Thermodynamics of a gas of hadrons with interaction using S-matrix formalism

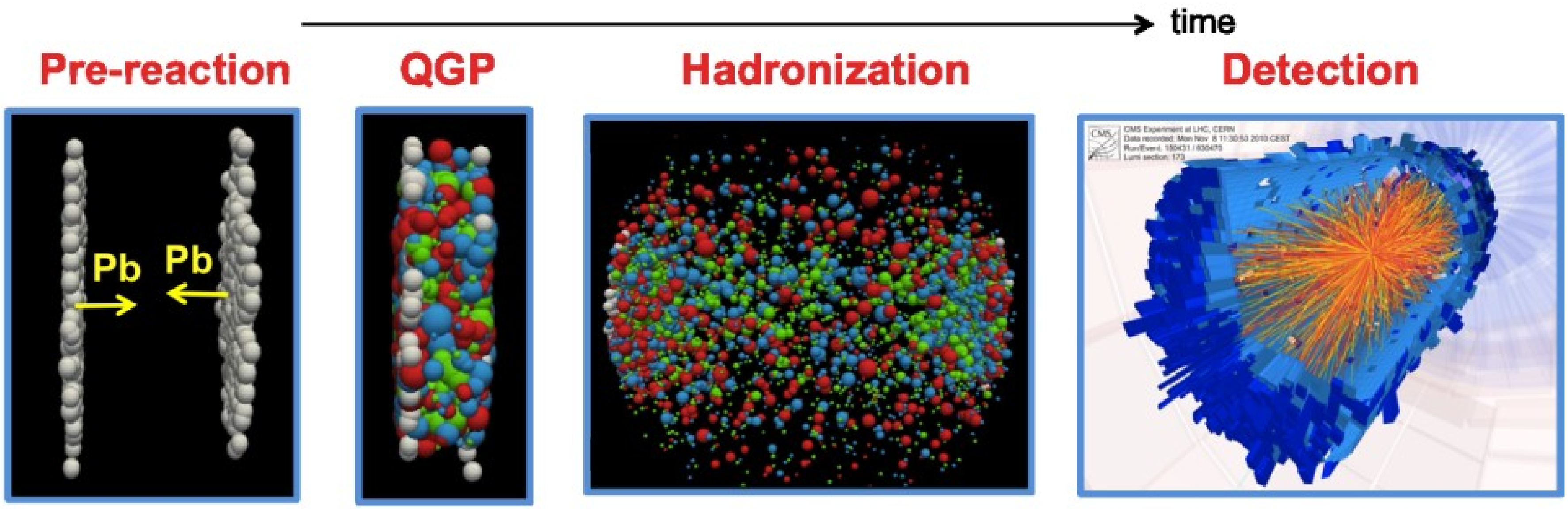
Ashutosh Dash, Subhasis Samanta and Bedangadas Mohanty

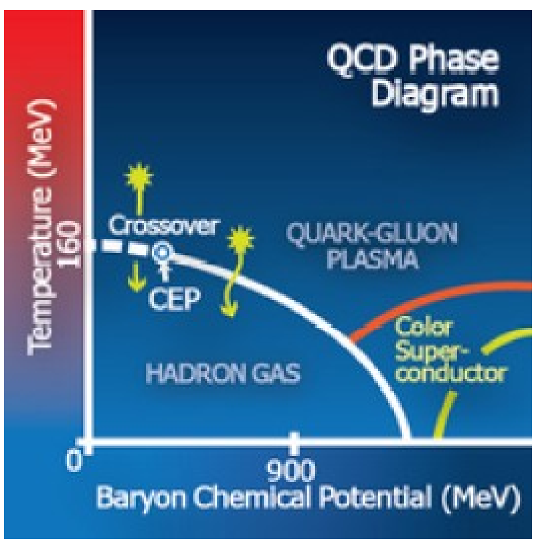
National Institute of Science Education and Research, HBNI
Jatni, India

Outline

- Motivation
- S-matrix formalism for HRG
- Results
- Summary

Relativistic heavy ion collisions





Major Goals

- Map QCD phase diagram in terms of T and μ_B
- Serach for QCD critical point.

HRG models have been used to study hadronic phase

Ideal Hadron Resonance Gas

- Non-interacting system consisting of all hadrons and resonances assuming point particles upto cutoff mass M< 2.5 GeV.
- System is assumed to be in thermal and chemical equillibrium.
- System can be described by a grand canonical partition function:

$$\ln Z = \sum_i \ln Z_i$$

$$\ln Z_i = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2 dp \ln[1 \pm \exp(-(E_i - \mu_i)/T)]$$

where the upper/lower sign stands for baryons/mesons.

- Results of thermodynamic observable depend on the cutoff mass M

Van der Waals HRG

$$\left(P + \left(\frac{N}{V}\right)^2 a\right)(V - Nb) = NT$$

$$P(T,n) = \frac{NT}{V - bN} - a\left(\frac{N}{V}\right)^2 = \frac{nT}{1 - bn} - an^2$$

where n is the number density of particles.

- VDW HRG model has interaction through the parameters a and b
- The free parameter a and b are fixed either by reproducing the properties of the ground state of nuclear matter or by fitting lattice QCD EOS at zero chemical potential.

(PRC 91, 064314 (2015), PRC 97, 015201 (2018))

Classical virial expansion

$$P = \frac{NT}{V}\left(1 + \frac{NB(T)}{V} + \frac{N^2C(T)}{V^2} + ...\right)$$

- The first term in the expansion corresponds to an ideal gas.
- The second term is obtained by taking into account the interaction between pairs of hadrons and subsequent terms involve group of three, four... hadrons.
- B, C, ... are called second, third etc. virial coefficients.

The second virial coefficient:

$$B(T) = \frac{1}{2}\int(1 - e^{-U_{12}/T})dV$$

where U_{12} is the two body interaction energy.

Relativistic virial expansion

The partition function:

$$\ln Z = \ln Z_0 + \ln Z_{\rm int} = \ln Z_0 + \sum_{i_1, i_2} z_1^{i_1} z_2^{i_2} b(i_1, i_2)$$

$$b(i_1, i_2) = \frac{V}{4\pi i} \int \frac{d^3p}{(2\pi)^3} \int d\varepsilon \exp\left(-\beta(p^2+\varepsilon^2)^{1/2}\right) \left[A\left\{S^{-1}\frac{\partial S}{\partial \varepsilon} - \frac{\partial S^{-1}}{\partial \varepsilon} S\right\}\right]_c$$

(Phys. Rev. 187, 345 (1969))

- Here z_1 and z_2 are the fugacities of the two hadronic species.
- The labels i_1 and i_2 refer to channel of the S-matrix which has initial state containing particles $i_1 + i_2$.
- We ignore the contribution from bound states.

$\ln Z_0$

Contains only stable hadrons

$\ln Z_{\rm int}$

Scattering between two stable hadrons

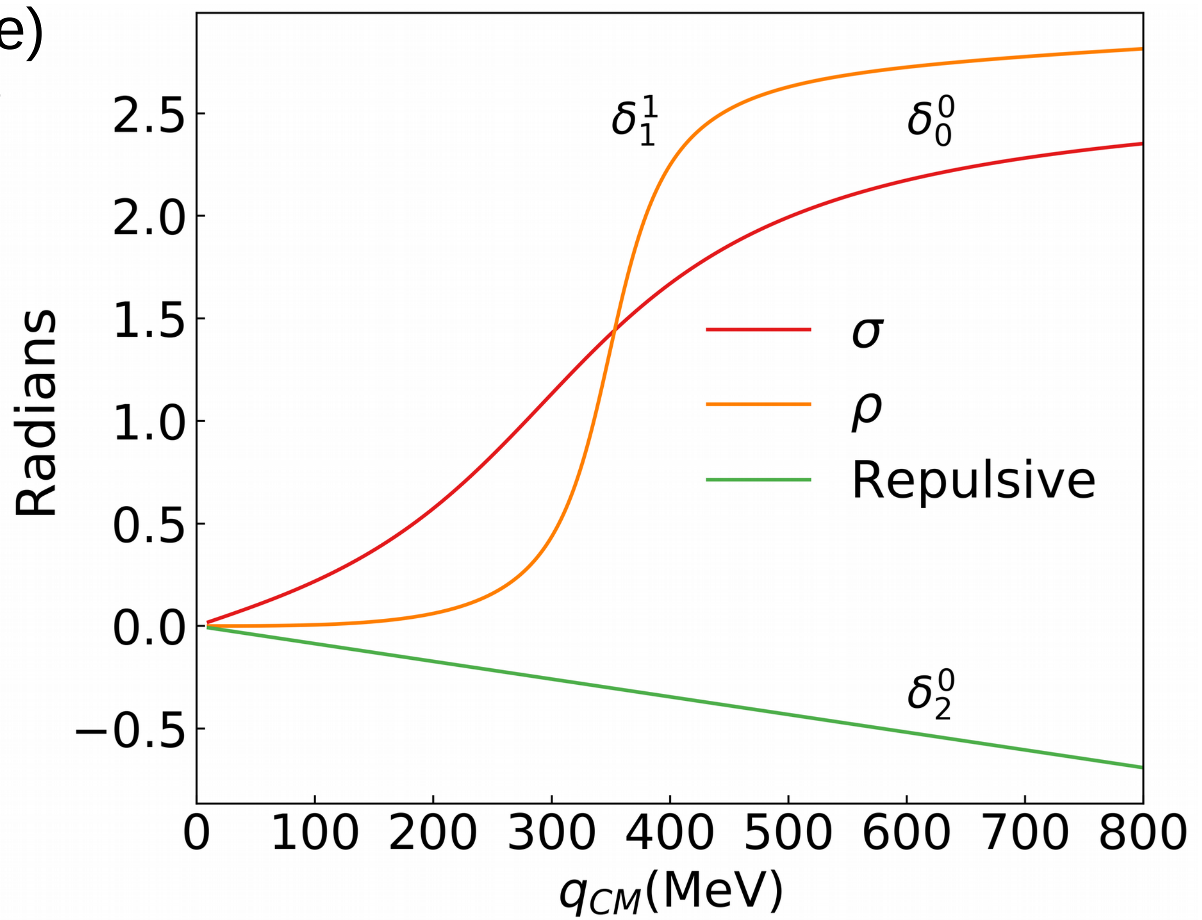
The second virial coefficient is :

$$b_2 = \frac{b(i_1 = i_2 = 1)}{V} = \frac{1}{2\pi^3 \beta} \int_M^\infty d\varepsilon \varepsilon^2 K_2(\beta\varepsilon) \sum_{l,I}{}' g_{I,l} \frac{\partial \delta_l^I(\varepsilon)}{\partial \varepsilon}.$$

Scattering phase shifts

Interaction is attractive (repulsive) depending whether $\partial\delta_l^I(\epsilon)/\partial\epsilon$ is positive (negative)

- For a very narrow resonance:
  $\delta_l^I \sim \Theta(\epsilon - m_R)$
  $\partial\delta_l^I/\partial\epsilon \sim \pi\delta(\epsilon - m_R)$
  $Z_0 = Z_{\rm int}$
- Narrow resonances behave like stable hadrons of mass m_R.
- This is fundamental premise of **ideal HRG**.



$\pi - \pi$ phase shifts versus center-of-mass momentum
(Phys. Rep. 227, 331 (1993))

K-matrix formalism (Attractive interaction)

Scattering matrix: $S = I + 2iT$

K-matrix: $K^{-1} = T^{-1} + iI, K = K^{\dagger}$

Resonances appear as sum of poles in K-matrix formalism

$$K_{ab\to R\to ab} = \sum_R \frac{m_R \Gamma_{R\to ab}(\sqrt{s})}{m_R^2 - s},$$

(PRC88, 044917 (2013), PRC97, 055208 (2018))

Partial wave decomposition of S-matrix:

$$S_l = \exp(2i\delta_l) = 1 + 2iT_l$$

Compute phase shift using the relation

$$\delta_l = \arctan K_l$$

Thus we need mass and partial decay width of a given resonance R to construct the phase shift.

Features of K-matrix formalism

- K-matrix preserves unitarity of S-matrix both for single and multiple resonances which Breit-Wigner (BW) parmaterization doesn't.
- Neatly handles both overlapping and non-overlapping resonance unlike BW parametrizat-ion.

Experimental phase shifts

Repulsive interactions

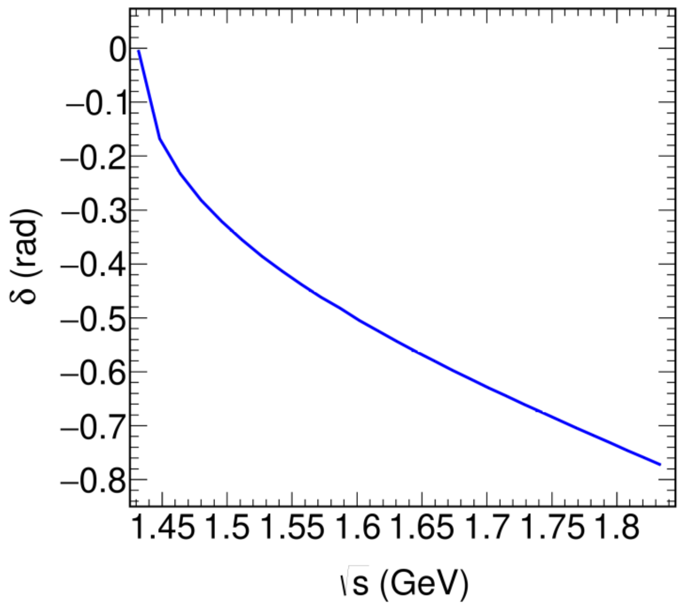
- πN interaction: $\Delta(1620)$, $\Delta(1910)$ etc.
- KN interaction: $\Sigma(1660)$, $\Sigma(1915)$ etc.
- $\pi\pi$ interaction: δ_0^2

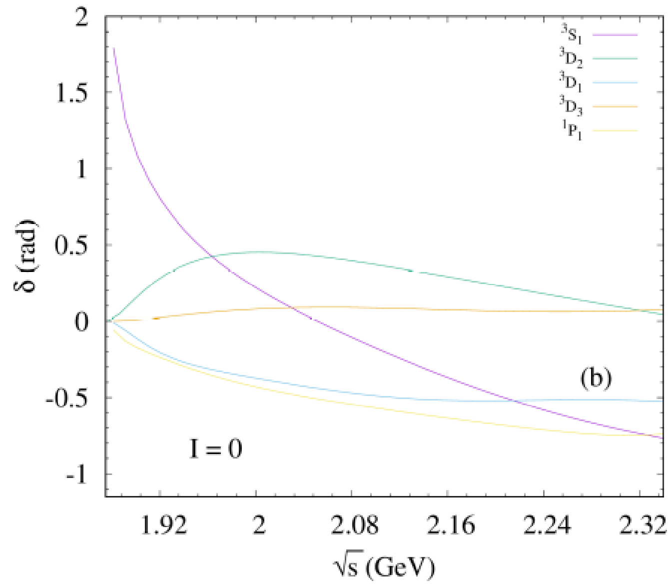
(PRD83, 074004 (2011))

Interactions involving |B|=2

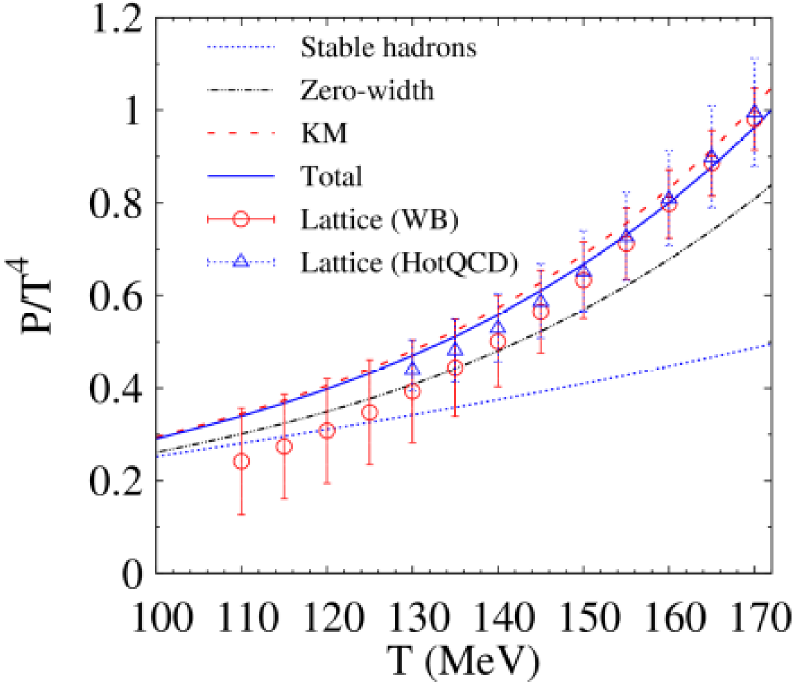
- NN interaction (|B|=2) all available data for l<7.
- Includes both attraction and repulsion.

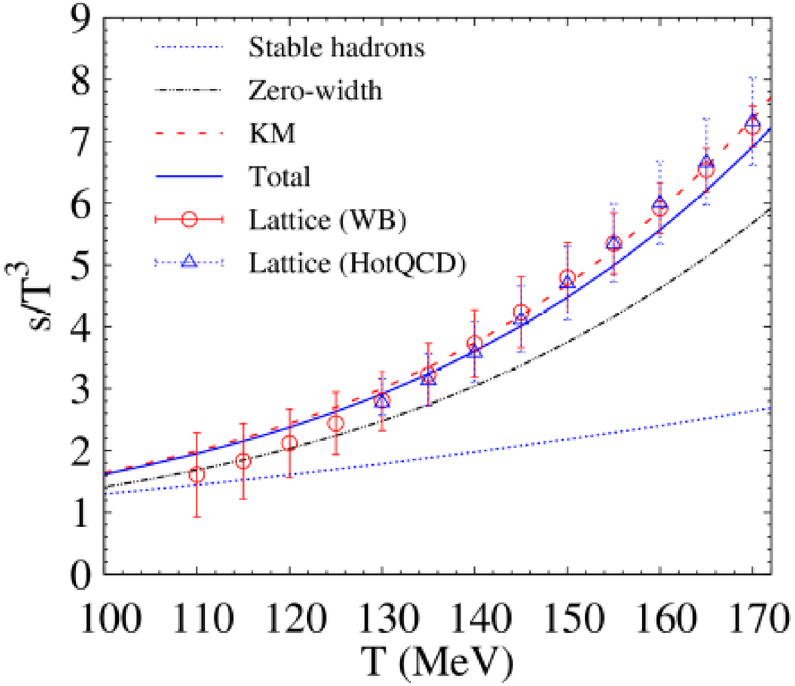
(PRC94, 065203 (2016))

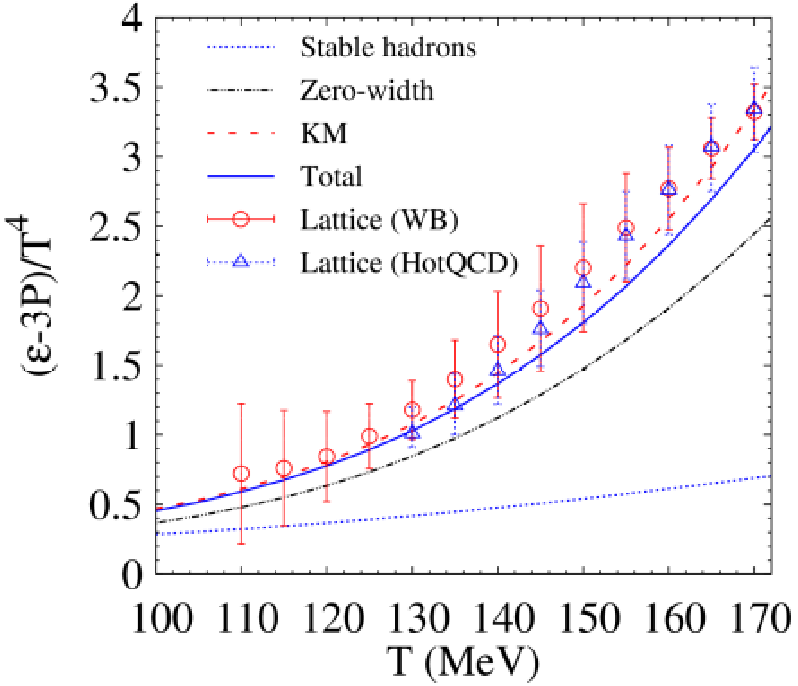




Results



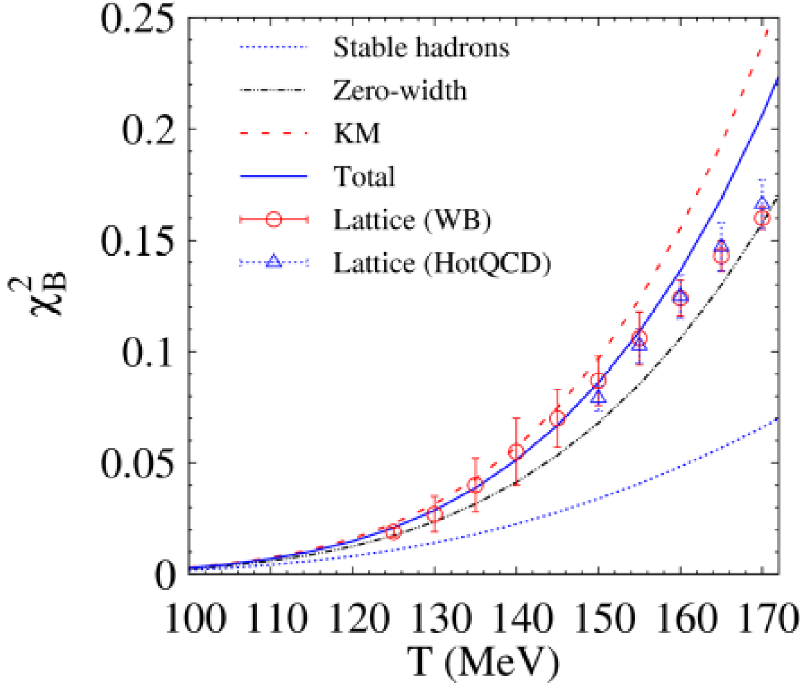


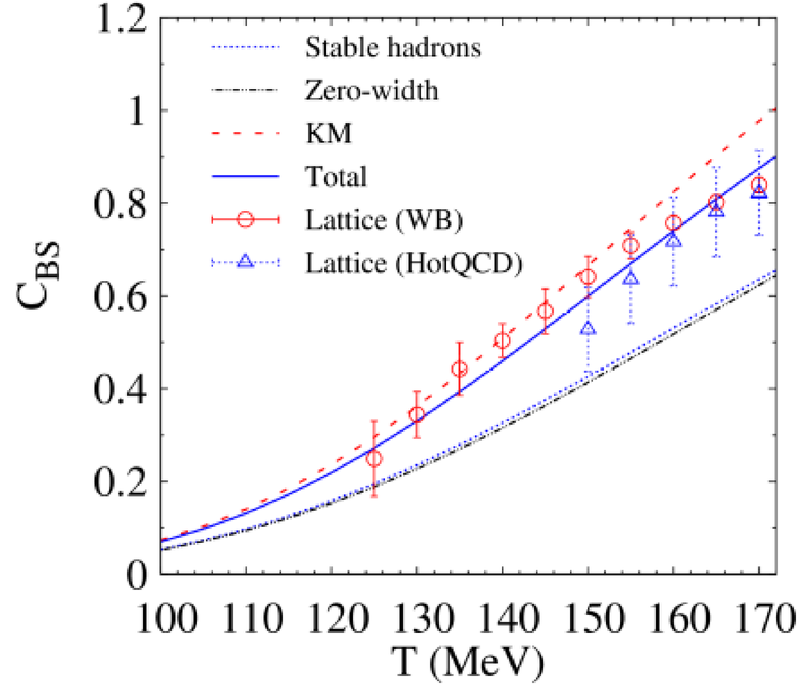


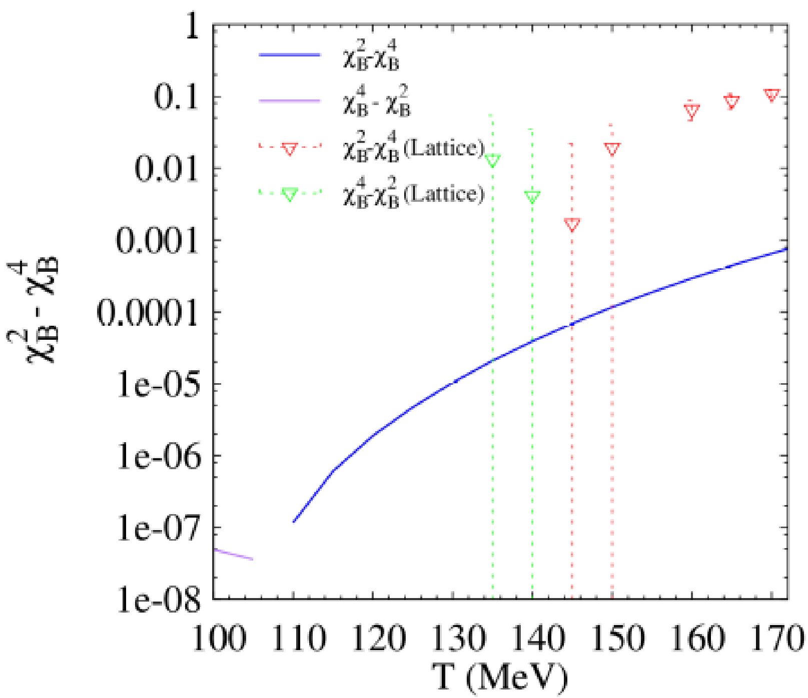
- The effect of attractive interaction through KM approach increases the value of all thermodynamic observables compared to zero-width.
- Repusive interaction (Total) cancel some part of the attractive components.
- For a complete cancellation, Total would match zero width.

(PRC97, 055208 (2018), arXiv:1806.02117 [hep-ph] AD, S Samanta, B Mohanty)

Results







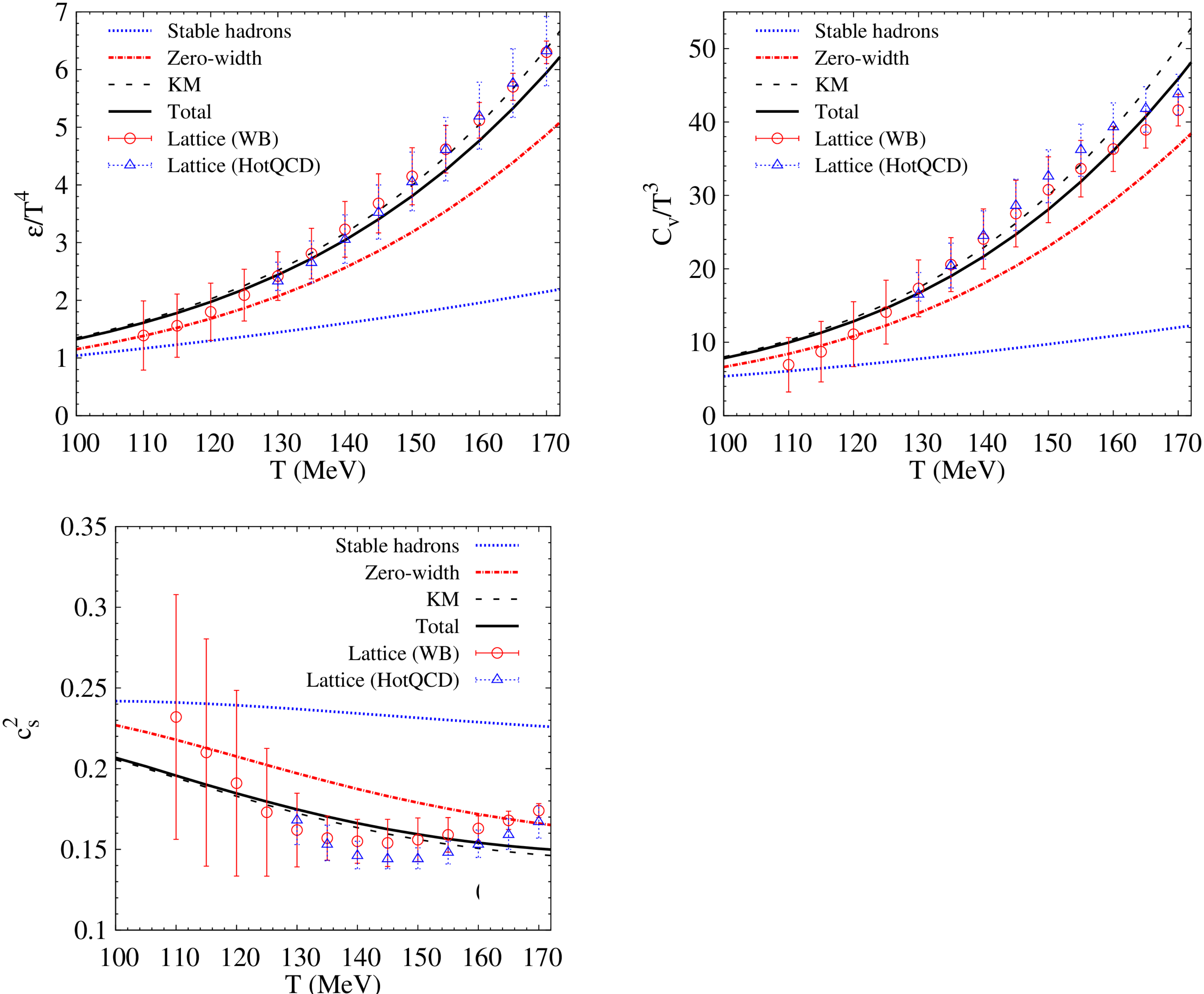
- The inclusion of repulsive interaction better describres the observable χ_B^2.
- C_{BS}= $-3\chi_{BS}^{11}/\chi_S^2$
- $\chi_B^2 - \chi_B^4$ > 0 for T>110 MeV and $\chi_B^2 - \chi_B^4$<0 T<110 MeV.

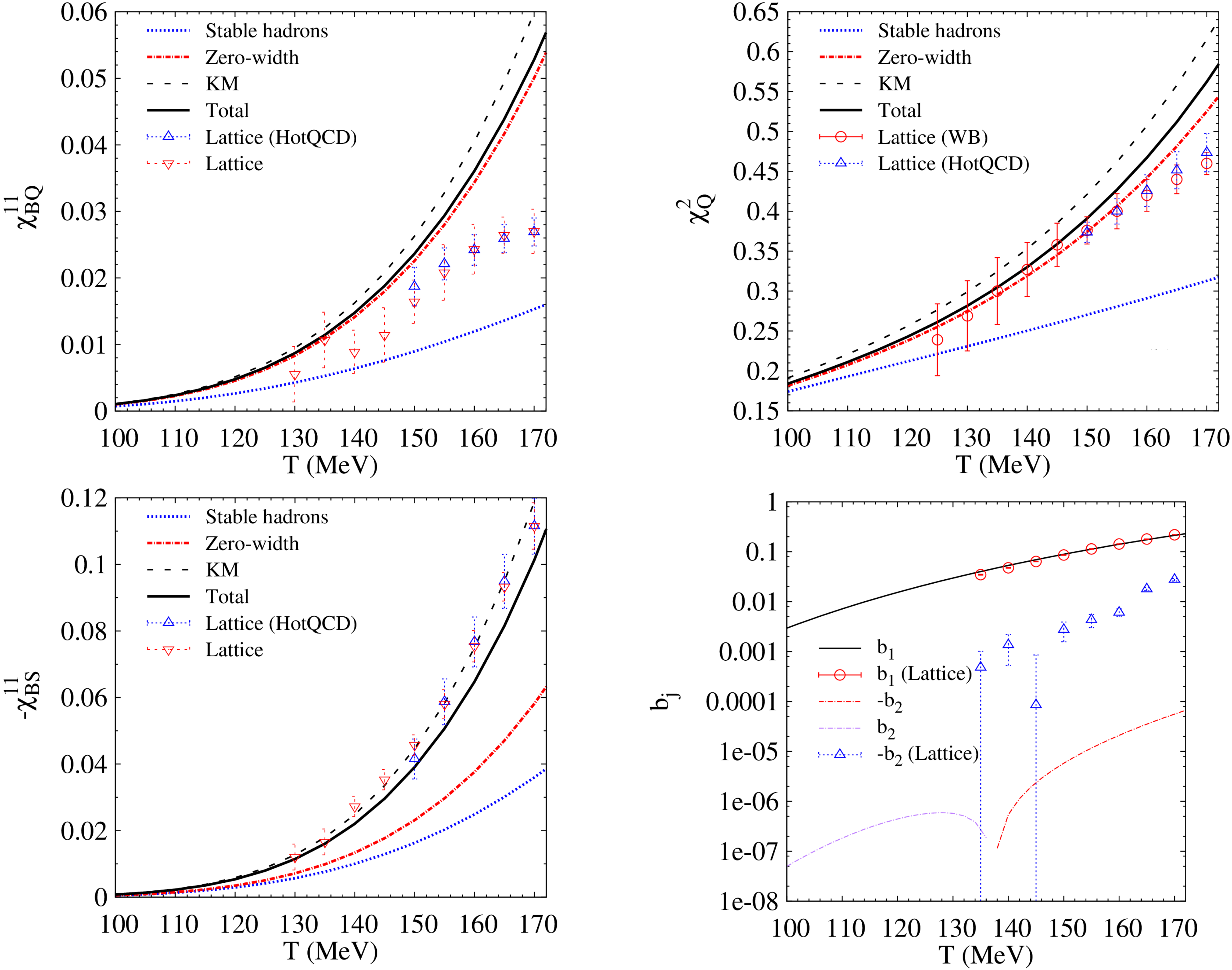
(PRC97, 055208 (2018), arXiv:1806.02117 [hep-ph]
AD, S Samanta, B Mohanty)

Summary

- HRG model is extended using S-matrix formalism.
- Interacting part of the partition function depends on the derivative of the phase shift.
- Attractive part ⟶ K-matrix
- Repulsive and NN interactions ⟶ Experimental phase shifts
- Good agreement between EOS in S-matrix with LQCD data.
- Non-zero value of $\chi_B^2 - \chi_B^4$.
- Good agreement for C_{BS} (without adding extra resonances) and LQCD data.

Thank You


Stable hadrons
Zero-width
KM
Total
Lattice (WB)
Lattice (HotQCD)
ε/T^4
C_V/T^3
c_s^2
T (MeV)



Stable hadrons
Zero-width
KM
Total
Lattice (HotQCD)
Lattice
χ^{11}_{BQ}
T (MeV)
Lattice (WB)
χ^{2}_{Q}
$-\chi^{11}_{BS}$
b_1
b_1 (Lattice)
$-b_2$
b_2
$-b_2$ (Lattice)
b_j