Chiral kinetic theory and magnetic effect

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What is chiral kinetic theory?

Relativistic Boltzmann equation

$$(v^\mu \partial_\mu + v^\mu F_{\nu\mu} \partial_{p_\nu}) f = C[f]$$

widely used in plasma physics

Transport coefficient: shear viscosity, etc..

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Relativistic Boltzmann equation with quantum anomaly?

Anomaly matching and effective theory

't Hooft ('80)

If UV theory has an anomaly,
IR theory has the same anomaly.

Anomaly matching and effective theory

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UV theory

QCD

Chiral anomaly: $\partial_\mu j_5^{3\mu} = C\boldsymbol{E} \cdot \boldsymbol{B}$

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Effective theory

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Chiral anomaly: $\partial_\mu j_5^{3\mu} = C E \cdot B$

Effective theory

Vacuum

Chiral perturbation theory

Wess-Zumino term

$\pi^0 \to 2\gamma$

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Effective theory

Vacuum	Manybody systems	
Chiral perturbation theory	Hydrodynamics	Kinetic theory
Wess-Zumino term	Anomalous transport	Berry curvature
$\pi^0 \to 2\gamma$	Chiral magnetic effect (CME) Chiral vortical effect (CVE)	Equilibrium: CME, CVE Nonequilibrium: ?

Chiral Kinetic theory

Son, Yamamoto ('12) Stephanov, Yin ('12)

cf. Chang and Niu ('95)

Hamiltonian $H = \boldsymbol{\sigma} \cdot \boldsymbol{p}$

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$$\Rightarrow Hu_{\pm} = \pm |\boldsymbol{p}| u_{\pm}$$

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When $\boldsymbol{p}$ is time independent:

Wave function

$$\psi(t) = e^{-i|\boldsymbol{p}|t} u_{+}$$

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When $\boldsymbol{p}$ is weakly time dependent:

Wave function

$$\psi(t) \approx e^{-i|\boldsymbol{p}|t - i\int^{t} dt' \boldsymbol{a}\cdot\dot{\boldsymbol{p}}} u_{+}$$

Berry connection: $\boldsymbol{a} := -u_{+}^{\dagger} i \nabla_{p} u_{+}$

Berry('84)

Chiral Kinetic theory

Son, Yamamoto ('12) Stephanov, Yin ('12)

cf. Chang and Niu ('95)

$$S = \int dt(\dot{\boldsymbol{x}} \cdot \boldsymbol{p} + \dot{\boldsymbol{x}} \cdot \boldsymbol{A} - |\boldsymbol{p}| - A_0 - \dot{\boldsymbol{p}} \cdot \boldsymbol{a})$$

Action with Berry connection: $\boldsymbol{a} := -u_+^\dagger i \nabla_{\boldsymbol{p}} u_+$

Chiral Kinetic theory

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Action with Berry connection: $a := -u_+^\dagger i \nabla_p u_+$

Classical EOM $\dot{x} = \hat{p} + \dot{p} \times \Omega$

$$\dot{p} = \dot{x} \times B + E$$

$$\Omega = \nabla_p \times a = \frac{\hat{p}}{2p^2}$$

Chiral Kinetic theory

Son, Yamamoto ('12) Stephanov, Yin ('12)

Chiral kinetic equation (CKE)

$$(\partial_t + \dot{\boldsymbol{x}} \cdot \nabla_x + \dot{\boldsymbol{p}} \cdot \nabla_p) f = 0$$

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Current

$$j = \int_p f\hat{\boldsymbol{p}} + \boldsymbol{E} \times \int_p f\boldsymbol{\Omega} + \boldsymbol{B}\int_p f\hat{\boldsymbol{p}} \cdot \boldsymbol{\Omega}$$

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Hall effect Chiral magnetic effect

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Hall effect Chiral magnetic effect

Anomaly

$$\partial_\mu j^\mu = \frac{1}{4\pi^2} \boldsymbol{E} \cdot \boldsymbol{B}$$

Lorentz covariance?

$$S = \int dt(\dot{\boldsymbol{x}} \cdot \boldsymbol{p} + \dot{\boldsymbol{x}} \cdot \boldsymbol{A} - |\boldsymbol{p}| - A_0 - \dot{\boldsymbol{p}} \cdot \boldsymbol{a})$$

The action looks not Lorentz invariant.

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1) Energy shift: $|\boldsymbol{p}| \to |\boldsymbol{p}|(1 - \boldsymbol{\Omega} \cdot \boldsymbol{B})$

Son, Yamamoto ('13)

Lorentz covariance?

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1) Energy shift: $|\boldsymbol{p}| \to |\boldsymbol{p}|(1 - \boldsymbol{\Omega} \cdot \boldsymbol{B})$

Son, Yamamoto ('13)

2) Modified Lorentz transform

Chen, Son, Stephanov, Yee, Yin ('14)

$$\delta \boldsymbol{x} = \boldsymbol{\beta} t + |\boldsymbol{p}| \boldsymbol{\beta} \times \boldsymbol{\Omega} \qquad \delta \boldsymbol{p} = \boldsymbol{\beta} \epsilon + |\boldsymbol{p}| (\boldsymbol{\beta} \times \boldsymbol{\Omega}) \times \boldsymbol{B}$$

Several approaches

Hamiltonian formalism

Son, Yamamoto ('12)

Semi-classical path integral

Stephanov, Yin ('12)

World line formalism: Mueller, Venugopalan ('17) ('18)

Field theoretical approach

High density effective theory: Son, Yamamoto ('13)

On-shell effective theory: Manuel,Torres-Rincon ('13) ('14)

Carignano, Manuel,Torres-Rincon ('18)

Wigner function: Gao,Liang,Pu,Wang,Wang ('12), Chen, Pu, Wang, Wang ('13)

Wu, Hou, Ren('17)

Huang, Shi, Jiang, Liao, Zhuang('18)

CVE: Gao, Pang, Wang ('18)

Kadanoff-Baym: YH, Shi Pu, Yang ('16) ('17), YH, Yang ('18)

QFT approach

Propagator (Wigner function)

$$S^{<}(p,X) = \int d^4 s e^{is\cdot p} \langle \psi^\dagger(y)\psi(x)\rangle U(x,y)$$

$$S^{>}(p,X) = \int d^4 s e^{is\cdot p} \langle \psi(x)\psi^\dagger(y)\rangle U(x,y)$$

where $X = \frac{x+y}{2}$ $s = x - y$

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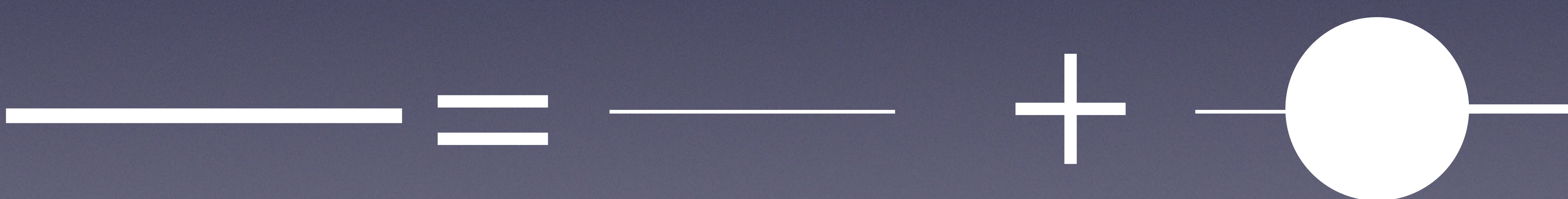
EOM (Schwinger-Dyson equation)



$$\sigma^{\mu}(p_{\mu} - A_{\mu}) \star S^{<} = \frac{-i\hbar}{2}(\Sigma^{<} \star S^{>} - \Sigma^{>} \star S^{<})$$

EOM Up to order hbar

$$\sigma^\mu\left(p_\mu + \frac{i\hbar}{2}\Delta_\mu\right)S^< = \frac{-i\hbar}{2}(\Sigma^< S^> - \Sigma^> S^<)$$

where $\Delta_\mu = \partial_\mu + F_{\nu\mu}\frac{\partial}{\partial p_\nu}$

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Chiral kinetic equation (CKE)

$$\Delta_\mu S^{<\mu} = \Sigma^<_\mu S^{>\mu} - \Sigma^>_\mu S^{<\mu}$$ where $S^{<\mu} = \frac{1}{2}\mathrm{tr}\sigma^\mu S^<$

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$$S^{<\mu} = 2\pi\epsilon(p\cdot n)\left[\delta(p^2)(p^\mu + \hbar S_n^{\mu\nu}\mathscr{D}_\nu) + \hbar p_\nu \tilde{F}^{\mu\nu}\delta'(p^2)\right]f$$

YH, Shi Pu, Yang ('16) ('17)

spin: $S_n^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\frac{p_\alpha n_\beta}{p\cdot n}$ $\qquad \mathscr{D}_\mu f = \Delta_\mu f + \Sigma_\mu^< f - \Sigma_\mu^> \bar{f}$

Talk by Amping Huang (Parallel II.3)

Lorentz invariance

Chen, Son, Stephanov ('15), YH. Pu, Yang ('16)

Talk by Jian-Hua Gao (Parallel II.1)

$$S^{<\mu} = 2\pi\epsilon(p\cdot n)\Big[\delta(p^2)(p^\mu + \hbar S_n^{\mu\nu}\mathscr{D}_\nu) + \hbar p_\nu \tilde{F}^{\mu\nu}_{\alpha\beta}\delta'(p^2)\Big]f$$

$S^{<\mu}$ is Lorentz covariant

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$S^{<\mu}$ is Lorentz covariant

f is not Lorentz scalar

$$f \to f + \hbar \frac{\epsilon^{\nu\mu\alpha\beta} p_\alpha n'_\beta n_\mu}{2(p\cdot n)(p\cdot n')}\mathcal{D}_\nu f$$

Application

(local) Equilibrium

Current: $J^{\mu} = 2\int \frac{d^4p}{(2\pi)^4} S^{<\mu}(p, X)$

➡ $\boldsymbol{J} = n\boldsymbol{u} + \sigma_B \boldsymbol{B} + \sigma_\omega \boldsymbol{\omega}$

CME CVE

Dissipative current

CKE with relaxation time approximation

Gorbar, Shovkovy, Vilchinskii, Rudenok, Boyarsky, Ruchayskiy ('16)

Chen, Ishii, Pu, Yamamoto ('16)

YH, Pu, Yang ('17)

$\nabla\mu, \nabla T$ correction

$$\delta J = C_1 \boldsymbol{E} \times \nabla\mu + C_2 \boldsymbol{E} \times \nabla T + C_3 \nabla\mu \times \nabla T$$

$$C_i \sim \tau_R$$

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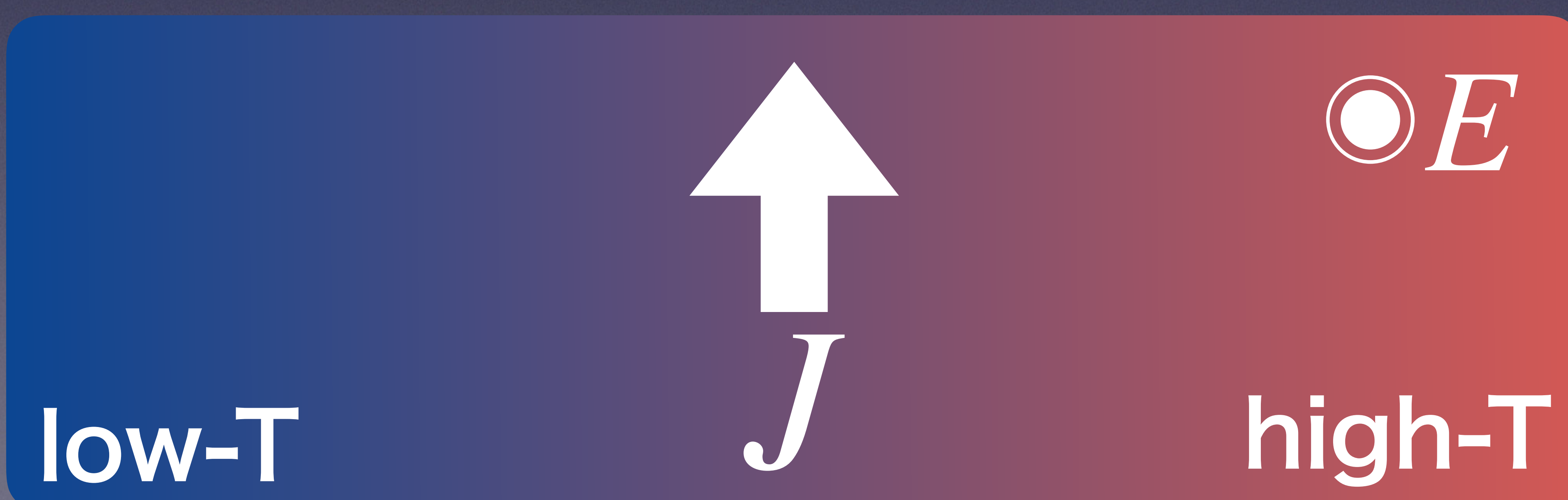

Dissipative current

YH, Yang ('18)

Shear and bulk correction

$$\delta J^i = C_4 \pi^{ij} B_j + C_5 \pi^{ij} \omega_j$$

$$+ C_6 (\nabla \cdot \boldsymbol{u}) B^i + C_7 (\nabla \cdot \boldsymbol{u}) \omega^i$$

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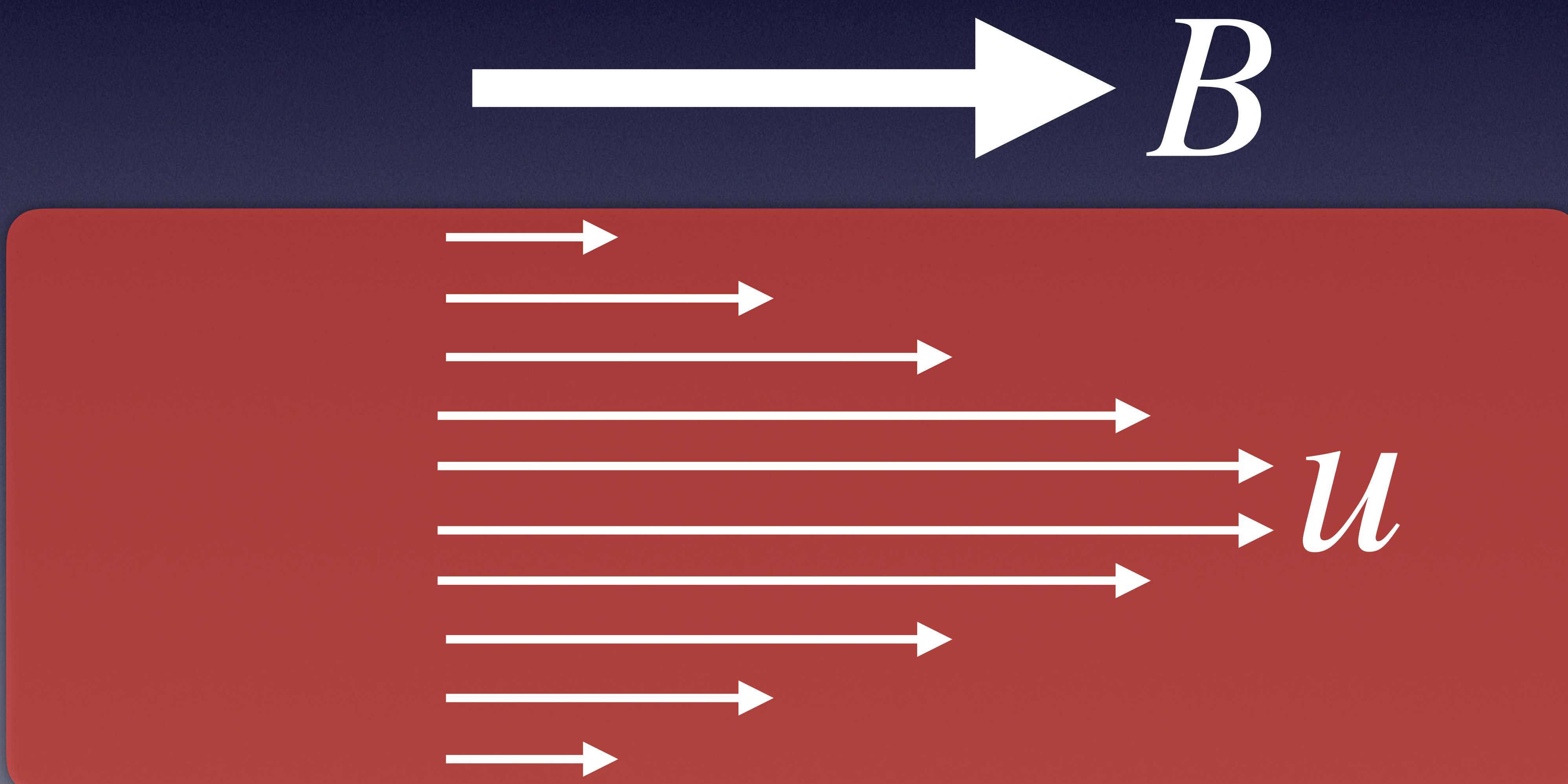

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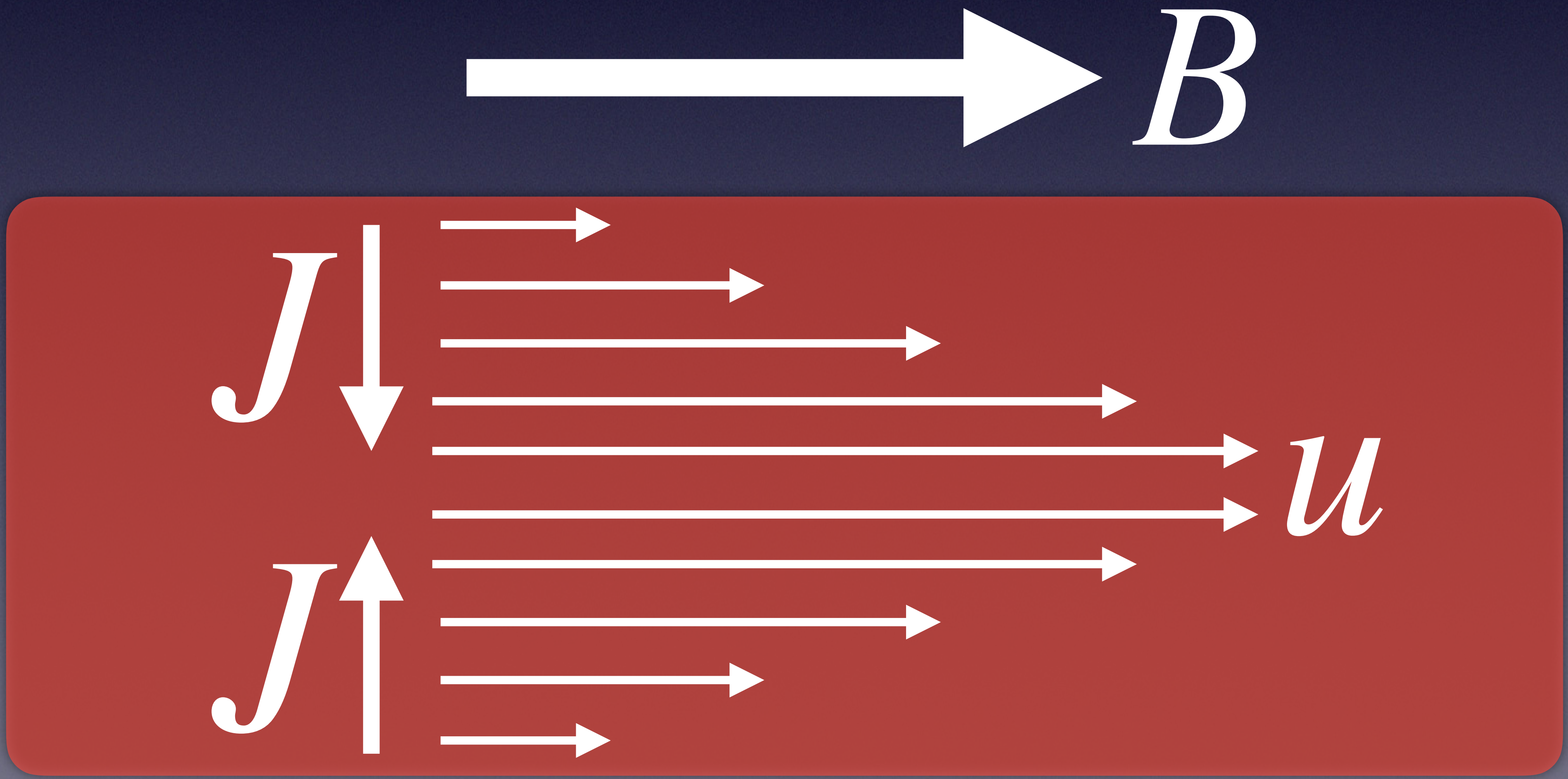
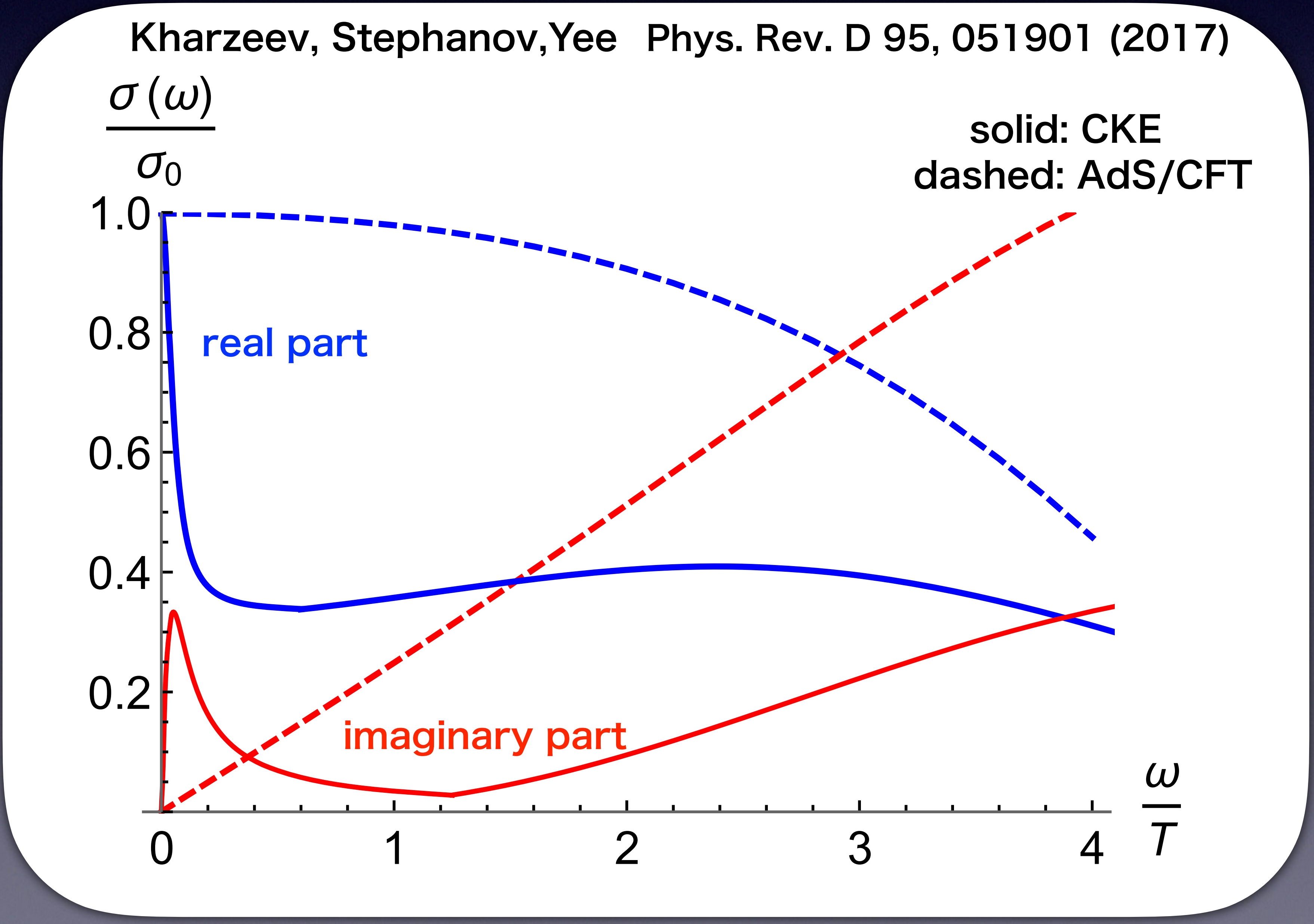


Out of equilibrium transport

Kharzeev, Stephanov,Yee('17)

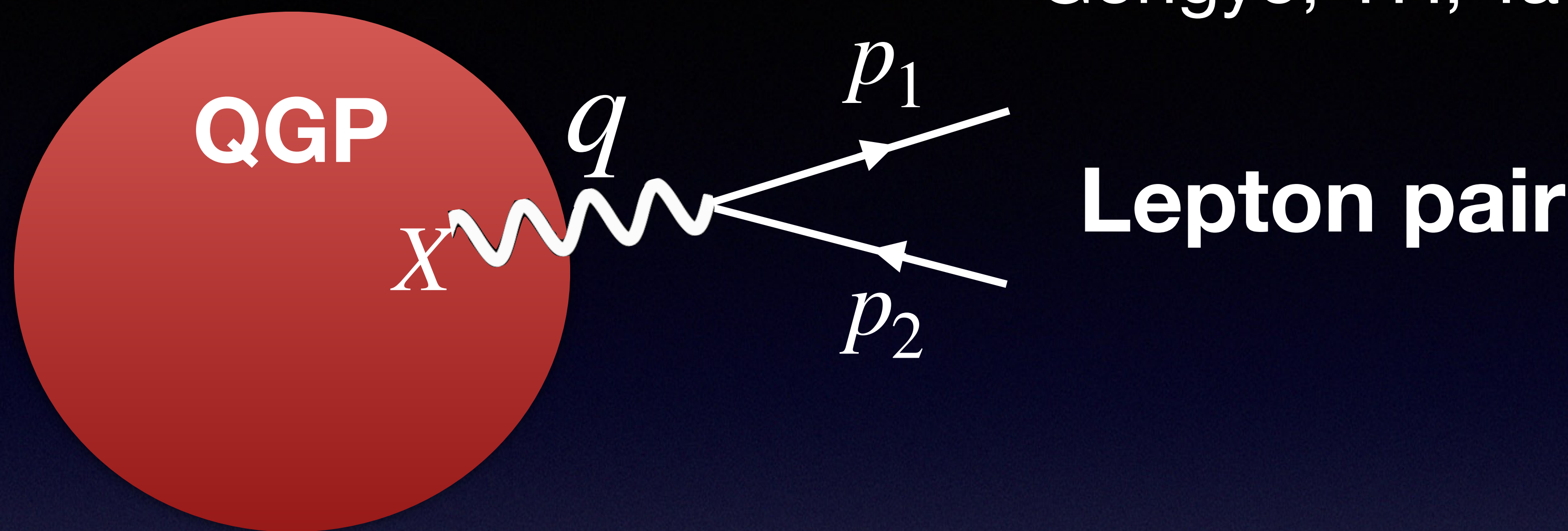
CKE with relaxation time approximation

ω dependence of CME $\sigma(\omega) = \sigma_0\left(1 - \frac{2}{3}\frac{\omega}{\omega + i\tau_R^{-1}}\right)$



Dilepton production

Gongyo, YH, Tachibana ('18)



Photon polarization funciton

$$\Pi^{<\mu\nu}(X,q) = \int d^4s e^{iq\cdot s}\langle j^\nu(X-s/2)j^\mu(X+s/2)\rangle$$

Dilepton production rate

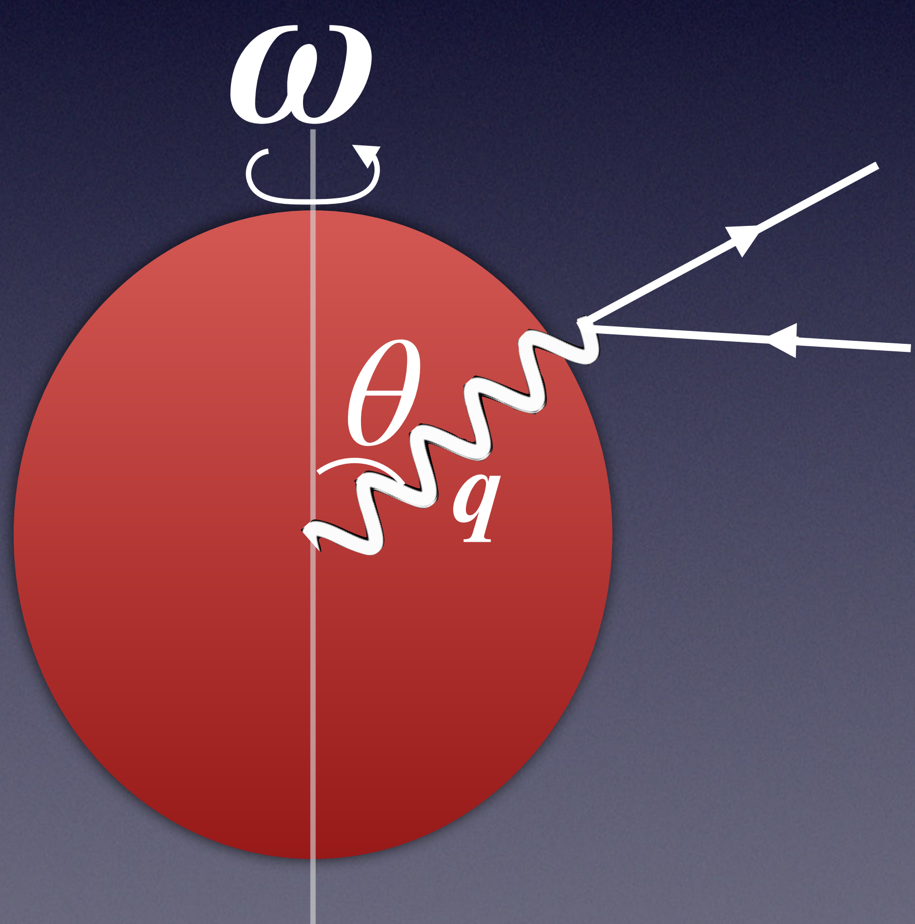
$$\frac{d\Gamma}{d^4q} = -\frac{\alpha}{24\pi^4}\Pi^{<\mu}{}_{\mu}(q,X)$$

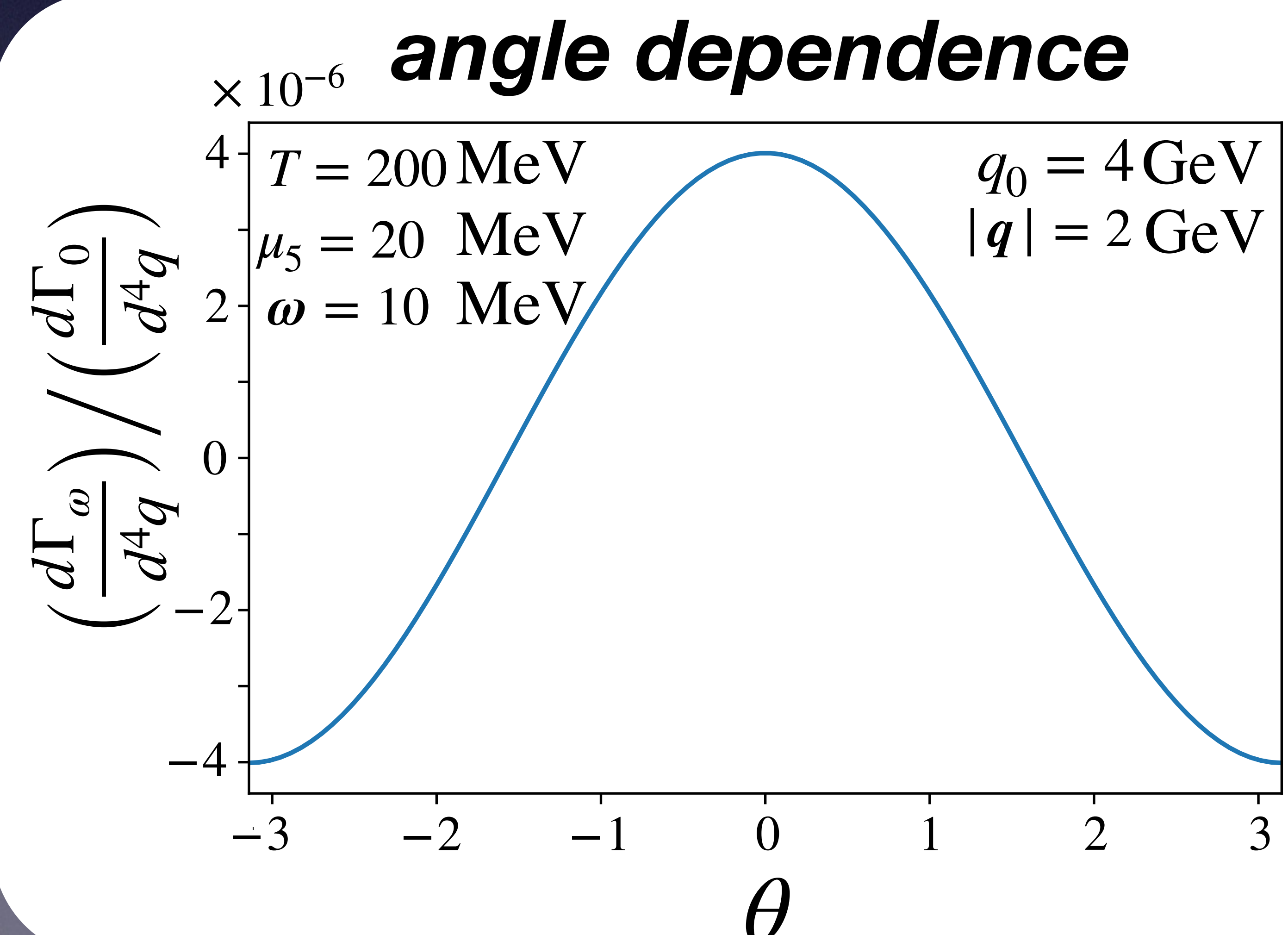
Di-lepton production

Gongyo, YH, Tachibana('18)

$$\frac{d\Gamma}{d^4q} = \frac{d\Gamma_0}{d^4q} + \frac{d\Gamma_\omega}{d^4q} \quad \textbf{with} \quad \frac{d\Gamma_\omega}{d^4q} = (\boldsymbol{\Omega}_\gamma \cdot \boldsymbol{\omega})C(q)$$

$$\boldsymbol{\Omega}_\gamma = \frac{\hat{\boldsymbol{q}}}{|\boldsymbol{q}|^2}$$





Puzzle?

CKE from on-shell effective theory

Carignano, Manuel, Torres-Rincon ('18)

$$\left(\Delta_0 + (1 + \boldsymbol{B}\cdot\boldsymbol{\Omega})\hat{\boldsymbol{q}}^i\Delta_i + \left(\frac{1}{2}\epsilon^{ijk}E^j\Omega^k - \frac{1}{4}B_\perp^i\right)\Delta_i\right)f = 0$$

reproduces consistent anomaly $\partial_\mu j_5^\mu = \frac{1}{3}\frac{1}{2\pi^2}E\cdot B$

CKE by Son, Yamamoto ('12), YH, Pu, Yang ('17)

$$\left(\Delta_0 + (1 + \boldsymbol{B}\cdot\boldsymbol{\Omega})\hat{\boldsymbol{q}}^i\Delta_i + \epsilon^{ijk}E^j\Omega^k\Delta_i\right)f = 0$$

where $\Delta_\mu = \partial_\mu + F_{\nu\mu}\frac{\partial}{\partial p_\nu}$

reproduces covariant anomaly

$$\partial_\mu j_5^\mu = \frac{1}{2\pi^2}E\cdot B$$

Summary

Chiral kinetic theory:

Effective theory
reproducing chiral anomaly

Novel dissipative anomalous transports are found.

Application to HIC and cond-mat

Quarks have mass. What is mass correction to CKE?

Mass correction to CVE cf. Flachi, Fukushima ('17), Lin, Yang ('18)

Talk by Lixin Yang (Parallel II.1)

$$j_5^\mu = \Big(\frac{T^2}{6} - \frac{m^2}{4\pi^2}\Big)\omega^\mu$$

Analysis with collisions without relaxation time approximation