left[\ln(1+e^{-\beta(E_q-\tilde{\mu})}) + \ln(1+e^{-\beta(E_q+\tilde{\mu})})\right]$$

with cut-off parameter Λ

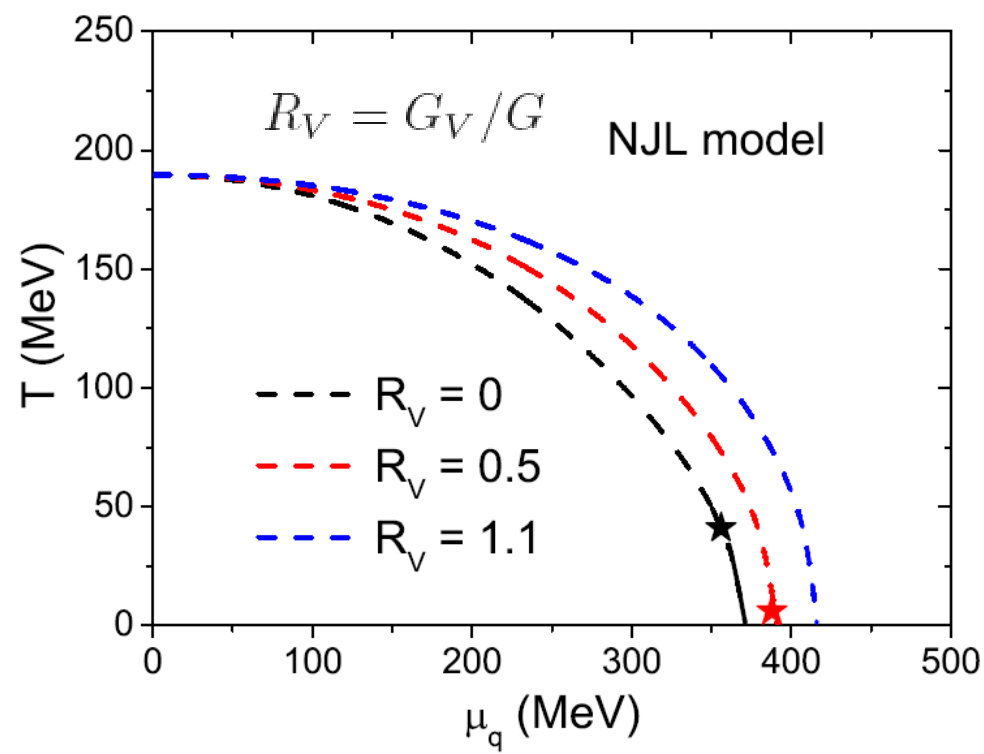
$$\Omega_{zero} = -2N_c \sum_{q=u,d,s} \int \frac{d^3p}{(2\pi)^3} E_q \qquad E_q = \sqrt{p^2 + M_q^2}$$

Quark mass (Quark condensate)

$R_V = G_V / G$



NJL Phase diagram (chiral transition)



NJL model with polyakov loop

Polyakov loop:

$$L(x) = P\exp\left[-ig\int_0^\beta dx_4 A_4(x, x_4)\right]$$

A_4: gauge field

$$l = \frac{1}{N_c} Tr[L]$$

Thermal potential

$$\Omega = \Omega_{cond} + \Omega_{quark} + \Omega_{zero} + \Omega_{polyakov}$$

$$\Omega_{quark} = -2T \sum_{q=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left[\ln\det(1 + Le^{-\beta(E_q - \tilde{\mu})}) + \ln\det(1 + L^+ e^{-\beta(E_q + \tilde{\mu})})\right]$$

$$\Omega_{polyakov} = -bT\left\{54e^{-a/T} l\bar{l} + \ln\left[1 - 6l\bar{l} - 3\left(l\bar{l}\right)^2 + 4\left(l^3 + \bar{l}^3\right)\right]\right\}$$

Taken from Kenji Fukushima, PRD (2008)

$$\frac{\partial\Omega}{\partial\phi_q} = \frac{\partial\Omega}{\partial l} = \frac{\partial\Omega}{\partial\bar{l}} = 0$$

Polyakov loop: order parameter of deconfinement

The expectation value of the Polyakov loop and its correlation in the pure gluonic theory can be written as [65–67]

$$\Phi = \langle \ell(\boldsymbol{x}) \rangle = \mathrm{e}^{-\beta f_{\mathrm{q}}}, \qquad \bar{\Phi} = \langle \ell^{\dagger}(\boldsymbol{x}) \rangle = \mathrm{e}^{-\beta f_{\bar{\mathrm{q}}}}, \tag{4}$$

$$\langle \ell^{\dagger}(\boldsymbol{x})\, \ell(\boldsymbol{y}) \rangle = \mathrm{e}^{-\beta f_{\bar{\mathrm{q}}\mathrm{q}}(\boldsymbol{x}-\boldsymbol{y})}. \tag{5}$$

Here, the constant f_{q} ($f_{\bar{\mathrm{q}}}$) independent of $\boldsymbol{x}$ is the excess free energy for a static quark (anti-quark) in a hot gluon medium[3]. Also, $f_{\bar{\mathrm{q}}\mathrm{q}}(\boldsymbol{x}-\boldsymbol{y})$ is the excess free energy for an anti-quark at $\boldsymbol{x}$ and a quark at $\boldsymbol{y}$.[4]

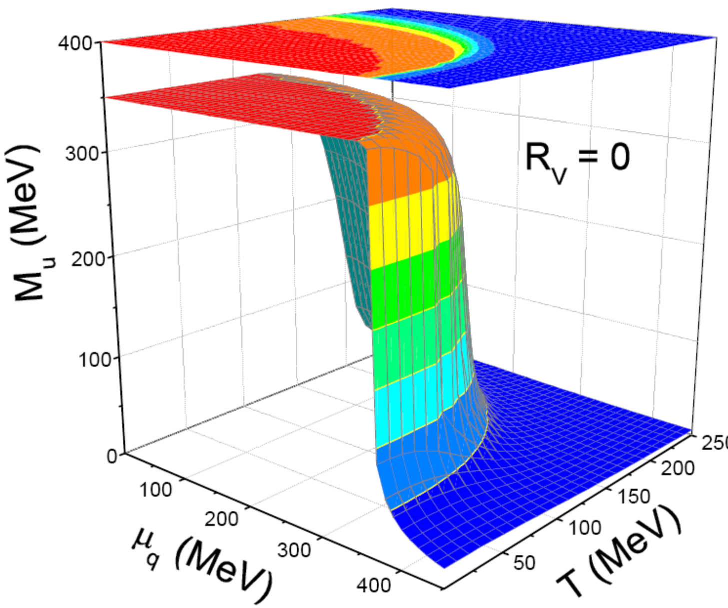
Kenji Fukushima and Tetsuo Hatsuda, Rep. Prog. Phys. 2011

Table 1. Behaviour of the expectation value and the correlation of the Polyakov loop in the confined and deconfined phases in the pure gluonic theory.

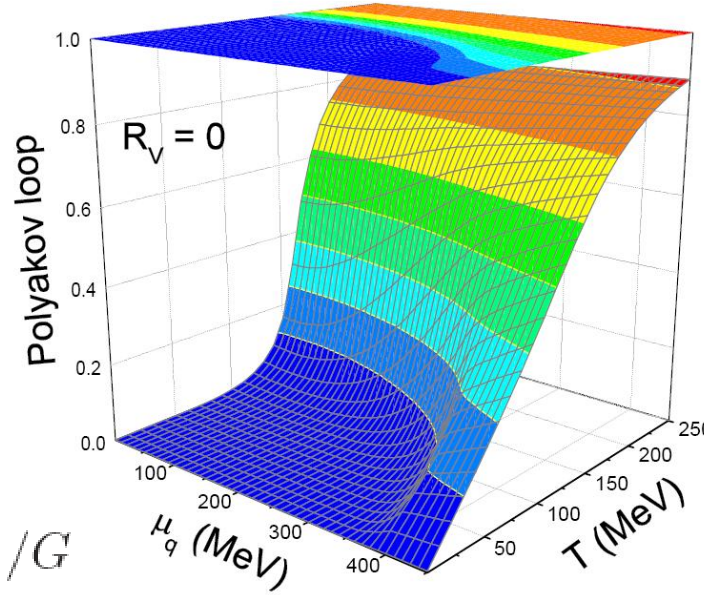
	Confined (disordered) phase	Deconfined (ordered) phase
Free energy	$f_{\mathrm{q}} = \infty$	$f_{\mathrm{q}} < \infty$
	$f_{\bar{\mathrm{q}}\mathrm{q}} \sim \sigma r$	$f_{\bar{\mathrm{q}}\mathrm{q}} \sim f_{\mathrm{q}} + f_{\bar{\mathrm{q}}} + \alpha \dfrac{\mathrm{e}^{-m_{\mathrm{M}} r}}{r}$
Polyakov loop ($r \to \infty$)	$\langle \ell \rangle = 0$ $\langle \ell^{\dagger}(r) \ell(0) \rangle \to 0$	$\langle \ell \rangle \neq 0$ $\langle \ell^{\dagger}(r) \ell(0) \rangle \to \lvert\langle \ell \rangle\rvert^2 \neq 0$

Quark mass/condensate (chiral transition)

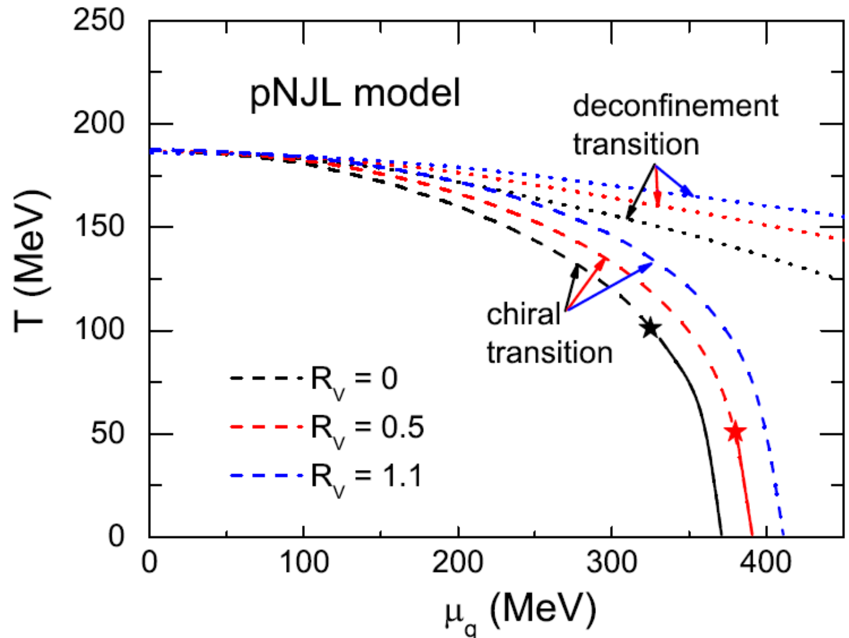
Polyakov loop (deconfinement transition)



$$R_V = G_V/G$$



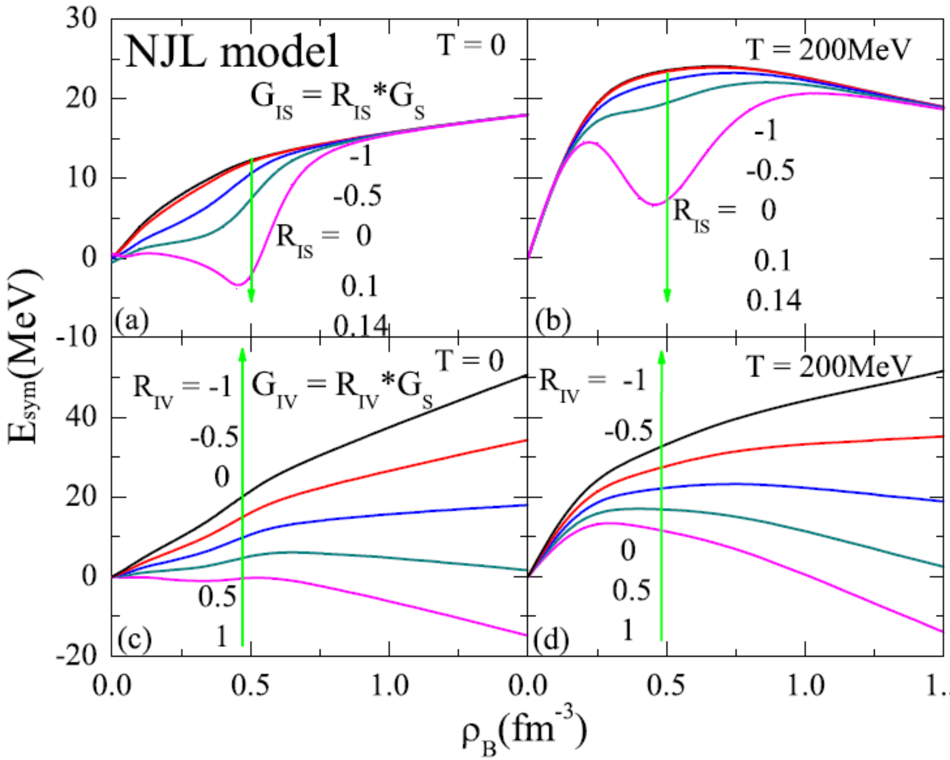
pNJL Phase diagram Chiral&Deconfinement

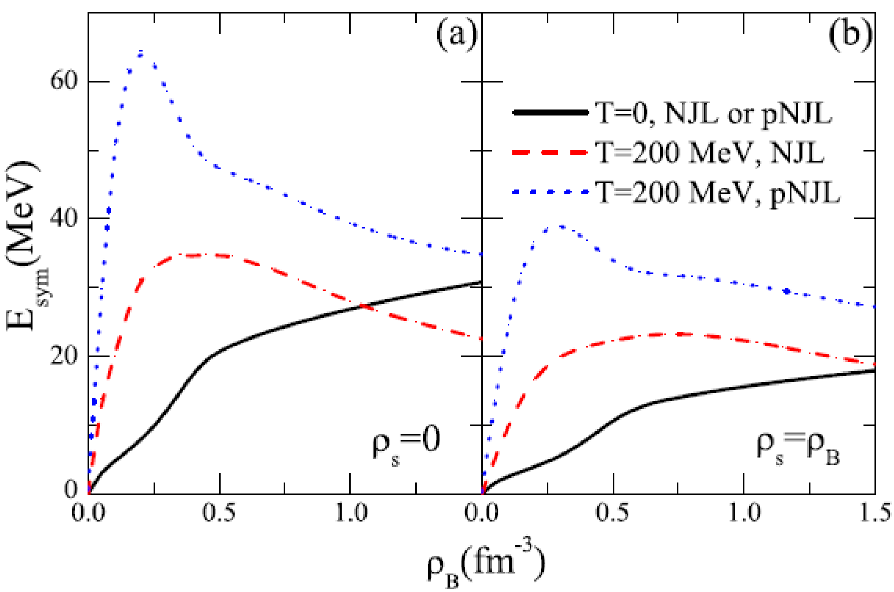


Symmetry energy from NJL model

$$E(\rho_B,\delta,\rho_s) = E_0(\rho_B,\rho_s) + E_{\rm sym}(\rho_B,\rho_s)\delta^2 + \vartheta(\delta^4). \quad E_{\rm sym}(\rho_B,\rho_s) = \frac{1}{2!}\frac{\partial^2 E(\rho_B,\delta,\rho_s)}{\partial\delta^2}\bigg|_{\delta=0}$$

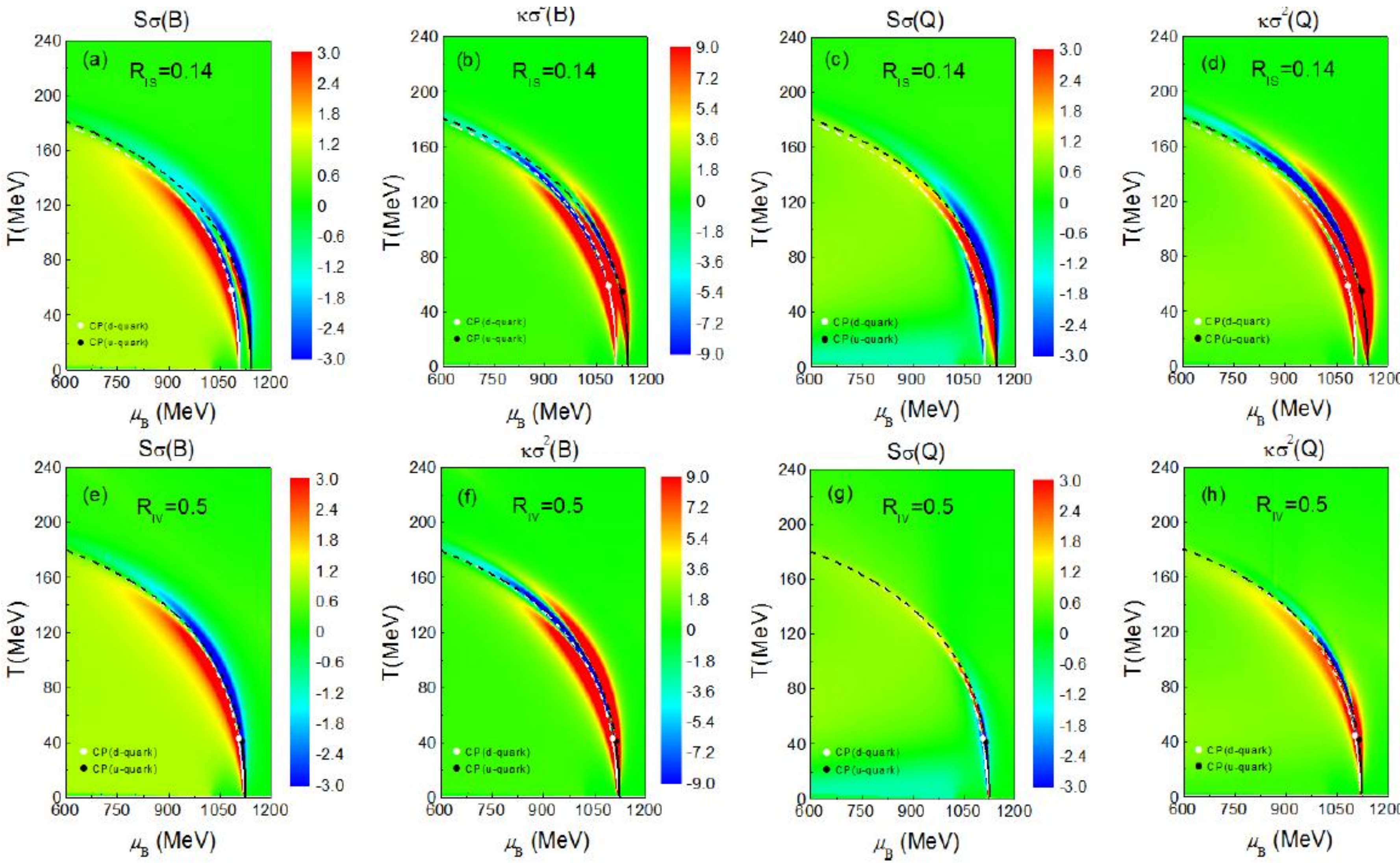
$$\begin{aligned}\varepsilon_{\rm NJL} &= -2N_c \sum_{i=u,d,s}\int_0^\Lambda \frac{d^3p}{(2\pi)^3} E_i(1-f_i-\bar{f}_i) \\ &- \sum_{i=u,d,s}(\tilde{\mu}_i - \mu_i)\rho_i + G_S(\sigma_u^2+\sigma_d^2+\sigma_s^2) \\ &- 4K\sigma_u\sigma_d\sigma_s + G_V(\rho_u^2+\rho_d^2+\rho_s^2) \\ &+ G_{IS}(\sigma_u-\sigma_d)^2 + G_{IV}(\rho_u-\rho_d)^2 - \varepsilon_0.\end{aligned}$$





H. Liu, JX, L.W. Chen, and K.J. Sun, PRD (2016)

Isospin effect on susceptibility from pNJL

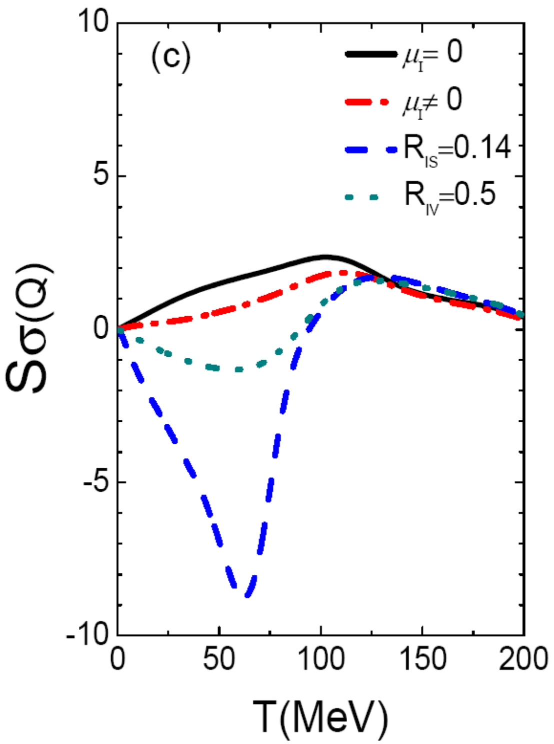


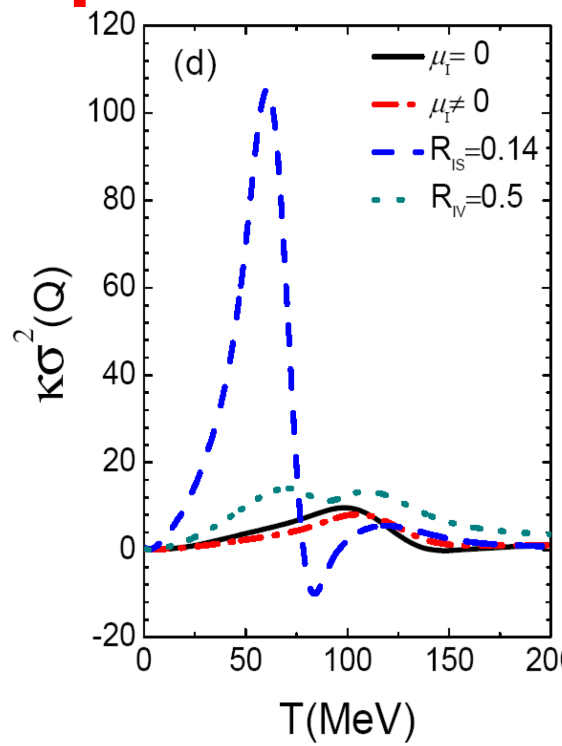
$$\chi_X^{(n)} = \frac{\partial^n(-\Omega/T)}{\partial(\mu_X/T)^n} \qquad S\sigma = \frac{\chi^{(3)}}{\chi^{(2)}}, \quad \kappa\sigma^2 = \frac{\chi^{(4)}}{\chi^{(2)}}$$

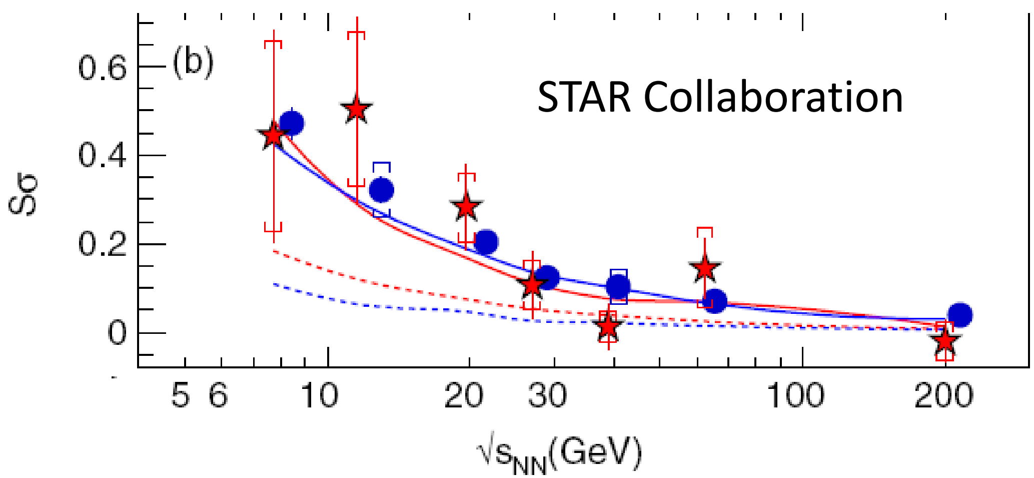
$$\mu_I = -0.293 - 0.0264\mu_B \text{ (MeV)}$$

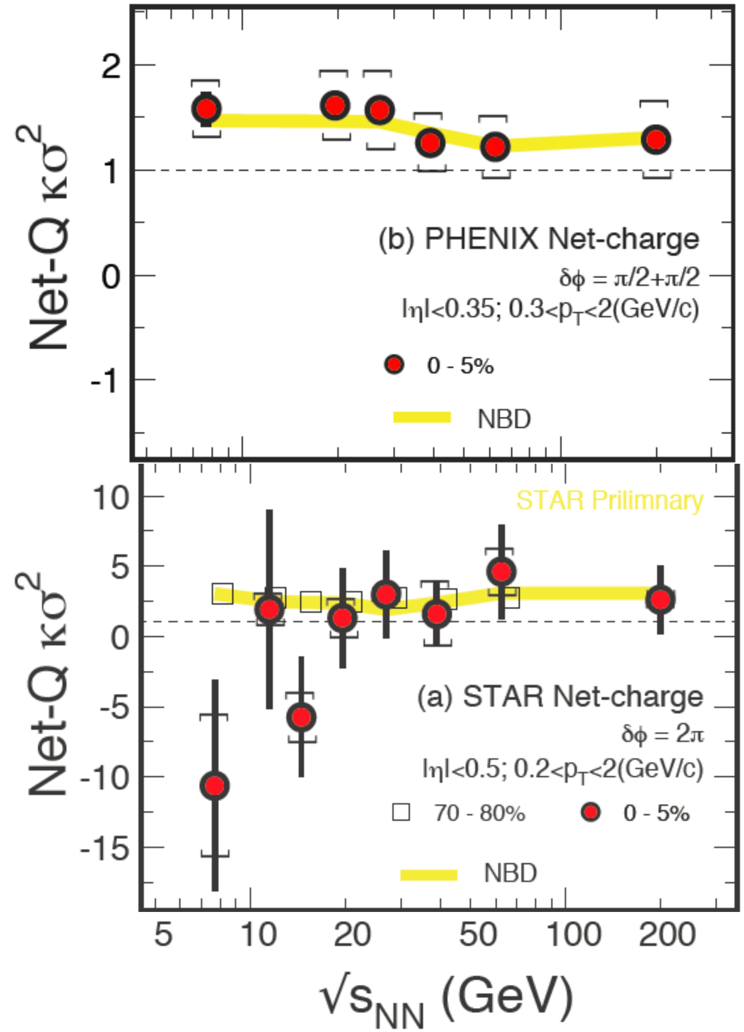
H. Liu and JX, arXiv: 1709.05178

What if the chemical potential line is very close to the phase boundary (0.98)









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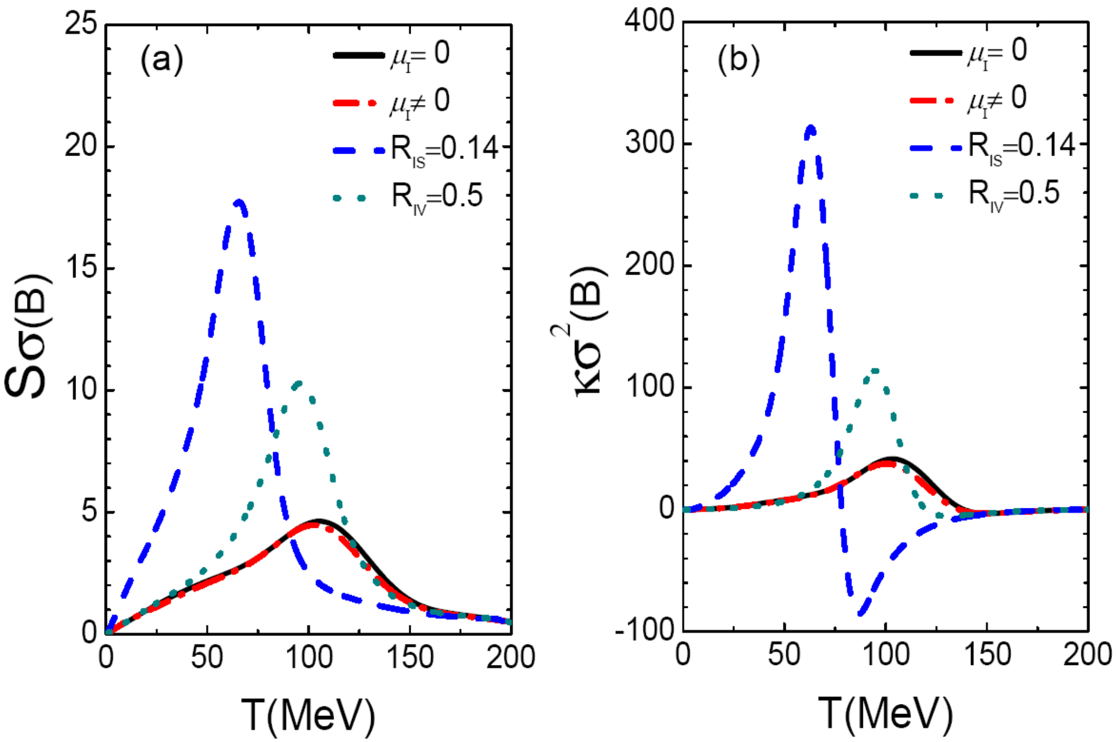

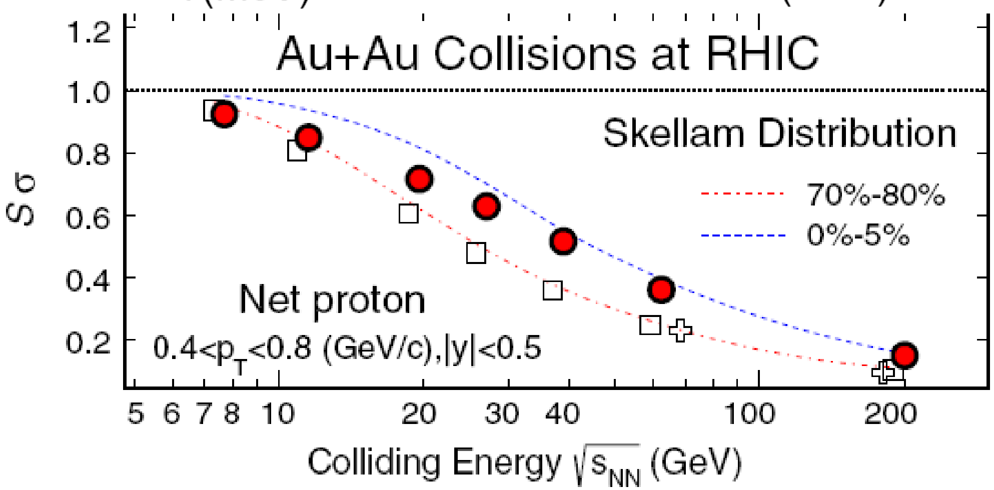

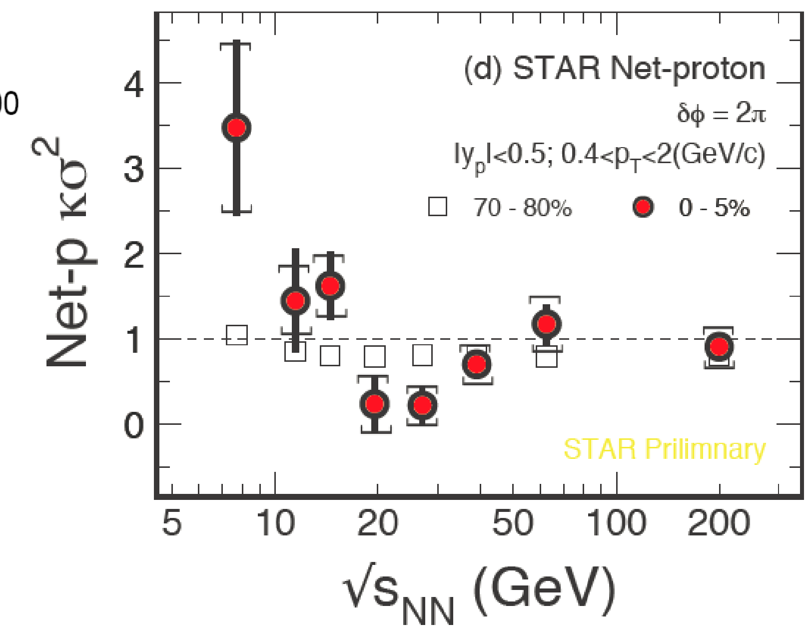

Dynamical coalescence with Wigner function

$$f_M(\boldsymbol{\rho}, \mathbf{k}_\rho) = 8 g_M \exp\left[-\frac{\boldsymbol{\rho}^2}{\sigma_\rho^2} - \mathbf{k}_\rho^2 \sigma_\rho^2\right],$$

where

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2), \qquad \mathbf{k}_\rho = \sqrt{2}\frac{m_2 \mathbf{k}_1 - m_1 \mathbf{k}_2}{m_1 + m_2},$$

$$\langle r^2 \rangle_M = \left| \langle Q_1 (\mathbf{r}_1 - \mathbf{R})^2 + Q_2 (\mathbf{r}_2 - \mathbf{R})^2 \rangle \right| = 3 \frac{|Q_1 m_2^2 + Q_2 m_1^2|}{(m_1 + m_2)^2} \sigma_\rho^2$$
$$= \frac{3}{2} \frac{|Q_1 m_2^2 + Q_2 m_1^2|}{\omega m_1 m_2 (m_1 + m_2)},$$

$Q_1 = Q_2 = 1/2$

$$f_B(\boldsymbol{\rho}, \boldsymbol{\lambda}, \mathbf{k}_\rho, \mathbf{k}_\lambda) = 8^2 g_B \exp\left[-\frac{\boldsymbol{\rho}^2}{\sigma_\rho^2} - \frac{\boldsymbol{\lambda}^2}{\sigma_\lambda^2} - \mathbf{k}_\rho^2 \sigma_\rho^2 - \mathbf{k}_\lambda^2 \sigma_\lambda^2\right],$$

$$\boldsymbol{\lambda} = \sqrt{\frac{2}{3}}\left(\frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2} - \mathbf{r}_3\right), \qquad \mathbf{k}_\lambda = \sqrt{\frac{3}{2}} \frac{m_3(\mathbf{k}_1 + \mathbf{k}_2) - (m_1 + m_2)\mathbf{k}_3}{m_1 + m_2 + m_3}.$$

$$\langle r^2 \rangle_B = \left| \langle Q_1 (\mathbf{r}_1 - \mathbf{R})^2 + Q_2 (\mathbf{r}_2 - \mathbf{R})^2 + Q_3 (\mathbf{r}_3 - \mathbf{R})^2 \rangle \right|$$
$$= \frac{3}{2} \frac{|Q_1 m_2 m_3 (m_2 + m_3) + Q_2 m_3 m_1 (m_3 + m_1) + Q_3 m_1 m_2 (m_1 + m_2)|}{\omega m_1 m_2 m_3 (m_1 + m_2 + m_3)},$$

$Q_1 = Q_2 = Q_3 = 1/3$